MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 42/1973

Geometry of Banach spaces

The meeting was conducted by: Prof.J.Lindenstrauss (Jerusalem), Prof.A.Pełczyński (Warszawa).
The scientific program includes 24 lectures. They were presented in the same order as the abstracts are. The lectures deal with the following topics: subspaces of general Banach spaces, weakly compact sets and WCG spaces, Orlicz spaces, random variables in Banach spaces, operators on Banach spaces.

Teilnehmer

E.Asplund, Odense
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H.U.Hess, Erlangen
J.Hoffmann-Jørgensen, Aarhus
R.C.James, Claremont
W.B.Johnson, Columbus
N.J.Kalton, Swansea
W.Kelz, Erlangen
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Ch.Stegall, Binghampton
A.Szankowski, Odense
J.Tischer, Erlangen
St.Troyansky, Sofia
H.Wittmer, Erlangen
P.Wojtaszczyk, Warszawa

ABSTRACTS OF LECTURES

R.C.JAMES: A counter-example to the l_1^3-problem .

A counter-example is given for the conjecture that each non--reflexive Banach space has three-dimensional subspaces nearly isometric to l_1^3. Specifically, a non-reflexive Banach space X is given for which there is a $\delta > 0$ such that if $\|x\| = \|y\| = \|z\| = 1$, then there is a choice of signs for which $\|x \pm y \pm z\| \leq 3 - \delta$. To describe X, let a bump be a sequence $\{x_n\}$ for which there is an interval I and a number a such that $x_n = 0$ if $n \notin I$ and $x_n = a$ if $n \in I$. For a number $\lambda < 1$ but nearly 1, let $[\![\ ]\!]$ be a functional whose domain is the set of all sequences of real numbers with finite support which have representations as $\sum_{i=1}^{r} \xi^i$, where each ξ^i consists of m_i disjoint bumps with equal absolute altitudes a_i, the quotient of the absolute altitude of a bump in ξ^i and the absolute altitude of a bump in ξ^j for $i \neq j$ is λ^p for some integer $p \neq 0$, and $[\![x]\!] = \left(\sum_1^r m_i a_i^2\right)$. For each arbitrary sequence x with finite support, let

$$\|x\| = \inf\left\{\sum_1^n [\![x^k]\!]\right\},$$

where the inf is over all representations $x = \sum_1^n x^k$ with x^k in the domain of $[\![\ ]\!]$. The counter example X is the completion of this normed space.

W.J.DAVIS: Some remarks on the l_1^n-problem (joint with W.B. Johnson and J. Lindenstrauss) .

Every non-reflexive B-space contains l_1^n if and only if whenever X is non-reflexive, there exists a space Y, finitely represented in X with Y^{**}/Y non-reflexive. The technique is based on the argument of R.C. James /"Uniformily non-square Banach spaces" /. Two classes of spaces are introduced : the α^{th} dual of X where α is any even ordinal number and, for any non-reflexive space, a separable function space W, finitely represented in X which has non-separable dual. In the first class, if J is the quasi-reflexive space of James, then $J = J + J$, and $J \quad /J$ is infinite dimensional. In the second class, there are spaces $W \quad l_1$.

CH.STEGALL: Separable Banach spaces with non-separable duals.

The lecturer's results about separable Banach spaces with non--separable duals are discussed along with a recent example of R.C. James. Last year the lecturer proved that if X is a separable and X^* is not, then X^* does not have the Radon-Nikodym property. This summer R.C. James has constructed a Banach space B, such that B^* is separable and B^{**} non-separable, but B^* has no subspace isomorphic to l_1 / he has recently shown that every infinite dimensional subspace of B has a subspace isomorphic to l_2/. The lecturer has shown that B^* also has the following properties : B^* is pre-weakly compact; B^* is sequentially dense in B^{***} ; B^{***} has the Radon-Nikodym property and is weakly compactly generated. The main property is that B^{**}/B is isometric to the Hilbert space of the dimension continuum. / The above results about B were also proved by J. Lindenstrauss/

Y.BENYAMINI: An example of Tzirelson of a reflexive, infinite dimensional space which does not contain subspaces isomorphic to l_p.

