MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 35/1975

"Mathematical Methods in Celestial Mechanics"

24.8. bis 30.8.1975

Nach dreijährigem Unterbruch fand die fünfte Tagung über Himmelsmechanik unter der Leitung von E. Stiefel (Zürich) und V. Szebehely (Austin) statt (frühere Tagungen: 1964, 1966, 1969, 1972).
Wiederum hatte die Tagung internationalen Charakter: Knapp die Hälfte der 32 Wissenschaftler entstammten dem deutschsprachigen Raum, während die übrigen aus Frankreich, Belgien, dem Vereinigten Königreich und vor allem aus den USA kamen.

Das wissenschaftliche Programm zeichnete sich durch die Breite seines Spektrums aus, die charakteristisch ist für die Himmelsmechaniktagung: Hier treffen reine Mathematiker (die mit subtilen mathematischen Methoden die Gleichungen der Himmelsmechanik analysieren) mit den angewandten Wissenschaftlern der NASA und ESA (die die Daten für Start und Unterhalt von Satelliten ermitteln) zusammen.

Es ist ein wesentliches Ziel dieser Tagung den Austausch zwischen diesen Gruppen zu fördern.

Einen Höhepunkt bedeutet es jeweils, wenn der Altmeister der Himmelsmechaniker, Professor O. Volk (Würzburg), in Form eines Abendvortrages, Einblick in seine historische Forschungsrabeit gibt. Diesmal galten seine Ausführungen den Newton'schen Bewegungsgleichungen.

Schliesslich sei angefügt, dass die einzigartige Atmosphäre des mathematischen Forschungsinstitutes Oberwolfach von allen Teilnehmern anerkannt und hoch geschätzt wurde.

Teilnehmer

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Vorträge:

K. AKSNES, A. FRANKLIN: Orbital Analysis of Jupiter I-IV from Light Curves of Mutual Events.

A world-wide observing campaign has provided a large collection of photometric observations of mutual occultations and eclipses of the Galilean satellites (Jup. I-IV) during the favorable apparition in 1973-74. Theoretical light curves are brought into close agreement with the observed ones by adjusting the predicted positions of the satellites and their radii. For the radii of Europe, Ganymede, and Callisto, the author's have derived the preliminary mean values and standard errors 1525 ± 30 km, 2622 ± 43 km, respectively. From a single, well-observed, light curve the relative position of the two satellites involved can be deduced to 100 km or better.

The results show that the two most recent orbital theories for the Galilean satellites, by Sampson (1910) and by de Sitter (1931), both suffer from longitude errors of 1000 km or more for current times, while about half as large errors may exist in the latitudes computed from Sampson's theory. De Sitter's theory, on the other hand, agrees remarkably well with the observed latitudes, at least for 10 and Europa.

R.F. ARENSTORF: Families of Periodic Solutions in the N-Body Problem. A Survey of Recent Results and Existence Proofs.

A brief historical review of the major contributions to the problem of existence of particular solutions as mentioned in the title was given, leading to the results of O. Perron and of M. Crandall for more than three bodies.

Most of these solutions represent "superpositions" of circular Keplerian motions. Since 1963 the author has added to these results classes of periodic solutions representing "superpositions" of circular with elliptic Keplerian motions for N = 3 and for N = 4. Methods of proof, and ideas which help overcome the inherent degeneracies for the latter periodic motions of mixed type, were outlined.

J. BAUMGARTE: A New Time Element for a General Time Transformation

The paper introduces a new time element to be used with a general time transformation for satellite equations of motion. The purpose of this time element is to reduce the growth of the numerical errors with respect to the time integration. It is characteristic for the new time element, that it does not depend on the independent variable s. Together with the differential equation for the time element τ here also exists the differential equation for the physical time t. This equation is supplied by control term which preserves the relation between the time element and the other dependent variables in an asymptotical stable manner. Numerical experiments show accuracy improvements by using this time element.

D.G. BETTIS: Optimal m-Fold Runge-Kutta Methods

Recurrent power series methods are extremely applicable to problems in celestial mechanics since the Taylor coefficients may be expressed by recurrence relations. However, as the number of Taylor coefficients increases, as is often necessary because of accuracy requirements, the computing time grows prohibitively large. In order to avoid this unfavourable situation, Dr. E. Fehlberg introduced, in 1960, Runge-Kutta methods that use the first m Taylor coefficients obtained by recursive relations, or some other technique.

