MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Unter der Leitung der Herren T. tom Dieck (Saarbrücken), D.B.A Epstein (Coventry) und K.Jänich (Regensburg) fand die diesjährige Tagung über Topologie in Oberwolfach statt. Fast alle Vorträge behandelten die Problemkreise der Theorie der Mannigfaltigkeiten, der verallgemeinerten Kohomologietheorien und der topologischen Methoden in der algebraischen K-Theorie.

<u>Teilnehmer</u>

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F.R.Beyl, Heidelberg
E.H.Brown, Waltham
S.R.Bullet, Coventry
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A.Dold, Heidelberg
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P.Klein, Bonn
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I.Madsen, Aarhus
J.Mäkinen, Helsinki
K.H.Mayer, Dortmund
W.Metzler, Frankfurt
W.Neumann, Bonn
T.Petrie, Oxford, Bonn, Rutgers
P.Orlik, Madison
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B.J.Sanderson, Coventry
L.Smith, Aarhus
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R.Vogt, Saarbrücken
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K.Wirthmüller, Saarbrücken
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Vortragsauszüge

D.D. Baird: A New Family of Generalised Homology Theories.

Let BP be the Brown-Petersen spectrum. $\pi_*(BP)$

$\pi_*(BP) = Q_p[v_1, \ldots, v_n, \ldots]$ $|v_n| = 2(p^n - 1)$ where Q_p is the integers localized at p (sometimes written Z(p)). For each $n \geq 1$ define a ring $\pi_*(E^{(n)})$ by:

$\pi_*(E^{(n)}) = Q_p[v_1, v_2, \ldots, v_n, v_n^{-1}]$

and make this into a $\pi_*(BP)$ module where v_{n+k} has the zero action for each $k \geq 1$.

Define a functer $E_*^{(n)}(-)$ from the stable category:

$E_*^{(n)}(X) = \pi_*(E^{(n)}) \otimes_{\pi_*(BP)} BP_*(X)$.

Theorem (i) $E_*^{(n)}$ is a homology theory for each $n \geq 1$.

(ii) $\exists$ a natural transformation $\phi_{n,k} : E_*^{(kn)} \longrightarrow E_*^{(n)}$ which on $\pi_*(E^{(kn)})$ annihilates $v_{n+1}, \ldots, v_{n+k-1}$ and sends v_{kn} to the appropriate power of v_n.

(iii) The maps $\phi_{n,k}$ allow us to form an inverse system of spectra and it has inverse limit BP.

The proof is entirely algebraic and uses certain cohomology operations form $BP^*(BP)$. The action of these operations on $H_*(BP)$ is known and the work of the proof is to translate this information to all $BP_*(BP)$ - comodules.

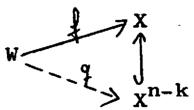
For small n $E_*^{(n)}(E^{(n)})$ is small enough that one can hope to do homological algebra over the coalgebra structure. It is easier to work with $\dfrac{E_*^{(n)}(E^{(n)})}{(p, v_1, \ldots, v_{n-1})}$ considered as a Hopf Algebra over $Z/(p)[v_n, v_n^{-1}]$ and in fact we neglect the grading and identify v_n with the identity.

In the resulting Hopf Algebra over Z/(p) the relation $\lambda^{p^n} = \lambda$ holds for all elements λ. $E_*^{(1)}$ is essentially complex K-theory and here all the calculations have been carried through, so that we know the E_2-term of the Adams Spectral Sequence. This collapses and the E_∞ -term is associated with the maps in an appropriate category of fractions of the stable category.

Baues, H.J.: Ueber Quotienten von Produkten von Suspensionen.

Ausgangspunkt der folgenden Betrachtung ist der zelluläre Approximationssatz für instabile Abbildungen:

Satz 1: Seien W,X CW-Komplexe mit dim $W \leq n$ und X k-zusammenhängend d.h. $\pi_i(X)=o$ für $i \leq k$.
Sei $f: W \longrightarrow X$ eine Abbildung mit $Sf \simeq 0$, dann gibt es eine Abbildung q, so daß das folgende Diagramm homotopiekommutativ wird:



,wobei X^{n-k} das (n-k) Skelett von X bezeichne.

Kor: Sei $T_n = S^1 \times S^1 \dots \times S^1$ der n-dim. Torus mit der Produktzellenzerlegung von $S^1 = e_o \cup e_1$. Es bezeichne # a die Anzahl der Elemente von $a \subset \bar{n} = \{1,2,\dots,n\}$. Für das k-Skelett T_n^k von T_n erhält man folgende Tatsachen:

1) $$T_n^{2\nu-1} / T_n^{\nu-1} \simeq \bigvee_{\substack{a \subset \bar{n} \\ \nu \leq \# a \leq 2\nu-1}} S^a \qquad \text{für } \nu \geq 1$$

2) Es gibt eine Abbildung

$$\bigvee_{\substack{c\in\overline{n}\\ 2\nu\leq\#c\leq 3\nu-1}} S^{\#c-1} \xrightarrow{\varphi} \bigvee_{\substack{a\in\overline{n}\\ \nu\leq\#a\leq 2\nu-1}} S^{\#a}$$, so daß

$T_n^{3\nu-1} / T_n^{\nu-1} \simeq C_\varphi$ (= Abbildungskegel von φ) gilt.

