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①Diffentialgeometrie in Grossen [illegible]

22.-28. Mai 1983

Discrete Reflection Groups in Hyperbolic Geometry

A discrete reflection group in hyperbolic space H^n is a discrete group of isometries generated by the reflections w.r.t. finitely many hyperplanes of H^n. Such a group gives rise to a fundamental polyhedron in H^n whose dihedral angles are natural, i.e., of the form $\frac{\pi}{p}$, $p \in \mathbb{N}$, $p \geq 2$. Conversely, a polyhedron with natural angles generates a discrete reflection groups.

In spherical and euclidean geometry, discrete reflection groups have been classified by Coxeter, but in hyperbolic geometry such a classification is out of sight. In this talk, I discuss a construction which gives rise to numerous new examples of hyperbolic finite volume polyhedra with natural angles.

We start with an orthoscheme (i.e., a simplex $(P_0, \ldots, P_n)$ such that $\operatorname{span}(P_0, \ldots, P_i) \perp \operatorname{span}(P_i, \ldots, P_n)$ for all i). If P_0 and P_n lie inside or on the quadric defining the hyperbolic structure, then this orthoscheme has finite volume. If P_0 (or P_n, or both) lie outside the quadric, we introduce the polar hyperplane of P_0 (or P_n, or both) and so we get a truncated (or doubly truncated) orthoscheme of finite volume. In good cases the polyhedra so obtained have natural dihedral angles. The classification of the good cases yields: continuous families for $n=2$, infinite families for $n=3$, finitely many cases for $4 \leq n \leq 9$, nothing at all for $n \geq 10$ (the classification for $n=4$ has not yet been completed).

The trigonometry involved is related to the pentagramma mirificum.

Hans-Christoph Im Hof

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Pinching and Betti members

Starting from the "sphere and rigidity theorems" (Berger), restricting ourself to the real cohomology of the manifold, and then considering the first non trivial dimension, we prove the following semicontinuity theorem:

Theorem: there exists an $\varepsilon > 0$ such that if (M, g) is a four dimensional connected and k-pinched riemannian manifold with $k > \frac{1}{4} - \varepsilon$ then $b_2(M) \leq 1$; one can take $\varepsilon = 2.5\ 10^{-4}$

The main tools that are used in the proof of this theorem are the Weitzenböck formula for harmonic 2-forms and a Sobolev inequality by Ilias.

Dominique Hulin

Diffeomorphisms with prescribed eigenvalues

Given a diffeomorphism of Riemannian manifolds $\varphi: (M, g) \rightarrow (N, h)$, one can compute the eigenvalues of the pullback of h with respect to g. These eigenvalues have physical meaning in the context of continuum mechanics. We consider the problem, given positive C^∞ functions $\lambda_1(x) \ldots \lambda_n(x)$ on M, of showing the existence of a diffeomorphism φ that realizes these eigenvalues. As it turns out, this is a problem that involves a nonlinear hyperbolic system of partial differential equations. The proof of our result (that such diffeomorphisms exist provided $\lambda_i(x) \neq \lambda_j(x)$ for $i \neq j$ and all x) involves a Nash-Moser argument.

Another application of this reasoning yields the existence on any C^∞ three-dimensional Riemannian manifold of an atlas of C^∞ coordinate charts such that all the coordinate systems are triply orthogonal (i.e, the metric tensor is diagonal at all points). Both these results had been known in the analytic category — our contribution is the extension to C^∞. (Joint work with Deane Yang)

Dennis DeTurck

Eigenvalue expansions and volume functions

The talk is concerned with the functions u_k, v_k, appearing as coefficients in Minakshisundaram - Plejel asymptotic expansion for the eigenvalues of the Laplacian,

$$U(t,\cdot) := \Sigma e^{-\lambda_i t}\varphi_i^2 \sim (4\pi t)^{-\frac{n}{2}}\{u_0 + tu_1 + t^2u_2 + \dots\} \quad (t\to 0)$$

and in the Taylor power series representing the volume function of geodesic balls,

$$V(t,\cdot) = \frac{(\pi t^2)^{n/2}}{(n/2)!}\left\{v_0 - \frac{t^2}{n+2}v_1 + \frac{t^4}{(n+2)(n+4)}v_2 - \dots\right\}.$$

We explain the universality of these coefficients, and the independence of dimension n as a result of the multiplicative properties of the "heat kernel" U, and of a certain Transform of the surface area function of geodesic spheres,

$$\hat{S}(s,\cdot) := \int_0^\infty e^{-\frac{s^2t^2}{2}}\frac{\partial V}{\partial t}\,dt = \int_{\xi\in M_m} e^{-\frac{s^2|\xi|^2}{2}}(\exp^* vol)_\xi$$

$$= (2\pi)^{n/2} s^{-n}\{v_0 - s^{-2}v_1 + s^{-4}v_2 - s^{-6}v_3 + \dots\}.$$

We then present a partial solution to the problem of determining coefficients of arbitrary order by computing a dominant part of their expression in terms of basic curvature invariants (τ = scalar curvature, ρ = Ricci curvature):

$$u_k = \frac{1}{6^k k!}\left(\tau^k - \frac{k(k-1)}{5}|\rho|^2\tau^{k-2} + \dots\right)$$

$$v_k = \frac{1}{6^k k!}\left(\tau^k + \frac{4k(k-1)}{5}|\rho|^2\tau^{k-2} + \dots\right)$$

Dr. Zvi Har 'El
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Twistor constructions of harmonic maps.

1st order constructions. Let N be an oriented Riemannian n-manifold, and $Q(N) \xrightarrow{\pi} N$ the Grassmannian of oriented 2-planes. The fibre is Kählerian: the complex quadric Q_{n-2}. We define the vector subbundle $\Pi \subset TQ(N)$ as follows: Each $q \in Q(N)$ is an or'd Euclidean plane in $T_{\pi(q)}N$, and therefore a complex line L_q. The space Π_q is the subspace of $T_q Q(N)$ spanned by the lift of L_q to the horizontal subspace, and the vertical $T^V_q Q(N)$. Those components have complex structures J^H_q and J^V_q. Define complex structures on Π:

$$J_1 = \begin{cases} J^H \text{ on } T^H Q(N) \\ J^V \text{ on } T^V Q(N \end{cases} ; \qquad J_2 = \begin{cases} J^H \text{ on } T^H Q(N) \\ -J^V \text{ on } T^V Q(N) \end{cases}$$

Theorem (Eells-Salamon CR Paris 1983) The correspondence $\varphi \rightsquigarrow \tilde{\varphi}$ (Gauss lift of φ

$$\begin{tikzcd} & Q(N) \arrow[d] \\ M \arrow[ur, "\tilde{\varphi}"] \arrow[r, "\varphi"'] & N) \end{tikzcd}$$

is a bijection between conformal harmonic maps and J_2-holomorphic maps $\tilde{\varphi}$. (Exclude φ constant and $\tilde{\varphi}$ vertical)

Examples $N = S^n$ with $n=3$ (Lawson's examples)
$n=4,6$ (Bryant's examples).

$N = \mathbb{R}P^n$ (Calabi examples)
$N = \mathbb{C}P^n$ (Eells-Wood examples)

James Eells.

Relativistic solar system experiments

Experiments which are presently discussed by relativists can very effectively be described without using local coordinates.
The Schwarzschild geometry describes the exterior of a spherically symmetric star (or planet). Its curvature operator has an eigenbasis of decomposable 2-forms from which all eigenvectors and eigenvalues of the Jacobitensors can be obtained. Gravitational redshift is expressed in terms of timelike variations of nullgeodesics; spacelike Jacobifields tangent to the lightcones describe the various distance measurements. This gives the redshift – observed size – observed luminosity relations (of each relativistic model). There are enough Killing fields to reduce the geodesic equation to one first order equation. From this the deflection of light and the time delay of light which passes close to the sun is deduced. The Jacobi equation along the worldline of a planet on a circular orbit has constant coefficients; it gives the perihel advance predicted by relativity. Gyroscopes are described by parallel vectorfields so that relativistic effects on gyroscopes circling the sun are immediate. The rotation of the sun (being small) is treated by linearizing the Kerrmetric at zero angular momentum which gives a symmetric 2-tensorfield solving the linearized Einstein equations along the Schwarzschild geometry. The first derivative of this field gives the linearized contribution to the connection, the second derivative gives the curvature correction caused by the rotation of the sun; this allows to discuss the influence of this rotation on the above effects.

H. Karcher

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A sharp Four dimensional isoperimetric inequality

Consider the following conjecture

Conjecture: Let M be a compact subdomain of a complete simply connected riemannian manifold of nonpositive curvature. Then

$$Vol(\partial M)^n \geq n^{n-1} \alpha(n-1) \, Vol(M)^{n-1}$$

with equality holding if and only if M is isometric to a flat ball ($\alpha(n)$ represents the volume of the unit n-sphere).

The conjecture was proved for $n=2$ by Beckenbach and Radó in 1933. In the talk we show that the conjecture is true for $n=4$. In fact we derive the best *known* constants $c(n)$ for all n such that

$$Vol(\partial M)^n \geq c(n) \, Vol(M)^{n-1}$$

although $c(n) = n^{n-1} \alpha(n-1)$ only for $n=4$.

Christopher B. Croke

The total (absolute) curvature of knotted surfaces

The total absolute curvature of a compact smooth submanifold imbedding $f: M^n \to \mathbb{R}^N$ can be defined by

$$\tau(f) = \mathcal{E}_L\, \mu(\pi_L \circ f) \qquad (1)$$

where π_L is orthogonal projection into a line L, μ is the number of nondegenerate critical points, and $\mathcal{E}_L$ is the expectation or mean over all lines through $0 \in \mathbb{R}^N$.

For closed curves: $\tau(f) = \int |\rho\, ds| / \pi$.

For closed surfaces in $\mathbb{R}^3$: $\tau(f) = \int |K\, d\sigma| / 2\pi$.

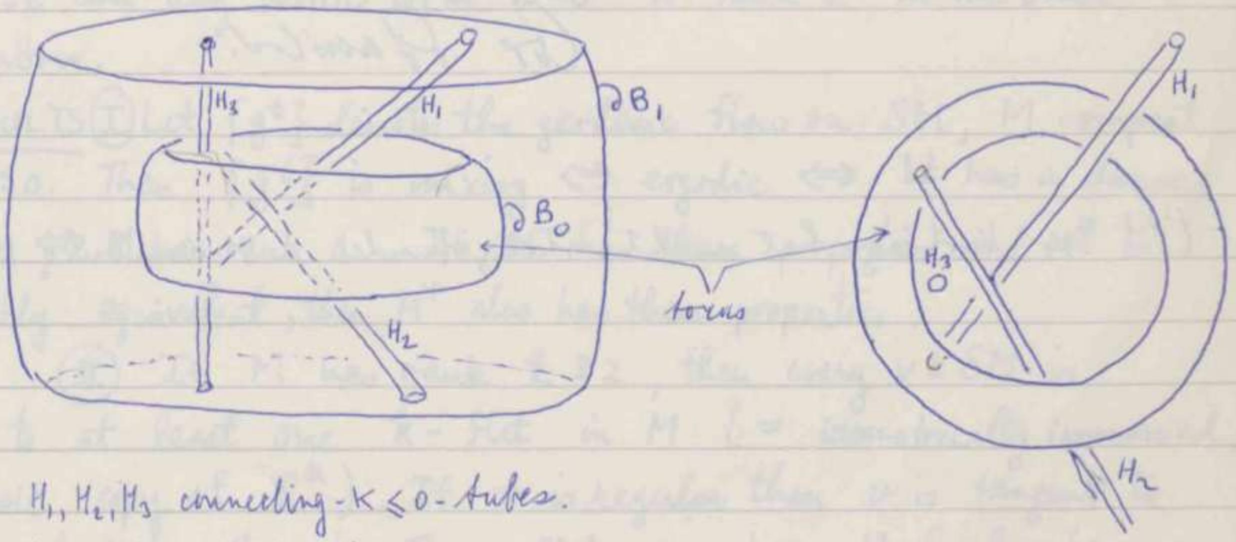
By Morse theory

$$\tau(f) \geqslant \beta = \text{sum of the Bettinumbers of } M$$

For knotted curves $\tau(f) \geqslant 4$ (Milnor-Fary) and even $\tau(f) > 4$ (Milnor) (by Langevin-Rosenberg-Meeks-Morton)

For knotted surfaces $\tau(f) \geqslant \beta + 4$, $\beta = 2 + 2\,\text{genus}$. (2)

W. Meeks and myself examined the possibility $\tau(f) = \beta + 4$ for knotted surfaces in $\mathbb{R}^3$. For the torus and the surface~~s~~ of genus 2 this is not possible, but for genus $\geqslant 3$ there are examples. Here is one for $g = 3$



H_1, H_2, H_3 connecting $-K \leq 0-$ tubes.

$\partial B_0, \partial B_1$ convex surfaces

Nicolaas H. Kuiper

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Decomposition of the curvature tensor in the almost Hermitian geometry and applications.(*)

For the curvature tensor R in the almost Hermitian geometry the decomposition $R=\sum_{i=1}^{10} p_i(R)$ under the action of the unitary group holds good. The classical Weyl's tensor is decomposed in the following way: $C(R)=C_1(R)+C_2(R)+p_5(R)+p_6(R)+p_7(R)+p_9(R)+p_{10}(R)$ (all components are orthogonal). It is given an analogous of this tensor: $C^*(R)=C_1^*(R)+C_2^*(R)+p_5(R)+p_6(R)+p_7(R)+p_8(R)+p_{10}(R)$, (all components are orthogonal). We have: $\rho\circ C(R)=0$ and $\rho^*\circ C^*(R)=0$. $C_1^*(R)\perp\sigma_2$, $C_1(R)\perp\sigma_1$. It is proved a theorem for the globality of the Kählerian defect $\Delta_R(p,E^4)=K_R(p,E^4)-K_R^*(p,E^4)$ in the class of AK_2-manifolds. The AHM with constant type are characterized by $p_i(R)=0$, $i=5,6,7,8,9,10$ and the AHM with conformal type – by $p_i(R)=0$, $i=6,7,8,9,10$.

Gr. Stanilov.

(*) Der Vortrag hat nicht stattgefunden. M. Hamburg

Compact Manifolds of Nonpositive Curvature

joint work of M. Brin, W. Ballmann, P. Eberlein and R. Spatzier

Problem To classify compact manifolds of nonpositive sectional curvature in some reasonable sense.

Question Given M compact with $K \leq 0$ and no Euclidean factor, can one find a finite cover M' that splits as a Riemannian product of locally symmetric spaces and spaces with "rank" $= 1$
(There is much evidence that the answer to this question is yes)

Rank of a manifold Recall that if Y is a parallel Jacobi vector field perpendicular to a geodesic γ in any Riemannian manifold N, then $K(Y, \gamma')(t) \equiv 0$. For a unit vector v tangent to a compact manifold M with $K \leq 0$ we define $r(v) =$ dimension of the space of Jacobi vector fields along γ_v. Then define

$$r(M) = \text{rank of } M = \inf_{v \in SM} r(v).$$

Moreover call v *regular* if $r(v) = r(M)$. Note that this definition of rank agrees with the usual definition for locally symmetric spaces. Also $r(M_1 \times M_2) = r(M_1) + r(M_2)$. Spaces of rank 1 behave geometrically like ~~spa~~ spaces with $K < 0$ but they may look very flat. For example, any compact surface of genus $g \geq 2$ and any metric with $K \leq 0$ is rank 1 in the sense defined above.

RESULTS (I) Let $\{g^t\}$ denote the geodesic flow on SM, M compact with $K \leq 0$. Then $\{g^t\}$ is mixing $\Leftrightarrow$ ergodic $\Leftrightarrow$ it has a dense orbit in $SM \Leftrightarrow M$ has rank 1. If M has these properties and M^* is homotopically equivalent, then M^* also has these properties

(II) If M has rank $k \geq 2$, then every $v \in SM$ is tangent to at least one k-flat in M (= isometrically immersed, totally geodesic copy of $\mathbb{R}^k$). If v is regular then v is tangent to exactly one k-flat Remark: These flats even have Weyl chambers. The conjecture is that any such M must be a product manifold (or finitely covered by one) or a locally symmetric space.

(III) ~~If M has rank $k \geq 2$~~

(over)

Ⅲ Every immersed k-flat is a limit of immersed k-tori in the case $k \geq 2$ (cf. the analogous situation in locally symmetric spaces) In particular $\pi_1(M)$ contains many different free abelian groups of rank k.

Corollaries : 1) Every compact M with nonpositive sectional curvature has its closed geodesics dense in the space of all geodesics (no assumption on rank)

2) If $N(t)$ = the number of free homotopy classes of closed curves that contain a closed geodesic of length $\leq t$, then $N(t)$ grows exponentially in t.

3) There exists an open $\{g^t\}$ invariant subset $U \subseteq SM$ on which there exist k-1 continuous first integrals if $k \geq 2$. In particular $\{g^t\}$ does not have a dense orbit if $k \geq 2$ (cf. Ⓘ above)

Patrick Eberlein

Classification of finite group actions on compact 3-dimensional manifolds

joint work with S.T. Yau, Peter Scott, Leon Simon

The problem is related to a conjecture of Thurston which states that every prime compact 3-manifold admits a geometric structure and that finite group actions are conjugate to a group of isometries of the geometric structure. This is a rather loose statement in that the manifold may break up into geometric pieces but the statement gives the idea. Methods of proof that deal with this question use minimal surface theory and hyperbolic geometry. Particular questions which are solved are

(1) Finite group actions on $\mathbb{R}^3$ are conjugate to linear actions.

(2) if M has a geometric structure and M is not base on S^3 or H^3 then every finite group action is "geometric".

William H Meeks

Bifurcations of closed geodesics

Consider a manifold M, and let $\mathcal{G}(M;I)$ denote the set of all one-parameter families g_μ of metrics on M, parametrized by $\mu \in I$, $I \in \{\mathbb{R}, \mathbb{R}/\mathbb{Z}\}$. Let c_0 be a given closed geodesic of g_0. Under some regularity conditions, we have a family c_μ of closed geodesics of g_μ passing through c_0. But there are some unavoidable cases, where this will not be the case. Such geodesics are bifurcational. To classify all the typical bifurcations, we consider the Poincaré maps P_μ, which will be symplectic mappings. Then we give a complete classification of the bifurcations of periodic points of symplectic mappings P_μ, depending on a 1-dimensional parameter. This classification will be only up to nondegeneracy conditions. Next we prove a local perturbation theorem, asserting that by a small perturbation of the metric around c_0, we can achieve, that P_μ ~~is of~~ satisfies the nondegeneracy conditions needed above. Finally, we prove the main

Theorem: there is a residual set $\mathcal{G}'(M;I) \subset \mathcal{G}(M;I)$ such that for any $g_\mu \in \mathcal{G}'(M;I)$ and any closed geodesic c_0 of a g_{μ_0}, c_0 lies either on a regular family or on a bifurcational family, which corresponds to a nondegenerate bifurcation of P_μ.

Thus, given a generic curve of metrics on M, we can describe all the unavoidable local bifurcation phenomena.

Matthias Hamburg

Some universal factorizations of differentiable functors.

Smooth groupoïds and smooth functors between them occur everywhere in global Geometry, and their algebraico-differential significance and structure are much richer than in the very special case of Lie group morphisms.

For instance smooth functors may describe such various situations as: a Lie group action, a maximum or globalizable local transformation group in the sense of Palais, a one-parameter group, a principal fibration, a cocycle, etc...

Those functors arising from a group (or groupoïd) action will be called "*actors*".

By describing algebraic properties of functors by means of suitable diagrams, we are able to give a very natural "smooth" version of algebraic concepts such as faithfulness, functorial *equivalence*, etc..., and we can then state: "every faithful functor factorizes through an equivalence and an actor", which we call its "*universal activation*".

The (purely diagrammatic though rather sophisticated) proof unifies all the specific constructions (Palais globalization, homogeneous spaces, principal bundles, Haefliger structures) and of course covers many new situations; it avoids any local triviality assumption.

This "universal activation" is an invariant of the class of the smooth functor up to an equivalence at the source. On the other hand, the "*transverse structure" of a foliation* may be described by the class of its smooth holonomy groupoïd up to functorial equivalences, a concept which turns out to be equivalent to the notion of equivalence recently considered by Skandalis-Haefliger (and in the special case of pseudo-groups by Van Est-Haefliger) in a maybe less natural way.

Jean PRADINES
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Examples of complete manifolds with positive Ricci curvature

We study the geometry and topology of certain real algebraic varieties and construct a series of manifolds $V_- = V_-(\ell, p, q)$, $V_0 = \partial V_-(\ell, p, q)$ with the following properties: For integers $\ell \geq 3$, $p > q > 0$, $p - q$ sufficiently large but bounded and $p + q$ sufficiently large, V_0 is a closed $(q-1)$-connected manifold with negativ Euler number $\chi = 2(2-\ell)$ ~~and~~ for p, q odd and positive Ricci curvature. The interior of V_- is an open $(q-1)$-connected manifold admitting a complete metric with positive Ricci curvature not having the homotopy type of any closed manifold. In particular V_- does not admit a metric of nonnegative sectional curvature. Similar examples of closed manifolds are expected, but not known to exist. Candidates are manifolds of type V_0 above and certain complete intersections as for example the cubic in $\mathbb{C}P^4$ having $c_1 > 0$ and $\chi < 0$. This is joint work with D. Gromoll.

Wolfgang T. Meyer

Foliations and metrics

A foliation $\mathcal{F}$ on a Riemannian manifold M is called harmonic, if all leaves of $\mathcal{F}$ are minimal submanifolds of M. The terminology is motivated by the fact that this geometric property is characterized by the harmonicity of the projection $\pi : TM \to Q$ to the normal bundle Q, viewed as a Q-valued 1-form. Riemannian foliations of this type on a compact and oriented manifold are further characterized as the critical points of an energy functional on the space of foliations. In this talk we discuss examples and geometric properties of such foliations (joint work with F. Kamber)

Ph. Tondeur

Compact conformally flat hypersurfaces

E. Cartan proved in 1917 that every conformally flat hypersurface in E^{n+1}, $n \geq 4$ locally (i.e. in the neighborhood of each non-umbilic point) is the envelope op a one-parameter-family of hyperspheres. Here we consider the corresponding global problem and study compact conformally flat hypersurfaces. In particular we determine the intrinsic conformal structure of such a hypersurface: Every compact conformally flat immersed hypersurface in E^{n+1}, $n \geq 4$ is conformorphic to a Schottky-manifold. By A Schottky-manifold ~~we mean~~ is a conformally flat manifold constructed in the following way: Start with a sphere S^n and cut out an even number of spherical holes $B_1, \cdots, B_k, B_1', \cdots, B_k'$. Then identify in $S^n - \bigcup_{i=1}^{k}(B_i \cup B_i')$ the boundar~~y~~ies ~~of~~ $\partial B_i'$ with ∂B_i by means of ~~a~~ Moebiustransformations $f_i : S^n \to S^n$ ~~for $i=1, \cdots, k$~~.

U. Pinkall

Pinching Theorems for the Diameter

A report was given on some recent results of D. Brittain. Let Ric be the average sectional curvature. Then we have:

Theorem A: There exits an $\varepsilon(n, \max K, \mathrm{vol}) > 0$ such that M^n with $\mathrm{Ric} \geq 1$, $d(M) \geq \pi - \varepsilon$ is homeomorphic to S^n

Theorem B: There exits an $\varepsilon(n, \max K) > 0$ such that M^n with $K \geq 1$ and $d(M) \geq \pi - \varepsilon$ is diffeomorphic to S^n

These results were motivated by:

Bonnet-Myers: $\mathrm{Ric} \geq 1 \Rightarrow d(M) \leq \pi$

Cheng (75): $\mathrm{Ric} \geq 1, d(M) = \pi \Rightarrow M$ isometric to S^n $(K \equiv 1)$

Grove-Shiohama: (75) $K \geq 1$, $d(M) > \pi/2 \Rightarrow M$ homeomorphic to S^n

Itokawa-Shiohama: (82), ex. $\varepsilon(n, \min K) > 0$ s.t. $\mathrm{Ric} \geq 1$, $\mathrm{vol}(M^n) \geq \mathrm{vol}(S^n)(1-\varepsilon)$
$\Rightarrow M$ homeomorphic to S^n.

Wolfgang Ziller

Free Isometric Actions on Compact Lie Groups and Manifolds of Positive Curvature

Consider the manifold $M_6 = T_2 \backslash U(3) / T_1$ where

$$T_1 = \left\{ \begin{pmatrix} a & & \\ & a & \\ & & \bar{a} \end{pmatrix} ; a \in S^1 \right\}, \quad T_2 = \left\{ \begin{pmatrix} b & & \\ & c & \\ & & 1 \end{pmatrix} ; b, c \in S^1 \right\}$$

M_6 is a manifold since $T_1 \times T_2$ is acting freely. We showed that there is a left invariant metric on $U(3)$ with respect to which $T_1 \times T_2$ acts isometrically such that the induced metric on M_6 has positive sectional curvature. M_6 is simply connected and not homotopically equivalent to any known space of positive curvature. We indicated the proof of the following uniqueness theorem: Let G be a compact simple Lie group with left invariant metric, which is right invariant w.r. to some maximal torus. Let U be a compact subgroup of $G \times G$ acting ~~free~~ isometrically without fixed points. If G/U has $K > 0$ and even dimension, then G/U is diffeomorphic to a homogeneous space of positive curvature or to M_6.

J. H. Eschenburg

Closed geodesics on manifolds with $\pi_1 = \mathbb{Z}$

Let M be a compact Riemannian manifold with fundamental group $\pi_1(M) \cong \mathbb{Z}$. We prove that the number $n(l)$ of geometrically distinct closed geodesics of length $\leq l$ grows like the prime numbers, so

$$\liminf_{l \to \infty} \left(n(l) \frac{\log l}{l} \right) > 0 .$$

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The proof consists in reducing the problem first to ~~an~~ a topological and then to an algebraic problem which finally can be solved. This is joint work with N. Hingston.

V. Bangert

Group actions, Morse theory and the double mapping cylinder

A 1-connected manifold M is called <u>$\mathbb{Q}$-elliptic</u> provided $\dim \pi_*(M) \otimes \mathbb{Q} < \infty$. The following gluing construction leads to a new class of $\mathbb{Q}$-elliptic manifolds:

<u>Thm</u> (Grove-Halperin) If M can be written as $M = D_0(A) \cup_S D_1(B)$, where $D_0(A)$ and $D_1(B)$ are disc bundles with common sphere bundle S, then M is $\mathbb{Q}$-elliptic iff S is $\mathbb{Q}$-elliptic.

<u>Corol.</u> All 1-connected manifolds of cohomogeniety 1 are $\mathbb{Q}$-elliptic.

A proof of this can be given by applying Morse theory to the relative loop space $\Omega(M,S)$. It is possible to give a complete list of the possible $\mathbb{Q}$-homotopy types of $\Omega(M,S)$ that occur in the more general setting of double mapping cylinders $M = {}^{S \times I}/_{f_0, f_1}$ with the (rational) homotopy fibers of $f_0 : S \to A$, $f_1 : S \to B$ being spheres (for general CW-type spaces). An amusing consequence of the above is that the complement of a non-$\mathbb{Q}$-elliptic submanifold N of a $\mathbb{Q}$-elliptic manifold M does not admit a structure of a disc bundle, - in particular it cannot admit a complete metric of non-negative sectional curvature.

Karsten Grove

Hypersurfaces with a constant higher mean curvature.

The r-th mean curvature H_r of a hypersurface M in a riemannian mfd. $\tilde{M}$ is (up to a constant factor) the r-th elementary symmetric function of the principal curvatures $k_1,\dots,k_m$. There are given several characterizations of complete or compact hypersurfaces in a space of constant sectional curvature c, $\tilde{M}=\tilde{M}(c)$, which have a constant fixed $H_r \neq 0$ and fulfill the intrinsic non strict lower bound curvature condition $K \geq \max\{0,c\}$. The results generalize the classical Liebmann/Süß-theorems and also recent results of (e.g.) Cheng/Yau, Nomizu/Smyth and U. Simon. The proofs are based on certain elliptic partial differential operators generalizing the Laplace/Beltrami-operator.

R. Walter

Isometry-invariant geodesics on S^2.

Let M be a compact simply connected Riemannian manifold and $A: M \to M$ an isometry. A geodesic $c: \mathbb{R} \to M$ is called isometry invariant if $c(t+1) = Ac(t)$. Such geodesics were first studied by K. Grove. We prove:

Let $A: S^2 \to S^2$ be an orientation preserving diffeomorphism of finite order $\neq 2$. Then for a generic A-invariant metric on S^2 there will be infinitely many A-invariant geodesics.

Note that if we take the standard metric on S^2 and if A is a rotation, then there is only 1 nontrivial A-invariant geodesic: the equator.

The proof consists of two steps. First the Birkhoff-Lewis fixed point theorem implies the existence of infinitely many, for a generic metric with an elliptic A-invariant geodesic. Next we use the equivariant Morse theory to conclude that if all A-invariant geodesics

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are hyperbolic, then there must be infinitely many.

N. Hingston

Hypersurfaces with prescribed Gaussian curvature

Let $\varphi : E^{n+1} \to \mathbb{R}$ be a given function. Under what conditions there exists a closed hypersurface F in E^{n+1} with prescribed genus and such that the Gauss curvature $K_F(X) = \varphi(X)$, $X \in F$? This question was raised by S.T. Yau in "Problem section", Seminar on Differential Geometry, Annals of Math. St., v. 102 (1982).

The following theorem gives a partial answer to this question:

Suppose a) $\varphi(X) > 0$, $X \in E^{n+1} \setminus \{0\}$;

b) $\varphi(X) \in C^m(E^{n+1} \setminus \{0\})$, $m \geq 3$;

c) there exist two numbers R_1 and R_2, $0 < R_1 \leq 1 \leq R_2 < \infty$ such that $\varphi(X) > |X|^{-n}$ when $|X| < R_1$ and $\varphi(X) < |X|^{-n}$ when $|X| > R_2$;

d) $\frac{\partial}{\partial \rho}(\rho^n \varphi(u,\rho)) \leq 0$, $\rho \in [R_1, R_2]$, where $X = (u, \rho)$ are the spherical coordinates in E^{n+1}.

Then there exists a closed convex hypersurface F in E^{n+1} such that

i) F is a graph of a radial function $\rho(u) > 0$, ~~$u \in S$~~ over a unit sphere $S^n \subset E^{n+1}$;

ii) $\rho(u) \in C^{m+1,\alpha}(S^n)$, $\alpha \in (0,1)$, and if φ is analytic then ρ is analytic;

iii) the Gaussian curvature of F is given by $\varphi(u, \rho(u))$;

IV) F is unique up to a homothetic transformation.

The proof is based on a study of a nonlinear elliptic equation of Monge-Ampère type on S^n which the function g must satisfy. [redacted]

V. Oliker

Duality theorems for Riemannian foliations
(joint work with Ph. Tondeur)

About 25 years ago, B.L. Reinhart (AJM 1959) asserted that the basic complex $\Omega_B^\cdot(\mathcal{F})$ of a Riemannian foliation $\mathcal{F}$ on a closed manifold with oriented normal bundle satisfies Poincaré duality, i.e. $H^r(\Omega_B(\mathcal{F})) \cong H^{q-r}(\Omega_B(\mathcal{F}))$, $q = \operatorname{codim} \mathcal{F}$. In 1981, Y. Carrière produced a counterexample to this assertion, namely a Riemannian flow $\mathcal{F}$ transverse to the fibre of a fibration $T^2 \to M^3 \to S^1$ satisfying $H^0(\Omega_B(\mathcal{F})) \cong \mathbb{R}$, $H^2(\Omega_B(\mathcal{F})) = 0$, $q = 2$. On the other hand, the flow $\mathcal{F}$ in the above example is not geometrically taut, i.e. M^3 does not admit a metric for which the leaves of $\mathcal{F}$ become minimally immersed submanifolds of M^3 (the flow is not geodesible). It turns now out that tautness of $\mathcal{F}$ is precisely the condition needed to establish Poincaré duality for $H(\Omega_B(\mathcal{F}))$.

More generally we say that a foliation $\mathcal{F}$ is geometrically tense if there exists a Riemannian metric on M for which the leaves of $\mathcal{F}$ become submanifolds of constant (= parallel) mean curvature, i.e. $\kappa = \operatorname{Tr} W \in \Omega_B^1(\mathcal{F})$, W the Weingarten operator. The mean curvature form κ is then a closed basic 1-form and one obtains two pairs of mutually adjoint operators $(d_B, d_\kappa^{-\bar{*}})$ and $(d_\kappa^-, d_B^{\bar{*}})$ relative to a suitable metric on $\Omega_B^\cdot(\mathcal{F})$ (d_B = ext. diff. in Ω_B, $d_\kappa^- = d_B - \kappa \wedge$, $\bar{*}$ = star operator in $\Omega_B^\cdot$ defined by the transverse Riemannian metric). Using the transversally elliptic operator $\Delta_B = d_B d_\kappa^{-\bar{*}} + d_\kappa^{-\bar{*}} d_B$, we can then prove the following theorem(s).

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<u>Theorem</u>: Let $\mathcal F$ be a Riemannian foliation with oriented normal bundle on a closed oriented manifold M^n ($q = \operatorname{codim}\mathcal F$). Then the following statements are equivalent:

(i) $\mathcal F$ is <u>tense</u> (<u>resp.</u> taut);

(ii) $\exists$ a bundle-like metric on M, for which $d_\kappa^{-*}\nu \in \Omega_B^{q-1}(\mathcal F)$ (<u>resp.</u> $\Delta_B \nu = 0$), where ν ($\in \Omega_B^q(\mathcal F)$) is the invariant transversal volume form of $\mathcal F$;

(iii) there exists a volume form $\omega_0 \in \Gamma\Lambda^p L(\mathcal F)$ ($p = \dim\mathcal F$) representing the induced orientation on $\mathcal F$ such that the pairing $\Psi(\alpha,\beta) = \int_M (\alpha\wedge\beta)\wedge\omega_0$, $\alpha\in\Omega_B^r$, $\beta\in\Omega_B^{q-r}$ induces a non-degenerate pairing

$$\Psi_* : H^r(\Omega_B^\cdot, d_B)\otimes H^{q-r}(\Omega_B^\cdot, d_\kappa^-) \longrightarrow \mathbb R \;, \quad r = 0,\dots,q \;,$$

i.e. the basic cohomology $H(\Omega_B^\cdot, d_B)$ is dual to the basic <u>twisted</u> cohomology $H^\cdot(\Omega_B^\cdot, d_\kappa^-)$ (<u>resp.</u> $H(\Omega_B^\cdot, d_B)$ satisfies <u>Poincaré Duality</u>).

(Observe that $d_\kappa^- = d_B$ in the taut case!)

Several applications of this theorem were discussed:

(a) $H^0(\Omega_B, d_\kappa^-)$ consists of solutions of the 1st order PDE $d_B f = \kappa f$. Hence either $H^0(\Omega_B, d_\kappa^-) = 0$ or $H^0(\Omega_B, d_\kappa^-) = \{ f\in\Omega_B^0 / \kappa = d_B \log f \} \cong \mathbb R$.

For a tense R-foliation one has therefore:

$\mathcal F$ taut $\Longleftrightarrow$ $H^q(\Omega_B, d_B)\cong\mathbb R$ $\Longleftrightarrow$ $[\kappa] = 0 \in H^1(\Omega_B, d_B)$ relative to a suitable metric on M.

(b) Foliations $\mathcal F$ with <u>compact</u> leaves: taut $\Longleftrightarrow$ $\mathcal F$ locally stable $\Longleftrightarrow$ $\mathcal F$ R-foliation (Rummler)

Thus the base space B (= leaf space) of a locally stable compact foliation (which is a Satake mf.) satisfies Poincaré Duality in de Rham cohomology.

(c) Foliation cycles of a taut R-foliation are homologically unique up to a constant >0 factor. This applies in particular to the fol. cycles defined by compact leaves.

A conjecture was made concerning a transversal signature theorem for taut R-foliations.

FWKamber.

An integrability condition for simple Lie groups
(joint work with Min-Oo)

The Maurer-Cartan equation, $d\omega + [\omega, \omega] = 0$, is the well known integrability condition for a local Lie group structure. Pinching theorems deal with the following question: What can be said if the integrability condition $\Omega = d\omega + [\omega, \omega] = 0$ is satisfied only up to a certain degree, i.e. $\|\Omega\|$ is small in a suitable norm? Since the definition of ω requires a global parallelism of the manifold M on which it is defined, its assumption is rather restrictive. Another possibility to define a Lie algebra structure in every tangent space of M is simply to define a tensor field $T : TM \otimes TM \to TM$ which restricts to a Lie algebra bracket in each tangent space. In 1965 Nomizu asked for the integrability condition for T. In case the Lie algebra $\mathfrak{g}$ used as model is simple the answer is as follows: T defines a pseudo-riemannian metric $\langle , \rangle$. Let D denote its Levi-Civita connection, and define $dT(X,Y,Z) = (D_X T)(Y,Z) + (D_Y T)(Z,X) + (D_Z T)(X,Y)$.

Theorem 1. If $\mathfrak{g}$ is simple, rank $\mathfrak{g} \geq 2$, then $dT = 0$ if and only if either M is locally isometric to the Lie group G with Lie algebra $\mathfrak{g}$, or M is flat.

The proof relies on the following version of Berger's theorem on holonomy groups.

Theorem 2. If $\mathfrak{g}$ is simple, rank $\mathfrak{g} \geq 2$, and $\beta : \mathfrak{g} \wedge \mathfrak{g} \to \mathfrak{g}$ satisfies $[X, \beta(Y,Z)] + [Y, \beta(Z,X)] + [Z, \beta(X,Y)] = 0$ (Bianchi equation), then $\beta = \lambda [\,,\,]$, wher $\lambda \in \mathbb{R}$ and $[\,,\,]$ = Lie bracket of $\mathfrak{g}$.

In case $\mathfrak{g}$ is compact and simple, Theorem 2 is a special case of Simons' result on holonomy systems.

Ernst Ruh

Parallel Gauss maps and rigidity aspects of minimal submanifolds.

We reported on joint work with M. DAJCZER. Starting point is the observation that hypersurfaces in euclidean spaces (and spheres) with constant relative nullity have a representation by the inverse of the Gauss map on the normal bundle of its image ('Gauss parametrization'). This has many interesting applications. A main result is: Any complete minimal hypersurface M^n in $\mathbb{R}^{n+1}$ is rigid as minimal submanifold in $\mathbb{R}^{n+p}$ for any $p \geq 1$, provided $n \geq 4$ and M^n has no euclidean factors $\mathbb{R}^{n-2}$ or $\mathbb{R}^{n-3}$. Local rigidity as minimal hypersurfaces can be completely described in terms of 'associated families' of minimal immersions and 'superminimality' of their Gauss image, which we define in general for certain ('circular') Kaehler manifolds in real spaces. Such Kaehler manifolds play a surprising role also in the congruence problem for isometric submanifolds with parallel Gauss maps which we analyze fairly completely. There are various other results in this context. For example, all Kaehler hypersurfaces of real euclidean space can be classified essentially in terms of superminimal surfaces in spheres.

Detlef Gromoll

Almost negative curvature on S^3

This is an example by Gromov of a smooth Riemannian metric on the 3-sphere with diameter = 1 and with ~~k~~positive upper curvature bound arbitrarily close to zero. The example is obtained by replacing small tubular neighbourhoods of great circles on the standard S^3 with suitable copies of $\mathring{S}^2 \times S^1$, where $\mathring{S}^2$ is the two-sphere with a small circular disc removed. The procedure is uniquely restricted to threemanifolds. It is not clear, wheater such metrics also exist on S^n for $n \geq 4$.

Peter Buser.

Relations between transverse structure of a foliation and its characteristic classes.

Characteristic classes of a foliations admits natural lifts to cohomology groups of the classifying space of the corresponding (transverse) holonomy groupoid. The lifts depend on the transverse structure of the foliation and not on the foliation itself. This is, in short, the „philosophy" of connections between characteristic classes and the transverse structure.

We prove two results of this kind:

<u>Thm. 1</u>. If a codimension-q foliation F on X admits a transverse k-field $(X_1, \ldots, X_k)$ of infinitesimal automorphisms such that F and the fields span a codimension-(q-k) foliation, say F', then the characteristic homomorphism α_F of F admits a factorization

$$H(WO_q) \to H(WO_{q-k}) \xrightarrow{\alpha_{F'}} H^*(X).$$

<u>Corollary</u>. α_F annihilates $\ker(H(WO_q) \to H(WO_{q-k}))$.

(Thm. 1 ~~pr~~ has been proved independently by Cordero and Masa)

<u>Thm 2.</u> If a foliation F admits a family of submersions on $\mathbb{R}^q$ such that the transition maps have (locally) constant Jacobians, then κ_F annihilates $\ker(H(WO_q) \to H(\widetilde{W}O_q))$, where $\widetilde{W}O_q := WO_q/(c_1)$

<u>Remark</u>. In codimension 1 such an F is simply a transversally affine foliation; the G-V class of F is then 0.

Proof of the two theorems is based on a decomposition of exotic classes into „elementary blocks" corresponding to the generators $c_1, \dots, c_q$ and $y_1, y_3, \dots$ of WO_q. In the case of thm. 1 we have $c_i = 0$ and $y_j = 0$ for $i,j > q-k$, whereas in the case of thm. 2. the conditions imposed on F is simply the solution of the equation $c_1 \equiv 0$.

<u>Remark</u>. The equality $y_1 \equiv 0$ holds iff F admits (up to ±1) a transverse volume form.

G. Andrzejczak.

<u>On a conjecture of Ossermans on the volume of the generalized Gauss map</u>

For an immersed n-manifold in $\mathbb{R}^{n+p}$ one has two natural Gauss maps γ_1: unit normal bundle $\to S^{n+p-1}$, $\gamma_2 : M \to G_p(\mathbb{R}^{n+p})$. Then for the (suitably normalized) volumes of the images of γ_1 and γ_2, denoted $\tau(f)$ (= total absolute curvature) and $\sigma(f)$, Osserman conjectured in his lecture at the Chern symposium 1979

$$\tau(f) \le \sigma(f)$$

for any $n \geq 2$. The conjecture is true.

D. Ferus

Minimal submanifolds of spheres.

Conjecture. Let (M,g) be a closed, 1-conn., oriented 2-manifold with curvature K, let $s \in \mathbb{N}$. Define the constants $\kappa(s) = \frac{2}{s(s+1)}$. Let $\tilde{x}: M \to S^N(1)$ be an isometric minimal immersion. Then

~~$\kappa(s+1) \le \kappa($~~[illegible]~~$) \le \kappa(s+1)$~~ $\kappa(s+1) \le K \le \kappa(s)$

implies $K = \kappa(s)$ or $\kappa(s+1) = K$, and (M,g) is a 2-sphere of curvature K.

Theorem. (a) The conjecture is true for $s = 1$ (Lawson $N = 4$, Simon et al. N arbitrary). (b) The conjecture is true for $s = 2$ (M. Kozlowski & U. Simon).

The proof extends a method from Coll. Math. 1979 (K. Benko, U. Simon et al.)

Theorem. Let M be closed, conn., oriented, $\dim M = n$, let $\tilde{x}_t: M \to S^N(1)$ be a 1-param. family of isom. minimal immersions. Let g_t be the family of corr. metrics and $g := g_0$ be of constant curvature. Let $j: S^N(1) \hookrightarrow \mathbb{R}^{N+1}$ be the canonical embedding and let ξ_t be the mean curvature vector of $x_t = j \circ \tilde{x}_t$ with corresponding second fundamental forms $II(\xi_t)$, $\xi := \xi_0$. Then $\delta II(\xi) = 0$ (δ first variation) implies that the family $\hat{x}_t$ of infinitesimal deformations is trivial. The proof uses results on deformations of Codazzi-tensors of V. Oliker and U. Simon

U. Simon

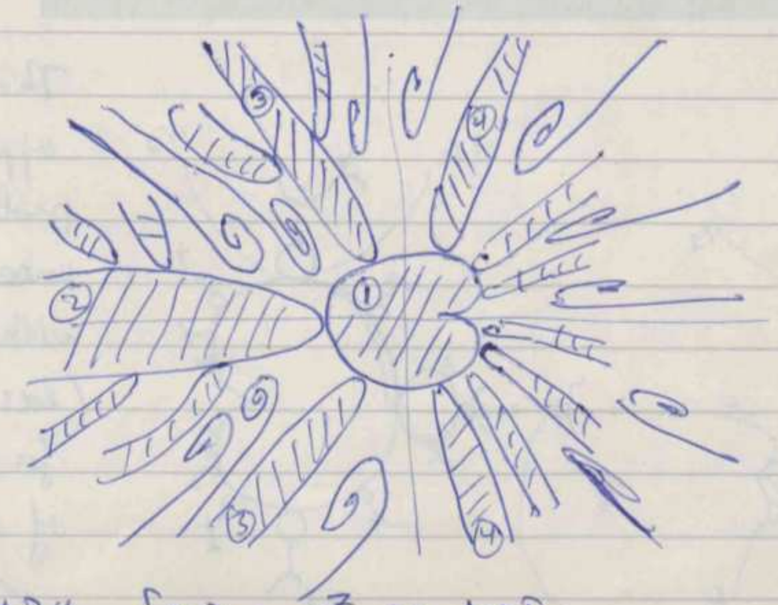
Dynamical Systems

29. May - 4. June. 1983

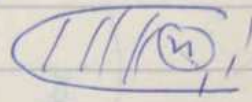
Dynamics of $z \longrightarrow \lambda \exp(z)$. R L DEVANEY.

Consider the family of maps $f_\lambda(z) = \lambda e^z$. When $\lambda = 1$, the Julia set of f is the entire complex plane. Using symbolic dynamics, one can show that there is a unique periodic point corresponding to each repeating sequence of integers, $s_0 s_1 \ldots s_n$, where s_j indicates that the orbit enters the s_j^{th} fundamental domain. Moreover, to each allowable sequence, there exists a curve of points in $\mathbb{C}$ which share the same itinerary.

For other λ-values, the bifurcation diagram in the λ-plane is:



where:

① 

= λ's for which there is a period n sink

② 
= λ's for which $J = \mathbb{C}$

COROLLARY $f(z) = e^z$ IS NOT structurally stable

RL Devaney

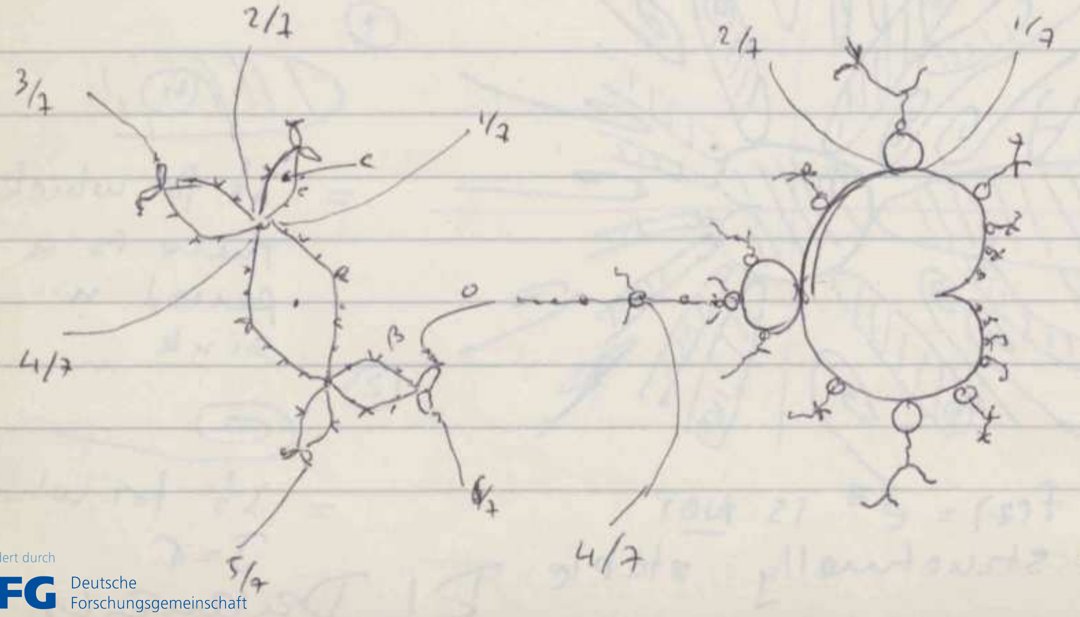
A. Douady and J. H. Hubbard.

Dynamics of $P_c : z \longmapsto z^2+c$

Plan

Set $K_c = \{ z \mid P_c^n(z) \not\to \infty \}$. Then $0 \in K_c \Rightarrow K_c$ is connected, $0 \notin K_c \Rightarrow K_c$ is a Cantor set. The Mandelbrot set is $M = \{ c \mid 0 \in K_c \}$. For each c, let φ_c conjugate P_c to $P_0 : z \longmapsto z^2$ at nbd ∞. If $c \in M$, φ_c provids the conformal mapping : $\mathbb{C} - K_c \longrightarrow \mathbb{C} - \bar{D}$. For $c \notin M$, $\varphi_c(z)$ is defined if $\eta(z) > \eta(0)$, where η is the Green function of ∞ in $\bar{\mathbb{C}} - K_c$. In particular $\Phi(c) = \varphi_c(c)$ is defined. The map $\Phi : \mathbb{C} - M \longrightarrow \mathbb{C} - \bar{D}$ is the conformal mapping. For $\theta \in \mathbb{R}/\mathbb{Z}$, $R_\theta(M) = \Phi^{-1}(\{ r e^{2\pi i \theta} \}_{r>1})$ is the external ray of M of angle θ. It is not known that M is locally connected (this would imply generic hyperbolicity for instance), but one can prove that each $R_\theta(M)$ with $\theta \in \mathbb{Q}/\mathbb{Z}$ has a limit in M. The ~~following~~ lecture was about the proof of the following theorem, which is the main step in this direction for rationals with odd denominator.

Thm: Let c such that P_c has a rational indifferent periodic point α : $f^k(\alpha) = \alpha$, $(f^k)'(\alpha) = \rho = e^{2\pi i p/q}$, Say $P_c^{kqn}(c) \to \alpha$. In K_c, the point α has a finite number of external argument of the form $p_i / 2^{kq} - 1$. Let θ and θ' be These of those arguments which are adjacent to the petal that contain c. Then θ and θ' are ~~the~~ external arguments of c in M.



The Mandelbrot set appears in lots of problems apparently unrelated, sometimes with its external rays (ex: Newton method for polynomials of degree 3)

A. D.

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·Forced vibrations of some Hamiltonian systems

Abbas BAHRI and Henri BERESTYCKI

In this talk we report on the following result.
Let $V \in C^2(\mathbb{R}^N, \mathbb{R})$ satisfy the assumption

(V) $\quad 0 < V(x) \leq \theta \, V'(x) \cdot x \quad \forall x \in \mathbb{R}^N, \; |x| \geq R_0$

for some $R_0 > 0$ and where $0 < \theta < 1/2$.

<u>Theorem:</u> For any given $f \in L^2_{loc}(\mathbb{R}, \mathbb{R}^N)$ which is T-periodic, there exist infinitely many T-periodic solutions (forced oscillations) of the system $\ddot{x}(t) + \operatorname{grad} V(x(t)) = f(t)$.

The method of proof rests on constructing critical values for the "autonomous" functional $I^*(x) = \frac{1}{2}\int_0^T |\dot{x}|^2 - \int_0^T V(x(t))$ via a mini-max formula (Liusternik-Schnirelman type theory) such that they persist for functionals $I(x)$ "near" to I^* but not necessarily symmetric (i.e. S^1-invariant). The solutions we obtain have arbitrarily large amplitude (i.e. $\|x\|_{L^\infty}$). We also obtain related results for more general systems

$$\ddot{x}(t) + V'_x(t, x) = 0$$

or

$$\dot{z} = J H'(z) + f(t)$$

where $z: \mathbb{R} \to \mathbb{R}^{2N}$, $H \in C^2(\mathbb{R}^{2N}, \mathbb{R})$ and $f: \mathbb{R} \to \mathbb{R}^{2N}$ is given T-periodic.

A dissipative version of the Poincaré-Birkhoff geometric theorem

A. Chenciner

The proof of Poincaré's last geometric theorem given by Birkhoff in 1925 does not use the preservation of area but only the purely topological "intersection property" (see also the recent work of P. Carter). We point out that the same proof gives a theorem concerning the existence of fixed points in 1-parameter families of homeomorphisms of the annulus which are distortions and satisfy one half of the intersection property for each extreme value of the parameter.

As in Rüssmann's proof of the invariant curve theorem (K.A.M.) preservation of area appears here as a codimension one condition.

This result has been used to prove the existence of periodic orbits of "high" periods not a priori belonging to an invariant curve in Hopf bifurcations of codimension greater than one.

Smooth conjugation of diffeomorphisms of the circle with diophantine rotation number

J.C. YOCCOZ.

Th 1 Let $f \in \mathrm{Diff}^k_+(\mathbb{T}^1)$, $\rho(f) = \alpha \in \mathbb{T}^1 - \mathbb{Q}/\mathbb{Z}$, s.t

i) $\exists C, \beta \geqslant 0$, s.t, $\forall p/q \in \mathbb{Q}$, $|\alpha - p/q| \geqslant \frac{C}{q^{2+\beta}}$

ii) $k \in \mathbb{N}$, $k > 2\beta + 1$, $k \geqslant 3$

Then, there exists $h \in \mathrm{Diff}_+^{k-1-\beta-\varepsilon}(\mathbb{T}^1)$ s.t $f = h R_\alpha h^{-1}$

Th 2 $\forall \alpha \in \mathbb{T}^1 - \mathbb{Q}/\mathbb{Z}$, the set of smooth diffeomorphisms smoothly conjugated to R_α is dense in the set of diffeomorphisms with rotation number α.

Th3 there is no real analytic Denjoy Counterexample .

Lagrangian intersections: proof of a conjecture of V. I. Arnold.

Marc Chaperon

Theorem. Let λ denote the Liouville form of the cotangent bundle $T^*\mathbb{T}^n$, and let $j : \mathbb{T}^n \longrightarrow T^*\mathbb{T}^n$ be an embedding such that

(i) $j^*\lambda$ is exact;

(ii) there exists a smooth path j_t, $0 \leq t \leq 1$, of embeddings $\mathbb{T}^n \to T^*\mathbb{T}^n$ such that j_0 is the null 1-form O_M on $M = \mathbb{T}^n$ and $j_t^* d\lambda = 0$, $0 \leq t \leq 1$.

Then $j(\mathbb{T}^n) \cap O_{\mathbb{T}^n}(\mathbb{T}^n)$ contains at least $n+1$ points, and at least 2^n if they are transversal intersection points.

This implies the Conley-Zehnder theorem, and was conjectured by V. I. Arnold in 1965. The proof uses variational methods, and is very similar to Conley-Zehnder's.

Geodesics on noncompact manifolds

Let M be a complete noncompact Riemannian manifold. The talk gave a survey on results on the following questions: Do there (or do there not) exist geodesics of the following types on M:

Closed geodesics, bounded geodesics, oscillating geodesics, in both directions divergent geodesics?

The case of negative curvature was not treated. While I did not hesitate to make topological assumptions on M I tried to keep the (sometimes necessary) geometric assumptions down to a minimum. One of the presented results says

Theorem: Assume M is homeomorphic to the plane $\mathbb{R}^2$. Then there exists a divergent geodesic $c: \mathbb{R} \to M$.

This result has an application in Wojtkowski's work on the existence of oscillating geodesics.

V. Bangert

A Denjoy example in the Annulus

G. R. Hall

A Denjoy map on the circle $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ is a homeomorphism $f: \mathbb{T} \to \mathbb{T}$ such that there exists a point $\theta \in \mathbb{T}$ with

1) $\theta \in$ interior $W^s(\theta, f)$
2) $\omega(\theta)$ is not periodic

where $W^s(\theta, f) = \{\eta : \text{distance}(f^n(\theta), f^n(\eta)) \to 0 \text{ as } n \to \infty\}$ $\omega(\theta)$ is the usual ω-limit set and a set is periodic if every point in the set is periodic. Denjoy's Theorem states that such maps can not be C^2-diffeomorphisms.

We construct a C^∞ diffeomorphism F from the annulus $A = [0,1] \times \mathbb{T}$ into itself such that $\omega(A)$ is a circle (which is Lipschitz in the sense that it satisfies a cone condition) and $F|_{\omega(A)}$ is a Denjoy map (i.e. $\exists \theta \in \omega(A)$ with $\theta \in$ interior $W^s(\theta, F|_{\omega(A)})$ while $\omega(\theta)$ is not periodic).

On the Spectrum of Schrödinger Operators

We consider the stationary Schrödinger equation (*) $Ly = -y'' + q(x)y = \lambda y$ on the real line, where q is quasi-periodic with basic frequencies $\omega = (\omega_1, \dots, \omega_d)$. We suppose that ω is diophantine, and $q \in \mathcal{Q}^a(\omega)$, that is, q extends to an analytic function on its hull.

