MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 19/1973

Gruppentheorie

13.5. bis 19.5.1973

Die diesjährige Tagung stand unter Leitung von Prof.Dr.K. Gruenberg (London) und Prof.Dr.W. Gaschütz (Kiel). Es nahmen ungefähr 40 aus- und inländische Gruppentheoretiker an der Tagung teil. Besonders hervorzuheben ist die große Zahl der Gruppentheoretiker aus den USA, deren Besuch in Oberwolfach auf Grund ihrer derzeitigen Teilnahme am "Gruppentheoretischen Jahr" in Coventry möglich wurde. Behandelt wurden Themen aus den verschiedenen Bereichen der Gruppentheorie, so daß sich ein Überblick über die Forschungen auf diesem Gebiet ergab.

Teilnehmer

R.Baer, Zürich
H.Bass, Paris
G.Baumslag, z.Z. Coventry
R.Bieri, Zürich
D.Blessenohl, Kiel
J.Cossey, z.Z. Coventry
K.Doerck, Mainz
A.Dress, Bielefeld
B.Fischer, Bielefeld
P.Fong, z.Z. Coventry
R.Göbel, Würzburg
D.Gorenstein, z.Z. London
K.Gruenberg, London
N.D.Gupta, Winnipeg
B.Hartley, Coventry
Chr.Hering, Tübingen
B.Huppert, Mainz
N.Ito, z.Z. Coventry
S.Janko, Heidelberg
K.Johnsen, Kiel
O.Kegel, London
M.Lazard, Paris
S.Mckay
T.A.Peng, z.Z.Coventry
I.Reiner, Urbana
K.W.Roggenkamp, Bielefeld
K.Schaller, Kiel
R.Schmidt, Kiel
M.Smith, St.Louis
U.Stammbach, Zürich
S.E.Stonehewer, Coventry
R.Strebel, Zürich
B.A.F.Wehrfritz, London
H.Wielandt, Tübingen
J.S.Wilson,
W.J.Wong, z.Z. Coventry
G.Zappa, Florenz

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Vortragsauszüge

H.BASS: Growth of finitely generated groups

The background of the following conjecture is discussed:

Let G be a group with finite generating set S. Let $\gamma(m)$ denote the number of elements of G expressible as words of length $\leq m$ in S. Suppose γ dominates no exponential function AB^m, $A > 0$, $B > 1$. Then G contains a nilpotent subgroup of finite index.

G.BAUMSLAG: Metabelian groups

The purpose of my talk was to indicate how to prove the

Theorem: Every finitely generated metabelian group can be embedded in a finitely presented metabelian group.

R.BIERI: Gruppen mit homologischer Dualität

G heißt eine D-Gruppe (duality group) der Dimension n, wenn es einen "dualisierenden Modul" C und einen "Fundamentalzyklus" $e \in H_n(G;C)$ gibt, derart,daß die cap-Produkt Abbildung

$$(e \frown -): H^k(G,A) \longrightarrow H_{n-k}(G;C \otimes A)$$

für jeden G-Modul A und für alle $k \in \mathbb{Z}$ ein Isomorphismus ist. Die PD-Gruppen (Poincaré duality groups) sind diejenigen D-Gruppen, für welche die unterliegende abelsche Gruppe von C unendlich-zyklisch ist.

Wenn eine Gruppe G eine endliche projektive Auflösung $P_* \twoheadrightarrow Z$ zuläßt, dann ist (a) $H^k(G; \mathbb{Z}G) = 0$ für $k \neq n$ und (b) $H^n(G, \mathbb{Z}G)$ torsionsfrei, (notwendig und) hinreichned dafür, daß G eine D-Gruppe ist. Damit gewinnt man

einen Überblick über alle (endlich präsentierbaren) D- Gruppen der Dimension ≤ 2.
Beispiele in höheren Dimensionen können auf Grund von Extensionssätzen oder mit Hilfe von gewissen verallgemeinerten freien Produkten mit amalgamierten Untergruppen konstruiert werden.