3) Für die Lusternik - Snirelman Kategorie gilt

$$\text{cat } T_n^r / T_n^{k-1} \leq \frac{r}{k} .$$

Das Korollar zu Satz 1 läßt sich auf Produkte von Suspensionen verallgemeinern.

Brown, E.H.: Higher order cohomology operations and characteristic classes.

The joint paper with S. Gitler, "A spectrum whose cohomology is a certain cyclic modul over the Steenrod algebry", (Topology August 1973) was described and indicated how these results were motivated by the conjecture that every n-manifold immerses in $\mathbb{R}^{2n-\alpha(n)}$ where $\alpha(n)$ is the number of 1's in the dyadic expression of n.

Bullett, S.R.: Formal groups and cobordism rings generated by manifolds with $Z/_p$ - singularities.

Summary: $N^*(-) \longrightarrow\!\!\!> H^*(-;Z/_2)$ (Thom). We look for an analogous theory $V^*(-)$ mapping onto $H^*(-;Z/_p)$. First recall certain properties $N(BZ/_2) \cong [[X]]$. We may choose

$X = e_N(\xi)$ [the N^*-Euler class of the canonical line bundle over $BZ/_2$]. Multiplication $Z/_2 \times Z/_2 \longrightarrow Z/_2$ induces $N^*[[X]] \longrightarrow N^*[[Y,Z]]$, $X \longmapsto F_n(Y,Z)$. Boardman, Quillen and others showed that (N^*, F_n) is isomorphic to the universal "$Z/_2$ formal group".

I consider ring theorie $h^*(-)$ mapping onto $H^*(-;Z/_p)$. These have $h^*(BZ/_p) \cong h^*[[\beta_h]] \otimes E[\alpha_h]$ (where E denotes exterior algebra) and $m: Z/_p \times Z/_p \longrightarrow Z/_p$ induces:

$$h^*[[\beta_h]] \otimes E[\alpha_h] \longrightarrow h^*[[\beta_h', \beta_h'']] \otimes E[\alpha_h', \alpha_h'']$$
$$\alpha_h \longmapsto F_1\left(\binom{\alpha_h'}{\beta_h'}, \binom{\alpha_h''}{\beta_h''}\right)$$
$$\beta_h \longmapsto F_2\left(\binom{\alpha_h'}{\beta_h'}, \binom{\alpha_h''}{\beta_h''}\right)$$

I define the notion of a "$Z/_p$-formal group" so that such a pair $(h^*, F = \binom{F_1}{F_2})$ is one. [under the extra restriction that F_2 is independent of α_h' and α_h'', which corresponds to choosing $\beta_h \in$ image of $i^*: h^*(BS^1) \longrightarrow h^*(BZ/_p)$]

<u>Theorem:</u> Every $Z/_p$- formal group law has a logarithm.

<u>Corollary:</u> The universal $Z/_p$-formal group has ground ring $A_v \cong P[x_{2i}; i \neq p^j - 1] \otimes E[z_{2i+1}; i \neq p^j - 1] \otimes P[y_{2i}]$. (P denotes polynomial algebra

In an attempt to realise this geometrically we are led to define a cobordism theory ${}^{(\infty)}V^*(-)$ of U-manifolds with singularities, where the singularity strata have normal bundles with fibre $\underbrace{Z/_p * \ldots * Z/_p}_{n\text{-fold join}} * pt$ and structure group $Z/_p \wr \Sigma_n$. The universal $Z/_p$-bundle has natural geometrie classes $\binom{\alpha_v}{\beta_v} \in \binom{{}^{(\infty)}V^1(BZ/_p)}{{}^{(\infty)}V^2(BZ/_p)}$

and these gererate a $Z/_p$-formal group. Hence there is a canonical $\phi : A_v \longrightarrow {}^{(\infty)}V^*$.

<u>Theorem:</u> ϕ is injective. Thus ${}^{(\infty)}V^*(-)$ "carries" the universal $Z/_p$ formal group law.

Hauschild, H.: <u>Bordismentheorie stabil gerahmter G-Mannigfalitgkeiten und stabile äquivariante Homotopie.</u>

Es sei G kompakte Liegruppe, V eine G-Darstellung und M eine kompakte C^∞-differenzierbare G-Mannigfaltigkeit. $\phi_V^n(M)$ bezeichne die Menge der Homotopieklassen von Bündelisomorphismen

$$\begin{array}{ccc} M\times\mathbb{R}^n\oplus T(M) & \longrightarrow & M\times\mathbb{R}^n\oplus M\times V \\ & \searrow \quad \swarrow & \\ & M & \end{array}$$

Man hat Abbildungen $\phi_V^n \longrightarrow \phi_V^{n+1}$, welche einfach durch Addition eines trivialen Bündels von links gegeben seien. $\phi_V(M)=\lim\limits_{\overrightarrow{n}} \phi_V^n(M)$ heißt die Menge der stabilen V-Rahmenstrukturen auf M. Mit $\omega_V^G(-)$ bezeichnen wir die aus obigen Mannigfaltigkeiten entstehende äuqivariante Bordismentheorie.

Die Einpunktkompaktifizierungen der G-Darstellungen bilden ein äquivariantes Spektrum; es liefert die stabile äquivariante Homotopietheorie $\pi_*^G(-)$. Die Pontrjagin-Thom Konstruktion liefert eine nat. Transformation $i:\omega_*^G(-) \longrightarrow \pi_*^G(-)$.

