MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 10|1973

Wahrscheinlichkeitstheorie

18. 3. bis 24. 3. 1973

Die diesjährige Tagung über Wahrscheinlichkeitstheorie stand unter der Leitung von W.Bühler (Mainz) und W.v.Waldenfels (Heidelberg).
Neben den Übersichtsvorträgen und der Vorstellung spezieller Ergebnisse aus dem Bereich der Wahrscheinlichkeitstheorie standen auch einige Vorträge aus dem Gebiet der Mathematischen Statistik, die den Themenkreis abrundeten und den gegenseitigen Kontakt zwischen W.-Theorie und Statistik förderten.
Außerdem war ein Halbtag für die Diskussion über "teaching of model building" reserviert. Diese Diskussion ging aus von der Vorstellung spezieller Modelle und einem Erfahrungsbericht von D.G.Kendall (Cambridge) über einen Kurs, den er in Cambridge in Zusammenarbeit mit Biologen, Archäologen und anderen Anwendern durchgeführt hatte.

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Vortragsauszüge

H. DINGES: Theory of stopping sequences

P is a fixed positive contraction of $L^1(E,\varrho)$ (or a submarkov-kernel); μ,ν denote σ-finite measures, η monotone limits of σ-finite measures, all absolutely continuous with respect to ϱ.
Notation: If there exists a Markov process $X_0, X_1, \dots$ and a stopping time τ with $\mathcal{L}(X_0) = \mu$, $\mathcal{L}(X_\tau) = \nu$, $\sum_{k=0}^{\infty} (X_k; \tau > k) = \eta$ we write $\mu \underset{\eta}{\mapsto} \nu$ (Starting with μ one reaches ν using η)
Remark: If $\mu \underset{\eta}{\mapsto} \nu$ then η satisfies the "Poisson-equation" $\eta + \nu = \eta P + \mu$. - The main tool to study the relation $\mu \underset{\eta}{\mapsto} \nu$ is the concept of a stopping sequence. Def: $m = (\mu; \mu_0, \mu_1, \dots)$ is called a stopping sequence if $\mu_0 \leq \mu$, $\mu_k \leq \mu_{k-1} P$ for $k = 1, 2, \dots$
Importent and in a precise sense extreme devices to construct stopping sequences are the filling-scheme and the flooding-scheme. For instance: a) The filling-scheme yields the minimal solution of the Poisson-inequality $\eta + \nu \geq \eta P + \mu$
b) If $\mu \underset{\eta}{\mapsto} \nu$ with η σ-finite, then the flooding-scheme yields a stopping time with mimimal variance.

A. DVORETZKY: Sums of dependent random variables

The sufficiency part of the results on limiting distributions of triangular arrays of sums of independent random variables

is carried over to the general dependent case via conditioning on the preceding row sum. The method extends to random variables assuming values in groups other than the real or complex numbers.
Under certain restrictions, e.g. martingale structure, much of the necessity part can also be carried over. Under suitable conditions one can deduce also convergence of the sequence of partial sums to suitable limiting stochastic processes, e.g. brownian motion.

G.K. EAGLESON: The central limit problem for martingales

Consider a triangular array of random variables whose rows are martingale difference sequences. That is, for each $n = 1,2,\ldots$ we have random variables $X_{n1},\ldots,X_{nk_n}$ on a probability space $(\Omega,\mathcal{F},P)$ with sub-σ-fields $\mathcal{F}_{n0} \subset \mathcal{F}_{n1} \subset \ldots \subset \mathcal{F}_{nk_n}$ of $\mathcal{F}$ such that X_{nk} is $\mathcal{F}_{nk}$-measurable and $E(X_{nk} | \mathcal{F}_{n,k-1}) = 0$ a.s. for $k = 1,\ldots,k_n$.
Sufficient conditions for the row sums of such arrays to converge in distribution to an infinitely divisible law will be given. These conditions are analogous to the classical conditions for the case when the X_{nk}, $k = 1,\ldots,k_n$, are independent. Applications and generalizations of the results will also be discussed.