J.COSSEY: Normal Fitting Classes

If X and Y are Fitting classes $XY = \{G: G/G_X \in Y\}$ is also a Fitting class: if Y is normal, so is XY. If H is the smallest normal Fitting class, we consider the problem: if $X \neq Y$, can $XH = YH$. Partial results can be obtained: among them are:

1) $XH = S \longleftrightarrow X$ is normal (S = all soluble Groups)

2) $XH = H \longleftrightarrow X = \{1\}$

3) $X^*H = Y^*H \longleftrightarrow X^* = Y^*$

(* is the closure operation on Fitting classes introduced by Lockett)

Normal Fitting classes are rarely closed under other closure operations: we have for X a normal Fitting class: 1) $X^* = S$ (Lockett); 2) $QX = S$ (Lockett); 3) $sX = S$ (Gaschütz-Blessenohl); 4) $\tau_0 X = S$; 5) if X is also a Fischer class, $X = S$ (Makan).
However, NX is Frattini closed, and $NX \neq S$ if $X \neq S$.

A.DRESS: The Permutaion Class Group of a Finite Group

Analogously to the projective class group one can define the permutation-class-group of a finite group π as the group of equivalence classes of direct summands of integral permutation-modules modulo permutation-modules. It is shown, that this group behaves nicely w.r.t. localization and completion, which then is used to prove that - contrary to the projective class group - ist isn't always a torsion group.

P. FONG: Centralizers of p-elements in finite groups

Conjecture I: Let p be a prime. Then there exists a function $f(c)$ with the following property: If G is a finite group, $x \in G$ has order p, and $c = |C_G(x)|$, then $|G:O_{p',p}(G)| \leq f(c)$.

Conjecture II: Let p be a prime. Then there exists a function $g(c)$ with the following property: If G is a finite simple non-abelian group, τ an automorphisms of order p of G, and $c = |C_G(\tau)|$, then $|G| \leq g(c)$. We remark that II) $\longrightarrow$ I).

Evidence for conjecture II can be seen in the following

Theorem: Let p be a prime. Then there exists a natural integer N with the following property. If G is a simple non-abelian group of a alternating or Chevalley type, τ is an automorphism of order p of G, and $c = |C_G(\tau)|$, then $|G| \leq c^N$.

R. GÖBEL: Homomorphisms of cartesian products

Let $C(M)$ be the cartesian product of a set M of groups. If $f \in C(M)$, then f is a map from M into the elements: $fX \in X$ for all $X \in M$. The product is defined bei components fX.

If G is a group and M is a set of groups, we define:

$\mathrm{Hom}(C(M),G) = \{\tau \in \mathrm{Hom}(C(M),G) \; , \; | \; S \leq M, \; |M:S| \leq \aleph_0, \tau|_{C(S)} = 0\}$

We consider: $\mathrm{Hom}(C(M),G) = \Delta \, \mathrm{Hom}(C(M),G)$ (1)

(1) has been shown for a set M of infinite cyclic groups Z and $G = Z$ by E. Specker (Port. Math. 9 (1950), $|M| \leq \aleph_0$) and E.C.Zeeman (J.L.M.S. 30 (1955)).

Theorem A: (1) is true for arbitrary sets M of groups, and groups G such that abelian subgroups of G are free abelian.

Lemma B: Let $M = \{X\, Y ,\ X \neq 1 \neq Y \neq Z_2\}$ be an infinite (countable) set of groups. There is an epimorphism from $C(M)$ onto Z_n for all n. (Therefore (1) is not true for G = finite cyclic !) From A. and B. we derive the following theorem, which answers a questin of R.Baer's : Let e be a class of groups, closed under cyclic subgroups. If e-groups are torsionfree, then every e-group $\neq 1$ has Z as epimorphic image (less than Qe = e). It is equivalent: (1) e^{-Q} is cartesian closed.

(2) e = all groups , or e = {1} or e = {Z} .

D.GORENSTEIN: Classification of finite simple groups

In this paper, which was obtained jointly with Koichiro Harada, we determine the general structure of all finite groups satisfying the given condition and, in particular, all such simple groups. By a result of MacWilliams a 2-group which possesses no normal elementary abelian subgroups of order 8 satisfies the hypothesis of our theorem. Hence as a corollary of our result, we obtain a complete classification of all simple groups whose Sylow 2-subgroups do not possess a normal elementary subgroup of order 8 (i.e. in which $SCN_3(2)$ is empty in the Thompson terminology).

