MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 37/1972

Topologie

3.9. bis 16.9.1972

Unter der Leitung der Herren T. tom Dieck (Saarbrücken), D.B.A. Epstein (Coventry) und K. Jänich (Regensburg) fand die jährliche Tagung über Topologie in Oberwolfach statt. Fast alle Vorträge und Diskussionen behandelten die drei Problemkreise Homotopietheorie, verallgemeinerte Homologie- und Kohomologietheorien, darunter besonders die Kobordismentheorie, und Theorie der Mannigfaltigkeiten.
Wieder sorgten besonders die zahlreichen ausländischen Gäste für einen regen Gedankenaustausch.

Teilnehmer

M. Boehme, Berlin
W. Bos, Konstanz
Th. Bröcker, Regensburg
S. Buoncristiano, Coventry
D. Burghelea, Bukarest
G. Burde, Frankfurt
K. Delinić, Heidelberg
T. tom Dieck, Saarbrücken
A. Dold, Heidelberg
D.B.A. Epstein, Coventry
D. Erle, Dortmund
M. Fuchs, East Lansing
D. Gibbs , Bonn
H. Hauschild, Saarbrücken
J.C. Hausmann, Genf
W. Jaco, Houston
K. Jänich, Regensburg
J.P. Jouanolon, Straßburg
M. Klingmann, Heidelberg
J.F. Kraus, Heidelberg
K. Lamotke, Köln
D. Lehmann, Lille
A. Liulevicius, Chicago
P. Löffler, Saarbrücken
G. Luke, Oxford
G. Luszig, Coventry
H.J. Munkholm, Odense
W. Neumann, Princeton
E. Ossa, Bonn
D. Puppe, Heidelberg
V. Puppe, Heidelberg
E. Rees, Oxford
C.A. Robinson, Coventry
C.P. Rourke, Coventry

Gefördert durch DFG Deutsche Forschungsgemeinschaft

H.A. Salomonsen, Aarhus
H. Scheerer, Heidelberg
G.P. Scott, Liverpool
B. Schellenberg, Bonn
L.C. Siebenmann, Orsay
E. Stamm, Toronto
R. Stöcker, Bochum
A. Van de Ven, Leiden
E. Vogt, Heidelberg
R. Vogt, Saarbrücken
G. Wassermann, Regensburg
G. Wilson, Oxford
G. Wolff, Konstanz
U. Würgler, Heidelberg
H. Zieschang, Bochum
M. Zisman, Paris

Vortragsauszüge

S. BUONCRISTIANO: Geometrical interpretation of coefficients in bordism.

If $h^*(-)$ is a cohomology theory, the usual definition of $h^*(-)$ with coefficients in an abelian group G is the following: $h^m(X;G) = h^{m+4}(X \wedge LG)$, where LG is a cohomology-Moore-space of type (4,G).

The difficulties arising from such definition are of two kinds: (a) LG does not exist for every G, (b) $h^*(X;-)$ is not always a functor in G.

The purpose of this talk was to show that, if $h^*(-)$ is a geometric cohomology theory, i.e. it is dual to a homology theory, $h_*(-)$, representable by means of singular cycles, it is possible to give a geometric definition of $h^*(-;G)$ in such a way that $h^*(-;G)$ is defined for every abelian group G and is functorial in G. The method consists of introducing suitable singularities into the cycles representing the theory $h_*(-)$ and it works to give also: (a) a functorial definition of $h^*(-;G)$ when G is an R-modul over the commutative ring R; the corresponding universal-coefficient-theorem is given by a spectral sequences; (b) a functorial definition of $h^*(X;F)$, where F is any sheaf over X.

D. BURGHELEA: Differential graded algebras, Chern Weil construction, and applications in topology and analysis. (two lectures)

For any differential graded algebra A over k (commutative ring with unit of characteristic o) with o-component A^o, one defines a ring homomorphism ch : $K_o(A^o) \to H^{even}(A)$, the Chern character, which is natural with respect to morphisms of D.G.A.'s

One applies this construction to prove:

1) Splitting theorems in complex analysis.
2) Cohomology properties of Stein Spaces.
3) Combinatorial invariance of the real Pontrjagin classes.
4) The rationality of the real Pontrjagin classes defined using curvature in differential geometry.
5) The existence of Chern classes in cohomology for complex analytic spaces which are locally complete intersections.

An important role is played by the "flat cochains"-algebra defined by Whitney in his book "Geometric integration theory".

T. TOM DIECK: The Burnside ring and equivariant (co-)homology. Let A(G) be the Burnside ring of finite G-sets of a finite group G. Then A(G) is additively the free abelian group on G/H, where H runs through a complete set of representives for the conjugacy classes of subgroups H of G. Let t_*^G be an equivariant homology theory and assume that one also has t_*^H for subgroups H of G together with restriction homomorphism $r: t_*^G \to t_*^H$ and transfer homomorphisms $\tau: t_*^H \to t_*^G$. Under reasonable circumstances the maps $\lambda_{G/H} := \tau r: t_*^G \to t_*^G$ make t_*^G into a module over A(G).

A. Dress has determined the prime ideal structure of A(G). In particular the localizations $A(G)_{(o)}$ and $A(G)_{(p)}$ split into a direct sum of smaller rings generated by a set of minimal orthogonal idempotents. These idempotents split $t_{*(o)}^G$ or $t_{*(p)}^G$ into smaller theories. We determine these theories. Here we describe the rational case. Let

$$r_H : t_*^G \to t_*^H \to t_*^H[F_\infty, F_{prop}]^{NH/H}$$

be the composition of the restriction with the "restriction to the fixed point set" and then dividing out the action of $N_G H/H$ and let

$$R: t_*^G \to \bigoplus_{(H)} t_*^H[F_\infty, F_{prop}]^{NH/H}$$

be the sum of the r_H where (H) runs through the conjugacy classes of subgroups. Then R is a rational isomorphism and the splitting of $t_{*(o)}^G$ induced by this isomorphism is the same as the one coming from the idempotents in $A(G)_{(o)}$. For $t_{*(p)}^G$ one gets a direct summand for every conjugacy class (H) with $[N_G H:H] \not\equiv 0(p)$ which can also be described by suitable families of isotopy groups. The same method applies to cohomology, e.g. equivariant K-theory.