An example constructed by Tzirelson was presented. This is an example of a reflexive Banach space with an unconditional basis which does not contain any symmetric basic sequence. In particular it does not contain c_0 or any l_p. The same is true about the conjugate of the space. This gives a counter-example for a long standing problem.

ST.KWAPIEN: On the modulus of smoothness and convexity and the Rademacher averages of trace class S_p (after N. Tomczak) .

We prove that the modulus of smoothness and convexity in S_p

space is of the same order as in L_p space. More exactly if ϱ_X / resp. δ_X / denotes the modulus of smoothness / resp. of convexity/ of a Banach space X, then for each $1 \leq p \leq \infty$ we can find constants C_1, C_2, C_3, C_4 such that

$$C_1\, \varrho_{L_p}(\tau) \leq \varrho_{S_p}(\tau) \leq C_2\, \varrho_{L_p}(\tau) \quad \text{and} \quad C_3\, \delta_{L_p}(\tau) \leq \delta_{S_p}(\tau) \leq C_4\, \delta_{L_p}(\tau)$$

These complete results of McCarthy and Dixmier. Another results are following: if $p \leq 2$ /resp. $2 \leq p < \infty$ / then there exists a constant C such that for each $A_1, \ldots, A_n \in S_p$ there holds

$$\left(\int_0^1 \left\|\sum A_i r_i(t)\right\|^2 dt\right)^{\frac{1}{2}} \geq \text{/resp.} \leq \text{/}\ C\left(\sum \|A_i\|^2\right)^{\frac{1}{2}}$$

Here $r_i(t)$ denotes the Rademacher system. These again show that S_p behaves similary to L_p spaces.

H.P.ROSENTHAL: The hereditary problem for weakly compactly generated Banach spaces.

A Banach space is said to be weakly compactly generated / WCG/ if it has a weakly compact subset which generates the space. It is proved that there exists a probability measure μ on some measurable space and a closed linear subspace Y of $L^1(\mu)$, which is not WCG. Since $L^1(\mu)$ is obviously WCG, this solves the heredity problem for WCG spaces in the negative. The particular Y constructed is the span of a certain family of independent random variables and has additional special properties such as the following one : the unit ball of Y^*, in its weak* topology, is homeomorphic to a weakly compact subset of some Banach space. This discovery also led to the following topological result: a compact Hausdorff space is homeomorphic to a weakly compact subset of some Banach space if and only if it has a point separating σ-point-finite family of open F_σ's. Some recent remarkable results in logic are applied to show the undecideability of certain other problems concerning WCG Banach spaces.

J.LINDENSTRAUSS: Some remarks on WCG spaces (joint work with W.B. Johnson) .

A simple example is presented of a Banach space X such that X' is WCG and has an unconditional basis without X being WCG. This space has also the property that it has a separating sequence of functionals without being isomorphic to a subspace of l_∞ . The following theorem is proved : Let X be a Banach space such that X^* is WCG and $X \subset Y$ with Y WCG, then X itself is WCG.

Y.BENYAMINI: Constants of simultaneous extension of continuous functions.

We show that if S is the unit ball of a non-separable Hilbert space with its weak topology, then for every $\lambda > 1$ there exists a compact Hausdorff space K_λ containing S, such that the constant of simultaneous extension from $C(S)$ to $C(K_\lambda)$ is exactly λ . This gives a negative answer to a problem raised by Corson-Lindenstrauss and Pełczyński, as to whether these constants have to be /odd/ integers.

D.DACUNHA-CASTELLE: On embeding between Orlicz spaces.

We study the problem of embedding a symetric basis space into an Orlicz space of functions on $(0,1)$, using probability tools. The result is the following: If $B \hookrightarrow L_F$ then $B \sim C_\cap D$

C is the space of sequences spanned by a sequence of normed functions in L_F with disjoint support and D is a space of sequences spanned by exchangeable random variables.

