MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 31/76

Modulfunktionen in mehreren Variablen

18.7. bis 24.7. 1976

Auf der von den Herren M. Eichler (Basel) und H. Klingen (Freiburg) geleiteten Tagung wurden neue Resultate aus der Theorie der Modulformen und Funktionen in mehreren Variablen und dem damit zusammenhängenden Problemkreis behandelt.
Die Vorträge befassten sich unter anderem mit folgenden Themen:
Zetafunktionen, welche den Modulgruppen und den Darstellungen von SL_n und GL_n zugeordnet sind, Darstellungstheorie dieser Gruppen überhaupt, Basen der Scharen der Modul- bzw. Spitzenformen gegebenen Gewichts, Singularitäten und topologische Struktur von Modulmannigfaltigkeiten, Struktur von Körpern von Modulfunktionen, Kurven auf Hilbertschen Modulflächen und zahlentheoretische Anwendungen der geometrisch-algebraischen Sätze der Theorie.

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Vortragsauszüge

(In chronologischer Reihenfolge.)

I. I. PJATETSKI-SHAPIRO : Classification of bounded homogeneous domains.

The classification of bounded homogeneous domains is based on a one-to-one correspondence between the set of these domains and a special class of real Lie algebras.A detailed exposition is in the author's book "Automorphic functions and the geometry of classical domains." (Gordon & Breach,New York,London,Paris 1969).

H. COHEN : Modular forms in two variables in relation to Shimura's theorem on forms of half integral weight.

Let K be a real quadratic field of discriminant D , $\mathfrak{O}_K$ the ring of integers and δ the different.For any modular form

$$f = \sum_{n\geq 1} a(n)\cdot q^n \in S_k(\Gamma_0(N), \chi)$$,where k is an integer ≥ 3 ,set

$$E_f^K(z_1,z_2) = \sum_{\substack{\nu \gg 0 \\ \nu\in \delta^{-1}}} \exp(2i\pi(\nu z_1+\nu' z_2)) \cdot \sum_{\substack{d\,|\,(\nu\delta) \\ d\in \mathbb{N}^*}} d^{k-1}\chi(d)\left(\frac{4D}{d}\right) a\left(N_{K/\mathbb{Q}}\left(\frac{\nu\delta}{d}\right)\right) .$$

Theorem . E_f^K is a Hilbert modular form of weight k and character $\chi\circ N_{K/\mathbb{Q}}$ on a certain congruence subgroup of $SL_2(\mathfrak{O}_K)$ of level dividing $2N/(4,N,D)$.
The proof uses in an essential way the theorem of Shimura on forms of half integral weight and a combinatorial characterization of Hilbert modular forms using forms in one variable and normal derivatives.

H. SAITO : On lifting of automorphic forms.

Let F be a totally real algebraic number field which satisfies the following conditions : i) $F/\mathbb{Q}$ is a prime cyclic extension of degree l .ii) $[E:E_+] = 2^l$ (E = group of units of F and E_+ = totally positive units) and the class number of F equals one. iii) F is tamely ramified over $\mathbb{Q}$.
Let $\mathfrak{O}$ be the maximal order of F and Γ be the subgroup of $GL_2(\mathfrak{O})$ consisting of elements with totally positive determinants.
Let R_F be the Hecke ring over F.Then R_F acts on the space of Hilbert cusp forms $S_k(\Gamma)$.Fixing the embedding of F into R and a generator γ of $Gal(F/\mathbb{Q})$,we can define a linear operator T_γ of $S_k(\Gamma)$.Using this T_γ and the action of R_F,a subspace $SS_k(\Gamma)$ of $S_k(\Gamma)$ can be defined,which is stable under the action of R_F and can be seen as R_F-module.By the condition on F,the conductor of F/Q is a prime $q \equiv 1 \pmod{l}$,and there are l-1 characters χ_i (i= 1...l-1) of order l .We can make $R_{\mathbb{Q}}$ act on the spaces of cusp forms $S_k(SL_2(\mathbb{Z}))$ and $S_k(\Gamma_0(q), \chi_i)$.
On the other hand we can define a homomorphism $\lambda : R_F \longrightarrow R_{\mathbb{Q}}$, and by this λ,these spaces may be seen as R_F-modules.
By comparing the modules we obtain the
Theorem : If $k \geq 4$, then:

$$SS_k(\Gamma) \simeq S_k(SL_2(\mathbb{Z})) \oplus S \qquad \text{(as } R_F\text{-modules)}$$
$$\bigoplus_i S_k(\Gamma_0(q), \chi_i) \simeq S \oplus S$$

Similar results can be proved for the cases of higher levels. The theorem is a generalization and a refinement of Doi - Nagunama's result on lifting of automorphic forms for quadratic extensions $F/\mathbb{Q}$.