W. EBERL: Bemerkungen zum Waldschen Entscheidungsmodell

Bericht über die Diplomarbeit von H. Stadler (Wien), in der das sequentielle Entscheidungsmodell von A. Wald mit Hilfe des bekannten Satzes von C. Ionescu Tulcea eine vereinfachende Darstellung erfährt. Die Risikofunktion wird zu einem Funktional auf einer zweiparametrigen Familie von stochastischen Prozessen. Diese Darstellung ist der Ausgangspunkt für Existenzsätze und Lösungsalgorithmen.

E. EBERLEIN: Erzeugersätze und strikte Ergodizität von Strömungen

Für meßbare Strömungen $(T_t)_{t\in\mathbb{R}}$ auf einem Lebesgueschen Maßraum $(\Omega,\mathfrak{X},m)$ gilt der folgende Erzeugersatz: Ist $(T_t)_{t\in\mathbb{R}}$ aperiodisch, so existiert ein abzählbarer Erzeuger von endlichem Typ. - Wir geben zwei Anwendungen dieses Satzes an: (1) Einbettung von aperiodischen Strömungen in den Raum der Pfade eines stationären Prozesses $(X_t)_{t\in\mathbb{R}}$ mit abzählbarem Zustandsraum. (2) Einbettung von aperiodischen Strömungen in ein metrisches Kompaktum von Lipschitzfunktionen. Die zweite Anwendung wird benutzt, um den folgenden Satz zu beweisen: Jede ergodische Strömung ist isomorph zu einer strikt ergodischen Strömung.

H. ENGMANN: Characterization of L^1_B-contractions by inequalities

Given families $x = \{x_k : k\in K\}$ and $y = \{y_k : k\in K\}$ of (Bochner-) integrable bounded random variables with values in a reflexive Banach-space one can introduce functionals D^1_x, D^1_y, D_x, D_y on special families of integrable bounded r. v. with values in the dual of the given B-space such that the following holds.

Theorem: There exists a linear transformation T on the integrable bounded r. v. with $\|T\|_1 \leq 1$, $\|T\|_\infty \leq 1$, $Tx_k = y_k$ for all $k\in K$ iff $D^1_x \geq D^1_y$ and $D_x \geq D_y$ (pointwise)

Similar statements for transformations T fulfilling only $\|T\|_1 \leq 1$ or only $\|T\|_\infty \leq 1$ are given. The main steps for proving results of the above type are indicated.

H. FÖLLMER: The representations of a semimartingale

Consider a real-valued stochastic process $X = (X_t)_{t\geq 0}$ over a system $(\Omega,\underline{\underline{F}},\underline{\underline{F}}_t,P)$. Call it a semimartingale if $\sup \sum_{i=1}^n E\left|X_{t_i} - E\left[X_{t_{i+1}} \middle| \underline{\underline{F}}_{t_i}\right]\right| < \infty$ where the supremum is taken over finite sequences $0 \leq t_0 < \ldots < t_n$. Under some regularity conditions on $(\Omega, \underline{\underline{F}}, \underline{\underline{F}}_t)$ a semimartingale X induces a finite signed measure P^X on the σ-field $\underline{\underline{P}}$ of previsible sets in $\Omega \times (0,\infty]$. This measure is used to derive some of the main decomposition theorems for semi- and supermartingales. In order to show how the measure enters into a refined analysis of the limit behavior of stochastic processes over $(\Omega, \underline{\underline{F}}, \underline{\underline{F}}_t, P)$, the following "boundary minimum principle" is derived: If a supermartingale X satisfies $X \geq -M$ and $\lim_{t \nearrow \infty} X_t \geq 0$ P^M-a.s., where M is a martingale ≥ 0, then we may conclude $X \geq 0$.

P. GÄNSSLER: Zur Konvergenz empirischer Verteilungen

Es wird darüber berichtet, wie sich mit Hilfe eines Charakterisierungssatzes für μ-uniforme Klassen $\mathcal{C} \subset \mathcal{A}$ (bzgl. der mengenweisen Konvergenz von Maßen $\mu \in ca_+(X,\mathcal{A})$) bisher bekannte Sätze vom Glivenko-Cantelli-Typ wiedergewinnen lassen, sowie das folgende Resultat (von W. Stute in Bochum):
Sei $\mu \in ca_+(\mathbb{R}^k, \mathcal{B}_k)$ so, daß $\mu \ll \mu_1 \otimes \ldots \otimes \mu_k$ für geeignete $\mu_i \in ca_+(\mathbb{R}, \mathcal{B})$, dann gilt: Ist $(f_n)_{n \in N}$ eine Folge unabhängiger identisch verteilter zufälliger Variabler $f_n: \Omega \to \mathbb{R}^k$ über einem W-Raum $(\Omega, \mathcal{F}, P)$, so folgt:

$$P_*(\{\omega \in \Omega : \lim_{n \to \infty}(\sup_{C \in \mathcal{C}_k} |\mu_n^\omega(C) - \mu(C)|) = 0\}) = 1$$