The spectral gaps of L are precisely the intervals of constancy of the rotation number $\alpha(\lambda)$ of q, and in a gap, $\alpha(\lambda) = \frac{1}{2}(j,\omega)$ for some $j \in \mathbb{Z}^d$ ("gap labelling"). We show: If $\mu = \frac{1}{2}(k,\omega)$ is sufficiently large and badly approximable by all other resonances $\frac{1}{2}(j,\omega)$, $j \neq k$, then the gap $[\lambda_-, \lambda_+] = \alpha^{-1}(\mu)$ is generically open, and (*) has Floquet solutions $e^{i\mu x}(\chi_1 + x\chi_2)$, $e^{i\mu x}\chi_2$, $\chi_1, \chi_2 \in \mathcal{Q}^a(\omega)$, for $\lambda = \lambda_\pm$. If $\lambda_- = \lambda_+$, then *all* solutions are of the form $e^{i\mu x}\chi$, $\chi \in \mathcal{Q}^a(\omega)$. This complements a result by Dinaburg and Sinai.

For λ in the resolvent set there always exist Floquet solutions $e^{i\mu x}\chi_1$, $e^{i\mu x}\chi_2$.

J. Moser and J. Pöschel

Asymptotics of high iterates of maps of the interval

Under certain conditions, high order iterates of a map of the interval approach a quadratic function. In this work in collaboration with P. WITTWER, advantage is taken of this observation to give a simple proof of the Feigenbaum theory of p-tupling cascades when p becomes very large, together with an explicit asymptotic estimate (exact in the leading order) of the relevant quantities.

The operator $\mathcal{N}_p$ is defined by

$$\mathcal{N}_p f(x) = \frac{1}{\lambda_f} f^p(\lambda_f x) , \quad \lambda_f = f^p(0)$$

on a set of even functions, analytic in $\{z \in \mathbb{C} : |z| < p\}$ whose restrictions to $[-1, 1]$ have the property: $\lambda_f < 0$, $J_0 = [\lambda_f, -\lambda_f]$, $J_k = f(J_{k-1})$, $k = 1, \dots, p$; $J_2 < J_3 < \dots < J_p \subset J_0$, $f''(x) < 0$ It is proved that, for sufficiently large p, $\mathcal{N}_p$ has a fixed point. g_p verifying $|g_p(z) - \psi(z)| < \text{Const}\, 4^{-p}$ for $|z| < p$, $\psi(z) = 1 - 2z^2$ $D\mathcal{N}_p$ has, at this point an unstable direction with eigenvalue δ_p, $\delta_p \sim \frac{128}{3\pi^2} 16^{2-p}$, and $\lambda_p \sim -\frac{\pi}{8} 4^{2-p}$.

J.-P. Eckmann & H. Epstein

Motion across Cantor Sets.

Invariant Cantor sets of Aubry and Mather form barriers to iterates of an area preserving twist map. The barrier can be penetrated at the gaps. A single iterate of the map interchanges an area in the plane equal to Mather's ΔW, with a given definition of curves joining the ends of the gaps. The theory has applications to particle confinement in Tokomaks. (Research with J. Meiss)

Robert MacKay and Ian Percival

~~Subordinate Šil'nikov bifurcations near some singularities of vector fields having low codimension.~~

~~On $\mathbb{R}^3$ consider a vector field with the origin as a singular point with eigenvalues 0 and $\pm i\alpha$ ($\alpha > 0$). This singularity~~

Non-integrability of the 1:1:2-resonance.

Consider the Hamiltonian system defined by the function $F = \sum_{k\geq 2} F^{(k)}$, $F^{(k)}$ homogeneous of degree k, in Birkhoff normal form ($\{F^{(2)}, F^{(k)}\} = 0$), with three degrees of freedom. Assume that $F^{(2)} = (q_1^2 + p_1^2) + (q_2^2 + p_2^2) + 2(q_3^2 + p_3^2)$. The third order term can be brought into the form

$$F^{(3)} = q_3(\beta_1(q_1^2 - p_1^2) + \beta_2(q_2^2 - p_2^2)) + 2p_3(\beta_1 q_1 p_1 + \beta_2 q_2 p_2),$$

with $\beta_1 \geqslant \beta_2 \geqslant 0$. If $\beta_1 > \beta_2 > 0$ then the system is not formally integrable. That is, if $\hat{G}$ is a formal power series which Poisson commutes with the Taylor series $\hat{F}$ of F, then $\hat{G}$ is a function of $F^{(2)}$ and $\hat{F}$. Also, if $\beta_1 = \beta_2 > 0$ then for $F^{(4)}$ in a non-void open subset of homogeneous polynomials of degree 4, the system exhibits homoclinic spirals and corresponding wild behaviour as described by Devaney.

Hans Duistermaat.

Topological instability in $\mathbb{R}^4$

We give an example of a symplectic diffeomorphism F of $\mathbb{R}^4$ which fixes the origine and is, at this point, $\mathcal{C}^\infty$-tangent to a mapping of the following form in polar coordinates: $F_0 : (r_1, \theta_1, r_2, \theta_2) \rightsquigarrow (r_1, \theta_1 + \lambda_1(r_1), r_2, \theta_2 + \lambda_2(r_2))$. F has the property that there exist a $z \neq 0$ such that $0 \in \overline{\{F^n(z), n \in \mathbb{N}\}}$.

Raphaël Douady

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Normal form for matrices near diagonal

In certain cases, a dense set of resonances occurs in a linear problem, and complicates the spectral theory. Two examples are the discrete Schrödinger operator with almost periodic potential

$$(1) \quad (H\psi)_j = \psi_{j+1} + \psi_{j-1} - 2\psi_j + \mu q_j \psi_j = \lambda \psi_j$$

q_j an almost periodic sequence $\qquad \psi \in \ell^2(\mathbb{Z})$

and the periodic Schrödinger operator with time periodic electric field,

$$(2) \quad (i\tfrac{\partial}{\partial t} - \Delta)\psi + (\vec{\mathcal{E}}(t)\cdot x + \tfrac{1}{\mu}V(x))\psi = 0 \qquad \psi \in L^2(\mathbb{R}^n)$$

$$\vec{\mathcal{E}}(t+\tau) = \vec{\mathcal{E}}(t) \qquad V(x+\gamma) = V(x) \quad \text{for } \gamma \in \Gamma \text{ a lattice}$$

For $\mu \gg 1$ it is natural to attempt to conjugate (1) and (2) to diagonal form via a unitary transformation G, whereupon one has that (1) has entirely pure point spectrum, and/or (2) admits plane wave solutions (the Brouillin basis $e^{i(k+m)x} e^{\frac{2\pi i}{\tau}nt}$ is used). A ~~K~~ modification of the Kolmogorov-Arnol'd-Moser technique can be used to address problem (2), and can be used to address the inverse spectral problem for (1), generating a class of examples with pure point spectrum.

Walter Craig

Invariant Circles for Area Preserving Twist Homeomorphisms

A twist diffeomorphism of the annulus is one which satisfies $f(x,y) = (x', y')$ if and only if $y = h_1(x,x') = \partial h(x,x')/\partial x$ and $y' = -h_2(x,x') = -\partial h(x,x')/\partial x'$, where h is an auxillary function called, in classical mechanics, a generating function. G. D. Birkhoff showed that an invariant circle for such a diffeomorphism which goes around the annulus is the graph of a Lipschitz function. Aubry, et. al., showed that for an orbit $\{\cdots (x_i, y_i), \cdots\}$ on an invariant circle, $W_{m,n}(x) = \sum_{i=m}^{n-1} h(x_i, x_{i+1})$ is a maximum for variations of x subject to x_m and x_n fixed. This is valid for all integers $m < n$. Let $\Delta W_{p,q}$ be the difference of the actions of a Birkhoff max orbit and a Birkhoff minimax orbit of type p/q. Then, for ω irrational, $\Delta W_\omega = \lim_{p/q \to \omega} \Delta W_{p,q}$ exists and there is an invariant circle of rotation number ω if and only if $\Delta W_\omega = 0$.

Subordinate Šil'nikov bifurcations near some singularities of vector fields having low codimension

On $\mathbb{R}^3$ consider a vector field with the origin as a singular point with eigenvalues 0 and $\pm i\alpha$ $(\alpha>0)$. This singularity has codimension two, so consider generic two-parameter unfoldings of it. Densely in a C^2 open subclass of such unfoldings there exist subordinate (or secondary) codimension one bifurcations of Šil'nikov.

In this Šil'nikov bifurcation there is a homoclinic orbit of a saddle point with non-real eigenvalues. This orbit induces a dynamical complexity comparable to the Smale horseshoe. The Šil'nikov bifurcation moreover is C^1 persistent.

Its occurrence as a subordinate bifurcation in our local problem, however, is a flat phenomenon: due to 'formal integrability' of the central singularity, close to it, it can be cancelled completely with an infinitely flat perturbation. Nevertheless there is some persistence of this Šil'nikov phenomenon further away from the central singularity. This proves an earlier statement of Guckenheimer.

Similar results hold in the conservative (divergence zero) case where our central singularity has codimension one.

Henk Broer
(joint work with Gert Vegter)

Rigidity and Stability of Cocycles over Dynamical Systems

A. Katok

We consider the so called over a discrete-time dynamical system $f: X \to X$
~~system~~

$$h(x) = \varphi(f(x)) - \varphi(x) \qquad (1)$$

where h is a given function from a fixed class $\mathcal{H}$ (e.g. continuous, C^2, C^∞, real-analytic) and we look for a solution φ from another fixed class Φ. Corresponding equation for a continuous time system is $h(x) = (D\varphi)(x)$ where D is the differential operator generated by the flow. We say that the space $\mathcal{H}$ is Φ-rigid (correspondingly Φ-stable) if for any $h \in \mathcal{H}$ there is a constant h_0 such that for $h - h_0$ equation (1) has a solution $\varphi \in \Phi$ (correspondingly the set of all $h \in \mathcal{H}$ for which (1) can be solved in Φ is closed). Let μ be an f-invariant measure. The space $\mathcal{H}$ is called Φ-effective (with respect to μ) if for every $h \in \mathcal{H}$ the existence of a measurable solution φ of (1) implies that $\varphi \in \Phi$

The only known example of C^∞ rigidity appears for toral rotations with not well rationally approximable rotation numbers. We conjecture that this is the only case.

We show that neither stability nor effectiveness can take place for continuous functions. This remains the case for C^∞ functions if f admits an abnormally fast periodic approximation. Known cases of stability include Anosov systems (C^1- Livshitz theory, C^∞ only for geodesic flows - Guillemin and ~~Kazhdan~~ Kazhdan) where the solvability of (1) follows from the vanishing of the sums of values over all periodic orbits, and several strictly ergodic algebraic systems including affine maps of the torus and geodesic flows on compact surfaces of constant negative curvature. In the last two cases the solvability of (1) in the C^∞ category is equivalent to vanishing of infinitely many f-invariant distributions which are not measures.

Some properties of resonant differential equations

We consider analytic germs of resonant differential equation in $\mathbb{C}^2$:

$$(1) \quad \omega = y(p+\dots)\,dx + x(q+\dots)\,dy = 0$$

where p and q are positive integers.

We prove that the classification of such equations, through analytic diffeomorphisms, is equivalent to the classification, up to analytic conjugacy, of local diffeomorphisms of the complex line $\mathbb{C}$, with linear part $z \longmapsto e^{2i\pi p/q} z$. The relation between the two problems is made by mean of the holonomy of the separatrices of (1). Moreover, we describe the "moduli space" of equivalence classes as a set of one dimensional, non Hausdorff, complex manifolds, which are the leaf space (or orbit spaces) of these diff. equation (or diffeos.).

J. Martinet (joint work with J.P. Ramis).

Analytic Invariants for germs of vector fields.

J.P. FRANÇOISE

The group of germs of analytic diffeomorphisms of $\mathbb{C}^n,0 \to \mathbb{C}^n,0$ acts on the set of analytic germs of vector fields at $\mathbb{C}^n,0$ which fix 0. We study invariants for this action which are constant functions on the orbits.

An invariant is said to be analytic if it depends analytically of the coefficients of the Taylor developments of the vector fields. We say ~~that~~ a family of analytic invariants is a complete system if it allows to separate the orbits. We define the reduced vectorfield and prove that if the reduced vector field is 0 then there can not exist complete system of analytic ~~vector~~ invariants near the vectorfield. This allows to give a new interpretation of a classical result of C. L. Siegel about the divergency of an hamiltonian vector field near its normal Birkhoff form.

Rigidity of the Centralizers of Diffeomorphisms
by Jacob Palis Jr

Several results showing that generically the centralizers of Axiom A diffeomorphisms have trivial centralizers (they are just powers of the map). This is done for C^∞ diffeomorphisms of a compact manifold.
Similar questions can be posed for volume preserving or sympletic transformations.
The results were obtained jointly with J. C. Yoccoz

Hyperbolic Invariant Sets for Twist Maps

The purpose of this talk is to observe that the arguments of Aubrey, Le Daeron, and André give examples of hyperbolic invariant Cantor sets in area preserving monotone twist maps. This gives a positive answer to Katok's question about the existence of positive Lyapunov exponents.

Daniel Goroff

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Self Similarity of Invariant Circles

1. Breakup of invariant circles for area preserving twist maps
Numerical work using the criteria of Greene and Mather suggests that given ω Diophantine, the boundary between systems with a smooth circle of rotation number ω and those with no circle of rotation number ω is a codimension 1 surface. In the case of noble frequencies, self-similarity of systems on the critical surface suggests it is the stable manifold of a certain fixed point of a renormalisation operator in the space of commuting pairs of a.p. maps

2. Boundary of Siegel domains
Numerical work indicates self similarity of the boundary of Siegel domains of arbitrary rotation number. This suggests there is an attracting 2-D set under a renormalisation operator, on which the motion is equivalent to a shift on doubly infinite continued fractions.

Robert MacKay

HOMOCLINIC BIFURCATION AND SPURIOUS SOLUTIONS OF NONLINEAR ELLIPTIC BOUNDARY VALUE PROBLEMS

It is a striking fact in the numerical analysis of nonlinear elliptic boundary value problems that numerical approximation schemes typically generate spurious solutions. These are solutions which are perfect solutions of the approximation scheme but are by no means approximate solutions to the given boundary value problems. For finite difference approximations it is possible to associate with the numerical approximation scheme a discrete-time dynamical system which is parametrized by the mesh size of the discretization. The boundary value conditions are reflected in this setting by orbits which are consistent with intersection properties of certain submanifolds. We discuss as a particular case nonlinearities which generate a hyperbolic structure for the dynamical system and observe that as we change the meshsize the homoclinic structure undergoes an odd-type bifurcation which by means of the λ-lemma gives then rise to a bifurcation of spurious solutions. For proofs we use in an essential way an underlying involution structure, a topological index (which is an intersection #) for transverse homoclinic points together with a very special model for the nonlinearity, which is close to a PL-function. By these means we obtain an infinite sequence (as the meshsize goes to zero) of bifurcation for spurious solutions.

Heinz Otto Peitgen

The Conley index and solutions of parabolic equations

Consider a smooth bounded domain $\Omega \subset \mathbb{R}^n$. Let $A(\cdot, D)$ be a uniformly strongly elliptic linear differential operator on Ω, and let $B(\cdot, D) = (B_1(\cdot, D), \ldots, B_m(\cdot, D))$ be lin. boundary operators ($2m$ ($m \geq 1$) being the order of $A(\cdot, D)$). Suppose that $(A(\cdot, D), B(\cdot, D))$ is formally self-adjoint and satisfies all assumptions of the theory of Agmon, Douglis and Nirenberg. Finally, let $f : \bar{\Omega} \times \mathbb{R} \to \mathbb{R}$ be a cont. function, locally Lipschitzian in $s \in \mathbb{R}$, unif. for $\xi \in \bar{\Omega}$. Assume that for large $|s|$, the graph of f lies strictly between two consecutive eigenvalue problems

$$(1) \begin{cases} A(x, D)\, u(x) = \lambda\, u(x) & \text{on } \Omega \\ B_i(x, D)\, u(x) = 0 & \text{on } \partial\Omega, \quad i = 1, \ldots, m. \end{cases}$$

Consider the following parabolic BVP

$$(2)\begin{cases} \frac{\partial u(t,x)}{\partial t} + A(x,D)\,u(t,x) = f(x,u(t,x)) & x\in\Omega \\ B_i(x,D)\,u(t,x) = 0 & x\in\partial\Omega,\quad i=1,\dots,m. \end{cases}$$

By using an extension of Conley's index theory to noncompact spaces we obtain the existence of equilibria of (2) and some heteroclinic orbits of (2) joining such equilibria. This is done under various hypotheses on f near the origin, comprising both the nonresonance and the resonance case.
Some of the results generalize earlier results of Amann-Zehnder who were the first to apply the Conley index to elliptic equations.

Krzysztof Rybakowski

A Morse lemma for Poisson commuting functions.

Let $(h_1, -, h_n)$ be C^∞-functions defined in a nghbd of 0 in $\mathbb{R}^{2n}$. ~~We assume~~ such that $h_j(0)=0$ and $dh_j(0)=0$ for all j. We assume that i) the h_j's are Poisson commuting with respect to the canonical 2-form $\Omega_0 = \sum dx_j \wedge dy_j$ on $\mathbb{R}^{2n}$, and ii) $h_j = \sum_{i=1}^{n} b_{ij} q_i$ $\forall j$ with $q_i = x_i^2+y_i^2$ $\forall i$ and the matrix $B=(b_{ij})$ satisfies a strong non-degeneracy condition: each $k\times k$-minor of B is non-singular.
Thm: Under the above assumptions, there exist a C^∞

differ $\phi: \mathbb{R}^{2n}, 0 \circlearrowleft$, $d\phi(0) = I$, and k functions $\psi_1, \ldots, \psi_k$, defined in a ~~neighbourhood~~ nbhd of 0 in $\mathbb{R}^n$, such that

$$h_j \circ \phi = \psi_j(q_1, \ldots, q_n) \quad \forall j.$$

Moreover, if $k = n$, we can assume ϕ to be symplectic.

Håkan Eliasson

Proof of a conjecture of V.I. Arnold

It is shown that a symplectic diffeomorphism of $T^2 = \mathbb{R}^2/\mathbb{Z}^2$ which leaves the center of gravity invariant possesses at least 3 fixed points.

C. Conley and E. Zehnder

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Spektral Theory of Ordinary Differential Operators

June 5 to June 11, 1983

Maximal accretive extensions of ordinary differential operators

Let $\tau u = -(pu')' + qu$ on $[a,b)$, $-\infty < a < b \leq \infty$, where the coefficients p, q are real-valued and satisfy the conditions:

(i) $p(x) > 0$, $1/p \in L^1_{loc}[a,b)$, (ii) $q(x) \geqslant \delta > 0$, $q \in L^1_{loc}[a,b)$.

The expression τ is therefore regular at a and at b it is assumed to be singular and to satisfy the Strong-Limit-Point and the Dirichlet conditions. In other words, for $\phi \in \Delta$, where

$$\Delta := \{\phi : \phi \text{ and } \phi^{[1]} := p\phi' \in AC_{loc}[a,b),\ \phi, \tau\phi \in L^2(a,b)\}$$

we have $\lim\limits_{x\to\infty} \phi(x)\phi^{[1]}(x) = 0$ and $p^{1/2}\phi', q^{1/2}\phi \in L^2(a,b)$. Let $T := \tau \upharpoonright \Delta$ and $T_0 = \tau \upharpoonright \Delta_0$, where

$$\Delta_0 := \{\psi \in \Delta : \psi(a) = \psi^{[1]}(a) = 0\}.$$

Then T_0 is closed and densely defined in $L^2(a,b)$ and $T_0^* = T$. Also, on setting

$$D[u,v] := \int_a^b \{pu'\bar{v}' + qu\bar{v}\} \quad , u,v \in \Delta$$

we get that $(T_0u, u) = D[u,u] \geqslant \delta \|u\|^2$, $u \in \Delta_0$ and hence T_0 is accretive. The subject of the talk is the characterization of all the maximal accretive extensions of T_0. This is achieved by means of the Phillips theory in which the problem is related to obtaining the maximal negative subspaces of the so-called "boundary space".

Theorem 1. An operator S is a maximal accretive extension of T_0 if and only if its adjoint S^* is the restriction of T to

$$D(S^*) = \{u \in \Delta : \bar{\beta}u(a) + \bar{\alpha}u^{[1]}(a) + 2D[u,\phi] = 0\}$$

for some $\alpha, \beta \in \mathbb{C}$ and $\phi \in H_1$, the completion of Δ with respect to $D^{1/2}[\cdot,\cdot]$, which satisfy $re(\alpha\bar{\beta}) + D[\phi,\phi] \leq 0$.

On using a result of R.C. Brown and A. Krall the operators S in Theorem 1 are given explicitly by

Theorem 2. For some $\lambda \in \mathbb{C}$, S is the restriction of the expression σ defined by

$$\sigma u := -\left\{(u - 2\lambda\phi)^{[1]} + 2\lambda\int_a^t q\phi\right\}' + qu$$

to the space

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$$D(S) = \{u : u - 2\lambda\phi \text{ and } (u-2\lambda\phi)^{[1]} + 2\lambda\int_a^b q\phi \in AC_{loc}[a,b),\ u, \sigma u \in L^2(a,b)$$
$$\text{and } u(a+) = \lambda\alpha,\ (u-2\lambda\phi)^{[1]}(a+) = -\lambda\beta\}$$

There is an analogous result when $\tau u = \sum_{i=0}^{n} (-1)^i [p_{n-i} u^{(i)}]^{(i)}$ with p_{n-i} real

WD Evans

Spectral Theory of Ordinary Differential Operators

This work is concerned with a spectral theory, for linear operators on Banach spaces, which applies to operators with conditionally convergent spectral expansions, as opposed to the case of self-adjoint operators on Hilbert spaces, which have unconditionally convergent spectral expansions. The notion of a well-bounded operator, due to D.R. Smart, and developed by Smart and J.R. Ringrose is used. Such an operator (on a reflexive Banach space) has a one-parameter family of projections which decompose the operator, and which permit the development of a Riemann-Stieltjes integration theory, in general only conditionally convergent.

If L is an ordinary differential operator on $L^p(-\pi, \pi)$ generated by a Birkhoff regular two-point boundary value problem, and if the spectrum is simple, then the resolvent is well-bounded. This yields the conditional convergence of the eigenfunction expansion. For even order operators (with an extra condition), strongly continuous semigroups are generated.

Further results are given on a functional calculus and abstract developments in the theory of well-bounded operators.

Harold E. Benzinger (in collaboration with Earl Berkson and Alastair Gillespie)

On the completeness of the system of eigenfunctions of irregular eigenvalue problems in $L^2[0,1]$

We consider several special cases of eigenvalue problems on the segment $[0,1]$ of the form

$$l(y,\lambda) = y^{(n)} + \sum_{\nu=1}^{n} p_\nu(x,\lambda)\, y^{(n-\nu)} = 0 \qquad (1)$$

$$U_j(y,\lambda) = \sum_{\nu=0}^{n-1} \left[\alpha_{j\nu}(\lambda)\, y^{(\nu)}(0) + \beta_{j\nu}(\lambda)\, y^{(\nu)}(1)\right] = 0 \quad , \ 1 \le j \le n \qquad (2)$$

where $p_\nu(x,\lambda) = \tilde{p}_\nu \lambda^\nu + \sum_{k=0}^{\nu-1} p_{\nu k}(x)\,\lambda^k$ and $\alpha_{j\nu}(\lambda)$ and $\beta_{j\nu}(\lambda)$ are polynomials in λ.
Extending results of Eberhard (Math. Z. 146), Shkalikov (Funkts. Anal. Prilosz. 10) and Vagabov (Sov. Mat. Dokl. 23) we show that the systems of eigen- and associated functions of certain classes of irregular eigenvalue problems of type (1) and (2) are complete in $L^2[0,1]$.

Examples show that the assumptions used in the proof cannot be weakened.

Gerhard Freiling

The essential spectrum of closed differential operators

For a closed, densely defined linear operator T let $\sigma_e(T) = \{\lambda \in \mathbb{C} : R(T-\lambda)$ is not closed$\} = \{\lambda \in \mathbb{C} : T-\lambda$ has a singular sequence$\}$. For T any closed operator generator by a differential expression $\tau y = \tau_m y^{(m)} + \tau_{m-1} y^{(m-1)} + \cdots + \tau_1 y' + \tau_0 y$ such that $A_0(\tau) \subset T \subset A_1(\tau)$ where A_0 & A_1 denote the minimal and maximal operators, respectively, we find conditions on the coefficients τ_j such that $\sigma_e(T)$ lies to the right of some straight line. This line need not be vertical. As a corollary we find conditions on τ_j so that $\sigma_e(T) = \phi$. This work is based on some joint work with W. D. Evans, R. T. Lewis & M. K. Kwong.

Anton Zettl

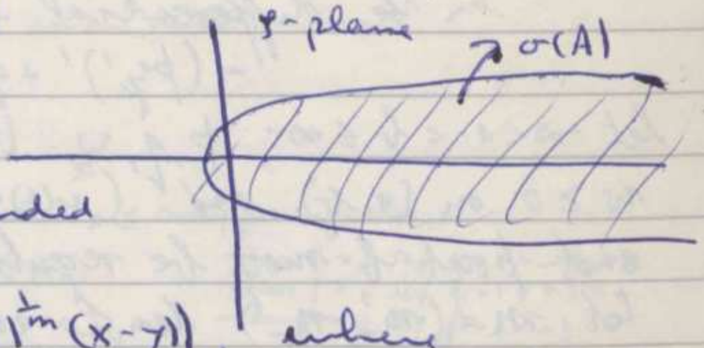
Semigroups Generated by Ordinary Differential Operators

$$\text{Let } A = (-1)^m \frac{d^m}{dx^m} + \sum_{k=0}^{m-1} b_k(x) \frac{d^k}{dx^k}$$

$$\left(b_k(x) \in L^\infty(\mathbb{R}) + L^{r_k}(\mathbb{R});\quad \frac{1}{r_k} + k < m\right)$$

be an ordinary diff. operator on the real line or on any subinterval of it. We discuss certain properties of the semigroup e^{-tA} generated by A. We start with the resolvent:

<u>Theorem</u>: The spectrum of A is contained in a "parabolic domain" about the positive real axis. If $\zeta \notin \sigma(A)$ (being outside this domain), then $(\zeta - A)^{-1}$ has a kernel bounded by



$$|K_\zeta(x,y)| \le C|\zeta|^{\frac{1}{m}} H(|\zeta|^{\frac{1}{m}}(x-y)),$$ where

$$H(x) = \begin{cases} |x|^{-t} & ; |x| \ge 1 \\ 1 & ; |x| \le 1 \end{cases},$$ where t can be chosen as any number greater than 1.

The semigroup is then

$$e^{-tA} = \frac{1}{2\pi i}\int_\Gamma (\zeta - A)^{-1} e^{-\zeta t}\, d\zeta,$$

with Γ an appropriate contour about $\sigma(A)$. We have

<u>Corollary</u>: As $t \to 0$, we have

$$\lim_{t \to 0} e^{-At} f(x) = f(x)$$

almost everywhere in x, and in all L^p spaces, $1 \le p < \infty$.

<u>Corollary</u>: The semigroup e^{-At} is infinitely smoothing in the scale of L^p spaces (i.e. instantaneously maps all L^p spaces into L^∞); but smoothing properties in the scale of Sobolev spaces depends strongly on the smoothness of the coefficients $b_k(x)$.

Mark A. Kon

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The asymptotic form of the Titchmarsh-Weyl m-coefficient

In the differential equation

$$-(py')' + qy = \lambda w y \quad \text{on } [a, b)$$

let $-\infty < a < b \le \infty$; $p, q, w : [a, b) \to R$; $p^{-1}, q, w \in L_{loc}[a, b)$; $w \ge 0$ on $[a, b)$ and $\int_a^x w(t)\,dt > 0$ $(x \in (a, b))$; $\lambda \in C_1$. The end-point b may be regular, limit-point or limit-circle. Let $m = (m_+, m_-)$ denote any T-W coefficient, i.e. $m_\pm : C'_{1\pm} \to C_{1\pm}$, $m_\pm \in H(C'_\pm)$, $m_\pm(\lambda) = \bar{m}_\mp(\bar{\lambda})$ $(\lambda \in C'_\pm)$ so that

$$\int_a^b w|\theta + m_\pm \varphi|^2 = \text{im}[m_\pm(\lambda)]/\text{im}[\lambda] \qquad (\lambda \in C'_{1\pm})$$

Here θ, φ are solutions of (*) determined on $[a, b) \times C_1$ by $\theta(a,\lambda) = 0$, $(p\theta')(a,\lambda) = 1$ $\varphi(a,\lambda) = -1$, $(p\varphi')(a,\lambda) = 0$

The asymptotic form of $m_\pm$ for large $|\lambda|$ can be determined, with λ away from the real axis, in a large number of cases. For example if in (*), additionally,

$$p \ge 0 \quad \text{and} \quad \int_a^x |w(t) - k\,p^{-1}(t)|\,dt = \underline{o}\left(\int_a^x p^{-1}(t)\,dt\right) \quad (x \to a)$$

for some $k > 0$ then

$$m_\pm(\lambda) = k^{-1/2} \cdot i/\sqrt{\lambda} \cdot \{1 + \underline{o}(1)\} \qquad (|\lambda| \to \infty)$$

provided λ is kept away from the real axis. The dominant term $i/\sqrt{\lambda}$ is the T-W coefficient for the case when $p = 1$, $w = 1$ and $q = 0$ on $[0, \infty)$ i.e. $-y'' = \lambda y$

W N Everitt

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On the eigenvalues and eigenfunctions of non definite elliptic operators

Let us consider the eigenvalues (e.v) and eigenfunctions (ef) of the following problem defined on Ω, a bounded domain in $\mathbb{R}^n$ $n \geq 1$

$$(P) \begin{cases} Lu \equiv -\Delta u + cu = \lambda g u \\ u/_{\partial\Omega} = 0 \end{cases}$$

where c and g are real bounded $^{\vee}$measurable functions; c may be negative.

g is in L^1, changes sign and is such that
$\text{meas}\,\{x \in \Omega / g(x) = 0\} = 0$
$\text{meas}\,\{x \in \Omega / g(x) > 0\} > 0$
$\text{meas}\,\{x \in \Omega / g(x) < 0\} > 0$

$D(L) = H^2(\Omega) \cap H^1_0(\Omega)$.

This problem is called non definite because L is non necessarly positive and $D_- = \{u \in D / (gu, u) < 0\} \neq \phi$.

In the one dimensional case this problem has been studied by Richardson and Mingarelli.

In such a problem, complex ev may occur.

I E.V. and E.F.

Thm 1. There exist an at most finite number of distinct, non real ev.

Thm 2. There exist an at most finite number of distinct positive ev whose (at least one for each) associated ef is in D_-.

Thm 3. There exist an infinite and countable set of positive eigenvalues $\lambda_j \xrightarrow[j \to +\infty]{} +\infty$.

II Principal eigenvalues (p.e.v).

def. λ_0 is a pev of (P) is λ_0 is an ev of (P) whose associated ef does not change sign.

when L is positive, existence of pev is well known (Manes Micheletti, Fleming, Brown...)

Let us denote by $\lambda^* = \inf\limits_{u \in D_+} \dfrac{(Lu, u)}{(gu, u)}$ where $D_+ = \{u \in D / (gu, u) > 0\}$.

prop If there exists λ_0 pev, and if $\lambda^* > -\infty$ $\Rightarrow \lambda_0 \leq \lambda^*$.

If $\lambda^* = -\infty$ then there exists no pev.

Thm 3. If $\lambda^* > -\infty$ and if $((L - \lambda^* g) u, u) \geq 0$ $\forall u \in D \Rightarrow \lambda^*$ is a pev.

Thm 4 If L is non negative and if there exists φ_1 s.t $L\varphi_1 = 0$

then if $\varphi_1 \in D^-$, λ^* is positive and is the only positive pev.

if $\varphi_1 \in D^+$ 0 is the only non negative ev.

Thm 5. Suppose that there exist $u \in D_+$ $(Lu, u) < 0$; if there exists a real number k such that $L + c + kg$ is positive then (P) admits 2 p.e.V.

J. Fleckinger (Toulouse France) (in collaboration with A Mingarelli - ottawa)

A simplified characterization of the boundary conditions which determine J-selfadjoint extensions of J-symmetric (differential) operators.

Various authors, including N.A. Zhikhar and I.W. Knowles, have looked at the problem of characterizing the boundary conditions which determine J-selfadjoint extensions of J-symmetric operators such as the minimal operator generated by

$$\tau y = \sum_{i=0}^{n} (-1)^{n-i} (p_i y^{(n-i)})^{(n-i)}$$

where each p_i is a complex-valued function. All the work to date has required the regularity field of this J-symmetric operator to be non-empty, a difficult condition to check in applications. We give here a more general but shorter and simpler method of proof, which yields the same results, but requires no such assumption. This answers several open questions concerning an example of J. B. M^c^Leod (1962), namely

$$-y''(x) - 2i\, e^{2(1+i)x} . y(x) = \lambda\, y(x) \quad ; x \in [0, \infty)$$

David Race

A property of matrices arising in the asymptotic theory of quasi-differential equations

The following theorem is proved:

<u>Theorem</u> Let the $N \times N$ matrix A have N simple eigenvalues μ_r and let A have the form $\quad A = EQ \quad$ (*)

where (a) E is the symmetric matrix with $e_{ij} = 1$ $(i+j = N+1)$; other $e_{ij} = 0$

(b) Q is a symmetric matrix with

$$q_{ij} = 0 \quad (N+3 \le i+j \le 2N)$$

and q_{ij} $(i+j = N+2)$ all non-zero. Let

$$m_r = (Ev_r)^t v_r ,$$

where v_r is an eigenvector of A with first component unity. Then

$$m_r = p'(\mu_r) \Big/ \prod_{i+j=N+2} q_{ij} \,, \qquad (**)$$

where $p(\mu)$ is the characteristic polynomial $\det(\mu I - A)$.

The significance of the quantity m_r is, first, that it arises in the diagonalization of A: $A = T\, dg(\mu_r)\, T^{-1}$. From (*), we have the orthogonality relation $(Ev_r)^t v_s = 0$ $(r \neq s)$ and hence T^{-1} has the rows $(Ev_r)^t / m_r$. Thus T^{-1} can be read off directly from (**) and the form of T. Second, the matrix A which occurs in the quasi-derivative formulation of

$$\sum_{k=0}^{n} D^k p_k D^k y + \sum_{k=0}^{m} \cancel{q_k}\left(D^{k+1} q_k D^k + D^k q_k D^{k+1}\right) y = 0 \qquad (\dagger)$$

has the form (*) and, in this case, (**) is

$$m_r = \begin{cases} p_n\, p'(\mu_r) & (n > m) \\ 2q_m\, p'(\mu_r) & (m \geqslant n) \end{cases} .$$

Now m_r occurs in the asymptotic theory of (†) where, under certain conditions on the p_k and q_k, there are solutions

$$y_r(x) \sim m_r^{-1/2}(x) \exp\left(\int_X^x \mu_j(s)\, ds\right) \qquad (\dagger\dagger)$$

generalizing the classical Liouville-Green forms for second-order equations. The appearance of the factor $m_r^{-1/2}$ in (††) was first noted by Fedorjuk in Trans Moscow Math Soc. (1966) in the case where all $q_k = 0$.

The question is raised whether there is any formula corresponding to (**) when (*) is replaced by a more general product $A = PQ$ of symmetric matrices.

M. S. P. Eastham (London).

Auxiliary Polynomials and Asymptotic Formulas for Solutions of a Singular Linear Ordinary Differential Equation.

Consider the equation (1) $\sum_{r=0}^{n} \alpha_{n-r}(x) y^{(r)} = i^{-n} \lambda x^{-b} y$, where q is a positive integer, $\alpha_0(x) \equiv 1$, $\alpha_{n-r}(x)$ is holomorphic for x in a sector S of the complex plane, $0 < x_0 \leq |x| < \infty$, and $\alpha_{n-r}(x) \sim \sum_{k=0}^{\infty} \alpha_{n-r,k}\, x^{-k}$ as $x \to \infty$. Let $I_0(\mu) = \sum_{r=0}^{n} \alpha_{n-r,0}\, \mu^r - i^{-n} \lambda \delta_{nq}$. For certain symmetric operators, $I_0(\mu) = i^{-n} H_0(i\mu)$, where H_0 has real coefficients. Let $\beta = s - it$ be a zero of $I_0(\mu)$ of multiplicity m. Let $f_1, \dots, f_m$ be part of a basis for (1) corresponding to β. Let $n^+(n^-)$ be the number of the f_j in L^2 for $\Im\lambda > 0 (< 0)$. For $q = 1$ it is known that $n^+ = n^-$ if $s \neq 0$ or if $s = 0$ and m is even, but if $s = 0$ and m is odd, then $n^+ - n^- = -1(+1)$ for $H_0^{(m)}(t) > 0 (< 0)$. The latter condition makes $n^+ - n^-$ switch sign over consecutive t of odd m. The following two questions are considered for $q \geq 2$: (a) Can $n^+ - n^- \neq 0$ and retain sign for consecutive t? (b) Is $|n^+ - n^-| \geq 2$ possible?

Richard C. Gilbert

A limit-point criterion for symmetric and J-symmetric second-order differential expressions

On $I = [a, b)$, $-\infty < a < b \leq \infty$, consider $My = R^{-1}(-(Py')' + Qy)$ and assume that $P, Q, A : I \to \mathbb{M}_s$ (complex $s \times s$ matrices) are measurable, $P(t)$ is invertible, $A(t) > 0$ a.e. on I, $P^{-1}, Q, A^2 \in L^1_{loc}(I)$, $P^+ = P$, $Q^+ = Q$, $R = A^+A$ where either $+$ denotes the complex conjugate transpose or the transpose. In the first case the minimal operator T_0 associated to M in $H = \{y : I \to \mathbb{C}^s \mid y^* A^2 y \in L^1(I)\}$ is symmetric, in the second case T_0 is J-symmetric where $Jy = A^{-1}\overline{Ay}$ is a conjugation in H.

<u>Theorem.</u> Assume that $K > 0$ and $0 \leq \beta < 1$ are constants, $W, B, U, Q_j : I \to \mathbb{M}_s$, $s_j : I \to \mathbb{R}$ $(1 \leq j \leq m)$, $\tau, v : I \to [0, \infty)$ are measurable and $\tau, B \in L^1_{loc}(I)$, $s_j, Q_j, v^2 P^* W \in AC_{loc}(I)$, $U > 0$, $U^2 = \operatorname{Re} W$, $B \geq 0$, $AB = BA$, $Q_j(t)$ are hermitian,

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$r \notin L^1(I)$, $r|(AU^+)^{-1}| \le Kv$, $rv|A_s^{-1}P^*WU^{-1}| \le K(1+\int_a^t r)$ where $A_s = (B^s A^{2(1-s)} + A^2)^{1/2}$, $|A_s^{-1}(v^2P^*W)'U^{-1}| \le Kv$, $v^2|A_s^{-1}P^*WA^+| \le K(1+\int_a^t r)$, $B - v^2 \mathrm{Re}(P^*WQ) \le KA^2 + \sum_{j=1}^{m} s_j Q_j'$, $r|s_j A_s^{-1} Q_j A_s^{-1}| \le K(1+\int_a^t r)$, $|s_j' A_s^{-1} Q_j A_s^{-1}| \le K$, $|s_j A_s^{-1} Q_j P^{-1} U^{-1}| \le Kv$. Then $d(\lambda) \le s$ for every $\lambda \in \mathbb{C}$ where $d(\lambda) = \dim\{y \in H \mid My = \lambda y\}$. If additionally T_0 is symmetric, then every real polynomial in M is limit-point.

Hubert Frentzen

Constructive method of least-squares solutions of a closed linear operator.

Suppose that $A \subset H_1 \oplus H_2$ (H_i: Hilbert space) is a closed linear operator whose range is not closed. Let $g \in H_2$ be given. We will consider the following two problems: (i) Construct a least-squares solution, u, of $A(x) = g$ in such a way that it is "stable" in a small change of g. (i) (Perturbation problem). Suppose that for $\varepsilon \geq 0$, $g_\varepsilon \in H_2$ and $A_\varepsilon \subset H_1 \oplus H_2$ is a closed operator such that $\| A - A_\varepsilon \| \le \varepsilon$ ($\|$ denotes a suitable operator norm) and $\| g - g_\varepsilon \| \le \varepsilon$ ($\|$ denotes the norm in H_2). Let x_ε be a least-squares solution of $A_\varepsilon(x) = g_\varepsilon$. Does $\{x_\varepsilon\}$ converges (in a suitable topology) to a least-squares solution of $A(x) = g$? In a contrete case, A might have been an integral operator in L_2-space, or A is a singular ordinary differential operator with $0 \in \sigma_e(A)$. We will study the problems by the Tikhonov's method of stable regularization. The main topology to be used will be the graph topology of A or that of the regularizer of A. This is a joint work with M. Z. Nashed

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A limit-point criterion for even order differential expressions.

We consider the general even order symmetric ordinary differential expression (ODE) of form

$$\sum_{j=0}^{n}(-1)^j(p_jy^{(j)})^{(j)} + i\sum_{j=0}^{n-1}(-1)^j\{(q_jy^{(j)})^{(j+1)}+(q_jy^{(j+1)})^{(j)}\} \text{ on } I:=[1,\infty)$$

where $p_j \in C^j_{\mathbb{R}}(I)$ $(j=0,\dots,n)$ and $q_j\in C^{j+1}_{\mathbb{R}}(I)$ $(j=0,\dots,n-1)$. To prove the limit-point-criterion we use the theory of relatively bounded perturbations. The perturbation theorem for ODE is:

<u>Theorem 1:</u> Let M, N ODE of form (1), with M and $M+N$ regular (i.e. the leading coefficient is positive). Assume further there exists $\beta\in\mathbb{R}$ such that for all $f\in C_0^\infty(I)$ we have $\|Nf\|_2^2\le\|Mf\|_2^2+\beta\cdot\|f\|_2^2$ then $\mathrm{def}(M+N)\le\mathrm{def}(M)$. ($\mathrm{def}(M):=\mathrm{def}(T_0(M))$.)

With this theorem we perturb the ODE $M_0y=(-1)^n(t^{\alpha_n}y^{(n)})^{(n)}+(-1)^\ell(ct^{\alpha_\ell}y^{(\ell)})^{(\ell)}$ $n,\ell\in\mathbb{N}_0$, $n\ge 2$, $n>\ell$, $c>0$, $\alpha_n,\alpha_\ell\in\mathbb{R}$, $\alpha_n-2(n-\ell)<\alpha_\ell$ on using

<u>Lemma 1</u> $\forall\varepsilon>0\ \exists K, b_{j\ell}>0\ \exists b\in I\ \forall f\in C^\infty((b,\infty))$

$$\|M_0f\|_2^2\ge(1-\varepsilon^2)\int_I t^{2\alpha_n}|f^{(2n)}|^2+\sum_{j=2\ell}^{2n-1}\int_I b_{j\ell}\,t^{\frac{1}{n-\ell}((j-2\ell)\alpha_n+(2n-j)\alpha_\ell)}|f^{(j)}|^2-K\|f\|_2^2.$$

By a theorem of Kauffman (1977) ~~the~~ M_0 is in the limit-point-case. We get a result which generalizes a result of Schultze (1981). Also the method used here is a generalization of the method which was developped by Schultze (1981). Bernhard

Powers and roots of ordinary differential operators

Let $L=\Sigma_0^N(-1)^iD^ip_iD^i$, $D=\partial/\partial x$, $p_N>0$, $p_i\geq 0$, $p_0\ge\varepsilon>0$. Suppose that each p_i is a finite sum of real multiples of real powers of x, and that $\exists j \ni$ degree $p_i-2i<$ degree p_j-2j for all $i\neq j$. We calculate the deficiency index in $L_2[1,\infty)$ of R, where R is any polynomial in L with real coefficients. If, for example, $\exists\varepsilon>0\ni(Rf,f)\ge\varepsilon(f,f)\ \forall f\in C_0^\infty(1,\infty)$, then $d(R)$ = deficiency index of R = no of linearly independent solutions to $Rf=0$ which lie in $L_2[1,\infty)$. It is known that any ~~the~~ two polynomials in L of the same degree have the same deficiency index, so we need only consider $d(L^n)$. It is not hard to show that $d(L^n)=nN$ if degree $p_j-2i\le 0$. It is known that [illegible]. $d(L)=N$.

Theorem. Suppose degree $p_j - 2j > 0$. Then

i) $d(L^n) = nN$ iff $j = 0$.

ii) if $j > 0$, $\lim_{n \to \infty} \frac{d(L^n)}{n} = N + j$.

The above ~~[illegible]~~ theorem points out that very reasonable R may not be limit-point, in the sense that boundary conditions at ∞ in addition to square-integrability may be necessary to define self-adjoint operators in $L_2[1, \infty]$. Therefore, we consider some such boundary conditions which may be expressed in a nice form, and use them to calculate the domain of the square root of the Friedrichs extension of the restriction of R to $C_0^\infty(1, \infty)$, for a very general class of positive R.

R. M. Kauffman

The structure of Green's function: a simple proof of a theorem of M. V. Keldyš.

Let E, F be $\mathcal{B}$-spaces, $\mathcal{U} \subset \mathbb{C}$ be an open subset, $\mu \in \mathcal{U}$, $T \in H(\mathcal{U}, L(E,F))$, i.e. a holomorphic operator function on $\mathcal{U}$ to $L(E,F)$. $y \in H(\mathcal{U}, E)$ is called a root function (RF) of T in μ if $y(\mu) \neq 0$ and $(Ty)(\mu) = 0$. Let $\nu(y)$ be the order of the zero of Ty in μ. A set of root functions $y_1, \dots, y_r$ is called a canonical system (CSRF) if $y_1(\mu), \dots, y_r(\mu)$ are a basis of $N(T(\mu))$ and $\nu(y_j)$ is the maximum of all $\nu(y)$ where y is any root function of T in μ such that $y(\mu) \notin \operatorname{span}\{y_1(\mu), \dots, y_{j-1}(\mu)\}$.

Thm. Assume that T^{-1} is holomorphic in $\mathcal{U} \setminus \{\mu\}$ and has a pole in μ. Let $y_1, \dots, y_r$ be a CSRF of T in μ. Set $m_j := \nu(y_j)$. We assert:

1) There exist uniquely polynomials $v_j : \mathbb{C} \to F'$ of degree $< m_j$ such that the function

$$T^{-1} - \sum_{j=1}^{r} (\lambda - \mu)^{-m_j} y_j \otimes v_j$$

is holomorphic in μ.

2) The $v_1, \dots, v_r$ are a CSAF of T^* in μ and $\nu(v_j) = m_j$.

3) The biorthogonal relationships

$$\frac{1}{\ell!}\frac{d^\ell}{d\lambda^\ell}\langle(\lambda-\mu)^{-k}T' y_i, v_j\rangle\Big|_{\lambda=\mu} = \delta_{ij}\,\delta_{m_j-k,\ell}$$

$$(k=1,2,\dots,m_i;\ \ell=0,1,\dots,m_j-1;\ i,j=1,2,\dots,r)$$

hold.

This theorem is due to Keldyš (1951, 1971), Gohberg and Sigal (1970). On this conference a simple direct proof was given, cf. a common paper with M. Möller.

Then the theorem is applied to boundary eigenvalue problems for ordinary linear differential equations. It yields the representation of the singular part of Green's function in terms of eigenvectors and associated vectors of T and T^*. The coefficients of the differential equations as well as those of the boundary conditions are allowed to depend on the parameter holomorphically. For special results cf. Langer (1939) and Cole (1961, 1964) (normal problems, i.e. poles of order 1) and Krall 1975 (diff. eq. linear in λ, bound. cond.'s independent of λ).

Reinhard Mennicken

Differential Equations and the Riemann Zeta Function

The main aim of the lecture is to present some of the consequences of a recently developed algorithm that associates a unique differential equation of the form $z'' - s\,b(x)\,z' + s^2 c(x)\,z = 0$ with certain Euler products (which include $\zeta(s)/\zeta(2s)$, $1/\zeta(s)$) of the form

$$R(s) = \prod_{n=1}^{\infty}\left(1 + \frac{a_n}{p_n^s}\right)$$

Some consequences include new formulae for the zeta function, the possibility of a proof of the prime number theorem via ODE asymptotics, and some connections with the automorphic wave equation and the theory of Hecke operators are discussed.

Ian Knowles.

On a Von Neumann Factorization For Some Selfadjoint Differential Operators

Let L, L_0 be the maximal and minimal operators induced in the space $L^2_w(a,b)$, $-\infty < a < b \le \infty$ by the quasiderivative expression $w^{-1} y^{[2n]}$ where w p_0^{-1}, $p_1 \dots p_n$ are locally integrable, $w > 0$, $p_0, \dots, p_n$ are nonnegative and additionally $p_n \ge \epsilon > 0$ on $[a,b)$. Let $\mathcal{H}$ be $\overset{n}{\underset{i=0}{X}} L^2(a,b)$ with the usual inner product. Define $\mathcal{L} : L^2_w(a,b) \to \mathcal{H}$ by $\mathcal{L}y = (p_0^{1/2} y^{(n)} \dots p_n^{1/2} y)^t$. Let $\mathcal{L}_0'$ be the "preminimal" restriction. We compute $\mathcal{L}^*$ $\mathcal{L}_0'^*$ and $\mathcal{L}_0'^{**} = \bar{\mathcal{L}}_0' := \mathcal{L}_0$. We show that $\mathcal{L}_0^* \mathcal{L}_0$ $\mathcal{L}^* \mathcal{L}$ are selfadjoint restrictions of L defined on cores of $\mathcal{L}_0$ and $\mathcal{L}$ and explicitly determine their structure. For example $D(\mathcal{L}_0^* \mathcal{L}_0) = \{ y \in D(L) : y^{[i]}(a) = 0$ $0 \le i < n$; $\int^b \sum_{i=0}^{n} p_i |y^{(n-i)}|^2 < 0$ $D(f, \bar{y})(b^-) = 0$, $\forall f \in D(\mathcal{L}_0) \}$ These operators are the Friedrich extensions representing the forms $\|\mathcal{L}_0 y\|^2$ $\|\mathcal{L} y\|^2$; from them the Dirichlet inequalities follow. The apparatus is also pertinent to the Dirichlet Index problem. — The statement that the index is minimal $\Leftrightarrow$ the "limit-pointness" of $\mathcal{L}_0$ i.e. dim $D(\mathcal{L}) / D(\mathcal{L}_0) = n$. The index can also be shown to be invariant under a class of ± bounded perturbations of the form. We also obtain that the index of a class of 4th order operators is "2" under hypotheses complementing those of T. Read

Finally (using a somewhat different apparatus) we sketch how the theory goes over to the neg. coeff. case. and discuss the "dual Dirichlet inequalities" which appear to be new – e.g., on $[1,\infty)$, $n=1$

$$\int_1^\infty |-(p_0^{1/2} z_1)' + p_1^{1/2} z_2|^2 \geq \mu_0 \, |\sup_{\substack{\{y : \int p_0|y'|^2 + p_1|y|^2 = 1\} \\ p_0 y'(a)=0}} [z_1 p_0^{1/2} y' + z_2 p_1^{1/2} y]|^2$$

and $\mu_0 = \inf \operatorname{spec} \{\mathcal{L}^* \mathcal{L}\}$

Richard C Brown

Selfadjoint subspace extensions of a symmetric subspace in Pontrjagin spaces, part I and II

Let S be a symmetric subspace in a Hilbertspace $(\mathcal{H}, (\,,\,))$. Let A be a selfadjoint subspace in a Pontrjagin space $(\mathcal{K}_\kappa, [\,,\,])$ of index κ. Then $(A, \mathcal{K})$ is a selfadjoint extension of $(S, \mathcal{H})$ if $\mathcal{H} \subset \mathcal{K}$, $[\,,\,]|_{\mathcal{H}\times\mathcal{H}} = (\,,\,)$ and $S \subset A$. It is called closely connected if $(\operatorname{span}\{\mathcal{H} \cup \{R_A(l)h \mid h \in \mathcal{H}, l \in \mathbb{C}\setminus\mathbb{R}\}\})^c = \mathcal{K}_\kappa$, where $R_A(l) = (A-l)^{-1}$, $l \in \rho(A)$ is the resolvent of A. If $(A, \mathcal{K}_\kappa)$ is a closely connected selfadjoint extension of $(S, \mathcal{H})$ then $R(l) = P R_A(l)|_{\mathcal{H}}$ is called the generalized resolvent of A for S and $T(l) = R(l)^{-1} + l$ is called the family of Štrausextensions associated with A of S. Theorems are stated which characterize all generalized resolvents for S and all Štrausextensions of S. The main feature is that: the kernel $K_R(l, \lambda) = \dfrac{R(\lambda)^* - R(l)}{\bar{\lambda} - l} - R(\lambda)^* R(l)$ has κ negative squares and $T(l)$ has κ negative squares also, ie. that for all $n \in \mathbb{N}$, $l_i \in \mathbb{C}\setminus\mathbb{R}$, $\{f_i, g_i\} \in T(l_i)$, $i = 1, .., n$ the matrix $M = (M_{ij})$ with $M_{ij} = \dfrac{(g_j, f_i) - (f_j, g_i)}{\bar{l}_i - l_j}$ has at most κ negative eigenvalues and for at least one such choice exactly κ negative eigenvalues. In the lectures we have given applications to (pairs of) differential operators. The lectures are a report on joint work with H. Langer (Dresden)

Aad Dijksma

Henk de Snoo

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Bounds for the Essential Spectrum of some Differential Operators

It is known, both for the ordinary differential expression $\tau_1 y = -(py')' + qy$ on $[a,\infty)$ and for the partial differential expression $\tau_2 y = -\Delta y + qy$ on $\mathbb{R}^n$ that if there exist $g(x)$ and $r > 0$ such that $\tau g = 0$ and $g(x) > 0$ for $|x| > r$, then $\inf \sigma_{ess}(T) \ge 0$, where T is the Friedrichs extension of the minimal operator. On the other hand, if no such g and r exist, then $\inf \sigma_{ess}(T) \le 0$. Thus we can obtain bounds on the essential spectrum by investigating the existence or nonexistence of positive solutions. We do this using the following tool.

THEOREM (a) $\tau_1 y = 0$ has a positive solution $\Leftrightarrow q = u + Q'$ where $u \ge Q^2/p$.

(b) $\tau_2 y = 0$ has a positive solution $\Leftrightarrow q = u + \operatorname{div} Q$ where $u \ge |Q|^2$.

Using this we obtain a number of results including the following extension to $\mathbb{R}^n$ of a result of Kwong and Zettl (1982)

THEOREM: Let $f(x) > 0$, $r_0 > 0$, $F(r) = \frac{1}{\omega_n}\int_{r_0 \le |x| \le r} \frac{|\nabla f|^2}{f}\,dx$, $f^*(r) = \frac{1}{\omega_n}\int_{|x|=r} f(x)\,ds$ where ω_n is the area of the unit sphere in $\mathbb{R}^n$. Fix $0 < \varepsilon < 1$ and define $E(\lambda) = \{r : -\int_{r_0 \le |x| \le r} qf\,dx - \frac{1}{4\varepsilon}F(r) \ge \lambda\}$. If $\exists \{\lambda_n\}_{n=1}^{\infty}$, $\lambda_n \to \infty$ as $n \to \infty$ and $A > 0$ such that $(\lambda_n + A)(1-\varepsilon)\int_{E(\lambda_n)} (f^*)^{-1}\,dr \ge 1$, then there is no positive solution of $\tau_2 y = 0$ on any set of the form $|x| \ge r$.

This result contains a result of Schmincke for $n \ge 3$ (Arch Rat. Mech Anal 1981) using $f(r) = r^{2-n}$ but can also be used to investigate non radial expressions.

EXAMPLE: In $\mathbb{R}^2$, $\tau y = -\Delta y + \frac{c\,x \sin r}{r^{\alpha}}\,y$, $\alpha < 2$. Here $\int_{|x| \le r} q(x)\,dx = 0$ for each $r > 0$. However the theorem above can be applied with $f(x) = 2 + \cos\theta$, where we use the usual polar coordinates in $\mathbb{R}^2$: $x = r\cos\theta$, $y = r\sin\theta$.

Thomas T. Read

Estimates of μ, the least limit point of the spectrum

Two theorems of Hinton and Lewis (1975) on sufficient conditions for $\mu = \infty$ for a real symmetric operator of order $2n$ are shown to ~~be~~ remain valid when the pointwise constraints $p_j(x)/x^{2j} \ge -C$ are weakened, a la Brinck (1959), to average constraints $\int_J (p_j(t)/t^{2j})\,dt \ge -C$ for all intervals J of length ≤ 1. A similar replacement is expected to be valid in $\mu=\infty$ criteria of Müller-Pfeifer (1977) and Read (1982).