Optimal m-fold Runge-Kutta methods are introduced. Inbedded methods of order (m+3) [m+4] and (m+4) [m+5] are presented which have coefficients, that produce minimum local truncation errors for the higher order pair of solutions of the method, as well as providing a near maximum absolute stability region.

It is emphasized that the methods are formulated such that the higher order pair of solutions is to be utilized. These optimal methods are compared to the existing m-fold methods for several test problems. The numerical comparisons show that the optimal methods are more efficient. It is stressed that these optimal methods are particularly efficient when m is small.

V.R. BOND (presented by O. GRAF): Development of Poincaré-Similar Elements and their Numerical Applications.

A new set of element differential equations for perturbed two-body motion is derived. These elements are canonical and are similar to the classical Poincaré elements, which have time as the independent variable. The Hamiltonian is extended into phase space by introducing the total energy and time as canonically conjugated variables. The new independent variable is, to within an additive constant, the eccentric anomaly. These element differential equations are compared to the Kustaanheimo-Stiefel (KS) element differential equations, which also have the eccentric anomaly as the independent variable. For several numerical examples, the accuracy and stability of the new set is equal to that of the KS solution. This comparable accuracy result can probably be attributed to the fact that both have the same time element and very similar energy elements. The new set has only eight elements compared to ten elements for the KS elements.

R.A. BROUCKE: On the Characteristic Exponents of the General Three-Body Problem.

The characteristic exponents of the general three-body problem have some remarkable properties which make the problem basically different from the restricted problem or any other dynamical system. They are associated with the eigenvalues of an 8 x 8 monodromy matrix. The monodromy matrix is symplectic while the matrix of the variational equations is skew-symplectic. The 8 eigenvalues of the monodromy are two reciprocal pairs and 4 unit roots. The unit roots correspond to the energy and angular momentum integrals.

The other four roots define two stability indices, rather than a single one in the restricted problem. This results in six types of unstable orbits and one region of stable periodic orbits. The variational equations have three explicitly known solutions, due to the energy integral, the angular momentum integral and the arbitrary scale factor of the problem. Because of these solutions, some remarkable eigenvectors and a principal vector of order 2 of the fundamental matrix can also be determined explicitly.

M. C. ECKSTEIN: Approximate Semi-Analytical Solution for Orbit-Attitude Maneuvers of Spin Stabilized Satellites

The mutual interaction of orbit- and attitude maneuvers of spin stabilized satellites was investigated by application of the "Two Variable Expansion Procedure" to the Euler equations. The resulting semi-analytical solution describes both the short periodic nutations and the long term attitude variations as well as the linear accelerations which influence the orbit. Some applications of the solution are shown by means of a few examples.

B. GARFINKEL: A Theory of the Trojan Asteroids

The paper constructs an analytical long-periodic solution for the case of 1:1 resonance in the restricted problem of three bodies. An intermediate Hamiltonian is furnished by the previously solved Ideal Resonance Problem. The perturbations are calculated by the method of Lie-series, the short-periodic terms are removed from the solution by equating the mean eccentricity to zero, and the calculation of the time-dependence is reduced to the inversion of a hyperelliptic integral.

The domain of the solution is a horseshoe-shaped region bounded by the inequalities $|\theta| > 2\, m^{1/3}$ and $|r-1| < m^{1/3}$. Here r and θ are the polar coordinates in a rotating coordinate system, and m is the mass-parameter. The solution excludes the internal resonances defined by $T_\ell = j\, T_s$, where j is an integer and T_s and T_ℓ are the short and the long periods, respectively.

O.F. GRAF, Jr.: A Canonical Theory for the Elimination of Short and Intermediate Period Terms from the Problem of a High Altitude Earth Satellite.

The problem of a satellite in motion about an oblate earth (J_2 problem) has been successfully solved by G. Scheifele by introducing Delaunay Similar (DS) elements in an extended phase space with the true anomaly as independent variable. A complete first order solution was established through the use of the canonical formalism of von Zeipel.