(Dabei bezeichnet $\mathcal{C}_k$ die Gesamtheit aller konvexen Borelschen Mengen in $\mathbb{R}^k$, μ_n^ω die zu $f_1(\omega), \ldots, f_n(\omega)$ gehörige empirische Verteilung und P_* die zu P gehörige innere Wahrscheinlichkeit)

F. HAMPEL: Robust estimation

The statistical theory of robust estimation deals with problems arising from the observation that stochastic models, when applied to real data, hold only approximately rather than exactly. To take a comparatively simple example, the assumption of normally distributed measurement errors, leading to the method of least squares, was introduced by Gauss only to let the arithmetic mean appear optimal. Moreover, the central limit theorem as well as experience suggest only approximate normality, and Tukey has shown that even slight deviations from exact normality lead to considerable deterioration of arithmetic mean and standard deviation. Huber and the speaker developed various theoretical approaches yielding robustified maximum likelihood estimates, in which outliers either "brought in" or eventually even "thrown out" in theoretically optimal and practically very successfull ways.

H. HERING: Markov branching processes with an arbitrary set of types

$(X,\mathfrak{a})$ any measurable space, $(\hat{X},\hat{\mathfrak{a}})$ corresponding population space, $(\hat{x}_t, P_{\hat{x}})$ a homogeneous Markov branching process in $(\hat{X},\hat{\mathfrak{a}})$. For every bounded, $\mathfrak{a}$-measurable, real function ξ define $\check{\xi}(\hat{x})=0$ if $\hat{x}=\Theta$ (Θ denotes "the empty population"), $\check{\xi}(\hat{x})=\sum_{i=1}^{n} \xi(\hat{x}_i)$ if $\hat{x}=\langle \hat{x}_1,\dots,\hat{x}_n\rangle \in \hat{X}$, n>0. Suppose that $\varrho=\lim_{t\to\infty}(\sup_{x\in X} E_{\langle x\rangle}\check{1}(\hat{x}_t))^{1/t}$, $E_{\langle x\rangle}\check{\varphi}(\hat{x}_t)=\varrho^t\varphi(x)>0$, $\varphi^*[E_{\langle\cdot\rangle}\check{\xi}(\hat{x}_t)=\varrho^t\varphi^*[\xi]$, $\varphi^*[\varphi]=1$, $\sup_{x\in X}|E_{\langle x\rangle}\check{\xi}(\hat{x}_t)-\varrho^t\varphi^*[\xi]\,\varphi(x)|=O(\alpha^t)$, $\alpha<\varrho$, t>0. Then it can be proved under conditions compatible with diffusion branching models and involving moments of at most second order: If $\varrho<1$, then $\lim_{t\to\infty}\varrho^{-t}P_{\hat{x}}(\hat{x}_t\neq\Theta)=\gamma\check{\varphi}(\hat{x})$, $\lim_{t\to\infty}P_{\hat{x}}(\check{\chi}_{A_\nu}(\hat{x}_t)=n_\nu,\nu=1,\dots,n\,|\,\hat{x}_t\neq\Theta)$ $=P^*(\check{\chi}_{A_\nu}(\hat{x}')=n_\nu,\nu=1,\dots,n)$, where $\gamma>0$, χ_A indicator function of $A\in\mathfrak{a}$, P^* a probability measure on $\hat{\mathfrak{a}}$ not depending on $\hat{x}$. If $\varrho=1$ and $P_{\hat{x}}(\check{1}(\hat{x}_t)=\text{const}>0)=0$, then $\lim_{t\to\infty}tP_{\hat{x}}(\hat{x}_t\neq\Theta)=\varphi(\hat{x})/\mu$ and $\lim_{t\to\infty}P_{\hat{x}}(t^{-1}\check{\chi}_{A_\nu}(\hat{x}_t)\le y_\nu,\nu=1,\dots,n\,|\,\hat{x}_t\neq\Theta)=1-\exp\{-(\sqrt{n}/\mu)\min_{1\le\nu\le n}(y_\nu/\varphi^*[\chi_{A_\nu}])\}$, where $y_\nu\ge 0$, $A_i\cap A_j=\delta_{ij}A_i$, $\bigcup_{i=1}^{n} A_i=X$, and $\mu=(1/2t)\,\varphi^*[E_{\langle\cdot\rangle}[(\check{\varphi}(\hat{x}_t))^2-(\check{\varphi^2})(\hat{x}_t)]]=\text{const}>0$. If $\varrho>1$, then $E_{\hat{x}}|\varrho^{-1}\check{\xi}(\hat{x}_t)-\varphi^*[\xi]W|^2=(\check{1}(\hat{x}))^2 O(\beta^t)$, $P_{\hat{x}}(\lim_{t\to\infty}\varrho^{-t}\check{\xi}(\hat{x}_t)=\varphi^*[\xi]W)=1$, where $\beta<1$ and the distribution function of W has a continuous density except for a possible jump at 0.

