MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 14 / 1976

Mathematische Stochastik

28.3. bis 3.4.1976

Die Tagung stand unter der Leitung von P. Gaenssler (Bochum) und P. Révész (Budapest).

Der behandelte Themenkreis umfaßte mehrere Vorträge über neuere Entwicklungen der Theorie mehrdimensionaler empirischer Prozesse, über schwache und fast sichere Invarianzprinzipien und Gesetze vom iterierten Logarithmus für empirische Prozesse, sowie zahlreiche mehr anwendungsorientierte Vorträge über die Konstruktion stochastischer Modelle zur Beschreibung realer Situationen aus verschiedenartigen Bereichen der Naturwissenschaften. Ergänzt wurde dies durch Beiträge über neue Methoden der Wahrscheinlichkeitstheorie und Mathematischen Statistik.

Die regen Diskussionen zwischen den Vertretern der Wahrscheinlichkeitstheorie und jenen der Mathematischen Statistik, die auf dieser Tagung besonders intensiv geführt wurden, dokumentierten erneut die Notwendigkeit solcher gemeinsamer Tagungen über "Mathematische Stochastik".

Die deutschen Tagungsteilnehmer und die erfreulich zahlreich vertretenen ausländischen Gäste waren sich am Ende darin einig, fruchtbare wissenschaftliche Arbeit geleistet zu haben, die in vielfältiger Weise über die Tagung hinaus nachwirken wird. Hierzu hat nicht zuletzt die angenehme Atmosphäre des Mathematischen Forschungsinstituts Oberwolfach ganz entscheidend beigetragen.

Teilnehmer:

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H. von Weizsäcker, München
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R. Zielinski, Warschau
W.R. van Zwet, Leiden

Vortragsauszüge

O. BARNDORFF-NIELSEN: Distribution of sand particles

The mass-size distributions of eolian sand deposits laid down under steady wind conditions exhibit certain, highly striking regularities. The question of building statistical models for these phenomena was discussed. In particular, it was noted that observed distributions seem well fitted by a new, four parameter family of distributions with both tails asymptotically exponential.

M. CSÖRGŐ: The multivariate empirical process when parameters are estimated

In a recent paper (M. Csörgő - P. Révész, A strong approximation of the multivariate empirical process) we have shown that for d-dimensional i.i.d.r.v.'s $\underline{Y}_1, \underline{Y}_2, \ldots$ with a continuous distribution function satisfying a Lipschitz-type condition one can define a sequence of Brownian Bridges $\{B_n(\underline{x}), \underline{x} \in I^d\}$ and a Kiefer process $\{K(\underline{x},t), \underline{x} \in I^d, 0 \leq t < \infty\}$ such that

$$\sup_{\underline{y} \in R^d} |\sqrt{n}\,(F_n(\underline{y}) - F(\underline{y})) - B_n(T_d D_{\underline{y}})| \overset{a.s.}{=} O(n^{-\frac{1}{2(d+1)}} (\log n)^{3/2})$$

and

$$\sup_{\underline{y} \in R^d} |n(F_n(\underline{y}) - F(y)) - K(T_d D_{\underline{y}}, n)| \overset{a.s.}{=} O(n^{\frac{d+1}{2(d+2)}} \log^2 n),$$

where $F_n(\underline{y})$ is the empirical distribution function of the r.v.'s $\underline{Y}_1, \underline{Y}_2, \ldots, \underline{Y}_n$ and the region $T_d D_{\underline{y}}$ of the unit box I^d of R^d for any fixed $\underline{y}$ is $\{(a_1,a_2,\ldots,a_d): a_i \leq F_i(y_i|y_1,\ldots,y_{i-1}), i=1,2,\ldots,d\}$, with $F_1(y_1) = P\{Y_1 \leq y_1\}$, $F_i(y_i|y_1,\ldots,y_{i-1}) =$

$= P\{Y_i \leq y_i | Y_1 = y_1, \ldots, Y_{i-1} = y_{i-1}\}$.