N.J.NIELSEN: The Orlicz spaces L_M 0, .

It is well known fact that $L_p(0,\infty)$ is isomorphic to $L_p(0,1)$, and therefore it is natural to ask, whether this property characterises the

L_p spaces among symetric function spaces. In this lecture we investigate this question for the class of Orlicz function spaces $L_M(0,\infty)$ and discuss the isomorphic properties of the spaces L_M O .
Let in the following M be a fixed Orlicz function and put
$C_M(0,\infty) = \overline{\text{conv}}\{N \in C(0,1) : \exists\ t \in R : N(x) = M(tx)\ M(t)^{-1}\}$
We prove the following results :

Theorem 1. The unite vector basis of an Orlicz sequence space l_N is equivalent to a sequence of functions in $L_M(0,\infty)$ with mutual disjoint supports if and only if N is equivalent to a function in $C_M(0,\infty)$.

Theorem 2. $x^p \in C_M(0,\infty) \Leftrightarrow x^p$ equivalent to a function in $C_M(0,\infty) \Leftrightarrow p \in [\alpha_M, \beta_M] \cup [\alpha_M^\infty, \beta_M^\infty]$, where the intervals are associated to l_M and $L_M(0,\infty)$ respectively.

Theorem 3. If $L_N(0,1)$ embeds isomorphically into $L_M(0,\infty)$, and $\max(\beta_M, \beta_M^\infty) < 2$, then there is a constant $K > 0$ so that $M(x) \leq K\ N(x)\ x \geq 1$.

Theorem 4. If $\max(\beta_M, \beta_M^\infty) < 2$ or $\min(\alpha_M, \alpha_M^\infty) > 2$ and $L_M(0,\infty)$ is reflexive and isomorphic to a symmetric function space X on $(0,1)$, then X is isomorphic to $L_M(0,1)$.

Theorem 5. Let M satisfy the conditions above and $\lim_{t\to\infty} M(tx)\ M(t)^{-1} = x^p$ for some $p \neq 2$, then $L_M(0,\infty)$ is isomorphic to a symmetric function space on $(0,1)$, if and only if M is equivalent to x^p.

Theorem 6. If $L_M(0,\infty)$ is iscmorphic to $L_M(0,1)$ then $\sup_s d(L_{M_s}(0,1), L_M(0,1)) < \infty$, where $M_s(x) = M(sx)\ M(x)^{-1}$ for $x \geq 1$.

From Theorem 6 it follows that the general question in the beginning has an affirmative answer, if the following problem is solved positively : If $\sup_s d(L_{M_s}(0,1), L_M(0,1)) < \infty$ does there exist a constant $k>0$ so that

$$k^{-1} M(x) \leq M_s(x) \leq kM(x) \text{ for } x \geq 1,\ 0 < s \leq 1.$$

G.DANKERT: Factorisation and products of Orlicz spaces.

To arbitrary Orlicz functions $M_1,\dots,M_n$ a function Q can be constructed such that $Q(x_1,\dots,x_n)\leq\sum_{i=1}^{n} M_i(x_i)$ for $0\leq x_i<\infty$, with equality for particular n-tuples $(x_1,\dots,x_n)$ (generalised Young inequality). M_i^{-1} denotes the inverse function of M_i. L_{M_i} is the Orlicz space generated by M_i over a σ-finite measure space, furnished with the Orlicz norm and $\coprod_{i=1}^{n} L_{M_i} := \{ f_1\cdots f_n \mid f_i\in L_{M_i}\}$.
Let M_o be an Orlicz function, then the following conditions are equivalent: (i) $L_{M_o}\subset\coprod_{i=1}^{n} L_{M_i}$; (ii) $\exists\ K>0$: $Q(x)\leq M_o(Kx)$ for all x mod μ; (iii) $\exists\ L>0$: $LM_o^{-1}(x)\leq M_1^{-1}(x)\cdots M_n^{-1}(x)$ for all x mod μ.
In this case there exists an $N>0$ and for each $f\in L_{M_o}$ exist $f_i\in L_{M_i}$ with $f=f_1\cdots f_n$ and $\prod_{i=1}^{n}\|f_i\|_{M_i}\leq N\|f\|_{M_o}$. Inverting "$\subset$" resp "$\leq$" in (i),…,(iii) yields conditions (i'),…,(iii') equivalent to (iv') $\exists\ N>0$: $\|\prod_{i=1}^{n} f_i\|_{M_o}\leq N\prod_{i=1}^{n}\|f_i\|_{M_i}$ for all $f_i\in L_{M_i}$.