J. GROSCHE : Über Fundamentalgruppen von Quotientenräumen Hilbert-Siegelscher Modulgruppen.

Let K be a totally real number field of degree m, $\mathfrak{O}$ its ring of integers and $\Gamma(n,K,q)$ the principal congruence subgroup of level q in the Hilbert-Siegel modular group. Congruence subgroups are denoted by Δ. Let H_n be the Siegel half plane, and $\overline{H_n^m}$ the extension of H_n^m, such that $\overline{H_n^m}/\Delta$ is the compactification of H_n^m/Δ. The problem is to determine the fundamental groups $\pi(H_n^m/\Delta)$ and $\pi(\overline{H_n^m}/\Delta)$.

Theorem: Let Δ_f, resp. $\overline{\Delta_f}$ be the subgroups of Δ, generated by the elements of Δ, which have a fixed point (f.p.) on H_n^m, resp. $\overline{H_n^m}$. Then $\pi(H_n^m/\Delta) \simeq \Delta/\Delta_f$ and $\pi(\overline{H_n^m}/\Delta) \simeq \Delta/\overline{\Delta_f}$.

As an application of the theorem some fundamental groups (compact and noncompact cases) were computed.

K. SHIH : Conjugations of arithmetic automorphic function fields

A theorem due to Doi and Naganuma states that the conjugation of a Shimura curve is a Shimura curve. The talk dealt with the higher dimensional generalization of their result. The main result is the following : The conjugation of a Shimura canonical system of models is again a canonical system of models.

W. L. BAILY : Special arithmetic groups and Eisenstein series.

In seeking for "natural" arithmetic groups, we are led to consider special arithmetic subgroups of a simply-connected semisimple $\mathbb{Q}$-group G, which in the case of G being associated with a rational tube domain have a direct relation with Eisenstein series. Then these E-series are automorphic forms for the special arithmetic group Γ, generate the field of automorphic functions for a (possibly) larger discrete subgroup Γ^{*} of Hol(T) (T = the tube domain) normalizing Γ, and have Fourier expansions with rational Fourier coefficients. It may be conjectured on the basis of results of Siegel, Maaß, Karel et al., that the coefficients of one such E-series have bounded denominator.

The main result of the talk was that in many cases, including all these where $G_{\mathbb{R}}$ operates on a tube domain, there are only finitely many classes of special arithmetic groups with respect to $\mathbb{Q}$-rational outer automorphisms. It would be interesting to consider arithmetic and, possibly, moduli problems in connection with this set-up.

H. L. RESNIKOFF : Singular automorphic forms.

Denote the $\mathbb{C}$-linear space of Siegel's modular forms of rank s and weight w by (Γ_s,w), and let $\Phi:(\Gamma_s,w) \longrightarrow (\Gamma_{s-1},w)$ denote Siegel's Φ-operator. $f \in (\Gamma_s,w)$ is a cusp form if $f \in \operatorname{Ker}\Phi$. Let Y denote the open cone of positive $s\times s$ matrices and $\overline{Y}$ its closure. If $(\Gamma_s,w) \ni f(z) = \sum a(n)\exp(i\pi\,\mathrm{tr}(nz))$, then f is cusp $\Leftrightarrow$ $n \mapsto a(n)$ is supported on Y. There is a complementary concept: $f \in (\Gamma_s,w)$ is a singular form if $n \mapsto a(n)$ is supported on $\overline{Y}-Y$.

Theorem 1 : If $f \in (\Gamma_s,w)$, then f is singular $\Leftrightarrow$ $w \in \{\frac{r}{2} | 0\leq r<s, r\in\mathbb{Z}\}$.