K.GRUENBERG: A formula for the minimum number of generators of a finite group

Let $d(G)$ denote the minimum number of generators of the finite group G. For each $p \mid |G|$ and each irreducible F_pG-module M let $E = \mathrm{End}_G|M|$, $M \cong r_M E$ (direct sum of r_M copies of E) and $H'(G,M) \cong s_M E$. If ζ_M is defined to be 1 when $M = F_p$ and 0 otherwise, then

$$d(G) = pr(G) + \max_M \lceil s_M/r_M + \zeta_M \rceil ,$$

where $\lceil \alpha \rceil$ denotes the smallest integer $\geq \alpha$ and $pr(G)$ is defined thus: If $1 \longrightarrow R \longrightarrow F \longrightarrow G \longrightarrow 1$ is a minimal free presentation of G and $R/R' = A \oplus P$, with P projective and A having no projective summand, then $P \oplus Q \cong (QG)^{pr(G)}$.

N.GUPTA: Torsion in free $M_{(c)}$-groups

An $\underline{\underline{M}}_{(c)}$-group is a group which satisfies the identical relation $$[[x,y],[u,v],z_1,\ldots,z_c] = 1 .$$

If F is the free group of countably infinite rank then $F/[F'',cF]$ is the free $\underline{\underline{M}}_{(c)}$-group, where $[F'',cF] = [F'',(c-1)F,F]$ for $c \geq 2$ and $[F'',1F] = [F'',F]$. It is shown that for all $c \geq 1$, the free $\underline{\underline{M}}_{(c)}$-group has elements of order 2. The case $c = 1$ is due to C.K. Gupta and the case $c = 2$ was established jointly with Frank Levin.

B.HARTLEY: Modules over locally finite groups

Complementation conditions for submodules of certain classes of modules over locally finite groups, will be discussed.

B.HUPPERT: Konstruktion der Spiegelungsgruppe H_4

Konstruktion der ang. Gruppe als Kranzprodukt der Schurschen Überlagerung von A_5 mit Z_2, mit vereinigten Zentren. Hinweis auf eine merkwürdige Realität gewisser Darstellungen.

N.ITO: Factorizable groups

A finite group G is called factorizable if G contains two non-identity elements a and b such that $G = C(a)C(b)$. The fundamental question for a factorizable group G is : "Is G not simple?" Special cases are verified affirmatively.

S.JANKO: On thin simple groups

A finite group X is said to be "thin" if every 2-local subgroup X_o of X has the following property:

(i) Each odd order Sylow subgroup of X_o is cyclic.

The aim of this work is to classify all finite simple thin groups. Here we shall talk about the case that a minimal counterexample possesses a non-2-constrained 2-local subgroup.

K.JOHNSEN: 2-Gruppen vom Rang 2

Es wurde der folgende Satz vorgetragen:

Sei G eine endliche 2-Gruppe, $rg(G) = 2$, $\Omega(G) = G$.

Dann gilt eine der folgenden Aussagen:

(a) $G \cong V_4$

(b) $G \cong D_{2^n}$, $n \geq 3$

(c) $G \cong Z_4 \curlyvee D_{2^n}$, $n \geq 3$

(d) $G \cong Q_{2^m} \curlyvee D_{2^n}$, $m, n \geq 3$

S.MC KAY: Calculation of p-groups using isologisms and group invariants

If $\underline{\underline{V}}$ is a variety and G any group, $V(G), V^*(G)$ will denote the verbal and marginal subgroups. Given $N \triangleleft G$, define $N\, V^* G$ = smallest normal subgroup of G inside N such that in the factor group the image of N is marginal for $\underline{\underline{V}}$. If $1 \longrightarrow R \longrightarrow F \longrightarrow G \longrightarrow 1$ is a presentation of G then $\underline{\underline{V}}M(G)$ is the G-module $R \cap V(F) / [RV\, F]$ and is an invariant of G.

Groups E,M are $\underline{\underline{V}}$-isologic if $\exists$ isomorphisms $\vartheta : E/V(E) \longrightarrow M/V^*(M)$, $\phi : V(E) \longrightarrow V(M)$ such that if $\psi : E \longrightarrow M$ induces ϑ then $v(g_1 \psi_1 - g_r\psi) = v(g_1 - g_r)\phi \;\; \forall g_1 - g_r \in G, v \in V$.

Given a group E in a $\underline{\underline{V}}$-isologism class with G as $\underline{\underline{V}}$-marginal factor, there is a natural procedure for constructing G-epimorphism $\alpha : \underline{\underline{V}}M(G) \longrightarrow V^*(E) \cap V(E) = B$ say.