H. HEYER: Einbettung stark wurzelkompakter W-Maße in Halbgruppen

Das Thema ist der Wahrscheinlichkeitstheorie auf algebraisch-topologischen Strukturen zuzuordnen.

Es wird ein halbgruppentheoretischer Zugang zum Problem der Einbettung von unendlich teilbaren Wahrscheinlichkeitsmaßen in vag stetige Einparameter (Faltungs) Halbgruppen vorgestellt, welcher gestattet, die klassischen Einbettungssätze für stark wurzelkompakte W-Maße auf beliebigen lokalkompakten Gruppen zu verallgemeinern.

L. JONES: A generalization of the main ergodic theorem.

Theorem: Let T be a linear operator from X into X where X is any linear space in the world. Suppose $\{T^n x\}_0^\infty$ is a conditionally compact set in the weak topology of X for each $x \in X$. Then if n_i is any strictly increasing sequence of integers of positive lower density, $T_n = \frac{1}{n}\sum_1^n T^n$ behaves in the limit, for each x, as if the space X were finite dimensional.

D.A. KAPPOS: Generalized probability with applications to the quantum mechanic

The probability theory is inadequate for the description of a quantum mechanical system Υ mainly because the statements (events) of such a system Υ fail in general to form a Boolean σ-algebra. The algebraic structure of the logic $\mathcal{L}(\Upsilon)$ of Υ is supposed to be an orthomodular σ-lattice isomorphic to the lattice $\mathcal{L}(H)$ of all closed subspaces of a separable Hilbert space H. Such a logic is introduced first and studied by J.v. Neumann. A generalized abstract probabilistic formulation of this theory is due to G. Bodiou, V.S. Varadarajan, S.P. Gudder.
We can introduce over $\mathcal{L}$ the concept of a random variable in the same way as in my book "Probability algebras and stochastic spaces" (Acad. Press 1969).

M. KEANE: Random walks on locally compact groups

A year ago I gave a lecture in Oberwolfach at the probability meeting on random walks on nilpotent Lie groups. This year a survey of known results on recurrence and renewal of random walks on locally compact groups was given. In particular, an example of a 2-dimensional Lie group with no recurrent random walk was stated, and new theorems on recurrence and renewal in the solvable Lie case, due to Y. Guivarch and myself, were presented. The following problem remains open: If G is a locally compact group with normal closed subgroup H such that G/H is compact, is the existence of recurrent random walks on G equivalent with the existence of recurrent random walks on H ?

D.G. KENDALL: Hunting the neolithic "quantum"

It has been suggested that some of the principal linear dimensions of the neolithic monuments of Western Europe are integer multiples of 1.66m. I have carried out an extensive test of this, using an amalgam of techniques: the spline transform, Monte Carlo, the empirical characteristic function, and modern likelihood theory. The results lend a modest degree of support to the "quantum hypothesis". The techniques are of very general application, and pose difficult problems in stochastic process theory.

D.G. KENDALL: Teaching "model building" in Cambridge

Some two years ago a number of us at Oberwolfach agreed to experiment with the teaching of "model building" to mathematicians, and then to report our experiences. This I am now ready to do.