The Lipschitz-type condition is imposed on the latter F_i and guarantees that $B_n(T_d D_{\underline{y}})$ and $K(T_d D_{\underline{y}}, n)$ are defined. Using the above results, a strong approximation of the multivariate empirical process is obtained, when unknown parameters are estimated via maximum likelihood methods. Also, under a given sequence of alternative hypotheses, a representation of the weak limit of the multivariate empirical process is obtained, when estimated parameters satisfy fairly general conditions.

L. DAVIES: The Hausdorff measure of the zero set of certain stationary processes

It is shown that the exact measure function for the Hausdorff measure of the zero set of the stationary Gaussian process X(t) with spectral density function

$$f(\lambda) = a \frac{2\alpha\Gamma(\alpha+1/2)}{\Gamma(1/2)\Gamma(\alpha)} (\lambda^2 + a^2)^{-(\alpha+1/2)}$$

is given by $\psi(h) = h^{1-\alpha} (\log (- \log h))^{\alpha}$.

H. DINGES: Sequentieller Vergleich von Behandlungsmethoden

Die folgende Situation wird diskutiert:
Zwei Methoden, eine Krankheit zu behandeln, sind vorgeschlagen worden. In einer Versuchsphase werden Patienten nach einem Plan mit der einen oder der anderen Methode behandelt; dann wird entschieden, welche Methode allgemein verwendet werden soll. Die Erfolge sind Zufallsgrößen, etwa

$$X_1, X_2, Y_1, X_3, Y_2, Y_3, Y_4, \ldots , X_{k+1}, X_{k+2}, X_{k+3}, \ldots X_N ,$$

wenn nach m Versuchen mit der ersten Methode (X) und n Versuchen mit der zweiten (Y) die Entscheidung für die erste gefallen ist (N = m+n).
Da die Verteilungen $\mathcal{L}(X)$, $\mathcal{L}(Y)$ unbekannt sind, ist ein überzeugender Plan schwer anzugeben. Die Anzahl der Versuchskaninchen, d.h. derjenigen Patienten, die mit der schlechteren Methode behandelt wurden, soll klein sein. Der folgende Weg, auf einen guten Plan zu kommen, wurde vorgeschlagen:

Betrachte unabhängige Brown'sche Bewegungen X_s und Y_t, wo

$$\mathcal{E}_{H_o}(X_s) = (c + \frac{a}{2}) \cdot s \ , \quad \mathcal{E}_{H_o}(Y_t) = (c - \frac{a}{2}) \cdot t$$

$$\mathcal{E}_{H_1}(X_s) = (c - \frac{a}{2}) \cdot s \ , \quad \mathcal{E}_{H_1}(Y_t) = (c + \frac{a}{2}) \cdot t$$

mit a bekannt (a > 0, d.h. "X ist besser"), c unbekannt. Beende die Versuchsphase, nachdem der X-Prozeß bis zur Zeit S beobachtet wurde, der Y-Prozeß bis zur Zeit T, wenn

$$Z(S,T) = \frac{T}{S+T} \cdot X_S - \frac{S}{S+T} Y_T$$

zum ersten Male größer wird als eine Konstante w. (Die Wahrscheinlichkeit, sich für die falsche Methode zu entscheiden, ist dann gleich $\frac{1}{1+e^{2aw}}$).

Steuere den Prozeß

$$(Z(\tau),\ S(\tau),\ T(\tau))$$

gemäß

$$(dS,dT) = d\tau(\frac{\lambda}{\alpha^2}\ ,\ \frac{1-\lambda}{(1-\alpha)^2}),\ \alpha = \frac{t}{s+t}\ ,\ \tau = \frac{s \cdot t}{s+t}.$$