Theorem. If E,M are two such groups with corresponding α, β then E,M are $\underline{\underline{V}}$-isologic $\Longleftrightarrow$ there is an automorphism of G inducing a map: $\ker \alpha \xrightarrow{\text{onto}} \ker \beta$.

Theorem. If α is such a surjection satisflying a condition(*) (which ensures G is the mariginal factor) and if $1 \longrightarrow R \longrightarrow F \longrightarrow G \longrightarrow 1$ is a presentation of G then let $E = F/[RV^*F] / \ker \alpha$. E is a representative of the class corresponding to α, and so is E/T for any $T \triangleleft E$ such that $T \cap V(E) = 1$. The representatives can be obtained in this way. $\underline{\underline{A}}_p$, giving this and further results to $\underline{\underline{A}}_p$ given G of order p^n, and B elementary abelian of order p^m with $\alpha : \underline{\underline{V}}M(G) \longrightarrow B$ as above, there is a unique group E of order p^{m+n} in the $\underline{\underline{A}}_p$-isologism class corresponding to α , and other representations of this class are all of the form $E \times C$ for C abelian and exponent p . This leads to a method for constructing p-groups.

I.REINER: Class Groups of Integral Group Rings

Let R be the ring of algebraic integers in an algebraic number field K , and let Λ be an R-order in a semisimple K-algebra A . Denote by $Cl\ \Lambda$ the locally free class group of Λ . If Λ' is a maximal R-order in A containing Λ, there is a surjection $Cl\ \Lambda \longrightarrow CL\ \Lambda'$, whose kernel will be denoted by $D(\Lambda)$. Using Jacobinskii formula for $Cl\ \Lambda$, it is shown that if G is a finite p-group,then $D(ZG)$ is also a finite p-group. Induction theorems for $CL(RG)$ are also discussed.

(joint wrote with Prof. S. Ullom)

K.W.ROGGENKAMP: Relationenmodule endlicher Gruppen

Let G be a finite group with integral argumentation ideal g . For a set of prime: π , $|\pi| > 1$, $\mathbb{Z}_\pi g$ has a projective cover iff G has a cyclic π-Hallsubgroup and a normal π'complement, in case G is solvable; if $\pi = \pi(G)$, then the result holds without the latter hypothesis.

For a solvable group G , the following conditions are equivalent: (i) $\mathbb{Z}$ is not a Hellermodule, (ii) g decomposes (iii) there exists a chain $1 \triangleleft H \triangleleft T \triangleleft G$ such that:

T and G/H are Frobeniusgroups, or G is a Frobeniusgroup.

Denote be $d(G)$ resp. $\mu_{\mathbb{Z}G}(g)$ the minimal number of generators of G resp. g as $\mathbb{Z}G$-module. Then

$$d(G) \leq \max_p d(S_p) + 1 + d(G) - \mu_{\mathbb{Z}G}(g) ,$$

when S_p denotes a p-Sylowsubgroup of G .

K.-U.SCHALLER: Sylowisatoren

Ein Sylowisator einer p-Untergruppe R einer endlichen auflösbaren Gruppe G ist eine Untergruppe S von G , die maximal ist in G mit der Eigenschaft, daß R in ihr eine p-Sylowgruppe ist. Es wird eine Abschätzung der p-Länge für Sylowisatoren angegeben, nämlich:

Ist $R \trianglelefteq P$ für eine p-Sylowgruppe P von G und $|R: R \cap \Phi(P)| = p^n$, so haben die Sylowisatoren von R in G höchstens p-Länge n + 1 .

Der Beweis benutzt, daß die Sylowisatoren von R zusammen mit P in Untergruppen G_o von G liegen, in denen R Deck-Meide-Eigenschaft für Hauptfaktoren hat.

R.SCHMIDT: Finite groups with modular subgroup lattice

The following improvement on Iwasawa's theorem on modular p-groups was given.

Theorem. Let G be a non-hamiltonian p-group with modular subgroup lattice.

a) If $x \in G$, $o(x) = \exp G$, and $< x > \triangleleft G$, then there is an element $t \in G$ and a natural number s (which is > 1 if $p = 2$) such that $G = < C_G(x), t >$, $C_G(x)$ is abelian, and $a^t = a^{1+p^s}$ for all $a \in C_G(x)$.

b) If all cyclic subgroups of maximal order in G are non-normal, then there is an abelian normal subgroup N in G such that for any cyclic subgroup X of maximal order in G there is an $x \in X$ and an s as in a) such that $G = \langle N, x \rangle$ and $a^x = a^{1+p^s}$ for all $a \in N$. Furthermore, if U is a subgroup of G such that $U \ X = 1$ and $U^x = U$, then $U \leq N$.
The proof of the theorem does not use Iwasawa's theorem so that we get a new proof of this result.