Satz: Ist G eine kompakte Liegruppe, deren Zusammenhangskomponente der Eins ein Torus ist, so ist i ein Isomorphismus.

Es existiert ein Beispiel einer Liegruppe für welche i nicht isomorph ist. Vermöge i übertragen sich die geometrischen Methoden zur Berechnung von $\omega_*^G(-)$ auf $\pi_*^G(-)$. Da $\pi_*^G(-)$ universell unter allen darstellbaren äquivarianten Homologietheorien ist, übertragen sich so die Methoden der äquivarianten Bordismentheorie auf beliebige darstellbare Homologietheorien.

Max Karoubi: Localisation of quadratic forms.

Let A be a ring with involution and L(A) be the Grothendieck group of the category of f.g. projective modules provided by a non degenerate form in the sense of Wall. If S is a multiplicative set invariant by the involution, one would like to compare the groups L(A) and $L(A_S)$ and also the associative Bass groups $L_1(A)$ and $L_1(A_S)$. Then one can prove the following exact sequence

$$L_1(A) \longrightarrow L_1(A_S) \longrightarrow U(A,S) \longrightarrow L(A) \longrightarrow L(A_S)$$

which generalize in some sense the well kwon exact sequence of Bass in algebraic K-theory (so-called sequence of localisation)

$$K_1(A) \longrightarrow K_1(A_S) \longrightarrow K(A,S) \longrightarrow K(A) \longrightarrow K(A_S)$$

The definition of U(A,S) is to elaborate to give it here. Roughly speaking one considers S-torsion modules over A of homological dimension one together with a "quadratic form" $q : M \longrightarrow A_S/A$ (for $A = Z$, $S = Z - \{0\}$ it is the usual notion of quadratic form in a finite group).

As corollaries of this result, one can show that the relative term U(A,S) does not change by taking $\hat{A} = \varinjlim_{s} A/s$ instead of A. So one finds in a purely algebraic way the Mayer-Vietories exact sequences of Wall and Bak. As applications one can find many of the classical results about the classification of quadratic forms on Dedekind rings and number fields using essentially the periodicity theorem in hermitian K-theory. If S^n is the sphere in k^{n+1} where k is an algebraically closed field, one can show as another consequence that the L-theory of S^n is periodic of period 8 as the usual real K-theory of the classical sphere.

Klein, P.: Non varishing homology of the free loop space.

$\Lambda(X) = \{\omega : S^1 \longrightarrow X\}$ of a space X is called the free loop space of X. We try to compute $H^*(\Lambda(X))$ with coefficients in $\mathbb{Q}$ as good as possible . The main tool is the study of the fibration $\Omega(X) \longrightarrow \Lambda(X) \longrightarrow X$.

We assume always $\pi_1(X) = 0$, $H_*(X,Z)$ of finite typ and X a CW-Komplex.

Theorem: Let $H^*(X)$ be an almost free algebra, that is it decomposes as $\bigotimes_{i=1}^{n} A_i$, where each of the A_i is generated by exacty one element.

Then $H^*(\Lambda(X)) = \bigotimes_{i=1}^{n} B_i$, where the B_i only depend on the corresponding A_i. The structure of the B_i is as follows:

a) $F_n[g]=A_i$ free $\Rightarrow B_i = F_{n-1}[x] \quad F_n[y]$

b) $T_{n,K}[y]=A_i$ truncated pol. algebra $\Rightarrow B_i$ has infinitly many generators and a lot of relations.

Theorem: Let $h^n : H^n(X) \longrightarrow \pi^n(X) = \mathrm{Hom}_{\mathbb{Z}}[\pi_n(X);Q]$ be deduced from the Hurewicz homeomorphism.

For each $x \in H^n(X)$ with

a) $x^2 = 0$

b) $h^n(x) \neq 0$

there is an algebra retract of $H^*(\Lambda(X))$ isomorphic to $H^*(\Lambda S^n)$.

The following theorem tells, when this is fulfilled:

Theorem: Let $H^n(X)$ be the first non vanishing cohomology modul then

$(\mathrm{Ker}\ h^m : H^m(X) \longrightarrow \pi^m(X)) = D^m(X)$ (= decomposable elements.)

if $m \leq 3n-2$.

Remark: The space $X = \Lambda S^{2k}$ gives an counter example that the equality does not hold for $m > 3n-1$.

Kosniowski, C.: Equivariant bordism ring of $Z/_p$-manifolds with isolated fixed points.

Let $Z/_p$ be the cyclic group of order p(= prime number).
Let $Z[X_1, X_2, \ldots, X_{p-1}]$ be the polynominal ring in p-1 generators.
If S^{2n-1} is a free linear unitary sphere then we can define the monomial of degree n.

$$\gamma(S) = or(S)\, X_{j1} \cdot X_{j2} \cdot \ldots \cdot X_{jn} \in Z[X_1, X_2, \ldots, X_{p-1}].$$

(Extend the action to $\mathbb{R}^{2n}$, let $j_1, j_2, \ldots, j_n$ be the rotation numbers which are uniquely determind and satisfy $1 \leq j_k \leq p-1$.
Finally $or(S) = +1$ or -1 depending on whether the complex structure induced on $\mathbb{R}^{2n}$ coincides with C^n or $C^{n-1} \oplus \bar{C}$).
We described explicitely a graded subring

$$B = \sum_{n \geq 1} B_n \subset Z[X_1, X_2, \ldots, X_{p-1}] \quad \text{so that}$$

Theorem: $\sum_i S_i^{2n-1} \underset{\text{freely}}{\sim} 0 \Leftrightarrow \sum_i \gamma(S_i^{2n-1}) \in B_n$.

