MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Distributionen , Convolutionen
und partielle Differentialgleichungen

8. 6. bis 12.6.1975

Unter der Leitung von Prof. Wloka (Kiel) und Prof. Zielezny (New York) fand im Oberwolfacher Institut eine Tagung über Distributionen, Convolutionen und partielle Differentialgleichungen statt.

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Vortragsauszüge

ALBRECHT, E.: Lokale Operatoren

Ein bekannter Satz von J. Peetre (1960) besagt, daβ jeder lokale lineare Operator $T : D(\Omega) \longrightarrow C(\Omega)$ (andere Zielräume sind ebenfalls möglich) schon ein lokalendlicher Differentialoperator mit Koeffizienten im Zielraum ist. Für die Räume $D^{(M_p)}(\Omega)$ von ultradifferenzierbaren Funktionen im Sinne von Roumieu wird gezeigt:

Satz: Sei $\{M_p\}_{p=0}^{\infty} \subset \mathbb{R}_+$ eine Folge mit den Eigenschaften:

(i) $\left\{\frac{1}{p}\frac{M_p}{M_{p-1}}\right\}$ ist monoton wachsend

(ii) Es gibt $A > 0$ und $H > 0$, so daβ für alle $p \in \mathbb{N} \cup \{0\}$: $M_p \leq A \cdot H^p \cdot \min\limits_{0 \leq q \leq p} M_q \cdot M_{p-q}$

(iii) $\sum\limits_{p=1}^{\infty} \frac{M_{p-1}}{M_p}(\log p)^s < \infty$ für ein $s > 1$.

Dann ist für jeden lokalen linearen Operator $T: D^{(M_p)}(\Omega) \to C(\Omega)$ (andere Zielräume sind auch möglich) die Einschränkung $T: D^{(K_p)}(\Omega) \longrightarrow C(\Omega)$ für $K_p = \sqrt{p!\, M_p}$ ein stetiger Ultradifferentialoperator mit Koeffizienten in $C(\Omega)$.

Der Beweis verwendet ein Resultat von C. Roumieu (1962) über Ultradistributionen mit Träger in nur einem Punkt und eine Stetigkeitsaussage für in einem verallgemeinerten Sinne lokale Operatoren, die auch in der Spektraltheorie Anwendungen hat.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

BENGEL, G.: <u>Singular Supports of Convolutions</u>

For the wave front set of a distribution T: WF(T) we show that the inclusion

$$WF(T*S) \subset \{(x,\xi);\ x = y+z,\ (y,\xi) \in WF(T), (z,\xi) \in WF(S)\}$$

holds. This has as a consequence the theorem:

Given $T \in \mathcal{E}'(\mathbb{R}^n)$ such that co sing supp $(T*S) =$ $=$ co sing supp T + co sing supp S for every $S \in \mathcal{E}'(\mathbb{R}^n)$, then the fibre of WF(T) over every extreme point of co sing supp T consists of all $\mathbb{R}^n \setminus \{0\}$. If Ω is a relatively compact open subset of $\mathbb{R}^n$ with smooth boundary, $T = \chi_\Omega$ the characteristic function of Ω, then there are $S \in \mathcal{E}'(\mathbb{R}^n)$ such that co sing supp $(T*S) \neq$ co sing supp T + co sing supp S.
An example shows that the condition on T is necessary but not sufficient.

BERENSTEIN, C.A.: <u>Fourier Integral Representations of Solutions to Difference-Differential Equations in n-Variables</u>

The relation between "interpolation" problems in $\mathbb{C}^n$ (i.e. given an analytic function defined on an analytic variety of codimension 1 one wants to extend it with prescribed growth conditions) and Fourier representations of solutions of convolution equations was discussed. One can understand this way the need to "group terms" in Schwartz's theorem on mean periodic functions (n=1) or in the classical theorems on difference-differential equations (Leont'ev, Titchmarsh, etc.). As a corollary of a general interpolation theorem (joint work with B.A. Taylor) such a representation is derived for solutions of difference-differential equations in n-variables. It was suggested that it could be used to find the uniqueness class of arbitrary difference-differential equations in n-space.