P.ENFLO: Factorisation of L_p-spaces.

It is known that if $C(0,1)\sim X+Y$ then $X\sim C(0,1)$ or $Y\sim C(0,1)$. For l_p and c_o it is even known that every complemented infinite-dimensional subspace is isomorphic to the whole space. We prove here that if $L_p(0,1)\sim X+Y$ then $X\sim L_p(0,1)$ or $Y\sim L_p(0,1)$. For $p>1$ this is done by constructing a reproduction of the Haar system in $L_p(0,1)$ on which one of the maps $L_p(0,1)\longrightarrow X$, $L_p(0,1)\longrightarrow Y$ given by the projection is itself an isomorphism. Thus X or Y has a subspace isomorphic to $L_p(0,1)$. By the construction this subspace is complemented and by a result of Pełczyński $X\sim L_p(0,1)$ or $Y\sim L_p(0,1)$.
For p=1 a somewhat different approach is used.

W.B.JOHNSON: Subspaces of L_p which embed into l_p (joint work with T. Odell) .

If X is a subspace of $L_p (= L_p[0,1])$ for $2 < p < \infty$ and no subspace of X is isomorphic to l_2, then X is isomorphic to a subspace of l_p. This main result and previously known facts yield that a separable $\mathcal{L}_p$ space $(1 < p < \infty)$ either is isomorphic to l_p or contains a subspace isomorphic to l_2. For $1 < p < 2$, a weaker version of the main result is proved: If X is a subspace of L_p for $1 < p < 2$ and there is a constant M so that every normalized basic sequence in X is M eqaivalent to the unite vector basis of l_p, then X is isomorphic to a subspace of l_p provided X admits an unconditional finite dimensioaal decomposition.

B.MAUREY: Caractérisation d'une classe d'espaces de Banach par des propriétés de séries aléatoires vectorielles.

On caractérise les espaces de Banach E qui ne contiennent pas de l_n^∞ uniformément / resp: pas de l_n^1 uniformément /, c'est a' dire que la borne inférieure des distances de l_n^∞ / resp: l_n^1 / aux sous-espaces de dimension n de E tend vers l'infini avec n. On désigne par $(r_n(t))$ la suité des fonctions de Rademacher sur $[0,1]$. Pour que E ne contienne pas de l_n^∞ uniformément / resp: pas de l_n^1 uniformément /, il faut et il suffit qu' il existe un nombre reél p, $1 < p < \infty$, et une constante C tels que l'on ait pour toute suite (x_n) de vectéurs de E :

$$\left(\sum \|x_n\|^p\right)^{1/p} \leq C \int \left\|\sum x_n \, r_n(t)\right\| dt$$

$$\left(\text{resp: } \int \left\|\sum x_n \, r_n(t)\right\| dt \leq C \left(\sum \|x_n\|^p\right)^{1/p}\right)$$

Ces résultats sont dus à G. Pisier et moi meme.

J.HOFFMANN-JØRGENSEN: Types and cotypes of B-spaces.