The above two theorems are also strengthened to give upper and lower bounds $\mu \leq \mu_E$ or $\mu \geq \mu_E$ where, as in Eastham (1970), μ_E is the least limit point of a corresponding Euler Operator.

Herbert Kurss.

Odd order selfadjoint differential expressions with positive real powers as supporting coefficients

We consider the odd order differential expression

$$My = \frac{i}{2}\sum_{\varkappa=0}^{m}{}' (-1)^{\rho_\varkappa}\{(\tilde{q}_{\rho_\varkappa} y^{(\rho_\varkappa+1)})^{(\rho_\varkappa)} + (\tilde{q}_{\rho_\varkappa} y^{(\rho_\varkappa)})^{(\rho_\varkappa+1)}\}$$

on $I=[1,\infty)$, where $m \in \mathbb{N}_0$, $0 \leq \rho_0 < \rho_1 < \dots < \rho_m = n$, $\tilde{q}_{\rho_\varkappa} = a_\varkappa t^{\alpha_\varkappa}$ with $a_\varkappa > 0$, $a_m = 1$ and $\alpha_\varkappa \in \mathbb{R}$ subject to the conditions

if $m>0$: $\quad 2 > \frac{\alpha_\varkappa - \alpha_{\varkappa-1}}{\rho_\varkappa - \rho_{\varkappa-1}} > \frac{\alpha_{\varkappa+1} - \alpha_\varkappa}{\rho_{\varkappa+1} - \rho_\varkappa}$ $(\varkappa = 1, \dots, m-1)$ if $m>1$

resp. $2 > \frac{\alpha_1 - \alpha_0}{\rho_1 - \rho_0}$ if $m=1$.

This expressions can be perturbed by

$$Ny = \frac{i}{2}\sum_{j=0}^{n}{}' (-1)^j\{(q_j y^{(j+1)})^{(j)} + (q_j y^{(j)})^{(j+1)}\} + \sum_{j=0}^{n}{}' (-1)^j (p_j y^{(j)})^{(j)}$$ where:

$$q_j^{(k)} = o\left(t^{\frac{1}{\rho_\varkappa - \rho_{\varkappa-1}}\{(\rho_\varkappa - j)\alpha_{\varkappa-1} + (j-\rho_{\varkappa-1})\alpha_\varkappa\} - \frac{k}{2}\frac{\alpha_\varkappa - \alpha_{\varkappa-1}}{\rho_\varkappa - \rho_{\varkappa-1}}}\right)$$

for $\varkappa \in \{1, \dots, m\}$ such that $2\rho_{\varkappa-1}+2 \leq 2j+1-k \leq 2\rho_\varkappa + 1$ and $k=0,\dots,j+1$

$$q_j^{(k)} = o\left(t^{\alpha_0 - 2\rho_0 + 2j - k}\right)$$ for $2j+1-k = 0, \dots, 2\rho_0+1$ and $k=0,\dots,j+1$

$$p_j^{(k)} = o\left(t^{\frac{1}{\rho_\varkappa - \rho_{\varkappa-1}}\{(\rho_\varkappa - j + \frac{1}{2})\alpha_{\varkappa-1} + (j-\rho_{\varkappa-1}-\frac{1}{2})\alpha_\varkappa\} - \frac{k}{2}\frac{\alpha_\varkappa - \alpha_{\varkappa-1}}{\rho_\varkappa - \rho_{\varkappa-1}}}\right)$$

for $\varkappa \in \{1, \dots, m\}$ such that $2\rho_{\varkappa-1}+2 \leq 2j-k \leq 2\rho_\varkappa + 1$ and $k=0,\dots,j$

$$p_j^{(k)} = o\left(t^{\alpha_0 - 2\rho_0 + 2j - 1 - k}\right)$$ for $2j-k = 0, \dots, 2\rho_0+1$ and $k=0,\dots,j$

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For $M+N$ the following result is obtained:

If $\alpha_0 < 2\rho_0+1$, then $M+N$ is in the limit-point case and
$\sigma_e(T_0(M+N)) = \mathbb{R}$

If $\alpha_0 > 2\rho_0+1$, then $M+N$ has equal deficiency indices
satisfying: $n+1 \leq \operatorname{def}(M+N) \leq 2n+1-\rho_0$
and $\sigma_e(T_0(M+N)) = \emptyset$

Bernd Schultze

"SPECIAL FUNCTIONS AND DIFFERENTIAL EQUATIONS IN THE COMPLEX DOMAIN"

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Über ein neues Additionstheorem für Sphäroidfunktionen

Unter einem Additionstheorem für Sphäroidfunktionen versteht man die Entwicklung einer in gestreckt-rotationselliptischen Koordinaten separierten Lösung der Schwingungsgleichung $\Delta u + k^2 u = 0$ nach Lösungen der Schwingungsgleichung, die in einem anderen Koordinatensystem des gleichen Typs separiert sind. Hier wurde der Fall betrachtet, daß das zweite Koordinatensystem durch eine Translationsbewegung aus dem ersten hervorgeht, wobei jedoch verschiedene Exzentrizitäten erlaubt sind. Die Entwicklung lautet dann

$$S_n^{m(j)}(\xi_0;\gamma_0)\, ps_n^m(\eta_0;\gamma_0^2)\exp(im\varphi_0) = \qquad (j=1,2,3,4)$$

$$= \sum_{t=-\infty}^{+\infty}\sum_{l=|t|}^{\infty} A_l^t\, S_l^{t(j)}(\xi;\gamma)\, ps_l^t(\eta;\gamma^2)\exp(it\varphi)$$

mit Koeffizienten

$$A_l^t = \frac{2l+1}{2}\frac{(l-t)!}{(l+t)!}\frac{i^{l-2n}}{2\pi}\int_{-1}^{+1}\left[\exp(i\gamma\beta\eta)\right]J_{t-m}\left(\gamma\alpha(1-\eta^2)^{\frac{1}{2}}\right)ps_n^m(\eta;\gamma_0^2)\,ps_l^t(\eta)\,d\eta.$$

Das Resultat wird erzielt durch Fourier-Entwicklung und durch Entwicklung nach dem Biorthogonalsystem ps. Die zuletzt auftretenden Koeffizienten werden mit Hilfe eines Integralsatzes und durch Untersuchung des asymptotischen Verhaltens als Sphäroidfunktionen identifiziert.

Stefan [illegible], Konstanz

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Heun's Equation, Heun Functions, and their Applications.

This is a talk by a physicist in the language of theoretical physics. The rôle of ordinary second-order linear differential equations in physics is considered with special reference to the gap that exists in the solution of applied problems that can be reduced to the hypergeometric equation or one of its special or confluent cases and those in which fuchsian equations with four or more regular singularities appear. This is on the one hand attributed to the increased mathematical complexity (three-term vs. two-term recurrence equations; linear Fredholm integral equations vs. ~~defin~~ integral representations by definite or contour integrals; two-parameter eigenvalue problems), on the other hand to the strong fixation ~~of~~ ~~the~~ on the study of those special cases which are important in the solution of the wave equation or the Laplace equation (Lamé fctns., Mathieu fctns., spheroidal functions). This has lead to the neglect of other problems in which Heun's Equation and Heun function come up.

Three typical examples ~~are given~~ for the occurrence of Heun's equation in physics outside the abovementioned fields are given:

(i) Diffusion with drift, governed by

$$\frac{\partial C}{\partial t} = \nabla(D\nabla C + \mu C \nabla U) \tag{1}$$

[$C = C(\underline{r},t)$ = concentration; D, μ (constant) = diffusivity, mobility; $U(\underline{r})$ = potential of drift force] The transformation

$$C = F(\underline{r},t)\exp\{-\frac{\mu}{2D}U(r)\} \tag{2}$$

takes (1) into a Schrödinger-type equation. Even for the simplest problems of interest the separation of the Schrödinger equation ~~[illegible]~~ leads to Heun's equation (or confluent cases). Particularly challenging problems arise because (2) may artificially introduce an essential singularity, which ~~may~~ has to be compensated in the final solution. Examples: $U = \frac{A \sin\varphi}{r}$, $U = \frac{A \sin \varphi/2}{r^{1/2}}$ (polar coordinates)

(ii) Diffusion with inhomogeneous diffusion coefficient (e.g. $D = D(x)$ in one-dimensional diffusion in a plate). A problem arising from the diffusion of a surface layer into a semispace under the influence of irradiation is discussed.

(iii) Perturbation theory arising from solitons under an external force. These leads to a two-parameter eigenvalue problem of Heun's equation. The eigenvalue chart could be constructed on the basis of the general theory given in the book of Meixner, Schäfke, and Wolf (Lecture Notes in Mathematics 837) except from the physically particularly important case where of the four singularities $0, 1, a, \infty$ two ~~[illegible]~~ almost coincide ($a \to 1$). Here Rayleigh-Schrödinger 2nd order perturbation theory not connected to the theory used for the general problem had to be employed.

Ein „Riemann-Symbol für (konfluente) Fuchssche Dgln. 2. Ordnung.

Schreibt man die Dgl. mit den n endlichen einfachen Singularitäten, den Indizes dort, mit den Exponenten und n geeignet (!) eingeführten Konstanten auf, so ergeben sich ausserordentlich einfache und vielfältige Invarianz- und Transformationseigenschaften. Sie gestatten, viel Bekanntes und Neues durchsichtig zu erhalten.

F.W. Schäfke (Konstanz)

Power series expansions for the Stokes' multipliers of certain differential equations

We consider a "hypergeometric system", i.e. an equation of the type

(1) $$z x' = (z\Lambda + A_1)x, \quad A_1 = [a_{kj}],\ 1 \le k,j \le n,$$

where Λ is a diagonal matrix having all distinct diagonal entries $\lambda_1, \dots, \lambda_n$. Reinhardt Schäfke and, independently, W. B. Jurkat, D. A. Lutz and myself proved that the Stokes' multipliers of (1) can be found explicitly in terms of <u>characteristic constants</u> c_{jk} $(1 \le j,k \le n)$ arising from the equation

(2) $$(tI - \Lambda)\frac{d}{dt}y = (\varrho I - A_1)y$$ (with a complex ϱ)

as follows: At each λ_k, $1 \le k \le n$, there exists a unique solution of (2) of the form

(3) $$y_k(t) = \sum_0^\infty f_k(\mu)\, t^{\mu+\varrho-\lambda_k'} / \Gamma(1+\mu+\varrho-\lambda_k'),$$

and at λ_j, $1 \le j \le n$, we have

$$y_k(t) = y_j(t)\, c_{jk} + \mathrm{reg}(t-\lambda_j).$$

The constants c_{jk} are entire functions in the off-diagonal terms of A_1; for example c_{21} may be expanded as

$$c_{21}/\Gamma(1+\varrho-\lambda_2') = a_{21}\, c(\lambda_2'; \varrho)$$
$$+ \sum_{p=1}^{\infty} \Big\{ \sum_{k_1,\dots,k_p=1}^{n} a(2,k_p,\dots,k_1,1)\, c(\lambda_{k_1},\dots,\lambda_{k_p}; \lambda_{k_1}',\dots,\lambda_{k_p}', \lambda_2'; \varrho)$$

(in case we normalize $\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_1' = 0$), where $a(k,k_p,\dots,k_1,j) = a_{kk_p} \cdot a_{k_pk_{p-1}} \cdots a_{k_1j}$ in case $k \ne k_p$, $k_p \ne k_{p-1}, \dots, k_1 \ne j$, and $= 0$ otherwise, and the functions $c(\lambda_1,\dots,\lambda_{p-1}; \lambda_1',\dots,\lambda_p'; \varrho)$ (for $p \ge 1$) arise from the characteristic constants of a system (1) but with A_1, aside from the diagonal elements, having only ones in the subdiagonal and zeros

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elsewhere. For $|\lambda_j| > 1$, $1 \le j \le n$, these functions are equal to

$$e^{i\pi(\lambda'_p - \varrho)}\, \Gamma(\lambda'_p - \varrho)\, b(\lambda_{p-1}^{-1}, \dots, \lambda_1^{-1}; \lambda'_p + p, \dots, \lambda'_1 + 1)$$

with b recursively given by the formulas

$$(4) \begin{cases} b(\beta_1) = 1/\Gamma(\beta_1),\ b(w_1; \beta_1, \beta_2) = \sum_0^\infty (\beta_2)_\nu\, b(\nu+\beta_1)\, w_1^{\nu+1}; \\ b(w_1, \dots, w_j; \beta_1, \dots, \beta_{j+1}) = \sum_0^\infty (\beta_{j+1})_\nu\, b(w_1, \dots, w_{j-1}; \beta_1 + \nu, \dots, \beta_j + \nu)\, w_j^{\nu+1} \end{cases}$$

(for $j \ge 2$).

I am glad that it was brought to my attention by people familiar with special functions that the functions given by (4) should be related to some generalized hypergeometric functions!

Werner Balser

Global Simplification of Singularly Perturbed Linear Ordinary Differential Equations in the Complex Domain.

First we surveyed globally block-diagonalization theorems of Braaksma [SIAM J. Math. Anal. 1971] and Gingold and Hsieh [Funk. Ekv. 1983].

Next consider an n by n differential system

$$(1) \qquad \varepsilon^h Y' = \{D(x,\varepsilon) + \varepsilon^\theta R(x,\varepsilon)\} Y, \qquad ' = \frac{d}{dx}$$

for $x \in \mathcal{D}$: simply-connected domain, $\varepsilon \in \mathcal{S}_c = \{\varepsilon \mid 0 < |\varepsilon| < c, |\arg \varepsilon| \le \frac{\delta\pi}{2h}, 0 < \delta < 1\}$

$D(x,\varepsilon) = \operatorname{diag}\{d_1(x,\varepsilon), \cdots, d_n(x,\varepsilon)\}$, $R(x,\varepsilon) = (r_{jk}(x,\varepsilon))$, $j, k = 1, 2, \cdots, n$, $r_{jj} \equiv 0$.

Def. The system (1) is said to be a globally almost diagonal system (G. A. D. S) in $\mathcal{D}$ if there exists an n by n matrix $P(x,\varepsilon) \in C^1(\mathcal{D} \times \mathcal{S}_{c_1})$ $(0 < c_1 \le c)$ $\ni \lim \|P(x,\varepsilon)\| = 0$, as $\varepsilon \to 0$ in $\mathcal{S}_{c_1}$, uniformly for $x \in \mathcal{D}$ and

$$Y = (I + P(x,\varepsilon)) Z$$

reduces (1) to

$$\varepsilon^h Z' = D(x,\varepsilon) Z.$$

i.e. (1) has a fundamental matrix of the form

$$Y = (I + P) \exp\left\{\varepsilon^{-h} \int^x D(t,\varepsilon)\, dt\right\}.$$

Assumption 1. (i) $D(x,\varepsilon)$ and $R(x,\varepsilon)$ are holomorphic and bounded in $\mathcal{D} \times \mathcal{S}_c$; (ii) There are two fixed points on $\partial\mathcal{D}$ $\ni$ for $\forall x \in \mathcal{D}$, $\exists\, \Gamma_x$ associated to x connecting x to both a and b, and

$$D_{jk}(x,t,\varepsilon) := \operatorname{Re}\left\{\varepsilon^{-h} \int_t^x [d_j(s,\varepsilon) - d_k(s,\varepsilon)]\, ds\right\}$$

are defined for all $t \in \Gamma_x$, $j, k = 1, 2, \cdots, n$;

(iii) For every pair j, k ($j \neq k$; $j, k = 1, 2, \cdots, n$), $\exists$ two numbers m_{jk} and $\hat{m}_{jk}$ $\ni$ either $\hat{m}_{jk} \le D_{jk}(x, t, \varepsilon) \le m_{jk}$ for $\forall t, x \in \mathcal{D}$, $\varepsilon \in S_c$ or if $D_{jk}(x, t, \varepsilon)$ is unbounded, then

$$D_{jk}(x, t, \varepsilon) \le m_{jk} \text{ or } \hat{m}_{jk} \le D_{jk}(x, t, \varepsilon) \text{ for } \forall x, t \in \mathcal{D}, \ \varepsilon \in S_c.$$

Let

$$r(\varepsilon) = \sup_{\Gamma_x} \int_{\Gamma_x} \| R(t, \varepsilon) \| dt$$

We proved the following

Thm 1. Assume that (1) Assumption I is satisfied; (2) $\theta > \frac{h}{2}$; (3) $\exists\ \mu > 0 \ni \quad |d_j(x, \varepsilon) - d_k(x, \varepsilon)| \ge \mu$, $j, k = 1, 2, \cdots, n$, $j \neq k$; for $\forall x \in \mathcal{D}$, $\varepsilon \in S_c$. Then (1) is a G.A.D.S and $\|P(x, \varepsilon)\| = O(\varepsilon^{\sigma})$ uniformly on $\bar{\mathcal{D}}$ as $\varepsilon \to 0$ in S_{c_1} ($0 < c_1 \le c$) with $\sigma = \min\{\theta, 2\theta - h\}$

Thm 2. Assume that (1) Assumption I holds; (2) $\theta = h$; (3) $r(\varepsilon) = o(1)$ as $\varepsilon \to 0$ in S_c. Then (1) is a G.A.D.S. where $\|P(x, \varepsilon)\| = O(r(\varepsilon))$ uniformly on $\bar{\mathcal{D}}$ as $\varepsilon \to 0$ in S_{c_1}, ($0 < c_1 \le c$).

The method used in proving these theorems can be applied to study uniform asymptotic integration and deficiency indices problems.

Po-Fang Hsieh (Kalamazoo, MI. U.S.A.)

(joint with Harry Gingold, Morgantown, W. Va.)

Connection Problems for Differential Equations of Rank Two or More.

The lateral connection problem for systems of linear differential equations of the form

$$x' = \left(z^{r-1}\sum_{0}^{\infty} A_\nu z^{-\nu}\right)x$$

(where A_0 has all distinct eigenvalues) can be transfered to a connection problem for a system of some associated functions. These functions by given by convergent local expansions that are constructed using the formal solutions and are, loosely speaking, the inverse Laplace Transforms of the formal solutions.

When $r=1$, Balser, Jurkat, and Lutz (S.I.A.M. Jour. Math. Anal. (1981)) have shown that the associated functions have only regular-type singularities in the finite complex plane. When $r \geq 2$ the singularities can be of ~~the~~ an irregular type, however, their singularities are of a relatively simple nature. Namely, they are convolutions of functions having regular-type singularities with ones that are Laplace transforms of certain exponential polynomials. In this talk, it was explained how the associated functions are constructed, for example, when $r=2$ and in the simplest case when the singularities are of the regular-type. The complete discussion can be found in Balser-Jurkat-Lutz, J. Math. Anal. Appl (1982).

Donald A. Lutz
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The eigenvalues of Mathieu's and the spheroidal wave equation for complex values of their parameters

Mathieu's equation, and the spheroidal wave equation for some fixed angular wavenumber, provide two instances of problems in which the eigenvalue λ depends on some second parameter, which we shall label as q. Using solutions of the Liouville-Green type, that are constructed on the assumption that $|\lambda|$ is large, approximate relations between λ and q are obtained. These relations appear to be valid uniformly throughout the complex q-plane. They match the known expansions for both large and small values of q, and also predict correctly the doubly infinite array of locations, in the complex q-plane, of square-root branch points at which specific pairs of eigenvalues (and their associated eigenfunctions) coalesce.

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Connection Problems for logarithmic Solutions

We shall show two formulas relating to connection problems, when differential equations have logarithmic solutions.

First, we consider the Birkhoff system

$$(*) \quad tX' = (A_0 + A_1 t + \cdots + A_q t^q)\, X$$

Let $X(t) = (x_0(t), x_1(t), \dots, x_\gamma(t)) = \sum_{m=0}^{\infty} g(m, \rho_0)\, t^{m+\rho_0+J}$ be an n by $(\gamma+1)$ matrix solution of $(*)$ near the regular singular point $t=0$. Here J is a $(\gamma+1)$ by $(\gamma+1)$ shifting matrix, whence t^J represents logarithmic functions. And let $Y(t)$ denote an n by n formal matrix solution at another singular point $t=\infty$. Then we have the central connection formula

$$X(t) \sim Y(t)\, T_{S_N} \quad \text{as } t \to \infty \quad \text{in } S_N$$

where the sectorial domains S_N $(N \in \mathbb{Z})$ cover the whole Riemann surface and each matrix of Stokes multipliers T_{S_N} consists of n vectors for $k=1,2,\dots,n$ chosen according to the sector S_N from the $(\gamma+1)$-dimensional vectors $(T^k_{l0}(\rho_0), T^k_{l1}(\rho_0), \dots, T^k_{l\gamma}(\rho_0))\exp(2\pi i p(J+\rho_0-\mu_k))$ $(l=1,2,\dots,q$, $p \in \mathbb{Z})$. The μ_k are characteristic exponents of formal solutions.

As to the Stokes multipliers, we obtain the following

<u>Theorem 1</u>

$$T^k_{li}(\rho_0) = \frac{1}{i!}\frac{\partial^i}{\partial \rho_0^i}\left[T^k_{l0}(\rho_0)\right] \qquad (i=1,2,\dots,\gamma)$$

This theorem implies that the Stokes multipliers for logarithmic solutions $x_i(t)$ $(i=1,2,\dots,\gamma)$ can be given by derivatives of those of a non-logarithmic solution $x_0(t)$ with respect to the characteristic exponent ρ_0. We may say that the above theorem is just "the Frobenius theorem in the large".

Second, we consider the hypergeometric system

$$(**) \quad (t-B)\, X' = AX$$

where $B = \operatorname{diag}(\underbrace{\lambda_1, \dots, \lambda_1}_{n_1}, \underbrace{\lambda_2, \dots, \lambda_2}_{n_2}, \dots, \underbrace{\lambda_p \cdots \lambda_p}_{n_p})$ and A is a constant matrix.

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Since this differential equation is invariant under the transformation $X = CY$, C being a block-diagonal matrix $C = \mathrm{diag}(C_1 \oplus C_2 \oplus \cdots \oplus C_p)$, we may assume without loss of generality that the block diagonal elements A_{ii} $(i=1,2,..,p)$ of $A = (A_{ij})$ are Jordan canonical matrices. For simplicity, we here assume that $A_{ii} = \rho_i + J_i^*$, J_i^* being a transposed matrix of a shifting matrix J_i, and moreover assume that A is similar to $\mathrm{diag}(\nu_1, \nu_2, \ldots, \nu_m)$. Then we have p logarithmic matrix solutions

$$X_i(t) = \sum_{m=0}^{\infty} G_i(m)\,(t-\lambda_i)^{m+\rho_i+J_i} \qquad (i=1,2,..,p).$$

For such a set of solutions, which must be suitably determined, we have

Theorem 2

$$\det[X_1(t), X_2(t), \ldots X_p(t)] = \frac{\prod_{i=1}^{p} (-1)^{\frac{n_i(n_i-1)}{2}} (\Gamma(\rho_i+1))^{n_i}}{\prod_{k=1}^{n} \Gamma(\nu_k+1)} \prod_{i=1}^{p} (t-\lambda_i)^{n_i \rho_i}$$

This formula, which we call the extended Gauß-Kummer's formula, plays an important role in the calculation of connection coefficients and monodromy groups.

Mitsuhiko Kohno
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Spectra of Jacobi matrices and orthogonal polynomials

I will ~~pre~~ discuss the relationship between spectra of positive definite Jacobi matrices and orthogonal polynomials. I will present a method to obtain the spectral measure of a bounded Jacobi matrix from the asymptotic properties of the associated orthogonal polynomials. These polynomials are orthogonal with respect to the spectral measure. Applications to specific problems will be mentioned. These problems are from the areas of: Birth and death processes and Queueing theory.

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On the reduction of the pole order of a linear differential equation at a singular point.

We consider the system of n linear differential equations

$$(*) \quad (Dy)(x) := x y'(x) - A(x) y(x) = 0 \ , \quad A(x) = x^{-s} \sum_{v=0}^{\infty} x^v B_v \ ,$$

where the power series either is convergent in a neighbourhood of $x=0$ or merely a formal power series, and $s := s(A) \in \mathbb{N}$. We want to determine $\ell(A) = \min \{ s(\tilde{A}) : \tilde{A} = T^{-1} A T - x T^{-1} T' \ , T$ meromorphic at $x=0$, $\det T(x) \not\equiv 0 \}$. We use the quantities ϱ_{r+1} $(r \in \{0,1,\ldots,s\})$ introduced by Gérard and Levelt in 1973, which are invariant with respect to meromorphic transformations of $(*)$. The question is, how to determine the ϱ_{r+1} practically. For this, we introduce certain matrices $A^{[r]}_{n}(\lambda, \kappa)$ which are derived from the Laurent series expansion of $A(x)$ and contain two linear complex parameters λ, κ. Let $d^{(r)}$ be the maximum defect number of all $A^{[r]}_{n}$. Then the main result is that $\varrho_{r+1} = n(s-r) - d^{(r)}$ and, consequently

$$\ell(A) \leq r \quad \text{iff} \quad d^{(r)} = n(s-r) \ .$$

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Integral representations for products of Lamé functions by use of fundamental solutions

We show that $v(s,t) = Q_\nu (k^2 \, \mathrm{sn}\, s \, \mathrm{sn}\, s_0 \, \mathrm{sn}\, it \, \mathrm{sn}\, it_0 - {k^2}/{k'^2} \, \mathrm{cn}\, s \, \mathrm{cn}\, s_0 \, \mathrm{cn}\, it \, \mathrm{cn}\, it_0 + {1}/{k'^2} \, \mathrm{dn}\, s \, \mathrm{dn}\, s_0 \, \mathrm{dn}\, it \, \mathrm{dn}\, it_0)$ is the fundamental solution of the elliptic partial differential equation $\frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial t^2} + \nu(\nu+1) k^2 (\mathrm{sn}^2 it - \mathrm{sn}^2 s) u = 0$ when Q_ν is Legendre's function of the second kind, and sn, cn, dn are Jacobi's functions with respect to the modulus k. This observation leads to a general representation formula for products of Lamé functions. Our result is

$$2\pi i\, E(s_0)\, F(it_0)\, E(it) = W[E,F] \int_{-2K}^{2K} v(s,t,s_0,t_0)\, E(s)\, ds$$

where E and F are Lamé functions of the first and second kind, respectively, and $W[E,F]$ is the Wronskian of E and F.
Especially, we generalize and improve an integral representation for external ellipsoidal harmonics mentioned by Erdélyi, Magnus, Oberhettinger [1955]. We also obtain expressions for some multipliers appearing in the representation formulae which are considerably simpler than those given by Shail [1980]

H. Volkmer
Essen

Coexistence of singly-periodic solutions of a doubly-periodic equation

The equation considered is that of LAMÉ: $\frac{d^2w}{dz^2} + (h - \nu(\nu+1)k^2 sn^2(z,k))\, w = 0$ (*)
whose coefficient has periods $2K, 2iK'$. The standard theory of periodic d.e. shows that for any ν, there are eigenvalues $h(\nu)$ such that (*) has solutions of period $4K$, in the usual 4 classes characterised by (i) being even or odd about $z=0$ and (ii) having $2K$ as period or anti-period.

Erdélyi's transformation $z = K+iK'-i\zeta$ shows also that for any ν, there are eigenvalues $h'(\nu)$ giving solutions of imaginary period $4iK'$; these fall in the 4 classes characterised by being (i) even or odd about $K+iK'$ and (ii) having $2iK'$ as period or anti-period.

It is shown, by consideration of appropriate curves in the (ν,h) plane, that a real-periodic solution of any of the 4 classes can ~~coincide~~ coexist (for appropriate value-pairs (ν,h)) with an imaginary-periodic solution of any of the 4 classes. Of the 16 combinations, there are 8 for which the real-periodic and imaginary-periodic solutions coincide, thus forming a doubly-periodic solution. This happens only for integer n and the solutions are the Lamé polynomials. In the other 8 combinations the real-periodic solution and the imaginary-periodic solution are independent; indeed, one is even, the other odd, about $z=K$.

The values of r giving rise to the second situation are not integral and do not appear to have any particular form. They can be found as simultaneous solutions of two transcendental equations, each involving r and h.

As an example, the lowest value of r producing coexistence of a real period $2K$ solution, even about $z=0$, with an imaginary period $4iK'$, even about $K+iK'$, was computed to be $r=0.7044$.

Felix M. Arscott

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Formal and convergent solutions of singular equations

G. Bengel – R. Gérard.

Let $\hat{\sigma} = A[[x_1, x_2, \cdots x_n]] = A[[x]]$ the ring of formal power series with coefficients in a valued (complete) ring A. $\hat{\sigma} = A\{x\}$ the ring of convergent series.

$$P : \hat{\sigma}^q \longrightarrow \hat{\sigma}^q$$

$$u \longmapsto Pu = \sum_{m=0}^{+\infty} \left(\sum_{\substack{k_0 \leq k \leq m \\ |k| \geq 0}} P_k(m-k) u_{m-k} \right) x^m$$

where $k_0 \in \mathbb{Z}^n$, $P_k(l)$ a matrix valued function of l, $P(\sigma^q) \subset \sigma^q$. We first give several examples of such operators including singular partial differential operators. We then give conditions for P such that $Pu = f$ has an analytic solution and solve non linear equations

$$Pu = F(x,u), \quad F \text{ analytic}$$

near the origin of $\mathbb{C}^n \times \mathbb{C}^q$.

For applications we treat also the cases with parameters and give a proof of a theorem of S. Kaplan.

R. Gérard (University of Strasbourg)

Generalization of Phragmén-Lindelöf Theorem

Theorem (Ching-her Lin):

"Let $S_j = \{\varepsilon\,;\ a_j < \arg\varepsilon < b_j,\ \ 0<|\varepsilon|<\rho_0\}$, $j = 1, 2, \cdots, \nu$, be sectors in the ε-plane such that

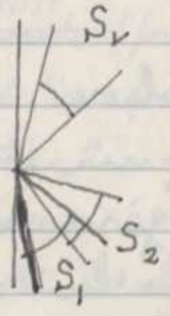
$S_1 \cup S_2 \cup \cdots \cup S_\nu = \{\varepsilon;\ |\arg\varepsilon| < \frac{\pi}{2\alpha},\ \ 0<|\varepsilon|<\rho_0\}$, $\alpha > 1$,

$S_1, \cdots, S_\nu$ intersect consequtively but no three of them intersect.

Let $\phi_1(\varepsilon), \cdots, \phi_\nu(\varepsilon)$ be functions of ε. Assume that

(1) ϕ_j is holomorphic in S_j and continuous on $\overline{S_j} - \{0\}$;

(2) $|\phi_j(\varepsilon)| \leqq A \exp(c/|\varepsilon|)$ in S_j for some $A>0$ and $c>0$;



$|\phi_{j+1}(\varepsilon) - \phi_j(\varepsilon)| \leqq M_0$ in $S_j \cap S_{j+1}$;

$|\phi_1(\varepsilon)| \leqq M_0$ for $\arg\varepsilon = -\frac{\pi}{2\alpha}$;

$|\phi_\nu(\varepsilon)| \leqq M_0$ for $\arg\varepsilon = \frac{\pi}{2\alpha}$; for some $M_0 > 0$.

Then $|\phi_j(\varepsilon)| \leqq M$ in S_j, $j = 1, \cdots, \nu$, for some $M>0$.

Yasutaka Sibuya (Univ. of Minn.)

Gevrey classes in the theory of difference equations

G.K. Immink

~~I consider~~ My talk is concerned with n-dimensional systems of linear difference equations of the form: $y(s+1) - A(s)\,y(s) = f(s)$, where both A and A^{-1} are meromorphic at ∞ and f is holomorphic at ∞. In my PhD thesis I have proved some existence theorems for holomorphic solutions of such equations with a ~~given~~ prescribed asymptotic behaviour in appropriate sectors of the complex plane. In particular I have looked for solutions belonging to certain Gevrey-classes. One of the main results is the following:

Let $H_j(R) = \{ s \in \mathbb{C} : -\frac{\pi}{2} + j\pi < \arg s < \frac{\pi}{2} + j\pi \,;\ |s| \geq R \}$, $R > 0$, $j = 0,1,2,3$.
Suppose that neither 0 nor π is a 'singular direction' of the (homogeneous) equation: ~~$y(s+1) - A(s) = y$~~ $y(s+1) - A(s)\,y(s) = 0$.
If R is sufficiently large, then there exist matrix functions F_j, holomorphic in $H_j(R)$ $(j = 0,1,2,3)$ such that:

1. F_j and F_j^{-1} belong to a certain Gevrey class of holomorphic matrix functions with an asymptotic expansion.
2. $F_j(s+1)^{-1} A(s) F_j(s) = A^c(s)$ in $H_j(R)$, where $A^c(s)$ denotes a canonical (or normal) form of $A(s)$.

Connection coefficients between a regular and an irregular singular point of a linear ordinary differential equation

I consider equations of the form

$$y'(z) = \left(\frac{1}{z} A_0 + \frac{1}{(z-1)^2} B + \frac{1}{z-1} A_1 + G(z) \right) y(z)$$

where A_0, A_1, B are $n \times n$ matrizes, $G(z)$ holomorphic in $|z| < r$, $r > 1$.

0 is a regular singularity, we have solutions of the form

$$y_0(z) = z^{\kappa} \sum_{k=0}^{\infty} z^k d_k$$

1 is an irregular singular point. If $B = \mathrm{diag}(\lambda_1, \dots, \lambda_n)$ with

$$\lambda_j - \lambda_k \notin \mathbb{R}$$

we have uniquely determined solutions $y_j(z)$, defined in

$$H = \{ z \mid |z-1| \text{small}, |\arg(1-z)| < \tfrac{\pi}{2} \}$$

and that

$$y_j(z) \sim e^{\lambda_j \frac{1}{1-z}} (1-z)^{\kappa_j} \Big(\overset{j^{\text{th}}\text{ unitvector}}{\overset{\downarrow}{e_j}} + (1-z) c_{j1} + \dots \Big) \qquad \underset{j=1,\dots,n}{(H \ni z \to 1)}$$

Then the connection problem arises

$$y_0(z) = \sum_{j=1}^{n} \gamma_j y_j(z) \qquad (z \in H, |\arg z| < \tfrac{\pi}{2})$$

The connection coefficients γ_j ~~the~~ are computed by a limit-formula from the quantities d_k.

R. Schäfke

Stability and Identification of formal invariants at an irregular singular point

This lecture presented results of a paper of D. Lutz and me. We consider a general equation with pole at 0

$$\text{(D)} \quad z^{s+1} y' = A(z) y \qquad A(z) = \sum_{\nu=0}^{\infty} A_\nu z^\nu \qquad A_\nu \ n \times n \text{ matrices}$$

It is known that it has a formal fundamental solution

$$H(z) = F(z^{1/p}) z^J \exp(Q(z^{-1/p}))$$

where $Q(z^{-1/p})$ is a diagonal matrix, whose entries are polynomials in $z^{-1/p}$ (p integer), J is a constant matrix and $F(z^{1/p})$ is a formal meromorphic transformation.

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The quantities $Q(z^{-1/p})$, J are a set of invariants of (1) with respect to formal meromorphic transformations.
The aim of our paper ist to find explicit relations between $Q(z^{-1/p})$ and the coefficients A_r and to find out, how stable $Q(z^{-1/p})$ is with respect to ~~transformations~~ perturbations

$$z^{s+1}y'(z) = (A(z) + z^N B(z))y \qquad N \text{ integer}, \ B(z) \text{ arbitrary analytic}$$

R. Schäfke

Doubly-Periodic Floquet Theory

In this talk we develop a theory for differential equations with doubly-periodic coefficients analogous to the classic Floquet theory for differential equations with singly-periodic coefficients. Unlike the classical theory the role of the exponent ν of the differential equation is fundamental. If ν takes integral values the analogous theory is well known and goes back to the work of Hermite (1877). When ν is rational the theory depends essentially on whether a certain number theoretic conjecture of Arscott and Wright (1969) is true. This talk resolves the conjecture and brings the doubly-periodic Floquet theory to some degree of completion.

B.D. Sleeman.

University of Dundee
Scotland

Asymptotics and deficiency indices for certain pairs of differential expressions

Let $M = \prod_1^n (xD - a_j) - \mu x^\alpha \prod_1^m (xD - b_j)$, $D = \frac{d}{dx}$, $0 \le m < n$.
Solutions of $My = 0$ have been studied by Meijer, Kohno and Ohkohchi who solved the connection problem in the general case except for certain singular cases. We give an outline of the

method of Meijer and show how the restrictions can be removed.

Next we consider perturbations of M with the property that their solutions have the same behavior as $x \to +\infty$ as those of M. This is used to determine deficiency indices related to $L_1 y = \lambda L_2 y$ where L_1, L_2 are formally symmetric, one them is positive and L_1, L_2 are perturbations of operators $x^\gamma M_1$, $x^\delta M_2$ where M_1 and M_2 are of the same type as M.

B.L.J. Braaksma (Groningen)

The solution of 2nd order ODEs with an irregular singular point of rank 1 by series in terms of confluent hypergeometric functions

In the theory of Special Functions of Mathematical Physics expansions of "higher" special functions (as Mathieu-functions, Spheroidal Functions, Lamé-Functions) in terms of "simpler" special functions (as hypergeometric and confluent hypergeometric functions) are well known and often applied. The aim of this lecture was to show, how such series, -especially in terms of confluent hypergeometric functions-, can be applied even for the global representation of solutions of a quite general ODE with an irregular singular point at ∞ of rank 1. These series describe the full analytic behavior of the solutions they represent, that is: the transformation behavior when turning around ∞ as well as the asymptotic behavior when going to ∞ in appropriate sectors. Also, the closed connection of the presented results to a paper of W. Jurkat, D. Lutz and A. Peyerimhoff in the Journ. of Math. Anal. and Appl. Vol. 53, 1976 was pointed out.

TH. KURTH & D. SCHMIDT
(Konstanz) (Essen)

Numerische Behandlung von Eigenwertaufgaben.

12. – 18. Juni 1983.

The Construction of convergent Intermediate Hamiltonians for Multi-electron systems

A convergent variant of the Fox construction of intermediate problems utilizing infinite-rank perturbations are presented for multiparticle Hamiltonians having potentials in $L^2 + [L^\infty]_\varepsilon$. Besides convergence criteria, some observations are offered concerning computational strategies and extensions to Rollnik class potentials.

Christopher Beattie
University of Arizona, USA

An Elementary Proof of the Monotony of the Temple Quotients

An elementary - rather straightforward - proof of monotony of the sequence of the Temple quotients is given

Karel Rektorys,
Technical University Prague,
Czechoslovakia

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Zur Anwendung der Theorie positiver Operatoren auf Eigenwertaufgaben mit gewöhnlichen Differentialgleichungen.

Auf eine Klasse allgemeiner Eigenwertaufgaben der Form $My = \lambda Ny$ mit gewöhnlichen Differentialoperatoren M und N der Ordnung 4 bzw. 2 wird das Reduktionsverfahren von J. Schröder angewendet und dazu benutzt, den Operator $M^{-1}N$ als positiven Operator nachzuweisen. Mit der Theorie positiver Operatoren ergibt sich dann die Existenz einer nichtnegativen Eigenfunktion, die zu dem kleinsten positiven Eigenwert des Ausgangsproblems gehört. Zur Einschließung dieses Eigenwerts wird der Quotientensatz für positive Operatoren benutzt und mit einem von W. Held angegebenen Quotienteneinschließungssatz verglichen.

Peter P. Klein
Techn. Universität Clausthal

Perturbation theorems for generalized eigenvalue problems

Given a pair $Z = (A, B)$ of complex $n \times n$ - matrices, $(\alpha, \beta) \neq (0,0)$ is called eigenvalue of Z if there is $x \neq 0$ with $\beta Ax = \alpha Bx$. The influence of a perturbation of Z on the eigenvalues is studied. Considering two pairs Z, W as points of the Grassmann manifold $G_{n,2n}$ and its eigenvalues as points in $G_{1,2}$, the projective complex plane, the distance of the spectra, measured in the chordal metric in $G_{1,2}$, is bounded by some distance of the matrix pairs in $G_{n,2n}$. Analogs of the Bauer-Fike theorem, Henrici's theorem and the Hoffman-Wielandt theorem can be obtained, from which the classical results ($B = I$, $\beta = 1$) can be derived via a limiting process. The results were obtained jointly with J. Sun (Lin. Algebra Appl. 48, 341-357 (1982)).

L. Elsner
Universität Bielefeld

Eine Variante des Lanczos-Verfahrens

Die Berechnung von Eigenfrequenzen und Eigenschwingungsformen von Systemen mit Hilfe der Methode der finiten Elemente führt auf allgemeine Eigenwertaufgaben $Ax = \lambda Bx$ mit symmetrischen, schwach besetzten Matrizen A und B hoher Ordnung. Das Lanczos-Verfahren scheint in seiner klassischen Form Schwächen aufzuweisen, ist aber nach geeigneten Modifikationen ein sicherer und sehr effizienter Algorithmus. Nach einer Idee von P. Waldvogel wird der Lanczos-Algorithmus auf das inverse, spektralverschobene Eigenwertproblem angewandt, wobei pro Lauf nur eine bestimmte Gruppe von Eigenwerten/Eigenvektoren berechnet werden soll. Bei kleiner Gruppengrösse sind nur wenige Lanczos-Schritte erforderlich, sodass auf die Nachorthogonalisierung verzichtet werden kann. Die Eigenvektoren werden schliesslich durch inverse Vektoriteration bestimmt, womit gleichzeitig die Information über die Vollständigkeit des Spektrums anfällt. Grössere Beispiele illustrieren die Arbeitsweise.

H. R. Schwarz
Universität Zürich (Schweiz)

Auswertung eines Flugschwingungsversuches (Inverses Eigenwertproblem)

Das zugrunde gelegte Gleichungssystem für das harmonisch erregte Flugzeug im Fluge ist $(-\omega^2 M + i\mathcal{C} + \mathcal{S})\, z = f$ mit den folgenden Bezeichnungen: ω = Kreisfrequenz der Erregung.

$M, \mathcal{C}, \mathcal{S}$ = $n \times n$ Matrizen ohne Symmetrie.

$f = (f_1, \dots, f_n)^T$ = erregender Vektor, d. h. f_j = komplexe Kraftamplitude im Strukturpunkt Nr. j.

$z = (z_1, \dots, z_n)^T$ = komplexe Strukturantwort, d. h. z_j = komplexe Ausschlagamplitude im Strukturpunkt Nr. j.

Gegeben: $z(\omega)$ = Strukturantwort bei Erregung mit konstantem Kraftvektor bei Eigenkreisfrequenzen nahe der Resonanzen.

Ziel: Parameteridentifikation.

Normalerweise löst man ein Gleichungssystem, in dem neben anderen linearen Unbekannten die n komplexen Eigenwerte, die physikalisch interessant sind, als nichtlineare

Unbekannte vorkommen. Das Besondere an der neuen Methode ist, daß in den zu lösenden Gleichungen stets nur ein Eigenwert vorkommt und dieser die einzige nichtlineare Unbekannte ist. Näheres in: "Parameteridentifikation bei Strukturen mit benachbarten Eigenfrequenzen, speziell bei Flugschwingungsversuchen", Z. Flugwiss. Weltraumforsch. 6 (1982), 4, 80-90.

Helmut Wittmeyer
Saab-Scania A.B. Linköping
Schweden

Effizienter Zweigwechsel ohne Berechnung des Verzweigungspunktes:

Das Verzweigungsverhalten von Randwertproblemen gewöhnlicher Differentialgleichungen wird numerisch untersucht. Insbesondere werden neue Methoden angegeben, mit deren Hilfe Zweigwechsel durchgeführt werden können. Die Verfahren berechnen zunächst eine Näherung zu einer Lösung auf dem abzweigenden Zweig. Hierzu sind lediglich 2 Lösungen auf dem "alten" Zweig zu ermitteln, der Verzweigungspunkt wird nicht benötigt. Ausgehend von der Näherung wird durch Lösen eines speziell geeigneten Randwertproblemes eine Lösung auf dem "neuen" Zweig berechnet. Die Verfahren liegen in einem FORTRAN-Programmpaket implementiert vor. Umfangreiche numerische Tests bei schwierigeren Beispielen belegen die Effizienz und Robustheit der Verfahren. Für den Fall von Systemen nichtlinearer Gleichungen wurden analoge Methoden entwickelt.

R. Seydel,
TU München

Nichtlinearisierung von Eigenwertaufgaben

Durch Elimination eines Teils der Variablen geht das lineare Eigenwertproblem $Kx = \lambda Mx$ (K, M positiv definit, $x \in \mathbb{R}^n$) über in das nichtlineare Eigenwertproblem $T(\lambda)u = (K_0 - \lambda M_0 - R\,D(\lambda)R^T)u = 0$ ($u \in \mathbb{R}^m$, $m << n$) mit einer rationalen Diagonalmatrix $D(\lambda)$. Es wird gezeigt, daß sich einige der kleinsten Eigenwerte von $T(\lambda)u = 0$ als minmax-Werte eines Rayleighfunktionals von T charakterisieren lassen. Hieraus ergibt sich eine Fehlerabschätzung für die Eigenwerte des reduzierten Problems $K_0 u = \lambda M_0 u$. Ferner ist ein rasch konvergentes Verfahren für das nichtlineare Problem anwendbar.

Heinrich Voß, Essen

Approximations to Periodic Solutions of a Duffing Equation

With P. Miletta, we look for solutions of fixed period T of Duffings equation:

$$\ddot{u} \pm \omega^2 u + \gamma u^3 = k \sin \Omega t)$$
$$u(0) = u(T),\ \dot{u}(0) = \dot{u}(T).$$

If we consider the subclass satisfying $u(0) = u(T/2) = 0$, the problem can be written as $Au - \gamma u^3 = f$, where $Au = -\nu^2\ddot{u} \mp \omega^2 u$, $u(0) = u(\pi) = 0$ ($\nu = 2\pi/T$), and $f = -\sqrt{\frac{\pi}{2}}\,k\,u_j$ in the Hilbert space $h = \{f \in L^2(0,\pi) \mid f(\frac{\pi}{2} - z) = f(\frac{\pi}{2} + z)\}$. Here $Au_i = \lambda_i u_i$ with $u_i = \sqrt{2/\pi}\,\sin(2i-1)z$. We approximate by $P^k u = \sum_{i=1}^{k} \alpha_{2i-1} u_i$, where $P^k A P^k u - \gamma P^k N(P^k u) = P^k f$. Since u^3 is "reproducing relative to $\{u_i\}$", we obtain an explicit nonlinear algebraic equation system, which is being investigated numerically. In the case of $-\nu^2\ddot{u} + \omega^2 u + |\gamma| u^3 = f$ our equation involves a maximal cyclically monotone operator, which is coercive, and a unique solution exists. Convergence follows by a result of R. Göthel. Error bounds and a variational inequality are discussed.

N. Bazley
Köln

Eigenvalue localization for irreducible matrices. –

The union of all Cassini ovals (the Cassini region) of a matrix A lends better possibilities for the localization of eigenvalues of A than the Gerschgorin region does. Examples are presented, where the separation of eigenvalues of A by its Cassini region is more efficient than that by the Gerschgorin region. Further, a characterisation of the class of irreducible matrices A is given for that the boundary of the Cassini region contains eigenvalues of A but there are some Cassini ovals of A, the boundaries of that do not contain these eigenvalues. It is the class of matrices for which a Theorem published by A. Brauer (1952) does not hold (Rein, X. J., 1967). As an example, a matrix from this class is shown and its Cassini region is discussed. A version of the corrected Theorem mentioned above is formulated.

Olga Pokorná, Prag

Bounds for the radially symmetric shape of a confined plasma in the unit circle.

The question of finding the shape of a confined plasma in a toroidal tokomak machine with cross-section $\Omega \subset \mathbb{R}^2$ leads to the following eigenvalue problem with free boundary:

<u>Given</u> $\Omega \subset \mathbb{R}^2$, $\partial\Omega$ smooth, $\lambda > 0$, $I > 0$,

<u>Find</u> $u \in C^1(\Omega)$, $\varkappa \in \mathbb{R}$ and $\Omega_p \subset \Omega$ such that:

$$-\Delta u = \left\{ \begin{matrix} \lambda f(x,u) & , x \in \Omega_p \\ 0 & , x \in \Omega \setminus \Omega_p \end{matrix} \right\} , \quad u = \left\{ \begin{matrix} 0 , & \text{on } \partial\Omega_p \\ \varkappa , & \text{on } \partial\Omega \end{matrix} \right\} ,$$

$$u > 0 \text{ in } \Omega_p \quad , \quad -\int_{\partial\Omega} \frac{\partial u}{\partial n} ds = I .$$

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For the ase $\Omega =$ unit circle a method is developed to include the radially symmetric solutions and, at the same time, to prove their existence. The method of "cone iteration" is then used to improve the bounds. Numerical results are presented.
The results are joint work with K.-H. Hoffmann, Augsburg.

Jürgen Sprekels, Augsburg.

Über ungerade, periodische Lösungen bei gewöhnlichen Differentialgleichungen

Es ist nicht schwer zu zeigen, daß die Differentialgleichung $y'' + N(y,y') = k\cdot\sin\Omega t$, $N:\mathbb{R}^2\to\mathbb{R}$, $N(-x_1,x_2) = -N(x_1,x_2)$ $\forall x_i\in\mathbb{R}, i=1,2$, unter zusätzlichen Regularitätsbedingungen an N, genau eine ungerade, periodische Lösung der Form $y(t) = \sum_{\nu=1}^{\infty} a_\nu \sin \Omega\nu t$ hat, sofern Ω "groß genug" gewählt wird. Wir beschäftigen uns hier mit folgenden zwei Problemen:

1. Wie findet man Näherungsformeln zur möglichst genauen Bestimmung der eindeutig bestimmten Lösungen für "großes" Ω? (mit E. SANDERS)
2. Was läßt sich über die Lösungsanzahl für "kleines" Ω aussagen?

Das 1. Problem wird behandelt, indem man die Lösung $y(t)$ in eine Potenzreihe um $\omega = \left(\frac{2\pi}{\Omega}\right) = 0$ entwickelt. Dann wird mittels einer Integraldarstellung von $y(t)$ eine Rekursionsformel für die Taylorkoeffizienten hergeleitet. Diese läßt sich mit Hilfe der symbolischen Mathematik (REDUCE) auswerten. Eine Verallgemeinerung für die Dgl. $y^{(2n)} + N(y,y',\dots,y^{(2n-1)}) = s(t)$, $N:\mathbb{R}^{2n}\to\mathbb{R}$, $N(-x_1,x_2,-x_3,\dots,x_{2n}) = -N(x_1,\dots,x_{2n})$ $\forall x_i\in\mathbb{R}, i=1,\dots,2n$, $s(t) = \sum_{i=1}^{\infty} b_i \cdot \sin\Omega i t$ ist möglich.

Das 2. Problem wird anhand der forcierten Pendelgleichung $y'' + \sin y = \sin\Omega t$, $y(0) = y(\omega/2) = 0$ illustriert. Neben dem Verzweigungsverhalten der Lösungen werden einige Bemerkungen, die sich aus den Rechnungen ergeben, dargestellt.

Ludger Brüll, Köln

Isoperimetric inequalities in the clamped membrane problem.

It has been conjectured many years ago that the level curves $u = \text{const.}$ of the first eigenfunction of the clamped membrane are convex for convex domains. This result can be obtained in R^2 using the maximum principle. This techniques gives in addition lower and upper bounds for the curvature $k(x)$ of the level curves in terms of u and $|\text{grad}\, u|$. These bounds may in turn be used to improve Payne and Stakgold's result: $|\text{grad}\, u|^2 + \lambda u^2$ takes its maximum value at a critical point of u for convex domains D. Similar results are possible for the torsion problem.

Gérard Philippin, Université Laval, Québec

Über Eigenwerte symmetrischer Membranen

Anhand einiger Beispiele (reguläres Sechseck; die Hälfte bzw. ein Viertel davon; David-Stern) wird gezeigt, dass aus isoperimetrischen Sätzen (Pólya, Szegö, M.T. Kohler) und einfachen Monotoniebetrachtungen brauchbare A priori-Schranken für Eigenwerte erhalten werden können. Wichtig ist es dabei, den Abbildungsradius des symmetrischen Gebietes im Mittelpunkt zu kennen. – Ein Beispiel von symmetrischen Membranen mit demselben ersten Eigenwert. – Ein Beispiel eines elementaren Verhältnisses zwischen Abbildungsradien.

Joseph Hersch (ETH-Zürich)

PROGRESS ON ESTIMATION OF ENERGY LEVELS FOR MULTI-ELECTRON ATOMS

With D. M. Russell we seek good rigorous lower bounds to the lower energy levels corresponding to the nonrelativistic, spin free Hamiltonian for an atom with N electrons. Hulthén potentials are used in the effective field method to obtain an explicitly solvable base problem for the Fox-Aronszajn method of intermediate problems with displacement of essential spectra. This base problem yields considerably higher initial approximations to the energy levels than the traditional Coulombic base problem.

W. M. Greenlee
University of Arizona

Lower bounds for eigenvalues in linear elasticity.

Considered is the eigenvalue problem

$$\Delta \underline{u} + \sigma \operatorname{grad} \operatorname{div} \underline{u} + \lambda \underline{u} = 0 \text{ in } \Omega \quad , \quad \underline{u} = 0 \text{ on } \partial\Omega ,$$

where Ω is a bounded region in $\mathbb{R}^N$ ($N=2$ or $N=3$), and where $\underline{u} = (u_1, \dots, u_N)$, $u_j \in H_0^1(\Omega) \cap H^2(\Omega)$.

Upper bounds for the eigenvalues λ_k ($k = 1, 2, \dots$) are obtained by the Rayleigh-Ritz method. It is shown that the eigenvalues $\lambda_k^{(h)}$ of a properly defined discrete problem (finite differences, grid constant h) give lower bounds for the λ_k in the following way:

$$\lambda_k^{(h)} / \left(1 + \frac{h^2}{\pi^2} \lambda_k^{(h)}\right) \leq \lambda_k .$$

The lower bounds converge to λ_k as $h \to 0$. The method is closely related to those of J. Hersch (1955 and 1963) and of H. F. Weinberger (1956, 1958) resp. Kuttler, J. R. (1970) which, however, do not apply to systems of differential equations with mixed derivatives.

W. Velte
University of Würzburg

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Spectral analysis of non-selfadjoint elliptic operators of mathematical physics.

Spectral analysis and Laplace transformation of many nonstationary problems of mathematical physics lead to analysis of non-selfadjoint elliptic problems parametrically dependent on the transform parameter p. It has been shown that eigenvalues and eigenfunctions of these problems are analytic functions of p in $p_0^+ = \{p \mid \mathrm{Re}\, p \geq b \geq 0\}$. The set of eigenfunctions is complete. Zero values of eigenvalues coincide with eigenvalues according to the definition of nonlinear eigenvalues. We have analysed their limit points which are important from the point of view of convergence of solutions and an analysis of nonstationary problems in spaces of analytic functions valued in Sobolev spaces and in weighted anisotropic Sobolev spaces, we have introduced.

For a numerical analysis a generalization of the finite element method has been proposed.

J. Mika
Comenius University, Bratislava

Hartree-Fock- und diskrete Newtonverfahren für die Eigenwertberechnung der Schrödinger-Gleichung

Durch Anwendung von Variationsmethoden auf geeigneten Unterräumen erhält man nach den üblichen Separationsansätzen (Radialfunktionen. Kugelflächenfunktionen) und nach ausführlichen Zwischenrechnungen nichtlineare Eigenwertsysteme für gewöhnliche Differentialgleichungen. Diese wurden mit diskreten Newton-Verfahren gelöst. Dabei ist darauf zu achten, daß das auf (0,∞) definierte Randeigenwertproblem durch Reduktion auf [ε, b], ε>0, und durch Differenzenverfahren auf zwei Stufen approximiert wird. Mit den berechneten Wellenfunktionsnäherungen wurden die Energieniveaus als Rayleigh-quotienten berechnet

Universität Hamburg

Alternativsätze und Eigenwertschranken

Zur Berechnung von Schranken für Eigenwerte zu Eigenvektoren mit gewissen Eigenschaften (z.B. Nichtnegativität) kann man sich der aus der Theorie der linearen Ungleichungen bekannten Alternativsätze bedienen. Hierbei ermittelt man alle diejenigen Parameterwerte λ, für die z.B. das System

$$(A-\lambda I)^T(A-\lambda I)\,x = \Theta,\quad Cx \geq \Theta,\quad Cx \neq \Theta$$

sicher nicht lösbar ist. Die erhaltenen Schranken werden mit denen des Einschließungssatzes von Collatz (1942) verglichen. Verallgemeinerungen werden diskutiert.