K. KRICKEBERG: Statistik von Hyperebenenprozessen

Sei G lokalkompakte abelsche Gruppe, mit abzählbarer Basis, Z ein von 2.Ordnung stationärer Punktprozeß in G, beschrieben durch die Anzahl $Z(A,\omega)$ der Punkte, die bei der Realisierung ω in $A \subset G$ fallen. Das 2.Momentenmaß ν von Z, definiert in G^2 durch $\nu(A \times B) = E(Z(A)Z(B))$, hat dann die Darstellung $\nu = r\tau_0 + {}_G\!\int \tau_u K(du)$, wobei τ_u Bild des Haarschen Maßes τ in G vermöge der Abb. $x \mapsto (x, x+u)$ von G in G^2, K Radonsches Maß in G. Die Aufgabe ist, die Intensität r und das Spektrum, d.h. die Fouriertransformierte $\hat{K}$ von K, als Maß auf der Charaktergruppe

G zu schätzen. Diskutiert werden folgende Spezialfälle: 1) $G = S_1$, die Rotationsgruppe des Einheitskreises, $d\tau = (2\pi)^{-1}dx$, also $\hat{G}$ die additive Gruppe der ganzen Zahlen vermöge der Identifizierung $n \leftrightarrow (x \mapsto e^{inx})$. - 2) $G = R$ (add. Gruppe), $d\tau = (2\pi)^{-1/2}dx$. - 3) G direkte Summe $R \oplus S_1$, also $G = R \oplus N$. Dieser Fall ergibt sich bei der Untersuchung gerichteter Geraden in der Ebene, die durch Parameter p (Abstand vom Ursprung) und ϑ (Winkel gegen die Ordinate) dargestellt werden.

P.A. MEYER: Some new results in the theory of Markov processes

Let X_t be a Markov process which satisfies the "right side" hypothesis, with (P_t) as transition semigroup, M a homogeneous closed random set in $]0,\infty[\times\Omega$. One sets $D(\omega)=\inf\{t>0:(t,\omega)\in M\}$, $\varphi_t(x,f)=E^x[f\circ X_t, t<D]$, $F=\{x:P^x\{D=0\}=1\}$, $D_s=\inf\{t>s:(t,\omega)\in M\}$. Let $M^{\rightarrow}$ be the set of all left endpoints of contiguous intervals. One splits $M^{\rightarrow}$ into two parts, the interesting one being $M^{\rightarrow}_{\pi} = M^{\rightarrow}\cap\{X_t \in F\}$. In their recent paper "Last exit decomposition and distributions" Getoor and Sharpe compute sums of the following kind:

(1) $$E^{\mu}\Big[\sum_{s\in M^{\rightarrow}_{\pi}} Z_s\, e^{ps}\int_s^{D_s} e^{-pu} h\circ X_u\, du\Big]$$, Z_s right continuous

and adapted. Their main result being as follows: their exists a continuous additive functional (A_t) carried by F, for each $x\in F$ an entrance law (unbounded) $\hat{\varphi}(x,\cdot)$ for (φ_t) such that, if one sets $\hat{V}_p(x,\cdot)=\int_0^\infty e^{-ps}\,\hat{\varphi}_s(x,\cdot)ds$ then (1) equals

(2) $E^{\mu}\big[\int_0^\infty Z_s\hat{V}_p(X_s,h)dA_s\big]$. It is shown how the equality of kind (1) and (2) can be extended to compute sums of the kind

$$E^{\mu}\Big[\sum_{s\in M^{\rightarrow}_{\pi}} Z_s C\circ\theta_s\Big] = E^{\mu}\Big[\int_0^\infty Z_s \hat{E}^{X_s}[C]dA_s\Big]$$, using the new theory

of Lévy systems, due to Benveniste and Jacod, and the theory of the "incursion process" due to Maisonneuve.