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CARMICHAEL, R.D.: <u>Representations and distributional boundary values of analytic functions</u>

We present results in which functions that are analytic in tubes over open convex cones obtain distributional boundary values in the distribution spaces $\mathcal{S}'$, $\mathcal{Z}'$, and λ', which are the distributional Fourier transform spaces of $\mathcal{S}'$, $\mathcal{D}'$, and Λ_∞, respectively. We obtain representations of the analytic functions in terms of a distributional Fourier transform and in terms of the Fourier-Laplace transform of the inverse Fourier transform of the distributional boundary value, and we obtain other information concerning the analytic functions in terms of the boundary values. Extensions to functions analytic in tubes over open disconnected cones are obtained. We then consider functions of polynomial growth which are analytic in tubes ofer open convex cones and represent such functions as Cauchy and Poisson integrals as well as a Fourier-Laplace integral, and we extend our results to function analytic in tubes over open disconnected cones. We then consider our Cauchy integral representation restricted to 1-dimension and show that the representation becomes in terms of a new type of Cauchy integral which is in fact an equivalence class of analytic functions defined by a classical Cauchy integral. We then develop from this a "Cauchy integral" for elements of $\mathcal{S}'$ and show that the analytic functions having $\mathcal{S}'$ boundary values can be recovered from the boundary values by this "Cauchy integral" and conversely.

CIORANESCU,J.: <u>Ultradistributionen in der Spektraltheorie von Operatoren</u>

In 1960 C. Foias introduced the vector-valued distributions into the study of spectral properties of operators.

Later Tillmann defined the class of self-adjoint operators on a Banach-space - these are the operators with the resolvent being the indicatrix function of a distribution. We show that the ultradistributions also can be used in the theory of operators because the fundamental function spaces of Roumieu are "admissible algebras"; so we define the class of generalized scalar operators in the ultradistribution-sense. The study of this class is related with a special class of quasinilpotent operators which we call $\{M_p\}$ -quasinilpotent.

The case of selfadjoint operators in the ultradistribution-sense can be completely characterized using the representations of ultradistributions as boundary values of holomorphic functions.

DOSTAL, M.: <u>Asymptotic Properties of certain Fourier Integrals</u>

Asymptotic expansion of integrals of the type

$$\int_{\mathbb{R}^n} e^{i\tau g(x)} f(x)\,dx \qquad \left(g(0) = 0 = \sum_{i=1}^{n} \left(\frac{\partial g}{\partial x_i}\right)^2(0),\ f \in C_0^\infty,\right.$$

$$\left.\text{supp } f \text{ in a nbhd. of } 0,\ g \text{ real valued},\ g \in C^{(N)}\right)$$

for $|\tau| \longrightarrow \infty$ are discussed. Under additional conditions on g in the neighborhood of its degenerate critical point 0 the asymptotic expansion is explicitly desired, and the coefficient of the main term is geometrically interpreted. This generalizes the previous results of E. Hlawka and others. Applications to number theory and harmonic analysis are given. Connections with the work of Arnol'd and Malgrange are also discussed.

DRESELER, B.: Räume von Testfunktionen und Sätze vom Bochner - Eberlein - Schoenberg Typ