Ulrich Eckhardt
Universität Hamburg

Eine einheitliche Herleitung von Einschließungssätzen für Eigenwerte

Für eine Klasse allgemeiner EWAn $M\varphi = \lambda N\varphi$, $\varphi \in D(M)$ wird ein einfaches Prinzip für Aufstellung von Einschließungssätzen für Eigenwerte geschildert; es besteht darin, Verfahren, welche die Berechnung oberer Schranken für eine geeignete Norm der Lösung gewisser linearer Gleichungen $N w = h$ (bzw. $\tilde{M} \tilde{w} = \tilde{h}$) ermöglichen, mit einem grundlegenden Einschließungssatz zu kombinieren. – Inhalt: I. Einführung und grundlegende Sätze. II. Die klassischen Einschließungssätze. III. Einschließung ohne Iteration $Mu = Nv$. IV. Einschließungssätze ohne Vergleichsfunktionen. V. Beispiele.

Julius Albrecht, T.U. Clausthal

Eigenvectors estimates and application to some problems of structural engineering

Upper and lower estimates for the error in eigenvectors approximation are presented in connection with eigenvalue problems for elastic structures.
New upper estimates are given and compared with previous ones.
They appear to depend less critically on the precision of the corresponding eigenvalue bounds.
The lower estimates of the error allow to get a better definition of the approximation.

Manfredi Romano

Istituto di Scienza delle Costruzioni – Univ. di Catania

Sloshing Eigenvalues

We discuss two systemetic methods for finding lower bounds to two-dimensinal problems of sloshing of fluids. This problem is unusual in that the eigenvalue appears only in the boundary condition on the horizontal part $\partial_1 G$ of the boundary. These methods, based on ideas of intermediate problems, make use of transformations of the problems into the form $Au - \lambda \sigma u = 0$ in $L^2(\partial_1 G)$, where

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A is an explicitly spectrally resolvable operator and σ is the operator of multiplication by a bounded non-negative function. Excellent numerical bounds (five to ten significant figures for the first five to 20 eigenvalues) are given for eigenvalues of a half-dozen typical problems.

David Fox, Johns Hopkins Univ.

Approximate Solutions to Nonlinear Operator Equations

In a Hilbert space $\mathcal{H}$ we consider operator equations of the form

$$Lx + N(x) = f \qquad (1)$$

where L is a linear self-adjoint operator, N non-linear and reproducing with respect to a complete orthonormal system of eigenvectors of L. Approximations to solutions to (1) are obtained by the Rayleigh-Ritz method where by we consider the equation

$$P^k L P^k x + P^k N(P^k x) = P^k f \qquad (2)$$

where P^k is the projection on the subspace of $\mathcal{H}$ spanned by the first k-eigenvectors of L. Equation (2) is assumed to have a solution x^k. Using a varient of Cesari's Alternative Method. we show that a certain mapping associated with (2) has a fixed point x in a neighborhood of x^k. Under appropriate conditions it is shown that. x is a solution to (1). The diameter of the neighborhood can be measured using the reproducing property of N to give error bounds on $\|x - x^k\|$. An application is given to the equation $x'' + x + \frac{1}{2}x^3 = \frac{1}{2}\sin\frac{3}{2}t$.

Peter Mlette G.
Depto de Matematicas y CC.
Universidad de Santiago
Santiago Chile

Die Anwendung komplementärer Extremalprinzipe bei der Berechnung von Eigenwertschranken

D sei ein reeller Vektorraum; $M(\cdot,\cdot)$ und $N(\cdot,\cdot)$ seien symmetrische Bilinearformen auf D; $M(\cdot,\cdot)$ sei positiv definit. Betrachtet wird die folgende Eigenwertaufgabe: Gesucht sind $\varphi \in D$, $\lambda \in \mathbb{R}$ mit $\varphi \neq 0$ und der Eigenschaft, daß $M(u,\varphi) = \lambda N(u,\varphi)$ für alle $u \in D$ gilt. Zur Berechnung von Schranken für die Eigenwerte solcher Aufgaben kann das Lehmann-Maehly-Verfahren nur dann herangezogen werden, wenn man Paare (v_i, w_i) kennt derart, daß $v_i \in D$, $w_i \in D$ und $M(u,w_i) = N(u,v_i)$ für alle $u \in D$ gilt. Es wird gezeigt, daß man diese Bedingung mit Hilfe komplementärer Extremalprinzipe wesentlich abschwächen kann. Das so erhaltene Verfahren zur Berechnung von Eigenwertschranken läßt sich auf viele Aufgaben, die mit dem ursprünglichen Lehmann-Maehly-Verfahren nicht behandelt werden können, mit Erfolg anwenden. Präsentiert werden numerische Resultate zu Aufgaben aus den folgenden Gebieten: Beulen eingespannter Rechteckplatten unter Druck und Schub, hydrodynamische Stabilität, Schwingungen elastisch gelagerter Platten, rhombische Membranen, Stekloffsche Eigenwertaufgaben, Schlingerfrequenzen.

Friedrich Goerisch, TU Clausthal, Institut für Mathematik

Combinatorics and Algebraic Groups

20 - 25 June

Introduction to Schur modules

To reproduce Lascoux' resolution of determinantal ideals in char 0, a definition of Schur and coSchur modules over arbitrary commutative rings R was ~~defined~~ given and discussed. For a matrix $A = (a_{ij})$ of 0's and 1's, and any free R-module F, a Schur map $d_A : \Lambda_A F \to S_{\tilde{A}} F$ was defined, where $\Lambda_A F = \Lambda^{a_1} F \otimes \dots \otimes \Lambda^{a_s} F$, $S_{\tilde{A}} F = S_{b_1} F \otimes \dots \otimes S_{b_t} F$, $a_i = \sum_j a_{ij}$, $b_j = \sum_i a_{ij}$. The image of d_A is denoted $L_A F$ and called the Schur module of shape A. When A is the matrix corresponding to a skew partition λ/μ, the module $L_{\lambda/\mu} F$ is universally free. (A similar construction involving the divided power algebra defines coSchur modules K_A, $K_{\lambda/\mu}$.) These modules are used to reproduce the terms of Lascoux' resolution, but over $\mathbb{Z}$ there is torsion in homology. This leads to the study of $\mathbb{Z}$-forms of rational representations of $GL(F)$. Examples connecting these $\mathbb{Z}$-forms with Ext and Giambelli identities lead to construction of "resolutions" of $L_{\lambda/\mu}$ by means of sums of tensor products of exterior powers. These resolutions should be constructible by means of iterated mapping cones (a complete description was given for 2-rowed shapes), but since terms of the resolution are not projective, one needs a proof of existence of maps on which to build mapping cones. This problem is solved by looking at resolutions of coSchur modules in terms of tensor products of divided powers. These latter modules are projective over the Schur algebra, so iterated mapping cones may be constructed. Invoking the involution between divided and exterior powers, the corresponding mapping cone constructions for Schur modules could also be effected.

David Buchsbaum, Brandeis University

Syzygies of determinantal ideals

Let X be an affine space of all $m \times n$ matrices over a field K of characteristic zero and let Y_r be a set of all matrices of rank $\leq r$ in X. Y_r is called a determinantal variety and its coordinate ring O_{Y_r} is isomorphic to $K[T_{ij}] / I_{r+1}(T)$ where $T = (T_{ij})$ is a generic $m \times n$ matrix of indeterminates and the ideal $I_{r+1}(T)$ is generated by $(r+1)$-order minors of T.

The problem discussed in the talk consists in finding an explicit minimal free resolution of $\mathcal{O}_{Y_r}$ over $\mathcal{O}_X$. Attempts of many mathematicians to solve the problem were presented in the chronological order. Emphasis was put on geometric construction of A. Lascoux allowing to ~~[illegible]~~ find components of a minimal resolution and a recent work of P. Pragacz & J. Weyman that also contains an explicit construction of differentials. Lascoux' method is based on a geometrical construction of a desingularization Z of Y_r in a suitable Grassmannian G. Z is a complete intersection in G, i.e. $\mathcal{O}_Z$ has a simple resolution over $\mathcal{O}_G$ which is given by a Koszul complex $K^\bullet$. By using a spectral sequence of hypercohomology associated to $K^\bullet$ and Bott's theorem one finds components of a minimal resolution of $\mathcal{O}_{Y_r}$ over $\mathcal{O}_X$ as sums of certain Schur functors. Pragacz & Weyman's construction describes the resolution as a total complex associated with certain double complex. At first one constructs rows of the double complex using trace and evaluation maps between Schur complexes. Differentials in rows are of degree 1. Then one completes the picture by defining maps of degree $r+1$ between consecutive rows. Exactness of the complex is proved by applying the acyclicity lemma.

Tadeusz Józefiak, Toruń, Poland

Combinatorics and representations of $GL(n, \mathbb{C})$

The characters of the polynomial representations of $GL(n, \mathbb{C})$ or $SL(n, \mathbb{C})$ are symmetric functions in the eigenvalues of $A \in GL(n, \mathbb{C})$ known as Schur functions. They have a combinatorial definition involving Young tableaux which leads to many connections between combinatorics and representation theory. One instance of this connection is to the enumeration of plane partitions, a generalization due to P. A. MacMahon of the classical theory of partitions. The Weyl character formula leads immediately to most of the basic results in this area. Moreover, an elegant generating function for a certain class of plane partitions can be obtained by decomposing the restriction of certain representations of $so(2n+1, \mathbb{C})$ to the Levi subalgebra $gl(n, \mathbb{C})$. Another connection between combinatorics and representation theory arises from the problem of decomposing the virtual character $\det \prod_i (1 - z_i \, \mathrm{ad}\, x)/(1 - w_i \, \mathrm{ad}\, x)$ of $SL(n, \mathbb{C})$ as $n \to \infty$. An explicit decomposition into irreducibles is found which can be applied to the computation of generalized exponents of $SL(n, \mathbb{C})$, the q-Dyson conjecture, and related problems.

Richard Stanley
M.I.T.

Enumerative Geometry and Embeddings.
Joint with C. Procesi.
Let G be a semisimple adjoint group, $\sigma : G \to G$ an order 2 automorphism, $H = G^\sigma$ the group of elements fixed by σ. We set $\underline{g} = \mathrm{Lie}\, G$, $\underline{h} = \mathrm{Lie}\, H$, $\ell = \mathrm{rk}\, G/H$ $m = \dim \underline{h}$. We can consider $\underline{h}$ as a point in the Grassmann variety $G_m(\underline{g})$ of m-dimensional

subspaces of $\underline{g}$. The action of G on $G_m(\underline{g})$ induced by the adjoint action allows us to define a G-variety $X = \overline{G\underline{h}} \subseteq G_m(g)$. It is easily seen that $H = \mathrm{Stab}\,\underline{h}$ so X is an embedding of G/H. X has many pleasant properties:

a) X is smooth projective
b) $X - G\underline{h} = \bigcup_{i=1}^{\ell} S_i$, where S_i is a smooth divisor which is a orbit closure for each $i = 1, .., \ell$.
c) The S_i's meet transversaly
d) Each orbit closure in X is of the form $S_I = \bigcap_{i \in I} S_i$ for some $I \subset \{1, .., \ell\}$ so in particular it is smooth by property c); furthermore $\bigcap_{i=1}^{\ell} S_i$ is the unique closed orbit in X.

Two special cases of this construction have be classically studied in the field of enumerative geometry.
In the first case $G = \mathbb{P}Gl(n+1) \times \mathbb{P}Gl(n+1)$ and $\sigma(g, g') = (g', g)$ for any $(g, g') \in G$. In this case X is the variety of complete collineations of $\mathbb{P}^n$
In the second case $G = \mathbb{P}Gl(n+1)$ and σ is the involution on G induced by the involution σ' on $Gl(n+1)$ defined by $\sigma'(g) = {}^t g^{-1}$
In this case the variety X is the variety of complete quadrics

Corrado De Concini
Università di Roma II.

Constant term identities related to root systems

A survey of various conjectures relating to constant terms in Laurent polynomials constructed from root systems, of which the following (generalizing "Dyson's conjecture" (1962)) is the simplest to state: let R be a reduced root system, W its Weyl group, d_i $(1 \leq i \leq l)$ the degrees of the fundamental polynomial invariants of W; also for each $\alpha \in R$ let e^{α} be the corresponding formal exponential. Then we conjecture that the constant term in the Laurent polynomial $\prod_{\alpha \in R} (1-e^{\alpha})^k$ (where k is a positive integer) should be $\prod_{i=1} \binom{kd_i}{k}$. This is true for all values of k when R is of classical type (A_n, B_n, C_n, D_n); also for all R when $k = 1, 2$. For R exceptional and $k \geq 3$ it remains an open ~~conjecture~~ question. More details may be found in "Some conjectures for root systems", SIAM J. Math. Anal., 13 (1982) 988 – 1007.

IGMacdonald
Queen Mary College, London

Cohen-Macaulayness and shellability of Bruhat order and buildings.

Algebraic groups are related to the combinatorics of posets and simplicial complexes in at least two important ways: via the "Bruhat" ordering and via Tits buildings. On the other hand posets and complexes are related to commutative rings via the construction of Hochster and Stanley, alias the "discrete rings" in the theory of Hodge algebras. In this talk we attempted to describe a few basic facts about these connections and about the use of shellability for establishing Cohen-Macaulayness out of combinatorial structure.

Anders Björner
University of Stockholm

Geometry of G/P

In this talk, we gave a survey of "Standard Monomial Theory" as developed in G/P I – V. We also mentioned the various applications of the standard Monomial Theory. Standard monomial theory for a semi-simple algebraic group, G, consists in the construction of an explicit basis for $H^0(G/B, L)$ (where B is a Borel subgroup and L is a positive line bundle on G/B) as a generalization of the classical Hodge Young theory. The construction is done using the Schubert calculus. The construction consists of the following steps

Step 1: Construction of an explicit basis for $H^0(G/P, L)$ (more generally for $H^0(X, L)$, where P is a maximal parabolic subgroup, ~~and~~ L is ample generator of Pic(G/P) and X a Schubert variety in G/P).

Step 2: Notion of monomials in the basis elements being standard.

Step 3: Proof of the fact that standard monomials on X of deg. m give a basis of $H^0(X, L^m)$.

Step 4: Using the theory for G/P, one obtains the theory for G/Q, where Q is any parabolic subgroup.

The above problem has been solved for parabolic subgroups $Q = \cap P_i$ where P_i is such that the associated fundamental weight ω_i satisfies, $|\langle\omega_i, \alpha^\vee\rangle| \leq 2$ for all roots α; and also for the parabolic subgroups P_2 (and P_1) of a group of type G_2. Among the various applications of the standard monomial theory, one striking application is the determination of singular locus of a Schubert variety.

V. Lakshmibai
University of Michigan (Ann Arbor)

Schubert Functors

The ring of symmetric polynomials $\mathbb{Z}[a,b,\dots]^{W_n}$, W_n being the symmetric group on n elements, can be considered as the ring of representations of the symmetric group, or the linear group, or also the cohomology ring of the Grassmann variety. Its natural basis, the Schur functions, can be generalized in different manners: they can be considered as sums of Young tableaux (⇒ in the "plactic ring") or looked at as functors on modules (Schur functors). Another generalization comes from the action of W_n on $\mathbb{Z}[a,b,\dots]$. Essentially, for the group W_2, there are three actions

$$\mathbb{Z}[a,b] \ni f(a,b) \rightsquigarrow f(b,a)$$
$$f(a,b) \rightsquigarrow [f(a,b)-f(b,a)]/a-b$$
$$f(a,b) \rightsquigarrow [a f(a,b) - b f(b,a)]/a-b$$

On can interpolate between these three actions and extend them to W_n. Now, the action of W_n on the special polynomial $a^{n-1}b^{n-2}c^{n-3}\dots$ gives a family of polynomials ("Schubert polynomials") which contains the Schur function. The Schubert polynomials can be lifted to the plactic ring, giving sums of tableaux with flags of alphabets, or can be considered as functors of flags of modules, in connexion with the study of Schubert varieties in the flag manifold.

Another extension applies to the ring of reduced decompositions in the symmetric group (i.e. $\mathbb{Z}[W_n]$, with the multiplication

$$\begin{aligned} w * w' &= \text{usual multipl. } w\cdot w' \quad \text{if } \ell(w)+\ell(w')=\ell(ww') \\ &= 0 \quad \text{otherwise} \end{aligned}$$

), and this ring can also be considered as a quotient of the nilplactic ring (a non commutative ring very similar to the ring of tableaux)

A. Lascoux and M.-P. Schützenberger

Laboratoire d'Informatique Théorique, Paris

Hodge Algebras and Cohen-Macaulayness.

(Ref: Hodge Algebras, by C. DeConcini, D. Eisenbud, and C. Procesi, Asterisque, 1982).

A Hodge Algebra A over a ring R (commutative, noetherian, ...) on a poset H governed by an ideal of monomials $\Sigma \subset \mathbb{N}^H$ is a (Commutative) algebra A generated by H, such that 1) the monomials in H that are not in Σ (the standard monomials) form a basis for A, and such that 2) if N is an element of the minimal generating set for Σ, and $N = \sum r_i M_i$ is its unique expression as an R-linear combination of standard monomials M_i, then for each i and each $x \in H$ dividing N formally, there is an $x_i \in H$ dividing M formally such that $x_i \leq x$.

The simplest Hodge algebra on H governed by Σ is the discrete algebra $A_0 = R[H]/(\Sigma)$, where R[H] is the polynomial ring and (Σ) is the ideal in R[H] generated by monomials in Σ. The significance of the condition 2) above is that it makes A a deformation of A_0 in a nice way; probably there are even weaker conditions that do this.

Note that if $I \subset H$ is an order ideal ($x \in I$, $y \leq x \Rightarrow y \in I$) then A/I is again a Hodge algebra; this is the significance of allowing arbitrary partial orders on H.

The notion of a Hodge algebra abstracts the notions of "standard monomials" found in G/P (Lakshmibai, Musili, Seshadri) and elsewhere. Because of the deformation idea above, properties of interesting Hodge algebras (reducedness, Cohen-Macaulay-ness, etc.) can often be deduced from properties of the discrete algebras, which are subject to a very penetrating combinatorial study (Reisner, Hochster, Stanley, Bjorner, Baclawski, Garsia, ...).

David Eisenbud

Representations of general linear groups.

We discuss the unipotent representations of the finite general linear groups $G_n = GL_n(q)$ over a field K whose characteristic does not divide q.

Let M^λ be the permutation module of KG_n on the parabolic subgroups corresponding to the partition λ of n. The module S^λ is defined to be the subset of M^λ which equals the intersection of the kernels of all KG_n-homomorphisms which map M^λ into some M^μ for which $\mu \rhd \lambda$. If K has characteristic zero, then S^λ is irreducible, and more generally S^λ has a unique irreducible image D^λ. As λ runs over partitions of n, D^λ runs over a complete set of inequivalent irreducible unipotent KG_n-modules. The matrix which records the composition multiplicities of the D^μ's in the S^λ's is part of the decomposition matrix of KG_n, and is lower unitriangular.

The remarkable feature is that the representation theory of the symmetric group (and perhaps even the theory of Weyl modules) appears to be the case "$q = 1$" of this theory.

Gordon James
Sidney Sussex College, Cambridge.

Introduction to standard monomials

The classical representation theory of the linear group, as developed by I. Schur, has a tight connection with invariant Theory.
This connection brings forth the role of determinantal varieties. In order to intepret the picture of A. Young, giving bases of representations of the symmetric group by standard diagrams, in the previous setting it is best to use Hodge's approach to the postulation formula for the Grassmann variety. In this picture the standard tableaux appear related to the natural

geometric ordering of Schubert cells.

In fact the Schubert varieties correspond bijectively to the Plücker coordinates and the monomials $a_1 a_2 — a_s$ in the Plücker coordinates, such that $a_i \le a_{i+1}$ (in the corresponding ordering of Schubert cells) are a basis for the projective coordinate ring of the Grassmannian.

The determinantal varieties appear then as affine parts of Schubert varieties and the previous theory of standard monomials becomes the Theory of Double Standard Tableaux (as in Doubilet-Rota-Stein). Thus one can go back to the representation theory and start a connection which has very large possibilities of generalizations.

Claudio Procesi

Univ. Roma.

Infinite-dimensional groups and their flag varieties.

A Lie algebra (possibly ∞-dimensional) is called integrable if it is generated by locally-finite elements (then it is spanned by them). Given an integrable Lie algebra $\underline{g}$, we associate to it a group G as follows. Denote by S the set of all locally-finite elements of $\underline{g}$. We call a $\underline{g}$-module V integrable, if every $x \in S$ acts locally-finitely on V. Then G is a group, ~~on~~ ~~generators~~ (generated by symbols $\exp x$,) $x \in S$, with relations between $\exp x$ ~~[illegible]~~ $= 1 + x + \frac{x^2}{2!} + \ldots$ ~~which hold in the~~ ~~intersection of kernels~~ in all integrable representations of $\underline{g}$.

Given a generalized Cartan matrix A, we denote by $G(A)$ the group associated to the Kac-Moody Lie algebra $\underline{g}(A)$. Then $G(A)$ has a structure of a Tits system and one can study the associated flag varieties, Schubert varieties, highest weight representations, etc. One of the consequences is the description of the compact form $K(A)$

in terms of generators and relations and the study of homology of $K(A)$.

At the end the group GL_∞ (or rather its central extention) was discussed, along with application to soliton solution of KP-equation.

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Victor Kac, MIT.

Polar Representations.

Let $G|V$ be a rational representation of a linear reductive algebraic group G over $\mathbb{C}$ ~~and~~ vectorspace V. If $v \in V$ lies on a closed G orbit let

$$c_v = \{x \in V \mid \mathfrak{g}.x \subseteq \mathfrak{g}.v\},$$

where $\mathfrak{g} = L.A(G)$. The representation is called polar if ~~for~~ for some c_v henceforth called a Cartan subspace, $\dim c_v = \dim V/G$, where V/G is the affine variety corresponding to the ring of invariants $\mathbb{C}[V]^G$. If $G|V$ is polar then all Cartan subspaces are G conjugate; all closed G orbits meet a given Cartan subspace c, and all orbits through c are closed. Intersection of a closed G orbit with c is a W orbit, where the Weyl group W is a finite group. Via restriction one obtains $\mathbb{C}[V]^G \simeq \mathbb{C}[c]^W$. In case G is connected W is generated by unitary reflections, and hence $\mathbb{C}[V]^G$ is a polynomial algebra. ~~These~~ Polar representations include adjoint actions, representations associated to symmetric spaces, θ groups and irreducible visible representations.

Ref: J. Dadok, V. G. Kac, Polar representations

Invariant algebras stable under straightening

There are only a few results in the invariant theory of non-reductive groups: a) Nagata's counterexample. b) The converse of the invariant theorem for reductive groups (Popov). c) Zariski's theorem: When a group G acts on a finitely generated algebra A and trdeg $A^G \leq 2$, then A^G is finitely generated. d) Grosshans' criterion: Let $H \leq Sl_n$ act on the polynomial algebra $k[X] \simeq k[X_{ij} \mid 1 \leq i,j \leq n]$ by left translation and let $k[X]^H$ be finitely generated. Then for any finitely generated algebra A on which a reductive group G, $H \subseteq G \subseteq Gl_n$, acts rationally, A^H is finitely generated. So a good substitute for Hilbert's 14th problem is: Find the „Grosshans subgroups" of a reductive group G. For a regular (= normalized by a maximal torus) unipotent subgroup U of Gl_n a necessary and sufficient condition is given for the stronger property, that $k[X]^U$ is spanned by the invariant standard bitableaux. This proves the Grosshans property for a large class of regular unipotent subgroups of Gl_n.

Klaus Pommerening, Johannes-Gutenberg-Universität Mainz

Nilpotent triangular matrices.

A nilpotent endomorphism x of a vector space V with $\dim(V) = n$ is characterized by a partition $\lambda(x:V) = (\lambda_1, \dots, \lambda_r)$ of n. Let $\mathcal{Y}(x)$ be the set of the x-invariant flags $F_* = (F_0, F_1, \dots, F_n)$ in V. A natural discrete invariant of a flag $F_* \in \mathcal{Y}(x)$ is the system of partitions $\tau(x, F_*) = (\tau[p,q])_{p \leq q}$ with $\tau[p,q] = \lambda(x : F_q/F_p)$. We give a representation of τ by a strictly upper triangular matrix A of zeros and ones. Such a matrix is called a typrix.

Let F_* be the standard flag in K^n. If x is a strictly upper triangular matrix of order n, then $F_* \in \mathcal{Y}(x)$, so we have a system of partitions $\tau(x, F_*)$ and a typrix $A(x)$, say. The following rules hold:

1) $x = 0 \Leftrightarrow A = 0$.

2) x is regular (i.e. $x^{n-1} \neq 0$) $\iff$ $a_{ij} = 1 \quad \forall\, i<j$.

3) If $1<p<n$, then $A = \begin{pmatrix} A(x:F_p) & * \\ 0 & A(x:V/F_p) \end{pmatrix}$.

Example.

$$A(x) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 1 & 1 & 0 \\ & & & 0 & 1 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix} \iff x = \begin{pmatrix} 0 & a & b & c & d & e \\ & 0 & 0 & g & h & i \\ & & 0 & j & k & l \\ & & & 0 & m & n \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix} \text{ with } \begin{cases} a \neq 0,\ j \neq 0,\ m \neq 0 \\ ag + bj = 0 \\ \begin{vmatrix} g & h & i \\ j & k & l \\ 0 & m & n \end{vmatrix} = 0 \end{cases}$$

$$A(x) = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 1 \\ & & & 0 & 1 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix} \iff x = \begin{pmatrix} 0 & a & b & c & d & e \\ & 0 & 0 & 0 & h & i \\ & & 0 & 0 & k & l \\ & & & 0 & m & n \\ & & & & 0 & 0 \\ & & & & & 0 \end{pmatrix} \text{ with } \begin{cases} a \neq 0,\ m \neq 0 \\ ah + bk + cm \neq 0 \\ \begin{vmatrix} k & l \\ m & n \end{vmatrix} \neq 0 \end{cases}$$

Question. If we call these sets of typrices C_1 and C_2, respectively, is it true that C_2 is contained in the closure of C_1? The answer is no. For all matrices in C_1 also satisfy the equation $\begin{vmatrix} b & h & i \\ -a & k & l \\ 0 & m & n \end{vmatrix} = 0$, and that is not true for C_2.

Overview of results. There is a combinatorial injection between the standard tableaux and the typrices, say $S \mapsto A$, such that $Y(x, A)$ is dense in the irreducible component $Y(x)_S$ of $Y(x)$. A typrix A is called very acceptable if it satisfies certain highly involved combinatorial inequalities. Theorem: all occurring typrices are very acceptable. Theorem (H. Bürgstein): If $n \leq 6$ and $\#(K) > 2$, all very acceptable typrices occur.

Table of number of typrices:

n	2	3	4	5	6	7
# very acceptable typr.	2	5	16	61	274	1419
# standard tableaux	2	4	10	26	76	232
# B-orbits in $\underline{u}$	2	5	16	61	$273+\infty$	$\infty + ..$

Wim Hesselink, Groningen.

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Shape algebras and applications

A review of the shape functor constructions for several typical cases (GL_n, $SO(2l+1)$, G_2) was given; the strategy leading to these constructions was then described, and its connection with Kostant's theorem on the kernel of a Cartan product. An alternative multiplication on the shape-algebra for $GL(E)$ ~~[illegible]~~ was described, and an application to the plethysm problem was given.

Jacob Towber

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MEASURE THEORY

June 26 to July 2

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Realcompactness and measurecompactness of the unit ball in a Banach space.

The realcompactness and measure-compactness of a Banach space in its weak topology have been of interest in connection with the theory of integration in the Banach space. Similar properties can be investigated for the unit ball of the Banach space. Several examples are worked out to illustrate these properties. The unit balls of l_∞ / c_0 and the long James space $J(\omega_1)$ are not realcompact. The unit ball of l_∞ is not measure-compact. It can be conjectured that the ball is realcompact (or measure-compact) if and only if the whole space is. I expect that this is false, but I do not know of a counterexample.

G. A. Edgar, Ohio State Univ.

Separate and joint measurability

The problem: Let (Ω, Σ, μ) be a complete probability space, and let (Y, Ξ, ν) be a Radon probability space. Let $f: \Omega \times Y \to \mathbb{R}$ which is a measurable in the first variable, and continuous in the second variable. When is f $\mu \hat{\otimes} \nu$ measurable?

1- D. Fremlin constructed an escample (under CH) where f fails Fubini' theorem. No absolute example is known to me.

2 - On the other hand, the answer is yes if Y is small, say metrizable.

3 - Consider the set $Z_f \subset \mathbb{R}^{\Omega}$, which is the ~~image~~ sets of maps $f(\cdot, t)$ for $t \in Y$. Z_f is a pointwise compact set of measurable functions.

Definition. A set $Z \subset \mathbb{R}^{\Omega}$ is stable, if given $\alpha < \beta$ and $A \in \Sigma$, $\mu A > 0$, there is $n > 0$ with

$$(\mu^{2n})^*(\{(s_1, \ldots, s_n, t_1, \ldots, t_n) \in A^{2n}; \forall i \leq n,\ f(s_i) \leq \alpha,\ f(t_i) \geq \beta\})$$
$$< (\mu A)^{2n}$$

Main result. If Z_f is stable, f is jointly measurable.

4 - In practice it is very easy to check that a set is stable. A general result is a follows:

Axiom L: $[0,1]$ is not the union of < card $\mathbb{R}$ closed negligible sets.

Theorem. (Axiom L) Assume (Ω, Σ, μ) is perfect. Assume $Y \overset{!}{=}$ support ν and that the map $\omega \to f(\omega, \cdot)$ from Ω to $L^1(\nu)$ is measurable. Then the set Z_f is stable.

Note: The assumption is equivalent to the fact There is g on $\Omega \times Y$ with g measurable and $\forall \omega$, $g(\omega, t) = f(\omega, t)$ for ν.a. each t. The point is that these negligible sets a-priori depend wildly on Ω. ~~However, which shows that f is measurable~~. This also shows that the main result is optimal under Axiom L.

5- A good property is

Theorem Assume that f as above is jointly measurable. Then for each ε, we have a family $A_n \times B_n$, $A_n \in \Omega$, $B_n \in \Xi$ with $\mu \times \nu (\bigcup_n A_n \times B_n) = 1$, and such that f has osculation $\leq \varepsilon$ on $A_n \times B_n$ for each n.

6- The above results give an easy route to the following.

Theorem. Let G be a compact group and $f \in L^\infty(G)$. Under axiom L, the following are equivalent:

a) $\forall \varphi \in L^\infty(G)^*$, the map $t \to \varphi(L_t f)$ is measurable (of course $L_t f(u) = f(tu)$)

b) $\forall \theta$ character on $L^\infty(G)$, $t \to \theta(L_t f)$ is measurable.

c) $\forall \rho$ left invariant lifting of G, $\rho(f)$ is Riemann measurable.

The main point is that if S denotes the spectrum of $L^\infty(\mu)$ and ν is canonical measure, the map $g : G \times S \to \mathbb{R}$ given by $g(t, \theta) = \theta(L_t f)$ is measurable in t by hypothesis, continuous in θ, so satisfies the hypothesis of the Th. of 4. It is hence jointly measurable, and it is not hard to conclude from 5.

M. Talagrand, Paris VI

Quelques aspects de la théorie des mesures coniques.

Soit E un espace vectoriel en dualité séparante avec un autre F. Soit $h(E,F)$ le treillis de fonctions sur E engendré par F. Une $\mu \in M^+(E,F)$ est une forme ≥ 0 sur le

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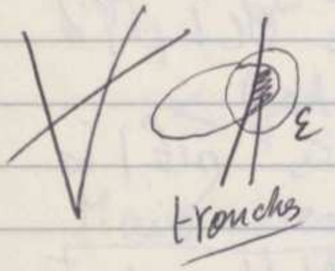
treillis $h(E)$. soit $r(\mu)$ l'élément de $\hat{E}$ (complété de E pour $\sigma(E,F)$) tel que $\mu = \varepsilon_{r(\mu)}$ sur F.
soit
$$K_\mu = \{ r(\lambda) : 0 \le \lambda \le \mu \}.$$
on dit que μ est portée par un cône $X \subset E$ si $\mu(f)$ ne dépend que de $f/_X$.
Ces notions sont dûes à G. Choquet.

Th1 : soit X un cône convexe dans un espace vectoriel. on suppose que X est complet pour une distance d. soit H un sous-espace de formes affines sur X, uniformément continues pour d sur X. on suppose que toute pseudo-base de X (ie $f^{-1}(1) \cap X$ pour $f \ge 0$ sur X) est telle que tous ses sous convexes admettent des tranches de diamètre arbitrairement petits. Alors, avec des hypothèses de séparabilité, toute μ sur X (pour la dualité avec H) vérifiant $K_\mu \subset X$ est donnée par une mesure de Radon sur $X \setminus 0$.



Le cas où les intervalles d'ordre X sont compacts permet de dire que toute μ sur X vérifiant $r(\mu) = x$ est telle que $K_\mu \subset X$.

Th2 : X est un cône convexe $\subset$ E e.l.c.s. soit $H \subset E'$ un sous-espace vectoriel.
on suppose que toute partie $\sigma(X,H)$ compacte de X est bornée dans E.
on suppose que E admet une base de voisinages de zéro $\mathcal{V}$ telle que $(V \in \mathcal{V}) \Longrightarrow (V \cap X$ fermé pour $\sigma(X,H))$
on suppose que les intervalles d'ordre de X sont $\sigma(X,H)$ compacts.
on suppose quelques condition de séparabilité sur X.

Alors, si toute $\mu \in M^+(X, H)$ est donnée par une mesure de Radon sur $X \setminus 0$ pour $\sigma(X, H)$ alors, pour toute suite (x_n) de X telle que $\sum x_n = x$ on a $\sum p(x_n) < \infty$ pour chaque semi norme p continue sur E.

C'est un résultat d'application sommante: l'identité de X est absolument sommante.

application: B Banach. $E = B'$, $H = B \subset E' = B''$

alors $X \subset B'$, et $\sigma(X, B)$ complet vérifie le théorème ssi la jauge j de $X \cap B'_1$ vérifie:

$\exists f$ sur $X - X$, linéaire, avec $f \leq j \leq kf$ sur X.

Il est équivalent de demander pour un cône $\subset$ Banach que ses mesures coniques soient localisable (pour la $\| \|$), (dual B')
ou que les mesures vectorielles à valeurs dedans, soit données par une $\int$ de Bochner.

R. Becker Paris VI

Some remarks on σ-fields and measurable functions.

Let Σ be a σ-field and $A \subset \Sigma$
let $\sigma(A)$ be a smallest σ-field which contains an A.
B.V Rao posed the following question:
Does there exist a σ-field Σ
s.t. $\forall_{A \subset \Sigma} \left(\sigma(A) = \Sigma \Rightarrow \exists_{a \in A} \sigma(A \setminus \{a\}) = \Sigma \right)$
(it means that Σ does not have minimal

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generator)
The following Theorem answers this question

Theorem 1. (Aniszczyk , Frankiewicz)

(1) Let $B(NS_{\omega_1})$ be a σ-field generated by non stationary subsets of ω_1 Then B does not have minimal generator [$a \in NS_{\omega_1}$ if $\exists$ closed unbounded set disjoint with a]

(2) Assume CH. then

(a) $P(\omega_1)$ (all subsets of ω_1) does not have minimal generator
(under assumption MA + ¬CH $P(\omega_1)$ has minimal generator (Silver) 1974

(b) σ-field of Lebesgue measurable sets (on $\mathbb{R}$) and σ-field of subsets of $\mathbb{R}$ having the Baire property does not have the minimal generator

Theorem 2 If ZFC is consistent then ZFC + $MA_{\sigma\text{-linked}}$ + $\diamondsuit_{c=\omega_2}$ a Boolean algebra of Lebesgue measurable sets modulo null sets (LM/Δ) is not embeddable into $P(\omega)/fin$.

Theorem 3. If ZFC is consistent then ZFC + $MA_{\sigma\text{-linked}}$ + $\diamondsuit_{c=\omega_2}$ + σ-field of Borel subset on Cantor set is not embeddable into $P(\omega)/fin$.

Problem. Assume LM/δ is embeddable into $P(\omega)/fin$. Is ~~there~~ it true that under this assumption there is a Borel lifting for Lebesgue measurable sets.

(Aniszczyk, Frankiewicz)

Theorem 4 Under assumption MA
$P(\omega)/\Delta_d$ is isomorphic to
$P(\omega)/\Delta_l$ where

$$\Delta_d = \{ A \subset \omega \mid \overline{\lim_n} \frac{|A \cap n|}{n} = 0 \}$$

$$\Delta_l = \{ A \subset \omega \mid \overline{\lim_n} \sum_{\substack{a \in A \\ a \leq n}} \left(\frac{1}{a+1}\right)(\ln n)^{-1} = 0 \}$$

(Under CH Just + Krawczyk)

R Frankiewicz

Ergodicity of Cartesian Products via Triangle Sets.

Let T, S be non-singular, invertible ergodic transformations of the unit interval. When is $T \times S$ ergodic and when is $T \times S^{n(x)}$ ergodic (power-skew product $n : [0,1] \to \mathbb{Z}$ $(x,y) \mapsto (Tx, S^{n(x)}y)$).

Using the definition T is ergodic if $\forall$ pairs A, B of measurable sets of positive measure $\exists$ $n > 0$ $\ni$ $T^n A \cap B$ has positive measure, one would like to study $T \times S$ on rectangle sets $A \times C$, $B \times D$. However, this does not seem sufficient, : But I have no example of ergodic Transformations T, S to show this.

Def: $F \subset [0,1] \times [0,1]$ is a triangle set if (msbl of pos. msr)

1) $\exists$ $A \subset [0,1]$ $F \subset A \times [0,1]$

2) $\mu(Q_\varepsilon) > 0$ $\forall$ $1 > \varepsilon > 0$ where $Q_\varepsilon = \{ y : \mu(F \cap (A \times \{y\})) \geq (1-\varepsilon)\mu(A) \}$

Results such as the following Thm 2 are obtained.

Thm 1. every msbl set (of pos. msr) in the ~~plane~~ unit square is a disjoint union of triangle sets

Thm 2 if T, S satisfy property 3 Then $\overset{T\times S}{\cancel{S\times T}}$ is ergodic.
where $N_S(A,B) = \{n>0 : T^nA \cap B$ is of pos. msr$\}$
$N_T^\varepsilon(A,B) = \{n \geq 0 : \mu(T^nA \cap B) \geq (1-\varepsilon)\mu(A)\mu(B)\}$
property 3 : is $N_T^\varepsilon \cap N_S(C,D) \neq \emptyset$.

Ibid
Stanley J. Eigen

Extremal families of probability measures

The talk is a survey on some questions about the extremal structure of convex sets H of probability measures.
Let Ω be a Polish space with its Borel σ-algebra. Let $H \subset \mathcal{P}rob(\Omega)$ be convex.

Q1: Is H a Choquet set, i.e. a) is $H = \{r(\pi) : \pi \in \mathcal{P}(ex H)\}$ when $r(\pi)(B) = \int \nu(B)\,\pi(d\nu)$. b) in part a) π is uniquely determined by $r(\pi)$ iff $\mathbb{R}_+H$ is a lattice cone. Here a simple sufficient condition is that H is of the form $H = \bigcap_{n=1}^{\infty}\{\mu : \int f_n\,d\mu \leq a_n\}$ where (f_n) is a sequence of Borel fn. and $(a_n) \in \mathbb{R}^{\mathbb{N}}$. (v. Weizsäcker-Winkler '79).

Q2: Characterize ex H : Here we explain the Martin-boundary for Brownian motion following the ideas of P. Martin-Löf and Dynkin. It is pointed out that for other (possibly infinite dimensional) diffusion processes the analogue questions ~~gas~~ are open and interesting.

Q3: Does there exist a measurable map $\varphi : \Omega \to ex H$ such that $\nu\{\omega : \varphi(\omega) = \nu\} = 1$ for all $\nu \in ex H$?
Answer : Yes if (1) H is a Choquet set (2) $\mathbb{R}_+H - \mathbb{R}_+H$ is a sublattice of all signed measures on $(\Omega, \mathcal{A})$ and (3) ex H is $\sigma(\mathcal{M}(\Omega), C_b(\Omega))$ - σ-compact. No, if (3) is surpressed. This is due to Preiss (contained in a paper of Mauldin - Preiss - v Weizsäcker) in Ann of Prob.

Q4: Find a computational algorithm for π based on statistical data on r(π). ~~[illegible]~~ Here I report on the ~~[illegible]~~ Dr thesis of W. Kriger in Kaiserslautern. It turns out that general methods to construct Choquet type measures lead to practical answers for problems in latent structure analysis ~~e.g.~~ or simply finding out the proportion of visitors of a swimming pool from Kaiserslautern ~~[illegible]~~ under random ~~sta~~ weather conditions.

Heinrich v. Weizsäcker, Kaiserslautern

28/6-83: Critical exponents in the classical theory ~~on~~ of moments.

The main result is: For each $\lambda \in [1,2]$ there exists a probability μ on $\mathbb{R}$ with moments of all orders such that ($\mathcal{P}$ is space of polynomials!)

$$\begin{cases} 1\le p<\lambda \Rightarrow \overline{\mathcal{P}} = L^p(\mu) \\ \lambda<p \Rightarrow \overline{\mathcal{P}} \neq L^p(\mu). \end{cases}$$

That is each such λ may occur as „critical exponent".

Several related problems were discussed in particular whether λ may be >2!

Jens Peter Reus Christensen

Non Singular Ergodic Transformations.

Let $(X, \mathcal{B}, \mu)$ be a Lebesgue probability space and $\mathcal{G}(X)$ the group of nonsingular transformations of $(X,\mathcal{B},\mu)$ onto itself. On $\mathcal{G}(X)$ put the coarse topology; i.e $T_n \to T$

coarsely if $\| U_{T_n} f - U_T f \| \to 0 \ \forall f \in L^1(X)$, where $U_T : L^1(X) \to L^1(X)$ is the L^1-isometry ~~defined~~ associated to T, $(U_T f)(x) = f(Tx) \cdot \frac{d\mu T}{d\mu}(x)$ for $f \in L^1(X)$.

With this topology $\mathcal{G}(X)$ is a complete metrisable space.

Thm 1. The transformations $T \in \mathcal{G}(X)$ such that the skew product extension $T^* : X \times \mathbb{R} \to X \times \mathbb{R}$ [where $T^*(x,t) = (Tx, t + \log \frac{d\mu T}{d\mu}(x))$ and $X \times \mathbb{R}$ has product measure $d\mu \times e^{-t} dt$] is ergodic on $(X \times \mathbb{R}, \mu \times e^{-t} dt)$ form a dense G_δ subset of $\mathcal{G}(X)$ with the coarse topology.

U. S. Prasad

Measure theory and "amarts"

Let $\mathcal{A}_n$ be a sequence of algebras of subsets of a space Ω with $\mathcal{A}_1 \subset \mathcal{A}_2 \subset \cdots$, $\mathcal{A} = \bigcup_{n=1}^{\infty} \mathcal{A}_n$. Let $\theta_n : \mathcal{A}_n \to E$, E Banach space, be a sequence of additive set functions of bounded variation s.t. (i) $\lim_{n\to\infty} \theta_n(A) = \theta(A)$ exists for all $A \in \mathcal{A}$; (i') $\theta : \mathcal{A} \to E$ is of bounded variation (ii) there exists a sequence $\nu_n : \mathcal{A}_n \to [0, \infty[$, of additive set functions with $(\nu_{n+1} | \mathcal{A}_n) \leq \nu_n$ and $\nu_n(\Omega) \to 0$ If $\lambda : \mathcal{A} \to [0, \infty[$ is countably additive then $\theta_n(A) = \int_A f_n \, d\lambda + \theta_n'(A)$ where $\theta_n' \perp \lambda$ and $f_n \in L_E^1$ provided that E has RNP. We can prove that $f_n \to f$ a.e. (λ) where $\theta(A) = \int_A f \, d\lambda + \theta'(A)$, $\theta' \perp \lambda$ The relationship of this theorem (proven in Manuscripta Math. Vol. 4 (1971) and Lec. Notes in Maths. Vol. 541 (1976)) to certain other convergence theorems including those concerning "amarts" are discussed.

S D Chatterji

Combinatorial and geometric problems in measure theory.

Let $x_1 > x_2 > \dots \; x_n > \dots \quad x_n \to 0$ be an arbitrary sequence of positive numbers tending to 0. Is it true that there always is a set of positive measure E which does not contain a subsequence similar to E? This is an old problem of mine and I offer 100 dollars for a proof or disproof.

Is it true that there is an absolute constant C so that if E is a set in the plane of measure $> C$ then E contains the vertices of a triangle of area 1? Perhaps the best value of C is given by a circle the inscribed equilateral triangle of which has area 1. If E has infinite measure the result is easy.

Let c be given and $r > r_0(c)$. Let E be in $|z| < r$ and the measure of E is $> c r^2$. Is it then true that E contains the vertices of an equilateral triangle of side > 1? Perhaps Furstenberg proved this. Straus further asked: The conclusion perhaps remains true if we only assume that the measure of E is larger than $r f(r)$ or perhaps only $C r^2$?

Székely a young Hungarian mathematician conjectured that if E is a set so that the intersection of E with the circle $|z| < r$ has measure $> c r^2$ for all $r > r_0$ then the set E realises all sufficiently large distances? (i.e. to every d, $0 < d < \infty$ there are two points z_1 and z_2 in E whose distance is d).

P Erdős 1983 VI 29

Some remarks on invariant liftings

The following results were discussed: If G is a non-discrete locally compact group, then there exists no left-invariant Borel lifting. G admits a bi-invariant lifting iff for each $x \in G$, $C_G(x) = \{y \in G : xy = yx\}$ is open in G. A connected locally compact group admits a bi-invariant linear lifting iff it is amenable. For $X = \mathbb{R}^n$, G the group of affine transformations of determinant 1, there is no G-invariant linear lifting on X (with respect to Lebesgue measure)

Viktor Losert, Wien

Complementation & Conjugation

We present a characterisation of minimal complements for structures generated by a finite partition, showing by example its failure for countable partitions. The characterisation has a reformulation in the case of two-fold partitions involving Borel embeddings, as well as an application to the problem of when the union of Blackwell sets is again Blackwell.

The analogous problem of maximal conjugation is looked at, and some partial results involving 0-1 transition kernels and measurable selectors are presented.

Rae Michael Shortt,
Houghton.

Tensor products of Banach spaces:

The talk reports on some recent results concerning the injective and projective tensor products (denoted $X \check{\otimes} Y$ and $X \hat{\otimes} Y$ respectively) of two Banach spaces X and Y.

Recent examples show that X and Y can have the RNP and be weakly sequentially complete while $X \hat{\otimes} Y$ contains c_0 and hence fails these properties.

Moreover, the following theorem was proved recently (cf. Acta Mathematica, to appear) to answer a conjecture of Grothendieck: Any B-space E of cotype 2 (resp. separable) can be embedded isometrically into a space X (resp. separable) such that $X \check{\otimes} X = X \hat{\otimes} X$ and such that both X and X^* are of cotype 2, and X/E has the RNP and the Schur property. Related results are discussed concerning the possibility of embedding in a similar way any arbitrary B-space E into a $\mathcal{L}^\infty$-space. (cf. a joint paper with J. Bourgain, in preparation).

Gilles Pisier, Paris 6.

Random Homeomorphisms.

Several methods of constructing homeomorphisms of [0,1] onto itself were discussed. Two of these were specifically investigated. The first construction is as follows. First, the value of the homeomorphism at 1/2 is chosen with uniform distribution over [0,1). Next, the value at $1/4$ is chosen with uniform distribution from 0 to the value already chosen at $1/2$ and independently the value at $3/4$ is chosen at random from the interval from the value at $1/2$ to 1. Continue this process. This defines a probability measure P on the homeomorphisms of $[0,1]$ which fix 0 and 1. The second method can be derived from the first by taking the average right translate of P with respect to P: $P_a(E) := \int_H P(Eg)\,dP(g)$. It turns out that the measure P_a is also derived from a "point" process. So, both P and P_a have the important property that one can make computer experiments to obtain information about their properties. Both P and P_a give each nonempty open set positive measure. Besides this they have certain other "natural" invariance properties. P and P_a are both invariant under "time reversal". P_a is invariant under inversion whereas P is not. Most importantly, P is invariant under scaling between $i/2^n$ and $i+1/2^n$. This means that one gets P back when one scales the conditional distribution of P given the values at $i/2^n$ and $i+1/2^n$ onto the homeomorphisms of the interval. On the other hand P_a does not have this property anywhere.

it was shown that P almost all homeomorphisms have derivative 0 at 0 whereas P_a almost all homeomorphisms have upper rt. derivative +∞ at 0 and lower rt. derivative 0 at 0. Finally, the structure of the fixed point set of P and P_a random homeomorphisms were discussed and several point-sets of computer generated P and P_a random homeomorphisms were exhibited.

R. Daniel Mauldin
NTSU, Denton Texas
Siegfried Graf
NTSU, Denton Texas
and Univ. Erlangen-Nürnberg

On the planar ~~plainar~~ representation of a measurable subfield

This talk sketched a simpler ~~the~~ proof of a slightly sharper version of the planar representation theorem of Rokhlin (1949) and Maharam (1950). Let (Ω, B, m) be a Polish measure space, with a σ-finite completed Borel measure ~~m~~ m, and let a be a countably σ-generated subfield of the Borel field ~~set~~ B. Then there is a measure-preserving isomorphism of almost all of Ω onto a ~~a~~ certain Borel subset ~~set~~ Z of the plane, taking B to the relative Borel sets of Z and a to the relative vertical Borel cylinders, on which the measure is planar Lebesgue measure ~~[illegible]~~ on an ~~the~~ ordinates set ~~set~~ ~~at a countable set~~, x, together with a sequence of linear sets ~~in the lines~~

~~[illegible]~~, the measures being absolutely continuous with respect to linear Lebesgue measure, ~~except~~ and ~~for~~ countably many atomic ~~on each line~~ points. The proof uses 2 devices (both known): (a) the use of two Marczewski functions to embed Ω suitably in the plane, (b) the fact that a function $f(x, y)$ that is Borel measurable in x for fixed y and monotone and continuous from the right in y for each fixed x is Borel measurable.

Dorothy Maharam
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The Radon-Nikodym Theorem for Positive Operators.

The main purpose of the talk was to discuss the following Radon-Nikodym type factorization theorem for positive operators defined on vector lattices.

Let L and M denote Dedekind complete vector lattices and let $\mathcal{L}_m(L,M)$ denote the Dedekind vector lattice of order-continuous order bounded linear transformations of L into M. A local operator on a vector lattice is a positive linear transformation that leaves invariant all the bands of the underlying space. The family of all such densely defined linear operators on L is denoted by $\mathrm{Orth}^{\infty}(L)$. By a Radon-Nikodym type factorization theorem we mean a factorization of the form $S = T\circ\pi$, where S, T are positive order continuous linear transformations and π is a local operator. If μ and ν are countably additive measures, then $d\nu = f d\mu$ holds if and only if ν is absolutely continuous w.r.t. μ and the Radon-Nikodym derivative f plays the role of the local multiplication operator. For positive operators we have the following result. Let L & M be as above and $0 \leq S, T \in \mathcal{L}_m(L,M)$. If T has the Maharam property i.e. maps intervals onto intervals, then the following conditions are equivalent: (i) S is contained in the band generated by T; (ii) S is absolutely continuous w.r.t. T, i.e., for all $0 \leq u \in L$, Su is contained in the band generated by Tu;

(iii) there exists a local operator $\pi \in \mathrm{Orth}^{\infty}(L)$ such that $S = T \circ \pi$.

A dual form of this results leads to a factorization theorem for linear lattice homomorphisms generalizing a result of Kutateladze. For spaces L and M of measurable functions the result relates to earlier results of D. Maharam-Stone.

Since every order bounded linear operator from L into M may be uniquely extended to a larger space E containing L having the Maharam property and the order continuity property the above Radon-Nikodym type factorization theorem has a wide application range analogous to the classical R-N theorem for measures.

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Radon Measures - with W. F. Pfeffer.

The first part of the talk gave consequences of the results announced in W. F. Pfeffer's talk. These are:

Theorem Let X be (i) weakly θ-refinable or (ii) (MA+¬CH) metalindelöf. Then every complete Radon measure in X is decomposable, and every Radon measure in X is Maharam. [decomposable = strictly localizable, Maharam = localizable].

Example (CH). There is a metalindelöf space X and a Radon measure in X which is not Maharam.

The second part compared various approaches to obtaining sufficient conditions for a (say, completely regular) space to be a Radon space. These are:

(i) Every open subset of X is Souslin-$\mathcal{K}$ (actually implies every open subset is σ-compact) ⇒ X is Radon.

(ii) X is Souslin ⇒ X is Radon (as in Schwartz's book)

(iii) X is hereditarily weakly θ-refinable, has no discrete subsets of measurable cardinality, and is universally Radon measurable.

(i) and (ii) are not directly related, but both are subsumed by (iii), which is the most general result presently known. In some situations (for example, when dealing with Eberlein compacts), its full generality is needed.

R. J. Gardner
presently at U.P.M., Dhahran, Saudi Arabia.

Measure and Integral - a new gambit : A procedure to construct the Daniell integral extension and the Baire integral extension of a pre-integral with swiftly convergent sequences taking on the role traditionally played by Cauchy sequences. Unlike Cauchy sequences, the swiftly convergent sequences converge almost everywhere dominatedly and almost uniformly. If (I, L) is a pre-integral define $\{f_n\}_m$ in L to be swiftly convergent if $\sum_n \|f_{n+1} - f_n\| < \infty$, and define a set N to be null if $N \subset \{x : \sum_n |f_{n+1} - f_n|(x) < \infty\}$ for some swiftly convergent $\{f_n\}_n$. Let L^1 be the class of real valued functions f which are a.e. limits of swiftly convergent sequences $\{f_n\}_n$ in L, and $I'(f) = \lim_n I(f_n)$. Then I' is well defined on L'; ~~and~~

Theorem 1. I' is an integral extension of I and L' is norm complete, ordercomplete (for $\geq$ a.e.) and null complete.

Theorem 2 : If L_1 is the $\overset{\text{sub}}{\wedge}$family of L-Baire functions in L^1 and $I_1 = I' / L_1$ then (I_1, L_1) is an integral extension of (I, L) and it is the smallest integral extension. The space L_1 is norm complete and order complete.

T.P. Srinivasan

R

Remarks on some borel structures

I) Let I be the unit interval and let I! be the group of all permutations of I. It is known the following

Theorem (B.V. Rao, Coll. Math. 21(1970)).

a) There are two separable σ-fields $\mathcal{A}_1, \mathcal{A}_2$ on I such that for every $p, q \in I!$ the σ-field $p(\mathcal{A}_1) \cap q(\mathcal{A}_2)$ is not countably generated (c.g.);

b) There are two separable σ-fields $\mathcal{A}_1, \mathcal{A}_2$ on I such that the σ-field $\mathcal{A}_1 \cap \mathcal{A}_2$ does not contain any non-trivial c.g. σ-field.

Assuming CH we can strengthen the theorem to the following

Theorem (CH). There are separable σ-fields $\mathcal{A}_1, \mathcal{A}_2$ on I s.t. $\forall p, q \in I!$ $p(\mathcal{A}_1) \cap p(\mathcal{A}_2)$ does not contain any non-trivial c.g. σ-field.

(A c.g. σ-field $\mathcal{A}$ is non-trivial iff it is generated by a countable partition)

II) We offer a very short proof of a recent theorem of R.M. Shortt (1982) that if A is an analytic non-borel set in R such that R∖A is totally imperfect then A is not isomorphic with any product $A_1 \times A_2$ of two uncountable (analytic) spaces A_1 and A_2

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Duality theorems for marginal problems

Given Hausdorff spaces X_i and tight prob.-measures μ_i on their Borel σ-algebras $\mathcal{B}(X_i)$ together with a function $g : X = \prod_{1\le i\le n} X_i \to \mathbb{R}$ (which for this abstract is assumed to be bounded), the following two problems are investigated:

(MP) maximize $\mu^*(g)$, where μ is a tight prob.-measure on $\mathcal{B}(X)$ with marginals $\pi_i(\mu) = \mu_i$ for $1 \le i \le n$,

(DP) minimize $\sum_{1\le i\le n} \mu_i(f_i)$, where the functions f_i are μ_i-integrable with the property $\sum_{1\le i\le n} f_i \circ \pi_i \ge g$.

Assuming the spaces X_i to be compact metrizable and the function g to be continuous, by use of the theorems of Hahn-Banach and Riesz it is not hard to show the „duality theorem" $S(g) = I(g)$ where

$$S(g) := \sup\{\mu^*(g) : \mu \text{ as in (MP)}\},$$
$$I(g) := \inf\{\textstyle\sum_{1\le i\le n} \mu_i(f_i) : f_i \text{ as in (DP)}\}.$$

A thorough examination of S and I, however, proves them to have the properties of a capacity with respect to the lattice $\mathcal{C}^u$ of upper semicontinuous functions on X. Therefore, by first showing $S(g) = I(g)$ for $g \in \mathcal{C}^u$ one obtains this duality theorem for all $\mathcal{C}^u$-analytic functions – a result which holds without special topological assumptions and may be carried over to all $\bigotimes_{1\le i\le n} \mathcal{B}(X_i)$- ~~$\mathcal{B}(X_1)$~~ measurable functions g.