It follows that $\Phi: (\Gamma_s,\frac{r}{2}) \longrightarrow (\Gamma_{s-1},\frac{r}{2})$ is injective if $s > r$; there are no cusp forms of "singular weights". Consequently

$\ldots(\Gamma_s,\frac{r}{2}) \subset (\Gamma_{s-1},\frac{r}{2}) \subset \ldots \subset (\Gamma_r,\frac{r}{2})$ is a descending sequence of finite dimensional vector spaces. Hence

$$\exists\, s_o(r) : s > s_o \Rightarrow (\Gamma_s,\tfrac{r}{2}) \simeq (\Gamma_{s_o(r)},\tfrac{r}{2}) .$$

Theorem 2 : If $s > r$, then a basis for $(\Gamma_s,\frac{r}{2})$ is given by the theta functions $\Theta(z,S) = \sum_{N\in\mathbb{Z}^{s\times r}} \exp(i\pi\,\mathrm{tr}(zN'SN))$ where S runs through a complete set of representatives of the unimodular equivalence classes of positive symmetric even integral matrices of rank r and determinant one.

(Proved by the author for $s \geq 2r$; subsequently by E. Freitag for $s > r$)

Cor. $s > r$ and $8 \nmid r$ $\Rightarrow (\Gamma_s,\frac{r}{2}) = \{0\}$.

Cor. $s > r$ and $r = 8k \Rightarrow \dim_{\mathbb{C}}(\Gamma_s,4k)$ = class number of the quadratic forms whose matrices S are described in the theorem.

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E. FREITAG : Der Körper der Siegelschen Modulfunktionen.

Let $K \supset \mathbb{C}$ be an algebraic function field of transcendental degree N.A rational differential form of degree ν over K is an alternating expression

$$\omega = \sum f_{j},\ldots,f_{j}\cdot dg_{j}\wedge\ldots\wedge dg_{j} \; ; \quad f_{i}\;,\; g_{i} \in K$$

An irreducible algebraic variety X together with an isomorphism $K \xrightarrow{\sim} K(X)$ is called model of K.The form ω is called regular, if it is holomorphic on the regular locus X_{reg} of every model. With $g_{\nu}(K)$ we denote the dimension of the space of all regular ν-forms over K.

Theorem. Let $K(\Gamma_n)$ be the field of all Siegel modular functions. Then $g_{N-1}(K(\Gamma_n)) > 0$ if $n \equiv 1 \pmod 8$ and $n \neq 9$; we have put $N = \frac{1}{2}(n(n+1))$.

Cor. $K(\Gamma_n)$ is not generally rational.

The proof uses the construction of Γ_n-invariant holomorphic alternating (N-1)-forms on the Siegel half plane,which involves subdeterminants of the matrix

$$\delta = (\,\delta_{\nu\mu}\,)\;, \qquad \delta_{\nu\mu} = (e_{\nu\mu}\cdot \partial/\partial z_{\nu\mu}) \qquad e_{\nu\mu} = \begin{cases} 1 & \nu = \mu \\ \frac{1}{2} & \nu \neq \mu \end{cases} .$$

Such subdeterminants have been used by Gårding (Ann.math. 1947). By means of these differential operators one also obtains an new access to the theory of singular modular forms.All results given in the talk of H. Resnikoff can also be derived within the framework of this theory.

M. OZEKI : Basis problem for modular forms and related topics.

Let L be a positive definite even integral lattice.We use the symbol $[x_1\ldots x_n]$ to denote the matrix whose elements are given by (x_i,x_j),where (,) means the metric on L ($x_i \in L, 1\leq i,j \leq n$). The theta series attached to L is defined as

$$\Theta_n(z,L) = \sum_{x_1\ldots x_n \in L} \exp(i\pi tr([x_1\ldots x_n]z)) \qquad (z \in H_n) \quad .$$

Let Γ be a subgroup of finite index in Γ_n (Siegel modular group of degree n).We denote by $M_{r/2}(\Gamma,v)$ the space of modular forms with group Γ ,weight $\frac{r}{2}$ and multiplier system v.

Basis problem : Is $M_{r/2}(\Gamma,v)$ spanned by theta-series $\theta_n(z,L)$, where L runs through some suitable lattices ?

Theorem : $M_k(\Gamma_2)$ is spanned by theta-series $\theta_2(z,L)$, where the L are lattices of determinant one $\Leftrightarrow$ $k \equiv 0 \pmod 4$.