Für eine Funktion $g^\lambda(z,s,t)$, die man interpretieren kann als die erwartete Zeit, die der schlechtere Prozeß noch beobachtet werden muß, wenn man schon in (z,s,t) angelangt ist, erhält man eine Gleichung

$$0 = \frac{1}{2}\ g_{oo} + a \cdot g_o^\lambda + \frac{\lambda}{\alpha^2}\ (e^{-2az} + g_1) + \frac{1-\lambda}{(1-\alpha)^2}\ (1 + g_2).$$

g_i bedeutet die Ableitung, λ beschreibt die Versuchsstrategie. Für das optimale λ hat man

$$0 = \frac{1}{2} \cdot g_{oo} + a \cdot g_o + \min\ (\frac{e^{-2az} + g_1}{\alpha^2},\ \frac{1+g_2}{(1-\alpha)^2}).$$

Computer-Rechnungen stehen noch aus. Man kann wohl hoffen, daß sich ein überzeugender Plan λ ausrechnen läßt.

R.M. DUDLEY: Chi-squared tests with estimation in a submanifold

For 2 x 2 contingency tables and a few other cases one can do exact tests with programmable pocket calculators.

For general χ^2 tests with estimated parameters (composite hypotheses), the results of Birch (Ann. Math. Statist. 35, 1964, 817-824) and Dzhaparidze and Nikulin (Theor. Prob. Appls. 19, 1974, 851-863) can be formulated for C^1 submanifolds in terms of a Riemann metric. So, instead of the rather difficult iteration needed to find a multinomial maximum likelihood estimator, one can use simpler ungrouped MLE's, as in testing for normality, and apply the Dzhaparidze-Nikulin correction, which is in effect only the first step of the iteration.

D. DUGUÉ: Analysis of variance in the large

Analysis of variance according to Sir R.A. Fisher is founded on the following equalities between tensorial products of matrices:

$$\bigotimes_{i=1}^{p} I_{n_i} = \bigotimes_{i=1}^{p} \left(I_{n_i} - \frac{1}{n_i} M_{n_i} + \frac{1}{n_i} M_{n_i}\right) = \sum_{j=1}^{2^p} Q_j ,$$

Q_j being the 2^p different matrices tensorial products of terms of the form $\frac{1}{n_j} M_{n_j}$ or $I_{n_j} - \frac{1}{n_j} M_{n_j}$.

I_{n_j} is the $n_j \times n_j$ unit matrix and M_{n_j} is the $n_j \times n_j$ matrix all terms of which are 1. This equality can be used with multivariate variables, with stochastic processes integrals and with von Mises Smirnoff integrals connected with independance tests. That needs new tables which are not yet calculated.

J. DURBIN: Kolmogorov-Smirnov tests when parameters are estimated

Suppose that $x_1 \leq \ldots \leq x_n$ is an ordered sample of independent observations from a distribution with distribution function F(x) and that we wish to test the null hypotheses H_o: $F(x) = F(x,\theta)$, where F is continuous in x and θ is a vector of unknown parameters. Let $\hat{\theta}$ be an asymptotically efficient estimator of θ, let $t_j = F(x_j,\hat{\theta})$ for $j = 1,\ldots,n$ and let $F_n(t)$ be the sample distribution function of the

t_j's, i.e. the proportion of the values $t_1,\ldots,t_n \leq t$ for $0 \leq t \leq 1$.

Let D_n^+ be the one-sided Kolmogorov-Smirnov statistic

$$D_n^+ = \sup_{0\leq t\leq 1} [F_n(t) - t].$$

Then $\sqrt{n}D_n^+$ does not have the same limiting distribution as when θ is known. The following procedures for constructing valid tests are considered:

1. A quasi-reflection technique
2. The half-sample device
3. Calculation of the distribution function of D_n^+ for finite n.