Es sei G eine lokalkompakte abelsche Gruppe und $\hat{G}$ die Dualgruppe von G. Der Satz von Bochner - Eberlein - Schoenberg (BES) charakterisiert dann die Fourier - Stieltjes - Algebra $\mathfrak{F}\mathcal{M}^1(G) = B(\hat{G})$, wobei $\mathfrak{F}$ die Fourier - Stieltjes - Transformation ist und $\mathcal{M}^1(G)$ die Convolutionsalgebra der beschränkten Radonmaße auf G. Diesem klassischen Charakterisierungsproblem wird ein äquivalentes Testproblem zur Seite gestellt, das in einem Fortsetzungssatz, den man durch Dualisieren einer Version des Satzes von Helly - Hahn - Banach erhält, eine allgemeine funktionalanalytische Behandlung erlaubt. Als Anwendungsbeispiele dieses Satzes werden Sätze vom BES Typ für die Fourier-Stieltjes - Transformation auf unendlich-dimensionalen lokalkonvexen Räumen und für die Fourier - Stieltjes - Transformation zu den Eigenfunktionen eines Sturm - Liouville - Operators und ein Interpolationssatz für Hardy - Räume über symmetrischen Räumen angegeben.

FENSKE, Ch.: A bifurcation theorem for a class of Fredholm mappings

X denotes a real Banach space, $F(X) := \{T \in L(X) \mid \lambda I - T$ is a Fredholm operator for all $\lambda \in (-\infty, 0]\}$. If $\Omega \subset X$ is open, $\mathfrak{F}(\Omega)$ denotes the set of all C^2-mappings $f: \bar{\Omega} \longrightarrow X$ such that $df(x) \in F(X)$ for all $x \in \Omega$. There is a degree theory for proper mappings $f \in \mathfrak{F}(\Omega)$ (see [2]). Although this degree does not possess all properties of Leray-Schauder degree, it is strong enough to prove the following bifurcation theorem of Rabinowitz type (cf. [3]): Let Ω be open in $\mathbb{R} \times X$. Suppose $f: \Omega \longrightarrow X$ is a C^2-mapping such that: i) for all $(\lambda, x) \in \Omega$ $I - D_2 f(\lambda, x) \in F(X)$, ii) $f(\lambda, 0) = 0$ for all λ such that $(\lambda, 0) \in \Omega$, iii) there is $T \in L(X)$ such that $D_2 f(\lambda, 0) = \lambda T$ for all λ such that $(\lambda, 0) \in \Omega$. Suppose $(\lambda_0, 0)$ is in Ω, and λ_0 is a characteristic value of T of odd multiplicity. Let $\mathcal{S}$ denote the closure of $\{(\lambda,x) \in \Omega \mid f(\lambda,x) = x \neq 0\}$ and $\mathcal{C}_{\lambda_0}$ the connected component of $\mathcal{S} \cup (\lambda_0,0)$ containing $(\lambda_0,0)$. Then $\mathcal{C}_{\lambda_0}$ is either not compact or contains $(\lambda,0)$ for $\lambda \neq \lambda_0$. This result has also been proved by E.N. Dancer [1]. If addition $f \in C^3(\Omega)$, λ_0 is simple and moreover a regular bifurcation point and if $\sigma(T)\setminus\{\lambda_0^{-1}\} \subset \{\lambda \in \mathbb{C} \mid \operatorname{Re}\lambda < \lambda_0^{-1}\}$, then one may prove as in Sattinger [4] that the supercritical solution branches are stable whereas the subcritical solution branches are unstable.

[1] Dancer, E.N., Boundary value problems for ordinary differential equations on infinite intervals, Proc. London Math. Soc. (to appear).

[2] Eisenack, G., Fenske, C., Fixpunkttheorie, Bibliographisches Institut, Mannheim (to appear in 1976).

[3] Rabinowitz, P.R., Some global results for nonlinear eigenvalue problems, J. functional analysis 7 (1971) 487-513.

[4] Sattinger, D.H., Stability of bifurcating solutions by Leray-Schauder degree, Arch. Rat. Mech. Analysis 43 (1971) 154-166.

FLORET, K.: Well-located Subspaces of Distribution-Spaces

A subspace F of an inductive limit-space $(E,\tau) = \operatorname{ind}_{\alpha\to} (E_\alpha,\tau_\alpha)$ is called well-located if $(F,\tau)' = (\operatorname{ind}_{\alpha\to}(F\cap E_\alpha,\tau_\alpha))'$.