Let U_*^p be the bordismring of unitary $Z/_p$-manifolds with isolated fixed points. The subring consisting of those manifolds with no fixed points is isomorphic to U_* - we denote its image in U_*^p by pU_*. An element of pU_* is $p\cdot N$, where $p\cdot N$ is p copies of some $N \in U_*$ and $Z/_p$ acts on $p\cdot N$ by permutating the p copies.
We can define a ring homomorphism

$$\gamma : U_*^p \longrightarrow Z[X_1, X_2, \ldots, X_{p-1}]$$

by $\gamma(M) = \Sigma\, \gamma(S(N_Q))$ where the sum is taken over the fixed point set of M and $S(N_Q)$ is the sphere of a tubular neighbourhood of Q. The Kernel of γ is precisely $p.U_*$.

Theorem: $U_*^p/_{pU_*} \cong B = \sum_{n\geq 1} B_n$.

Finally we mention the techniques:

Let $\sum_\alpha A_\alpha X_{\alpha_1} X_{\alpha_2} \ldots X_{\alpha_n} \begin{pmatrix} \alpha=(\alpha_1, \alpha_2 \ldots, \alpha_n) \\ 1 \leq \alpha_i \leq p-1 \end{pmatrix}$ be a homogeneous polynomial. Let

$$\sum_\alpha A_\alpha X_{\alpha_1}, X_{\alpha_2} \ldots X_{\alpha_n} = \sum_{r\geq 0} \sum_\alpha A(\alpha,r)\cdot p^r X_{\alpha_1} \ldots X_{\alpha_n}$$ with

$0 \leq A(\alpha,r) \leq p-1$ then the following statements are equivalent:

a) $\sum_\alpha A_\alpha X_{\alpha_1} \ldots X_{\alpha_n} \in B_n$

b) $\Sigma A_\alpha X_{\alpha_1} \ldots X_{\alpha_n} = \gamma(M)$ for some $M \in U_*^p$.

c) $\Sigma A(\alpha,r)\cdot p^r . X_{\alpha_1} \ldots X_{\alpha_n} \in B_n$ for all $r\geq 0$

d) $\Sigma A_\alpha \dfrac{f(\omega^{\alpha_1} - 1, \omega^{\alpha_2} - 1, \ldots, \omega^{\alpha_n} - 1)}{\prod_j (\omega^{\alpha_j} - 1)} \in Z[\omega]$

for all symmetric homogeneous polynomicals f $[\omega = \exp(\frac{2\pi i}{p})$.

e) $\Sigma A(\alpha,r) \dfrac{f(\alpha_1, \alpha_2, \ldots, \alpha_n)}{\prod \alpha_j} = 0 \bmod p$ for all f of degree $< n - r(p-1)$ and all $r \geq o$.

(The equivalence of (b) and (d) was proved by T. tom Dieck.)

Kreck, M.: <u>Eine Invariante für stabil parallelisierte Mannigfaltigkeiten.</u>

Sei M^{4K-1} eine komp. or. diff. Mf, $\partial M = \phi$; α eine Trivialisierung des stabilen Tangentialbündels STM. Sei N^{4K}, so daß $\partial N = M$.

Dann können wir das Vektorbündel $STN/_{\alpha} \longrightarrow N/M$ betrachten.

<u>Definition:</u> $\delta(M,\alpha) := \text{sign } N - L(p_i(STN/\alpha))\,[N/M]$,

wobei sign N die Signatur von N ist und $L\,(p_i(STN/\alpha)) \in H^*(N/M)$

das Hirzebruch L-Polynom in den Pontrjaginklassen $p_i(STN/\alpha)$ ist.

$\delta(M,\alpha)$ hängt nicht von Nab.

I) Um die δ-Invariante zu berechnen, definieren wir eine Bordismusgruppe $\Omega_{\pi 0}^{4K-1}$ von stabil paralleliersten Mf (M,α):

<u>Definition:</u> $(M,\alpha) \sim (M',\alpha') \Leftrightarrow \exists\,(N,\beta)$ mit

(i) $\partial(N,\beta) = (M,\alpha) + (-M',\alpha')$ und

(ii) sign $N=0$

$\delta : \Omega_{\pi 0}^{4K-1} \longrightarrow Q$ ist ein Homomorphismus.

<u>Theorem:</u> $\Omega_{\pi 0}^{4K-1} \cong Z \oplus Z_{m(2K)/_{n(2K)}} \oplus$ Kern e

$$\delta(a,x,y) = \frac{a}{n(2K)} .$$

Dabei ist n(2K) der Nenner von $\frac{B_K}{4K}$ (B_K Bernoulli Zahl) und

m(2K) der Nenner von $a_K \dfrac{2^{2K-2}(2^{2K-1}-1)}{K} B_K$; e ist die Adamsinvariante $e:\Omega_{\pi}^{4K-1} \longrightarrow Q/Z$

(Ω_{π}^{4K-1} gewöhnliche Bordismusgruppe von stabil parallesierten Mf)

$\bar{\delta} = \delta \bmod 1$ ist auch ein Homomorphismus $\bar{\delta}:\Omega_{\pi}^{4K-1} \longrightarrow Q/Z$

<u>Theorem:</u> $\bar{\delta} = -a_K\, 2^{2K+1}\,(2^{2K-1}-1) \cdot e$

II) Betrachte Objekte $S^1 x S^1 \longrightarrow E(A)$, $E(A)$ Faserbündel über S^1 ($E(A) \downarrow S^1$)

mit Faser $S^1 x S^1$ und $A \in SL(2;Z)$ charakteristische Abbildung.