C. GRILLENBERGER: Minimale Mengen mit vielen invarianten Maßen

Wir betrachten die Frage, in wie einfache kompakte dynamische Systeme (X,T) ein maßtheor. dyn. System $(\mathcal{X},T,m)$ eingebettet werden kann. Als einfach ("minimal") sehen wir ein kompaktes X mit Homomorphismus T an, wenn X keine kompakte, T-invariante echte Teilmenge besitzt. Ein minimales X ist strikt ergodisch, wenn nur ein (ergodisches) Wahrscheinlichkeitsmaß μ_X auf X existiert. Der Satz von Jewett-Krieger sagt: Zu jedem ergodischen $(\mathcal{X},T,m)$ gibt es ein isomorphes strikt ergodisches (X,T,μ_X). Hansel zeigt für jedes $(\mathcal{X},T,m)$ Isomorphie zu einem System (X,T,μ), so daß X vollständig in (disjunkte) strikt ergodische Teile zerfällt. Eine entgegengesetzte Art der Einbettung ist die folgende:

Zu aperiodischem $(\mathcal{X},T,m)$ mit einer Bedingung über die Endlichkeit der Entropie gibt es $\emptyset : (\mathcal{X},T,m) \to (X,T,\mu)$, so daß X minimal und: 1. Die Menge der ergodischen Maße auf X ist abgeschlossen; 2. Die Entropie hängt stetig vom Maß ab. 3. $\overline{\{\emptyset m' | m' << m\}}$ ist die Menge der invarianten Maße auf X. (2. und 3. drücken in gewissem Sinne aus, daß es auf X nicht viel mehr invariante Maße gibt als auf $\mathcal{X}$).

Der konstruktive Teil des Beweises liefert auch minimale Mengen mit weitgehend vorgegebener Struktur der invarianten Maße, z.B. endlich viele ergodische Maße mit gegebenen Entropien.

J. HOFFMANN-JØRGENSEN: Measures which agree on balls

Let E be a Banach space with norm $\|\cdot\|$, and μ and ν two probability measures on E, which agree on all sufficiently large balls. If E belongs to one of the following classes of spaces, then $\mu = \nu$:

(1) $\|\cdot\|$ is smooth (i.e. Gateaux differentiable except at 0)

(2) $E = L^p$ $(1 < p \leq \infty)$

(3) $E = L^1(m)$, m non-atomic and σ-finite

(4) $E = C(T)$, T topological space

(5) E is a real B-algebra (commutative), satisfying $\|x\|^2 \leq \|x^2+y^2\|$ for all $x,y \in E$

These results are corollaries to a theorem stating that $\hat{\mu}(X^*) = \hat{\nu}(X^*)$ for all $X^* \in \overline{\text{span}}\,(F)$, whenever F is a closed simplicial face of the set of normalized outwards pointing normals to a point on the unit sphere.

O. KALLENBERG: Some applications of Feller's dominated variation

Dominated variation of a monotone function f means boundedness from above or below of quotients of the form f(cx)/f(x) as x tends to zero or infinity. A systematic theory was developed by Feller in 1965-69, generalizing the Karamata theory for regularly varying functions. The aim of the present talk is to illustrate how a systematic use of the Karamata-Feller theory unifies and improves a variety of results in different areas of analysis and probability. The problems treated include Tauberian type theorems for Laplace transforms and characteristic functions, criteria for recurrence of symmetric random walks, the rate of decrease of concentration functions of convolution powers, the time to extinction of critical Galton-Watson processes, and the stability of critical spatially homogeneous cluster fields.

D.G. KENDALL: A Stochastic Automaton - "Making eyes for goldfish"

For some time the writer has been interested in reconstructing maps from fragmentary material (cf. Phil. Trans. Roy. Soc. London (A)

279 (1975) 547). Recent work in this programme has concentrated on (i) the use of strictly local data (e.g. actual adjacencies, rather than "Wilkinson distances"), and (ii) locally processed strictly local data. The solution to (i) appears to lie in a new proposed "Tertiary treatment of Ties" in the Shepard-Kruskal MD-SCAL algorithm. A first version of this, called CTR, has at my request kindly been built into his Cambridge version of MD-SCAL by Professor Robin Sibson. I have used this to reconstruct maps from strictly local data with gratifying success- and it can also be adapted to meet the needs of a new approach to Policy Selection devised by Professor Patrick Rivett. Something will be said about these and other applications of TTT. Problem (ii) arises in connexion with an important task in neurophysiology, pointed out to me by Mr. R.A. Hope and others at the National Institute for Medical Research at Mill Hill, London. Here I have preferrred to abandon MD-SCAL and to use instead a stochastic automaton which reacts to strictly local data and processes it in a local manner, ultimately to yield a form of map-reconstruction which, if the process can be perfected, will represent an important conceptual step forward in the physiolcgical context (the automaton is NOT intended to model in detail what actually happens in (e.g.) the Goldfish. This last work has been developed in close collaboration with Hope and his colleagues, to whom the writer is much indepted.