F. PAPANGELOU: On the conditional intensity of point processes

Given a point process in a locally compact, 2nd countability, Hausdorff space Γ, let N(Q) be the random number of points the process throws in Q and $E(N(Q)|\mathcal{F}_{Q^c})$ the conditional expectation of $N(Q)$ given the σ-field of events occurring in Q^c. Suppose $\Delta^{(1)}, \Delta^{(2)}, \ldots$ is a sequence of countable partitions of Γ, each consisting of bounded sets and such that: (i) every element of $\Delta^{(n)}$ is a finite union of elements of $\Delta^{(n+1)}$ and (ii) for every $x \in \Gamma$ and every neighbourhood U of x there is a $J \in \bigcup_{n=1}^{\infty} \Delta^{(n)}$ such that $x \in J \subset U$. Theorem: If $E(N(Q)^2) < \infty$ whenever Q is bounded, then for every $I \in \bigcup_{n=1}^{\infty} \Delta^{(n)}$

$$\lim_{n\to\infty} \sum_{J \in \Delta^{(n)}, J \subset I} E(N(J) | \mathcal{F}_{J^c}) = W(I)$$

exists a.s. and in the mean and determines a random measure in Γ. Further $E(W(I)|\mathcal{F}_{I^c}) = E(N(I)|\mathcal{F}_{I^c})$ a.s.
A second theorem states necessary and sufficient conditions for a stationary line process with finite variance to be a doubly stochastic Poisson process.

W. PHILIPP: Empirische Verteilungsfunktionen und Gleichverteilung mod 1

$X_1, X_2, \ldots$ Folge unabhängiger Zufallsvektoren im $\mathbb{R}^r$, alle gleichverteilt auf $[0,1]^r$, $\mathcal{S}$ eine Klasse meßbarer Mengen $S \subset [0,1]^r$, $A(N,S)$ Anzahl der Indizes $n \leq N$ mit $X_n \in S$, $D_N(\mathcal{S}) = \sup_{S \in \mathcal{S}} |N^{-1} A(N,S) - V(S)|$, wobei $V(S)$ das L-Maß von S ist. Es wird ein Gesetz vom iterierten Logarithmus in der Form

$$P\left\{ \limsup_{N\to\infty} \frac{N\, D_N(\mathcal{S})}{\sqrt{2N \log\log N}} = \frac{1}{2} \right\} = 1$$

besprochen für verschiedene Klassen $\mathcal{S}$.
Weiter werden Sätze vom Berry-Esseen'schen Typ besprochen. Für $r=1$ gilt z.B. folgender Satz: $F(x)$ nicht fallend auf $[0,1]$ mit $F(0)=0$, $F(1)=1$ und $G(x)$ mit $G(0)=0$, $G(1)=1$, $|G(x)-G(y)| \leq M|x-y|$ für alle $0 \leq x,y \leq 1$, dann gilt für jedes ganze $m \geq 1$

$$\sup_{0 \leq x \leq 1} |F(x)-G(x)| \leq \frac{4M}{m+1} + \frac{4}{\pi} \sum_{h=1}^{m} \left(\frac{1}{h} - \frac{1}{m+1}\right) |\hat{F}(h) - \hat{G}(h)|$$

wobei $\hat{F}(h) = \int_0^1 \exp(2\pi i h x)\, dF(x)$ und $\hat{G}(h)$ analog. Für $r \geq 2$ gilt ein analoger Satz.

P. REVESZ: On the limit theorems of the Robbins-Monro process

Let $M(x)$ be a monotonically increasing function with $M(\Theta)=0$. Our aim is to find the root Θ, when we can measure the value of $M(x)$ only together with some random error Y_x. Let X_1 be an arbitrary initial point and construct the sequence $X_{n+1}=X_n-\frac{1}{n}Z_n$ where $Z_n=M(X_n)+Y_{X_n}$. The sequence X_n will be called Robbins-Monro process. Under some conditions Chung has proved that the limit distribution of $\sqrt{n}(X_n-\Theta)$ is normal law if $M'(\Theta)>\frac{1}{2}$. We investigate the limit properties when $M'(\Theta)\leq\frac{1}{2}$. The moments of X_n are also studied.

M. SKOLNICK: <u>The construction and analysis of genealogies</u>

Three aspects of the above problem were briefly discussed. A method of constructing genealogies, and the nature of the resulting male descent groups for Parma Valley, Italy was presented (see Skonick, Moroni, Cannings, and Cavalli-Sforza, 1971 and references therein). The possibility of analyzing such a process as a Galton-Watson process with dependence relationships was discussed. A second approach was mentioned - computer simulation of human populations (see Skolnick and Cannings, 1972). Results from these simulations with implications for possible mechanisms of population regulation and social stability were briefly mentioned. The third aspect discussed was the need for probabilists to construct analytical models which could generalize the results obtained from simulations and data analysis.