Auf E(A) hat man eine Klasse von Trivialisierungen, so daß die δ-Invariante unabhängig von einer speziellen Trivialisierung ist.

Schreibe dafür $\delta E(A)$. Die Funktion $SL(2;Z) \longrightarrow Q$ ist gleich
$A \quad A \longrightarrow \delta(E(A))$
einer von W. Meyer betrachteten und berechneten Funktion.

Wir betrachten die Situation $A \in SL(2;Z)_p$, $SL(2;Z)_p$ die Kongruenzuntergruppe mod p. Dann ist die kanonische $Z_p \times Z_p$ Aktion auf $S^1 \times S^1$ mit der Operation A verträglich und induziert deshalb eine $Z_p \times Z_p$ Aktion auf $E(A)$.

Theorem: $A \in SL(2;Z)_p$; $\delta E(A) = \frac{1}{p^2-1} \sum_{\substack{g \in Z_p \times Z_p \\ g \neq 0}} \alpha(g, E(A))$.

Um $\alpha(g,E(A))$ zu berechnen, betrachten wir die Funktion

$$\Delta^p_g : SL(2;Z)_p \longrightarrow Q$$

$$A \longmapsto \delta E(A) - \alpha(g,E(A)).$$

Δ^p_g ist ein Homomorphismus und stimmt für p=2 mit einer von C. Meyer definierten zahlentheoretischen Funktion ϕ^2_g bis auf einen Faktor überein.

Theorem: $\Delta^2_g = -4\,\phi^2_g$.

Madsen, I.: Oriented topological bordism groups.

Let Ω_*^{Top} (Ω_*^{SO}) denote the oriented cobordism ring of topological (smooth) manifolds. From transversality we know that $\Omega_\sigma^{Top} = \pi_*(MSTOP)$ in dimensions other than 4 where MSTOP is the Thom spectrum. It is well kwon that MSTOP becomes a sum of Eilenberg-Maclane spectra when localized at the prime 2. In particular,

$$\pi_*(MSTOP) \otimes Z_{(2)} = H\,(BSTOP;\ Z(2)),$$

where BSTOP is the space classifying stable topological bundles.

Consider the fibrations

$$Top/_O \longrightarrow BSO \longrightarrow BSTOP,$$
$$^G/Top \longrightarrow BSTOP \longrightarrow BSG,$$

where BSG is the classifying space for stable spherical fibrespaces. The space $G/_{Top}$ (which classifies surgery problems) has been succesfully analysed by Sullivan. In particular (when localized at the prime 2),

$$G/_{Top} = \prod_{n\geq 1} K(Z_{(2)},4n) \times \prod_{n\geq 1} K(Z/_2, 4n-2)$$

Using either of the two fibrations above it is easy to compute $H_*(BSTOP;Q)$ whence $\Omega_*^{Top}\otimes Q$,

$$\Omega_*^{Top}\otimes Q = P\{[CP^{2n}]\,|\,n\geq 1\},$$
$$\Omega_*^{Top}\otimes Q = P\{[M^{4n}]\,|\,n\geq 1\}.$$

Here $M^{4n} = M_o^{4n} \cup_{\Sigma^{4n-1}} C\,\Sigma^{4n-1}$, M_o^{4n} the Milner manifold of index 8 (for n=1 the index is 16) constructed by plumbing together disc tangent bundles of S^{2n}($\partial M_o^{4n} = \Sigma^{4n-1}$ is the homotopy sphere generating $bP_{4n}\subset\Gamma_{4n-1}$, $n>1$). Consider the mapping

$$BSO \times {}^G/_{Top} \longrightarrow BSTOP \times BSTOP \longrightarrow BSTOP.$$

The homology of the domain is kwon, in particular

$$H\,(BSO \times G/_{Top};\ Z_{(2)})/Torsion = P\{Q_n\,|\,n\geq 1\}\otimes\Gamma\{b_n\,|\,n\geq 1\}$$

with deg $a_n = 4n =$ deg b_n. ($\Gamma\{\}$ is the divided power algebra)

<u>Theorem:</u> $H\,(BSTOP;Z_{(2)})/Torsion = P\{a_n\,|\,\alpha(n)<4+\nu(n)\}\otimes\Gamma\{b_n\,|\,\alpha(n)\geq 4+\nu(n)\}$, where $\alpha(n)$ is the number of non-zero terms in the dyadic expansion of n and $\nu(n)$ the 2-adic valuation on n($n=2^{\nu(n)}\cdot$ odd)

<u>Corollary:</u> $\Omega_*^{Top}/Torsion\otimes Z_{(2)} = P\{[CP^{2n}]\,|\,\alpha(n)<\nu(n)+4\}\otimes$ $\Gamma\{[M^{4n}]\,|\,\alpha(n)\geq\nu(n)+4\}$.