In this work, the DS theory is extended to the problem of an earth satellite that is perturbed by the sun and moon, and also J_2. All three effects are assumed to be the same order of magnitude. Since the external body terms depend explicitly on time, the time element appears as an additional angle variable. The Hamiltonian is expressed in DS-elements with the eccentric anomaly being used as a noncanonical auxiliary variable. A more general solution to the first von Zeipel equation allows simultaneous elimination of short and intermediate period terms (terms associated with the period of the external body). This solution will be valid for large eccentricities. The canonical transformation to mean elements is defined by a generating function that is a series involving Bessel coefficients.

D.C. HEGGIE: Redundant Variables and Regularisation in the Three-Body Problem

It has been shown (Celes. Mech. 10, 217, 1974) that the equations of motion for the three-body problem may be cast into a form which is regular for collisions between any pair of bodies. The method proceeds by two stages, which are:

1) the introduction of redundant variables
2) the application of the KS-transformation.

The present contribution gives a different treatment of the first stage, in a manner related to the work of Broucke and Lass (Celes. Mech. 8, 5, 1973)

J. HENRARD: On the Artificial Satellite Theory

Some of the basic ideas of an analytical orbiter theory which is being developed by Hubert Claes in Namur are presented.

The theory is based on the Lie transform technique and is expressed in a closed form up to second order. The inclusion of additional terms of the third order (expanded in power series of the excentricity) is considered.

Special attention is being given to the choice of the elements and to the final form of the theory. Three main criteria are used. The removal of the virtual singularities of small inclination and eccentricity. The simplicity of the final form of the theory once the elements have been given their numerical values. The numerical stability of the evaluation of the theory.

G. JANIN: Decay of a Highly Eccentric Satellite

During the reentry phase of a highly eccentric satellite there is a theoretical possibility offered by Celestial Mechanics where a small increase in the perigee altitude during the last revolutions of the satellite can allow it to describe several more revolutions.

Reentry of HOES I predicted for October 28, 1975 is near such a situation.

D.J. JEZEWSKI: K/S Two-Point-Boundary-Value Problems

A method of developing the missing general K/S (Kustaanheimo/Stiefel) boundary conditions is presented, with use of the formalism of optimal control theory. As an illustrative example, the method is applied to the K/S Lambert problem to derive the missing terminal condition. The necessary equations are then developed for a solution to this problem with both the fictitious time, s, and the generalized eccentric anomaly, E, as the independent variables. The latter formulation, requiring the solution of only one nonlinear, well-behaved equation in one unknown, E, results in considerable simplification of the problem. This simplification is possible because the energy equation, in the E- formulation, is separable.

A.H. JUPP: Further Investigations in the Atmospheric Drag Problem

This short note is concerned with the motions of artificial satellites in an atmosphere. It is a relatively simple procedure to set up the equations for the variations of the elements; their subsequent analysis may be approached in several ways. One very useful method is to develop the right-hand sides of these equations as functions of the eccentric anomaly, and then to integrate the equations over a single revolution of the orbit. The changes in the elements in one period can then be found. It is shown here that one particular term in this analysis, which seems to have been neglected in earlier theories on this type, can be significant and should therefore be retained in the analysis for some satellite motions. The term is most significant for near-circular orbits of light balloon-type satellites.

Some consequences of the rotation of the atmosphere are also noted.

U. KIRCHGRABER: Error Bounds for Perturbation Methods

There are many papers dealing with the problem of error bounds for perturbation methods (development with respect to a small parameter, method of averaging, stroboscopic method etc.). The majority of these bounds, however, is very pessimistic and does not really reflect the qualities of the underlying perturbation method. In this paper a new attempt is made to overcome this problem. By using a new comparison theorem and the higher order approximations the author is led not only to upper bounds, but to lower bounds as well.

Mme. L. LOSCO: On a Generalization of the Lagrangian Equations which permits an Extension of the KS-Transformation

This paper is composed of two parts, the first one established by M. Langlois, the other part by L. Losco. First is made the study of Poincaré's equations, which are Lagrangian equations when use is made of some quasi-coordinates. One application of these equations is very interesting when some coordinates are ignorable in the Lagrangian. A theorem of reduction is obtained with invariant relations. KS is of this kind. Then are constructed matrizes which generalize KS. There are matrizes of coordinates and quasi-coordinates, which allow to apply the theorem of reduction previously obtained. The general motion, helicoidal motion, of a rigid body in R^n-space allows to obtain such matrizes, just as KS corresponds to a rotation in R^4.