H. Kellerer (Munich)

Invariant Daniell Integrals E. G. F. Thomas

Let X be a Hausdorff space and let L be a sublattice of the vector lattice of real continuous functions on X. We consider localizable Daniell integrals μ on L, i.e. Daniell integrals definable by Radon measures on X by the formula $\mu(\varphi) = \int \varphi \, dm$. Then, if G is a group of homeomorphisms of X leaving L invariant, it is claimed that, under appropriate hypotheses, the invariance of μ under the action of G implies the quasi-invariance of a certain class of Radon measures on a quotient G-space Y of X. Conversely every quasi-invariant measure class on Y can be ~~thus~~ obtained in this way from some G invariant triple (X, L, μ).

A TENSOR PRODUCT VECTOR INTEGRAL

Let X and Y be Banach spaces. An integration theory of X-valued functions with respect to a Y-valued measure λ is given. To achieve the completeness of the space of integrable functions, we need consider those functions with values in a locally convex Hausdorff space which contains a copy of the space X. Let α be a cross norm on the tensor product $X \otimes Y$. A function f with values in W is called λ-integrable if there exist $c_i \in X$, measurable sets E_i, $i = 1, 2, \ldots$, such that $\{c_i \otimes \lambda(F_i)\}_{i=1}^{\infty}$ is unconditionally summable in the completed space $X \otimes_\alpha Y$ for every measurable set $F_i \subset E_i$ and

X such that if $w' \in W'$, then $\langle w', f \rangle$ can be expressed as the sum of the absolutely summable sequence $\{\langle w', c_i \rangle \chi_{E_i}\}_{i=1}^{\infty}$.

This integral can be applied to have the Fubini theorem of scalar-valued functions with respect to the product of two Banach space-valued measures.

Susumu OKADA, Flinders Univ.
Australia

Two problems connected with Kantorovič distance.

1. Let $\{X_t, t \in T\}$, $X_t : (\Omega, \mathfrak{A}, P) \to E$ be a family of random E-valued variables, where E is a complete separable metric space $(E, \rho(\cdot,\cdot))$, and let $\{\mu_t, t \in T\}$ be the corresponding family of their distributions: $\mu_t = P X_t^{-1}$. Such a family $\{X_t, t \in T\}$ is called Kantorovič set if for any $t_1, t_2 \in T$ the equality $E_P\, \rho(X_{t_1}, X_{t_2}) = \varkappa(\mu_{t_1}, \mu_{t_2})$ where $\varkappa$ denotes the Kantorovich distance.

A class of spaces (E, ρ) is described, for which for every given family of probability distributions $\{\mu_t, t \in T\}$ there exist a corresponding Kantorovič set of E-valued random variables.

2. Suppose that on a finite-dimensional Banach space $(\mathbb{R}^n, \|\cdot\|)$ two Borel probability measures μ and ν are given, each is absolutely continious with respect to Lebesgue measure, $\iint \|x-y\|\, d\mu\, d\nu < \infty$. Then there exist an optimal one-to-one plan of transport $\mu \to \nu$, i.e. such a Borel measure m_0, defined on $\mathbb{R}^n \times \mathbb{R}^n$ and concentrated on the graphic of a one-to-one measure preserving map $(\mathbb{R}^n, \mu) \longrightarrow (\mathbb{R}^n, \nu)$ that

its marginal distributions are μ and ν and
$\iint \|x-y\| \, dm_0 = \varkappa(\mu,\nu)$

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When Radon measures are saturated.

Let μ be a Radon measure in a Hausdorff space X with a concassage D. We show that if the space X is metacompact or metalindelöf and MA + $\neg$CH holds, then $\cup D$ is Borel, and paracompact whenever it is regular. It follows that under the above assumptions μ is saturated. Under CH, an example is given indicating that in metalindelöf spaces these results depends on the axioms of set theory. Applications to the decomposability of Radon measures are given.

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On multiple random measures and integrals

The goal is to study integrals of the form

$$I_n(f) = \int_0^1 \int_0^1 \dots \int_0^1 f(x_1,\dots,x_n)dM(x_1)\dots dM(x_n)$$

where $M(x)$ is the hom. process with independent increments determined by Lévy measure.

Basic questions: 1) What's the class of f's for which $I_n(f)$ exists?
2) What is the distribution of $I_n(f)$.

Some answers (old)

ad 1) Urbanik + Woyczynski 1967 : $I_1(f)$ exists $\Leftrightarrow f \in L_\Phi$ & $\Phi(s) \sim u^{-2} \int_0^{1/u} \frac{L(u)}{u^3} du$.

ad 2) by Ito's formula (W - brownian motion)

$$I_2(I_\Delta) = \int_0^1 \int_0^t dW(s)dW(t) = \int_0 W(t)dW(t) = \frac{1}{2}(W^2(1)-1)$$

in general

$$\int_{0 \le t_0 < t_1 \dots < t_n \in T} \int dW(t_1)\dots dW(t_n) = H_n(W(T),T)$$

where $H(t,s)$ are gener. Hermite polynomials (Cameron - Martin 194..)

<u>Applications</u>: Quantum field theory, statistics etc.

<u>Second</u> <u>order</u> <u>case</u>: (Rosiński, Sudga - Engel 1982)
LNM Proc. Clev. Conf. Mem AMS

Product measure $M_2(A) = M(A_1)\cdot M(A_2) \in L_1$ $A = A_1 \times A_2$, $F(A) = EM^2(A)$

$$\mu(B) = F(\pi(B \cap D)) + F \otimes F(B \setminus D)$$

where π is standard projection, D - diagonal.

<u>Thm</u>. μ is a control measure for M_2. Thus M_2 extends to countably additive measure.

Functions with $N(f) < \infty$ are M_2 integrable.

$$N(f) = \int f(t,t)dF(t) + \left[\iint_{I^2 \setminus D} |f(s,t)|^2 F(ds) F(dt)\right]^{1/2}$$

and

$$E \exp itI_2(f) = [d(2it,f)]^{-1/2}$$

where

$$d(\alpha,f) = 1 + \sum_n \frac{(-\alpha)^n}{n!} \int \dots \int \begin{vmatrix} f(t_1,t_1) \dots f(t_1,t_n) \\ \vdots \\ f(t_n,t_1) \dots f(t_n,t_n) \end{vmatrix} dt_1 \dots dt_n$$

(extension of Varberg's etc.)

<u>Stable case</u> (Szulga – Woyczynski, J. Mult. An 1983)

$$E\exp iuM(A) = e^{-|A||u|^p} \qquad 1<p<2$$

$$f(s,t) = \sum c_{kj}\,\phi_k(t)\phi_j(s) \qquad I_2(f) = \sum_{k,j} c_{kj} \underset{\substack{\shortparallel \\ X_k}}{\int \phi_k\, dM(s)} \underset{\substack{\shortparallel \\ X_j}}{\int \phi_j\, dM(t)}.$$

<u>Thm</u>. If (ϕ_k) is the Haar system normalized in L_p then if $\sum_{k,j} |c_{kj}|^{p/2} < \infty$ then $\sum c_{kj} X_k X_j = I_2(f)$ converges. a.s.

<u>Iterated</u> <u>integrals</u> (S. Cambanis + W.A. Woyczynski).

In discrete version.

$$Q_n = \sum_{j=1}^{n} \sum_{k=1}^{j-1} f(kj) M_k M_j \qquad (M_k) \text{ i.i.d stable.}$$

Q_n converges in probability iff

$$(*) \qquad \sum_{k=1}^{\infty} \sum_{j=1}^{k-1} |f(kj)|^p \Big(1 + \log \frac{1}{\sum_{j=1}^{k-1} |f(k,j)|^p}\Big) < \infty$$

The proof goes via ~~proof~~ lemma characterizing p-stable -radonifying operators $T = (f(k,j)) : \ell^q \to \ell^p$. [redacted]
The necessity of the above was noticed by Pisier.
For multiple integrals one gets higher powers of the logarithm.

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Some combinatorial properties of measures

For any measure μ we define its norm:

$$\|\mu\| = \min\{|X| : \mu(X) > 0\}$$

and compare it with additivity: add(μ)

Obviously add$(\mu) \leq \|\mu\|$.

Using somewhat stronger axiom then the existence of measurable cards we proved (using Method of Solovay) that it is consistent to have a real valued measure such that $2^\omega \geq \text{add}(\mu) < \|\mu\|$. This inspired the transfer of large cards to real valued Large, just changing in the definition of large cardinal binary measures to real valued measures. Trivialy large cardinal is real valued Large. Also Ulam's separation theorem holds for real valued large. The consistency of real valued large relative large can be proved essentialy using Solovay's method. The forcing argument is about the same and Solovays construction of real valued measure starting from binary one can be applied to all ultrafilters that witness k was large cardinal in the starting model, and thus prove it is real valued large in generic extension. A number of filter combinatory properties can be translated to measures, so that it makes sense to consider some clasification for real valued measures, analogusly to Rudin-Keisler klasification of ultrafilters. Kunen, unpublished, proved that product measure extension axiom consistency follows from the consistency of strongly compact card. also using Solovay's method. PMEA implies Normal Moore Space conjecture as well as number of other interesting properties. But this was very similar to what I call real valued compact cardinals.

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On reconstruction of convex bodies from a finite number of Steiner symmetrals.

Given a plane convex body K, there are several known results on the determination of K, known a finite number of Steiner symmetrals. The older (1962) is Giering's theorem, stating that given a convex body, there exists three directions such that they determine K in the set of all convex bodies. Gardner and McMullen (1980) proved that the set of directions $\mathcal{D}$ distinguish among convex bodies iff ~~they~~ $\mathcal{D}$ is not the linear image of diagonals of a regular n-gon. As a counterpart for this result, we prove that whenever $\mathcal{D}$ does not distinguish among convex bodies, there exists a collection $\mathcal{C}$, such that card $\mathcal{C} = \mathfrak{c}$ and $\mathcal{C}$ are not distinguished by $\mathcal{D}$. In particular, from the result of Gardner and McMullen, it follows that four conveniently choosen directions are enough. If we denote with $\mathcal{K}_*$ the family of all the convex bodies, it can be proved that the mapping $\delta : \mathcal{K}_* \to \mathcal{K}_*^4$ assigning to every K the quadruplet of its Steiner symmetrals, is continuous and that also $\delta^{-1} : \delta(\mathcal{K}_*) \to \mathcal{K}_*$ is continuous, so the problem of reconstructing K from $\delta(K)$ is well-posed. For sets, which are union of inscribed parallelograms having sides parallel to D_1 and D_2, a reconstruction procedure is presented.

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We consider modular functions ($m a\vee b + m a\wedge b = ma + mb$) on an abstract Lattice with values in a topological group. The analogue of the "Frechet-Nikodym distance" $d(A,B) = \mu(A \triangle B)$ in this setting is $d_m(a,b) = \sup |m(v) - m(u)|$ where the sup is taken over all u, v with $a\wedge b \leq u \leq v \leq a\vee b$. (in case the group has a quasinorm?, which we assume for this abstract) This is a (generalized) pseudometric on L for which the translations $a \mapsto a\wedge x$ and $a \mapsto a\vee x$ are contractions and $d(a\wedge b, b) = d(a, a\vee b)$. If M is a family of such modular functions for which ma_n increase uniformly for m in M, then the M-topology (generated by $\{d_m : m \in M\}$ coincides with the equi M-topology, generated by the distance $d = \sup_m d_m$. A consequence is a local Rybakov-type theorem, for B-space-valued modular functions on a distributive lattice. Several related problems are left open, even in the real case.

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Ergodic Theory and Truncated Limits

E is a Banach Lattice such that (A) There is a weak unit u, i.e., $u \in E_+$ and $u \wedge |f| = 0$ implies $f = 0$, and (B) Every norm-bounded increasing φ_k in E_+ converges (strongly) to $\varphi \in E$. For $f_n \in E_+$, $WTL f_n = \varphi$ (weak truncated limit of f_n is φ) means that $\forall k = 1, 2, \dots$ $f_n \wedge ku \xrightarrow{w} \varphi_k$, $\varphi_k \uparrow \varphi$. $WTL f_n = WTL f_n^+ - WTL f_n^-$. TL (strong truncated limit) is defined analogously. If $f_n \in E_+$ then $WTL f_n = 0 \Rightarrow TL f_n = 0 \Rightarrow f_n$ admits a subsequence $f_{n_i} = g_i + h_i$, $g_i, h_i \in E_+$, $\|g_i\| \to 0$ and $h_i \wedge h_j = 0$ if $i \neq j$. (Under the extra assumption that E^* has a quasi-interior point, $\sup \|f_n\| < \infty \Rightarrow \exists f_{n_i} = g_i + h_i$ with $g_i \xrightarrow{w}$ and $h_i \wedge h_j = 0$ if $i \neq j$, $g_i, h_i \in E_+$). The interest of WTL is that if $\sup_n \|f_n\| < \infty$ then WTL f_{n_i} exists for a subsequence f_{n_i}. WTL is unique, and $g_i \xrightarrow{w} g$, (W)TL $g_i = 0$ imply $g = 0$. If $E = L_1$ then (W)TL $g_n = 0 \Leftrightarrow g_n \to 0$ in measure on sets of finite measure. Let T be a positive linear operator on E; if WTL $f_n = \varphi$ and WTL $Tf_n = \psi$ then $T\varphi \leq \psi$. Hence if $\|f_n - Tf_n\| \to 0$ then $T\varphi \leq \varphi$. Let $A_n = (1/n)\sum_{i=0}^{n-1} T^i$, $\sup_n \|A_n\| < \infty$. Theorem 1 The following are equivalent: (i) $\exists$ a weak unit $\bar{u} \ni T\bar{u} = \bar{u}$. (ii) $\forall$ band projection $P \neq 0$, $PA_n u$ has no TL null subsequence (iii) $\forall H \in E^*_+$, $H \neq 0$ one has $\underline{\lim}_n H(A_n u) > 0$. If $\sup \|T^n\| < \infty$, A_n can be replaced by T^n. — The proof uses that if $\varphi = \text{WTL}\, A_{n_i} f$ then $T\varphi \leq \varphi$. Theorem 2. If $0 \leq \rho = T\rho$, P_ρ is the band projection on ρ, $f \in E_+$ then TL $P_\rho A_n f$ exists. — But unless one makes an assumption which implies that $\varphi = \text{WTL}\, A_{n_i} f$ is invariant, TL$A_n f$ need not exist. One such assumption is (C) $\forall h \in E_+$, $\alpha > 0$ there is $\beta = \beta(h, \alpha)$ such that if $0 \leq f \leq h$, $\|f\| \geq \alpha$, $g \in E_+$, $\|g\| \leq 1$, then $\|f + g\| \geq \|g\| + \beta$ — Theorem 3 Assuming also (C), if $\|T\| \leq 1$, $f \in E_+$ then $A_n f = g_n + h_n$, $g_n, h_n \in E_+$, $g_n \xrightarrow{s} \varphi$ with $T\varphi = \varphi$ and TL$h_n = 0$.

(Joint work with Mustafa Akcoglu)

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Let $1<p<\infty$, let $(X,\mathcal{F},\mu)$ be a Lebesgue measure space. Let $L_p = L_p(X,\mathcal{F},\mu)$. We observe that the arguments of T. Ando (Pacific J. Math. 17, 391-405, 1966) yield the following theorem, where, for each $f \in L_p$, f^* denotes the unique vector in $L_p^* = L_q$ such that $\|f\|_p^p = \|f^*\|_q^q = (f, f^*)$. Theorem Let $p \neq 2$ and let $M \subset L_p$. Then the followings are equivalent: (i) M is a closed linear manifold and M^* is also a linear manifold with $M^* = \{f^* \mid f \in M\}$. (ii) There is an $f \in M$ and a sub σ-algebra $\mathcal{G} \subset \mathcal{F}$ such that $M = \{gf \mid g \text{ is } \mathcal{G}\text{-measurable and } gf \in L_p\}$. (iii) M isometrically isomorphic to the L_p space of another measure space. — The implication (i) $\Rightarrow$ (ii) (which is the only non-trivial part of the theorem) is an example of the fact that if $p \neq 2$ then some pointwise properties of the elements of L_p can be formulated in terms of Banach-space conditions on L_p. Note that if $T: L_p \to L_p$ is a contraction then $M = \{f \mid Tf = f\}$ satisfies the conditions of the Theorem (most easily (i)). This gives Ando's theorem.

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BOUNDEDNESS FOR UNIFORM SEMIGROUP VALUED SET FUNCTIONS

LET $X = (X, \mathcal{U})$ BE A UNIFORM SPACE. A SUBSET $\mathcal{V} = \{V_m : m \in \mathbb{N}\}$ OF $\mathcal{U}$ IS CALLED A UNIFORM BOUNDING system if (i) every V_n is symmetric; (ii) If $n < m$, then $V_n \subseteq V_m$; and (iii) $V_n \circ V_m \subseteq V_{n+m}$. Let $B \subseteq X$. We say that B is $\mathfrak{V}$-bounded if there exist $n \in \mathbb{N}$ and a non-emity finite subset F of X such that $B \subseteq V_n[F]$. We say that B is bounded if, for every symmetric element $V \in \mathfrak{U}$, B is $\mathfrak{V}$-bounded for $\mathfrak{V} = \{V^n : n \in \mathbb{N}\}$. This important notion already appeared in Bourbaki's book on General Topology, where is indicated the good properties of this extension of the notion of total boundedness.

In particular, if X is a commutative Hausdorff topological group, this notion coincides with the notion of boundedness used by Kats and Musiak. If X is connected, then this notion coincides with the notion of "additive boundedness" used by Darst, Landers & Rogge, Traynor, Turpin, etc.

Let now X be a commutative Hausdorff uniform semigroup with neutral element 0, and let $\mathcal{R}$ be a ring of subsets of a fixed set T. Some of the results of this paper are:

1) Let $\mu : \mathcal{R} \rightarrow X$ be s-bounded and additive, and let $\mathfrak{V} = \{V_n : n \in \mathbb{N}\}$ be a uniform bounding system. Then μ is $\mathfrak{V}$-bounded.

This result generalizes well-known results of Kats and Musiak.

2) Let $\mathfrak{V} = \{V_n : n \in \mathbb{N}\}$ be a uniform bounding system and let (μ_n) be a sequence of X-valued s-bounded additive set functions defined on a σ-ring $\mathcal{R}$. If, for every $E \in \mathcal{R}$, the sequence $(\mu_n(E))$ converges to 0, then the μ_n are

uniformly bounded.

3) A generalization of the Nikodym uniform boundedness theorem.

4) A generalization of the following result of Dieudonné (1951): Let T be a compact Hausdorff space and let $\mathcal{M}$ be a family of regular Borel measures on T such that, for each open subset U of T, $\sup\{|\mu(U)| : \mu \in \mathcal{M}\} < +\infty$, then $\sup\{\|\mu\|(T) ; \mu \in \mathcal{M}\} < +\infty$.

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SELECTORS IN NONSEPARABLE SPACES

Let T be a set with a paving $\mathcal{M} \subset \mathcal{P}(T)$ closed to finite intersections. Let (X, ρ) be a metric space. Suppose $F : T \to X$ is a multimap that is weakly $\mathcal{M}_\sigma$-measurable with values that are nonempty, separable, ρ-complete, and totally-bounded relative to <u>some</u> metric on X (not necessarily complete). This latter property holds, for example, when the values of F are compact or when X is separable. Let $\mathcal{S}(F) = \{ f : T \to X \mid f \text{ is point-valued, } (\mathcal{M}^-)_\sigma\text{-measurable, and } f(t) \in F(t) \ \forall\ t \in T \}$. ($(\mathcal{M}^-)_\sigma$ = countable unions of differences of sets in $\mathcal{M}$.) The basic question is: When is $\mathcal{S}(F) \neq \emptyset$?

<u>THEOREM</u>. Suppose T is metrizable and $F : T \to X$ is as above.

(i) If $\mathcal{M}_\sigma$ = Borel sets of additive class $\alpha < \omega_1$, then F has a Borel selector of class $\omega \cdot \alpha$.

(ii) If $\mathcal{M}_\sigma$ = Souslin sets of T, then $\mathcal{S}(F) \neq \emptyset$.

(iii) If $\mathcal{M}_\sigma$ = (Souslin sets) $\cap$ (Co-Souslin sets), then $\mathcal{S}(F) \neq \emptyset$.

(iv) If $\mathcal{M}_\sigma$ = a countably generated σ-algebra on T (any set), then $\mathcal{S}(F) \neq \emptyset$.

This theorem is obtained in the following way: We first introduce an appropriate "nonseparable" analog of the familiar countable reduction property [shown by Maitra and Rao to be equivalent to the basic selection theorem of Kuratowski and Ryll-Nardzewski].

<u>DEFINITION</u> Fix a class $\mathcal{L} \subset \mathcal{P}(T)$ [e.g., the Souslin sets]. We say that a family $\mathcal{E} \subset \mathcal{P}(T)$ has an <u>$\mathcal{L}$-reduction</u> iff $\exists\ \mathcal{R} = \{ R_E : E \in \mathcal{E} \} \subset \mathcal{L}$ having the following properties:

(i) $R_E \subset E \ \forall\ E \in \mathcal{E}$,

(ii) $\mathcal{R}$ is disjoint and $\bigcup \mathcal{R} = \bigcup \mathcal{E}$,

(iii) $\mathcal{R}$ is $\mathcal{L}$-hereditarily-additive [i.e. whenever $L_E \subset R_E$, $L_E \in \mathcal{L}$, $\forall\ E \in \mathcal{E}$, then the union of any subfamily of $\{ L_E : E \in \mathcal{E} \}$ belongs to $\mathcal{L}$].

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REMARK. It can be shown that, if T is an analytic metric space, then $\mathcal{E}\subset\mathcal{P}(T)$ has a Borel-reduction iff $\mathcal{E}$ has a σ-discrete Borel set refinement.

THEOREM 1 (On selection). If, whenever $\mathcal{U}\subset\mathcal{P}(X)$ is open and locally finite, $\{F^{-1}(U) : U\in\mathcal{U}\}$ has an $(\mathcal{M}^-)_\sigma$-reduction, then $\mathcal{S}(F)\neq\emptyset$.

THEOREM 2 (On hereditary additivity). Every point-finite $\left\{\begin{matrix}\Sigma_\alpha\\ \text{Souslin}\\ \text{Co-Souslin}\end{matrix}\right\}$-additive family in T (metrizable) is $\left\{\begin{matrix}\Sigma_{\omega\alpha}\\ \text{Souslin}\\ \text{Co-Souslin}\end{matrix}\right\}$-hereditarily-additive.

REMARK (1) $\exists$ a point-countable $\mathcal{F}_\sigma$-additive $\mathcal{E}\subset\mathcal{P}(\text{reals})$ that has a non-analytic point selection [So Thm. 2 fails badly for point-countable families]

(2) It is consistent with ZFC that $\exists$ $X\subset\mathbb{R}^2$ and a disjoint $\mathcal{F}_\sigma$-additive $\mathcal{E}\subset\mathcal{P}(X)$ which is not $\mathcal{F}_\sigma$-hereditarily-additive.

THEOREM 3. (On reduction). If $\mathcal{E}\subset\mathcal{P}(T)$ is point-finite and $\mathcal{M}_\sigma$-hereditarily-additive, then $\mathcal{E}$ has an $(\mathcal{M}^-)_\sigma$-reduction.

The stated selection theorem now follows from Theorems 1, 2, and 3. □

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Hausdorff dimension of intersections of sets in n-space

Let A and B be Borel sets in $\mathbb{R}^n$ with Hausdorff dimensions $\dim A = s$ and $\dim B = t$. What can be said about the Hausdorff dimensions of the intersections $A \cap fB$ where f runs through the isometry group of $\mathbb{R}^n$? Some examples indicate that in general there is very little to say. But if we assume that t is integral and B is sufficiently nice, e.g. a C^1 submanifold or t rectifiable, then $\dim A \cap fB = s+t-n$ for many isometries f provided $s+t-n \geq 0$. For general Borel sets A and B we have to use a larger family of transformations; similar results hold if we replace isometries by similarities, that is, maps composed of translations, rotations and homotheties. Then $\dim A \cap fB \geq s+t-n$ for many similarities f. Equality does not hold in general, but it does under some extra conditions on B. For example, it suffices to assume that B has positive t dimensional lower density at all of its points.

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Gruppen- und vektorwertige s-beschränkte Inhalte

Im folgenden wird eine Methode zur Behandlung von gruppen- und vektorwertigen Inhalten angegeben, mit der sich zahlreiche Sätze einheitlich und mit einem Minimum an technischem Aufwand beweisen lassen. Eine wesentliche Rolle spielen dabei FN-Topologien.

Sei $\mathcal{G}$ eine vollständige Hausdorff-topologische Gruppe, R ein Boolescher Ring, u_s die feinste s-beschränkte FN-Topologie auf R und $(\tilde{R}, \tilde{u}_s)$ die vollständige Hülle von (R, u_s). Dann läßt sich jeder s-beschränkte Inhalt $\mu : R \longrightarrow \mathcal{G}$ in eindeutiger Weise stetig zu einem Inhalt $\tilde{\mu} : \tilde{R} \longrightarrow \mathcal{G}$ fortsetzen. Zur Untersuchung von s-beschränkten Inhalten $\mu : R \longrightarrow \mathcal{G}$ betrachtet man zuerst die Fortsetzungen $\tilde{\mu} : \tilde{R} \longrightarrow \mathcal{G}$ und überträgt dann die Ergebnisse auf die Restriktionen $\mu = \tilde{\mu} | R$. Daß die Untersuchung der Fortsetzungen $\tilde{\mu}$ einfacher ist als die von μ, liegt daran, daß $\tilde{R}$ eine (als Verband) vollständige Boolesche Algebra und $\tilde{\mu}$ τ-stetig ist (d.h. für jedes nach unten gerichtete System (x_γ) in $\tilde{R}$ mit $x_\gamma \downarrow 0$ gilt $\tilde{\mu}(x_\gamma) \longrightarrow 0$.

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Isolated and anti-isolated measures

Denote by X a nonempty set, by $\underline{\underline{R}}$ a σ-ring of subsets of X, by $\mathcal{M}(\underline{\underline{R}})$ the order complete vector lattice of all real-valued σ-additive measures on $\underline{\underline{R}}$ and by $\mathcal{M}$ a band of $\mathcal{M}(\underline{\underline{R}})$. The notion of an isolated and an anti-isolated measure with respect to $\mathcal{M}$ is introduced, and examples are given (if X is a Hausdorff space and $\mathcal{M}$ the set of all Radon measures on the relatively compact Borel sets of X, then $\mu \in \mathcal{M}$ is isolated iff it is atomical and it is anti-isolated iff it is atomfree). Another characterization of these measures is given in terms of a representation of $(X, \underline{\underline{R}}, \mathcal{M})$, and it is proved that the set of isolated and the set of anti-isolated measures form orthogonal bands of $\mathcal{M}$ the sum of which is $\mathcal{M}$.

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Some measure theoretic applications to the Pettis integral

What is an exact description of the difference between a function which is Bochner integrable and one which is Pettis integrable? Let (Ω, Σ, μ) be a probability space and S the Stone space of the measure algebra $\Sigma/\mu^{-1}(0)$ with $g \to \hat{g}$ denoting the $L^\infty(\Omega, \Sigma, \mu) \to C(S)$ isometry. If $f: \Omega \to T \subset K$ where K is compact and T is completely regular define the Stonian transform $\hat{f}: S \to K$ by $\hat{f}(s)$ is the unique point in K representing the multiplicative functional $\Gamma_s(\varphi) = \widehat{\varphi \circ f}(s)$, $\varphi \in C(K)$. Then $\hat{f}$ is continuous and may be used to prove the following. Let $\mathfrak{X}$ be a Banach space and $f: \Omega \to \mathfrak{X}$ a bounded weakly measurable function. Then $f = g + h$ where g is strongly measurable and h is purely weakly measurable—i.e., $\hat{h}(s) \notin \mathfrak{X}$ a.e. Moreover g and h are unique up to strong equivalence with $x^*h \cdot x^*g = 0$ a.e. Since a bounded strongly measurable function is always Bochner integrable it follows that the Pettis integrability of f depends only on that of h. Further since $\int_\Omega x^* h \, d\mu = \int_S x^* \hat{h} \, d\mu$ and $\hat{h} \notin \mathfrak{X}$ a.e. the question of Pettis integrability lies in whether or not $\hat{h}(S)$ "convexifies" back into $\mathfrak{X}$. For a perfect measure space the sets $\overline{\text{conv}\, \hat{f}(\text{clopen sets})}^{w^*} \cap \mathfrak{X}$ are norm separable and allow the proof. But M. Talagrand has extended the result to the general case: f is Pettis iff $\overline{\text{conv}\, \hat{f}(a)}^{w^*} \cap \mathfrak{X} \neq \square$ $\forall$ clopen a. This then leads thru a duality argument to the equivalence of (a) f is Pettis integrable (b) If $x_\alpha^* \xrightarrow{\mathfrak{X}} x^*$ then if $x_\alpha^* f = 0$ a.e, then $x^* f = 0$ a.e. (c) If $x_\alpha^* \xrightarrow{\mathfrak{X}} x^*$ then if $x_\alpha^* f \leq 1$ a.e., then $x^* f \leq 1$ a.e. Moreover if $\hat{f}(S)$ lies in the Baire 1 class for $\mathfrak{X}$ in $\mathfrak{X}^{**}$, then f is Pettis integrable. If, on the other hand, $\hat{f}(S) \subset \nu\mathfrak{X} \setminus \mathfrak{X}$, where $\nu\mathfrak{X}$ is the real compactification of $\mathfrak{X}$ (in $\sigma_{\mathfrak{X}^*}$), then f is not Pettis integrable. Finally, if $\{x^* \hat{f} : \|x^*\| \leq 1\}$ is weakly compact in $C(S)$, for f purely weakly measurable, then f is not Pettis integrable. This same set is weakly compact for a strongly measurable function.

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~~Mu~~ Bilinear Integration of Multifunctions.

The purpose of this paper is to consider some extensions and also an approximation of Lyapunov's theorem in terms of the bilinear m-integral of N. Dinculeanu. The integration is performed successively with respect to a non-atomic, a direct sum and a Darboux vector measure. The necessary counterexamples are provided.

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Gateaux differentiability and a class of topological spaces

Let $\mathcal{C}$ be the category of topological spaces defined by:
$K \in \mathcal{C} \Longleftrightarrow$
all C, S, T, V top. spaces where V is a Baire space, ~~and~~ $C \subseteq K \times S$,
and for all $\varphi : C \to T$ perfect
and all $\lambda : V \to T$ continuous
then there exists a selection
$(\psi(u), \xi(u))$, $u \in V$ and a dense G_δ set G, so that ψ is continuous at each point of V (ξ doesn't matter).

This category has nice permanence properties and contains, for example, the duals of Asplund spaces (in the weak* topology), Eberlein compacts, compact metric spaces and RNP sets. The important property of $\mathcal{C}$ is that if $\underline{X}^*$ (in the weak* topology) is in $\mathcal{C}$ then $\underline{X}$ is a weak Asplund space.

This is the first theorem that gives permanence properties of a large class of weak Asplund spaces.

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BOREL MEASURABLE SELECTORS AND THE RADON-NIKODYM PROPERTY

We discuss several applications of the theorem that an upper semi-continuous set-valued map from a metric space into the weak topology of a Banach space with its weak topology has a Borel measurable selector if the range is everywhere dentable or equivalently has, the Radon-Nikodym property.

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Law of Large Numbers in Banach Space

Let (Ω, Σ, μ) complete prob. space,

$f\cancel{g} : \Omega \to E$ Banach space (no measurably assumed)

On $\Omega^{\mathbb{N}}$, let $g_n(t) = \sum_{i \le n} \cancel{g} f(t_i)$ for $t = (t_i)$.

Th TFAE

a) $\mu^{\mathbb{N}}$ a.e $\lim g_n(t)$ exists in norm

b) f is Pettis integrable and a.e. $\lim g_n(t) = P\text{-}\int f$

c) f is Pettis integrable and

$$\int \| g_n(t) - P\text{-}\int f\,d\mu \| \, d\mu^{\mathbb{N}}(t) \longrightarrow 0$$

d) $\int^* \|f\|\,d\mu < \infty$ and the set

$Z = \{x^* \circ f;\ x^* \in E^*_1\}$ is stable, i.e.,

$\forall A \in \Sigma,\ \mu A > 0,\ \forall \alpha < \beta\ \exists n$

$$(\mu^{2n})^* \{(s_1, \dots, s_n, t_1, \dots, t_n) \in A^{2n};\ \exists h \in Z \quad \forall i \le n,\quad h(s_i) < \alpha,\ h(t_i) > \beta\} < (\mu A)^{2n}$$

<u>Corollary</u> A countable sequence (C_n) of Σ is not a Glivenko-Cantelli class $\Longleftrightarrow$ $\exists A,\ \mu A > 0$ $\forall n$, for almost all choices $t_1, \dots, t_n \in A$, each subset of $\{t_1, \dots, t_n\}$ is the trace on $\{t_1, \dots, t_n\}$ of a set C_p.

M. Talagrand

Isometries of measure algebras

(joint work with S. GRAF)

Let $(X, \mathcal{A}, \mu)$ be a measure space, $\mathfrak{A} := \mathcal{A}/\mu$ be the associated measure algebra and $\mathcal{E} := \{a \in \mathfrak{A} : \mu(a) < \infty\}$. For $a, b \in \mathcal{E}$ the Nikodym distance of a and b is given by $d(a,b) := \mu(a \Delta b)$.

Question: What are the isometries of the metric space $(\mathcal{E}, d)$ and of certain subspaces of that space.

Thm 1: If μ is σ-finite, then $T: \mathcal{E} \to \mathcal{E}$ is an isometry iff there is a measure preserving Boolean automorphism Φ of $\mathfrak{A}$ and $a_0 \in \mathcal{E}$ s.t. $T(a) = \Phi(a) \Delta a_0 \quad \forall a \in \mathcal{E}$.

Thm 2: If X is Polish and μ a σ-finite Borel measure on X then $T: \mathcal{E} \to \mathcal{E}$ is an isometry iff there is a bimeasurable measure preserving bijection $F: X \to X$ and $A_0 \in \mathcal{A}$, $\mu(A_0) < \infty$ s.t.
$T[A] = [F(A) \Delta A_0]$ for all $A \in \mathcal{A}$, $\mu(A) < \infty$.

Now let X be always Polish and μ be a locally finite Borel measure on X (μ is then a σ-finite Radon measure). Let $\mathcal{A}$ be the Borel-σ-algebra, $\mathcal{K} := \{K \subseteq X : K \text{ cpt}\}$, $\mathcal{F} := \{F \subseteq X : F \text{ closed}, \mu(F) < \infty\}$, $\mathfrak{K} := \mathcal{K}/\mu$ and $\mathfrak{F} := \mathcal{F}/\mu$.

Call $F: X \to X$ an almost homeomorphism (mod μ) iff:
1) F is a bimeasurable bijection, 2) There is $Y \in \mathcal{A}$, $\mu(X \setminus Y) = 0$, Y invariant under F s.t. $F|_Y$ is a homeomorphism of Y.

Then we have:

Thm 3: Suppose that μ is a diffuse measure (i.e. $\mu\{x\} = 0 \ \forall x \in X$)
a) $T: \mathfrak{F} \to \mathfrak{F}$ is an isometry iff there is a measure preserving almost homeomorphism $F: X \to X$ s.t. $T[A] = [F(A)] \ \forall A \in \mathcal{F}$
b) Suppose, in addition, that X is locally compact. Then $T: \mathfrak{K} \to \mathfrak{K}$ is an isometry iff there is a measure preserving almost homeomorphism $F: X \to X$ s.t. $T[A] = [F(A)] \ \forall A \in \mathcal{K}$

G. Mägerl, Math. Inst. Univ. Erlangen-Nürnberg

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Error Asymptotics and Defect Corrections

July 3 to July 9, 83

Willi Schönauer, E. Schnepf, K. Raith, Univ. Karlsruhe:
Numerical Engineering: Experiences in designing PDE software with selfadaptive variable step size / variable order difference methods.

We want to develop robust and efficient general purpose software for the solution of arbitrary nonlinear systems of elliptic and parabolic PDE's in a rectangular domain. The relative accuracy is prescribed and the method must choose itself the optimal grid and order independently in all coordinate directions t, x, y, z. There must be selected also the optimal solution method, within a given scale of methods, for the solution of the resulting linear system for the computation of the Newton-Raphson correction. The key to the solution method is the use of families of difference formulae. The discretization error is determined by the difference of difference formulae of these families. The error equation tells us how to choose the grids and orders and how to stop the Newton-Raphson iteration. The Newton-Raphson correction and the discretization error define the stopping criterion for the iterative solution of the linear equations. A polyalgorithm selects the solution method for the linear equations by the comparison of normalized convergence factors. An essential condition is that the resulting program must be fully vectorizable for vector computers (Super computers). The whole solution process is a continuous compromise between robustness and efficiency which quite naturally contradict each other. There is discussed the sequence "method, algorithm, program" from the point of view of numerical engineering.

W. Schönauer

On a combination of defect corrections with adaptive finite element methods applied to singularly perturbed differential equations.

First, a simple idea will be presented having the aim to improve numerical solutions of linear problems. This approach is based on a-posteriori error estimates which, in a certain sense, monitor the error improvement. On the other hand, realistic a-posteriori error estimates allow an adaptive computation of the numerical approximations, so that a combination of both aspects leads to adaptive defect correction methods. For an example of a linear, singularly perturbed o.d.e., the a-posteriori error estimates associated with a finite element method will be given and numerical results will be presented. This approach can be extended to nonlinear problems, provided that initial approximations (numerical or asymptotic ones) are available. For a rather general class of nonlinear singularly perturbed o.d.e.'s, the (linear) equations of the defect corrections and the corresponding a-posteriori error estimates will be given. Again, the latter allow an adaptiv computation of the defect correction terms.

H.-J. Reinhardt, Goethe-Univ., Frankfurt

Local defect correction and domain decomposition techniques

The usual defect correction methods for elliptic problems work with discretizations in the whole domain. We describe a 'local defect correction', when a second discretization is defined only locally. One multi-grid version of this method is a well-known local mesh-refinement technique. If there are several separated refined regions they are coupled by coarse grids. On the other hand we can formulate the elliptic problem as a set of two equations in two overlapping subdomains with additional boundary conditions for the new interior boundaries. We discuss the solution of the corresponding discrete systems of equations by a multi-grid process, in which the major part of the computations can be performed simultaneously in every subdomain.

W. Hackbusch, Kiel

Extrapolation in a convection-diffusion equation with a boundary layer

We examine a typical example of a convection-diffusion equation with a boundary layer,

$$u_x = \nu (u_{xx} + u_{yy}), \qquad x > 0,\ y > 0,$$

with $u = 0$ for $x > 0, y = 0$ and $u = 1$ for $y > 0, x = 0$. This problem has a parabolic boundary layer near the x-axis if ν is a small, positive number. A typical computation in practice uses central differences with Δy much smaller than Δx. We find that if one is only interested in computing the drag $\partial u / \partial y$ on the boundary $y = 0$, then the error depends on Δx only as $O(\beta_x^2 / p^{5/2})$ where β_x is the horizontal cell Reynolds number, $\Delta x / (2\nu)$, and $p = x/(2\nu)$. The error in computing $\partial u / \partial y$ depends on Δy, however, as an expansion in even powers of the vertical cell Reynolds number $\beta_y = \Delta y / (2\nu)$.

Gerald Hedström, Lawrence Livermore Laboratory

Iterated Deferred Corrections Implementations and its application to seismic ray tracing.

After an introductory historical discussion on implementations and supporting theory for iterated deferred corrections, both in ordinary and in partial differential equations, we describe in some detail our latest computer program PASVA4 (with M. Lentini) that solves nonlinear two-point boundary value, nonlinear, first order systems of equations with discontinuous right hand sides and additional parameters and algebraic equations. A non-trivial application to two-point seismic (or acoustic) ray tracing in an inhomogeneous, isotropic, piece-wise smooth media is described and some tri-dimensional results are shown in the form of computer graphics.

V. Pereyra, Universsidad Central
Caracas, Venezuela

Deferred/defect correction for stiff ordinary differential equations

A simple example is given illustrating that defect correction is a nontrivial generalization of deferred correction. The role of global error asymptotics and the meaning of stiffness are discussed, and then the existence of asymptotic expansions for stiff equations is considered. For purposes of deferred/defect correction it is necessary to consider variable stepsize. Error per step and error per unit step are compared. In particular, it is shown that local extrapolation, which is a generalized error per unit step, does not quite increase the order by one.

Robert D. Skeel
Univ. of Illinois at Urbana-Champaign

Asymptotic Expansions for Semi-linear Equations of Elliptic Type.

A class of finite difference schemes, due to H.-O. Kreiss, for a weakly coupled mildly nonlinear elliptic system of the type

$$-\Delta u_j(x) = f_j(x, u_1(x), \dots, u_m(x)) \quad , \quad 1 \le j \le m, \; x \in \Omega$$
$$u_j(x) = g_j(x) \qquad 1 \le j \le m, \; x \in \partial\Omega$$

where Ω is a bounded region in $\mathbb{R}^n$ is considered.
The schemes use the standard $(2n+1)$-point-approximation of the Laplacian combined with polynomial extrapolation of degree k near the boundary. The FD-scheme thus obtained is neither of monotone type nor symmetric. No conditions regarding the definiteness or the sign-pattern of $\partial_u(f_1, \dots, f_m)$ are imposed. The convergence of the FD-solutions to isolated solutions of the original system and the existence of asymptotic expansions are stated for $k \le 4$. Finally we report on numerical test in which the asymptotic expansions are exploited by a modified deferred correction method.

Harry Munz
Universität Tübingen

Mixed Defect Correction Iteration for the solution of a singular perturbation problem.

A numerical method (mixed defect correction) is described, for the solution of a two-dimensional elliptic singular perturbation problem. The method is an iterative process in which two discretizations are used: one with and one without additional artificial diffusion. The method works well for problems with interior- or boundary layers. The resulting discretization is stable and yields a 2nd order accurate approximation in the smooth parts of the solution, without using any special directional bias in the discretization method

Pieter W. Hemker
Math. Centrum, Amsterdam.

Asymptotic Expansions of the Global Error for the Implicit Midpoint Rule (stiff case).

A new stability result for the implicit midpoint rule is given. This new result gives estimates independent of the stiffness of the (scalar) differential equation. By means of this stability result one is able to obtain an asymptotic expansion in powers of the average step size for a stiff scalar linear problem (discretized by the implicit midpoint rule). In discretizing this problem we use a fine mesh in the boundary-layer region, a coarse mesh far away from the boundary-layer, and a gradual increase of the step size in between.

In this way an asymptotic expansion for the global error can be proved, provided that the number of gridpoints multiplied by the logarithm of the "stiffness" is small. This expansion is valid uniformly on the domain of integration.

M. van Veldhuizen
Vrije Universiteit, Amsterdam.

Error estimation by defect calculations in finite element discretizations

Some idéas on computable pointwise estimates of the error in finite element discretizations are presented. For the equation $a(u,v) = (f,v)$ where $a(\,,\,)$ is a bilinear form, $(\,,\,)$ an innerproduct a FEM discretization can be written $a(\Sigma c_i \phi_i, \phi_j) = (f, \phi_j)$, $j = 1,2,..N$ with $\phi_i, i=1,2..N$ a basis for the subspace in which we seek the approximate solution. The defect for equation j is defined as $R_j = a(w, \phi_j) - (f, \phi_j)$ where w is properly defined from the solution data $\underline{c} = (c_1, c_2, .. c_N)$ e.g. $w = \Sigma c_j \psi_j$ with $\psi_j, j=1,2,..N$ the basis of another FEM subspace. The talk concerns practical ways of computing R_j for some one and twodimensional problems.

Bengt Lindberg
KTH, Stockholm

Defect Correction and Stiff Ordinary Differential Equations

The B-convergence properties of certain Defect-Correction methods, based on the implicit Euler scheme and on the implicit midpoint rule, are discussed. It turned out, that full B-convergence results do not hold in this case; nevertheless it was possible to prove "Restricted B-convergence" for these methods, i.e. satisfactory global error bounds could be derived under the following assumption: The eigenvalues of the Jacobian $f_y(0, y_0)$ at the starting point are either "moderately sized" or satisfy a relation $h \cdot \mathrm{Re}(\lambda_i) << 0$

Reinh. Frank
(REINHARD FRANK)

Defect Correction in Adaptive Discretization

THIS is a subject discussed in the first half of W. Hackbuschs talk at this meeting. We consider the use of defect correction in a specific and applied way as a process for transferring local fine mesh accuracy to a perhaps global coarse mesh. We have in mind the pressure equation solution for IMPES methods applied to oil reservoir simulation where the finer mesh patches are placed at the injection and production wells (and perhaps at the moving fluid interfaces).

Steve McCormick
Colorado State University

Defect corrections on infinite intervals.

If the infinite interval is truncated to a finite one for the solution of a boundary value problem on R, new boundary conditions are needed. The boundary conditions given in the literature for general problems make necessary the use of very large intervals.

For the simple differential equation $-u''+\varkappa^2 u = g$ we give the exact discrete boundary conditions for the discrete Newton method with equidistant grid. Also an algorithm is presented which computes these boundary conditions along with the discrete Newton iteration from some basic "data". If these data are known the algorithm computes the discrete Newton iterates on the infinite grid exactly.

Bernhard Schmitt
Universität Marburg.

Iterative Residual Correction in Function Spaces

Part 1: Basic Methodologies (Lecture by Edgar Kaucher)

Part 2: Block Relaxation Formalism (Lecture by Willard Miranker)

We combine ~~thus~~ two recently developed approaches in computation to provide a methodology for self-validating numerics for function space problems (e.g.; differential equations, integral equations, ...)

(i) E-methods; the use of the method of residual correction to furnish existence, uniqueness and bounds of good quality for the solution of computational problems.

(ii) Functoids: Let M be a Banach with operations $\Omega = \{+, -, \cdot, /, \int\}$. A model for computer implementation of the structure $(M; \Omega)$ is defined by a semimorphism $S_N: M \to S_N(M)$. This induces a semimorphic algebraic structure $(S_N(M); S_N(\Omega))$ which is called a functoid.

~~(iii)~~ These talks deal with the method of residual correction in this setting.

In the context of functoids novel features and problems for iterative residual correction (IRC) emerge.

Part 1 of the lecture introduces the formalities of E-Methods (self-validating methods) and functoids. It then characterizes some typical phenomenon of IRC in floating point number spaces and develops analogues for IRC in the functoid. Examples then illustrate the ideas.

Part 2 of the lectures introduces a formal block relaxation process. The process is conducted on the infinite linear system which is the isomorphic counterpart of a linear

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functional equations in terms of a basis in a Hilbert space $\mathcal{H}$. A convergence criteria is developed. Examples relating the ~~two~~ parts of this lecture are given as well as some computational results.

Edgar Kaucher
Edgar Kaucher

Willard L. Miranker

Step-by-step stability in the numerical solution of shallow water equations

Shallow water equations - a nonlinear system of hyperbolic partial differential equations - describe flow problems in fluid dynamics. Applications are found in e.g. oceanography (water elevation due to storms) and meteorology (weather prediction). Numerical computations with these problems are frequently hampered by nonlinear instability phenomena, the so-called exponential blow ups. We will provide insight in the origin of these blow ups. Next we will provide insight in how to overcome the difficulties by means of the energy method stability analysis. We also propose approximation schemes, derived along the lines of this analysis, for which stability is guaranteed, despite the nonlinearities. Among others, an LOD-scheme which can be implemented such that only tridiagonal systems of linear algebraic equations need to be solved.

Jan G. Verwer
Mathematical Centre
Amsterdam.

A Quadraturformula Method for Integralequations with a logarithmic Singularity

The Integralequations treated are of the form $\int_0^1 k(x,x')u(x')dx' = f(x)$, $x \in [0,1]$ with $k(x,x') = \frac{c}{\pi} \ln |2 \sin \pi(x-x')| + \bar{k}(x,x')$, with $\bar{k}, f, u$ being 1-periodic and C^∞-smooth (or some variants). A "primitive approximation" is given by solving the system of linear equations $\sum_{\nu=0}^{N-1} h\, k(\lambda h, (\nu-\frac{1}{2})h)\, u_\nu = f(\lambda h)$, $\lambda = 0, \dots, N-1$, $h = \frac{1}{N}$, N odd and defining u_h by trigonometrical interpolation. For the underlying rough integration method an nonstandard asymptotic error expansion can be given, which – for each given order $O(h^p)$ – gives rise to an refined discretization method of this order. The properties of numerical stability are discussed and the theoretical results are tested by numerical experiments. It is indicated, that this method is a special case of a more general concept of a unified treatment of discretization of differential- and some classes of integral operators.

Helmut Brakhage, Universität Kaiserslautern

Sequential Defect Correction for High-Accuracy Algorithms

As was shown in the presentation of S. Rump, there exist algorithms in floating point arithmetic which compute the solution of algebraic problems to full floating point accuracy, almost irrespective of the condition of the problem. However, if the result intervals of such algorithms are fed into another such algorithm, a loss of accuracy occurs.

It is shown that the principles underlying these algorithms, viz. the representation of the approximations in staggered correction format and the exact computation of defects, may also be used for the coupling of such algorithms into one global high-accuracy algorithm. The strategy by which the required accuracy in the individual algorithms is achieved dynamically and automatically, comprises three passes through the sequence of algebraic problems: In ~~the~~ a first pass, the individual results are corrected to a preset accuracy, at the same time estimates of the relative condition numbers are determined. With the aid of these, the necessary accuracy is ~~achieved~~ obtained in the second pass. The third pass generates the required interval inclusions.

These ideas apply equally to the analytic defect correction algorithms described in the contributions of E. Kaucher and W.L. Miranker.

Hans J. Stetter

Hartree - Fock Methods

This talk is thought to be an introductory one to give the framework for the talks of B. Schmitt and R. Schwarz. It describes the way how one comes from the Schrödinger equation in wave-mechanics via the variational principle to the socalled Hartree-Fock equations (HFE) for the radial parts of the electron wavefunctions. The (HFE) are a coupled system of second order differential equations on a semi infinite interval with a singularity in the coefficients at $r = 0$,

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boundary conditions and orthogonality constraints. It is an eigenvalue problem as well. A short outline is given of methods recently used to solve this problem and some points are outlined where improvements could be gained especially using defect corrections.

Wolfgang Gross

Inclusion of the solution of linear and nonlinear equations

A synopsis have been given of new methods for solving algebraic problems with high accuracy. Examples of such problems are solving of linear systems, eigenvalue/eigenvector determination, computing zeros of polynomials, sparse matrix problems, computation of the value of an arbitrary arithmetic expression (in particular the value of a polynomial at a point), nonlinear systems, linear, quadratic and convex programming, over- and underdetermined linear systems etc. over the field of real or complex numbers as well as over the corresponding interval spaces.
The new algorithms are developed by means of a number of mathematical theorems and the corresponding algorithms which do verify the assumptions of the corresponding theorems on a computer. The appropriate computer arithmetic (developed by Kulisch and Miranker) is shortly described. All these algorithms based on our new methods have some key properties in common:

- every result is verified to be correct by the new algorithms
- the results are of high accuracy; the error of every component of the result is of the magnitude of the relative rounding error
- the solution of the given problem is verified to exist and to be unique within the computed error bounds
- the computing time of one of the new algorithms is of the same order as a comparable (pure) floating-point algorithm (the latter, of course, with none of the above features)

The key property of the new algorithms is that error control is performed automatically by the computer without any additional effort on the part of the user. The efficiency of the algorithms has been, for instance, demonstrated by inverting a Hilbert 21x21 matrix on a computer with 14 hex (17 dec.) mantissa. This is, after multiplying with a proper factor, the largest Hilbert matrix exactly storable on that computer. After automatically verifying, that this matrix is not singular, the inverse is included with least significant bit accuracy (lsba). That means, that the left and right bounds of all components of the inclusion are consecutive in the floating-point screen. Our experience shows, that our algorithms very often have the 'least significant bit accuracy' property.

Siegfried M. Rump

Newton methods for iterative refinement of eigenelements of linear operators

Let A be an $n \times n$ matrix in $\mathbb{C}^n$. To deal with close or multiple eigenvalues of total algebraic multiplicity m, we consider the nonlinear equation

$(*) \quad F(U) = AU - U(Y^H AU) = 0,$

where the unknowns are the m columns of the $n \times m$ matrix U normalized by $Y^H U = I$, where Y is a given $n \times m$ matrix. The columns of U span the invariant subspace M for A associated with the m eigenvalues of the $m \times m$ matrix $B = Y^H AU$. B represents $A_{\restriction M}$ in the adjoint bases (U, Y).

Starting from an approximate invariant subspace X for A, such that $Y^H X = I$, and applying Newton and modified Newton methods on $(*)$ yield several iterative schemes to refine on X. This is used in two ways:

(i) as a computational scheme,

(ii) as a means to derive a posteriori error bounds in terms of the n×m residual $R = AX - X(Y^H AX)$. The same method applies to a closed linear operator in a Banach space (integral or differential).

FChatelin

Françoise Chatelin

On multilevel iterative methods for integral equation of the 2nd kind and related problems

A unifying framework for multilevel iterative methods for integral equations of the second kind is presented. Particular cases include the methods of Brakhage, Atkinson, Hackbusch and Hemker and Schippers. Another particular case is the projection iterative method (Sokolov, Lucka, Kurpel) and the iterative aggregation method for systems of linear equations (Chatelin, Miranker, myself).

Jan Mandel

On the numerical solution of boundary value problems on infinite intervals

The error by truncating the infinite interval and the construction of asymptotic boundary conditions is discussed exemplarily for the problem $-y'' + \kappa^2 y = g$ on $(-\infty, \infty)$, where the inhomogeneity g has special properties as occuring in Hartree-Fock-theory. The construction of asymptotic boundary conditions is generalized to problems $-y'' + f(x) y = g$ on $(0, \infty)$, f having an asymptotic expansion at infinity, $f(\infty) \neq 0$.

Roland Schmary

Discrete Newton methods for the Bader Deuflhard methods in stiff initial value problems

Discrete Newton methods are defined as follows: Compute a discrete approximation ζ to the original (nonlinear) problem with exact solution z. Linearize the discrete problem at the solution $\zeta^0 := \zeta$ and compute the iterations ζ^l from $(F^h)'(\zeta^0)(\zeta^l - \zeta^{l-1}) = -$ (modified defect for ζ^{l-1}). Under certain, easily verifiable, conditions one finds the asymptotic expansion $\zeta^l(t) - z(t) = \sum_{j=(l+1)p}^{q_l} h^j e_{j,l}(t) + O(h^{q_l+1})$ in grid points t and for p, the order of the basic method. This approach is applied to the Bader-Deuflhard method for stiff initial value problems. The relations to other known results are discussed and experiments are reported

Klaus Böhmer
Universität Marburg

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~~in the [illegible] of [illegible] value problems on infinite interval.~~

~~The error by truncating the infinite interval and the evaluation of asymptotic boundary conditions is discussed numerically for the problem [illegible] where the eigenvalues y has several properties [illegible]. The construction of asymptotic boundary conditions is generalized to problems [illegible] of having an asymptotic expansion at infinity, $f(x) \neq 0$.~~

~~Roland [illegible]~~

~~Discrete Newton methods for the Bader-Deuflhard method in stiff initial value problems~~

~~Discrete Newton methods are defined as follows: Compute a discrete approximation of the original (nonlinear) operator with exact solution. Discretize the discrete problem at the points [illegible] and compute the iterations ξ^l from $F'(\xi^0)(\xi^l - \xi^{l-1}) = -F(\xi^{l-1})$ [illegible]. Under certain [illegible] one obtains the asymptotic expansion $\xi^l(h) - u(x) = \sum h^{2j} e_{jl}(x) + O(h^{[illegible]})$ [illegible] of the Bader method. This approach is applied to the Bader-Deuflhard method for stiff initial value problems. The relations to other known results are discussed and experiments are reported.~~

~~Hans [illegible]~~
~~Universität Freiburg~~

QUASIGROUPS and LOOPS

July 3 - July 9, 1983

The meeting is project-oriented and revolves around the plan of producing a two part text and monograph on quasigroups, related structures, and their applications. Seminar sessions and lectures are devoted to special parts of the projected book as well as the concept of the work as a whole. Emphasis is placed on cross-connections to other fields and applications of the theory, but the foundations of the subjects are reviewed, too, for the purposes of presentation in the book.