M.SMITH: <u>Ring Theretic Aspects of Group Rings</u>
One method of attack for problems in group rings of infinite groups is to embed the group "tightly" in a ring with a rich structure theory. Two types of embeddings have proved useful. The first applicable to any group in characteristic zero, is an embedding into an appropriate Banach algebra. The second approach is to use a ring of quotients. This technique is available in all characteristics, but nor for all groups. The ring of quotients itself may be interesting, as in the following result, whose proof exemplifies a technique frequently useful in group rings.
<u>Theorem</u>: Let G be a torsion-free nilpotent group, A a maximal abelian subgroup of G, and R any integral domain. Then RG has a ring of quotients which is a division ring having the field of quotients of RA as a maximal subfield.

S.E.STONEHEWER: <u>Groups of Finite Rank</u>
If A is an abelian group, let P denote the maximal periodic subgroup of A. Call A an A_1-group if A/P

has finite rank; and call A an A_4-group if in addition P is finite. Let pA_i denote the class of soluble groups with a finite series having factors in A_i. In 1951 Mal'cev proved that if $G \in pA_1$ and if N in the maximal priodic normal subgroup of G, then $G/N \in pA_4$. Also Mal'cev showed that such a group G possesses only finitely many conjugacy classes of maximal periodic subgroups, by proving that a pA_4-group possesses only, finitely many conjugacy classes of periodic subgroups. Mal'cev's argument was very long, using results from the theory of linear groups, classical representation theory and completions of groups. The object of this work* is to give a particularly short and elementary proof.

(* jointly carried out with T. Bowers)

R.STREBEL: <u>On the derived series of E-groups</u>

Let G be called an $\underline{E}$-group iff

(i) G_{ab} is torsionfree, and

(ii) G has a ZG-projective resolution of $\mathbb{Z}$

$$\cdots \longrightarrow P_2 \xrightarrow[S]{} P_1 \longrightarrow P_o \longrightarrow Z \longrightarrow 0$$

such that $1_Z \underset{\mathbb{Z}G}{\otimes} S_2$ is <u>injective</u>.

Examples of $\underline{E}$-Groups are

(i) any knot group, and more generally

(ii) any f. pr. group which has a presentation of the form

$$< x_1,\dots,x_t, y_1,\dots,y_d \ : \ x_1K_1(x_m,y_n),\dots,x_tK(x_m,y_n) >,$$

where K_i $(1 \leq i \leq t)$ has zero exponent sum on all the generators.

(iii) any countable, locally free group.

As results we have

(i) Every derived group $G^{(\alpha)}$ (α any ordinal) of an $\underline{E}$-group G is an $\underline{E}$-group.

(ii) If G is an $\underline{E}$-group then every $G^{(\alpha)}/G^{(\alpha+1)}$ is torsion free and the multiplier $H_2(G^{(\alpha)},Z)$ equals zero.

(iii) If G is an $\underline{E}$-group then $G/\bigcap_{\alpha} G^{(\alpha)}$ is an $\underline{E}$-group of cohomological dimension at most 2.

(iv) The derived length of an $\underline{E}$-group is $0,1,2$ a unit ordinal λ or $\lambda + 1$.

(v) A map $\phi : F \longrightarrow G$ from a free group into an $\underline{E}$-group which induces an embedding $\phi : F/F^{(j)} \rightarrowtail G/G^{(j)}$ for all j $(1 \leq j \leq \;)$.

The theory of $\underline{E}$-groups may be used to construct parafree one-relator groups G with free polynilpotent $G/(\ldots((G_{K_1})_{K_2})\ldots_{K_s})$ for any preassigned $(\ldots(K_1,K_2),\ldots K_s)$.

B.A.F.WEHRFRITZ: Divisible 2-subgroups in locally finite groups

We discuss the proof and implications of the following.

Theorem. Let G be a locally finite group satisfying the minimal condition on 2-subgroups and suppose that the centralizer of every involution of G contains a unique maximal divisible 2-subgroup. If G does not have a unique maximal divisible 2-subgroup, then one of the following two statements holds.