A. DOLD: The K-theory and the cobordism theory associated with a cohomological structure for vector bundles.

Let Vect = category of vector bundles over CW-spaces.

A cohomological structure in Vect is a contravariant functor Q: Vect $\to$ Sets together with a natural pairing $(QE_1)\times(QE_2) \to Q(E_1 \times E_2)$ satisfying certain axioms (homotopy, Mayer-Vietoris, additivity, suspension, multiplicative properties). Typical examples:

(i) $QE = \{\simeq$-classes of stable complex structures on $E\}$

(ii) $QE = \{h$-orientations of $E\}$, where h is a general cohomology theory.

A Q-bundles is then a pair (E,ε) where $E \in Ob($Vect$)$, $\varepsilon \in QE$. One can form the K-theory of Q-bundles, K_Q, and the (co-)bordism theories of Q-manifolds, Ω_Q, Ω^Q. If h is a cohomology theory then h-orientations of Q-bundles correspond to $\Omega_Q \to h$; also, in this case, one has $\Omega_Q \to \Omega_h$, $K_Q \to K_h$, and a commutative diagram

$$\begin{array}{ccc} \Omega_Q & \rightarrow & h \\ & \searrow \quad \nearrow & \\ & \Omega_h & \end{array} \quad ; \text{ in particular } \quad \begin{array}{ccc} \Omega_Q & \xrightarrow{id} & \Omega_Q \\ & \searrow \quad \nearrow & \\ & \Omega_{\Omega_Q} & \end{array}$$

Question: What is the composite $\Omega_{\Omega_Q} \to \Omega_Q \to \Omega_{\Omega_Q}$?

Another result concerns stable h-valued characteristic classes $\gamma : V_Q \to h$ for Q-bundles. It asserts that these factor as follows

$$\begin{array}{ccc} V_Q & \rightarrow & h \\ & \alpha \searrow \quad \nearrow & \\ & S_{\Omega_Q} & \end{array} ,$$

where α assigns to every Q-bundle the corresponding Ω_Q-spherical fibration, and $S_{\Omega_Q}(X) = \{S_{\Omega_Q}$-spherical fibrations over X$\}$. Further questions concern the universal role of cobordism-valued characteristic classes.

D.B.A. EPSTEIN: Manifolds folitated with all leaves compact.

Let M be a manifold of dimension m with all leaves compact of dimension q. The following theorem (known for some years to the experts) was proved: Let $\pi: M \to Q$ be the quotient map identifying each leaf to a point. The following conditions are equivalent:

1) Q is Hausdorff
2) π is a closed map
3) Let L be any leaf. Then L has arbitrarily small saturated neighbourhoods.
4) Let L be any leaf and U any neighbourhood of L. Then there is a neighbourhood V of L such that any leaf meeting V lies entirely inside U.
5) With respect to some Riemannian metric on M, the volumes of the leaves is locally bounded.
6) The same as 5) for any Riemannian metric
7) Given any leaf L, there is a finite subgroup F of O(m-q), depending on L, and a regular covering $\tilde{L}$ of L whose group of covering translations is F and a diffeomorphism $\varphi: U \cong \tilde{L} \times_F D^{m-q}$ of a neighbourhood U of L such that the quotient of any set of the form $\tilde{L} \times pt$ in $\tilde{L} \times_F D^{m-q}$ corresponds to a leaf in U.

It is conjectured that these conditions always hold if M is compact. Counterexamples are known if M is not compact.

M. FUCHS: An application of Segal's classifying space construction.

Let A be a set. Let $\underline{\underline{A}}$ be the category with the non empty finite subsets of A as objects and inclusions as morphisms. Define a formal covering to be a contravariant functor $U : \underline{\underline{A}} \to \underline{Top}$. For every formal cover U we define the category $\underline{\underline{X}}_U$ with objects: $(x,U(\sigma))$, where $x \in U(\sigma)$ and morphisms $f_{\sigma\tau} : (x,U(\sigma)) \to (f(x),U(\tau))$ whenever $\tau \subset \sigma$ and $U(\tau \subset \sigma) = f$. Let Segal's classifying space of the category $\underline{\underline{X}}_U$ be the space BX_U. If $\sigma \in$ ob $\underline{\underline{A}}$ consider all finite subsets of A containing σ and denote the corresponding full subcategory of $\underline{\underline{A}}$ by $\underline{\underline{A}}|\sigma$. $\{BX_{U|\sigma}\}_{\sigma \in \text{ob } \underline{\underline{A}}}$ forms a numerable cover of BX_U ($U|\sigma$ is U restricted to $A|\sigma$). The inclusions are cofibrations. $U(\sigma)$ is a strong deformation retract of $BX_{U|\sigma}$. One can use the mapping polyhedra $BX_{U|\sigma}$ to obtain e.g. theorems 1 and 4 in tom Dieck's paper: partitions of unity in homotopy theory. Comp. Math. Vol. 23(1971) pp.159-167. tom Dieck's theorems are also obtained for suitable domination instead of homotopy equivalences.

J.C. HAUSMANN: Embeddings of Homology spheres.