G. KERSTING: A weak convergence theorem with application to the Robbins-Monro-Process

We consider the asymptotic distribution of a sequence of random variables $(X_n)_{n \in \mathbb{N}}$, given by the recursion

$$X_{b+1} = X_n (1 - a_n^2 d(X_n)) + a_n Y_n,$$

where (Y_n) is a sequence of independent identically distributed random variables, $d : R \to R$ is a positive continuous function, and (a_n) is a sequence of positive numbers, going to zero. Two applications to the Robbins-Monro-Process are discussed, in which the function d will not be constant. Here the asymptotic distribution is no longer normal.

J.F.C. KINGMAN: The structure of certain stochastic models in genetics

Some models for the mutation of selectively neutral alleles in a population of fixed size N assume that each individual can be described by a point in $\mathbb{R}^d$, that her daughters are independently displaced from her position as a result of random mutation, and that the numbers of daughters born to the mothers in a given generation have a symmetrical multinomial distribution. In this case, there is a statistical equilibrium for the relative positions of the N points of a given generation, and equations may be written down to determine the properties of this equilibrium. These simplify if $N \to \infty$ and the probability of mutation is of order N^{-1}. De Finetti's theorem may be applied to the resulting joint distributions, giving an integral representation of a type which can be interpreted in terms of convergence in distribution of the empirical distribution as $N \to \infty$.

P. MAJOR: Reconstructing the distribution from partial sums of samples

Let us observe an infinite sequence $z_1 = r_1+\varepsilon_1$, $z_2 = r_2+\varepsilon_2, \ldots$ where $r_1, r_2, \ldots$ are the partial sums of i.i.d.r.v.s with some unknown distribution F(x). All we know about the ε_k-s (the errors) is that they are bounded by a function f(k). We are interested in the question whether F(x) can be recognized by these observations. P. Bártfai proved that the answer is in the affirmative if $f(k) = o(\log k)$ and $\int \exp(tx)\, dF(x) < \infty$ with some $t > 0$. The case when the moment-generating function may not exist is different.
We can construct two sequences $r_1, r_2, \ldots$ and $s_1, s_2, \ldots$ of partial sums of i.i.d.r.v.s in such a way that

$$P(\sup_n |r_n - s_n| \leq 1) = 1.$$

This implies that the problem in general can be solved only in the trivial case when $f(k) = o(1)$. Some further investigations show that the class of distributions that cannot be recognized even with $f(k)=1$ contains a lot of non-pathological distributions.

P. MANDL: The law of the iterated logarithm in controlled Markov chains

Es werden steuerbare Markovsche Ketten mit abzählbar vielen Zuständen und mit kompakter Parametermenge betrachtet. Bedingungen für die Gültigkeit der Gesetze der großen Zahlen und des iterierten Logarithmus für ein additives Funktional des Pfades werden angegeben. Die Voraussetzungen bestehen aus gewissen Ketten Liapunovscher Bedingungen und aus Annahmen über das asymptotische Verhalten der Steuerung. Die Resultate wurden mit Hilfe von Martingalen abgeleitet.