P. NACOZY: Numerical Aspects of Time Elements

Time elements are presented for use with the Sundman time transformation

$$dt = r^n dS$$

for n = 1, 3/2, 2. The case for n = 3/2 involves elliptic integrals and the new independent variable, S, is referred to here as the "intermediate anomaly". Numerical results are presented that show that the use of time elements increases the accuracy of the Sundman transformation by about one order of magnitude for a close Earth satellite. A discussion is given indicating the necessity of using stabilization techniques in addition to the use of a time transformation with time elements.

F. NAHON: Cartan's 11th Integral of the N-Body Problem

Cartan has discovered that N-body-problem equations admits a linear differential invariant form, which corresponds to a 11th first integral of the extended equations by use of the variable action.

The link with variational equations is studied, an an application is given to the triple close approach in the collinear three body problem, when the level of energy is h = 0.

H. RUESSMANN: On a New Proof of Moser's Twist Mapping Theorem.

A new proof of the analytic version of Moser's twist mapping theorem is given. Our proof is simpler than that in the book of Siegel-Moser, "Lectures on Celestial Mechanics" so far we are able to formulate the iteration process for constructing an invariant curve in the frame work of the classical Newton method, such that no coordinate transformations but only Jacobians near the unit matrix have to be inverted.

D. RUFER: Trajectory Optimization by making Use of the Closed Solution of Constant Thrust-Acceleration Motion.

The problem of fuel-optimal rendezvous and transfer maneuvers in a central gravitational field is considered. By using analytical results and a parametrization of the control functions, the original optimal control problem can be solved by a sequence of mathematical programming problems. After introducing KS-variables and piecewise-constant thrust-accelerations, all necessary trajectory integrations are performed in closed form. This optimization procedure allows the solution of a wide class of problems: The propulsion system may be thrust-limited or power-limited, one may consider rendezvous or transfer maneuvers with fixed or free final time.

E.A. ROTH: Perturbation of a Satellite Orbiter by the Oblateness of the Primary Planet

The perturbation of an orbiter around a large satellite of a giant planet (Jupiter, Saturn, Uranus or Neptune) produced by the oblateness of the planet is investigated. The perturbing force of the J_2-term (general case) and the J_4-term (special case of small eccentricity and inclination) is expanded in an appropriate form and the main term and the parallactic term are given explicitly. The variations of the orbital elements are derived using the stroboscopic method. An example shows that the perturbation of the orbit cannot be neglected.

D.G. SAARI: The N-Body Problem of Celestial Mechanics

In the last three years there has been a considerable amount of activity in the n-body problem of celestial mechanics. Some of the more important results are reviewed. The talk consists of three parts. The first has to do with recent developments concerning the evolution of Newtonian systems for large values of time. The second considers the possible configurations particles can assume in physical space, and in the third part the discussion is focused on singularities, collisions, and regularizations of n-body systems. (In particular, some unexpected problems concerning regularization of binary collisions in 4 body systems are mentioned.)

G. SCHEIFELE: A State Transition Matrix based on a new Analytical Satellite Theory.

In 1970 the author has proposed a new set of orbital elements which are canonical, use the extended phase space and are formulated with independent variables different from time. In the case where the independent variable is the true anomaly, a new artificial satellite theory based on these elements was developed. This new approach gives more accurate results than classical theories, mainly because a second integration of the energy-element (semi-major-axis) is not necessary for conservative perturbations.

Because of the very concise form of the analytical solution, an attempt was made to derive a J_2-perturbed State Transition Matrix. Numerical tests show a considerable improvement in the mapping of differences of the initial state vector above a two-body approximation.

E. STIEFEL: Near Parabolic Orbits

For near parabolic orbits the distinction between coordinates and elements disappears provided the KS-technique is used. In KS-variables a pure parabolic motion is described by linear functions. Advantage is taken from that fact for establishing numerical procedures in perturbed near parabolic cases.

V. SZEBEHELY: Relative Motion

Three theorems are presented regarding the analytical aspects of the relative motion of mutually non-interacting particles in force-fields influencing their motion. The first theorem is a generalization of Encke's method, the second a generalization of Sundman's regularizing transformation and the third generalizes the introduction of synodic coordinates.