A homology sphere Σ^n is a compact n-dimensional manifold with $H_*(\Sigma^n) \cong H_*(S^n)$, (where S^n is the standard sphere and H_* means the singular homology with integers as coefficients). We show that homology spheres behave like S^n for embeddings in S^{n+1}.

if $q \geq 3$. (C^∞, topological or piecewise linear category).

W. JACO: The Structure of Three-Manifold Groups.

A group G is said to belong to the class of groups $\mathfrak{M}$ if and only if there exists a 3-manifold M (possibly not compact and possibly not orientable) with $G \approx \Pi_1(M)$. A group $G \in \mathfrak{M}$ is called a 3-manifold group.

An outline of the proof of the theorem which lists all finitely generated groups in $\mathfrak{M}$ which are extensions of finitely generated groups by infinite groups was given. Using modifications of the techniques in this proof a listing of the groups which belong to a nontrivial variety of groups and may also belong to $\mathfrak{M}$ was obtained and a description of some of the properties that the integral group ring over certain groups in $\mathfrak{M}$ must enjoy was given.

The root structure for groups in $\mathfrak{M}$, the structure of centralizers of elements in groups in $\mathfrak{M}$ and the Hopfian and residual finiteness properties of groups in $\mathfrak{M}$ was discussed.

M. KLINGMANN: Curves on oriented surfaces.

If one wishes to separate spheres embedded in a manifold by diffeotopic displacement of one of them in the case of dimension excess 2 so that in the transversal case the intersection is a 2-dim. manifold one comes to ask the following question: Let F be an oriented surface and let $\alpha_1,\dots,\alpha_n,\beta_1,\dots,\beta_n$ be a basis of $H_1(F;\mathbb{Z})$. Can one attach a handlebody (in the sense of Seifert-Threlfall i.e. the interior of an ordinary embedding of F into $\mathbb{R}^3$) to F such that the basis $\alpha_1, \ldots, \beta_n$ is induced by the meridians and parallels of latitude of that handlebody? The answer is:

Prop.: A basis of $H_1(F;\mathbb{Z})$ can be represented by meridians and parallels of an attached handlebody iff the (skew-symmetric) intersection form takes its normal form on this basis.

If F is embedded in $\mathbb{R}^n$, $n \geq 5$, the same proposition holds for the attachment of a handlebody within $\mathbb{R}^n$. The situation is more complicated for embeddings $F \subset \mathbb{R}^4$. First of all one must restrict oneself to unknotted embeddings. But also for unknotted embeddings not every basis on which the intersection form takes its normal form is representable by the meridians and parallels of a handlebody attached within $\mathbb{R}^4$. From the embedding one has a second symmetric bilinear form, the "linking form", on $H_1(F;\mathbb{Z})$, and a basis is representable iff both the intersection and the linking form take their normal form on this basis.

K. LAMOTKE: Isolated critical points.

Let f be a holomorphic function in n+1 variables with an isolated critical point at 0 and $f(0) = 0$. For a small circle S in $\mathbb{C}$ centered at 0 and a small ball B in $\mathbb{C}^{n+1}$ centered at 0, $E = f^{-1}(S) \cap B$ is locally trivially fibred by f. The typical fibre F is a compact oriented 2n-manifold with boundary. Up to homotopy equivalence F equals a bouquet of n-spheres (Milnor, Ann. of Math. Studies no.61). For n even (similar results hold for n odd) there exists an ordered base $\beta_1, \beta_2, \ldots, \beta_k$ of $H_n(F, \mathbb{Z})$ with self-intersection $\langle \beta_j, \beta_j \rangle = (-1)^{n/2} \cdot 2$. Define the reflection s_j of H_n by $s_j(x) = x - (-1)^{n/2} \langle x, \beta_j \rangle \beta_j$. The monodromy of the fibration is the composition $s_k \circ \ldots \circ s_2 \circ s_1$. If the intersection form is definite, it follows from the classification of root systems that $(H_n(F), \langle -,- \rangle)$ is a direct sum of the classical root systems A_k, D_k, E_k. Tjurina's result (Izvestija Mat. 1968) can be generalized from 3 to n+1 variables: If $\langle -,- \rangle$ is definite, there exists a coordinate transformation, such that f is one of the following germs:

A_k : $x^{k+1} + y^2 + \sum_j z_j^2$, D^k : $x^{k-1} + xy^2 + \sum_j z_j^2$, E_6 : $x^4 + y^3 + \sum_j z_j^2$, E_7 : $x^3y + y^3 + \sum_j z_j^2$, E_8 : $x^5 + y^3 + \sum_j z_j^2$. Thus, direct sums don't occur. Another application of Tjurina's method yields the following result: Let $\mathfrak{O}$ be the germ at 0 of all holomorphic functions, (∂f) the ideal generated by $\frac{\partial f}{\partial x_0}, \ldots, \frac{\partial f}{\partial x_n}$. If there is an epimorphism $\mathfrak{O}/(\partial f) \to \mathfrak{O}/(\partial g)$, then $H_n(F_g)$ is isomorphic to a submodule of $H_n(F_f)$ with the restricted intersection form.

D. LEHMANN: Exotic characteristic classes.

One way of defining exotic classes for foliations (Bott, Haefliger and others) has been given by Bott by using methods of the type Chern, Simons. This method is generalized here to other geometrical problems as these which arise from foliations, for instance: call a bundle "flat" if its structural group may be reduced to a totally disconnected subgroup; may it actually be reduced to a finite one? knowing that a vector bundle is stably flat, is it flat? does there exist a bundle like metric on a foliated manifold? do there exist q infinitesimal automorphisms of a foliation of codimension q, globally defined, everywhere linearly independent and transverse to the leaves? does there exist a connection without curvature whose holonomy group is contained in a given subgroup of the structural group? ..... Obstructions are given for these problems to have a solution by defining "exotic classes". In the particular cases of the problems 3 and 4, these classes are the ones given by Bott-Haefliger. Many of the problems above are homotopical ones and could be theoretically treated by homotopy methods; but these methods are not at all computable in general; here the obtained obstructions are generally much weaker, but computable.