Motivated by counterexamples $\mathcal{D}(\Omega)$, Ω P-convex and not strongly P-convex, the following theorem was stated and proven:

Let (E_α) be an inductive net of reflexive Fréchet-spaces and F a closed subspace of $E = \operatorname{ind}_{\alpha\to} E_\alpha$. Then the following conditions are equivalent

(1) F is well-located

(2) $(F,\tau)'_\beta = (\operatorname{ind}_{\alpha\to}(F\cap E_\alpha,\tau_\alpha))'_\beta = \operatorname{proj}_{\leftarrow\alpha} (E_\alpha,\tau_\alpha)'_\beta/(F\cap E_\alpha)^\circ$

(3) E'_β/F^o is complete.

If, aditionally, $E = \mathcal{D}(\Omega)$

(4) $(F,\tau) = \underset{n\longrightarrow}{\text{ind}}\ (F \cap E_n, \tau_n)$.

v. GRUDZINSKI, O.: Construction of Fundamental Solutions for Convolutors

It is shown that Hörmander's method of construction of fundamental solutions for general differential polynomials can be used to construct fundamental solutions in distribution spaces where the topology of the space of testfunctions is defined by means of an analytic uniform (AU) structure. When applied to convolutors $f * : \mathcal{D}' \longrightarrow \mathcal{D}'$, where $f \in \mathcal{E}'$, this method gives fundamental solutions in $\mathcal{D}'$ if the Fourier transform $\hat{f}$ of f is slowly decreasing. If $\hat{f}$ is very (resp. extremely) slowly decreasing one obtains fundamental solutions of finite distributional order (resp. of exponential growth).

HANSEN, S.: Einbettungssätze für die Distributionsnräume $B_{p,k}$

Für die Räume $\overset{o}{B}_{p,k}(\Omega) := \overline{\mathcal{D}(\Omega)}^{B_{p,k}}$ ($B_{p,k}$ ist der in Hörmanders Buch über partielle Differentialgleichungen eingeführte Distributionenraum) werden notwendige und hinreichende Bedingungen für die Stetigkeit bzw. die Kompaktheit der Einbettung $\overset{o}{B}_{p,k_1}(\Omega) \hookrightarrow \overset{o}{B}_{q,k_2}(\Omega)$ angegeben, wobei insbesondere der Fall $p \neq q$ berücksichtigt wird. Die Stetigkeit (ebenso die Kompaktheit) der Einbettung hängt - wenn Ω beschränkt oder Ω ein offener, konvexer Kegel ist - bzgl. der Gewichtsfunktionen k_1 und k_2 nur von ihrem Quotienten $\frac{k_2}{k_1}$ ab.

HUSSEIN, D.: Hypoelliptic Convolution Equation in Beurling Spaces

Necessary and sufficient conditions are given for a convolution operator in a Beurling space to be hypoelliptic. Also, partial hypoellipticity with respect to some variables in Beurling spaces is considered, necessary and sufficient conditions for convolution operators to be partially hypoelliptic with respect to some variables are given. These results extend some work of Björck.

KÖRNER, J.: Darstellung Roumieu'scher Ultradistributionen als Randwerte holomorpher Funktionen

Folgende Sätze wurden bewiesen. Der Zerlegungssatz ist ein Hilfsmittel zum Beweis von b.

Satz: a) Jede Ultradistribution $T \in \mathcal{D}'\{M_p\}$ ist ein Randwert einer holomorphen Funktion f, die der Bedingung

(*) $\forall K \subset\subset \mathbb{R} \quad \forall L > 0 \quad \exists\, C(K,L) > 0$, so daß

$$\sup_{x \in K} |f(x+iy)| \leq C(K,L) \cdot \sum_p p!\, L^p M_p^{-1} |y|^{-p}$$

genügt.

b) Jede holomorphe Funktion $f \in H(\mathbb{C} - \mathbb{R})$, die (*) erfüllt, hat einen Randwert in $\mathcal{D}'\{M_p\}$.