J.P. VINTI: Newtonian Cosmology if G varies

Suppose G(t) has the modified Dirac form $A(k+t)^{-1}$, where t is the age of the universe and k is inserted to avoid a singularity in the two-body problem. With the concept of the isolated galaxies and clusters of galaxies as forming a homogeneous and isotropic fluid, the scale factor R(t) obeys, in the Newtonian case, $R^2\ddot{R}=-(4\pi/3)G(t)\rho_o$. This equation is for non-random motion, in which the separation S(t) of two galaxies at time t equals R(t) S_o, the subscript o referring to some epoch time τ. If G (t) has the behaviour postulated and x=k+t, then $R^2\ddot{R}=-B/x$, where $B=(4\pi/3)\ G_o\rho_o x_o$. The substitution R=xu leads to $x^2\dot{u} = \pm(2\ Bu^{-1}+c_1)^{1/2}$, where $c_1=(x_oH_o-1)^2-2\ Bx_o$. Here $H_o=\dot{R}_o$, Hubble's constant at epoch time. Given $R_o=1$ and $\dot{R}_o = H_o$, the equation has a unique solution, even with the $\pm$ signs. Theorems are developed to find the sign in any given case.

If $c_1 = 0$, R has a maximum if and only if $0<x_oH_o<1/3$. If so, the values at maximum are $R_m=p(3p-2)^{-1/3}$ and $x_m=x_op(3p-2)^{-1}$, where $p=1-x_oH_o$.

The fundamental equation shows that any extremum of R must be a maximum, so that for $x>x_o$, R must either $\to\infty$ or reach one maximum and then diminish to zero. Reaching R=0 at $x=x_1>x_o$ is thus necessary and sufficient that R has a maximum value. The various possible cases are then classified. Next, the inequalities involving x_oH_o and c_1 that constitute the necessary and sufficient conditions are derived when applicable, along with the equations for the values R_m and x_m at maximum.

The necessary and sufficient conditions lead to a boundary curve of ρ_{oc} versus H_o, ρ_{oc} being the value of ρ_o required to close the inverse; i.e., to achieve a maximum of R. Despite the uncertainty in $x_o = k+\tau$, the resulting numbers suggest that this Newtonian model, with varying G, probably corresponds to an open universe. The deceleration constant q_o ranges from about 1 to about 2.

O. VOLK: <u>Die sogenannten Newton'schen Differentialgleichungen der Physik in der analytischen Himmelsmechanik</u>

Die sogenannten Newton'schen Differentialgleichungen der Physik treten zum ersten Male <u>explizit</u> in der Himmelsmechanik auf, als die Basler Mathematiker Jakob I und Johann I Bernoulli, Jakob Hermann und L. Euler den Leibniz'schen "Calculus" ein festes rechtwinkliges Koordinatensystem und Polarkoordinaten (mit dem Nullpunkt in der Zentralkraft) in die Newton'sche Himmelsmechanik einführten.

J. WALDVOGEL: The Three-Body Problem Near Triple Collision

A theory of triple collision and the triple close encounter in the planar problem of three bodies is presented. The basic idea is to use the homothetic transformation

$$\underset{\sim}{x}_j = \delta^2 \tilde{\underset{\sim}{x}}_j \quad , \quad t = \delta^3 \tilde{t}$$

(Celest. Mech. 11, 429, 1975) for blowing up solutions of the three-body problem which are nearby Lagrangean triple collision solutions. By means of the theory of singular perturbations the close encounter is then related to parabolic motion in the three-body problem. One result is that a close triple encounter generally leads to the escape of one body with arbitrarily large asymptotic velocity.

K. ZARE: The Effects of Integrals on the Totality of Solutions of Dynamical Systems

Regions of possible motions are established for any three degrees of freedom dynamical system which possesses a time independent Hamiltonian or for any system which is reducable to that form by means of integrals of the motion using only extended point transformations with time independent generating functions. The method is applied to the problem of three bodies in a plane and surfaces of zero velocity are established. These are governed by the energy, angular momentum and the masses of the perticipating bodies. The analytical and geometrical properties of these surfaces provide interesting qualitative results such as conditions for boundedness and for the probability of escape of each body for given constants of the motion.

U. Kirchgraber, Zürich

J. Kriz, Zürich