A. LIULEVICIUS: <u>Immersions up to cobordism.</u>

We ask the question: given a compact m-dimensional manifold M_1^m, find the smallest natural number k such that there exists M_2 cobordant to M_1 such that M_2^m immerses into R^{m+k}. A complete answer was given for $m \leq 14$ by exhibiting the structure of $E^0\pi_*(MO)$ for $* \leq 14$, where $\pi_*(MO)$ is filtered by the images of the stable homotopy of MO(k). It was shown that: 1) the algebraically obvious generators of $\pi_*(MO)$ do not in general have the smallest filtration, 2) $E^0\pi_*(MO)$ is not a polynomial algebra (minimal counterexample in $* = 10$), 3) the filtration defined by $H_*(MO(k);Z_2)$ differs from the geometric filtration above (minimal example in dimension 14).

G. LUKE: <u>Pseudo-differential operators on Hilbert bundles.</u>

Let H be a Hilbert space. A pseudo-differential operator $P : C^{\infty}(\mathbb{R}^n, H) \to C^{\infty}(\mathbb{R}^n, H)$ is defined locally in terms of a symbol function $p : \mathbb{R}^n \times \mathbb{R}^n \to$ "bounded linear operators on H" by the usual formula. A symbol is said to be elliptic if 1) p is of order 0, 2) $p(x,0) - p(x,\xi)$ is a compact operator for all x and ξ, 3) $p(x,\xi)$ is a Fredholm operator for all x and ξ, 4) $p(x,\xi)$ is invertible for $|\xi|$ sufficiently large. If X is a compact manifold, a pseudo-differential operator on $C^{\infty}(X,H)$, defined locally in terms of elliptic symbols is a Fredholm operator and also possesses a symbol which defines an element of $K(T^*X)$. The analytic and topological indices of such an operator are equal.

Let X and Y be compact manifolds. As a consequence of the theory above, we get a map

$$K(T^*(X \times Y)) \to K(T^*X)$$

such that the following diagram commutes

$$\begin{array}{ccc} K(T^*(X \times Y)) & \to & K(T^*X) \\ & \text{index} \searrow \quad \swarrow \text{index} & \\ & \mathbb{Z} & \end{array}$$

G. LUSZTIG: <u>Infinite cyclic coverings.</u>

A theorem of Novikov states that for any compact smooth oriented manifold M^{4k+1} the class $L_{4k} \in H^{4k}(M)$ is a homotopy invariant. This is shown to have an analytic interpretation in terms of the index of a family of self adjoint elliptic operators parametrized by a circle (in the sense of Atiyah). In order to make the connection one has to develop an analogue of Hodge's theory on non compact manifolds which are infinite cyclic coverings of compact ones.

G. LUSZTIG: The discrete series representations of the general linear groups over a finite field.

In his proof of the Adams conjecture, Quillen constructs a map $BGL_n(F) \to BGL_\infty(\mathbb{C})$, F a finite field, which in the limit is a homology equivalence at primes $\nmid$ char. F. He used the Brauer's lifting of modular characters which makes the construction non-effective. We show how to construct explicitely this map by constructing representations of $GL_n(F)$. In particular we construct the discrete series representation of $GL_n(F)$ as module over the Witt vectors W(F) over F of rank $(q-1)(q^2-1) \ldots (q^{n-1}-1)$, q = card F. The definition field of this representation is

$$Q\Big(\sum_{\substack{\xi \in F_{q^n} \\ \mathrm{trace}_{(F_{q^n}/F_q)}\xi = 1}} \varphi(\xi)^{-1} \quad , \ \varphi(\xi)^{1+q+\ldots+q^{n-1}} \Big)$$

where $\varphi : F_{q^n}^* \to \mathbb{C}^*$ is a multiplicative embedding.

H.J. MUNKHOLM: <u>Collapse of the algebraic Eilenberg Moore spectral sequence for homogeneous spaces.</u>

<u>Theorem</u>: Let $\begin{array}{ccc} E' & \to & E \\ \downarrow & & \downarrow \\ B' & \to & B \end{array}$ be a fiber square with E,B,B' connected.

Take coefficients in an arbitrary ring R. Assume that H^*E, H^*B, H^*B' are polynomial algebras, and if char R = 2 assume that S_{q_1} vanishes on H^*B' and H^*E. Then the Tor-Eilenberg-Moore spectral sequence collapses and has no additive extension problem.

This is proved by studying an extension $\mathfrak{DASH}$ of the category $\mathfrak{DA}$ of differential graded algebra, $\mathfrak{DASH}(A,B) = \mathfrak{DA}(\mathfrak{FB}A,B)$ with $\mathfrak{F},\mathfrak{B}$ the cobar- and bar-construction, respectively.

The main tool in the study of $\mathfrak{DASH}$ is the category Ext^o_A of trivialized extensions of $A \in \mathfrak{DA}$. Here objects are diagrams

$$A \xleftarrow{\alpha} X \xrightarrow{h} X \xleftarrow{\rho} A$$

with $\alpha\rho = A$, $\rho\alpha = X - (dh+hd)$, $\alpha \in \mathfrak{DA}$, $\rho \in \mathfrak{DA}$, $h \in Hom$. Morphisms are slightly complicated, in order to obtain the following

<u>Main Lemma:</u> Ext^o_A has an initial object

$$A \xleftarrow{\alpha_A} \mathfrak{FB}A \xrightarrow{h_A} \mathfrak{FB}A \xleftarrow{\rho_A} A$$

with α_A the front adjunction.