A. $OG \neq \langle 1 \rangle$ and G/OG is either a Prüfer 2-group, or an infinite locally dihedral 2-group, or an infinite locally quaternion group.

B. G/OG has a unique minimal normal subgroup M/OG, G/M is an abelian 2'-group and $M/OG = PSL(2,F)$ for some quadratically closed, locally finite field F of odd characteristic.

H.WIELANDT: Subnormalität in noetherschen Gruppen

Eine Untergruppe A einer Gruppe G heiße projektiv-subnormal, A psnG, wenn aus $A \leq H \leq G$, $\phi \in \mathrm{Hom}\, G$ und Kern $\phi \neq 1$ stets $A^{\phi} \trianglelefteq\trianglelefteq H$ folgt. Erfüllt G die Maximalbedingung für Untergruppen, so gilt:

(a) Aus $A \in$ psnG, $1 \neq B \trianglelefteq\trianglelefteq H \leq G$ und $A \leq H$ folgt $\langle A,B\rangle \trianglelefteq\trianglelefteq H$;

(b) aus $A \in$ psnG, $1 \neq B \trianglelefteq\trianglelefteq H \leq G$ und $B \leq A \leq H$ folgt $A \trianglelefteq\trianglelefteq H$;

(c) psnG ist abgeschlossen gegen Bildung von Erzeugnissen ;

(d) aus $A \in$ psnG folgt $A \trianglelefteq\trianglelefteq \quad H =: \mathfrak{f}_G(A) \leq G$;

$$A \trianglelefteq\trianglelefteq H \leq G$$

(e) für A,B psnG gilt $\mathfrak{f}_G(A) = (B)$ genau dann, wenn A und B in $\langle A,B \rangle$ subnormal sind.

Hieraus ergeben sich verschiedene notwendige und hinreichende Bedingungen für Subnormalität in noetherschen Gruppen G.

Beispiel: Sei $A \trianglelefteq\trianglelefteq \langle A,A^t\rangle$ für jedes $t \in \{a^g \mid a \in A, g \in G\}$.

Dann ist $A \trianglelefteq\trianglelefteq G$.

J.S.WILSON: On $\overline{SI}$-Groups and $\overline{SN}$-Groups

A discussion of $\overline{SI}$-groups and $\overline{SN}$-groups, and of methods for getting certain group-theoretic counterexamples from easier ring -theoretic ones, led to the construction of a countable

$\overline{SI}$-group G having a serial subgroup H with a free quotient group of counably infinite rank; G is therefore certainly not an $\overline{SN}$-group. Since there are many known examples of $\overline{SN}$-groups which are not SI-groups, it follows that Problem VIII of the Kuros aund Cemikov survey article on generalized suluble and nilpotent groups has a negative answer. An example answering Problem X of this survey was mentioned briefly.

G.ZAPPA: Sylowizers in finite groups

Let G be a finite group, π a set of primes, π' the complementary set. A relation ρ between π-subgroups and π'-subgroups in called a T-relation if: 1) $R \rho C$, $a \in G \longrightarrow R^a \rho C^a$, 2) $R \rho C_1, R \rho C_2$, $\langle C_1, C_2 \rangle$ π'-group $\longrightarrow R \rho \langle C_1, C_2 \rangle$.

Let ρ be a T-relation, let G verify the condition $D_{\pi'}$ of P.Hall, and let R be a π-subgroup of G. A subgroup C of G is called on absolute ρ-complement of R if: 1) C is a π'-group, 2) $R \rho C$, 3) C is maximal under 1) and 2), it is called a relative ρ-complement in a Hall π'-subgroup H of G if: 1) $C \leq H$; 2) $R \rho C$; 3) C is maximal unter 1) und 2).

Theorem (J.Szep, G.Zappa). Let G be a finite group verifying $D_{\pi'}$, let P be a Hall π-subgroup of G and let R be a subgroup of P. Let ρ be a T-relation, and let C be an absolute ρ-complement of maximal order. Then the relative ρ-complements of R are conjugate (and are absolute ρ-complements) if and only if or every $a \in P$ is $R^a \rho C$. If $R \rho C$ is "R and C commute" and if G is soluble, it follows a theorem of Gaschütz on the sylowizers of a finite group.

K.Johnsen (Kiel)

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