Let E be a Banach space, (W,F,P) a probability space and ε_j a Bernouilli sequence defined on (W,F,P). Then we define :

$$C(E) = \left\{ (x_j) \;\middle|\; \sum_{j=1}^{\infty} \varepsilon_j x_j \text{ converges a.s.} \right\}$$

$C^p(E) = \{(x_j) \mid \sum \varepsilon_j x_j$ converges in $L^p(E)\}$ $\quad 0 \le p \le \infty$

$B(E) = \{(x_j) \mid \sum_{j=1}^{\infty} \varepsilon_j x_j$ is bounded a.s.$\}$

$B^p(E) = \{(x_j) \mid \sum_{j=1}^{\infty} \varepsilon_j x_j$ is bounded in $L^p(E)\}$ $\quad 0 \le p \le \infty$

$Tx = \sum_{j=1}^{\infty} \varepsilon_j x_j$ for $x = (x_j) \in C(E)$

$S(f) = (\mathbb{E}(\varepsilon_j f))_{j=1}^{\infty}$ for $f \in L^1(E)$

It is well known that $C(E) = C^p(E)$ $\forall\, 0 \le p < \infty$ and $B(E) = B^p(E)$, and that $T(C(E))$ is closed subspace of $L^p(E)$ for all $0 \le p < \infty$, and that $\lim_{j\to\infty} \mathbb{E}(\varepsilon_j f) = 0$ for all $f \in L^1(E)$. E is said to be

of type p if and only if $l^p(E) \subseteq C(E)$ $(\Leftrightarrow l^p(E) \subseteq B(E))$

of weak cotype q if and only if $C(E) \subseteq l^q(E)$ $(\Leftrightarrow B(E) \subseteq l^q(E))$

of strong cotype q if and only if $S(L^\infty(E)) \subseteq l^q(E)$ $(\Leftrightarrow S(L^{1+}(E)) \subset l^q(E))$

where $L^{1+}(E) = \bigcup_{p>1} L^p(E)$.

Theorem 1. Let $0 < r < \infty$, then the following statements are equivalent:

(i) E is of type p

(ii) $\exists\, C \ni (\mathbb{E}\|\sum_{j=1}^{n} \varepsilon_j x_j\|^r)^{\frac{1}{r}} \le C (\sum_1^n \|x_j\|^p)^{\frac{1}{p}}$ $\quad \forall_n,\ \forall x_1,\dots x_n$

(iii) $\exists\, C \ni \mathbb{E}\|X_1+\dots+X_n\|^p \le C(\mathbb{E}\|X_1\|^p+\dots+\mathbb{E}\|X_n\|^p)$

$\forall\, n \ge 1$, $\forall\, X_1,\dots,X_n$ independent r. v. of mean 0

(iv) $\sum_1^\infty X_n$ converges, whenever (X_n) are independent r.v. with mean 0 and $\sum_1^\infty \mathbb{E}\|X_n\|^p < \infty$

(v) $n^{-1}\sum_1^n X_j \longrightarrow 0$ a.s., whenever (X_n) are independent r.v. with mean 0 and $\sum_1^\infty n^{-p}\mathbb{E}\|X_n\|^p < \infty$

Theorem 2. Let $0 < r < \infty$ then the following statements are equivalent: (i) E is of weak cotype q

(ii) $\exists\, C \ni (\mathbb{E}\|\sum_1^n \varepsilon_j x_j\|^r)^{\frac{1}{r}} \ge C(\sum_1^n \|x_j\|^q)^{\frac{1}{q}}$ $\quad \forall_n\ \forall x_1,\dots,x_n$

(iii) $\exists\, C \ni \mathbb{E}\|X_1+\dots+X_n\|^q \ge C(\mathbb{E}\|X_1\|^q+\dots+\mathbb{E}\|X_n\|^q)$

$\forall\, n \ge 1$, $\forall\, X_1,\dots,X_n$ independent r.v. of mean 0

(iv) $\sum_1^\infty \mathbb{E}\|X_n\|^q < \infty$, whenever (X_n) are independent r.v. so that $\sum_1^\infty X_n$ converges a.s. and $\sup_n \|X_n\|$ is q-integrable

Theorem 3. Let E and F be B-space and $\langle\cdot,\cdot\rangle$ a normdeterming bilinea form on $E \times F$. If $p^{-1} + q^{-1} = 1$, then we have: E is of type p if and only if F is of strong cotype q.