Related to this result is the problem of distinguishing theta-series attached to two different lattices of the same genus.

U. CHRISTIAN : Berechnung des Ranges der Schar der Spitzenformen zur Modulgruppe zweiten Grades und Stufe $q > 2$.

The talk gave an exposition of the author's paper mentioned above which was published in J. reine angew. Math. 277, 130 - 154 (1975) .

W. QUAAS : Berechnung eines Integrals, das bei der Bestimmung des Ranges der Schar der Siegelschen Modulformen auftritt.

The computation of the dimension of the space of all Siegel modular forms of degree n and level q can be reduced to the computation of the dimensions for the spaces of cusp forms of all degrees $\leq n$. Cusp forms satisfy an integral equation which has as kernel a Poincaré series. From this integral equation results an integral formula for the dimension of the space of cusp forms. The evaluation of this integral employs Christian's reduction theory for symplectic matrices. To use this theory, the principal congruence group is split up in disjunct classes according to the behaviour of the characteristic polynomial and a rank condition. The talk presented the evaluation of the integral for the class

$\sigma_2 = \{M \in \Gamma(3,q) \mid \chi(M) = (x-1)^6, \mathrm{rank}(M-E) = 1\}$, where $q \geq 3$.

J.-I. IGUSA : Higher degree forms and Eisenstein series.

In the theory of quadratic forms we talk about theta series, Eisenstein series, zeta functions, etc. associated with quadratic forms. Does there exist a similar theory if we replace quadratic forms by higher degree forms ? More specifically, does there exist a generalization of the "Siegel formula"? If we use adelic language (as in Weil's acta papers), then it is perfectly easy to associate the above-mentioned series and functions to any higher degree form ; and the first major difficulty came from the proof of the convergence of the new types of Eisenstein series.Concerning this problem we now have criteria for the validity of a certain Poisson formula (not only for the convergence);the Poisson formula becomes the Siegel formula if the given form is quadratic.The criteria permit us to reproduce known cases and to treat new cases, e. g.,the case of "Freudenthal quartics".Finally in the case where the given form is "strongly non-degenerate",then the Eisenstein series behaves similarly as the classical Eisenstein series (as the variable approaches rational boundary points); and the relation between this property and the Dickson-Artin conjecture is mentioned.

D. ZAGIER : Curves on Hilbert modular surfaces.

Let $X = H^2/SL_2(\mathfrak{O}_K)$ be the Hilbert modular surface associated to a real quadratic field K of discriminant D.A skew-hermitian matrix is $A \in M_2(\mathfrak{O}_K)$ such that ${}^tA = -A'$ (where $'$ denotes the conjugation of $K/\mathbb{Q}$).To A we associate the curve

$F(A) = \{(z_1,z_2) \in H^2 \mid {}^t\binom{z_2}{1} A \binom{z_1}{1} = 0\}$ .Since $F(nA) = F(A)$ $(n \in \mathbb{N})$ we can assume A primitive (i.e. $\frac{1}{n}A \notin M_2(\mathfrak{O}_K)$ for $n>1$).For an integer $N > 0$,the union of the curves $F(A)$ for all primitive skew-hermitian matrices A with $\det A = N$ is invariant under $SL_2(\mathfrak{O}_K)$ and so determines a (possibly reducible) curve F_N on X. The curve F_N is non-empty iff $N \equiv \lambda\lambda' \pmod D$ for some $\lambda \in \mathfrak{O}_K$, and is non-compact iff $N = \lambda\lambda'$ for some $\lambda \in K$ .Each component of F_N has the form H/Γ (Γ = units of an order in some quaternion algebra).Using results of Shimura one can determine the number of components (typically a power of 2).All intersections of F_N,F_M are transversal(in particular,F_N has at most multiple points as singularities.) and F_N and F_M can meet only if there is a $x \in \mathbb{Z}$ with $x^2 < 4NM$ and $x^2 \equiv 4NM \pmod D$.Finally,the way F_N passes through the resolutions of the cusps of X is described completely in terms of the representations of N by reduced binary quadratic forms of discriminant D .