Zerlegungssatz: $g \in H(D)$, D Einheitskreis, mit

$$|g(w)| \leq \sum_p p!\, L^p M_p^{-1} (1-|w|)^{-p}$$

läßt sich zerlegen in $g = \sum_{p \geq 0} g_p$, wobei

$g_p \in H(D)$ und

$$|g_p(w)| \leq c \cdot p! \cdot (BL)^p M_p^{-1} (1-|w|)^{-p}$$

(c,B sind von g und p unabhängige Konstanten).

KRANZLER, St.K.: Existence Theorems for Systems of Convolution Equations

Let $\{S_i\}_{i=1}^{N}$ be a family of distributions, each having compact support in $\mathbb{R}_n$ and $\{\Omega_i\}_{i=0}^{N}$ be a corresponding family of open sets in $\mathbb{R}_n$ such that Ω_i + support $S_i \subset \Omega_0$ $(1 \leq i \leq N)$. Necessary conditions for the system $S_i * u = f_i$ $(1 \leq i \leq N)$ to have a distributional solution (resp. a C^∞ solution) u in $\mathcal{D}'(\Omega_0)$ (resp. $\mathcal{E}(\Omega_0)$) for $(f_1,f_2,\dots,f_N)$ in $\prod_{i=1}^{N} \mathcal{D}'(\Omega_i)$ (resp. $\prod_{i=1}^{N} \mathcal{E}(\Omega_i)$) is exhibited by making use of topological vector space methods and some techniques developed by Lars Hörmander. It is subsequently shown that for certain types of differential operators, these same conditions are also sufficient.

The associated problem of finding conditions on subspaces of test function spaces which imply that every sequentially continuous linear functional on the subspace admits a continuous linear extension to the entire space is also discussed.

LAWRUK, B. and TULCZYJEW, W.M.: Criteria for Partial differential Equations to be Euler-Lagrange Equations

Let V be a bounded contractible domain in $\mathbb{R}^n$ and let $u: V \longrightarrow \mathbb{R}^n$ be a C^∞-mapping. Nonlinear partial differential operators defined on such mappings are considered. Of particular interest, however, are such operators which appear in Euler-Lagrange equations. If the Lagrangian $L = L(\eta, \dots, \eta^k, \dots)$ is a C^p-function of $\eta^k = (\eta_{1k}, \dots, \eta_{nk})$, $k \in \mathbb{N}^n$. $|k| = k_1 + \dots + k_n \leq p$, then the necessary condition for the functional

$$\int_V L(u,\dots,D^k u,\dots)dx,\quad D^k u = \frac{\partial^{|k|} u}{\partial x_1^{k_1} \dots \partial x_n^{k_n}}$$

to have extremum at u is

$$\sum_{j=1}^{n} \left[\sum_{|k'|\leq p} (-1)^{|k'|} D^{k'} \frac{\partial L}{\partial \eta_{jk'}} (u,\ldots,D^k u,\ldots) \right] \Delta u_j = 0, \quad x \in V$$

for arbitrary Δu_j, and leads to the system of Euler-Lagrange equations

$$\sum_{|k'|\leq p} (-1)^{|k'|} D^{k'} \frac{\partial L}{\partial \eta_{jk'}} (u,\ldots,D^k u,\ldots) = 0, \quad j = 1,\ldots,m; \; x \in V,$$

in general of order 2p. The left hand side of this system is called the Euler-Lagrange operator corresponding to the Lagrangian L.

We formulate criteria for an operator $\lambda = (\lambda_1,\ldots,\lambda_m)$, $\lambda_j = \lambda_j(u,\ldots,D^k u,\ldots)$, $j = 1,\ldots,m$ to be the Euler-Lagrange operator corresponding to a Lagrangian L and also show the possibility of constructing L from λ. Further, we show that if every linearization of λ is a formally self-adjoint operator in V, then there is a Lagrangian L such that λ is the Euler-Lagrange operator corresponding to L.