Theorem 4. The following statements are equivalent : (i) $B^{\infty}(E) \subseteq c_0(E)$ (ii) $B^{\infty}(E) = C^{\infty}(E)$ (iii) $c_0 \not\subset E$

Theorem 5. The following statements are equivalent :

(i) $B(E) \subseteq c_0(E)$ (ii) $B(E) = C(E)$ (iii) $c_0 \not\subset L^p(E)$ for all $0 < p < \infty$ (iv) $c_0 \not\subset L^p(E)$ for some $0 < p < \infty$.

ST.KWAPIEN: A general form of a linear operator in the space of all measurable functions.

In the talk it is proved that the general form of a linear operator in the space of all measurable functions is the following one :

$$Ax(t) = \sum_{i=1}^{\infty} \varphi_i(t)\, x(\phi_i(t))$$

where $\{\varphi_i\}$ and $\{\phi_i\}$ are sequences which fulfills the conditions :

1/ $\{\varphi_i\}$ is a sequence of measurable functions such that $\varphi_i(t) \neq 0$ for infinitely many i on a set of measure 0

2/ $\{\phi_i\}$ is a sequence of measurable transformations such that counter image of a set of measure zero is of measure zero.

A.SZANKOWSKI: A proof of Dvoretzky's theorem.

The well known Dvoretzky's theorem says that for any k and $\varepsilon > 0$ there exists $n(k,\varepsilon)$ such that if X is an n-dimensional B-space with $n \geq n(k,\varepsilon)$ then there exists $E \subset X$, $\dim E = k$ and $d(E, l_2^k) \leq 1+\varepsilon$ (d denotes the Banach-Mazur distance). The original proof of this theorem is very difficult and not quite satisfactory. The present proof bases on the same ideas as the original one, but is much simpler and differs from that in the difficult part.

N.J.KALTON: Banach spaces of compact operators.

Let E and F be Banach spaces and let $K(E,F)$ be the Banach space of compact operators $T: E \longrightarrow F$. It is shown that if $K(E,F)$ contains a copy of l_∞, then either E contains a complemented copy of l_1 or F contains copy of l_∞. Under the additional assumption that E has an unconditional finitedimensional expansion of the identity it is shown that $K(E,F)$ is complemented in $L(E,F)$ if and only if $K(E,F) = L(E,F)$.

G.PISIER: A new proof of the Grothendieck inequality and an application to the space L^0, (joint work with B. Maurey).

Using Ky Fan's fixed point theorem, we generalise classical result on subspaces of L^q on which the p- and q-norms are equivalent $(0 < p < q < \infty)$, in order to obtain the:

Proposition. Let H be the $\sigma(l^\infty, l^1)$-compact set of the extremal points of the unit ball of l^∞. There exists a constant ρ and a number $\beta \in]0,1[$ such that : for every probability measure λ on H, there exists a probability measure Λ on H with the property :

$$\forall x \in \ell^1 \qquad \left(\int_H |\langle x,\xi\rangle|^2 d\lambda(\xi)\right)^{\frac{1}{2}} \leq \rho \inf\{c \geq 0 \mid \Lambda(\{|\langle x,\xi\rangle| > c\}) \leq \alpha\}$$

The Grothendieck theorem $(L(L_1, L_2) = \Pi_1(L_1, L_2))$ follows easily, as well as its strengthened version, due to B. Maurey : $L(L_1, L_2) = \Pi_0(L_1, L_2)$. Eventually, we deduce from the proposition that if (Ω, μ) is a probability space and if (X_n) is an unconditionally convergent series in $L^0(\Omega,\mu)$ then the series $\sum c_n X_n$ converges in $L^0(\Omega,\mu)$ whenever (c_n) is a bounded sequence of scalars.

T.FIGIEL: Factorisation of weakly compact operators and related topics.

