MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 38|1975

Random Vibrations and their Stability

17. 9. bis 20. 9. 1975

Die Tagung hatte das Ziel diejenigen Wissenschaftler aus der Mathematik, der Mechanik und der Regelungstechnik zusammenzuführen, die über ihre Fortschritte in der Theorie und in den Anwendungen dieser Theorie berichten konnten. Die dreitätige Veranstaltung stand unter der Leitung von F. Weidenhammer und W. Wedig von der Universität Karlsruhe. Es nahmen 35 Wissenschaftler aus 9 Nationen teil. Darunter war seit vielen Jahren erstmals wieder mit Professor Dr. Günter Schmidt, Berlin,DDR (Herausgeber der ZAMM) ein aktiv tätiger Kollege aus der Deutschen Demokratischen Republik. Leider nahm kein Wissenschaftler aus der UdSSR teil, denn der sehr bekannte Fachvertreter Professor Dr. V. V. Bolotin hat am Vortag des Tagungsbeginn seine Teilnahme und seine weiteren Vorträge in Karlsruhe, Stuttgart und Darmstadt krankheitshalber abgesagt. Dennoch waren alle Forschungsrichtungen angemessen vertreten, so daß lebhafte Diskussionen zustande kamen und die jüngeren Wissenschaftler Gelegenheit hatten die Forschungsprobleme dieser modernen Wissenschaft kennenzulernen.

Dem Thema der Tagung entsprechend wurden in 3 Tagen in 17 Fachvorträgen vor allem die Erfassung der Stabilitätseigenschaften als sogenannte Momentenstabilität und als Sample-Stabilität diskutiert. In beiden Richtungen wurden interessante Ergebnisse erzielt. In der praktisch wichtigen, aber in der Theorie äusserst schwierigen Frage der Auffindung der Verknüpfung der beiden Stabilitätsaussagen konnten leider noch keine endgültigen und ausreichenden Ergebnisse vorgestellt werden; dieses zentrale Problem konnte leider nur Gegenstand einiger Diskussionen sein. Man darf daher mit großem Interesse einer Fortschrittstagung entgegensehen, die in einigen Jahren den annähernd gleichen Kreis von Wissenschaftlern zusammenführen sollte und die dann hoffentlich über Fortschritte in dieser Frage berichten werden. Diejenigen Wissenschaftler, deren Interesse den Anwendungen der Theorie der stochastischen Prozesse in der Mechanik (z.B. Schwingungen, Fahrzeugdynamik), in der Physik (Wasserverschmutzung, Ozeanographie, Seismik) und der Regelungstechnik gehört, haben sich offensichtlich die Aufgabe gestellt, die sehr anspruchsvollen allgemeinen Theorien (von Stratonovitsch, Kashminski) für ihre Probleme aufzubereiten. Sie haben hierbei bemerkenswerte Erfolge erzielt und konnten ihre Näherungsrechnungen mit den Vertretern der Grundlagentheorien erfolgreich diskutieren.

Die überaus glücklichen Gegebenheiten des Institutes und der genius loci führten schnell zu jener aufgelockerten Atmosphäre, die wissenschaftliche und menschliche Kontakte ermöglichten, die über den Tag hinaus wirken werden. Die Konzentration auf eine Dauer von 3 Tagungstagen erwies sich als derzeitig dem Thema und dem Teilnehmerkreis angemessen und ließ noch Zeit für die traditionelle Schwarzwaldwanderung bei gutem Wetter.

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Vortragsauszüge

S.T.Ariaratnam, D.S.F. Tam

Sample Stability of Coupled Linear Stochastic Systems.

The sample stability of two-degree of freedom, non-gyroscopic systems subjected to stationary wide-band random parametric excitation of small intensity is examined. By approximating the response amplitudes to a Markov process governed by a pair of Itô equations, and using a procedure due to Khasminskii, a condition is derived for stability with probability one. The result is applied to the flexural-torsional sample stability of a thin simply-supported beam subjected to randomly varying end couples and compared with that for second moment stability.

L.Arnold

Stability of Linear Stochastic Differential Equations.