U. MÜLLER-FUNK: Sequentielle Rangtests für das Einstichproben-Symmetrie-Problem

Seien $X_1, X_2, \ldots$ u.i.v. Zufallsvariable, auf Grund derer die Nullhypothese H_o: $F(x) + F(-x) \equiv 1$ (F stetig) gegenüber Klassen nichtparametrischer Alternativen überprüft werden soll. Asymptotische Überlegungen (d.h. im Sinne von Alternativen nahe der Hypothese) motivieren sequentielle Rangtests, die auf den üblichen Rangstatistiken basieren. Mit Resultaten u. Techniken von Sen u. Ghosh läßt sich eine Chernoff-Savage-Darstellung für die Prüfgröße beweisen, aus der sich ein geeigneter funktionaler zentraler Grenzwertsatz ergibt. Angewandt auf erste Austrittszeiten kann damit die Limesverteilung der OC -Funktionen bestimmt werden. Durch den Nachweis der gleichgr. Integrierbarkeit der erwarteten Stichprobenumfänge (ASN) erhält man für diese einen Limesausdruck. Beide Approximationen sind von der Form, die man üblicherweise für OC -Funktionen u. ASN unter Normalverteilungsannahmen erhält.

M. MÜRMANN: Poisson point processes with exclusion

Let E be a locally compact space with denumerable base and μ be a positive Radon measure on E. We want to define Poisson point processes with respect to μ and excluded configurations of the following form: To each $x \in E$ there is attached a Borel set $I(x) \subset E$ such that all elements in I(x) are forbidden by the occurence of x. Evidently we have to claim $x \in I(y) \Longleftrightarrow y \in I(x)$. Since in general the set of allowed configurations has probability 0 of the Poisson point process with respect to μ, we specify the processes by its local conditional distributions as in the case of Gibbs measures. Sufficient conditions for the existence and for the uniqueness can be derived. Examples are the distribution of hard balls in Euclidean space and the distribution of percolation clusters in the case of finite clusters with probability 1 the exclusion being given by overlapping clusters. This subcritical case is equivalent to the existence of the corresponding Poisson point process with exclusion. Applications of this fact in both directions and generalizations to interacting systems and dynamic clusters as defined by Sinai were briefly mentioned.

W. PHILIPP: A functional law of the iterated logarithm for empirical distribution functions of weakly dependent random variables

Let $\{\eta_k, k \geq 1\}$ be a sequence of random variables uniformly distributed over $[0,1]$ and let $F_N(t)$ be the empirical distribution function at stage N. Put

$$f_N(t) = N(F_N(t) - t)\,(N \log\log N)^{-\frac{1}{2}},\ 0 \leq t \leq 1,\ N \geq 3.$$

For strictly stationary sequences $\{\eta_k\}$ with η_k a function of random variables satisfying a strong mixing condition and for $\eta_k = n_k x \bmod 1$ where $\{n_k, k \geq 1\}$ is a lacunary sequence of real numbers a functional law of the iterated logarithm is proven: The sequence $\{f_N(t), N \geq 3\}$ is with probability 1 relatively compact in $D[0,1]$ and the set of its limits is the unit ball in the reproducing kernel Hilbert space associated with the covariance function of the appropriate Gaussian process.

R.D. REISS: Statistical procedures for quantiles in certain non-parametric models

For the one-sided testing problem $q(P) \leq r$ against $q(P) > r$, where $q(P)$

is the q-quantile of an arbitrary distribution P, a sign test is uniformly most powerful (see Lehmann (1959), Example 8, pages 92-93). Dealing with symmetric distributions only, nonparametric tests of a better performance than the sign test are well known for the special case of the median. Since the symmetry condition seems to be very stringent, models are investigated which are defined by certain differentiability conditions with respect to the densities. In these models the sequence of sign tests (for increasing sample sizes) is still asymptotically efficient but the deficiency of this sequence is quickly increasing to infinity.

In the case that, roughly speaking, the second derivatives of the given densities are uniformly bounded, estimators are constructed which are specially suited to the model. The relative deficiency of the sample q-quantile with respect to these estimators is also quickly increasing to infinity when the sample size increases. Using critical regions derived from suitable estimators we find one-sided tests which attain the order of the best obtainable power.