The 2-primary torsion structure of Ω_*^{Top} is very complicated (but kwon!).

Metzler, W.: Ueber den Homotopietyp zweidimensionaler C-W-Komplexe und Elementartransformationen bei Gruppendarstellungen durch Erzeugende und definierende Relationen.

Rapaports Q-Transformationen sowie Q^*- und Q^{**}- Transformationen führen auf zweidimensionale Komplexe $K^{(i)}$, die homotopieäquivalent sind. (Q^*- Transformationen entstehen, indem außer Q - Transformationen noch freie Erzeugende zugelassen sind; und für Q^{**}-Transformationen darf überdies eine zusätzliche Erzeugende a mit Relation R=a eingeführt oder eleminiert werden.) Bei Q^*- Transformationen ergibt sich ein vorgeschriebener Isomorphismus $\pi_1(K) \to \pi_1(K')$. Durch Untersuchung zweidimensionaler Komplexe mit endlicher abelscher Fundamentalgruppe ($\neq Z_m$) wird gezeigt, daß es verschiedene Homotopietypen und verschiedene Q^{**}- Aequivalenzklassen gibt. Die Beweise benutzen Reidemeisters' Homotopieketten-ring und lassen sich mittels Fox-Kalkül algebraisieren. Insbesondere gibt es 2-dim. Komplexe K, K' mit $K \not\simeq K'$, $K \vee S^2 \simeq K' \vee S^2$.

Als Problem des einfachen Homotopietyps wird besprochen, daß die Menge $Wh^*(\pi)$ der Torsionswerte $\tau \in Wh(\pi)$, die sich durch Inklusionen $L \subseteq K$ 2-dim. endlicher C-W-Komplexe ergibt, vermutlich mit der durch 4-dim. PL - h - Cobordismen realisierbaren identisch ist. $Wh^*(\pi)$ läßt sich durch Bedingungen über Erzeugende und definierende

Relationen von $L \subseteq K$ algebraisch beschreiben; jedoch ist weder ein Element $\tau \neq 1$ aus einem $Wh^*(\pi)$ noch ein Beweis für $Wh^*(\pi) = 1$ bekannt.

Neumann, W.: Signature related invariants of (4k-1)-manifolds and periodicity of signature.

If $(\overline{N}^{4k-1}, \pi, N)$ is a n-fold cover of closed oriented manifolds, then for some $m>0$ the union $m(\overline{N}, \pi, N)$ bounds a covering $(\overline{M}^{4k}, \pi, M)$. A standard argument shows

$$\gamma(\overline{N}, \pi, N) := \frac{1}{m} (\text{sign } \overline{M} - n \text{ sign } M)$$

is a well defined invariant, we restrict to the situation N^{4k-1} and $\alpha \in [N, S^1] = H^1(N, Z)$ given and $N(n) \xrightarrow{\pi} N$ the pulled back covering

$$\begin{array}{ccc} N(n) & \longrightarrow & S^1 \\ \downarrow \pi & & \downarrow n \\ N & \xrightarrow{\alpha} & S^1 \end{array}$$

Define: $\gamma_n(N, \alpha) = \gamma(N(n), \pi, N)$.

Theorem: 1) There exists a homotopy invariant antisymmetric isometric structure over Z:

$\mathcal{H}_\alpha = (H_\alpha, \langle\rangle, t_\alpha)$ (i.e. $(H_\alpha, \langle\rangle)$ is an antisymmetric bilinear space over Z and

$t : H_\alpha \longrightarrow H_\alpha$ an isometry)

2) If α is "good" then $\gamma_n(N, \alpha)$ only depends on $\mathcal{H}_\alpha$, so is homotopy invariant.

3) $\exists\, c, d \in Z$, $d>0$ such that $\gamma_{n+d}(N, \alpha) = \gamma_n(N, \alpha) + c$ (equivalently: $\gamma_n(N, \alpha) = n \frac{c}{d} + p(n)$ with $p(n)$ periodic of period d).

<u>Corollary 1</u> If $M^{4k} \longrightarrow S^1$ is given s.t. $\partial M \longrightarrow S^1$ is "good", then $\exists\, e, d \in Z$, $d>0$ such that $\text{Sign } M(n+d) = \text{Sign } M(n)+e$

<u>Corollary 2</u> Similar periodicity for signature of isolated hypersurface singularities at zero of polynomials of the form $f_n(Z_o,\ldots,Z_k) = g(Z_o,\ldots,Z_{k-1})+Z_k^n$.

The definitions are as follows:

<u>Definition 1</u> Choose $\alpha : N \longrightarrow S^1$ smooth representing $\alpha \in [N, S^1]$ and choose $W = a^{-1}$ (regular value). The α is "good" if W can be chosen s.t. the obvious inclusions $W \hookrightarrow N-W$ indue $H_{2k-1}(W) \longrightarrow H_{2k-1}(N-W)$ injective.

<u>Definition 2</u> Let $\tilde{N} \longrightarrow N$ be the induced cover

$$\begin{array}{ccc} \tilde{N} & \xrightarrow{\tilde{\alpha}} & \mathbb{R} \\ \downarrow & & \downarrow \\ N & \xrightarrow{\alpha} & S^1 \end{array}$$

and $\hat{\alpha} \in H_{4k-2}(\tilde{N}, Z)$ the class represented by $\tilde{\alpha}^{-1}$ (regular value) $\subset \tilde{N}$

Define $<,> : H^{2k-1}(\tilde{N}) \otimes H^{2k-1}(\tilde{N}) \longrightarrow Z$; $<x,y> = x \cup y(\hat{\alpha})$.