M. SMORODINSKY: <u>Classification of measurable transformations</u>

$X=(\ldots,X_{-1}(\omega),X_0(\omega),X_1(\omega),\ldots)$ stationary process, def. on its path space, T shift transformation, i.e. $X_n(T\omega)=X_{n+1}(\omega)$. Given T we can produce back stationary processes by taking any measurable Y_0 and $Y_n(\omega)=Y(T^n\omega)$. $\mathfrak{F}(Z)$ denotes the sub-σ-field generated by Z, $\mathfrak{F}^n(Y)=\mathfrak{F}(Y_n,Y_{n+1},\ldots)$, ${}^n\mathfrak{F}(Y)=\mathfrak{F}(Y_{-n},Y_{-n-1},\ldots)$. Y is not predictable from its past if Y_0 is not measurable ${}^1\mathfrak{F}$. X generates a Kolmogorov (K) system if no Y is predictable from its past. It is enough to check that X itself has this property (Rokhlin, Sinai). Equivalent conditions are: $\mathrm{Tail}(X)=\bigcap_1^\infty \mathfrak{F}^n$ is trivial or $\mathfrak{F}(X_1,\ldots,X_m)$ is asymptotically independent of $\mathfrak{F}^{m+k}$ as $k\to\infty$. If X is i.i.d. it defines a Bernoulli (B) shift and Tail(X) is trivial. For every $0<h\leq\infty$ there is one such system (up to isomorphism) (Ornstein). If X is an irreducible Markov process

(finite state) then it is isomorphic to a B-shift if it is mixing (Friedman, Ornstein). If not, then it is isomorphic to a product of rotation of finite number of points with a B-shift (Adler, Shields, Smorodinsky). Since Ornstein constructed a K-system which is not a B-shift, the main problem now is to further classify K-systems.

P. TAUTU: <u>A note on Savage's sure-thing principle in treatment decisions</u>

L. Savage (1954) has defined the sure-thing principle as a condition of rationality for an individual faced with a set of alternatives. This note is concerned with a medical (therapeutics) application of the expected-utility theory in Savage's formulation. It is supposed that many medical decisions are under uncertainty. - Consider a weak preference pattern $\mathfrak{A}=\langle\mathcal{A},\pi,\tilde{\pi}\rangle$, where $\mathcal{A}$ is the set of actions, π is a binary preference relation and $\tilde{\pi}$ an indifference relation. The preference decision structure $\langle\mathcal{S},\mathcal{C},\mathcal{A},\pi\rangle$ is associated with a QP-structure $\langle\mathcal{S},\mathcal{B},\Pi\rangle$, where $\mathcal{S}$ is the set of states in the world, $\mathcal{B}$ is the algebra of subsets of $\mathcal{S}$, Π a preference relation between two events $A,B\in\mathcal{S}$, and $\mathcal{C}$ the set of consequences. An action $f\in\mathcal{A}$ is defined as a function attaching a consequence $a\in\mathcal{C}$ to each state of the world $s\in\mathcal{S}$. The sure-thing principle asks for an induced conditional ordering on $\mathcal{A}$ and an induced order on $\mathcal{C}$ (Savage's postulates 2 and 3).
A medical example is presented and discussed.

P. TAUTU: <u>Discussion on model building</u>

1. Outlines of possible mathematical models in cancer research are presented. 2. Different types of mathematical models (deterministic, stochastic, algebraic, combinatorial, etc.) in biology are briefly discussed. 3. The problem of the parameters of stochastic models (e.g., the Malthusian parameter) is pointed out. 4. A schema of mental processes in scientific research is produced.

F. TOPSØE: Compactness and tightness in $\mathcal{M}_t(X)$, a survey

Definition: X is a Prohorov space if $\mathcal{P} \subseteq \mathcal{M}_t^1(X;t)$ is relatively compact if and only if $\mathcal{P}$ is "tight" ($\forall_{\varepsilon} \ \exists_{K \text{ compact in } X} \ \forall_{P \in \mathcal{P}} \ PK > 1-\varepsilon$)

It will be natural to devote most of the time to the latest results on the class of Prohorov spaces, in particular to the fascinating results of David Preiss, Prague.

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