P. RÉVÉSZ: Three theorems on the multivariate empirical process

Let X_1, X_2,... be a sequence of independent r.v.'s uniformly distributed over the unit cube I^d of the d-dimensional Euclidean Space. Further let F_n be the empirical distribution function based on the sample $X_1, X_2, \ldots, X_n$ and let

$$\alpha_n(x) = n^{1/2}\,(F_n(x) - x_1 \cdot x_2 \cdot \ldots \cdot x_d) \quad (x = (x_1, x_2, \ldots, x_d) \in I^d)$$

be the empirical process. The properties of the stochastic set function $\alpha_n(A) = \int_A d\alpha_n$ are investigated when A runs over a class of Borel sets of I^d. Let $\mathcal{A}$ be the set of Borel sets of I^d having d-times differentiable boundaries. Then a large deviation theorem and a law of iterated logarithm are proved for $\sup_{A \in \mathcal{A}} \alpha_n(A)$. A strong invariance principle (uniform over $\mathcal{A}$) is also formulated.

K. SCHÜRGER: A class of interacting particles having internal states

The class of particles under consideration emerged from thinking about the cell cycle of biological particles which can also mutate (i.e. change their types). Hence let $S = \{1,...,s\}$ $(s \geq 1)$ and $K = \{1,...,k\}$ $(k \geq 1)$ denote sets of "states" and "types", respectively. The particles are located at the sides of Z^d - the d-dimensional square lattices. The time a particle of type i in state $j < s$ (an "(i,j)-particle") needs until the transition $j \to j+1$, is exponential with parameter $a_{ij} > 0$. An (i,j)-particle located at $x \in Z^d$ divides thereby giving rise to two (i',1)-particles with probability $d_{ii'}$, where $i' \in \{i-1,i,i+1\} \cap K$ and $d_{i,i-1}+d_{ii}+d_{i,i+1}$.
One of the two resulting particles stays at x, the other one chooses a side $y \in Z^d$ with $\|y-x\| = 1$ all possible choices being equiprobable. The interaction arises from the assumption that a particle already located at the chosen y is replaced by the second particle. There exists a Hunt process $\{\xi_t\}_{t \geq 0}$ having the above mentioned properties the states being "configurations" (this follows from a result of Holley, Liggett, 1972). Start with a finite nonvoid configuration ξ_o. Denote by $\tau_1(x)$ the first instant when $x \in R^d$ is occupied.
(x is said to be occupied if that $y \in Z^d$ for which $y_i-1/2 < x_i \leq y_i+1/2$, $1 \leq i \leq d$, is occupied). Main conjectures (suggested by a method of D. Richardson, 1973): There exists a norm $N(\cdot)$ on R^d equivalent to Euclidean norm such that if $\pi(\varepsilon,t)$ denotes the P_{ξ_o}-probability of the event $\{x|N(x) \leq (1-\varepsilon)t\} \subset \{x|\tau_1(x) \leq t\} \subset \{x|N(x) \leq (1+\varepsilon)t\}$, $\lim_{t\to\infty} \pi(\varepsilon,t) = 1$ for all $\varepsilon > 0$. This suggests: For some constant $\alpha > 0$ the number of particles in ξ_t is $\sim \alpha t^d$ P_{ξ_o}-a.s., $d \geq 1$.

V. STATULEVICIUS: Application of the method of semi-invariants to limit theorems and statistic of random processes

Let X_t, $t = \pm 0, \pm 1, \pm 2, ...$, be a random process and $\Gamma\{X_{t_1},...,X_{t_k}\}$ its correlation function of the k^{th} order, i.e. the simple semi-invariant of the random vector $(X_{t_1},...,X_{t_k})$.

1.) Estimators of $\Gamma\{X_{t_1},\ldots,X_{t_k}\}$ in terms of different regularity conditions of the process X_t are given.

2.) The asymptotic behavior of the multilinear forms

$$\xi_n = \sum_{1\leq t_2,\ldots,t_m\leq n} a(t_1,\ldots,t_m)\, X_{t_2}\ldots X_{t_m}$$ are investigated.

3.) Some applications for the asymptotic behavior of estimators $\hat{f}_k(\lambda)$ and $\hat{F}_k(\lambda)$ of the spectral densities $f_k(x)$ and spectral functions $F_k(\lambda)$, respectively, are given.