One can use the F_N to determine the type of X in Kodaira's "rough classification" of regular algebraic surfaces into four types; X is of general type for all but 32 values of D. The curves F_1, F_2, F_3, F_4 and (if $D \equiv 0 \pmod 3$) F_9 are exceptional curves on Y(D) = minimal desingularisation of $X \cup \{\text{cusps}\}$.
Conjecture : The surface obtained from Y(D) by blowing down these curves (and the elliptic fixed-point resolutions they meet) is minimal.
Finally, one can study the intersection of the curves F_N (or rather T_N, the curve obtained by dropping the condition of primitivity in the definition of F_N). These intersection numbers are finite sums of class numbers of definite binary quadratic forms.
<u>Theorem :</u> For D prime and $\chi \in H_2(X)$, the function

$$\varphi_\chi(z) = -\frac{1}{4}c_1\chi + \sum_{N=1}^{\infty}(\chi\cdot T_N)\exp(2\pi iNz) \qquad (z \in H)$$

(c_1 = 1st Chern class of χ) is a modular form of weight 2 and character $\left(\frac{D}{\cdot}\right)$ on the group $\Gamma_0(D)$.
The map $\chi \longmapsto \varphi_\chi$ from $H_2(X)$ to $M_2(\Gamma_0(D),\left(\frac{D}{\cdot}\right))$ seems to be the dual map (with respect to Poincaré duality and the Peterson product) of the Doi-Naganuma lifting $S_2(\Gamma_0(D),\left(\frac{D}{\cdot}\right)) \rightarrow S_2(SL_2(\mathfrak{O}_K))$.

G. VAN DER GEER : <u>Classification of Hilbert modular surfaces associated to principal congruence subgroups and questions of relative minimality.</u>

Let $Y_\pm(\mathfrak{A})$ be the Hilbert modular surface associated to the principal congruence subgroup $\Gamma(\mathfrak{A})$. Such a surface can be classified according to the rough classification of Kodaira. If Y_+ (+ means product of two upper half planes) is not rational, one might ask for a minimal model. Consideration of the surfaces of special type leads to the following
<u>Conjecture :</u> If D (= discriminant of the quadratic field) ≥ 13, $N(\mathfrak{A}) \geq 5$, then $Y(\mathfrak{A})$ is minimal.
For a proof in many special cases, one first observes that every non-singular rational curve has at least three points in common with the curves coming from the resolutions. Further,

one can attach integers $b_r(\mathfrak{X})$ to a configuration of curves $\mathfrak{X}$ such that the m-th pluri-genus $P_m(X)$ is $\leq b_r(\mathfrak{X})$ if $\mathfrak{X}$ occurs on X. The assumption that $Y(\mathfrak{A})$ is not minimal leads to new configurations $\mathfrak{X}$ by blowing down, such that $P_g(Y) > b_r(\mathfrak{X})$, contradiction.

U. KARRAS : <u>On deformations of cusp singularities of Hilbert modular surfaces.</u>

An isolated singularity (X,p) is rigid if every flat family (X_t), $t\in B$, which contains X is locally trivial in the sense that the total space Y = X_t is locally a product $X\times B$. Equivalently, every sufficiently near deformation X_t of X is isomorphic to X. A singularity of this type must therefore be a "generic" member of any family which contains it.

<u>Theorem :</u> (Freitag-Kiehl) Cusp singularities of dimension ≥ 3 are rigid.

<u>Theorem:</u> Let (s,p) be a two-dimensional cusp singularity with minimal resolution $\pi : M \to S$. Let A denote the exceptional set $\pi^{-1}(p)$. Then the simple elliptic singularities of degree $d = -A\cdot A$ are local deformations of (S,p).

The proof uses a deformation of the resolution "smoothing" the singular points of A. There results a regular one-parameter family (M_t) of strictly pseudoconvex non-singular complex surfaces with $\dim_{\mathbb{C}} H^1(M_t, \Omega_{M_t})$ = const. such that M_0 is a minimal resolution of (S,p) and the exceptional sets A_t of M_t, $t\neq 0$, are non-singular elliptic curves with $A_t\cdot A_t = A\cdot A$. The family can be blown down to a deformation of (S,p).

The second part of the talk dealt with cusp singularities which are complete intersections. Their defining equations can be computed and description be given of their semi-universal deformations. It turns out that all local deformations of a cusp (S,p) being a complete intersection are determined by subgraphs of the "Dynkin diagram" associated to (S,p).