Factor out the degeneracy:

$$H_\alpha := H^{2k-1}(\tilde{N}) / \{x \mid \forall y \in H^{2k-1}(\tilde{N}),\ <x,y>=o\},$$

and put

$$\mathcal{H}_\alpha := (H_\alpha,\ < >,\ t_\alpha)$$

where $t_\alpha : H_\alpha \longrightarrow H_\alpha$ is induced by the covening transformation $\tilde{N} \longrightarrow \tilde{N}$.

H_α is finitely generated.

Apart from the application above, this isometric structure seems to be otherwise a useful invariant of the manifold.

<u>Problem 1</u> Is $\det <,> = +1$? For "good" α the answer is "yes".

<u>Problem 2</u> Is every $\alpha \in H^1(N,Z|$ "good"?

Theorem 1 probably works without the goodness restriction, but the algebra becomes horrible, so an answer to problem 2 would be nice.

Orlik, P.: Equivariant branched covers.

Let G be a connected Lie group and A a finite central subgroup. Let G act properly on a manifold K of a suitable category (smooth) so that the isotopy groups are contained in A and generate it. Joint work with Philip Wagreich gives a classifikation for such actions in terms of the stratification the orbit space $K^* = K/G$ by images of the different isotropy types, certain orbit invariant subgroups and the derived G/A bundle $K/A \longrightarrow K^*$. Specific results are obtained if K^* is a manifold and $H_1(K^*;Z)$ is free. As an example all such algebraic actions of $G=S^1$ with orbit space $K^* = CP^n$ is computed.

Petrie, T.: Obstructions to transversality for compact Lie groups.

Let G be a compact Lie group. The notation of fiber homotopy equivalence is extended to the category of complex G-bundles as follows: Let N and M be two G-bundles over a G-space Y. A G-map $\omega = N \longrightarrow M$ is called a quasiequivalence, if ω is proper, fiber preserving and degree one on fibers. A fundamental question is: When is ω properly G-homotopic to a map γ, which is transvers regular to the zero section $Y \subset M$ written $\gamma \pitchfork Y$?

For simplicity let G be connected. Then its complex representation ring is an integral domain whose field of fractions is denoted by F(G). The integers Z are contained in F(G) as the ring of integral valued constant functions on G.

Theorem: Let $\omega : N \longrightarrow M$ be a quasiequivalence of G-bundles. There is an obstruction $\tau(\omega) \in F(G)/Z$ which vanishes, if ω is property G-homotopic to γ and $\gamma \pitchfork Y$.

Raymond, Frank: A topological classification of the quotient spaces of 3-dim'l Liegroups by uniform discrete subgroups.

A classification of closed 3-dimensional manifolds whose universal coverings admits a structure of a Liegroup (so that the covering transformations are a discrete subgroup) is given. In all cases but one of the solvable types the 3-manifolds admit an S^1-action and the classification can thus be given in terms of their invariants as an S^1-space (or equivalently in terms of their fundamental groups). Furthermore, the essential problem reduces to the case of manifolds whose universal covering "is" the universal covering of $SL(2,\mathbb{R})$ (simple case) since other cases have been more or less investigated. A typical example is as follows:

If Γ is a uniform discrete subgroup of $P\,SL(2,R)$, then $\Gamma \backslash PSL(2,\mathbb{R})$ is an oriented S^1-manifold (a Seifert fiber space) of type $\{g;\ b=2g-2;\ (\alpha_1,\alpha_1-1),\ldots,(\alpha_r,\alpha_r-1)\}$, where Γ has classical signature

$$\{g;\alpha_1,\ \ldots,\ \alpha_r\}.$$

That is, Γ acts, as a Fuchsian group, on the unitdisk with quotient a surface of genus g and r orbits of elliptic type of branching orders $(\alpha_1,\ldots,\alpha_r)$. It is not necessary to assume Γ is uniform; finitely generated (discrete) will suffice. Infact, much of the method does not depend upon being in dimension 3.

The technique also allows one to explicate all the invariants in terms of a "Bieberbach class" in $H^2(\Gamma;Z)$. This class is the pull-back of the cannonical class in $H^2(B_{PSL(2,R)};Z)$ induced by the map $B_\Gamma \longrightarrow B_{PSL(2,\mathbb{R})}$ arising form the inclusion $\Gamma \subset PSL(2,\mathbb{R})$. Of course this Bieberbach class also represents the central extension, $\pi_1(\Gamma \backslash PSL(2,\mathbb{R})$ of Z by Γ.

Thomas, CH.: Chern classes of extra-special p groups.

Let $R^{\gamma}_{2k}(G)$ and $R^{top}_{2k}(G)$ denote the filtrations on the complex representationring defined respectively by the operations $\{\gamma^i\}$ and the skeleta of a classifying space B_G. If G is extra special of order p^{2m+1}, G is a central extension of Z_p by an elementary abelian group of rank 2m, and there exist free actions by G on $S^{2k-1} \times \ldots \times S^{2k-1}$ (2m+1 factors, $k \equiv 0(p^m)$). Using the Thom isomorphism in K_G-theory one can prove.