Hala Pflugfelder
Temple University
Philadelphia, PA U.S.A

A survey on topological quasigroups and loops

In the course of discussing the content of a projected textbook and monograph on quasigroups, related structures and their applications, we proposed a chapter on the state of information on topological and analytical quasigroups and loops. We sketched a general frame for the position of non-associative binary algebra and its applications in geometry within topological algebra in general; and, based on this framework, we proposed a tentative table of contents for the chapter of the monograph to the extent it covers topological quasigroups and loops.

Notably, we sketched the contents of the following four subchapters:

I. The general background of the structure of topological quasigroups and loops (Definitions, morphisms and congruences, separation, connectivity, the translation groups as topological transformation groups, uniformities, universal covering, construction methods, example catalogue)

II. Algebraic hypotheses (power-associativity, di-associativity, idempotency, distributivity)

III. Analytical loops (Hudson's partial solution of Hilbert's Fifth Problem for loops, Moufang Lie loops and Malcev-algebras, diassociative topological loops)

IV. Topological double loops (Generalities, double loops with associative multiplication or addition, distributivity, classification via projectivities and collineations, characterisations on the multiplications of the classical Hurwitz algebras)

Karl H. Hofmann, 07-05-83
FB Math, THD Darmstadt

7-7-83

A Survey of Methods of Construction of loops and quasigroup

Methods of construction were subdivided into the following Categories:

1. Constructions of loops arising as extensions of one loop or quasigroup or another. (Constructions discussed include the direct product, constructions using quasi factor systems, Crossed extensions, quasidirect products, twisted direct products, and constructions using normal forms.)
2. Other constructions of new quasigroups from given quasigroups. These include the singular direct product, generalized singular direct product in various forms, generalized semidirect product and generalized twisted singular direct product.
3. Constructions by defining new operations on existing algebraic structures. Structures mentioned in the discussion include, quasigroups, loops, groups, rings, fields, ternary rings, vector spaces, and exterior algebras
4. Constructions which define algebraic operations on geometric structures
5. Constructions based on designs
6. Constructions which define algebraic operations on unstructured sets
7. Structures obtained using the right regular representation

Orin Chein
Temple University
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U.S.A.

A survey on local differentiable quasigroups and webs

The following topics were reviewed:

1. Foliations on a differentiable manifold.
2. A d-web $W(d, n, r)$, $d \geq n+1$, of codimension r on a differentiable manifold of dimension nr.
3. Local differentiable n-ary quasigroups Q_r connected with a web $W(n+1, n, r)$
4. Canonical expansions of finite equations of Q_r
~~(It was stressed~~
5. W-algebras of $W(3, 2, r)$
6. Fundamental tensors of Q_r
7. Closure conditions on $W(n+1, n, r)$ and corresponding algebraic identities in Q_r (It was stressed that only in multicodimensional cases $r \geq 1$ there is a perfect correspondence between special webs and special quasigroups while for $r = 1$ all special webs coincide although in the corresponding quasigroups different identities hold).
8. 1-parameter subquasigroup and conditions of existence of 1-parameter subloops and subgroups in any direction.
9. A four-web $W(4, 2, r)$ on $(2r)$-dimensional manifold and two corresponding orthogonal quasigroups.

The results presented are due to M. A. Akivis (Moscow, U.S.S.R.) and his students and to V. Goldberg (NJIT, U.S.A.)

Vladislav Goldberg, Department of Mathematics,
New Jersey Institute of Technology,
Newark, N.J. 07102, U.S.A.

p-rank and association schemes of Commutative Moufang loops of exp 3

We construct new multidimensional, MD association scheme on the planes of exp. 3 commutative Moufang loop $E_n = L_3 \times \mathbb{Z}_3^{n-4}$ using the homomorphism $E_n \to E_n / Z(E_n)$. This is only one known MD realizing equality in Bose-Srivastava inequality for MD's. We also calculate 3-rank of incidence matrix of above loop

M. Deza, CNRS, Paris

Beziehungen zwischen Loops und deren Geweben.

Es wurde eine Klassifikation von Loops, die auf einer Klassifikation der zugehörigen Gewebe beruht, vorgestellt. Auch wurde diskutiert, wie sich die Eigenschaften der Loops in den Gruppen der Projektivitäten und in der Gruppe der Kollineationen der zugehörigen Gewebe widerspiegeln.

Karl Strambach (Erlangen)

CUBIC HYPERSACE QUASIGROUPS

In his book ("Cubic form", North Holland P. C. (1974)) MANIN generalized the classical construction of an abelian group in the set of non-singular points of a projective plane curve. Starting from a cubic hypersurface of dimension ≥ 2, the three-place relation of collinearity gives rise, in a suitable quotient, to an exponent 6 CML (or equivalently to a totally symmetric quasigroup (E, ·) whose lose satisfies $x^2 \cdot yz = xy \cdot xz$ and $x^2 \cdot x^2 = x^2$. But it is still an open question whether this loop can be non associative. The contents of the report are: (1) algebraic presentation of the cubic hypersurface quasigroups; (2) Geometrical motivations: Lamé's theorem, admissible relations; (3) the main open problem.

Lucien BÉNÉTEAU (~~Toulouse, France~~)
Univ. P. Sabatier, 31.062 - Toulouse, France

COMMUTATIVE MOUFANG LOOPS AND RELATED GROUPOIDS

The trimedial quasigroups (TQs) are isotopic to Commutative Moufang Loops (CMLs). They allow a synthetic study of the distributive quasigroups and the so-called cubic hypersurface quasigroups. If E is a TQ or a CML, and if its rank ρ (or more generally: its central rank) is finite, then the central nilpotency class k, as well as the orders of the central factor and the classes of the derived congruence are finite; they can be give bounds depending only on n. Several descriptions of free objects are given, including some exterior algebra representations which are faithful when the rank is small enough.

Lucien BÉNÉTEAU
Univ. P. Sabatier, 31.062 TOULOUSE FRANCE

Commutative idempotent entropic (CIE) quasigroups and related groupoids

The special rôle of the dyadic numbers D for the variety of CIE-quasigroups was discussed, and the lattice of all varieties described. The integers together with one of the quasigroup operations of D plays a similar rôle for the variety of entropic symmetric spaces and the dyadic numbers in the interval [0,1] with another of the quasigroup operations of D a similar rôle for the variety of CIE-groupoids. In both cases the lattices of varieties and equational bases were described.

A. Romanowska
(TH Darmstadt,
Techn. University Warsaw, Poland)

Centrality

This aspect of quasigroup theory deals with appropriate generalisations of familiar group theory concepts such as centres, nilpotence, solubility, modules, etc. The fundamental notion is that of a central congruence on a quasigroup: a congruence containing the diagonal or equality congruence as a normal subquasigroup. Corresponding to abelian groups, one obtains the class of Z-quasigroups („zentral"), those Q for which Q×Q is a central congruence on Q. Structure theorems and algebraic applications for

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these are given. Quasigroups of prime order are classified using the classification theorem for finite simple groups. Theorems are obtained relating quasigroups and their multiplication groups.

J. Smith, TH Darmstadt

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HARMONIC ANALYSIS UND DARSTELLUNGSTHEORIE OF TOPOLOGICAL GRUPPEN 10 JULY - 16 JULI 1983

Hypoellipticity of Left Invariant Differential Operators on Certain Nilpotent Lie Groups

Let N be a two-step nilpotent Lie group with Lie algebra $\mathfrak{n}$; we may write $\mathfrak{n} = \mathfrak{n}_1 \oplus \mathfrak{n}_2$ (as a vector space), with $\mathfrak{n}_2 = [\mathfrak{n}, \mathfrak{n}]$. This grading induces a grading of $\mathcal{U}(\mathfrak{n})$; a typical element of $\mathcal{U}(\mathfrak{n})$ is $L = \sum_{j=0}^{m} L_j$, where L_j is homogeneous of degree j. We say that N is of type (H), or an (H)-group, if for every $l \in \mathfrak{n}^*$ with $l|_{\mathfrak{n}_2} \neq 0$, the $Ad^*(N)$-orbit of l is $l + \mathfrak{n}_2^{\perp}$. Thus the irreducible representation corresponding to l by Kirillov theory depends only on $\xi = l|_{\mathfrak{n}_2}$; we thus denote the infinite-dimensional irreducibles by π_ξ, $\xi \in \mathfrak{n}_2^* \setminus \{0\}$. The 1-dimensional representations ρ_η are parametrized by $\mathfrak{n}_1^*$. We say that L is transversally elliptic if $\rho_\eta(L_m) \neq 0$ for all nonzero $\eta \in \mathfrak{n}_1^*$. Rothschild and Grigis have proved the following theorem about transversally elliptic operators L on (H)-groups: let $d(\xi)$ denote the product of the "small" eigenvalues of $\pi_\xi(L^*L) \cdot |\xi|^{-m}$ (these eigenvalues are analytic near ∞, and an eigenvalue is small if it $\to 0$ as $\xi \to \infty$ through some path). Then L is strongly hypoelliptic $\Leftrightarrow$ L^*L is strongly hypoelliptic $\Leftrightarrow$ $d(\xi)$ is strongly hypoelliptic as a multiplier operator. This result can be clarified in one case. Suppose that $\mathfrak{n}$ has a subalgebra $\mathfrak{h}$ which is polarizing for every l with $l|_{\mathfrak{n}_2} \neq 0$. Then the following are equivalent: (a) L is strongly hypoelliptic; (b) L^*L is strongly hypoelliptic; (c) L^*L is hypoelliptic; (d) the distance from $\xi \in \mathfrak{n}_2^*$ to the nearest zero of $d(\zeta)$, $\zeta \in (\mathfrak{n}_2^*)_{\mathbb{C}}$, tends to ∞ faster than any multiple of $\log|\xi|$. (Note: "strongly hypoelliptic" means "microlocally hypoelliptic". The hard part of the proof is (c) $\Rightarrow$ (d); for this, one defines representations π_ζ, $\zeta \in (\mathfrak{n}_2^*)_{\mathbb{C}}$, and uses the fact that $\pi_\zeta(L^*L)$ has kernel for a sequence of ζ's whose imaginary parts grow slowly to construct a distribution $u \notin C^\infty(N)$ with $L^*Lu = 0$.

Laurence Corwin

Rutgers University, New Brunswick, New Jersey,
USA

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Positive-energy Representations of the Diffeomorphisms of the Circle

Let $\mathcal{D}$ be the group of orientation-preserving diffeomorphisms of the circle. With the C^∞ topology, $\mathcal{D}$ is a Fréchet Lie group with Lie algebra $\underline{d}$ the smooth real vector fields on the circle. We describe the construction of a family of continuous projective unitary representations of $\mathcal{D}$ which have the "positive-energy" property: the infinitesimal generator of the rotation subgroup of $\mathcal{D}$ is represented by a semi-bounded self-adjoint operator. These representations arise in connection with the natural action of $\mathcal{D}$ as automorphisms of the loop algebra on a simple finite-dimensional Lie algebra, and are realized in the "standard modules" for affine Kac-Moody algebras (joint work with Nolan R. Wallach)

Roe Goodman

Rutgers University, New Brunswick, New Jersey

Invariant Paley Wiener theorem for connected semi simple Lie groups.

Let G be a connected semi simple Lie group with finite center, K max. compact subgroup. $C_c^\infty(G,K)$ the convolution algebra of left and right K-finite elements in $C_c^\infty(G)$, $P_i = M_iA_iN_i$ $i = 1, .., R$ a set of representatives for the association classes of ~~parab~~ cuspidal parabolic subgroups (standard with respect to a fixed maximal one).

Let $(F_i)_i$ $i = 1, .., N$ a family of functions $F_i : (\hat{M}_i)_{L.SD} \times (\mathfrak{a}_i^*)_{\mathbb{C}} \to \mathbb{C}$

Here $(\hat{M}_i)_{L.SD}$: set of classes of limit of discrete series (with non degenerate data)

$(\mathfrak{a}_i^*)_{\mathbb{C}}$ = complexified dual of $\mathfrak{a}_i$ = Lie A_i.

Then there exists $f \in C_c^\infty(G, K)$ such that, $\operatorname{tr} \pi_{\delta,\lambda}(f) = F_i(\delta, \lambda)$ for every $i = 1, \dots, N$, $\delta \in (\hat{M}_i)_{L.S.D}$, $\lambda \in (\mathfrak{a}_i^*)_{\mathbb{C}}$, (where $\pi_{\delta,\lambda} = \operatorname{Ind}_{P_i \uparrow G} \delta \otimes e^{\lambda} \otimes 1_{N_i}$) if and only the (F_i) satisfies:

(i) F_i has finite support in $\delta \in (\hat{M}_i)_{L.SD}$

(ii) for fixed $\delta \in (\hat{M}_i)_{L.SD}$, $\lambda \to F_i(\delta, \lambda)$ is of Paley Wiener type.

(iii) $\forall w \in W_{A_i}$ $\quad F_i(\delta, \lambda) \equiv F(w\delta, w\lambda)$

(iv) if $M_i \subset M_j$, $\delta_i \in (\hat{M}_i)_{L.SD}$ and $\operatorname{Ind}_{P_i \cap M_j \uparrow M_j} (\delta_i \otimes e^{0} \otimes 1_{N_{ij}}) = \bigoplus_m \delta_j^m$ for some $\delta_j^m \in (\hat{M}_j)_{L.SD}$ (here $P_i \cap M_j = M_i A_{ij} N_{ij}$ and e^{0} is the trivial character of $\mathfrak{A}_{ij}$.) then $F_i(\delta_i, \lambda) = \sum_m F_j(\delta_j, \lambda)$ for every λ in $(\mathfrak{a}_j^*)_{\mathbb{C}}$

(joint work with Laurent Clozel). P. Delorme - Faculté des Sciences de Luminy, Marseille.

Convolution of "Measures" on the "Dual".

Suppose G is the semi-direct product of an abelian group A and a finite group D. The group D also acts on $\hat{A}$. Let Ω be a cross-section ~~of D finite~~ orbits in $\hat{A}$, equipped with the topology of $\hat{A}/D$. Then the C^*-algebra of G is isomorphic with a C^*-subalgebra of $C_0(\Omega, M_n)$ the continuous functions from Ω into the $(n \times n)$-matrices which vanish at infinity, where n is the order of D. The $(n \times n)$-matrix valued measures on Ω forms the dual (linear) of this $C_0(\Omega, M_n)$. The adjoint of the above mentioned isomorphism is a mapping of $C_0(\Omega, M_n)$ onto $B(G)$ with well understood kernel K. Using this, one can define the "convolution" of such matrix-valued measures on Ω. With sufficiently good conditions on the D-action on $\hat{A}$ one recovers the tensor product of two irreducible representations of G from this "convolution"

Keith E. Taylor
University of Saskatchewan, Saskatoon.

Exceptional discrete series for symmetric spaces.

Let G be a semisimple Lie group (connected, finite center). Let σ be a Cartan-involution and K the identity component of the fixpoint-group. Let τ be an involution such that $\sigma\tau = \tau\sigma$, and H the identity component of the fixpoint-group for τ. Now G/K is a Riemannian symmetric space and G/H is a symmetric space, which is non-Riemannian if H is non-compact. A discrete series representation for G/H is an irreducible subrepresentation of the regular representation of G in $L^2(G/H)$. – A certain duality is introduced among triples of the form (K, G, H). We denote it $(K,G,H) \longleftrightarrow (H^\circ, G^\circ, K^\circ)$. – Now fix compatible Iwasawa decompositions for H and G: $G = KAN$, $H = (K\cap H)(A\cap H)(N\cap H)$. We generalize Harish-Chandras integral formula for the spherical functions on G/K to the following:

$$⊛ \quad \psi_\lambda(x) = \int_{K\cap H} e^{\langle i\lambda-\rho,\, H(x^{-1}k)\rangle} dk \;, \quad x \in G \;;\; \lambda \in \mathfrak{a}^*_{\mathbb{C}}.$$

If $\operatorname{rank}(G/K) = \operatorname{rank}(H/H\cap K)$ then these functions describe "most" of the discrete series for G°/K°. The missing ones are called exceptional. An attempt to describe these for the case of $\operatorname{rank}(G/K) = 1$ by a formula like ⊛ was ~~discus~~ discussed in the lecture.

Mogens Flensted-Jensen
(Copenhagen)

A characterization of SIN-groups and groups with the one-sided Wiener property

Let G be a locally compact group. Then G is a SIN-group (i.e. there exists a neighbourhood basis at the identity of sets invariant under inner automorphisms) iff the following property is satisfied: every proper closed left ideal (p.c.l.i.) in $L^1(G)$ is annihilated by a uniformly continuous function (bounded, $\neq 0$). In the connected case the result is due to Henrichs and Sindlarck. As a consequence, G has the left Wiener property [L] (i.e. every p.c.l.i. in $L^1(G)$ is annihilated by a positive-definite function ($\neq 0$)) iff G is a SIN-group and symmetric (i.e. every modular p.c.l.i. in $L^1(G)$ is annihilated by a continuous positive-definite function ($\neq 0$)). This generalizes a result by Leptin: for G connected, $G \in [L] \iff G = \mathbb{R}^n \times K$ (K compact).

Viktor Losert (Wien)

Norms of zonal spherical functions and Fourier series on compact symmetric spaces

Let (U, K) be a symmetric pair of compact type and $X = U/K$. $L^2(X)$ decomposes under U into irreducible components H_λ, $\lambda \in \hat{X}$, which contain the zonal spherical functions φ_λ, $\varphi_\lambda(e) = 1$. Let d_λ denote the dimension of $\lambda \in \hat{X}$. Then $(\Phi_\lambda)_{\lambda \in \hat{X}}$, $\Phi_\lambda := d_\lambda^{1/2} \varphi_\lambda$, form an orthonormal system in $L^2(X)$. We prove new estimates for p-norms of Φ_λ and a Cohen type inequality. For the Dirichlet kernel $D_N = \sum_{|\lambda+\beta| \leq N|\beta|} d_\lambda \varphi_\lambda$, where $|\cdot|$ comes from the Killing form and $\beta = \sum_{i=1}^{l} \mu_i$, where the μ_i are the fundamental highest weights of the representations of class one, we get the estimate

$$\| D_N \|_1 \geq \text{const } N^{(\dim X - \operatorname{rank} X)/2},$$

which is sharp for $l = 1$.

Bernd Dreseler (Siegen)

On the deformation of K to $M\bar{N}$

Let $G = KAN$ be the Iwasawa decomposition of a real rank 1, finite centre, semisimple Lie group, and let M be the centralizer of A in K. For fixed $H \in \underline{a}$, we define a family $(\pi_t)_{t \in \mathbb{R}^1}$, $\pi_t : M\bar{N} \to K$, by $\pi_t(m\bar{n}) = k(\exp -tH\, m\bar{n} \exp tH)$, where k is the Cayley transform. These maps deform K into $M\bar{N}$. We obtain an approximation theorem, expressing matrix coefficients of $M\bar{N}$ as a limit, uniformly on compacta, of a sequence of the form $(f_\ell \circ \pi_{t_\ell})_{\ell \in \mathbb{N}}$. Here, the f_ℓ are suitably chosen matrix coefficients for K, and $t_\ell \to \infty$ is a sequence of numbers. This is joint work with F. Ricci.

A. H. Dooley (Kensington)

Local correspondence for representations of GL(2) and quaternions

This is a joint work with Wen-Ch'ing Winnie Li (Penn State U., USA).

We give a more direct and natural proof of the following theorem of Jacquet and Langlands (SLN 114): let F be a local field and G, resp G', be the groups $GL_2(F)$, resp the group of multiplicative quaternions; then there exists a bijection between the discrete series of G and the set of classes of irreducible representations of G' characterized by ~~the rel~~ $\pi \leftrightarrow \pi'$ if and only if $\mathrm{Tr}\,\pi + \mathrm{Tr}\,\pi' = 0$ on the elliptic set.

We attach to any irreducible representation π of G, resp π' of G', the following function of χ, a one dimensional representation of $F^\times$, and of ψ, a non trivial unitary character of F

$$\gamma_\pi(\chi, \psi) = \int_G \pi(x)\, \chi(Nx)\, \psi(Tx)\, |Nx|^{-1} d_\psi x$$

$$\text{resp} \quad \gamma_{\pi'}(\chi, \psi) = -\int_{G'} \pi'(x')\, \chi(Nx')\, \psi(Tx')\, |Nx'|^{-1} d_\psi x'$$

with N, T reduced norm and trace, d_ψ the self dual measure on $M_2(F)$, resp quaternions, with respect to the self-duality $\psi(Txy)$. We characterize the functions $\gamma(\chi, \psi)$ arising from representations of G and G'. The correspondence $\pi \leftrightarrow \pi'$ is $\gamma_\pi = \gamma_{\pi'}$.

Paul Gérardin
(Université Paris V

On the Continuity of Mackey's Extension Process

If N is a closed normal subgroup of a locally compact group H, and if L is an H-invariant irreducible representation of N, then it is known that there exists an irreducible representation ρ of H whose restriction to N is a multiple of L. The representation ρ is important in Mackey's theory, but it is not constructively defined and it is often difficult to determine ρ explicitly. In this talk we study the continuity properties of the assignment L to ρ. We investigate what kinds of continuity conditions are possible, and we prove a theorem for a special case.

Baggett
University of Colorado

Ideal structure of Beurling algebras on $[FC]^-$-groups

(joint work with E. Kaniuth and A. Kumar)

We generalize results of Domar for locally compact abelian groups to $[FC]^-$- resp. $[FD]^-$-groups, namely:

(A) $\sum_{n\in\mathbb{N}} \log \omega(x^n)/n^2 < \infty$, iff $L^1_\omega(G)$ is $*$-regular (G $[FC]^-$-group)

(B) In this case $\emptyset \subseteq \mathrm{Prim}_* L^1_\omega(G)$ is spectral (")

(C) If (i) $\omega(x^n) = O(|n|^\alpha)$ $(\alpha = \alpha_x > 0)$ and (ii) $\liminf \omega(x^n)/n = 0$ (Shilov's cond.), then $\{\ker \pi\} \subseteq \mathrm{Prim}_* L^1_\omega(G)$ is spectral (G $[FD]^-$-group)

In doing this we prove some propositions which might be of interest by their own rights:

Prop 1: G a separable $[FD]^-$-group, $\pi \in \hat{G}$. Then there exists $\rho \in \hat{H}$ (H a closed subgroup) $\dim \rho < \infty$ such that $\pi \sim U^{\rho}$.

Prop 2: The projection theorem for spectral sets holds, if smallest ideals exist.

Prop 3: Let A be a symmetric, locally regular Banach $*$-algebra and $E \subseteq \mathrm{Prim}_* A$ closed, then $j(E)$ exists.

Wilfried Hauenschild
(Paderborn)

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On stability of measures on locally compact groups and on vector spaces.

Let $(\mu_t)_{t>0}$, $\mu_0 = \varepsilon_e$, be a continuous convolution semigroup on a locally compact group G, and let $A \in \mathcal{D}'(G)$ be the generating distribution, i.e. $\langle A, f \rangle = \frac{d^+}{dt} \langle \mu_t, f \rangle |_{t=0}$.
A resp. (μ_t) is called stable w.r.t. a group of automorphisms $(\tau_t)_{t>0} \subseteq \mathrm{Aut}(G)$ $(\tau_t \tau_s = \tau_{ts}, t, s > 0)$, if
$(*)$ $\tau_t(A) = tA + X(t)$ for some primitive $X(t)$
(generating a group of point measures).

A is called semistable w.r.t. $\tau \in \mathrm{Aut}(G)$, $c \in (0,1)$, if $(**)$ $\tau(A) = cA + X$.

⌈ This are natural generalisations of the classical concept of stability. For $G = \mathbb{R}^d$, τ_t resp. $\tau \in GL(\mathbb{R}^d)$ these concepts coincide with the operatorstable resp. operator-semistable laws: See M. Sharpe, Trans AMS 136 (1969) 51-65 or R. Jajte, Studia Math. 61 (1977) 29-39 ⌋.

(1) If G is a nilpotent, connected simply connected Lie group, then via exp $G \cong \mathfrak{g} \cong \mathbb{R}^d$ and there is a 1-1-correspondence between stable [resp. semistable] distributions on G and operator-stable [operator semistable] distributions on $\mathfrak{g} = \mathbb{R}^d$. So in this case we are able to describe completely the generating distributions of all possible stable laws on G.

(2) If G is still a Lie group, but not necessarily nilpotent, then there exists a connected subgroup G_1 on which (τ_t) [resp. $\tau^k, k \in \mathbb{Z}$] operates contracting, such that A resp. (μ_t) is concentrated on G_1. G_1 is necessarily nilpotent. Therefore via (1) it is again possible to describe completely the generating distribution via the corresponding distribution of operator stable laws.

W. HAZOD

Dortmund

Composition series for algebras associated with nilpotent and solvable Lie groups.

The algebras in question are $C^*(G)$, $C^\infty_c(G)$ (and $U(\mathfrak{g}_c)$).

First we gave a very explicit character formula for exponential solvable Lie groups. The new part of our results had to do with the properties of certain elements in $U(\mathfrak{g}_c)$.

We next applied these new results to show that if G is nilpotent, connected and simply connected Lie, then $C^*(G)$ is with generalized continuous trace with respect to $C^\infty_c(G)$ (in some specific sense), with a finite composition series of $C^\infty_c(G)$.

Finally we applied our results about the elements in $U(\mathfrak{g}_c)$ to show that if G is solvable, connected and simply connected Lie, then $C^*(G)$ has a finite composition series giving rise to GCCR-subquotients, and such a composition series can even by performed by the aid of $C^\infty_c(G)$-functions.

As a digression in our talk we gave an explicit formula for the infinitesimal kernel $\ker d\pi$ of the differential $d\pi$ of an irreducible representation π of a connected, simply connected nilpotent Lie group. The formula should probably be viewed as an analogue of the Kirillov character formula.

Niels Vigand Pedersen
(Technical Univ. of Denmark)

Weak containment and tensor product of group representations

The following two problems are studied: I) When is the trivial representation 1 of a locally compact group weakly contained in a tensor product ($1 \prec \pi \otimes \rho$)?, II) What are the subduals of $\hat{G}$, where by definition a closed subset Γ of $\hat{G}$ is called a subdual if $\pi, \rho \in \Gamma \Rightarrow \bar{\pi} \in \Gamma$ and $\operatorname{supp} \pi \otimes \rho \subseteq \Gamma$. Theorem: Suppose that G is Lie projective and has a composition series $G = N_m \supseteq \ldots \supseteq N_0 = \{e\}$ of closed normal subgroups where the N_j/N_{j-1} may be abelian or compact. Then every subdual Γ of $\hat{G}$ is of the form $\Gamma = \widehat{G/N}$ for some closed normal subgroup N of G. In particular, this conclusion holds for almost connected amenable groups. Concerning I), several results are presented, one them being the Theorem: Let G be a discrete group such that every finitely generated subgroup of G contains a nilpotent subgroup of finite index (i.e. G has polynomial growth), then for any two unitary representations π and ρ of G, $1 \prec \pi \otimes \bar{\rho}$ iff $\operatorname{supp} \pi \cap \operatorname{supp} \rho \neq \emptyset$.

E. Kaniuth (Paderborn)

An order theoretical characterization of the Fourier transform.

The role of order structures in Harmonic Analysis is illustrated by several results.

a) The "biordered space" $(B(G), B(G)_+, P(G))$ is a complete isomorphism invariant of the locally compact group G (here $P(G)$ is the cone of all continuous positive definite functions on G, $B(G) := \operatorname{span} P(G)$ denotes the Fourier Stieltjes Algebra and $B(G)_+ = \{ u \in B(G) : u(x) \geq 0 \ (x \in G) \}$.

b) The Fourier transform is characterized as "biorder anti-isomorphism".

c) G is amenable if and only if for every $f \in C_0(G)$ there exists $u \in A(G)$ such that $f \leq u$. As a consequence, the ordered space $(A(G), A(G)_+)$ is an invariant, which is complete with respect to amenability ($A(G)$ denotes the Fourier Algebra)

Wolfgang Arendt (Tübingen)

The principal topic of my discussion has been
1) The canonical bijection between the set of all quasi-equivalence classes of normal representations and the underlying set of the primitive ideal space of any connected Lie group,
2) The generalization to an arbitrary connected and simply connected solvable Lie group of previous theories of Kirillov (nilpotent case) and type I solvable case (Auslander and Kostant). It is to be born in mind, however, that here the principal objective is not a geometric description of the unitary dual, but that of the normal part of the quasi-dual.

L. Pukanszky
(University of Pennsylvania, Philadelphia, Pa, U.S.A.)

The elements of bounded trace in the C^*-algebra of a nilpotent Lie group.

Let G be a nilpotent Lie group with Lie algebra $\mathfrak{g}$. Let $BT = \{\xi \in C^*(G) \mid \sup_{\pi \in \hat{G}} \mathrm{tr}|\pi(\xi)| < \infty\}$ be the twosided ideal in $C^*(G)$ of the elements of bounded trace. The following two questions concerning BT are discussed 1) What is the hull $h(BT)$ of BT in $\hat{G}$? Do there exist estimations of $\sup_{\pi \in \hat{G}} \mathrm{tr}|\pi(f)|$ for $f \in BT$? It turns out that $h(BT)$ is the set of all G-orbits whose dimensions are not maximal. As the functional calculus of Dixmier can be applied to Schwartz-class functions it is possible to construct many elements in $BT \cap \mathcal{S}(G)$. We give also an estimation of

$$\sup_{\pi \in \hat{G}} \mathrm{tr}\,\pi\{[(f\circ\exp)^* *_+ (f\circ\exp)]\circ\exp\}$$

if $(f\circ\exp)^\wedge \in C_c^\infty(\mathfrak{g}^*)$ and $\mathrm{supp}(f\circ\exp)^\wedge \cap \mathfrak{g}^*_{<\max} = \phi$ where $*_+$ denotes the abelian convolution on $\mathfrak{g}$ and $\mathfrak{g}^*_{<\max}$ the set of all $l \in \mathfrak{g}^*$ whose G-orbits do not have maximal dimension

Jean Ludwig (Bielefeld)

Some summability methods for eigen-expansions related to nilpotent Lie groups

Let

$$L = \sum_{j=1}^{k} (-1)^{\varepsilon_j} \frac{\partial^{d_j}}{\partial x^{d_j}} + \sum_{j=1}^{m} p_j^2 ,$$

where $d_j \equiv 2\varepsilon_j \mod 4$

and p_j are polynomials on $\mathbb{R}^k$. L is a positive essentially self-adjoint (on $\mathcal{S}(\mathbb{R}^k)$) operator on $L^2(\mathbb{R}^k)$.

Let

$$Lf = \int_0^\infty \lambda \, dE(\lambda) f$$

be the spectral resolution of L.

THEOREM There exists numbers N, M such that if $K \in C^N(\mathbb{R}^+)$ and $K(0) = 1$; and

$$\sup_{\lambda > 0} (1+\lambda)^M \left| \frac{d^j}{d\lambda^j} K(\lambda) \right| < \infty ,$$

then for every function $f \in L^p(\mathbb{R}^k)$, $1 \le p < \infty$,

$$\lim_{t \to 0} \int_0^\infty K(\lambda) \, dE(\lambda) f = f \quad \text{a.e. and in } L^p \text{ norm.}$$

Andrzej Hulanicki (Wrocław)
Joe W. Jenkins (Albany N.Y.)

Compact operators in unitary representations of L^1-group algebras

Let G be a locally compact group with C^*-algebra $C^*(G)$. Let $I \triangleleft C^*(G)$ be a primitive ideal and let $I^\wedge$ be the intersection of all closed, twosided ideals of $C^*(G)$, which strictly contain I. If I is the kernel of an irreducible $*$-repr. π of $C^*(G)$, such that $\pi(C^*(G))$ contains the compact operators, then we have $I^\wedge = \{x \in C^*(G) \mid \pi(x) \text{ compact}\}$. We give a necessary and sufficient condition for $I \cap L^1(G) \neq I^\wedge \cap L^1(G)$ and use this to

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give a simple proof of $I \cap L^1(G) \neq I^{\wedge} \cap L^1(G)$ for connected amenable groups and arbitrary primitive ideals $I \triangleleft C^*(G)$. We also obtain that $I \neq I^{\wedge}$ implies $I \cap L^1(G) \neq I^{\wedge} \cap L^1(G)$ if G is $*$-regular.
Especially our results give that a well known example of A. Guichardet, which is the semidirect product of two (separable) abelian groups, cannot be used to construct an example of an irreducible unitary repr. π s.t. $\pi(C^*(G))$ contains the comp. operators but $\pi(L^1(G))$ does not contain non-trivial compact operators.

J. Boidol, Bielefeld

On coefficients of L^p-representations
(joint work with M. Cowling)

In 1972 D. Dacunha-Castelle and J.L. Krivine showed, that the class of L^p-spaces, and some other classes of Banach spaces too, are closed under Banach space ultra products.

Our aim was to show how this fact may be used to construct certain representations of locally compact groups:

Let π be a strongly continuous representation of a locally compact group G by isometries of an L^p-space E. $(1<p<\infty)$.
Let $A_\pi = \{ t \,/\, t(x) = \sum_{n\in\mathbb{N}} \langle \pi(x)\xi_n, \eta_n \rangle, x \in G, \sum \|\xi_n\| \|\eta_n\| < \infty, \xi_n \in E, \eta_n \in E^* \}$ be the space of generalized coefficients of π and let B_π denote the dual of the normed algebra $\pi(L^1(G))$ (with the Operator norm
then there exists a representation π' of G on an L^p-space E', such that $B_\pi = A_{\pi'}$ and $B_\pi = B_{\pi'}$.

G. Fendler, Genova

Non-hypoelliptic boundary Laplacians on unbounded domains in $\mathbb{C}^n$

In this talk we first discussed how a general contractible, homogeneous, rational domain in $\mathbb{C}^n$ could be described in terms of nilpotent Lie groups in much the same manner that a homogeneous bounded domain is described interms of the Heisenberg group and Siegel domains of type I and II. We then used this description to study an analogue of the boundary Laplacian $\Box$ which was defined from the Levi-form in much the same way that the real Laplacian is defined for a real, pseudo-Riemannian manifold. We showed how to use group representation to completely solve the operator and obtained a regularity estimate for the real direction, despite the non-hypoellipticity of the operator.

Calderón-Zygmund kernels carried by linear subspaces of homogeneous nilpotent Lie algebras

Let N be a connected simply connected nilpotent Lie group with Lie algebra $\mathfrak{n}$, and assume N is parametrized by $\mathfrak{n}$ via exp. Assume further that N is homogeneous with respect to a family $D = \{D_r\}_{r>0}$

of dilations of N. Fix a linear subspace w of n supplementary to the center z of n. If k is a Calderón-Zygmund kernel on w, extend k to a tempered distribution K on n by $\langle K,\varphi\rangle=\langle k|\varphi|_w\rangle$; and define a linear operator T on $C_0^\infty(N)$ by $T\psi=\psi * K$, where K here is considered as a distribution on N. Under certain condition on either k or the structure of N, it is proved that T extends to a bounded operator on $L^2(N)$, thus generalizing in parts a result of Geller and Stein for the Heisenberg group.

D. Müller, Bielefeld

A characterisation of Riemannian pairs

(G,Γ) is called a Riem. pair if G is a con. Lie gr., Γ closed subgr. and there exist a G-invariant Riem. metric on G/Γ. More generally a subgroup Γ of a locally compact group is called neutral ($\Gamma \underset{n}{\leq} G$) if there exists a G-invariant uniform structure on G/Γ.

<u>Thm 1</u>: G con. Lie, $G \geq \Gamma$ closed, G acts effectively on G/Γ then $\Gamma \underset{n}{\leq} G \iff G \in [SIN]^\Gamma \iff \mathrm{Ad}\,\Gamma$ rel. comp on $\mathfrak{L}G \iff \mathrm{Ad}\,\Gamma$ rel. comp on $\mathfrak{L}G/\mathfrak{L}\Gamma \iff (G,\Gamma)$ Riem. pair

<u>Thm 2</u>: G^* con. Lie, $G^* \geq \Gamma^*$ comp., G^* eff. on G^*/Γ^*, G dense con. subgr. of $G^* \Rightarrow (G, G\cap\Gamma^*)$ Riem. pair ~~and~~ s.t. $G^*/\Gamma^* = G/_{G\cap\Gamma^*}$ and any eff. Riem. pair arises in this way.

<u>Thm 3</u>: G Lie gr., $G \geq \Gamma$ closed then $\Gamma \underset{n}{\leq} G \iff G/_{\ker G} \in [SIN]^\Gamma \iff (G_0, G_0\cap\Gamma)$ Riem. pair

In general define $C(G,\Gamma)=\{a\in G\ ;\ \Gamma a\Gamma/\Gamma,\ \Gamma a^{-1}\Gamma/\Gamma \text{ rel. comp in } G/\Gamma\}$ then $C(G,\Gamma)$ is a subgr of G containing Γ and $\Gamma \underset{n}{\leq} G$ iff $\Gamma \underset{n}{\leq} C(G,\Gamma)$ and $C(G,\Gamma)$ open in G.

Thm 4: G l.c., $G \geq \Gamma$ closed, G eff on G/Γ and $G = C(G,\Gamma)$ then $\Gamma \underset{\sim}{\leq} G \iff \Gamma$ rel. comp. in $\mathcal{H}(G/\Gamma)$ the group of homeomorphisms of G/Γ equipped with the Arens-topology. In this case $\overline{G} = G\overline{\Gamma}$, $\overline{G}/\overline{\Gamma} = G/\Gamma$ and $\overline{G}$ is l.c.

G. Schlichting, München

Symbolic calculus for 3-step nilpotent Lie groups.

We derive sufficient conditions for L^2-boundedness of convolution by a tempered distribution on a 3-step nilpotent Lie group. This is achieved by producing estimates on the symbol of the distribution, which is the Fourier transform of the pull-back of the distribution to the Lie algebra via the exponential map. By relating distributions on SH_n (a 3-step group with one-dimensional centre) to distributions on the $2n+1$-dimensional Heisenberg group, and discovering a connection between their symbols, we derive estimates involving the co-adjoint derivatives of symbols on SH_n^*. These estimates can then be applied to subgroups of SH_n.

G. Ratcliff, New Haven, CT.

On the Fourier inversion over Generalized H-Groups.

Let X_1, X_2, X_3 be 3 locally comp. abelian groups, $B: X_1 \times X_2 \to X_3$ a continous $\mathbb{Z}$-bilin. map. Let G the Generalized Heisenberg group defined by H. Reiter in CMH 1974. Let $\chi \in \hat{X}_3$, $\chi(B(x,y))$ bicharacter of $X_1 \times X_2$, σ_χ, σ^*_χ the associated morphisms resp. from $X_1 \to \hat{X}_2$, $X_2 \to \hat{X}_1$. Suppose they are top. isom. on their range (after killing the kernels). Then, let $\pi_{\dot{x}_1,\dot{x}_2,\chi}$ the unitary irred. cont. repr. of G, $\dot{x}_1 \in \hat{X}_1/\mathrm{Im}\,\sigma^*_\chi$, $\dot{x}_2 \in \hat{X}_2/\mathrm{Im}\,\sigma_\chi$, $\chi \in \hat{X}_3$. Let $\tilde{G} = X_1 \times X_2 \times X_3$, abelian group, and

$$\Lambda(\tilde{G}) = \{ f = f_1 + f_2 \mid f_1 \in C_{00}(\tilde{G}),\ \hat{f}_1 \in L^1(\hat{\tilde{G}}),\ f_2 \in L^1(\tilde{G}),\ f_2 \in C_{00}(\hat{\tilde{G}}) \}$$

then:

Th: for $\varphi \in \Lambda(\tilde{G})$, let $Tr\,\pi_{x_1,x_2,\chi}(\varphi)$ the integral over the diagonal of the kernel of this bounded operator. Then:

$$\phi(0,0,0) = \int_{\hat{X}_3} dx \,\|\mathring{\sigma}_x\| \int_{\hat{X}_1/\mathrm{Im}\,\sigma^*_x} d\dot{x}_1 \int_{\hat{X}_2/\mathrm{Im}\,\sigma_x} d\check{x}_2 \; \mathrm{Tr}\, \pi_{-\dot{x}_1, -\dot{x}_2, -x}(\phi)\,.$$

<u>Thz</u> $\|\pi_{\dot{x}_1,\dot{x}_2,x}(\phi)\|^2_{HS} = |\mathring{\sigma}_x|^{-1} \int_{\mathrm{Im}\,\sigma^*_x} dw^*_1 \int_{\mathrm{Im}\,\sigma_x} dw^*_2 \,\big|\hat{\phi}(-x_1+w^*_1, -x_2+w^*_2, -x)\big|^2$

<u>Corollary</u>: Let $W^2(\tilde{G})$ the Wiener space of $\tilde{G}$. Then there exists $C_\pi > 0$ such that $\|\pi(\phi)\|_{HS} \leq C_\pi \|\phi\|_{W^2(\tilde{G})}$ $\forall \pi \in \hat{G}$.

Marc Burger Lausanne.

Differentialoperatoren erster Ordnung und invariante Distributionen auf Exponentialgruppen

Es wurden lokale und globale Auflösbarkeitsfragen behandelt für Differentialoperatoren erster Ordnung mit kritischen Punkten. Mit Hilfe der Auflösbarkeitsresultate konnten dann für gewisse exponentielle Liesche Gruppen die unter der koadjungierten Darstellung invarianten Distributionen mittels der invarianten Maße auf den Bahnen charakterisiert werden; dies führte zu einer Charakterisierung der zentralen Distributionen durch die Charaktere irreduzibler Darstellungen.

Rainer Felix, Bielefeld

Fourier multipliers on Cartan motion groups

Let (G, K) be a Riemannian symmetric pair of compact type and $V \rtimes K$ the associated Cartan motion group, where $\underline{\mathfrak{g}} = \underline{\mathfrak{k}} \oplus V$ is the decomposition of the Lie algebra of G into its θ-stable and θ-reflective components. For each $\lambda > 0$, let $\pi_\lambda(v, k) = \exp_G\left(\frac{v}{\lambda}\right)k$ map $V \rtimes K$ into G. Then the <u>contraction</u> mappings π_λ provide a means of transferring the Fourier analysis of G to that of the motion group. In particular, it is possible to prove an exact analogue of the following Fourier multiplier theorem of de Leeuw: Suppose $1<p<\infty$ and Φ is a bounded continuous function on $(-\infty,\infty)$. For each $\lambda>0$, let $\Phi^{(\lambda)}$: $\Phi^{(\lambda)}(n) = \Phi(\frac{n}{\lambda})$ $(n\in\mathbb{Z})$ be a Fourier multiplier of $L^p(\mathbb{T})$, and suppose that $|||\Phi^{(\lambda)}|||_p \leq M$ for all sufficiently large λ. Then $\Phi \in M_p(\mathbb{R})$, and $|||\Phi|||_p \leq M$ also.

Garth Gaudry, Bedford Park. Aust.

Scattering Theory

(17. Juli – 23. Juli 1983)

Hybrid ray-mode methods for scattering and propagation

Wave propagation and scattering can be analyzed alternatively in terms of progressing (ray/wavefront) or oscillatory (mode/resonance) events. These alternative descriptions have complementary convergence properties in that the one can express the collective effect of the other. Rays or wavefronts characterize local, and modes or resonances global, features of the propagation or scattering process. A recently developed hybrid technique combines some progressing and some oscillatory constituents in uniquely defined combinations that may be chosen to exploit the best features of each. This is made possible by a rigorously formulated ray-mode equivalent. The method may also be interpreted to quantify the truncation error of a series of ray fields or a series of mode fields when a finite number of terms is taken for numerical computation. Illustrative examples include transient scattering by a cylindrical target, transient propagation in a layered medium, and time-harmonic propagation in various guiding environments.

L. B. Felsen, Polytechnic Institute of New York,
Farmingdale, NY USA

Hybrid Ray-Mode: How the Ansatz was Born

A staunch engineer
Of waves had much fear
He started with rays
But soon lost the trace.
He then switched to modes
But messed up the codes.
To help him to think,
He fixed up a drink.
Half whisky, half gin,
In a glass he poured in,
And took a long sip.
His mind made a flip:
The Ansatz -- Let wave
Like a Mixed Drink behave.
Use Ray-Modal Blend
(And that's how it went).

LF

Construction and calculation of solutions to exterior problems with the WKB- method.

Parametrices for time dependent problems and for time independent problems can be constructed with Fourier integral operators and with pseudo differential operators, respectively, but the expressions obtained are not very explicit. In the talk it has been shown how to construct a parametrix with the WKB- method, and how this parametrix can be used to construct the solution for time dependent and time independent problems using Neumann'series. The construction is not restricted to high frequencies. Some numerical examples are presented.

Hans - Dieter Alber, Bonn

Resonance in One Dimensional Potential Scattering

The quantum mechanical motion of a particle in $[0,\infty)$ is determined by its Hamiltonian operator $H = -\frac{d^2}{dr^2} + V(r)$, with Dirichlet boundary condition at 0. Physically, a resonance is indicated when the particle stays for an abnormally long time in some region $[0,R]$ where the potential $V(r)$ is influential. A mathematical model for this is a "resonance eigenfunction" ψ satisfying $H\psi = k^2\psi$, $\psi(0) = 0$ and $\psi(r) \sim e^{ikr}$ as $r \to \infty$, with $k = \kappa - i\eta$, $\kappa, \eta > 0$. Such a resonance eigenfunction grows exponentially, but can be cut off near R to yield $\varphi \in L^2([0,\infty))$ such that $\int_0^R |(e^{-iHt}\varphi)(r)|^2 \, dr \sim e^{-4\kappa\eta t}$ for $t > 0$, if η is small enough. If V has a potential barrier near R, and $E << \sup V(r)$ is an eigenvalue of H with Neumann boundary condition at R, then exists a resonance eigenfunction with $H\psi = k^2\psi$ and k^2 near E.

Richard Lavine, Rochester NY, USA

Convergence and stability of the T-matrix approach in scattering theory

A numerical method for solving exterior boundary value problems (of which the scattering problem is a particular one) is studied. The method (called the T-matrix approach) converges. The rate of convergence and stability of the method with respect to small perturbations in data depend on the choice of the basis functions. This dependence is analysed and numerical examples are discussed. Part of this work is joint with Prof. S. Ström and Dr. G. Kristensson Univ. of Göteborg.

<u>References</u>: A.G. Ramm J. Math. Phys. 23, (1982), 1123;
23, (1982), 2408.
G. Kristensson, A.G. Ramm, S. Ström, J. M. Phys. (to appear 1983 or 84)

A. G. Ramm Math. Dep. KSU, Manhattan KS 66506
USA

Scattering of ~~[illegible]~~ scalar waves in the Schwarzschild geometry of general relativity

I outline a class of problems which arise in the theory of black holes and which are amenable to methods used in scattering theory. This possibility relies on the observation that a light ray, say, in the Schwarzschild geometry, takes infinite ~~time~~ ("Killing"-) time t to reach the horizon. This situation can be generalised by considering static space-times whose spatial optical metric is geodesically complete. I give some general results based mainly on Wilcox's abstract scattering theory and then treat the Schwarzschild geometry as a special case. The challenge here for scattering theory lies in the fact that one has here _two_ asymptotic regions corresponding to infinity and the Schwarzschild horizon. I end with some conjectures.

Ref: R. Beig, Acta Phys. Austr. 1982

R. Beig. Theor. Phys. Inst.
Univ. of Vienna.

Dense Sets of Far Field Patterns in Acoustic Scattering

We consider the class of far field patterns corresponding to the classical boundary value problems of time-harmonic acoustic scattering. For the Dirichlet- and Neumann problems it is shown that the class of far field patterns corresponding to entire incident fields is not dense in L^2 if the wave number is an eigenvalue of the interior boundary value problem and at least one eigenfunction is an entire Herglotz wave function. Related results will also be given for the transmission boundary value problem

Andreas Kirsch (Göttingen)
David L. Colton (Delaware, U.S.A.)

On low-frequency asymptotics in electro-magnetic theory

By the use of two formal matrix differential operators

$$M = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i\varepsilon^{-1}\operatorname{curl} & 0 \\ 0 & i\mu^{-1}\operatorname{curl} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad N = \begin{pmatrix} 0 & 0 & i\operatorname{div}\mu\cdot & 0 \\ 0 & 0 & 0 & i\operatorname{grad} \\ i\operatorname{grad} & 0 & 0 & 0 \\ 0 & i\operatorname{div}\varepsilon\cdot & 0 & 0 \end{pmatrix},$$

it is possible to rewrite the complete system of Maxwell's equations in the time-harmonic case as

$$(1) \qquad (M-\omega)\begin{pmatrix} 0 \\ E_\omega \\ H_\omega \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i\varepsilon^{-1}K_\omega \\ i\mu^{-1}J_\omega \\ 0 \end{pmatrix}, \quad N\begin{pmatrix} 0 \\ E_\omega \\ H_\omega \\ 0 \end{pmatrix} = -\omega^{-1} N \begin{pmatrix} 0 \\ i\varepsilon^{-1}K_\omega \\ i\mu^{-1}J_\omega \\ 0 \end{pmatrix}$$

in a medium $G \subset \mathbb{R}^3$, the properties of which are described by ε, μ. The assumption that the obstacle $O = \mathbb{R}^3 \setminus G$ is a perfect conductor leads to boundary conditions

$$(2) \qquad n \times E_\omega = 0, \quad n \cdot \mu H_\omega = 0 \quad \text{on } \partial G, \ n \text{ normal to } \partial G.$$

Generalizing the notion of differentiability and acceptance of boundary values in the sense of L_2-distributions, (1), (2) can be transfered into operator equations of the type

$$(3) \qquad (\mathcal{M}-\omega)U_\omega = F_\omega, \quad \mathcal{N}U_\omega = -\omega^{-1}\mathcal{N}F_\omega,$$

where $\mathcal{M}, \mathcal{N}$ are selfadjoint operators with respect to the corresponding 'energy' inner product.

For the system (2) the limit $\omega \to 0$ is studied in the case that G is an exterior domain. Under appropriate assumptions on $(F_\omega)_\omega$, it can be shown that $(U_\omega)_\omega$ converges to a unique static solution as $\omega \to 0$.

Rainer Picard

Rational reflection coefficient and inverse scattering on the line by P C SABATIER

Inverse scattering for Schrödinger Equation on the line is studied for reflection and transmission coefficient that satisfy usual regularity conditions and are rational fraction of k. The origin is still a particular point but the potentials do not need to be cut at this point like in old studies. There are poles for both reflection coefficient in both upper and lower half k-plane. It is shown that the problem reduces to solving a linear algebraic system. A different algorithm, made of a sequence of Backlund-Darboux transforms, gives also the solution, and enables to study separately modification of both sides of the potential due to introduction of poles. This progressive undressing technique for the potential paves the way for many other studies.

P Sabatier

Spectral analysis of wave propagation in perturbed stratified media

by Yves Dermenjian and J. C. Guillot

~~Short range perturbation of~~

Let us consider the following operator

$$A_0 = -c^2(y)\,\Delta_{x,y} \qquad x \in \mathbb{R}^n,\ y \in \mathbb{R}$$

where

$$c(y) = \begin{cases} c_0 & y < 0 \\ c_1 & 0 < y < h \\ c_2 & y > h \end{cases}$$

with $0 < c_1 < c_0 \leq c_2$.

A_0 is a self adjoint ~~op~~ in the Hilbert space $L^2(\mathbb{R}^{n+1}, c^{-2}(y)\,dx\,dy)$
~~A_0~~ is ~~an absolutely cont~~ with domain $D(A_0) = H^2(\mathbb{R}^{n+1})$
A_0 is an absolutely continuous spectrum whose spectrum is $[0,\infty)$. Another caracteristic features of the spectral theory of A_0 is the existence of a sequence of thresholds $\{c_0^2 p_k^2\ k=1,\dots\infty\}$ where $p_k < p_{k+1}$ for every k and $p_1 \geq 0$.
One considers short range perturbations of A_0. Let $c(x,y)$ be a measurable positive function on $\mathbb{R}^{n+1}$ such that

$$0 < m \leq c(x,y) \leq M \quad \text{for a.e } (x,y) \in \mathbb{R}^{n+1}$$
$$c(x,y) - c(y) = O\left(\frac{1}{(1+|y|)^{1+\varepsilon}} \frac{1}{(1+|x|)^{1+\varepsilon}}\right)$$

Consider the following operator

$$A = -c^2(x,y)\,\Delta_{x,y}$$

A is a selfadjoint operator in $L^2(\mathbb{R}^{n+1}, c^{-2}(x,y)\,dx\,dy)$ with domain $D(A) = H^2(\mathbb{R}^{n+1})$
Let $R(z)$ (resp $R_0(z)$) be the resolvent of A (resp A_0) ($z \notin \overline{\mathbb{R}_+}$)
We then have the following theorems

Theorem 1

The limits

$$\lim_{\substack{z \to \mu \\ \pm \operatorname{Im} z > 0}} R_0(z) = R_0^{\pm}(\mu)$$

exist in the topology of $\mathcal{B}(L^{2,s_1,s_2}, L^{2,-s_1,-s_2})$ for every $\mu > 0$ and with $s_1 > \frac{1}{2}$ and $s_2 > \frac{1}{2}$

Theorem 2

The spectrum of A is $[0,\infty)$
All the eigenvalues of A in $(0,\infty) - \bigcup_{k=1}^{\infty}\{c_0^2 p_k^2\}$ form a discrete set of $(0,\infty) - \bigcup_{k=1}^{\infty}\{c_0^2 p_k^2\}$ and each such an eigenvalue has a finite multiplicity
For every $\mu > 0$ in $(0,\infty) - \bigcup_{k=1}^{\infty}\{c_0^2 p_k^2\}$ and distinct of an eigenvalue of A, the following limits

$$\lim_{\substack{z \to \mu \\ \pm \operatorname{Im} z > 0}} R(z) = R^{\pm}(\mu)$$

exist in the topology of $\mathcal{B}(L^{2,s_1,s_2}\ L^{2,-s_1,-s_2})$

Jean Claude Guillot

Gefördert durch Deutsche Forschungsgemeinschaft

A Scattering Problem for Maxwell's Equations

We consider Maxwell's equations in the exterior of a periodically moving (bounded) body which may be either a perfect conductor or a dielectric. The motion is assumed slower than the speed of signal propogation in vacuum. Energy is not conserved.

An abstract theory along the lines of that of Lax-Phillips is constructed which encompasses the above cases, as well as others. Within the abstract theory we find a scattering amplitude which has poles at certain scattering frequencies in the complex plane. These poles lead to a "near-field" expansion for solutions which may include exponentially growing modes. (Joint work with Walter Strauss)

Jeffery Cooper.

Scattering Theory for the wave equation in spaces of one and two dimensions.

The energy form for the wave equation (in free space) differs from that of higher dimensions in that data is only determined modulo constant functions: $\{c, 0\}$. Such data, i.e. $\{c, 0\}$ are invariant under the action of the free space solution operators. As a consequence the wave operators will not exist for most data. However it is possible to choose one data out of each coset for which the wave operators exist. The so defined wave operators can be shown to be complete for short range perturbations.

Ralph Phillips

Multiparticle quantum scattering theory

We give a time-dependent, geometrical proof of asymptotic completeness for three-particle quantum systems (including absence of singular continuous spectrum). The pair potentials are fairly general locally, the may contain both a short-range part (roughly $\sim |x|^{-(1+\varepsilon)}$ as $|x| \to \infty$) and a long-range part with $|\vec{\nabla} V_\ell(x)| \leq C\,(1+|x|)^{-3/2-\varepsilon}$ (all previously treated cases are covered by our analysis).

We emphasise the use of asymptotic observables to control the propagation of a scattering state in phase space under the interacting time evolution. On the absorbing subsets of the state space (charakterized by phase space properties) the asymptotic time evolution is simple and easy to control. Some auxiliary steps extend to higher particle numbers.

Volker Enss, Ruhr-Universität Bochum

On the distribution of poles of the scattering matrix

We consider the scattering of acoustic equation in $\mathbb{R}^3$ by a bounded obstacle $\mathcal{O}$. Let us denote by $\mathcal{S}(z)$ the scattering matrix. The problem I like to concern with is to find concrete relationships between geometric properties of obstacle $\mathcal{O}$ and the analytic properties of $\mathcal{S}(z)$. The result we like to show is the following

<u>Theorem</u>. Let $\mathcal{O} = \mathcal{O}_1 \cup \mathcal{O}_2$, $\overline{\mathcal{O}_1} \cap \overline{\mathcal{O}_2} = \phi$. Suppose that $\mathcal{O}_j$, $j=1,2$ are strictly convex.

Then there exists positive constants c_0, c_1 such that
(i) $\mathcal{S}(z)$ is holomorphic in $\{z; \operatorname{Im} z < c_0 + c_1\}$
$$- \bigcup_{j=-\infty}^{\infty} \{z; |z - z_j| \leq C(1+|j|)^{-1/2}\}$$
where $z_j = ic_0 + \frac{\pi}{d} j$, $d =$ distance $(\mathcal{O}_1, \mathcal{O}_2)$.
(ii) For each j, $\{z; |z - z_j| \leq C(1+|j|)^{-1/2}\}$ contains a pole of $\mathcal{S}(z)$.