ORTON, M.: <u>Singular integral Equations and Distributions</u>

Singular integral equations of the form

$$A(x)f(x) + B(x) \frac{1}{\pi} \text{Pr} \int_a^b \frac{f(t)}{x-t} dt = g(x) \quad \text{on } (a,b)$$

$$a(x) \int_a^x \frac{(x-t)^{\alpha-1}}{\Gamma(\alpha)} f(t)dt + b(x) \int_x^b \frac{(t-x)^{\alpha-1}}{\Gamma(\alpha)} f(t)dt = g(x) \quad \text{on } (a,b)$$

as well as certain mixed boundary value problems are formulated as distributional problems and shown to be equivalent to Hilbert-Riemann problems for analytic representations of distributions in $\mathcal{D}'(\mathbb{R})$. For such Hilbert-Riemann problems the following is shown:

Theorem 1: Let $G(x)$ be infinitely differentiable on $U = \mathbb{R} \setminus \bigcup_{k=1}^{m} \{x_k\}$ and assume that G has a "proper factorization" $G(x)\cdot\hat{k}(x+i0) = \hat{k}(x-i0)$. Then given any $g \in \mathcal{D}'(U)$ which can be extended to $\mathcal{D}'(\mathbb{R})$, there exists an analytic representation $\hat{f}(z)$ of $f \in \mathcal{D}'(\mathbb{R})$ whose boundary values satisfy

(1) $\hat{f}(x+i0) - G(x)\cdot\hat{f}(x-i0) = g(x)$ in $\mathcal{D}'(U)$.

Given g, there exists an analytic representation $\hat{K}_g(z)$ such that $G\cdot\hat{K}_g(x+i0) = \hat{K}_g(x-i0)$ on U and all solutions of the Hilbert-Riemann problem (1) are of the form

$$\hat{f}(z) = \Big\{[g(x)\cdot\hat{K}_g(x+i0)]^{\wedge}(z) + e(z) + \sum_{k=1}^{m}\sum_{j=0}^{N_k} a_{jk}(z-x_k)^{-j-1}\Big\} / \hat{K}_g(z)$$

for $e(z)$ entire analytic and arbitrary constants a_{jk}.

Conditions for G to have a proper factorization are given by:

Theorem 2: Let $G(x)$ be infinitely differentiable on U with $G = G_1\cdot G_2$. Assume G_1 and G_2 satisfy:

(i) G_1 is complex-valued with $|G_1(x)| = 1$ on U. G_1 and all of its derivatives have limits as $x \longrightarrow x_k^+$ and as $x \longrightarrow x_k^-$.

(ii) G_2 is real-valued with $G_2(x) > 0$ on U. For $k = 1,2,\ldots,m$ and $j = 0,1,\ldots$ there exist $a_{jk}^{\pm} \in \mathbb{R}$ such that

$$\lim_{x \longrightarrow x_k^{\pm}} (x-x_k)^{a_{jk}^{\pm}+\varepsilon} |G^{(j)}(x)| = \begin{cases} 0 & \text{for any } \varepsilon > 0 \\ \infty & \text{for any } \varepsilon < 0. \end{cases}$$

Then $G(x)$ has a "proper factorization".

As an application we derive the distributional solutions of a mixed boundary value problem and construct generalized eigenfunctions for the finite Hilbert-transform.

SCHMETS, J.: Discrete and e-additive measures

Let X be a completely regular and Hausdorff space and denote by $\mathcal{C}^b(X)$ the space of all continuous and bounded functions on X. On the space $M_d(X)$ of all discrete (the atomic measures would do the same) measures on X, put the topology $\tau_{\mathcal{H}}$ of uniform convergence on all equicontinuous and uniformly bounded subsets of $\mathcal{C}^b(X)$. Then the following general theorem "Let E be a locally convex topological vector space. Then the finest locally convex topology on its dual E' which is equivalent to the w*-topology on all equicontinuous subsets of E' is the topology of uniform convergence on the compact sets of the completion $\hat{E}$ of E " applied to $E = [M_d(X), \tau_{\mathcal{H}}]$ gives Wheeler's result that "the space $\mathcal{C}^b_{\beta_e}(X)$ is always strongly Mackey". This has been done with the collaboration of J.Zafarani.