W. NEUMANN: <u>Cutting and pasting of manifolds.</u>

Cutting and pasting of manifolds can be done along 2-sided 1-codimensional submanifolds. Cutting and pasting of oriented singular manifolds in a space X leads to a graded group $SK_*(X)$ with the following properties: $SK_n(X) = 0$ for n odd. $SK_n(pt) =$

$\mathbb{Z}$, $\mathbb{Z}\oplus\mathbb{Z}$ for $n \equiv 2, 4$ modulo 4. For X simply connected $SK_n(X) = SK_n(pt)$ and if $|\pi_1(X)| < \infty$ then $SK_n(X) = SK_n(pt) \oplus$ torsion. However $SK_n(X)$ in general can differ from $SK_n(pt)$ by much more than just torsion.

One tool in the calculations is the fact: if $\overline{SK}_n(X) := SK_n(X)/(\text{Bordism relations})$, then $SK_n(X) = \overline{SK}_n(X)\oplus\mathbb{Z}$ (n even), and $\overline{SK}_n(X) = \Omega_n(X)/\{[M,f] \in \Omega_n(X)|M \text{ can be fibred over } S^1\}$.

Another tool is Winkelnkemper's "Open Book Theorem". The "reduced group" $\mathrm{Ker}(SK_n(X) \to SK_n(pt))$ contains, for suitable X, complete obstructions for the multiplicativity of the signature problem.

<u>Conjecture</u>: $SK_n(X)$ only depends on $\pi_1(X)$ (true for $n \le 3$).

Details will appear in forthcoming notes by Kreck, Karras, Neumann and Ossa.

E. OSSA: <u>Report on a paper of Arnold on the topology of real algebraic curves.</u>

The paper in question appeared in: Functional Analysis and its Applications, vol.5, no.3 (1971).

Let $F \in \mathbb{R}[x,y,z]$ be a homogeneous polynomial of degree 2k which is irreducible and non singular over $\mathbb{C}$. Let $F_{\mathbb{C}} = \{[x : y : z] \in P_2(\mathbb{C}) | F(x,y,z) = 0\}$ and $F_{\mathbb{R}} = F_{\mathbb{C}} \cap P_2(\mathbb{R})$.

Then F is called an M-curve iff $F_{\mathbb{R}}$ has its maximum number $(2k-1)(k-1)+1$ of connected components; these components are called ovals, are homeomorphic to the circle and have a well-defined interior homeomorphic to a disc. Denote by p (resp. m) the number of ovals lying in the interior of an even (odd) number of other ovals, and assume that F is an M-curve. Arnold proves that $p - m \equiv k^2 (4)$. This result is obtained by taking the double cover of $P_2(\mathbb{C})$ ramified along $F_{\mathbb{C}}$ and studying the intersection form, the covering transformation and the complex conjugation on the middle homology of Y; $2k^2$ and $-2(p-m)$ are the signatures of these involutions. Along these lines he also gives a simple proof of the Petrowsky-inequality $|2(p-m)-1| \leq 3k^2 - 3k + 1$.

D. PUPPE: The stable homotopy category.

We give a construction of a category equivalent to Boardman's stable category which may have some advantages over other known approaches (Boardman's or the semisimplicial one). Let Sp be the "naive" category of (CW)-spectra. Objects are sequences of pointed CW-complexes X_n $(n \in \mathbb{Z})$ together with CW-embeddings $\xi_n : \Sigma X_n \subset X_{n+1}$ as "structure maps". Morphisms are sequences of pointed continuous maps $f_n : X_n \to Y_n$ which are (strictly) compatible with the structure maps. A "dense embedding" is a morphism $X' \to X$ in Sp such that $X'_n \to X_n$ is a CW-embedding and for any finite subcomplex $K \subset X_n$ there is an r such that $\Sigma^r K \subset X_{n+r}$ is contained in the image of X'_{n+r}. Now the stable category S is obtained from Sp by formally inverting all dense embeddings (category of fractions). There is a straightforward homotopy notion in S which leads to the stable homotopy category Sh. All the important properties of Boardman's corresponding categories can be established for S and Sh quite directly. In constructing smash products

Boardman's idea that the index set $\mathbb{Z}$ should be replaced by the set of finite dimensional subspaces of a euclidean vector space of countable dimension is used here, too. Except for this point our approach is very similar to the one by Adams in his Chicago lecture notes which have just become available.

E. REES: <u>The geometric dimension of certain bundles.</u>

If $x \in \tilde{K}(X)$, the geometric dimension of x is defined by g.-dim $x \leq k$ if there is a k-dimensional bundle ξ over X whose stable class is x. A necessary condition for g.-dim $x \leq k$ is that $c_i x = 0$ for $i > k$. The sufficiency of this condition was discussed.

<u>Example</u>: There exists $x \in \tilde{K}(\mathbb{H}P^2)$ $c_4 x = 0$ and g.-dim $x = 3$.

<u>Theorem</u>: If dim $X = 2n$ and $H^{2n}X$ has no elements of order 2, $x \in \tilde{K}(X)$, $c_n x = 0$ and $c_{n-1}x = 0$ then g.-dim $x \leq n - 2$ if

a) (E. Thomas) n is odd

b) n is even and $[X, S^{2n-1}] = 0$

<u>Corollary 1</u>: $x \in \tilde{K}\mathbb{C}P^n$, $c_n x = 0$, $c_{n-1}x = 0$ then g.-dim $x \leq n-2$

<u>Corollary 2</u>: There is a 2-dimensional bundle ξ over $\mathbb{C}P^4$ with $c_1\xi = ay$, $c_2\xi = by^2$ if and only if

$$b(b+1-3a-2a^2) \equiv 0(12)$$

<u>Theorem</u>: $x \in \tilde{K}\mathbb{C}P^n$ $c_i x = 0$ $i > k$ then g.-dim $x \leq k$ if

i) $k = n - 3$

or ii) $k = n - 4$ and $n \equiv 2,3(4)$

An optimistic conjecture would be that this theorem is true for all n,k.