Furthermore we show an example of $\mathcal{O}$ whose scattering matrix has a sequence of poles with imaginary parts converge to zero.

M. IKAWA. Osaka University.

An acoustical inverse problem and some recent numerical results.

At the last scattering theory symposium (1980) I had presented an inverse acoustical problem for determining the shape A(x) of the vocal tract from measurement of impulse response at the lips. [A(x) is the area of the cross-section of the vocal tract at a distance x behind the lips]. Since then, in collaboration with J.R. Resnick, I have conducted experiments and numerical computations for which I can report the following:

a) We can now measure the impulse response, compute the A(x) and display it on an oscilloscope in about 55 msec. This enables us to display a "movie" of the vocal tract at about

18 frames/sec.

b) The recovered area functions are accurate enough to enable synthesis of sentences of speech.

After a brief review of the basic mathematics, I will describe the numerical procedures we developed to achieve these results, — in particular the algorithms for regularizing the impulse response estimates, and fast algorithms for matrix inversion. [For details see Sondhi & Resnick, J. Acoust. Soc. of Am., vol 73, pp 985-1002]

Man Mohan Sondhi. Bell Labs., Murray Hill, N.J.,
U.S.A.

Geometric optics for media with spatial dispersion

With an integral constitutive relation for spatially dispersive media slowly varying in space and time and with Maxwell's equations a system of partial differential equations is established for the slowly varying parts of the electromagnetic field components.
The zeroth approximation leads to dispersion equation, polarization relations and Hamilton's ray equations, the first approximation to a transport equation (along the rays) for the scalar amplitude of the electromagnetic field. The solution accounts for focussing effects as well as for the influence of (small) gradients and time variations of the parameters of the medium.

Kurt Suchy, University of Düsseldorf

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Application of inverse scattering algorithms to nondestructive testing of materials with ultrasound.

Even though ultrasonic testing of materials relies on the propagation of elastic waves todays inverse scattering procedures do not account for mode conversion effects of longitudinal and transverse waves on the surfaces of scatterers: the formulation is essentially a scalar one. Two approaches are presently under concern: explicit inversion of Kirchhoff's integral under the assumption of the validity of Physical Optics or Born's approximation, yielding algorithms like POFFIS (Physical Optics Far-Field Inverse Scattering) or IBA (Inverse Born Approximation) in the frequency domain, or SAFT (Synthetic Aperture Focussing Technique) in the time domain, and the application of the backward wave propagation argument leading to Generalized Holography. It is shown that this concept can be interpreted as a spatial matched filter for planar scatterers, where the lack of axial resolution can be accounted for by a finite aperture. Extension to broadband signals improves axial resolution, but numerical experiments reveal a couple of essential drawbacks.

Karl-Jörg Langenberg
University of Kass

Spectral and Scattering Theory for Strongly Propagative Systems.

I consider strongly propagative systems. As is well known these systems contain as particular cases the equations of most of the wave propagation phenomena of classical physics, e.g. Maxwell equations, and the equations of acoustic and elastic waves in crystals.

First I give a theorem in the limiting absorption principle with long range interaction (non homogeneous system). Then I consider the short range case, and I obtain an eigenfunctions expansion theorem (distorted plane waves) and a representation of the wave operator in terms of the eigenfunctions, in particular existence and completeness follows. Then the analytic structure of the scattering operator is considered. I obtain a representation of the scattering matrix in terms of the generalized eigenfunctions and I prove that the scattering matrix has an extension to an function analytic in the upper complex plane. If furthermore the number of space dimensions is odd and the interaction (lack of homogeneity) decays exponentially it is proven that the scattering matrix extends to a meromorphic function on the complex plane with poles only in the lower half plane.

Ricardo Weder.
Univ. of México. México D.F.

Local time-decay for high-energy scattering states for the Schrödinger equation

Consider the Schrödinger operator $H := H_0 + V$ in the Hilbert space $L^2(\mathbb{R}^n)$ where $H_0 := \overline{(-\Delta) \upharpoonright C_0^\infty}$ and V is realvalued with $V \in C^{N+2+\frac{n}{2}}(\mathbb{R}^n)$; $|D^\alpha V(x)| \leq C_N (1+|x|)^{-|\alpha|-\varepsilon}$ for $|\alpha| \leq N+1$, $N \geq 4$, $N \in \mathbb{N}$.
Then we show for the weighted norms

$$\| e^{-itH} \chi(H) \| \leq C_N (1+|t|)^{-s+\frac{s}{2}}, \quad 0 \leq s \leq N$$

for $\chi \in C^\infty$ with $\chi(\lambda) = 0$ for $\lambda \leq t_0$ and $\chi(\lambda) = 1$ for $\lambda \geq 2t_0$ and t_0 suitable large.
This can be physically understood as meaning that for scattering states with high energy and which are "localized" near the origin at time zero, have a time-decay rate depending on the "localization"-weights and the smoothness of the potential V.
The proof uses an "approximative" complex dilation of the resolvent and a suitable limiting absorption argument for N-th power of the resolvent.

Hans Cycon, Technische Universität Berlin
Peter Perry, CALTEC

Phase-space analysis and N-body scattering theory (or micro-local analysis of propagation of particles).

The QM many-body scattering theory is one of those fields where the physical intuition is vivid but the mathematics is just too hard. In ~~the~~ my talk I have described some rigorous results and methods in this field due to Eric Mourre and myself. We prove the asymptotic completeness for certain quantum-mechanical many-body systems. We do it in three steps. On the first step we show that the particle propagate into certain rays in the phase-space of the system (the cotangent bundle of the configuration space). On the second step we construct a ~~phase~~ partition of unity on the phase space satisfying given conditions. This is purely classical problem. On the ~~certain~~ third step we use the quantized version of the phase-space partition of unity (i.e. ψ-differential partition of unity) to split a ~~the~~ time-dependent ~~of~~ solution of the Schrödinger equation: $e^{-iHt}\psi$ into terms "living" in different (and accordingly chosen) regions of the phase-space. Then using the result obtained on the first step we show that ~~th~~ each term has appropriate asymptotics as $t \to \pm\infty$. This method which we call the phase-space analysis can be applied to much more general systems including non-quantum scattering systems (e.g. el-magn. scattering). Finally, note that the idea of localizing in the phase-space was introduced into QM in the pioneering work of Volker Enss. Similar ideas were used in the Q Field theory by Glimm and Jaffe. In mathe
this technique was used for number of years under the name of micro-local and

I.M. Sigal, The Weizmann Inst. of Sci. Rehovot Israel

Completeness of Wave Operators in Relativistic Quantum Mechanics by Geometric and Algebraic Methods.

A combination of geometric and algebraic methods are used to prove asymptotic completeness for Schrödinger type equations with potential not vanishing at ∞ along hyperboloids in space time, and with the free Hamiltonian given by the (not bounded below) relativistic $(\text{mass})^2$ operator. The same method can be applied to study similar problem with the usual free Hamiltonian perturbed by "finger potenials" not vanishing at ∞ on unbounded domains in space. Or else one can study the case of a general pseodo-diff. operator for the free Hamiltonian perturbed by potentials of the usual kid, say compactly supported. The proof is based on the use of a modified form of local compactness and additional geometric properties of asymptotic scattering states which are needed to distinguish them from states "trapped" inside some hyperbaloid for all times.

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Determination of singularities of S-matrices

Consider the Schrödinger operators $H = -\Delta + V$, $H_0 = -\Delta$ in $L^2(\mathbb{R}^n)$. We concern ourselves here 2-body problems. Let S be the scattering operator and $\hat{S}$ the Fourier transform of S : $\hat{S} = \mathcal{F}_1 S \mathcal{F}_1^*$. As is well-known $\hat{S} = \int_0^{\oplus} \hat{S}(\lambda) d\lambda$ $(\lambda > 0)$. $\hat{S}(\lambda)$ is called the scattering matrix. In the short-range case, $\hat{S}(\lambda) = 1 + A$, where A is known to be compact. The first of our result is that A is an integral operator with kernel $A(\lambda : \omega, \omega')$ which is C^∞ off the diagonal. At the diagonal, we can see that $A(\lambda)$ has a singularities corresponding to the decay of potentials. In the long-range case, $\hat{S}(\lambda)$ can be never written as 1 + compact. In this case, however, if we regard $\hat{S}(\lambda)$ as a distribution on $S^{n-1}\times S^{n-1}$, we see that sing supp $\hat{S}(\lambda)$ is also contained in the diagonal. If $\omega\neq\omega'$, $|\hat{S}(\lambda:\omega,\omega)|^2$ (the differential-cross section) can be approximated by the differential cross-sections of short-range potentials. We can also reconstruct the potential from the high-energy asymptotic behavior of the scattering amplitude.

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H Kitada Tokyo University
Tokyo Japan.

Spectral representations and the asymptotic wave function for long-range perturbations of the d'Alembert equation

We consider the asymptotic behavior for time tends to infinity of acoustic waves which is governed by long-range perturbations of the d'Alembert equation. The principle of limiting absorption is proved for the reduced equation. Then spectral representations are obtained, and by use of them, the asymptotic wave function is constructed. It is a modified diverging spherical waves and approximates each energy finite solution.

The perturbation term of the problem is "very long range", So, it becomes necessary to determine a new radiation condition. An approximate phase function is constructed by solving a Riccati equation and the an eikonal equation.

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And our radiatión condition is defined by use of this phase function. It gives a natural generalizatión of the Sommerfeds original one.

K. Mochizuki Shinshu University
Matsumoto, Japan

Limiting Absorption for a Sum of Tensor Products

We consider a selfadjoint operator which can be written in the separated form

(1) $$H = H_1 \otimes I_2 + I_1 \otimes H_2 ,$$

where H_1 and H_2 are selfadjoint operators acting in Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively. The question we ask is the following: Given that H_1 and H_2 satisfy a limiting absorption principle in $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively. What are the conditions needed to guarantee that H defined by (1) satisfies a limiting absorption principle in $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$?

We give abstract results of an elementary nature which answers this question. These abstract results in turn can be used to provide very simple proofs of known results, as well as providing a framework for new results, which could be difficult to prove by other techniques. A key role is played by the operator version of the classical Privaloff-Korn theorem. Roughly speaking it says that if H is selfadjoint, $dE(\lambda)$ its spectral measure and $A(\lambda) = dE(\lambda)/d\lambda$ exists in some week operater topology in some open set $U \subseteq R$, and is Hölder continuous in U in the norm topology, then $R(\lambda \pm i\varepsilon) \rightarrow R^{\pm}(\lambda)$ in U in the norm topology and is Hölder continuous there.

We give a number of examples to illustrate the ease of application of the abstract results. For example limiting absorption for the n-dimensional Laplacian is immediately obtainable from the corresponding result for the 1-dimensional Laplacian. The latter result is immediate from the well known form of the 1-dimensional Green's function. Other examples are

$$-\frac{\partial^2}{\partial x_1} + x_1 - \Delta_{(x_2, x_3)}$$

or more generally

$$-\frac{\partial^2}{\partial x_1^2} + V(x_1) - \Delta_{(x_2, x_3)}$$

for a wide class of potential $V(x_1)$. Other applications may be made to half-space problems with boundary conditions, etc.

A. Devinatz, Northwestern Univ.
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Properties of the scattering matrix

Let $H = H_0 + V(\underline{r})$, where $H_0 = -\Delta$ in $L^2(\mathbb{R}^3)$ and $V(\underline{r})$ is a short range potential which may be arbitrarily singular near $\underline{r} = 0$. If $E_\pm$ are projections associated with the positive / negative parts of the spectrum of $A = \frac{1}{2}(\underline{P}.\underline{r} + \underline{r}.\underline{P})$ and λ is a positive energy, one has always

$$\lim_{\varepsilon \to 0} \| E_{|H_0 - \lambda| < \varepsilon} \; E_+ S E_- \; E_{|H_0 - \lambda| < \varepsilon} \| = 0 ,$$ where

$S = \Omega_+^* \Omega_-$ is the scattering operator. The corresponding result with + and − interchanged need not hold in general, but is a necessary and sufficient condition (assuming strong asymptotic completeness) for $\gamma_{a.c.}(\lambda) = 0$. (Recall that $\gamma_{a.c.}(\lambda) = \lim_{\substack{\varepsilon \to 0 \\ R \to 0}} \| E_{|\underline{r}|<R} \, E_{|H-\lambda|<\varepsilon} \, E_{a.c.}(H) \|$. Then $\gamma_{a.c.}(\lambda) = 0$ or 1, and $\gamma_{a.c.}(\lambda) = 1$ corresponds to simultaneous localisation of states, to abitrary accuracy within the a.c. subspace of H, in both position and total energy.) In particular, $S(\lambda)$ will be norm discontinuous in λ whenever $\gamma_{a.c.}(\lambda) = 1$.

Current work towards a converse of this result were described. It appears that a uniform power estimate in ε of $\| E_{|\underline{r}|<R} \, E_{|H-\lambda|<\varepsilon} \|$ will be sufficient to prove (Hölder) continuity of $S(\lambda)$ for a suitable class of potentials. Some indication of cases in which $S(\lambda)$ was discontinuous were given.

D.B. Pearson, Dept. of Applied Maths,
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Resonance phenomena in cylindrical and parallel-plane waveguides

We study the asymptotic behaviour of acoustic and electromagnetic waves, generated by given time-harmonic exterior forces with frequency ω, in the unbounded region between the parallel planes $x_3 = 0$ and $x_3 = 1$, and show that the principle of limiting amplitude is violated if $\omega = \pi n$ $(n = 1, 2, \ldots)$. For these values of ω, forces with compact support can be chosen such that the amplitudes of the waves increase with a logarithmic rate as $t \to \infty$. A spectral-theoretical discussion relates this phenomenon to singularities of the resolvent of the corresponding time-independent differential operator. Similar resonances (with growth rate $t^{1/2}$) occur in cylinders $Q \times (-\infty, \infty)$ and half-cylinders $Q \times (0, \infty)$ with arbitrary cross sections Q. The resonances react very sensitively to small perturbations. For example, in the case of the half-cylinder with Neumann boundary data $\partial u / \partial n = 0$ on the bottom, resonances occur if $\omega = \sqrt{\lambda_i}$ and λ_i is an eigenvalue of the $(n-1)$-dimensional Laplacian $-\Delta$ in Q. In contrast to this, the solution is bounded as $t \to \infty$ for all frequencies ω if we replace the Neumann condition on the bottom by an impedance condition $(\partial/\partial n - \alpha) u = 0$ with arbitrary $\alpha > 0$.

Peter Werner, Univ. Stuttgart

Asymptotic Behavior of Semigroups

Let $[W(t) : t > 0]$ be a semigroup with generator $-T$ in a Banach space or a Banach lattice X. We have in mind the example of the semigroup generated by the linear transport operator in the Banach lattice $L^1(\Omega \times S)$ and the example of the group generated by the Schrödinger operator in an L^2-Hilbert space. There exist sufficient conditions such that the spectrum of the generator has a strictly dominant eigenvalue. See KAPER, LEKKERKERKER, HEJTMANEK [1982] "Spectral Methods ~~of~~ in Linear Transport Theory", Birkhäuser-Verlag. In reactor theory, the desired asymptotic behavior would be $W(t) = e^{\lambda_0 t} P_0 + Z_0(t)(I - P_0)$ where λ_0 is the decay constant and the transient semigroup Z_0 has type less than the type of W. There is a connection to the problem of 1) scattering theory of the linear transport operator 2) the Schrödinger semigroup and 3) the spectral mapping theorem of the form $\exp(\sigma(-T)) = \sigma(\exp(-T)) \setminus \{0\}$.

There are two methods to get information about the asymptotic behavior: 1) We know the spectrum of $W(1)$ and 2) we know the spectrum of $-T$ and need some additional information. Some of such additional conditions are discussed.

Hejtmanek J.
Universität Wien, Österreich

On the Condition Number of Boundary Integral Operators in Scattering Theory

Brakhage and Werner, Leis and Panich suggested to reduce the exterior Dirichlet boundary value problem for the Helmholtz equation to an integral equation of the second kind which is uniquely solvable for all frequencies by seeking the solution in the form of a combined double- and single-layer potential. We present an analysis of the appropriate choice of the parameter coupling the double- and single-layer potential in order to minimize the condition number of the integral operator.

Rainer Kress, Univ. Göttingen

Darstellungstheorie endlicher Gruppen

(24. Juli – 30. Juli 1983)

Characters of π-separable groups.

If G is a finite p-solvable group, the irreducible Brauer characters of G (for the prime p) are all restrictions of ordinary characters to p-regular elements. It is possible to construct a set $\mathcal{Y}(G)$ of ordinary irreducible characters such that restriction maps $\mathcal{Y}(G)$ bijectively onto $\mathrm{IBr}(G)$, the set of irreducible Brauer characters. This construction of $\mathcal{Y}(G)$ is invariantly defined.

Now let π be any set of primes and assume G is π-separable. One can define (in a canonical way) a set $B_\pi(G) \subseteq \mathrm{Irr}(G)$ (with $\mathcal{Y}(G) = B_{p'}(G)$ for $\pi = \{p\}$). It is true that $|B_\pi(G)| = \#$ of π-classes of G. In fact, the set of restrictions of the B_π-characters to π-elements provide a π-analog of Brauer characters for π-separable groups. (The classical case being $\pi = p'$.) Much of Brauer's theory; decomposition numbers, defect groups etc can be made to work in this setting.

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Extensions of Representations over Normal Subgroups

Let G be a solvable group, M a maximal subgroup, and $\psi \in \mathrm{Irr}(M)$. If $\psi|^G$ is irreducible then why is this so? Let L be the intersection of conjugates of M and K/L be a chief factor of G. Then $G = MK$ and $M \cap K = L$. Let φ be an irreducible constituent of $\psi|_L$. Let T be the stabilizer in G of φ. If $T \leq M$ then $\psi|^G$ is nicely induced. If $T = G$ then stable Clifford theory gives good descriptions of $\psi|^G$. What can happen if $T \nleq M$ and $T \neq G$? Unfortunately, the answer is: almost anything. An analytical tool is proved to facilitate building examples.

Theorem: Let G be a group with normal subgroup L. Let φ be an irreducible character of L. There is a group G^* with normal abelian subgroup L^* and a linear character φ^* of L^* such that

(1) $G/L \simeq G^*/L^*$

(2) There are 1-1 correspondences $\mathrm{Irr}(I|\varphi) \longleftrightarrow \mathrm{Irr}(I^*|\varphi^*)$ (described by $\mu \mapsto \mu^*$) where $L \leq I \leq G$ and $I/L \simeq I^*/L^*$ by (1) and where $(\mu|^J, \nu|^J) = (\mu^*|^{J^*}, \nu^*|^{J^*})$ for $J \geq I_1, I_2$, $\mu \in \mathrm{Irr}(I_1|\varphi)$, $\nu \in \mathrm{Irr}(I_2|\varphi)$.

In essence, L may be assumed to be abelian. Using this theorem it is shown that, T can be almost any subgroup of G containing L and not in M.

Details on the next Garrison Keillor show.

Tom Berger
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On lower defect groups

Let F be a field, char $F = p \neq 0$, G a finite group, FG the group algebra with center ZFG. Decompose $ZFG = \Delta ZFG \oplus JZFG$ where ΔZFG is semisimple and $JZFG$ is the radical. Denote by $\delta : ZFG \to \Delta ZFG$ the corresponding projection.
Let $g \in G$ with p-factor g_p, conjugacy class K in G and p'-section S in $C_G(g_p)$.
Decompose $FG = FC_G(g_p)_{p'}g_p \oplus F[G \setminus C_G(g_p)_{p'}g_p]$ and $FG = FC_G(g_p) \oplus F[G \setminus C_G(g_p)]$ and denote by $\pi_g : FG \to FC_G(g_p)_{p'}g_p$ and $\sigma_g : FG \to FC_G(g_p)$ the corresponding projections.
Then $\pi_g(\delta(z)K^+) = \pi_g(\sigma_g(z)S^+)$.
For conjugacy classes K, L of G, $l \in L$ and $P \in Syl_p(C_G(l))$,
$$\delta(K^+)L^+ = \sum_{C \in Cl(G)} \frac{|L|}{|C|} |\{(c,k) \in C \cap C_G(l_p)_{p'}l_p \times K \cap C_G(l_p) : c^{-1}kl \in P\}| C^+.$$
This implies that for p-subgroups D, P of G and a p-section T of G the sum of multiplicities of P as a lower defect group in T of blocks B having a defect group contained in D can be described as the rank of a suitably defined $\mathbb{F}_p$-matrix.

Burkhard Külshammer, Universität Dortmund

Brauer Trees in Classical Groups (joint work with B. Srinivasan)

Let G be one of the groups $GL_n(q)$, $U_n(q)$, $SO_{2n+1}(q)$, or $Sp_{2n}(q)$. Let B be a cyclic r-block of G, where r is an odd prime and $r \nmid q$. Suppose B is a unipotent block, i.e. the non-exceptional characters in B are unipotent characters. The non-exceptional characters in B are then labeled by partitions or symbols λ. The Brauer tree of B, which is an open polygon, is completely described by combinatorial properties of the λ's. In the case G is $GL_n(q)$, $U_n(q)$, or $SO_{2n+1}(q)$, this implies a complete description of the tree of any cyclic r-block of G.

Paul Fong,
University of Illinois at Chicago

Some Mackey theorems for algebraic groups

A general Mackey imprimitivity theory is presented (joint work with E. Cline and B. Parshall, appearing in a recent issue of Math. Z.) and applied to give Mackey decomposition theorems in special cases for algebraic groups. In more detail, if H is a closed subgroup scheme over an algebraically closed field k of an algebraic group G, and V is a rational G-module over k, then V is induced from H iff V is the global sections of a sheaf $\mathcal{F}$ over G/H such that there is a quasicoherent action of the structure sheaf $\mathcal{O}_{G/H}$ on $\mathcal{F}$ and an action of G on $\mathcal{F}$ over G/H, all actions compatible. For finite groups this means nothing more than V is the direct sum $V_1 \oplus \cdots \oplus V_h$ of subspaces permuted transitively according to the action of G on G/H.

The proof of this criterion in the algebraic groups case is based on a generality of Grothendieck and Verdier valid in suitable functor categories more general than schemes (topoi).

Some applications of the resulting Mackey decomposition theorems obtainable in special cases are mentioned. These include a reformulation of the Andersen-Haboush proof of Kempf's vanishing theorem, and a recent result (in which the above theory was used only in a routine capacity) regarding the finite dimensional injective modules for a group scheme TG_r, where G_r is the scheme-theoretic kernel of the r th power of the Frobenius morphism of a semisimple group G split over $\mathbb{F}_p$ with split torus T. Specifically, a finite dimensional TG_r-module M is injective iff $M|_{T(U_\alpha)_r}$ is injective for each root group U_α.

Leonard Scott

Eulerian numbers and certain characters of the symmetric groups

H.O. Foulkes introduced in 75/76 certain in general reducible characters $\chi^{n,k}$ of S_n which have the following properties:

<u>Thm (Foulkes):</u> (i) $\chi^{n,k}(1) = A(n,k)$, the Eulerian number

(ii) $\chi^{n,k}(\pi)$ does only depend on the number $c(\pi)$ of cyclic factors of π

⋮

It was mentioned, among other things:

<u>Thm (cf [1]):</u> (i) The $\chi^{n,k}$ are linearly independent,

(ii) For each $\chi : S_n \to \mathbb{C}$ such that $\chi(\pi)$ depends only on the number $c(\pi)$ we have

$$\chi = \sum_k \frac{(\chi, \zeta^{(k+1, 1^{n-k-1})})}{\zeta^{(k+1, 1^{n-k-1})}(1)} \chi^{n,k}$$

⋮

So that for ex. $\chi(\pi) := m^{c(\pi)}$ satisfies $\chi = \sum_k \binom{m+k}{n} \chi^{n,k}$

<u>Reference:</u>

[1] A. Kerber / K.-J. Thürlings: Symmetrieklassen von Funktionen und ihre Abzählungstheorie II Bayreuther Math Schriften (in print)

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Extending Group Modules

If N is a normal subgroup of a finite group G, if $\mathcal{O}$ is a coefficient ring, and if M is a G-invariant $\mathcal{O}N$-module, then the G/N-graded endomorphism ring $\mathcal{E} = \mathrm{End}_{\mathcal{O}G}(M^G)$ of the induced $\mathcal{O}G$-module M^G yields an exact <u>Clifford Extension</u>:

$$X\langle \mathcal{E} \rangle : 1 \to U(\mathcal{E}_1) \longrightarrow \mathrm{Gr}U(\mathcal{E}) \longrightarrow G/N \to 1$$

which is known to split if and only if M is extendible to an $\mathcal{O}G$-module.

If ~~the~~ J_1 is the Jacobson radical $J(\mathcal{E}_1)$ of $\mathcal{E}_1$, then $J_1\mathcal{E} = \mathcal{E}J_1$ is a G/N-graded two-sided ideal of $\mathcal{E}$ with J_1 as its 1-component. So we may form the factor G/N-graded ring $\bar{\mathcal{E}} = \mathcal{E}/J_1\mathcal{E}$ and the associated <u>Residual Clifford Extension</u> $X\langle \bar{\mathcal{E}} \rangle$.

<u>Theorem:</u> <u>If</u> $|G/N|$ <u>is invertible in</u> $\mathcal{E}$ <u>and if every idempotent of</u> $\mathcal{E}/J(\mathcal{E})$ <u>can be lifted to an idempotent of</u> $\mathcal{E}$, <u>then any splitting for</u> $X\langle \bar{\mathcal{E}} \rangle$ <u>can be lifted to a unique</u> $(1+J_1)$<u>-conjugacy class of splittings for</u> $X\langle \mathcal{E} \rangle$. <u>Thus in this case</u> M <u>extends to an</u> $\mathcal{O}G$<u>-module if and only if</u> $X\langle \bar{\mathcal{E}} \rangle$ <u>splits</u>.

Since $\bar{\mathcal{E}}$ is a much "smaller" ring than $\mathcal{E}$, the extension $X\langle \bar{\mathcal{E}} \rangle$ is far more computable than $X\langle \mathcal{E} \rangle$. Hence the utility of this criterion for the extendibility of M.

Everett C. Dade

M-groups and symplectic modules

In the talk we have given a survey about recent developments (1980–1983) in the theory of: n-isolinism, M-groups, symplectic modules.

In particular we deal with:

1) the results in I. M. ISAACS' paper in Math. Zeitschr., 182 (1983), 205–221:

<u>Ex</u> A. There exists an M-group of order $2^{15}7^2$ in which the center, which is of order 2, is the unique maximum abelian normal subgroup.

<u>Th</u> B. Let $\mathcal{G}$ be an M-group of odd order in which every abelian normal subgroup is cyclic. Then $\mathcal{G}$ is supersolvable.

<u>Ex</u> C. Let p en q be odd primes with $q \mid p^2+1$. Then there exists an M-group of order $p^{10}q$ in which the center, being of order p^2, is the unique maximum abelian normal subgroup. This center is necessarily of type $C_p \times C_p$.

2) a theorem like Schur's and Clifford's, for symplectic modules, viz.:

<u>Th.</u> Let $\mathcal{G}$ be a finite group, $N \trianglelefteq \mathcal{G}$, $|\mathcal{G}/N| =$ odd prime q. Let $\mathbb{F}$ be a finite field. ~~Then there~~ Let V be a faithful non-singular symplectic irreducible $\mathbb{F}\mathcal{G}$-module. Then there exists a certain finite extension $\mathbb{K}$ of $\mathbb{F}$, and a faithful non-singular symplectic irreducible $\mathbb{K}\mathcal{G}$-module $W \subseteq V \otimes_{\mathbb{F}} \mathbb{K}$ such that at least one of the following properties is true.

1) $W_N = L_1 \perp \ldots \perp L_q$, $L_i \not\cong_{\mathbb{K}N} L_j$, any $i \neq j$ such that the L_i are non-singular symplectic ~~K~~ irreducible $\mathbb{K}N$-module, standing orthogonal to each other, with respect to the sympl form.

2) W_N is irreducible as $\mathbb{K}N$-module.

3) There exists a self-dual absolutely irreducible $\mathbb{K}\mathcal{G}$-module T, such that T_N is an irreducible $\mathbb{K}N$-module, and a 2-dimensional irreducible $\mathbb{K}\mathcal{G}$-module S, such that N is trivially represented on S, such that $T \otimes_{\mathbb{K}} S \underset{\mathbb{K}G}{\cong} W$.

3) some results on n-isolinism obtained by N. S. HEKSTER (Spring 1983)

R. W. van der Waall (Amsterdam)

Modular Representations from local-subgroup geometries via homology

Work of M. Ronan and S. Smith exhibits interrelations between finite geometries and modular representations for simple groups — both Chevalley and sporadic.

The analysis arose partly from the notion of "2-local geometries" introduced at Santa Cruz in 1979 by Ronan-Smith. These geometries are determined by local subgroups but can usually be exhibited by certain subspaces in a small-degree representation. It was observed that these spaces form a chain complex, determining homology representations.

The original results were obtained for Chevalley groups, using the parabolic subgroups (which provide the geometry of the building). For such a group G, we define a sheaf $\mathcal{F}$ (or G-equivariant coefficient system) by

terms (a k-space $\mathcal{F}_P$ for each parabolic P — k is the field of defn. of G)

connecting maps (when $P \leq P'$ are parabolics, $\mathcal{F}_P \to \mathcal{F}_{P'}$)

G-action ($\mathcal{F}_P \xrightarrow{g} \mathcal{F}_{(g^{-1}Pg)}$ for $g \in G$)

with suitable properties to define a chain complex. The most important example: if V is a kG-module, define the fixed-point sheaf $\mathcal{F}_V$ by:

term $\mathcal{F}_P = V^U$ (U = unipotent radical of P ; maps: $\leq$; G-action from module.

An earlier result shows {irreducible kG-modules} and {irreducible sheaves} are in bijection. Further when V is irreducible, the module $H_0(\mathcal{F}_V)$ is "locally constructed" and has head $\cong V$; in practice the unique maximal submodule is small. In applications, it is often useful to know $H_0(\mathcal{F}_V)$ — in some cases (like most "minimal-weight" modules), it co-incides with V.

For sporadic simple groups, a similar theory can be developed, with weaker but still fairly general results. The building is replaced by a chamber system, and the parabolics by stabilizers of geometric objects; the formalism of sheaves and homology is unchanged. For geometries which are "over $\mathbb{F}_p$" for some p (including most cases of interest) it is possible to mimic aspects of the sheaves of restricted-weight irreducibles of Chevalley groups; such sheaves and their homology appear to deliver much of the modular-irreducible theory in sporadic cases.

Stephen D. Smith.
U. Illinois — Chicago

A Proof of the P.A. Smith Theorem

In 1937 P.A. Smith showed that if the group G of prime order p acts (topologically) on the n-sphere S^n then the fixed point set is a mod p homology sphere. We reduce, as in the usual arguments, to considering G acting piecewise linearly on a finite simplicial complex X with the homology of S^n. The argument then gives detailed information about the structure of the mod p chain complex so that the theorem can be read off.

J. L. Alperin

Blocks, isometries, and sets of primes.

In this talk, we are concerned with the following situation:

G is a finite group, π is a set of primes, L is a subgroup of G, A is a union of π-sections of L such that:

i) Any two π-elements of A which are conjugate in G are conjugate in L.

ii) For each π-element $a \in A$, $C_G(a) = C_L(a)\, O_{\pi'}(C_G(a))$.

We are interested in class functions, ψ (complex valued) of L which satisfy: ψ vanishes on $L \setminus A$, and $\psi(abc) = \psi(ab)$ whenever $a \in A$ is a π-element, $b \in C_L(a)$ is a π-regular element, and $c \in O_{\pi'}(C_L(a))$.

In the above situation, there is a unique extension, ψ^0, of ψ, to a

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~~generalized chara~~ class function of G satisfying :

$\psi^\sigma|_L = \psi$, ψ^σ vanishes on π-sections of G which do not meet A, and $\psi^\sigma(abc) = \psi^\sigma(ab)$ whenever $a \in A$ is a π-element, $b \in C_G(a)$ is π-regular, and $c \in O_{\pi'}(C_G(a))$. Also $(\psi^\sigma, \psi^\sigma)_G = (\psi, \psi)_L$.

In the case that $C_L(a)$ is a π-group, for each π-element $a \in A$, Dade has shown that ψ^σ is a generalized character of G if ψ is a generalized character of L. Reynolds showed that the same is true when $C_L(a)$ has a normal π-complement for each π-element $a \in A$. When $\pi = \{p\}$ for a single prime p, Reynolds showed that if ψ is a linear combination of characters in $B_0^{(p)}(L)$, then $\psi^\sigma = \sum_{\chi \in B_0^{(p)}(G)} (\psi^G, \chi)\, \chi$, which makes it clear that in this case, ψ^σ is a generalized character, if ψ is.

We have proved the following results.

~~PROPOSITION~~ THEOREM 1 (In particular, an answer to a conjecture of Reynolds).

Let G, L, A, ψ be as above. Assume also that ψ is constant on π-sections. Then if ψ is a generalized character, so is ψ^σ.

COROLLARY 2

Let H be a finite group, K be a subgroup of H. Suppose that there is $K_0 \lhd K$ such that K/K_0 is a π-group. Suppose that any two π-elements of $K \setminus K_0$

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which are conjugate in H are conjugate in K, and that for each π-element a of $K \setminus K_0$, $C_H(a) = C_K(a)\, O_{\pi'}(C_H(a))$. Then there is a unique subgroup $H_0 \triangleleft H$ such that $H = H_0 K$ and $H_0 \cap K = K_0$.

THEOREM 3.

Suppose that G, L, A satisfy conditions i), above. Suppose further that for each π-element $a \in A$ there is a π'-subgroup $\theta(a) \triangleleft C_G(a)$ such that, for all $p \in \pi$, whenever $a, b \in A$ are π-elements with the <u>same</u> p'-part, x say, we have $\theta(a)^c \cap C_G(b) \leq \theta(b)$ for all $c \in C_G(x)$. ~~let ψ be a generaliz~~ Assume also that $C_G(a) = C_L(a)\, \theta(a)$ for each π-element $a \in A$.

Then if ψ is a generalized character of L which satisfies $\psi(abc) = \psi(ab)$ whenever $a \in A$ is a π-element, $b \in C_L(a)$ is π-regular, and $c \in \theta(a) \cap L$, ψ^σ is also a generalized character of G, where $\psi^\sigma|_L = \psi$, ψ^σ vanishes on π-sections which do not meet A, and $\psi^\sigma(abc) = \psi^\sigma(ab)$ whenever $a \in A$ is a π-element, $b \in C_L(a)$ is π-regular, and $c \in \theta(a)$.

THEOREM 4

Let G, L, A, ψ be as in the introduction. Assume that for each π-element $a \in A$ and each $p \in \pi$ we have $C_G(a_{p'}) = C_L(a_{p'})\, O_{\pi'}(C_G(a_{p'}))$. Then if ψ is a generalized character, so is ψ^σ.

Theorems 1 and 3 can be viewed as generalizations of the isometries of Dade and Reynolds. Theorem 4 as an extension of results of L. Puig.

Finally, we have:

THEOREM 5.

Let G, L, A, ψ be as in the introduction. Assume that $C_L(a)$ is π-soluble for each π-element $a \in A$, and that for each such a, each $p \in \pi$, $O_{p'}(C_L(a))$ has a normal π-complement. Then if ψ is a generalized character, so is ψ^σ.

The basic idea in the proofs of these theorems is to use some consequences of Brauer's characterization of characters to reduce to the case when the π-elements in A are p-elements for a fixed prime p and then to use techniques from block theory to finish the proofs (though theorem 4 admits a proof by ordinary character theory)

Geoffrey R. Robinson (University of Chicago).

Invariant Characters and Invariant Lattices

Suppose H is a normal subgroup of some finite group G and $\mathfrak{I}$ is an (absolutely) irreducible character of H which is invariant under G. Suppose $\mathfrak{I}$ is realizable over some finite extension K of the p-adics $\mathbb{Q}_p$ (p any prime). Let R be the ring of integers of K. Then $\mathfrak{M} = \mathfrak{M}(R, \mathfrak{I})$, the set of isomorphism types of RH-lattices affording $\mathfrak{I}$ is a (non-empty) finite set on which G (or $S = G/H$) acts as a permutation group. We are interested in finding fixed points, i.e. invariant lattices.

In general there will be no G-invariant lattice. Examples are provided by the exceptional characters in p-blocks with cyclic defect group. (Using results of Brauer-Dade-Plesken one can give a fairly complete description of the situation here.) However, we have the following (in joint work with U. Reinhardt):

Theorem. If R contains the p-th root of unity for odd p and the 4-th root of unity in case $p=2$, then there is a G-invariant RH-lattice affording $\mathfrak{I}$.

The proof is by using techniques of Clifford theory. One can define a cohomology class $\tilde{\omega} = \tilde{\omega}(\mathfrak{I}, G, p) \in H^2(S, \mathbb{Z})$ such that there is a G-invariant RH-lattice affording $\mathfrak{I}$ if and only if the ramification index of K over $\mathbb{Q}_p(\mathfrak{I})$ is a multiple of the order of $\tilde{\omega}$. From the theorem we obtain that $o(\tilde{\omega})$ divides $p-1$ in case p is odd and $p=2$ otherwise. It is easy to see that $o(\tilde{\omega})$ also divides $\mathfrak{I}(1)$ and $|S/S'|$.

The result is of some interest for Clifford theory in context with p-modular decomposition. It also might give some useful information on the variety $\mathfrak{M}$ itself.

Peter Schmid (Tübingen).

PPC- Groups

A finite group G is called a $PPC(p_1, .., p_n)$ - group, if all $\chi \in Irr_{\mathbb{C}}(G)$ have prime power degree and if the primes which occur are $p_1, \dots, p_n$. Then G is solvable if and only if $n \leq 2$.

Let G be a $PPC(p,q)$ - group. Then the following assertions hold:

(1) $3 \leq dl(G) \leq 5$ and $2 \leq n(G) \leq 4$.

(2) If $dl(G) = 5$, then G nearly is the semidirect product of $SL(2,3)$ with its standard module.

(3) $dl(G) = 4$ implies that $p = 2$ and q is a Fermat prime. Furthermore the abelian normal $\{p,q\}$-Complement is central.

Olaf Manz (Mainz)

Construction of almost split sequences

If A is a finite-dimensional algebra over a field k, A symmetric, & $M \in \mathrm{mod}\, A$ indecomposable non-projective left A-module, one knows how to construct an almost split sequence (3, below) as the pull-back sequence derived from a map $\theta \in (M, \Omega M)$ $(= \mathrm{Hom}_A(M, \Omega M))$. Here (1), (2) are minimal projective resolutions in $\mathrm{mod}\, A$.

$$\begin{tikzcd}
(1) & 0 \arrow[r] & \Omega M \arrow[r] & P_0 \arrow[r] & M \arrow[r] & 0 \\
(2) & 0 \arrow[r] & \Omega^2 M \arrow[r] & P_1 \arrow[r] & \Omega M \arrow[r] & 0 \\
(3) & 0 \arrow[r] & \Omega^2 M \arrow[r] & E \arrow[r] & M \arrow[r] \arrow[u, "\theta"] & 0
\end{tikzcd}$$

To find a 'good' θ, i.e. such that (3) is almost split, one uses a decomposition of $P_0 \cong \prod_{\nu=1}^{n} Ae_\nu$ (e_ν idempotents in A) to construct a linear form $T_\theta : \mathrm{End}\, M \to k$. Then θ is good iff 1. $T_\theta \neq 0$ & 2. $T_\theta(\mathrm{rad}\, \mathrm{End}\, M) = 0$

J. A. Green (Warwick)

Permutation groups with uniserial modules

P.M. Neumann raised the question whether there are transitive p-subgroups of the symmetric group S_{p^n} of degree p^n which have an exponent less than p^n but nevertheless act uniserially on their natural permutation module over a field of characteristic p. It is proved that the uniserial permutation groups just defined coincide with the p-groups acting p-uniserially in the sense of C.R. Leedham-Green and M.F. Newman on a free abelian group. Independently a quick proof for ~~the~~ uniseriality by the latter two authors is given for both cases. In particular, the uniserial p-subgroups of S_{p^n} of exponent smaller than p^n exist iff $n > p$.

W. Plesken (Aachen)

Blocks of Finite Groups with Radical Cube Zero

Let G be a finite group and k be an alg. closed field of char. $p>0$. Let B be a block algebra of kG with defect group D and let $J(B)$ denote the Jacobson radical of B. We obtained the following theorems;

Theorem 1. If $J(B)^3=0$ (and $J(B)^2\neq 0$), then
(1) $p=2$, D is a four group and B is isomorphic to the full matrix ring over kD of some degree or is Morita equiv. to kA_4, or
(2) $p=$odd, D is of order p and ~~its~~ the Brauer tree of B is a straight line segment. with $p-1$ or $P-1/2$ edges, and the exceptional vertex in an end point.

Theorem 2. Assume $p=2$/ Let U be the projective indecomp. kG-mod. with $U/\mathrm{Rad}\,U = k_G$. If Loewy length of U is 3, then a 2-Sylow subgp of G is dihedral.

Tetsuro Okuyama (Osaka)

Inner products on some Green rings

This is joint work with I. Reiten. Let T be a Brauer tree with e edges and multiplicity m at the exceptional vertex. Let A be a k-algebra to T, k a field, and

$$\begin{tikzcd}
\Omega: 0 \arrow[r] & \Omega_0 \arrow[r] & P_{2e-1} \arrow[r] \arrow[dr] & & \cdots & P_1 \arrow[rr] \arrow[dr] & & P_0 \arrow[r] & \Omega_0 \arrow[r] & 0 \\
& & & \Omega_{2e-1} \arrow[ur] & & & \Omega_1 \arrow[ur] & & &
\end{tikzcd}$$

a minimal projective resolution of a simple A-module corresponding to a non-exceptional endpoint of T. Let $Q_1, \ldots, Q_e$ be the indecomposable projectives and $Q_i^{\perp}$ the dual to Q_i with respect to the Cartan matrix in the rational Grothendieck group. Let $\overline{a(\Omega)}$ $(a(\Omega))$ be the rational Grothendieck group generated by $\{\Omega_i\}$ $(\{\Omega_i, Q_j\})$ with respect to direct sums. For $X, Y \in \{\Omega_i, Q_j\}$ we put $[X, Y] = P(X, Y)$, the A-homomorphisms that factor via projectives, and consider the bilinear form

$$\langle X, Y \rangle = [X, Y] - \sum_{i=1}^{e} [P_i, Y][X, P_i^{\perp}]$$

<u>Proposition</u>: Assume $T \neq \overset{1}{\bullet} \!-\! \overset{1}{\bullet}$

1.) $[\ ,\ ]$ is nondegenerate on $a(\Omega)$

2.) $\langle\ ,\ \rangle$ is nondegenerate on $\overline{a(\Omega)}$

3.) $\langle M, N \rangle = -\delta_{M,N} + \frac{m}{me+1} (\operatorname{sig} M)(\operatorname{sig} N)$, where $\operatorname{sig} \Omega_i = (-1)^i$. Hence $\langle\ ,\ \rangle$ is invariant under stable equivalence.

Applications to blocks with cyclic defect are given modularly and p-adically. Moreover, a Bäckström order to T is constructed.

Klaus Roggenkamp

Stuttgart

joint work with
C. Kratzer

Burnside ring, combinatorics and topology

The primitive idempotents e_S of the Burnside ring $\mathbb{Q}\otimes_{\mathbb{Z}}\Omega(G)$ of a finite group G can be expressed as linear combinations of the canonical basis $\{G/S\}$ of $\Omega(G)$. (Gluck, Yoshida). The coefficients involve the Möbius function of the lattice L of all subgroups of G, e.g. $e_G = \sum_{S\in L} \frac{\mu(S,G)}{|G:S|}\, G/S$.

Theorem 1: $|N_G(S):S|\ |S:S'|_0$ is the smallest integer n such that $ne_S \in \Omega(G)$.
(notation: $|S:S'|_0 = p_1 \cdots p_r$ if $|S:S'| = p_1^{n_1}\cdots p_r^{n_r}$)

This theorem is equivalent to the following result which gives divisibility properties of the Möbius function (analoguous to Brown's result about the poset of p-subgroups of G).

Theorem 2: $\frac{|G:G'|_0\ \mu(S,G)}{|N_G(S):S|} \in \mathbb{Z}$. In particular $\frac{|G:G'|_0\ \mu(1,G)}{|G|} \in \mathbb{Z}$.

The next result is basic for the computation of the Möbius function:

Theorem 3 (special case of a formula of Crapo): If $N \lhd G$,
$\mu(1,G) = \mu(N,G) \sum_{H\in\mathcal{C}} \mu(H,G)$ where $\mathcal{C}$ is a set of complements of N.

Corollary: a) $\mu(1,G)=0$ if N has no complement.
b) $\mu(1,G)=0$ if $\mu(1,G/N)=0$.

Now $\mu(1,G)$ is well known to be equal to the reduced Euler characteristic $\tilde{\chi}(S(G)) = \chi(S(G))-1$ of the simplicial complex S(G) associated to $L \setminus \{1,G\}$.
We are interested in the homotopy type of S(G).

Theorem 4: If G is soluble, if $1 \lhd G_{n-1} \lhd G_{n-2} \lhd \cdots \lhd G_1 \lhd G_0 = G$ is a chief series of G and if m_i is the number of complements of G_i/G_{i+1} in G/G_{i+1}, then S(G) has the homotopy type of a bouquet of $m_1 \cdots m_{n-1}$ spheres of dim $n-2$.

Corollary 1: $\mu(1,G) = \tilde{\chi}(S(G)) = (-1)^n m_1 \cdots m_{n-1}$

Corollary 2: If G is soluble, the following are equivalent: (i) $\mu(1,G) \neq 0$.
(ii) G_i/G_{i+1} has a complement. (iii) Every normal subgroup has a complement.
(iv) Every characteristic subgroup has a complement.

Further examples: $S(A_5)$ has the homotopy type of a bouquet of 60 circles S^1.
$S(PSL_2(7))$ has non-zero homology groups in dimension 1 and 2.
Moreover $\mu(1, PSL_2(7)) = 0$

Jacques Thévenaz (Lausanne
Switzerland)

Brauer Pairs in the General linear Group

Let l be a prime number, and let G be a direct product of general linear groups over finite fields with characteristic $p \neq l$. If S is any semi-simple subgroup of G, we set $C_G(S) = \prod_{\sigma \in I_G(S)} GL_{k_\sigma}(V_\sigma)$, and for σ in $I_G(S)$ we set $q_\sigma = |k_\sigma|$, $\varphi_{q_\sigma}(l)$ = order of q_σ in $(\mathbb{Z}/l\mathbb{Z})^*$.

We show that l-subpairs of G are indexed by triples (R, s, Δ), where R is an l-subgroup of G, s a semi-simple l'-element of G commuting with R, and Δ a map from $I_G(s)$ into the set of all Young diagramms, such that

(1) $|\Delta(\sigma)| \leqslant [V_\sigma^R : k_\sigma]$ and $\varphi_{q_\sigma}(l) \mid [V_\sigma^R : k_\sigma] - |\Delta(\sigma)|$

(2) $\Delta(\sigma)$ has no $\varphi_{q_\sigma}(l)$-hook.

The corresponding l-subpair is denoted by $(R, s, \Delta)_G$. One of the main result is that

$$(R', s', \Delta')_G \subset (R, s, \Delta)_G \iff \begin{cases} (1)\ R' \subset R \\ (2)\ (\exists g \in C_G(R'))\ (s' = s^g \text{ and } \Delta' = \Delta^g) \end{cases}$$

From that result (and from its proof) follow in particular:

(1) Fong-Srinivasan's classification of l-blocks of $GL_n(q)$ and classification of characters in blocks, extended without any change to the case $l = 2$,

(2) knowledge of images of blocks through Brauer morphisms,

(3) Structure of the "Brauer-category" of a block, which is equivalent to the Frobenius category of a subgroup of G with the same type.

The proof is based on a generalization of Curtis-type formulas for Deligne-Lusztig induction, and on a "geometrical" interpretation of deleting hooks.

(Joint work with Lluis Puig) Michel Broué

Ecole Normale Supérieure de Jeunes Filles

Paris.

On the height 0 conjecture

joint work with
T. R. Berger

It is shown that the "if"-part of Brauer's height 0 conjecture holds for any finite group provided it holds for quasi-simple groups; here a non-trivial perfect group is called quasi-simple if every proper normal subgroup is central.
More precisely, we prove

Theorem Assume that for all quasi-simple finite groups, the characters in blocks with abelian defect groups are all of height 0. Then the same is true for all finite groups.

An essential step in the reduction is contained in the following proposition and its corollary which may be of independent interest:

Proposition Let $G = DN$ be finite where D is a p-group and $N \trianglelefteq G$. Then:

(1) For any block b of N, there is precisely one block B of G covering b.

(2) If B has defect group D, then b is the only block of N covered by B.

(3) If B has defect group D and $\chi \in B$ is an irreducible character such that $\mathrm{vx}\, M =_G D$ for any RG-lattice M affording χ, then $\chi_{|N}$ is irreducible.

Corollary Let B be a block of G with abelian defect group D and let N be a normal subgroup of G. Then there exists a block b of N covered by B such that every irreducible character of b extends to DN.

Reinhard Knörr

Univ. Essen

BRAUER'S HEIGHT CONJECTURE (PART 1)

(PART 2 GIVEN BY DAVID GLUCK)

Let B be a p-block of a finite group G. Brauer's height conjecture states that every ordinary character of B has height zero if and only if D is abelian. If D is abelian and G is p-solvable, P. Fong showed that all $\chi \in B \cap \mathrm{Irr}(G)$ have height 0. We prove the converse for p-solvable G and here sketch a proof. In a minimal counterexample to this result, There is a group H (a factor group of G) that acts irreducibly and faithfully on a finite vector vector space V in such a way that each $v \in V$ is centralized by a Sylow-p-subgroup of H. Also $|H:H'| = p$, $p \nmid |H|$ and H' is the unique maximal ~~sb~~ normal subgroup of H. This severely restricts the structure of H and using this knowledge we are able to produce an appropriate character and yield a contradiction.

Restricting the structure of H is the key to the argument. For example, if H is solvable and V is primitive, then H' is cyclic or $|H'| = 8$. If H is solvable and V is imprimitive, then $p \leq 3$ and $d.l.(H) \leq 5$. The passage from solvable to p-solvable use the classification of simple groups.

Thomas R Wolf
Ohio University

Brauer's Height Conjecture for p-Soluble Groups, Part 2

This is a continuation of T. R. Wolf's talk on the same topic. In this talk we discuss some of the technical aspects of the proof, emphasizing the case that the module V (defined on the preceeding page) is imprimitive. We also indicate several ways in which the classification of simple groups is used.

David Gluck
Wayne State University

Products of Conjugacy Classes.

Definitions. G a finite group; $cn(G)$, the covering number of G, is the least integer m such that $C^m = G$ for every nontrivial conjugacy class C of G; $ecn(G)$, the extended c. n., is the least integer m such that $C_1 \cdot \ldots \cdot C_m = G$ for every collection of nontrivial conjugacy classes C_i of G; $cn(G)$ and $ecn(G)$ may not exist; k = number of conj. classes in G.

Results. 1) (Well known) $cn(G)$ and $ecn(G)$ exist iff G is nonab. simple.
2) (Y. Dvir) $ecn(A_5) = cn(A_5) + 1 = 4$; for $n \geq 6$, $ecn(A_n) = cn(A_n) + 1 = [\frac{n}{2}] + 1$.
3) (Arad, Chillag, Moran) $ecn(Sz(q)) = cn(Sz(q)) + 1 = 4$; $ecn(PSL(2,q)) = cn(PSL(2,q)) + 1 = 4$; $cn(G) = 2$ iff $G \simeq J_1$ or, possibly, ${}^3D_4(q)$, q odd; there exist infinite groups with $cn(G) = 2$.
4) (Arad, Herzog, Stavi) $cn(G) \leq \min\{\frac{1}{2}k(k-1), \frac{4}{9}k^2, 4|G|^{1/2}\ln|G|\}$; if every elt of G is a commutator, then $cn(G) \leq 2(k-1)$.
5) (Karni) $ecn(C_{\cdot 3}) = 5$, $cn(C_{\cdot 3}) = 3$; there exists conj. class C in $C_{\cdot 3}$ and other groups, with $C \not\subset C^2$, where $C_{\cdot 3}$ is the Conway sim. gp.

"Conjectures". (I) $cn(G) \leq k - 1$; (II) $ecn(G) = cn(G) + 1$; (III) $C^2 \supset C$ for every conj. class of G; (IV) $C^2 = G$ for some conj. class C of G;

(V) $C_1 C_2 \neq C_3$, where C_i are nontrivial conj. classes of G (G simple & I-V)
Remarks. (I) No counterex. found; (II) One counterex found, see (5); (III) Several counterex. found (see (5)); but true for A_n; (IV) No counterex. found; this is Thompson's conjecture; (V) No counterexamples found.

Marcel Herzog
Tel-Aviv University

Complexity of modules and periodic modules

Let R be a complete discrete valuation ring with quotient field k of characteristic 0, residue class field F of characteristic $p > 0$, $A \in \{R, F\}$ and G a finite group. AG-modules are f.g. and free over A. First, some properties of complexity are presented. Especially, an improved version of Green's lower bound for the p-part of the A-rank of an AG-module is given. Then irreducible RG-lattices are considered. Easy counterexamples show that the first guesses one might have on the R-forms of an irreducible character are not true, e.g. even R-forms for the same character that have the same vertex, do not necessarily have the same complexity. Some bounds for the complexity of an R-form for a given character are stated.

For periodic AG-modules with abelian vertices we get a better lower bound for the p-part of the A-rank than the one above. Furthermore, characters of periodic RG-lattices of odd period are $\mathbb{Z}$-linear combinations of characters of projective lattices, so they are zero on p-singular elements, and $|G|_p$ divides the rank. Because of this, irreducible periodic RG-lattices are always of even period (if k is assumed to be a splitting field).

Christine Bessenrodt
Univ. Duisburg
(University of Illinois at Urbana)

Periodic modules for $SL(2,2^n)$ generated by almost-split sequences.

Periodic modules seem to be of interest lately. For $G = SL(2,2^n)$, certain irreducible modules are periodic. By constructing almost-split sequences involving these modules and their syzygies, one can construct infinite families of periodic modules one for each periodic irreducible.

Tony Chanter, University of Warwick.

Lower Defect Groups with Modules

The major results (due largely to Brauer) on lower defect groups were summarized. The direction of more recent work by others was mentioned. The one shortcoming of the theory – no description of the multiplicity of D in a p-block of $N_G(D)$ – was cited. Brové's result handles p-blocks b with defect group D.

Thm (Brové) B a p-block of G with defect group D Set T equal to the stabilizer of a root block β of B in $C(D)$. Then the multiplicity of D as a lower defect group of D is the number of T-conjugacy classes of $Z(D)$.

The following result dealing with the general case was announced.

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Thm b a p-block of $N_G(D)$. Set T equal to the stabilizer of a root block β of B in $C(D)$. Then the multiplicity of D as a lower defect group of b equals, the number of vertex δD scott module components of $\beta_{\delta(D)}$ extended to δT, where $\delta: G \to G \times G$ is the diagonal map.

University of Fairfield.

On a conjecture of Brauer in case of linear groups

Let $k(B)$ ($k_0(B)$) be the number of ordinary irreducible characters (of height 0) in the r-block B of a finite group G. When you study examples there appear to be the following connections between these numbers and the structural properties of the defect group D of B

(I) $k(B) \leqq |D|$ (R. Brauer)

(II) $k_0(B) \leq |D : D'|$ (D' the commutator subgroup of D)

(III) $k(B) = k_0(B) \Leftrightarrow D$ abelian (R. Brauer)

(IV) $k(B) = |D| \Rightarrow D$ abelian.