We consider the stability of the undamped oscillator $\ddot{y}+f_t y=0$, where f_t is a stationary ergodic stochastic process. Put $x=(y,\dot{y})$, $x/|x| = (\cos\varphi,\sin\varphi)$, then $|x_t|^2 = |x_0|^2 \exp(tR_t)$, $R_t = \frac{1}{t}\int_0^t (1-f_s)\sin 2\varphi_s ds$, where $\dot{\varphi}_t = -f_t\cos^2\varphi_t - \sin^2\varphi_t$ (*). It is proved that, if $f_t \in I$, where the interval I is compact and either completely contained in the positive or in the negative half-line, then $R_t \to R \gtreqless 0$ with probability 1 $(t\to\infty)$, where $R = E(1-f_t)\sin 2\varphi_t^0$, φ_t^0 being the unique stationary ergodic solution of (*).

R.F.Curtain

Filtering for Infinite Dimensional Systems Excited by Poisson - White Noise

There is now a fairly complete theory for the filtering problem for linear infinite dimensional systems, where one assumes a Gaussian White Noise type disturbance in the system model and in the observations. Recently, in the finite dimensional stochastic control literature, there has been interest in problems involving jump processes. In particular, Kwakernaak considers the filtering problem for linear systems excited by Poisson White Noise and Gaussian observation noise, which he then uses to solve a river pollution problem. Although the river model is distributed, he approximates it by a suitable finite dimensional model in the usual manner.

In this paper a general filtering theory for infinite dimensional linear systems excited by Poisson White Noise and with observations corrupted by Gaussian White Noise is developed. The results are then applied to Kwakernaak's river pollution problem.

R. Grossmayer

Improved Bounds for the Reliability of Structures under Seismic Loadings

The reliability of structures, excited by a nonstationary random earthquake process, can be determined from the first-passage probability. Starting from lower and upper bounds for the first-passage probability, as proposed by Shinozuka, improvements are derived in two steps: First, a nonstationary enveloppe process is introduced, and the clumpsize of the barrier crossings is taken into account. Better results for the upper bound are obtained for lower barrier levels, when only those enveloppe crossings are considered, that remain below the barrier level one cycle earlier. Hence, the probability density $p(a_1, a_2, \dot{a}_2, t-\tau, t, t)$ will be derived. This second step is combined with a generalisation of Vanmarcke's result for "qualified enveloppe crossings" to nonstationary processes. The improved upper and lower bounds are demonstrated for different numerical examples.

U.G.Haussmann

Asymptotic Stability of the Linear Itô Equation in Infinite Dimensions with Multiplicative Noise

We consider the integral equation

$$(1)\quad X_t = U_t X_0 + \int_0^t U_{t-s} B(X_s)\,dw_s$$

which is a weak form of

$$X_t = X_0 - \int_0^t AX_s\,ds + \int_0^t B(X_s)\,dw_s$$

where $\{U_t\}$ is a strongly continuous semigroup on the separable Hilbert space K with generator - A, w_t is a Wiener Process on the Hilbert space H, and where B(x) is linear in x and assumes values which are continuous linear operators from H into K. Assuming that solutions to (1) exist, we give sufficient conditions for global exponential asymptotic stability of the second moment of X_t. Further conditions are then given for the sample paths to be asymptotic to zero.

A. Kistner

Zur Abschließung der Momentengleichungen linearer Systeme mit farbig verrauschten Parametern

Betrachtet werden lineare Systeme mit Parametererregung durch einen stationären Gaußschen farbigen Rauschprozeß mit beschränkten Realisierungen. Die Momente der Zustandsgrößen genügen bekanntlich deterministischen Systemen von gewöhnlichen Differentialgleichungen mit konstanten Koeffizienten. Diese Momentensysteme sind allerdings nicht abgeschlossen.
Unter Verwendung des Matrizanten wird gezeigt, daß das erste Moment der Zustandsgrößen einer abgeschlossenen inhomogenen linearen Differentialgleichung mit zeitvariablen Koeffizienten genügt. Für die Bestimmung der inhomogenen Terme wird eine Rekursionsformel angegeben. Die Momentengleichung wird unter verschiedenen Gesichtspunkten diskutiert, und die Ergebnisse werden an Hand eines Beispiels veranschaulicht.