THEOREM $R^{\gamma}_{2k}(G) = R^{top}_{2k}(G)$.

The same method shows that the filtrations coincide without restriction on k for finite abelian groups, and also applies to metacyclic p-groups.

Work in progress: We can give a cohomological proof of this theorem using the Serre-Hochschild spectral sequence. This appears to generalise to more general extensions $Z_p \rightarrowtail P_r \twoheadrightarrow P_{r-1}$, detecting (up to nilpotent elements) the cohomology of P_{r-1}, by its elementary abelian subgroups.

Tornehave, J.: Finite sets and BSG.

A well known result of Barrat, Priddy, Quillen, Segal etc. asserts that there is a homotopy equivalence of infinite loop spaces $\Omega B \coprod_{n=o}^{\infty} B\Sigma_n \simeq QS^o = \Omega^\infty S^\infty$. The space $\Omega B \coprod_{n=o}^{\infty} B\Sigma_n$ is constructed by the Segal-Anderson method from the category of finite sets and bijective maps equiped with disjoint union.

The following multipinative analogue can be proved. Let S be a multiplicative monoid of natural numbers, such that $1 \in S$ and $S \neq \{1\}$. We consider the category $E^S_\otimes$ of finite sets, whose cardinality belong to S, and bijecture maps with cross product as the "addition". Let $B^oE^S_\otimes$ be the associated infinite loop space, and $(B^oE^S_\otimes)_o$ its base point component. The degree 1 component SG of QS^o is kwon to be an infinite loop space (Boardman-Vogt). We let $SG[S^{-1}]$ denote the localization inverting the integers of S.

Theorem 1: $(B^oE^S_\otimes)_o \simeq SG[S^{-1}]$ as infinite loop spaces. From this and Quillen's work on algebraic K-theory of finite fields, we can dedence.

Theorem 2: For l an odd prime, there is a splitting of infinite loop spaces $SG_{(l)} \simeq Imj_{(l)} \times Cokerj_{(l)}$.

For the construction of this splitting choose a field F_q of order $q=p^a$, such that q generates the units of the l-adic intergers topologically. Let P_Θ be the category of finite dimensional F_q-vectorspaces and linear isomorphisms with direct sum. The corresponding Ω-spectrum is the spectrum defining Quillen algebraic K-Theory of F_q. Let $P_\otimes$ be the category of F_q-vector-spaces of dimension a power of p and linear isomorphisms with tensor product as the "addition". We have functors commuting with "addition"

$$P_\oplus \xrightarrow{j} E^{\{p^i\}}_\otimes \xrightarrow{e} P_\otimes$$, where j forgets the linear structure, and e is the free-functor. We form the associated infinite loop spaces and maps, restrict to base point components and localize at l, to get

$$I_m J_{(l)} \xrightarrow{j} SG_{(l)} \xrightarrow{e} I_m J_{(l)}$$

The composite is shown to be a homotopy equivalence and Thm. 2 follows with Coker $j_{(l)}$ being the fiber of l.

Waldhausen, F.: <u>Quillen K-theory of knot groups and one-relator groups.</u>

Let K(R) denote the space whose homotopy group give the K-theory of the ring R, that is $\pi_i K(R)=K_i(R)$, i=o,1,2,... . If R has either one of several structures that generalize the notion of free product, polynomial extension, and Laurent extension respectively, then there is a 'splitting theorem' for K-theory; it is a fibration relating K(R) to simpler spaces. Besides the more obvious implications, one has the following consequence.

Anderson has defind a functor spaces $\longrightarrow$ spaces, $X \longmapsto K(X,R)$, where K(pt, R) = K(R), and $\pi_* K(X,R)$ is a homology theory; together with a natural transformation $K(BG,R) \longrightarrow K(RG)$ when BG is the classifying space of a group G and RG its group algebra over the ring R.

<u>Thm.:</u> Let either $G=\pi_1 M$ where M is a submanifold of the 3-sphere, or let G be a torsion free one - relator group. Let R be regular noetherian.. Then $K(BG,R) \longrightarrow K(RG)$ is a homotopy equivalence.

These G have homological dimension $\leq$ 2. So by the appropriate spectral sequence, letting $K_i(RG) = K_i(R) \oplus \tilde{K}_i(RG)$, one has:

Cor.: Let G and R be as before then $\tilde{K}_0(RG) = 0$, $\tilde{K}_1(RG) = H_1(BG, K_0(R))$, and if $i \geq 2$ there is a short exact sequence

$$0 \longrightarrow H_1(BG, K_{i-1}(R)) \longrightarrow \tilde{K}_i(RG) \longrightarrow H_2(BG, K_{i-2}(R)) \longrightarrow 0.$$

These results will appear in a paper entitled "Algebraic K-theory of generalized free products."

Wirthmüller, K.: Duality in G-manifolds.

We consider stable G-equivariant homology and cohomology theories (G a compact Lie group). For theories defined by a suitable G-spectrum (e.g. a Thom spectrum) we show that the G-(co) homology groups of $G \times_H X$ can be interpreted as H-(co) homology groups of X. We introduce the concept of orientability of G-vector bundles and manifolds with respect to an equivariant cohomology theory and prove a duality theorem which implies an equivariant analogue of Poincare - Lefschetz duality.

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