E. REES: Brown's generalisation of the Kervaire invariant.

This talk was an exposition of E.H. Brown: Generalisations of Kervaire's invariant. Ann. Math. 95, p. 386.

C.A. ROBINSON: Stable homotopy theory over a space B.

We construct a category $\mathfrak{S}/B$ in which a kind of stable homotopy theory can be constructed, and we investigate an analogue of the Adams spectral sequence in this category. This has applications to lifting problems, and to the enumeration of immersions in the metastable range. Intuitively, we think of objects as "bundles" over a fixed space B with "fibre" a spectrum, and of morphisms as being "fibre-preserving maps".

Let $\mathfrak{S}$ be Boardman's category of CW spectra. For any objects F_1, F_2 of $\mathfrak{S}$, there is a simplicial set $\underline{\mathrm{Mor}}_{\mathfrak{S}}(F_1,F_2)$. If $F_1 = F_2$ this is a simplicial monoid whose invertible elements form a simplicial group $\underline{\mathrm{Aut}}_{\mathfrak{S}}\, F_1$. Let B be a fixed simplicial set, which for simplicity we assume to be connected.

Definition: An object of $\mathfrak{S}/B$ ("bundle of spectra over B") is a pair (F,ξ) where $F \in \mathrm{ob}\,\mathfrak{S}$ and ξ is a principal simplicial $\underline{\mathrm{Aut}}_{\mathfrak{S}}\, F$-bundle over B. A morphism from (F_1,ξ_1) to (F_2,ξ_2) is a section of the simplicial bundle with fibre $\underline{\mathrm{Mor}}_{\mathfrak{S}}(F_1,F_2)$ associated to the simplicial $(\underline{\mathrm{Aut}}_{\mathfrak{S}}F_1 \times \underline{\mathrm{Aut}}_{\mathfrak{S}}\, F_2)$-bundle $\xi_1 \times \xi_2$ over B.

In $\mathfrak{S}/B$ one can define homotopy and an invertible translation-suspension functor S_B. The homotopy category $(\mathfrak{S}/B)_h$ is additive, and triangulated with respect to S_B. We write $\{X,Y\}_B^*$ for the graded group of homotopy classes from a bundle X to a bundle Y.

Let p be a prime. We define on $\mathfrak{S}/B$ a cohomology theory $H^*(\ ;p)$ with values in an abelian category $A^*_{B,p}$-mod, by using all finite-dimensional semisimple representations of $\pi_1 B$ over $\mathbb{Z}/p$ as coefficients for "ordinary" cohomology. If $\pi_1 B = 0$, $A^*_{B,p}$-mod is equivalent to the category of graded modules over the Massey-Peterson algebra $H^*(B;\mathbb{Z}/p) \odot A^*_p$. If B is a point, this is the category of modules over the Steenrod algebra A^*_p.
Let Y_1, Y_2 be objects of the category $\mathfrak{S}/B$. Under certain restrictive finiteness conditions we construct a spectral sequence of Adams' type by realizing a minimal resolution in $A^*_{B,p}$-mod of $\underset{\sim}{H}^*(Y_2;p)$, and applying the functor $\{Y_1,\ \}^*_B$.
Theorem: Under these conditions, there is a convergent spectral sequence

$$\mathrm{Ext}^{s,t}_{A^*_{B,p}\text{-mod}}(\underset{\sim}{H}^*(Y_2;p),\underset{\sim}{H}^*(Y_1;p)) \underset{s}{\Longrightarrow} \{Y_1,Y_2\}^*_B/\text{torsion prime to p}$$

This spectral sequence can be used for practical calculations. The above extends work of J.-P. Meyer, J.F. McClendon, J.C. Becker and R.J. Milgram.

C.P. ROURKE: Representing homology classes.

We give a short proof of Thom's theorem on representing $\mathbb{Z}_2$-homology classes by manifolds. The proof uses about ¼ of the ingredients of Thom's proof at the expense of a spectral sequence argument. The method also gives a new representation theorem for $\mathbb{Z}_p$-homology for p prime by manifolds with a simple type of singularity (the "ring closure of twisted $\mathbb{Z}_p$-bordism"). This partially answers a question of Sullivan (in his M.I.T. notes).

H.A. SALOMONSEN: Embeddings of manifolds in the metastable range.

Let $f : M^n \to U^{n+k}$ be a mapping between smooth manifolds, M compact. We assume for simplicity that $\partial M = \emptyset$.

We considered the question: Is f homotopic to an embedding? – and if it is – to how many up to regular homotopy?

It was shown that, if f satisfies suitable transversality conditions, it is possible to associate to f a triple $\mathcal{E}(f) = (\bar{\Delta}^{n-k}, \delta f, F_f)$, where $\bar{\Delta}$ is the image of the double points of f, $\delta f : (\bar{\Delta}, \partial\bar{\Delta}) \to (W_f, M \times P^\infty)$, $F_f : \tau_{\bar{\Delta}} \oplus (\delta f)^* \psi_- \cong (\delta f)^* \psi_+$.