SPECK, F.: The characterization of the Fredholm Property for Generalized Convolution Operators on Sobolev Spaces by its Symbol

Let $\mathfrak{A}$ be a class of generalized convolution operators on Sobolev spaces

$$A: L^p_s \longrightarrow L^p_t \quad (1 < p < \infty, \ s,t \in \mathbb{Z}),$$

which are of local type, i.e. $\varphi \cdot A - A\varphi \cdot$ compactly for all $\varphi \in C^\infty(\overline{R^n})$. Then we have equivalence of the following facts:

(i) A is a Fredholm operator $\Longleftrightarrow \operatorname{ess\,inf}_A |\Phi_A| > 0$

where $A \longrightarrow \Phi A$ is a homomorphism from $\mathfrak{A}$ into $L^\infty(\Delta)$, $\Delta \subset \mathbb{R}^n \times \mathbb{R}^n$,

(ii) there is a criterium of such type for the local Fredholm property for operators of the more simple corresponding class of pure convolutions in L^p.

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As an example one can take an integro-differential operator $\sum_{|\nu|\leq s-t} A_\nu D^\nu$, whose coefficients A_ν are enveloping operators of sets $\{A_{\nu x}\}_{x\in\mathbb{R}^n}$ of singular and L^1-convolutions.

ZIELEZNY, Z.: Hypoelliptic convolution equations in $\mathcal{K}'_p$

Let $\mathcal{K}_p$, $p > 1$, be the space of C^∞-functions φ in $\mathbb{R}^n$ such that

$$\nu_k(\varphi) = \sup_{x\in\mathbb{R}^n, |\alpha|\leq k} |D^\alpha\varphi(x)e^{k|x|^p}| < \infty .$$

The topology in $\mathcal{K}_p$ is defined by the seminorms ν_k. The dual $\mathcal{K}'_p$ to $\mathcal{K}_p$ is the space of distributions which grow like $e^{a|x|^p}$, for some a (depending on the distribution).

Let $\mathcal{O}'_C(\mathcal{K}'_p:\mathcal{K}'_p)$ be the space of convolution operators in $\mathcal{K}'_p$. Further let $\mathcal{E}\mathcal{K}'_p$ be the space of C^∞-functions f such that

$$D^\alpha f(x) = 0(e^{b|x|^p}) \quad \text{as} \quad |x| \longrightarrow \infty$$

for all α and some b (depending on f).

A distribution $S \in \mathcal{O}'_C(\mathcal{K}'_p:\mathcal{K}'_p)$ is said to be hypoelliptic in $\mathcal{K}'_p$ if every solution $u \in \mathcal{K}'_p$ of the convolution equation $S * u = v$ is in $\mathcal{E}\mathcal{K}'_p$ if $v \in \mathcal{E}\mathcal{K}'_p$.

Theorem: The distribution $S \in \mathcal{O}'_C(\mathcal{K}'_p:\mathcal{K}'_p)$ is hypoelliptic in $\mathcal{K}'_p$ if and only if its Fourier transform $\hat{S}$ satisfies the following conditions:

(I) There are sonstants $A,B > 0$ such that
$|\hat{S}(\xi)| \geq |\xi|^{-B}$ for $\xi \in \mathbb{R}^n$, $|\xi| \geq A$

(II) $\dfrac{|\text{Im}\,\zeta|^q}{\log|\zeta|} \longrightarrow \infty$ as $\zeta \in \mathbb{C}^n$, $\hat{S}(\zeta) = 0$, $|\zeta| \longrightarrow \infty$,

where $\frac{1}{p} + \frac{1}{q} = 1$.

S. Hansen (Kiel)