W. STOUT: A weak invariance principle with applications to domains of attraction

An elementary probabilistic argument is given which establishes a "weak invariance principle" which in turn implies the sufficiency of the classical assumptions associated with the weak convergence of normed sums to stable laws. The arguments, which uses quantile functions (the inverses of distribution functions) exploits the fact that two random variables $X = F^{-1}(U)$ and $Y = G^{-1}(U)$ are, in a useful sense, close together when F and G are, in a certain sense, close together. Here U denotes a uniform variable on (0,1). By-products of the research are two alternative characterizations for a random variable being in the domain of partial attraction to a normal law and some results concerning the study of domains of partial attraction.

W. STUTE: A necessary condition for the convergence of the isotrope discrepancy

Given a sequence $\{X_i\}_{i\in\mathbb{N}}$ of i.i.d. $\mathbb{R}^k$-valued random vectors with distribution μ, the isotrope discrepancy $D_n^\mu(\cdot)$ is defined by $D_n^\mu(\cdot) := \sup_{C\in\mathcal{C}_k} |\mu_n^\bullet(C)-\mu(C)|$, where $\mu_n^\bullet$ denotes the empirical distribution and the supremum is taken over the class $\mathcal{C}_k$ of all convex measurable subsets of $\mathbb{R}^k$. It is proved that

$$(+) \sup_{C \in \mathcal{C}_k} \mu_c(e(C)) = 0$$

whenever $D_n^\mu(\cdot) \to 0$ as $n \to \infty$ $\mathbb{P}$-a.s.,

where μ_c denotes the nonatomic part of μ and $e(C)$ consists of all extreme points of $C \in \mathcal{C}_k$. Furthermore (+) and "$D_n^\mu(\cdot) \to 0$ $\mathbb{P}$-a.s." turn out to be equivalent in the case $k = 1,2$.

D. SZÁSZ: Renewal theory and multicomponent reliability systems

The method of the first step, well-known in renewal theory, makes it possible to prove that a sequence of point processes tends to the Poisson process via checking a simple compactness condition and some conditions concerning - maybe conditional - expectations only. We show a geometric picture that helps us in checking the latter conditions in case of a fairly general reliability system with more components. It is shown that these conditions follow from some new renewal theorems, which can be called "uniform renewal theorems".

R. ZIELINSKI: Global stochastic approximation

Let $\mathfrak{X}$ be an abstract set, $\{Y_x, x \in \mathfrak{X}\}$ a family of real r.v.'s and $F(x) = EY_x$ the regression function. Let ξ_n, $n=1,2,\ldots,$ be $\mathfrak{X}$-valued r.v.'s: u_n, $n=1,2,\ldots,$ real r.v.'s uniformly distributed in $[0,1]$ and let $w: R^1 \to [0,1-\delta]$, $\delta > 0$, be a decreasing function. Let x_1 be a $\mathfrak{X}$-valued r.v. and

$$x_{n+1} = \begin{cases} x_n & \text{on } \{u_n \leq w(Y_{x_n})\} \cup \{u_n > w(Y_{x_n}), Y_{\xi_n} \leq Y_{x_n}\} \\ \xi_n & \text{otherwise}. \end{cases}$$

Under rather general conditions the process x_n converges to the global maximum of the regression function.

W.R. VAN ZWET: Does efficiency mean more than it does?

Suppose that $\mathcal{T}$ and $\mathcal{T}'$ are statistical procedures for the same problem, that $\mathcal{T}$ is optimal and that $\mathcal{T}'$ has asymptotic relative efficiency 1 with respect to $\mathcal{T}$. Let N denote sample size and d_N the deficiency of $\mathcal{T}'$ with respect to $\mathcal{T}$. Unless certain symmetries are present (as e.g. in the one-sample location problem) one would expect d_N to be of the order $N^{1/2}$. However, it is becoming increasingly clear from the work of Pfanzagl on parametric problems and of Bickel and van Zwet on nonparametric problems that d_N will typically be of order approximately 1 rather than $N^{1/2}$ in many cases of interest.

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