Here W_f is defined as the pull-back

$$\begin{array}{ccc} W_f & \longrightarrow & U \times P^\infty \\ \downarrow & & \downarrow \text{diag.} \\ Ep(M) & \xrightarrow[Ep(f)]{} & Ep(U) \end{array} \quad (= Ep^\infty(U) = U \times U \times_{\mathbb{Z}_2} S^\infty)$$

and the bundles ψ_+ and ψ_- are defined as the pull-backs of $Ep(\tau_M)$ and $\tau_U \tilde{\otimes}$ Hopf bundle over W_f. In a similar way it is possible to associate to a homotopy $F = \{f_t\} : M \times I \to U$ a bordism $\mathcal{E}(F)$ from $\mathcal{E}(f_0)$ to $\mathcal{E}(f_1)$. We stated

Theorem: Let $f : M^n \to U^{n+k}$, $n \leq 2k - 3$, and let $\omega = (W^{n-k+1}, g_W, G_W)$ be a bordism from $\mathcal{E}(f)$. Then there exists a homotopy F of f such that $\mathcal{E}(F) = \omega$ up to homotopy rel. to $\mathcal{E}(f)$, i.e. $W = \bar{\Delta}(F)$ and there is a homotopy $g_W \simeq \delta F$ rel. $\bar{\Delta}(f)$ covered by a homotopy of the bundle isomorphisms.

It follows that for $n \leq 2k - 3$ the bordism class $[\mathcal{E}(f)] \in \Omega_{M-k}(W_f, M \times P^\infty; \psi)$, $\psi = \psi_+ - \psi_-$, vanishes iff f is homotopic to an embedding.

If $n \leq 2k - 4$ and $f : M \to U$ is a given embedding, then there is a bijection of {regular homotopy classes of embeddings

$g : M \to U$ in the homotopy class of $f\}$ with the {orbits of $\Omega_{n-k+1}(W_f, M\times P^\infty;\psi)$ under a certain action of $\pi_1(U^M;f)\}$.

B. SCHELLENBERG: Homotopy equivalences of 2-complexes.

There is a functor which associates to a CW complex X its fundamental group π_1 together with the $\mathbb{Z}[\pi_1]$-chaincomplex $C_*(\tilde{X})$; to a homotopy class $[f] : X \to Y$ the induced homom. θ on π_1 together with the homotopy class $[f_*]$ of the θ-linear chainmap f_*. This functor is bijective for 2-complexes. In dimension 2 we have: $\pi_1 X \cong \pi_1 Y$ implies that X and Y have the same homotopy type, after adding a number of 2-spheres to each. Problem: when does $X\vee S^2 \simeq Y\vee S^2$ imply $X \simeq Y$? If the number of 1-cells is 1, the answer is "always". If this number is 2, it may or may not be "always". The condition can be translated into one about finding elements in $Gl(\mathbb{Z}[\pi_1])$ with certain properties.

G.P. SCOTT: Compact submanifolds of 3-manifolds.

Let M be a 3-manifold. Then the following result is true.

Theorem 1: If $\pi_1(M)$ is finitely generated, then there is a compact submanifold N of M such that inclusion induces an isomorphism $\pi_1(N) \to \pi_1(M)$. From this result follows immediately

Theorem 2: If $\pi_1(M)$ is finitely generated, then $\pi_1(M)$ is finitely presented. This second theorem has also been proved by Shalen. Results in the direction of Theorem 2 or closely related to it have been proved by Jaco, Swamp, and Galewski,

Mollingsworth and McMillan.
These results tell us that every finitely generated subgroup of the fundamental group of a 3-manifold is finitely presented. They also give a method for introducing compact manifolds into the study of non-compact 3-manifolds.

L. SIEBENMANN: Topological Stratifications and Polyhedra.

Call a locally compact locally finite dimensional metrizable space X locally starlike if at any point x in X a sufficiently near-sighted observer might believe himself to be the center of the universe - i.e. x has an open neighbourhood U homeomorphic to the open cone on a compactum L, $U \approx cL \equiv L \times [0,\infty] / L \times 0$, with x corresponding to the cone point.

Conjecture: For such starlike X, the closed filtration $X \supset \dots \supset X^{(n)} \supset X^{(n-1)} \supset \dots$, $X^{(n)} = \{x \in X;$ x has no neighbourhood of the form $R^{n+1} \times A\}$, satisfies (i) $\forall n$; the 'stratum' $X^{(n)} - X^{(n-1)}$ is an n-manifold (ii) a certain stratified 'locally cone-like' property, see CMH 1972.

A filtration of X satisfying (i) and (ii) makes X a so-called CS set and assures some pleasant properties, e.g. an isotopy extension principle, countability of compact homeomorphism types ... . There are some curious CS sets. (1) A compact CS set with 2 non-empty strata (one a circle), that is locally triangulable but not triangulable. (2) A CS set (with 3 strata) that is not locally triangulable. (3) Sphere pairs (S^n, K^{n-2}), n even ≥ 6, that are locally triangulable but not triangulable. (4) A quotient space X of a locally smooth prime cyclic action on $S^n \times S^1$, n even ≥ 4, that is a CS set and non-triangulable. The constructions use non-compact invertible cobordisms: The proofs use torsion invariance and torus geometry (or meshing).

A. VAN DE VEN: Hilbert modular group and algebraic surfaces.

Let $k = L(\sqrt{p})$, p prime, $p \equiv 1(4)$. Then $SL_2(k)$ operates in a natural way on the product of the upper half plane $H = \{x \in \mathbb{C}, \text{Im}(x) > 0\}$ with itself. After compactifying with the cusps and desingularisation, the quotient becomes a non singular algebraic surface $H^{(p)}$, which is rational for p = 5, 13, 17, blown up elliptic K3 for p = 29, 37, 41, blown up elliptic (non rational, non K3, non torus, non ruled) for p = 53, 61, 73, and blown up of general type for p ≥ 89.