This is a report of the joint work with W.J. Davis, W.B. Johnson and A. Pełczyński. A simple construction is presented, which yields facts such as " Every weakly compact operator factors through a reflexive space", "Separable conjugate spaces embed into spaces with boundedly complete basis". Several, old and new, results are immidiate corollaries.

D.J.H.GARLING: Diagonal mappings between sequence spaces.

Some general results are obtained about r-nuclear, r-integral and r-summing mapings from one sequence space into another. These are used to give a nearly complete characterisation of such mappings from one l_p-space to another, extending results of Schwartz and Tong.

I.SINGER: On the extension of basic sequences to bases.

A. Pełczyński and H.P. Rosenthal have raised the problem whether for every basis sequence (y_n) in a Banach space E with basis there exists a permutation $(y_{\sigma(n)})$ which can be extended to (i.e. is a subsequence of) a basis of E. We show that the answer is negative, namely there exist even a Banach space E with a basis and a subspace $G \subset E$ with a basis, such that no basis of G can be extended to a basis of E.

D. VAN DULST: On semi-norming subspaces of a conjugate Banach spaces.

Let E be a separable Banach space, E^* its conjugate and V a w^*-dense subspace of E^*. Kadec and Pełczyński have shown that if the characteristic $r(V)$ of V is positive, then there exists on E an equivalent norm $|||\cdot|||$ for which the following two conditions are satisfied:

(K_1) $x_n \longrightarrow x_0$ for $\sigma(E,V) \Longrightarrow \underline{\lim}\, |||x_n||| \geq |||x_0|||$

(K_2) $x_n \longrightarrow x_0$ for $\sigma(E,V)$ and $\lim\limits_{n\to\infty} |||x_n||| = |||x_0||| \Longrightarrow |||x_n - x_0||| \longrightarrow 0$

Kadec has raised the question whether $r(V) > 0$ is a necessary condition for such a renorming to be possible. We show by an example that the answer to this question is negative.

ST.TROYANSKY: <u>On moduli of convexity of Orlicz spaces,</u>

(joint work with R.P. Maleev) .

Let for a Banach space X δ_X denote the modulus of convexity and ρ_X denote the modulus of smoothness.

Theorem 1. Let $M(t)$ be an Orlicz function such that l_M is a reflexive space. Then there exist Orlicz function $N(t)$, equivalent to $M(t)$ and positive constants c_M, C_M, k_M and K_M such that :

$$\ast \quad c_M \leq \delta_{l_N}(\varepsilon)\ \varepsilon^{-2}\left(\inf\left\{ M(au)\left(M(a)\right)^{-1},\ 0<a\leq 1,\ \varepsilon \leq u \leq 1\right\}\right)^{-1} \leq C_M$$

$$k_M \leq \rho_{l_N}(\varepsilon)\ \varepsilon^{-2}\left(\sup\left\{ M(au)\left(M(a)\right)^{-1},\ 0<a\leq 1,\ \varepsilon \leq u \leq 1\right\}\right)^{-1} \leq K_M$$

for $0<\varepsilon\leq 1$. Moreover, for every Orlicz function $Q(t)$ equivalent to $M(t)$, exist positive constants A_Q and B_Q such that

$$\delta_{l_N}(\varepsilon) \geq A_Q\ \delta_{l_Q}(\varepsilon) \quad \text{and} \quad \rho_{l_N}(\varepsilon) \leq B_Q\ \rho_{l_Q}(\varepsilon).$$

In particular for $M(t)=t^p$, $p>1$ we obtain $N(t)=t^p$ and $\ast$ becames the classical inequalities.

Theorem 2. If $2 \notin [\alpha_M, \beta_M]$ then for every Banach space X isomorphic to l_M there exist positive constants A_X and B_X such that

$$\delta_{l_N}(\varepsilon) \geq A_X\ \delta_X(\varepsilon) \qquad \text{and} \qquad \rho_{l_N}(\varepsilon) \leq B_X\ \rho_X(\varepsilon)$$

P. Wojtaszczyk / Warszawa /

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