F.Kozin

Stability of Undamped Oscillators with Random Parameters

In this lecture we present the results of analytical and simulation studies of the undamped oscillator $\ddot{x}(t)+(w^2+\epsilon g(t))\ x(t) = 0$, where g(t) is a physical (non-white) random function and ϵ is a small parameter. Techniques that have been recently developed by the author and his students are applied to study the regions of stability of the sample solutions in terms of the statistical properties of the random parameter. In particular it is now known that the region of sample stability of a linear stochastic system is determined by the parameter regions for which the limit $\lim_{p\to 0} \frac{1}{p} \log E\{\|x(t)\|^p\}$ is negative, where x(t) is the solution process of the undamped oscillator. This limit is studied for the undamped oscillator.
Simulation studies for determining the regions of stability of stochastic differential equations have always been difficult due to the problem of determining whether or not samples are stable, when the parameters are near the boundary of the stability region. The simulation studies presented in this lecture make use of a specific statistic to separate stable samples from unstable samples. Finally, the results of the analytic studies as well as the simulation studies are compared.

H.J. Kushner:

Approximations for Stability Computations for Diffusion Models

Consider the diffusion process $dx = f(x)dt + \sigma(x)dw$, with differential generator $\mathcal{L} = \sum_{i,j} a_{ij}(x) \frac{\partial^2}{\partial x_i \partial x_j} + \sum_i f_i(x) \frac{\partial}{\partial x_i}$, where $a(x) = \sigma(x)\sigma'(x)/2$. Frequently, stability questions cannot be answered directly via an available stochastic Liapunov function, and numerical calculations must be used. Various quantities of interest (yielding insight into the stability and qualitative proberties) are, among others

(1a) $P_x \{x_t$ reaches set A before B$\}$,

(2a) Calculation of an invariant measure, if one exists,

(3a) $P_x\{$total time in set A is $\geq \alpha$ in interval $[o,T]\}$,

Under certain smoothness conditions, (1a) - (3a) satisfy

(1b) $\mathcal{L}V(x) = 0$, $V(x) = 0$ on B, $V(x) = 1$ on A,

(2b) $\mathcal{L}V(x) - \gamma + k(x) \equiv 0$, where for any smooth $k(\cdot)$,
$\gamma = \int k(x)\mu(dx)$, where $\mu(\cdot)$ is an invariant measure,

(3b) $V_t + \mathcal{L}V(x) = 0$, $V(x,T) = 1$ if $x_{oT} \geq \alpha$, and zero otherwise, where the "augmented" state x_{Ot} satisfies $\dot{x}_{Ot} = I_{\{x_t \in A\}}$.

Various numerical techniques for calculating the solutions will be discussed, together with their probabilistic interpretation and convergence proofs.

H.Kwakernaak:

Periodic Linear Differential Stochastic Processes

Periodic linear differential processes are defined and their properties are analyzed. Equivalent representations are discussed, and the solutions of related optimal estimation problems are given. An extension is presented of Kailath and Geesey's results concerning the innovations representation of stochastic processes with a given covariance function.

L.Lambert:

Jump Phenomena in Nonlinear Control Systems Forced by Stochastic Signals.

The input/output-standard deviation characteristic of a randomly excited nonlinear control system computed by the quasi-linearization technique has multivalued sections under certain conditions. By means of a certain servosystem as example it can be proved, that in such cases abrupt decreases or increasses of the output variance do not occur - in contrast to the majority of publications. Nevertheless the amplitude swellings Booton ascertained can be confirmed. But they also arise, if the characteristic is unique. This means, that the multivaluation cannot be regarded as a necessary criterion for the swellings. Analysing the stability of the response of the excited nonlinear control system we are led to the question of the stability of a homogeneous parametrically excited stochastic differential equation. If the nonlinearity is a limiter, as performed here, a parameter alternates between two values at random times. On the one hand a criterion for almost sure stability for independent time intervals is specified. On the other hand a criterion for stability is derived for the case that the time intervals are determined by a narrow-band random process. For that case one can manage to specify a probability for the occurrence of the amplitude swellings.

G.Papanicolaou:

Asymptotic Analysis of Stochastic Equations and Applications

We consider some applications of asymptotic results on the equation

$$\frac{dx^{\epsilon}(t)}{dt} = \frac{1}{\epsilon} F(x^{\epsilon}(t), y^{\epsilon}(t)), \quad x^{\epsilon}(o)=x,$$

Where $\{y^{\epsilon}(t),\ t \geq o\}$ is a given process converging to white noise as $\epsilon \to o$. We describe the necessary assumptions on F and $y^{\epsilon}(t)$ for the validity of the asymptotics. We also consider, briefly, stability questions.