R. VOGT: Homotopy limits and applications.

Let $\mathfrak{C}$ be a small indexing category. A homotopy-$\mathfrak{C}$-diagram (h$\mathfrak{C}$-diagram) is a $\mathfrak{C}$-diagram of topological spaces up to coherent homotopies. Define a homotopy morphism (h-morphism) of h$\mathfrak{C}$-diagrams to be a morphism of h$\mathfrak{C}$-diagrams in the usual sense but only up to coherent homotopies. If one introduces a notion of homotopy between h-morphisms in the obvious way, the h$\mathfrak{C}$-diagrams and homotopy classes of h-morphisms form a category $\mathfrak{H}\mathfrak{C}$. Let $\mathfrak{J}$: Top → $\mathfrak{H}\mathfrak{C}$ be the functor sending a space X to the constant h$\mathfrak{C}$-diagram, i.e. the diagram having X at each vertex, all morphisms are the identity on X and all homotopies are trivial. Then $\mathfrak{J}$ has a left adjoint h-colim, the homotopy colimit functor, and a right adjoint h-lim, the homotopy limit functor . Given an h$\mathfrak{C}$-diagram D, then h-colim D has a filtration and h-lim D a cofiltration giving rise to a homology (cohomology) spectral sequence respectively a homotopy spectral sequence. E.g. let k^* be a generalized cohomology theory and X a based space such that $[X,-]^o$, the based homotopy classes of maps from X to -, is abelian group valued, then we have spectral sequences

$$E_2^{p,q} \cong \lim^{(p)} k^q(D) \Rightarrow k^{p+q}(\text{h-colim } D)$$

$$E_2^{p,q} \cong \lim^{(p)} [S^qX, D]^o \Rightarrow [S^*X, \text{h-lim } D]$$

where $\lim^{(p)}$ is the p-th right derived of lim. The first spectral sequence generalizes the Milnor-$\lim^1$-lemma or the Mayer-Vietoris-sequence, the second can be used to give a direct description of Sullivan's p-adic profinite completion of a topological space. As further applications of our constructions we obtain results about local homotopy equivalences or local dominations in the sense of Dold [Partitions of unity in the theory of fibrations] or tom Dieck [Partitions of unity in homotopy theory].

G. WILSON: <u>Characteristic numbers for G-bordism.</u>

Let G be a finite group, X a closed smooth free unitary G-manifold. Let

$$ch(X) : \mathbb{Z}[\gamma^i] \to K_*(BG)$$

be the 'characteristic numbers' of X [image of X under $\mu : U_*(BG) \to \mathrm{Hom}(\mathbb{Z}[\gamma^i]; K_*(BG))$].

Let R = complex character ring of G, I = ideal of characters zero at 1, S = a multiplicative subset of R with $S \cap I \neq \emptyset$. Then $S^{-1}K_G^*(\text{free } G\text{-spaces}) = 0$. If X is as above, dim X odd, and $X = \partial Y$, where Y is a unitary G-manifold (not necessarily free) we define

$$At(X) : \mathbb{Z}[Y^i] \to K_G(Y) \to S^{-1}K_G(Y) \xleftarrow{\cong} S^{-1}K_G(Y-X) \xrightarrow{!} S^{-1}R \to \frac{S^{-1}R}{R}$$

[K_G with compact supports: ! = Gysin map of $(Y-X) \to$ (point)]. The invariant $At(X)$ is of the kind described in Atiyah-Singer: Elliptic operators III.

<u>Theorem</u>: The invariants $ch(X)$, $At(X)$ coincide. More precisely, there is a commutative diagram

$$\begin{array}{ccc} & \overset{ch(X)}{\nearrow} & K_1(BG) \\ \mathbb{Z}[Y^i] & & \downarrow \alpha \\ & \underset{At(X)}{\searrow} & \dfrac{S^{-1}R}{R} \end{array}$$

, where

1) if there is a complex G-module V on which G acts freely except at 0, and S = {powers of $\lambda_{-1}(V)$}, α is an isomorphism,
2) for any G, if S = {powers of ($|G|$-regular representation)}, α is the inclusion of $\{x \in \frac{S^{-1}R}{R} | I^n x = 0 \text{ for some } n\}$; α is an isomorphism for G a p-group.

H. ZIESCHANG: <u>On finite groups of mapping classes of surfaces.</u>

<u>The problem</u>: Can a finite group of mapping classes of a surface be represented by a finite group of mappings? For the cyclic case it is proved by J. Nielsen, Acta Math. 75 (1942). For the general case it was stated by S. Kravetz, Ann. Acad. Fennicae A 278 (1959), but his proof turned out to have gaps. The methods of Nielsen can be generalized to solve the pro-

blem, if the "Spezialfall" does not occur, i.e. if in the extension of the surface group by the finite group will not contain two elements x,y of finite order, which are not from a cyclic group, such that x,y has finite order.

Corollary: A finite torsion free extension of a surface group is a surface group.

M. ZISMAN: Homotopy spectral sequence for -spaces.

Let X be a Δ-space i.e. a simplicial object in the category of topological spaces such that all inclusions made with degeneracies operators are cofibrations. G. Segal then proves the existence of a convergent spectral sequence $E^2_{p,q} = H_p H_q X \Rightarrow H_{p+q}|X|$ where $|X|$ stands for the geometric realization of X. Let us suppose now that the X_n are pathwise connected and X_1 is 1-connected. Under these conditions it is proved that there also exists a spectral sequence $E^2_{p,q} = \pi_p \pi_q X \Rightarrow \pi_{p+q}|X|$. As an application, one gives an algebraic proof of the fact that the canonical map $X_1 \to \Omega|X|$ is a weak homotopy equivalence when X is a Δ-special space in the meaning of G. Segal. The existence of the homotopy spectral sequence comes from a theorem of Quillen on bisimplicial groups and an analogue for bisimplicial sets.

R. Vogt (Saarbrücken)