These questions are discussed in case of $G = S_n$ (r arbitrary) $G = GL(n,q)$ or $U(n,q)$ ($r \neq 2$, $r \nmid q$.) The r-blocks of $GL(n,q)$ and $U(n,q)$ were described by Fong and Srinivasan (Inv Math, 1982) Using this it is of course possible to compute $k(B)$ and $k_0(B)$ for blocks of these groups. For S_n these numbers are known (see Math. Scand. 38 (1976), 25-42) A reduction theorem of Michler & Olsson (to appear in Math. Z.) allow you to conside only the principal r-block B of S_{wr}, $GL(we,q)$, $U(we,q)$ ($w \in \mathbb{N}$, e the minimal dim. of a group containing an element of order r)

For $n \in \mathbb{N}$ let $\pi(n)$ be the number of partitions of n and put

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$P(x) := \sum_{n\ge 0} \pi(n) x^n$. For $s, t \ge 0$ define integers $k(s,t)$ by ~~P~~ $P(x)^s = \sum_{t\ge 0} k(s,t) x^t$. Let $p^a T(q^e-1)$. Then

$$k(B) = \begin{cases} k(r,w) & \text{(symmetric gp)} \\ \sum_{(w_i)} \left(k\left(e + \frac{r^a-1}{e}, w_0\right) \prod_{i\ge 1} k\left(\frac{r^a-r^{a-1}}{e}, w_i\right) \right) & \text{(linear or unitary gp)} \\ \text{where } (w_i) \text{ runs through the sequences satisfying} \\ \sum_{i\ge 0} w_i r^i = w \end{cases}$$

and if $w = \sum t_i r^i$ is the r-adic decomposition of w then

$$k_0(B) = \begin{cases} \prod_{i\ge 0} k(r^{i+1}, t_i) & \text{(symm. gp)} \\ \prod_{i\ge 0} k\left(\left(e + \frac{r^a-1}{e}\right) r^i, t^i\right) & \text{(general linear or unit. gp)} \end{cases}$$

The defect groups are direct products of wreath products of cyclic r-groups and then it is easy to see that (I) & (II) above hold for these blocks if $k(s,t) \le s^t$ for $s \ge 2, t \ge 0$, which is (almost) true. Indeed:

Prop'n (Atkin) $k(s,t) \le s^t$ if $s \ge 3$ or $s = 2$ and $t \ne 2, 3, 4, 5, 6$.

The remaining cases are checked directly. The question (III) has already been checked and (IV) is easy for these blocks.

J. Olsson (Dortmund)

On projective resolutions for simple $SL_2(p^n)$-modules

Let B be a nontrivial p-block of the group $SL_2(p^n)$, and let Γ be the graph whose vertices are the irreducible B-modules and where the number of edges $S \to T$ equals $\dim \mathrm{Ext}^1_G(S,T)$. There is a covering graph $\underset{\sim}{\Gamma}$ for Γ which describes certain filtrations of the indecomposable projective modules in B. These filtrations are used to describe minimal projective resolutions of the simple modules in B and to find the dimension of $\mathrm{Ext}^r_G(S,T)$ for arbitrary r.

K. Erdmann (Oxford)

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Ext^1_G for irreducible modules over p-solvable group

The following theorem was proved which is a joint work with T. Okuyama.

Theorem Let p be a prime number and G a finite p-solvable group with a Sylow p-subgroup of order p^n. Let k be an algebraically closed field of characteristic p and S, T simple kG-modules. If $p^s \,\|\, \dim S$ and $p^t \,\|\, \dim T$, then we have

$$\dim_k \mathrm{Ext}^1_G(S,T) \leq \min\left\{ (n-s)\dim S/\dim T,\ (n-t)\dim T/\dim S \right\}$$

In particular we have $\dim_k \mathrm{Ext}^1_G(S,T) \leq 1$, provided one of $\dim S$ and $\dim T$ is "sufficiently smaller" than the other. Details will appear in "Comm. in Algebra".

Y. Tsushima (Osaka)

The variety of an indecomposable module is connected

Let G be a finite group and let K be an algebraically closed field of characteristic $p>0$. The ring $\mathcal{E}(K) = \sum_{n\geq 0} \mathrm{Ext}^{2n}_{KG}(K,K)$ is a finitely generated graded commutative K-algebra and has an associated affine variety $V(K)$. If M is a KG-module, let $J(M)$ be the annihilator in $\mathcal{E}(K)$ of $\mathrm{Ext}^*_{KG}(M,M)$, and let $V(M) = V(J(M))$ be the corresponding variety $V(M) \subseteq V(K)$. Let $\tilde{V}(M)$ be the associated projective variety. The main theorem is that if M is indecomposable, then $\tilde{V}(M)$ is connected in the sense that it cannot be written as the union of two disjoint closed sets.

The proof is based on the following lemma. Let $\zeta : \Omega^n(K) \to K$ be a non-zero homomorphism with kernel L. If $n > 0$ and if $\mathfrak{cl}(\zeta) \in J(M)$ then $L \otimes M \cong \Omega^n(M) \oplus \Omega(M) \oplus (\text{proj})$.

Jon F. Carlson (Athens, Georgia).

On the decomposition numbers of the finite general linear groups.

For the symmetric groups it is a well known theorem that all decomposition matrices in all positive characteristics have lower triangular form with one's on the diagonal.
Since the Weyl group of the full linear group $Gl_n(q)$ is isomorphic to the symmetric group S_n on n letters, it seems to be natural to ask, if a similar statement is true for the general linear groups.
So let $G = Gl_n(q)$, and let $2 \neq r$ be a prime not dividing q. Using the classification of the irreducible characters of G given by J.A. Green, and the classification of r-blocks of G given by P. Fong and B. Srinivasan, it is shown that the decomposition matrix of an r-block B of G has lower triangular form with one's on the diagonal, if the semisimple part s of B has the following property: r divides $q^{dg\Lambda} - 1$ for all elementary divisors Λ of s. In this case parts of the decomposition matrix of B may be described by decomposition matrices of certain Weyl groups. In particular this applies to all r-blocks of G, if r divides $q - 1$.

Richard Dipper (Essen)

On the induction and restriction of modular representations

This subject was discussed from the blocktheoretical point of view.

The following results were presented 1) Let M be an indecomposable KG-module in a block B of KG and let H be a subgroup of the finite group G. Assume that $DC_G(D) \subseteq H$, where D is a defect group of B. Then every block b of H with defect group D contains a component of the reduced module M_H, if $b^G = B$. For induction we have 2) If L is an indecomposable KH-module in the block b of H with vertex V such that $C_G(V) \subseteq H$, then every ind. component of L^G with vertex $\nsubseteq V \cap V^g$ for some $g \in G \setminus N_G(V)$ is contained in the block b^G. There are more results of this type. With assumptions on M better results are obtained. Using them, results of Brauer on flat blocks can be extended in several directions. We mention here such a result: Assume that a block B has a representation of dimension $d < p$. Let S be a Sylow p-subgroup of G. If $(d, |N_G(S)/SC_G(S)|) = 1$ then every subgroup H which contains the centralizer of its Sylow p-subgroup in G has exactly one block b with $b^G = B$. Our method also provides new conditions to a module to belong to the principal block. As a particular case a theorem of Cassey and Gaschütz is proved.

Arye Juhász (Rehovot, Israel)

Some permutation characters of the finite general linear group

Let p be a prime, $q = p^m$, $G = GL(n, q^2)$, $H = GL(n, q)$, $U = U(n, q^2)$. There is a one-to-one correspondence between U,U-double cosets in G and conjugacy classes in H, and between H,H-double cosets and conjugacy classes in U. The Hecke algebras associated to the induced characters 1_H^G, 1_U^G are commutative and hence these characters are multiplicity free. If F is the obvious Frobenius map acting on G, and F^* the twisted Frobenius map, the constituents of 1_U^G are precisely the F-fixed characters of G & the constituents of 1_H^G are the F^*-fixed characters. The characters in common to 1_U^G, 1_H^G are the real-valued constituents and their number is the number of real classes in H (= number of real classes in U).

R. Gow (Dublin)

Local Formulae for Cohomology.

Given a finite group G and prime p, Quillen's simplicial complex $\mathcal{Q}$ of elementary abelian p-subgroups of G is defined to be the simplicial complex whose n-simplices are the chains $E_0 \subset E_1 \subset \dots \subset E_n$ of non-trivial elementary abelian p-subgroups. The following result and some applications are discussed:

Theorem Let M be any finitely generated $\mathbb{Z}G$-module and $n \in \mathbb{Z}$. Then

$$\hat{H}^n(G,M)_p = \sum_{\sigma \in \mathcal{Q}/G} (-1)^{\dim(\sigma)} \hat{H}^n(G_\sigma, M)_p .$$

Here the suffix p denotes the Sylow p-subgroup, G_σ is the stabilizer of the simplex σ and the equation holds in the Grothendieck group of finite abelian groups with respect to direct sum decompositions. This suffices to determine the isomorphism type of the p-part of the cohomology of G. The theorem is deduced from a corresponding theorem on permutation modules.

Peter Webb

Diagrams for Modular Lattices

In modular representation theory, it is very useful to have a notation for writing down the lattice of submodules of a module. In my lecture, I described some recent work by J. Conway and myself, which to each modular lattice satisfying suitable finiteness conditions associates a diagram. This diagram usually contains considerably fewer vertices than the original modular lattice. The theorem stating that the modular lattice may be recovered from the diagram, depends on an identity for modular lattices; namely that if a and c are elements of a modular lattice, and b is minimal with respect to the condition $a \vee b \geq c$, then

$$b \wedge (a \vee \mathrm{Rad}(c)) = \mathrm{Rad}(b).$$

Dave Benson

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FUNCTIONAL ANALYSIS AND APPROXIMATION
(30.7 - 6.8.83)

Positive Commuting Perturbations of Selfadjoint Operators and Hyponormality

Let P be a selfadjoint operator on a separable, infinite dimensional Hilbert space. Then there exists a completely hyponormal operator T having a polar factorization $T = UP$, U unitary, and satisfying the condition that T^*T and TT^* commute, if and only if $P \geq 0$ and $\sigma(P)$ contains at least two points, 0 is not in $\sigma_p(P)$, and, whenever $\sigma_p(P)$ is not empty, neither $\sup \sigma_p(P)$ nor $\inf \sigma_p(P)$ belong to $\sigma_p(P)$ with a finite multiplicity.

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Shortest Path Algorithms for the Approximation by Nomographic Functions

In a compact domain $D \subset \mathbb{R}^2$ the functions $f \in C(D)$ are approximated in the uniform norm by the class of nomographic functions
$NOM := \{ w \in \ell_\infty(D) \mid w(s,t) = g(x(s)v(t) + u(s)y(t))$, x and y are arbitrary bounded functions$\}$
where u and v are given positive continuous functions, g is strictly monotone and continuous. This approximation problem is converted into the negative cycle problem in a properly chosen family of weighted directed graphs, and a version of the Ford-Bellman algorithm for finding shortest paths leads to constructive proofs of new characterization and existence theorems.

Special cases of NOM are well-known approximation subspaces in the theory of integral equations, functional equations, scalings of matrices, Goursat-type problems for the wave equation, and in bivariate approximation theory.

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Avereged moduli of smootness

The purponse of the paper is a new appraach for estimating the error in a large number of numerical methods as interpolation, approximation of functions by means of operators, quadrature formulae, network methods of solution of integral and differential equations, ec.

These new characteristics of functions are named averaged moduli of smoothness, or τ-moduli. They have sense for every bounded function and are an integral analogue of the classical moduli of continuity and smoothness for the uniform metric, so-colled ω-moduli.

The novelity in our approach is the different way of conveying an analogue of the uniform case.

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Strong approximation

The aim of the lecture was to present a generalization of Sunouchi's theorem. Our theorem reads as follows: If α, γ and p are positive numbers, and $0 < p\gamma < 1$ then $\sum c_n^2 n^{2\gamma} < \infty$ implies

$$\left\{ \frac{1}{A_n^\alpha} \sum_{k=0}^{n} A_{n-k}^{\alpha-1} |s_{\nu_k}(x) - f(x)|^p \right\}^{1/p} = o_x(n^{-\gamma}) \qquad \left(A_n^\alpha = \binom{n+\alpha}{n}\right)$$

almost everywhere for any increasing sequence $\{\nu_k\}$, where $s_n(x)$ denotes the n-th partial sum of $\sum c_n \varphi_n(x)$, and $\{\varphi_n\}$ is an arbitrary orthonormal system.

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GRAPH THEORY IN THE APPROXIMATION THEORY OF FLUID DYNAMICS

We solve the dimension and bases problems of Temam's book on the Navier Stokes equations, by means of concepts from graph theory. In this way a large number of "incompressibility subspaces" and other subspaces of finite element theory may be studied.

Karl Gustafson

A CLASS OF POSITIVE TRIGONOMETRIC SUMS

This is joint work with Professor Gavin Brown of The University of New South Wales. Let $(a_k)_{k=0}^{\infty}$ be a nonincreasing sequence of positive real numbers. One seeks reasonable conditions on a_k ensuring that

$$(1) \qquad \sum_{k=0}^{N} a_k \cos k\theta$$

and

$$(2) \qquad \sum_{k=1}^{N} a_k \sin k\theta$$

be positive for all $N \in \{1,2,\dots\}$ and $\theta \in]0,\pi[$. The examples $a_0 = 1$, $a_k = \frac{1}{k}$ for $k \geq 1$ go back to Jackson (1911) for (2) and W. H. Young (1913) for (1). Rogosinski and Szegő dealt with (1) and $a_0 = a_1 = \frac{1}{2}$, $a_k = \frac{1}{k+1}$ for $k \geq 1$. In 1958, Vietoris proved positivity for (1) and (2) for $a_0 = a_1 = 1$, $a_{2k} = a_{2k+1} = \frac{2k-1}{2k} a_{2k-1}$ for $k = 2,3,\dots$. This result contains Young's and Jackson's but not Rogosinski and Szegő's. We prove positivity of (1) for $a_0 = a_1 = 1$, $a_{2k} = a_{2k+1} = \frac{2k}{2k+1} a_{2k-1}$ for $k \geq 1$. The proof is somewhat involved and requires a new method. The series (2) for our a_k's is never positive everywhere in $]0,\pi[$ but is positive in $]0, \pi - \frac{1}{N}\pi]$.

Edwin Hewitt (also for Gavin Brown)

Reconstruction of Entire Harmonic Functions from Given Values.

According to a result of Carlson (1914), an entire function f of exponential type less than τ is uniquely determined by its values at the points $n\pi/\tau$ $(n=0,\pm1,\pm2,\ldots)$. The reconstruction of f from these values is of special interest to engineers (keyword "Sampling Theorem"). R.P. Boas observed that the situation changes if we consider an entire harmonic function of exponential type less than τ. He proved that such a function u is uniquely determined by its values at the lattice points $n\pi/\tau$, $(n+i)\pi/\tau$ $(n=0,\pm1,\pm2,\ldots)$ and asked for the reconstruction of u from these values. We give an answer to this question thereby improving on an earlier approach of Ching & Chui and also solving another problem of Boas. — (Joint work with R. Gervais and Q.I. Rahman).

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On condensation of singularities on a set of full measure.

Continuing previous work on uniform boundedness and condensation principles (with rates) by W. Dickmeis and R.J. Nessel another version of a condensation principle is given. As one application one now obtains for the Bernstein polynomials $B_n(f;x)=\sum_{k=0}^{n}\binom{n}{k}x^k(1-x)^{n-k}f(k/n)$:

THEOREM: For each $\alpha \in (0,2]$ there exists a function f_α satisfying the usual (generalized) Lipschitz condition of order α such that

$$\limsup_{n\to\infty} \frac{|B_n(f_\alpha;x)-f_\alpha(x)|}{[x(1-x)/n]^{\alpha/2}} \geq c > 0$$

for almost every $x \in [0,1]$.

This theorem shows that the rates of convergence $O([x(1-x)/n]^{\alpha/2})$ for the Bernstein polynomials on Lipschitz classes (of order α) as given by H. Berens and G. G. Lorentz are sharp in the sense that there exists (at least one) conterexample f_α for which they cannot be improved to $o([x(1-x)/n]^{\alpha/2})$.

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On maximal extensions of accretive operators in the plane

In 1962 Minty proved that for a real Hilbert space, say H, a monotone (accretive) operator $A \subset H \times H$ is maximally monoton exactly when there exists a $\lambda \in \mathbb{R}^+$ such that (and consequently for all $\lambda \in \mathbb{R}^+$) $I + \lambda A$ is a surjection of A onto H. In the Banach space setting the latter property is called m-accretiveness. In contrast to Minty's result Crandall and Liggett showed in 1971 that for $l_p(2)$, $1 \leq p \leq \infty$, the class of m-accretive operators coincides with the class of maximally accretive ones exactly when $p = 1, 2$, or ∞.

In joint work with Dr. Hetzelt we extended C. ; L.'s result as follows: Let X be a real, 2-dimensional, normed vector space with a strictly convex and smooth norm. If every accretive operator in

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X × X has a m-accretive extension then the norm generates an inner product.

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On generation of one-parameter operator groups.

Let $\alpha = (\alpha_t)_{t\in\mathbb{R}}$ be an appropriately continuous one-parameter group of continuous linear operators in a Banach space X. The analytic generator α_{-i} of α is defined as follows:

$$(x,y)\in \mathrm{graph}(\alpha_{-i}) \iff \left\{\begin{array}{l} \mathbb{R}\ni t \mapsto \alpha_t(x) \text{ has a} \\ \text{continuous extension on} \\ \{z\in\mathbb{C}\,;\, -1\leq \mathrm{Im}\, z\leq 0\}, \\ \text{analytical on the interior,} \\ \text{whose value in } -i \text{ is } y. \end{array}\right.$$

If X is Hilbert space and the α_t's are unitaries, then α_{-i} is an injective positive selfadjoint operator and α_t is the it'th power of α_{-i}. If X is a von Neumann algebra and the α_t's are $*$-automorphisms, then in the majority of the cases the spectrum of α_{-i} is the whole complex plane, so α_{-i} has bad spectral properties. In spite of this, acceptable characterisations can be given for analytic generators of one-parameter groups of $*$-automorphisms of von Neumann algebras. Such characterisations are useful in the quantum field theory.

László Zsidó
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Invariant function spaces connected with the holomorphic discrete series.

Joint work with Jonathan Arazy, Steve Fisher, Svante Janson, Steve Semmes, Per Nilsson. First I summarize the theory of Möbius invariant spaces of holomorphic functions in the disk, as developed by Arazy, Fisher, Rubel, Timoney, and others. Then I consider from a similar point of view more general group actions connected with the holomorphic discrete series. Finally assorted applications are given: Hankel operators (new proof and generalization of Peller's theorem [as well as the theorem of Janson–Wolff for Calderón–Zygmund commutators]), rational approximation, Shields's Riesz–Fejér inequality, Carleson measures etc.

Jaak Peetre
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Fourier transformation as integration with respect to vector measures

Let Γ be a l.c.a. group, $\mathcal{B}$ be the class of Borel subsets of Γ, $\mathcal{B}_m = \{B : B \in \mathcal{B} \,\&\, m(B) < \infty\}$, and $\hat{\Gamma}, \hat{\mathcal{B}}, \hat{m}, \hat{\mathcal{B}}_{\hat{m}}$ be the dual entities. For $B \in \mathcal{B}_m$, let $\xi(B) = \check{1}_B$, where $\check{1}_B(\lambda) = \int_B \lambda(t)\, m(dt)$, $\lambda \in \hat{\Gamma}$. Then ξ is a countably additive measure on $\mathcal{B}_m$ with values in each of the Banach spaces $L_{p'}(\hat{\Gamma})$, $1 \le p \le 2$. Write $\xi_{p'}$ for ξ, when the $L_{p'}$ topology is used on its range. Then the class $\mathcal{L}_{\xi_{p'}}(\Gamma)$ of $\xi_{p'}$ integrable functions on Γ satisfies the inclusion $L_p(\Gamma) \subseteq \mathcal{L}_{\xi_{p'}}(\Gamma)$ for $1 \le p \le 2$, and for all $f \in L_p(\Gamma)$, $\int_\Gamma f(t)\, \xi_{p'}(dt) = \check{f}$, the Fourier transform of f in the Hausdorff-Young sense. This generalizes the $p = 2$

were settled in 1969 [Abstract spaces & Approximation, P. L. Butzer & B. Sz-Nagy, Birkhauser, 162-182]. If the last equality is adopted as a definition of the FT $\check{f}$ of f, we get a unified theory of Fourier transformation for $1 \leq p \leq 2$, devoid of improper integration

We have $L_2(\Gamma) = \mathcal{L}_{\xi_2}(\Gamma)$. But for $1 < p < 2$ it transpires that $L_p(\Gamma) \subsetneqq \mathcal{L}_{\xi^+_{p'}}(\Gamma)$. This ~~creates~~ raises some interesting new questions.

P. Masani
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Product formulas for Bessel, Whittaker, and Jacobi functions via the solution of an associated Cauchy problem

An analytic proof of the product formulas for Bessel, Whittaker, and Jacobi functions is given which are due to Sonine (1880), Watson and Glaisher (1939/1981), and Koornwinder (1972), respectively. The proof is based on the approach of Delsarte (1936) to generalized translation operators via the solution of an associated Cauchy problem. The three systems of functions are eigenfunctions for different potential functions q of a Sturm-Liouville equation of the form $D^{\alpha}_{q,x} u_{\lambda}(x) + \lambda^2 u_{\lambda}(x) = 0$, where $D^{\alpha}_{q,x} = \frac{d^2}{dx^2} + \frac{2\alpha+1}{x}\frac{d}{dx} - q(x)$, $0 < x < \infty$, $\alpha \geqslant -\frac{1}{2}$, and $u_{\lambda}(0) = 1$, $u_{\lambda}'(0) = 0$. The generalited transeation of a function f is introduced as the solution of the Cauchy problem $(D^{\alpha}_{q,x} - D^{\alpha}_{q,y})\, u(x,y) = 0$, $u(x,0) = f(x)$, $u_y(x,0) = 0$. The main step in solving this problem is to determine the associated Riemann function. In all three cases, the characteristic boundary value problem for the Riemann function can be transformed into a normal form for which the solution is known. This leads to an explicit representation of the Riemann functions associated with the Bessel, Whittaker, and Jacobi differential operator by means of which the kernels of the corresponding translation operators are then calculated.

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On a new class of generalized functions introduced by J. J. Lodder

by Tom H. Koornwinder, Math. Centrum, Amsterdam

This is a report of work in progress joint with J. J. Lodder. In view of applications in quantum electrodynamics Lodder [1] developed a new class of generalized functions which is closed under multiplication, Fourier transform, differentiation and dilation, and which is symmetric in the sense that there is no longer a distinction between test functions and distributions. However, the proofs in [1] are still somewhat sketchy. In the lecture a more rigorous approach to Lodder's generalized functions will be discussed. Let PC be the smallest subspace of $\mathcal{S}'$ (space of tempered distributions) which contains all $x_\pm^\alpha (\log x_\pm)^k$ and $\delta^{(n)}$ and which is invariant under multiplication with elements of $\mathcal{S}$, translation and Fourier transform. On PC a multiplication can be defined which is noncommutative nonassociative and rather arbitrary. However, on a certain dual PC' of PC a~~n~~ canonical associative multiplication can be defined

[1] J. J. Lodder, Physica 116A (1982), 45-58, 59-73, 380-391, 392-403, 404-41

Some Extremal Problems With Constraints

Let D be the unit disk in the complex plane, and let σ be normalized Lebesgue measure on $\Gamma = \partial D$. Fix $0 \leq w \in L^\infty(\sigma)$.

Def. Given $h \in L^2(w\,d\sigma)$, by an optimal approximant to h we mean any $\tilde{k} \in H^2(D)$ such that $\int_\Gamma |h-\tilde{k}|^2 w\,d\sigma \leq \int_\Gamma |h-k|^2 w\,d\sigma$ for all $k \in H^2(D)$ such that $\|k\|_2 \leq \|\tilde{k}\|_2$.

Set
$$c_\lambda(z) = \exp\left(\frac{1}{2}\int_\Gamma \frac{e^{i\theta}+z}{e^{i\theta}-z}\log[w(e^{i\theta})+\lambda]\,d\sigma\right)$$
$$k_\lambda(z) = \frac{1}{z}\left[1-\frac{c_\lambda(0)}{c_\lambda(z)}\right]$$
for $z \in D$, $\lambda > 0$, and also for $\lambda = 0$ when meaningful.

Theorem. The optimal approximants to $e^{-i\theta}$ are given by

(i) $\{k_\lambda\}_{\lambda>0}$ if $w^{-1} \notin L^1(\sigma)$,

(ii) $\{k_\lambda\}_{\lambda\geq 0}$ if $w^{-1} \in L^1(\sigma)$.

This yields the following refinement of Szegö's infimum.

Theorem. For $\lambda > 0$,
$$\min\left\{\int_\Gamma |e^{-i\theta}-k|^2 w\,d\sigma : k \in H^2(D),\ \|k\|_2^2 \leq \exp\left(\int_\Gamma \log(w+\lambda)\,d\sigma\right)\int_\Gamma \frac{d\sigma}{w+\lambda} - 1\right\}$$
$$= \exp\left(\int_\Gamma \log(w+\lambda)\,d\sigma\right)\int_\Gamma \frac{w\,d\sigma}{w+\lambda}.$$
The unique extremal function is $k = k_\lambda$.

A similar result is obtained relative to Kolmogorov's infimum.

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Fixed Point and Implicit Function Theorems and their Applications

This paper discusses applications, to differential equations in Banach spaces, to optimization theory, to systems of partial differential equations, and to numerical functional analysis, of three fixed point theorems and a quasi-Newton iteration scheme associated with an implicit function theorem of functional analysis.

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Funktionalanalytische Aspekte der Radarortung und digitalen Signalübertragung.

Im Vortrag wird gezeigt, daß die Darstellungstheorie (d.h. die Mackey-Maschinerie, oder, in geometrischer Formulierung, die Kirillov-Korrespondenz) der reellen nilpotenten Heisenberg-Gruppe $\tilde{A}(\mathbb{R})$ im Schnittpunkt der Quantenmechanik, der Theorie der analogen Signale (Radarortung) und der Theorie der digitalen Signale (Abtasttheorem) liegt. Diese Theorie ermöglicht insbesondere die Untersuchung der Symmetrieeigenschaften der Radar-Unschärfeflächen. Mit Hilfe der unitären Oszillatordarstellung der metaplektischen Gruppe $Mp(1,\mathbb{R})$ lassen sich dann die zugehörenden erzeugenden Impulseinhüllenden explizit berechnen. Auf Anwendungen der harmonischen Analyse der endlichen nilpotenten Heisenberg-Gruppe in der numerischen

Mathematik und abschließend kurz hingewiesen.

Walter Schempp (Siegen).

Some embedding theorems for modular classes

1961 W. Matuszewska obtained results concerning connections between $\bigcap_{i=1}^{\infty} L^{\varphi_i}\langle a,b\rangle$, $\bigcup_{i=1}^{\infty} L^{\varphi_i}\langle a,b\rangle$ and $L^{\varphi}\langle a,b\rangle$ for Orlicz classes. These results were generalized 1974 and 1977 by A. Waszak and myself to Orlicz classes with functions φ depending on a general parameter ξ in place of index i, ξ running over a set Z. Now, the results are extended to the case of general concave and convex functionals in place of integrals over sets Z.

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Two of my favorite ways of obtaining asymptotics for orthogonal polynomials.

Improvements of the continuous and discrete Liouville-Steklov method for proving asymptotic formulas for orthogonal polynomials are discussed, and a short survey of recent asymptotic results is given

Go Bucks!

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Convolution Structures for Eigenfunction Expansions Arising from Regular Sturm-Liouville Problems

The eigenfunctions associated with a regular Sturm-Liouville problem behave "like" trigonometric expansions in many ways – for example there are various asymptotic estimates and equiconvergence theorems. In order to utilize the full machinery of harmonic analysis, however, it is necessary to have some substitute for the group structure so useful in arguments concerning trigonometric expansions. That substitute is a positive convolution which is ~~then~~ shown to exist, and then utilized to prove various maximal function inequalities.

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Extremalpolynome in der L^1- und L^2-Norm auf zwei disjunkten Intervallen.

Sei $-1<\alpha\leq\beta<1$. p_n bezeichne das Orthogonalpolynom bez. der Gewichtsfunktion

$$w(x) = \begin{cases} \sqrt{x-\alpha}\,/\sqrt{(1-x^2)(x-\beta)} & \text{für } x\in(-1,\alpha)\cup(\beta,1) \\ 0 & \text{"} \ x\in(\alpha,\beta) \end{cases}.$$

Für die Rekursionskoeffizienten von p_n wird eine Rekursionsrelation angegeben. Mit Hilfe des Orthogonalpolynoms p_n werden dann

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jene Polynome $P_n = x^n + \dots$ bestimmt, die bezüglich der L^2-Norm auf $[-1,\alpha] \cup [\beta,1]$ am wenigsten von Null abweichen.

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Green's Functions for the finite difference heat, Laplace, and wave equations

In this paper, representations are developed for the Green's functions for a partial difference formulation of an initial-value problem that includes the half-plane heat (diffusion), Laplace, and wave equations as special cases. Solutions of the partial difference equation are shown to be given by a discrete convolution that is analogous to integral representations for the continuous case. A convergence property relating each discrete Green's function to that of its associated partial differential equation is also presented.

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Subnormal suboperators and the subdiscrete topology

A suboperator is a bounded linear transformation from a subspace $\mathcal{H}_0$ of a Hilbert space $\mathcal{H}$ into all of $\mathcal{H}$; it is subnormal if it can be extended to a normal operator on $\mathcal{H}$. Principal problem: characterize subnormal suboperators. Subquestion: what is the closure (e.g., strong topology) of the set of all suboperators from $\mathcal{H}_0$ (fixed) into $\mathcal{H}$? The paper solves some related problems (but not the ones stated here — they are unsolved). Pertinent concept: the subdiscrete topology of operators on $\mathcal{H}$ is the specialization to $\mathcal{B}(\mathcal{H})$ of the

Tychonoff product topology of $\mathcal{H}^{\mathcal{H}}$, where the exponent is given the discrete topology. This circle of ideas has close connection with Bishop's theorem to the effect that the strong closure of the normal operators is the set of subnormal operators.

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Polynomial approximation on disjoint intervals

A general classical result of J. L. Walsh ensures the possibility of approximation of certain type of functions by polynomials on disjoint finite interval. Nevertheless, this result is not constructive, and, in general, it does not give concrete information on the order of convergence. Starting from a recent result of C. K. Chui and M. Hasson, we prove a convergence estimate for the set $[-b,-a]\cup[a,b]$ $(0<a<b)$, when besides analyticity, the function satisfies some smoothness condition on the boundary. The norm of the best approximating polynomials on $[-a,a]$ are also estimate from both sides. A generalization of the convergence theorem for more than two disjoint intervals of possibly different lengths is also given.

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$\sqrt{\delta}$

In a recent master's thesis, my student B. Hanzon has shown that there exist integrable functions f on $\mathbb{R}$ such that $f . f=\delta$. There are <u>no</u> real solutions! One can make supp f arbitrarily small. Solutions are constructed "by hand", starting with the periodic case, $\tilde{f}.\tilde{f}=\delta_{2\pi}$. A reasonable product definition for $\hat{f}.\tilde{g}$ there is via Fourier series: $\hat{f}.\tilde{g} \overset{\text{def}}{=} \sum_n \{\sum_k \hat{f}(k)\hat{g}(n-k)\}e^{inx}$ when this makes sense. Question: is there an "analytic" solution f of the equation $f^2=\delta$, that is, a solution given by an analytic expression?

J. Korevaar – Amsterdam

The necessity of a new kind of modulus of smoothness

A new kind of modulus of smoothness is introduced and applied to different approximation problems. It very much resembles the ordinary moduli of smoothness only the increment in it varies together with the variable (see below). The applications include the characterization of best polynomial approximation, exact estimates on the rate of approximation by positive or contraction operators, inverse theorems and the characterization of the K-functional between L^p and the corresponding weighted Sobolev space (with a given weight). As an illustration let us state the equivalence of $E_n[f]_{L^p[-1,1]} = O(n^{-\alpha})$ and $\|\Delta^r_{h\varphi} f\|_{L^p[-1,1]} = O(h^\alpha)$ $(\varphi(x) = \sqrt{1-x^2}\,;\ \alpha < r)$.

V. Totik (Szeged)

Subspace lattices connected with C_{11} - contractions

We say that a Hilbert space-contraction T belongs to the class C_{11} if for every non-zero vector h the limits $\lim_{n\to\infty} \|T^n h\|$ and $\lim_{n\to\infty} \|T^{*n} h\|$ are not equal to zero. C_{11}-contractions are close to unitary operators in the sense also that they are quasi-similar to unitary ones. We consider the hyperinvariant subspaces $\mathcal{L}$ of T such that $T|\mathcal{L} \in C_{11}$. The set of these subspaces is denoted by $\mathrm{Hyplat}_1 T$. The behaviour of $\mathrm{Hyplat}_1 T$ under quasi-similarity and its relation to Hyplat T is studied. Among others negative answer is given for a problem of Sz.-Nagy and Foiaş.

László Kérchy (Szeged, Hungary)

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The best harmonic approximant to a continuous function.

Suppose that f is bounded and continuous in a domain D in R^k. Then there exists a best harmonic approximant h to f in the uniform norm. If D is a Jordan domain, f is continuous in $\bar{D}$ and there exists an h which has a continuous extension to $\bar{D}$ then h is the unique best approximant and can be characterised in terms of the sets in $\bar{D}$ where $h-f$ assumes the extreme values $\mp m$. Examples show that if these hypotheses are relaxed in various ways the conclusions may fail. For instance h need not be continuous in $\bar{D}$ even if f is continuous in $\bar{D}$ and, if f is only bounded and continuous in D, h need not be unique. Further the characterisation can break down if D is the unit disk cut along the nonpositive real axis. The work is joint with D. Kershaw and T. J. Lyons.

W. K. Hayman (London, England)

Approximation of functions of two variables by means of algebraic polynomials.

Let D be a domain in the plane with the boundary Γ and let Γ be a finite union of arcs with continuous curvature and the angles in the join-points with are positive and less than π (with respect of the domain). For the best approximation of a function in $L_p(D)$ $(1\le p\le\infty)$ we state a direct theorem of Stečkin's type and a converse theorem of Salem-Stečkin's type. In this theorems we use a new moduli of functions of two variables. Some of the properties of these moduli and their conection with usualy moduli are given.

K. G. Ivanov (Sofia, Bulgaria)

The spectrum of the Laplacian for domains in hyperbolic space

This talk is concerned with the spectrum of the Laplace-Beltrami operator acting on domains with the finite geometric property and of infinite volume in real hyperbolic space $\mathbb{H}^{n+1}$. In such domains the Laplacian has a discrete spectrum in the interval $[0, (\frac{n}{2})^2)$ and an absolutely continuous spectrum in $[(\frac{n}{2})^2, \infty)$. Discrete subgroups of motions have fundamental domains of this sort where the lowest eigenvalue is closely related to the Hausdorff dimension of the limit set and the counting number for orbits. A lower bound on this value is obtained from the lowest eigenvalue for the Laplacian with free boundary conditions. Its existence or nonexistence is investigated as well as its continuity under deformations, especially degenerate types of deformations. It is shown that the Hausdorff dimension of the limit sets of a discrete group of motions in $\mathbb{R}^n$, $n \geq 3$) generated by inversions in a finite number of mutually exterior spheres can not be made arbitrarily close to n. These results were obtained in collaboration with Peter Sarnak.

Ralph Phillips

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Some Negative Results in Connection with Marchaud-Type Inequalities

Continuing our previous investigations on quantitative uniform boundedness principles, the present paper, which represents joint work with W. Dickmeis and E. van Wickeren, is concerned with some negative results in connection with Marchaud-type inequalities. The existence of the relevant counterexamples follows by means of a general theorem, given in terms of operators in Banach spaces. The method of proof essentially consists in a quantitative version of the familiar gliding hump method.

Rolf J. Nessel (RWTH Aachen)

Über die Konvergenz der Mitteln von orthogonalen Funktionen

Es sei $\lambda = \{\lambda_k\}_1^\infty$ eine monoton wachsende und ins Unendliche strebende Folge von positiven Zahlen. Weiterhin für ein K $(1 \leq K \leq \infty)$ sei $\Omega(K)$ die Klasse der orthonormierten Systeme $\varphi = \{\varphi_k(x)\}_1^\infty$ in $(0,1)$ mit $|\varphi_k(x)| \leq K$ $(x \in (0,1);\ k=1,2,\ldots)$. Für eine Folge $a = \{a_k\}_1^\infty$ und für eine positive ganze Zahl N setzen wir $\|a;K;\lambda;\| = \sup_{\varphi\in\Omega(K)} \left\{\int_0^1 \sup\left(\frac{1}{\lambda_n}\sum_{1}^{n} a_k\varphi_k(x)\right)^2 dx\right\}^{1/2}$. Es sei endlich $M(K;\lambda) = \{a : \lim_{N\to\infty} \|a;K;\lambda;N\| = 0\}$.

Man kann z.B. die folgenden Sätzen beweisen.

I. Gilt $a \in M(K;\lambda)$, dann besteht $\frac{1}{\lambda_n}\sum_{k=1}^{n} a_k\varphi_k(x) \to 0$ $(n\to\infty)$ für jedes $\varphi \in \Omega(K)$ in $(0,1)$ fast überall. Gilt aber $a \notin M(K;\lambda)$, dann gibt es ein $\Phi \in \Omega(K)$ derart, dass die Folge $\frac{1}{\lambda_n}\sum_{k=1}^{n} a_k\Phi_k(x)$ $(n=1,2,\ldots)$ in $(0,1)$ fast überall divergiert.

II. Für jedes K, $1 < K < \infty$ gilt $M(K;\lambda) = M(1,\lambda) \supsetneqq M(\infty)$.

K. Tandori (Szeged)

Ideals in C(X)

Let C(X) be the Riesz space and algebra of all real continuous functions on some Tychonov space X. The set of all order ideals (algebra ideals) is denoted by $\mathcal{O}$ ($\mathcal{A}$), whereas the collection of all order prime ideals (algebra prime ideals) is denoted by $\mathcal{OP}$ ($\mathcal{AP}$). Furthermore, let $\mathcal{OM}$ ($\mathcal{AM}$) be the set of all order maximal ideals (algebra maximal ideals). The following results hold:

1) $\mathcal{OM} \subset \mathcal{AM} \subset \mathcal{AP} \subset \mathcal{OP}$

2) $\mathcal{O} \subset \mathcal{A} \iff X$ pseudo-compact

(normal)

3) (Gillman-Henriksen, 1956) $\mathcal{A} \subset \mathcal{O} \iff C(X) = \{f^+\}^d + \{f^-\}^d \ \forall f \in C(X)$.

(Seever, 1967) $\iff C(X)$ has the σ-interpolation property

4) $\mathcal{OM} = \mathcal{AM} \iff X$ pseudo-compact

5) $\mathcal{AM} = \mathcal{AP} \iff C(X)$ Z-regular (Riesz space analogue of von Neumann-regular)

$\iff C(X)$ σ-laterally complete

6) $\mathcal{AP} = \mathcal{OP} \iff X$ finite

All these results can be generalised to f-algebras and (in a proper setting) to Riesz spaces. It is shown that the study of C(X) can be used to derive general Riesz space theorems. For instance, Seever's result generalises to: the Riesz space L has the σ-interpolation property $\iff$ L is normal + relatively uniformly complete. The result that R_π is an order ideal in L for all orthomorphisms π on L whenever L has the σ-interpolation property depends heavily on this theorem.

C. B. Huijsmans (Leiden)

Some recent results on the divergence of Lagrange interpolation

The two main theorems are as follows:

If $X = \{x_{kn}\} \subset [-1,1]$, $k = 1, 2, \dots, n$; $n = 1, 2, \dots$; is an interpolatory matrix, $L_n(f, X, x)$ is the Lagrange interpolatory polynomial of degree $\leq n-1$, $\omega(t)$ is a modulus of continuity,

$C(\omega) = \{ f ; \ \omega(f,t) = O(\omega(t)) \}$

$C^*(\omega) = \{ f ; \ \omega(f,t) = o(\omega(t)) \}$

where $\omega(f,t)$ is the modulus of continuity of $f(x) \in C[-1,1]$ then

Th.: If X is given and

$\underline{\lim\limits_{n\to\infty}} \ \omega(\tfrac{1}{n}) \ln n > 0$

then $\exists \ f \in C(\omega)$ s. t. $\overline{\lim} |L_n(f, X, x) - f(x)| \geq c$ on a dense set of second category in $[-1,1]$;

if $\lim\limits_{n\to\infty} \omega(\tfrac{1}{n}) \ln n = \infty$

then $\exists \ f \in C^*(\omega)$ s. t. $\overline{\lim\limits_{n\to\infty}} |L_n(f, X, x)| = \infty$ on a dense set of second category.

—·—

<u>Th</u> (joint work with J. Szabados). If $X \subset (-\infty, \infty)$, then $\exists \ F \in C(-\infty, \infty)$ (= F is pointwise continuous on $(-\infty, \infty)$) s. t. $\overline{\lim\limits_{n\to\infty}} |L_n(F, X, x)| = \infty$ a. e. on the real line.

This is the generalization of the Erdős - Vértesi theorem for infinite interval.

Péter Vértesi (Budapest)

An exponential representation of Hille-Yosida type for evolution operators.

We consider the time-dependent Cauchy problem in a Banach space X, as follows (all operators below being linear):

$$(1)\qquad \frac{du}{dt} = A(t)\,u\ ,\ s<t<T \qquad (s,t \in J =]s,T[\subset R),$$
$$u(s) = f\ ,\ f \in D(A(s)). \qquad (u \in C([s,T],X))$$

Let $B(X) = \{$all bounded everywhere defined operators on $X\}$; for any interval $I \subset R$, $\mathcal{F}_I = \{$all $B(X)$-valued functions on $I\}$; if $M \in \mathcal{F}_I$ (or $\in \mathcal{F}_{I_1}$, $I_1 \supset I$) we define $M_- : \mathcal{F}_I \to \mathcal{F}_I$, and D (mapping the differentiable elements of $\mathcal{F}_I$ into $\mathcal{F}_I$), as follows: $(M_- F)(t) = F(t)M(t)$, $(DF)(t) = F'(t)$, $\forall t \in I$. With this terminology our main result in the following:

Theorem. Let $J =]s,T[\subset R$. Consider (1) with all the $A(t)$ dissipative generators (in X), $t \mapsto R(\lambda, A(t))$ $B(X)$-continuous $\forall \lambda > 0$ fixed, and assume $\exists$ a space-time dense (i.e. $J \times X$ dense) set of initial values (s_i, f_i) from which start $W^{1,1}([s_i,T],X)$ solutions of (1), $u(t,s_i,f_i) = U(t,s)f_i$, where U is a contractive evolution operator $\{U(t,s)\}$ on X. Then U is uniquely determined; moreover $\forall \lambda > 0\ \exists\ J_\lambda =]s_\lambda, T_\lambda[$, $J_\lambda \uparrow J$ as $\lambda \to +\infty$, and $\exists\ t \mapsto A_\lambda(t) \in B(X)$, $A_\lambda(t)$ dissipative, with:

$$(2)\quad (2')\qquad A_\lambda(t) \to A(t) \text{ strongly on } D(A), \text{ for each } t \text{ in } J_\lambda,$$

$$(2'')\qquad U(t,s) = \lim_{\lambda\to\infty} \left(e^{(t-s)(A_{\lambda -} + D)} 1 \right)(s), \text{ strongly on } X,$$

where $\left[\left(e^{(t-s)(A_{\lambda -}+D)} 1\right)(s)\right] f$ denotes the limit in X-norm, as $N \to \infty$, of $\left[\left(\left(\sum_{n=0}^{N} \frac{(t-s)^n}{n!} (A_{\lambda -} + D)^n\right) 1\right)(s)\right] f$, $\forall f \in X$, (1 being the constant function with value the identity on X).

Moreover, for $A(t)$ constant $= A$, the $A_\lambda(t)$ in (2') reduce to the usual Yosida approximation, and (2'') reduces to the usual Hille-Yosida representation formula $U(t,s) = \lim_{\lambda\to\infty} e^{tA_\lambda}$ strongly on X.

G. LUMER (Mons)

Uniform Convergence of some poised problem of Hermite - Birkhoff interpolation.

In this paper we study a special Hermite - Birkhoff interpolation problem which is similar to (0, 2, 3) interpolation. It consists of finding a polynomial $p_n(X, x)$ of degree atmost $2n+1$ such that for arbitrary given set of real nodes

$$X: \quad -1 \leq x_n < \cdots < x_1 \leq 1$$

and arbitrary real numbers $f_1^0 \cdot f_n^0 ; f_i^2 , f_i^3$. $i = 1, 2, \ldots, n$,

$$p_n(X, x_i) = f_i^0 , \quad i = 1, n,$$

$$p_n^{(j)}(X, x_i) = f_i^j , \quad i = 1, \ldots, n ; \; j = 2, 3 .$$

We call it quasi-(0, 2, 3) interpolation. First we construct the interpolation in an explicit form and then prove a theorem which gives a sufficient condition under which the quasi-(0, 2, 3) interpolation for every $f \in C^2[-1, 1]$ converges uniformly on $[-1, 1]$. The error estimates for Tchebycheff nodes are derived. [a joint work with H. C. Tripathi].

R. B. SAXENA
(LUCKNOW, INDIA)

Interpolation of $H^1(\mathbb{R})$ and $H^\infty(\mathbb{R})$

A Banach space X is called an interpolation space of a Banach couple (X_1, X_2) if each admissible operator (i.e. $T/_{X_i}$ is a bounded operator on X_i, $i = 1,2$) is bounded on X. Calderón characterized the interpolation spaces for (L^1, L^∞) as Banach lattices of measurable functions which satisfy the property

$$(*) \qquad g \prec f \ \& \ f \in X \Longrightarrow g \in X \text{ and } \|g\|_X \le c \, \|f\|_X .$$

We show that the interpolation spaces for the Hardy spaces (H^1, H^∞) can be characterized as the Hardy spaces of all such X which satisfy $(*)$. The proof uses recent results of Peter Jones (on L^∞ estimates for solutions of $\bar\partial F = \mu$ where μ is a Carleson measure) and of Brudnyi-Krugljak (on K monotone spaces). A rephasing of the result is that the interpolation spaces for $(\operatorname{Re} H^1, \operatorname{Re} H^\infty)$ can be characterized as the spaces $\operatorname{Re} H(X) = \{f \in X : \text{the Hilbert transform of } f \in X\}$.

R. Sharpley
(Columbia, S.C.)

n-widths of Smoothness Spaces

In this lecture we survey the known results for the asymptotics of the n-widths of the unit ball in the Sobolev space W_p^α, $\mathcal{U}(W_p^\alpha)$, as measured in L_q, $1 \le p \le \infty$, $0 < q \le \infty$, $\alpha \ge \frac{1}{p} - 1/q$. Using the $\mathcal{C}_p^\alpha$ spaces introduced by DeVore and Sharpley, which agree with the Sobolev spaces for integer α and $1 < p \le \infty$, we can extend these results to include the case when $0 < p < 1$. Using embeddings between $\mathcal{C}_p^\alpha$ spaces and Besov spaces, this method also gives n-width results for the Besov spaces $B_p^{\alpha, q}$ when $0 < p < 1$.

S. D. Riemenschneider
(Edmonton, Alberta, Canada)

Spectral properties of positive operators

We prove without representation methods that a positive and band irreducible abstract kernel operator on a Dedekind complete Banach lattice has a strictly positive spectral radius. Having proved this, we remark that it is also possible to deduce the abstract versions of the theorems of Jentzsch and Frobenius (as they appear in chapter 19 of Riesz spaces II by A.C. Zaanen) without representation theory. This solves a problem posed by A.C. Zaanen.

J. J. Grobler
(Potchefstroom, South Africa)

Spline interpolation of power-dominated data

Let (x_k) be a bi-infinite knot sequence for which the mesh ratio is smaller than exponential order (in particular, the local mesh ratio must be finite, but the global mesh ratio may be infinite). Let ρ be a non-negative real number, and $(y_k)_{k \in \mathbb{Z}}$ be a (real or complex) data sequence for which $y_k = O(|x_k|^\rho)$ as $k \to \pm\infty$. We prove the existence and uniqueness of a spline function of any previously specified odd degree, with simple knots (x_k), which interpolates (y_k) (i.e. $S(x_k) = y_k$, $\forall k$) and which is dominated by the same power, namely $S(t) = O(|t|^\rho)$ as $t \to \pm\infty$. We may replace O by o throughout. Uniqueness is guaranteed by showing that, within our space of spline functions of power growth, there are no non-trivial null-splines, while the actual existence of a solution follows from a series representation for $S(\cdot)$ in terms of (y_k) and of a sequence of "fundamental splines which decay exponentially near $\pm\infty$. The results generalize theorems of Schoenberg and deBoor.

Joint work with M. Stieglitz (Karlsruhe). Dennis C. Russell
(York Univ., Toronto, Canada).

Exact quadrature identities for analytic and harmonic functions

One is concerned here with identities of the type

$$\int_\Omega u\,dx = \int u\,d\mu$$

where Ω is a domain in $\mathbb{R}^d$ and μ a measure with compact support in Ω. (In the most typical case μ is a finite linear combination of "delta" measures.) The identity is to hold for all u harmonic and integrable over Ω. (Simplest example: the mean-value formula for harmonic functions on a ball Ω). A new approach to such identities is presented, based on a (known) characterization of distributions in Ω that annihilate harmonic functions, that unifies many known results and leads to new ones, especially in the absence of boundedness assumptions on Ω.

H. S. Sh
KTH, Stockholm

Nontangential Maximal Functions & Bounded Mean Oscillation

Characterizations are obtained of the functions of bounded lower oscillation (BLO) in terms of the nontangential maximal functions of functions of bounded mean oscillation (BMO). As a corollary, one obtains a characterization of the harmonic functions in R^+_{n+1} whose traces belong to BLO $(\mathbb{R}^n)$.

Colin Bennett
Columbia, South Carolina

estimation of the regression function via orthogonal expansion

Our goal is to estimate a regression function $r(x) = \mathbb{E}(Y \mid X = x)$ $(0 \leq x \leq 1)$ from an independent sample of size n. We propose a sequence of estimators $\hat{c}_k$ of the Fourier coefficients $c_k = \int_0^1 r(x) \varphi_k(x) dx$ of $r(x)$ (where $\{\varphi_k(x)\}$ is a complete orthonormal sequence satisfying some regularity conditions) and prove that $\hat{c}_k \to c_k$ with probability one for any k as $n \to \infty$. We also investigate the L^2 distance between the estimate $r_n(x) = \sum_{k=1}^{N_n} \hat{c}_k \varphi_k(x)$ and $r(x)$ where N_n is a suitable sequence of integers.

P. Révész
Math. Inst. Budapest.

Über Wold-Zerlegung isometrischer Halbgruppen.

Es sei $\{T_s\}_{s\in S}$ eine isometrische Halbgruppe auf den Hilbert Raum $\mathcal{H}$ (S ist eine Untergruppe einer geeigneten Gruppe G). Die Wold-Zerlegung von Ion Suciu (1968) für $\{T_s\}$, besagt daß $\{T_s\}_{s\in S}$ aus drei Teilen besteht: $\{T_s^{(u)}\}_{s\in S}$ eine unitäre Halbgruppe; $\{T_s^{t}\}_{s\in S}$ ein Shift und $\{T_s^{(e)}\}_{s\in S}$ – der „evanescent" Teil. Unser Zweck ist weiter $\{T_s^{(e)}\}$ zu zerlegen. Um eine neue Zerlegung zu bekommen, benutzen wir ein weiteres Defektraum $\mathcal{L} := \mathcal{H}^\perp \ominus P_{\mathcal{H}^\perp} \bigvee_{g \notin S^{-1}g} U_g^* \mathcal{H}^\perp$. Anschließend bekommen wir eine Zerlegung folgender Art: $\mathcal{H} = (\mathcal{H}^{(u)} \vee \mathcal{H}^{(t)}) \oplus \mathcal{H}^{ne}$; wo alle diese Unterräume reduzieren $\{T_s\}$, beziehungsweise auf einen modifizierten

Shift, einen normalen Shift und auf eine isometrische „ultraevanescente" Halbgruppe. Diese ist eine Zusammenarbeit mit. Dr. Nic. Suciu.

D. Gaşpar
Univ. Timişoara

Martingales, bases, Hardy spaces and a.e. convergence

A survey on martingale Hardy spaces is given. Some new results are presented with respect to some special martingales with non-linearly ordered index set. Estimations with respect to such martingales are connected with the a.e. convergence of Walsh-Fourier series. A new example of a separable Banach space of VMO-type is given, which has not a Schauder basis.

F. Schipp
Eötvös L. University, Budapest

New methods for maximal convolution operators.

Let $\{k_j\}_1^\infty \subset L^1(\mathbb{R}^n)$ be a sequence of kernels. Define the operators K_j acting on $L^1(\mathbb{R}^n)$ by setting $K_j f = k_j * f$. Let $K^* f(x) = \sup_j |K_j f(x)|$. In order to prove the a.e. convergence of $K_j f$ one is led to proving the weak type (1,1) for the maximal operator K^*, i.e. $|\{x \in \mathbb{R}^n : K^* f(x) > \lambda > 0\}| \leq c \frac{\|f\|_1}{\lambda}$, c independent of λ, f. One useful theorem to do this in an effective way is the following: K^* is of weak type (1,1) exactly when K^* is of weak type (1,1) over finite sums of Dirac deltas, i.e. when

$$|\{x \in \mathbb{R}^n : \sup_j |\sum_{k=1}^{H} k_j * f(x)| > \lambda\}| \leq \frac{cH}{\lambda}$$

Several recent instances of the use of this theorem by Krogstadt, Carlsson, Guzmán, ... are given that show the power of the theorem to simplify and clarify some important theorems and to obtain new results

Miguel de Guzmán
Madrid, Spain.

The Cardinal Interpolation Series

If f is an entire function of exponential type $\leq \sigma\pi$, integrable over the real axis, then it has the representation

$$(1) \qquad f(t) = \sum_{k=-\infty}^{\infty} f\left(\frac{k}{\sigma}\right) \frac{\sin\pi(\sigma t-k)}{\pi(\sigma t-k)} .$$

If f is not such a function, then (1) may hold at least in the limit for $\sigma\to\infty$. The aim of the talk is to give some conditions implying

$$f(t_0) = \lim_{\sigma\to\infty} \sum_{k=-\infty}^{\infty} f\left(\frac{k}{\sigma}\right) \frac{\sin\pi(\sigma t_0-k)}{\pi(\sigma t_0-k)}$$

for a fixed $t_0\in\mathbb{R}$. Moreover, the connection between (1), the Poisson summation formula and the Cauchy integral formula is studied.

R. Stens (Aachen)

Differentiation in $\mathbb{R}^n$

Recently, E. M. Stein (Annals of Math. 1981) proved the following generalization of Lebesgue's theorem $(n=1)$:

<u>Theorem</u> If the weak gradient ∇f is locall in the Lorentz space $L_{n,1}(\mathbb{R}^n)$ then f can be redefined on a set of measure zero so as to be continuous and $f(x+h)-f(x)-\nabla f(x)\cdot h = o(|h|)$, $h\to 0$, a.e. x.

Stein's proof of this theorem relies heavily on techniques of harmonic analysis – Riesz potentials and singular integrals. With R. Sharpley, we present a simple proof of this theorem based along the lines of Lebesgue's theorem. Namely, we show that $\lim\limits_{Q\downarrow\{x\}} f_Q =: F(x)$ defines a continuous function F,

where the Q are cubes in $\mathbb{R}^n$ and $f_Q := \frac{1}{|Q|}\int_Q f$. In addition,

$$⊛ \qquad \frac{|f(x+h)-f(x)-\nabla f(x)\cdot h|}{|h|} \leq c\, M_n(\nabla f)(x)$$

where $M_n g(x) := \sup\limits_{Q\ni x} \dfrac{\|\chi_Q g\|_{L_{n,1}}}{\|\chi_Q\|_{L_{n,1}}}$. The classical differentiation of f now follows from ⊛ and the fact that M_n is of weak type (n,n)

R. DeVore (Col

A Lefschetz theorem on sections in projective manifolds

The theorem is proved: Be X a submanifold of $\mathbb{P}_n$, Y an algebraic set with not too large codimension in $\mathbb{P}_n$. Then certain homotopy groups of X and $X \cap Y$ are isomorphic.
This is a generalization of the Lefschetz theorem on hyperplane sections.
The proof is based on Morse theory. One considers how the homotopy type changes from $X \cap Y$ to X. The used Morse function is not differentiable but the minimum of differentiable functions.

Mathias Peternell

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for a fixed $n \in \mathbb{N}$. Moreover, the connection between (1), the Poisson summation formula and the Cauchy integral formula is studied. R. Stens (Aachen)

Differentiation in $\mathbb{R}^n$

Recently, E. M. Stein (Annals of Math. 1981) proved the following generalization of Lebesgue's theorem $(n=1)$:

Theorem. If the weak gradient of f is locally in the Lorentz space [illegible] then f can be redefined on a set of measure zero to be continuous and [illegible]

Stein's proof of this theorem uses Lorentz spaces, techniques of harmonic analysis, Riesz potentials and singular integrals. With R. Sharpley, we found a simple proof of this theorem based along the lines of Lebesgue's theorem. Namely, we show that [illegible] defines a continuous function [illegible]

where the Q are cubes in $\mathbb{R}^n$ and $f_Q := \frac{1}{|Q|}\int_Q f$ [illegible]

(*) [illegible]

where [illegible] the [illegible] differentiation of f [illegible]

[illegible] from (*) and the fact that [illegible]

R. DeVore (

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