W.O.Schiehlen:

Random Vibrations of Periodically Time-Varying Systems with Jumping States

Linear dynamical systems with periodically time-varying coefficients and periodically jumping states are treated. The stability and the steady-state responses are investigated using Floquet's theory and Ljapunov's reducibility. In particular random vibrations, e.g. responses to stochastic disturbances, are considered. The covariance matrix can be found either by numerical integration of the Ljapunov matrix differential equation or by solution of the algebraic Stein matrix equation. As an example, random vibrations of a magnetically levitated vehicle on an flexible guideway are computed and some results are shown.

G. Schmidt:

Parametrically Excited Random Vibrations

The differential equation of vibrations with a random parametric excitation of white noise type, linear and nonlinear damping and quadratic and cubic restoring terms is investigated. Such an equation describes for instance parametrically excited vibrations of curved bars and shells. Applying the Itô calculus and the Fokker-Planck-Kolmogorov equation, a variant of an iterative method of Stratonovich is used which is based as the method of integro-differential equations in deterministic and narrow-band random vibrations, on several independent small parameters. In the stationary as well as in the instationary case, probability densities of the amplitude are found by means of the function of the parabolic cylinder and the Whittaker functions. The results are compared with corresponding ones in the case of narrow-band random parametric excitation.

E.Vanmarcke:

Earthquake Response Prediction via Random Vibration Theory

Recent advances in random vibrations methodology to predict system response and performance during transient excitations such as earthquakes are reviewed. The starting point is a "first-order" description of the frequency content of a stationary random process in terms of several spectral parameters which depend on the first few moments of the spectral density function. One of these parameters is a dimensionsless measure of spectral bandwidth. Most important performance measures of a random motion (such as maximum values) are shown to depend almost solely on these spectral parameters. Examples of systems considered are linear, viscously-damped multi-degree-of-freedom structures and structure-equipment systems. A major advantage of the proposed method of solution is that it can easily be extended to nonstationary random processes whose frequency content can easily be described by evolutionary power spectra for which time-dependent spectral parameters may be computed. The results of practical applications of this analysis to earthquake response prediction are shown.

W.Wedig:

Random Vibrations of Multi-wheeled Vehicles and Continuous Systems - Some Applications of Itô's Integral

The investigation of randomly excited systems by means of stochastic Itô differential equations is based on the assumptions of independent initial state vectors and nonanticipating properties of dynamical systems. The entitled problems are examples, in which both assumptions are not satisfied.
To overcome this difficulty we make use of Itô's integral equations defined on the Wiener process respectively on the Wiener field, in order to investigate random vibrations of multi-wheeled vehicles respectively internal forces and deflections of statically loaded continuous systems.
Finally, a Wiener field process as a base model of stationary and homogeneous loading is introduced. In this dynamical case, the application of Itô's integral definition leads to integral equations which allow to determine covariance functions without any knowledge of the eigenfunctions of the continuous system.

J.L.Willems:

Stability of High Order Moments of Stochastic Systems

The conribution deals with the moment stability problem for stochastic systems described by the Itô differential equations of the type.

$$dx(t) = Ax(t)dt + \sum_i \sigma_i B_i x(t) dW_i(t)$$

where the processes W_i are normalized Wiener processes. In particular the dependence of the maximum allowable noise intensities σ_i on the order of the moment considered is discussed. Particular classes of systems are indicated for which the above mentioned dependence can be explicitly obtained. The question is discussed whether or not there exist noise intensities for which all moments are stable. A sufficient condition for this phenomenon to accour is given.
Finally some similar results are discussed for the coloured noise stochastic system

$$\dot{x}(t) = Ax(t) + \sum_i \sigma_i B_i x(t)\ f_i(t)$$

J.C.Willems:

Average Value Stability Criteria for Symmetric and Passive Systems

We will derive various average value stability criteria for systems described by ordinary differential equations with (possibly randomly) time-varying coefficients. It will be shown how one may use symmetry and passivity properties in order to develop a representation theory for certain classes of linear dynamical systems. The stability criteria then exploit these special representations.

W. Wedig (Karlsruhe)

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