W. F. HAMMOND : <u>Chern numbers of modular surfaces.</u>

The author reported on a paper which will appear in the Journal of the London Mathematical society.

It concerns "Hirzebruch proportionality" for properly discon-

tinous groups acting on 2-dimensional bounded symmetric domains with finite quotient volume in the case where the action has at most isolated fixed points.
The author noted that Professor D. Mumford had informed him privately of his result of the same nature for neat arithmetic groups.

H. COHN : Use of formal series in describing cusp singularities.

It became clear several years ago that some form of number theory was being used to compute the formal series of the local ring of cusp singularities of Hilbert modular forms. In 1972 the author published a paper describing the number theoretic manipulations by "support polygons" and followed this by a paper computing some of the local rings in 1976. The present talk considers the possibility of a "support polyhedron" for modular functions in three variables. The contrast between two and three dimensions presents certain peculiarities.In two dimensions every support polygon decomposes into unimodular sections connected by incidence numbers which are essentially freely prescribed.In three dimensions there are not necessarily unimodular solid angles (it requires concave subdivisions to accomplish unimodularity).Even so, when the incidence numbers are prescribed,they must obey a monodromy relation for embedding.Yet not every periodic structure leads to a cusp singularity.Finally,the periodic structure in two dimensions is convex; the convexity makes for identities involving class number.The periodic structure in three dimensions is "flat" on the average.

L. J. GOLDSTEIN : Zeta functions and Eichler integrals.

Let $\Gamma(f)$ be the principal congruence subgroup of $SL_2(\mathbb{Z})$ of level f, $n = 2m > 0$, $p \in \mathbb{C}[z]$, $\deg(p) = n$; if $\gamma \in \Gamma(f)$, set $p^{\gamma}(z) = j(\gamma,z)^n p(\gamma z)$. Let be $h \in M_{n+2}(\Gamma(f))$ (automorphic forms of weight $n+2$ with respect to $\Gamma(f)$) and define

$$G_p(z) = \int_{z_0}^{z} (h(u)p(u))\,du \quad .$$

Further, let $H(z)$ be an Eichler integral associated to $h(z)$, and let $S(\gamma,z)$ be its period polynomial under the action of $\gamma \in \Gamma(f)$. Finally, set
$S(\gamma,z) = (1,z,\dots,z^n)S(\gamma)$, $p(z) = (1,z,\dots,z^n)P$ and let
$P_n \in SL_{n+1}(\mathbb{Z})$ be such that
$(u^n,vu^{n-1},\dots,v^{n-1}u,v^n)P_n{}^t(z^n,wz^{n-1},\dots,w^n) = (uv-zw)^n$.

Theorem. Assume that $p^\gamma = p$. Then

$$G_p(\gamma z) = G_p(z) + n!\,{}^tP\,P_n^{-1}\,S(\gamma) .$$

The theorem was applied to give an explicit formula for the value of the zeta function of a ray class $\mathfrak{x}$ mod $\mathfrak{f}$ at the integer 1-k (k=1,2...) in the case of a real quadratic field. The formula immediately yields the fact that this value is rational.

I. I. PJATETSKI-SHAPIRO : Zeta-functions for adèle groups.

The classical Eichler-Shimura theorem shows that Hasse-Weil zeta-functions of modular curves are essentially Jacquet-Langlands zeta-functions for GL_2 .
We can conjecture that Hasse-Weil zeta-functions for arbitrary algebraic varieties can be expressed by zeta-functions of cusp forms on adèle groups.
The talk was devoted to zeta-functions for GL_n , which will be the subject of a forthcoming paper of Jacquet, Shalika and the author.

M. E. NOVODVORSKY : Zeta-functions associated to automorphic forms.

The direct part of Hecke theory for split orthogonal groups over global fields was discussed. The definition of L-functions was based on Whittaker functions corresponding to the characters of some subgroups including non-unipotent ones; the uniqueness theorem for these Whittaker models provides Euler decompositions of the corresponding zeta-functions. Meromorphic continuations and functional equations are obtained with the help of Poisson summation similarly to the case of GL_n .In some cases Eisenstein series are necessary.The results (and the difficulties) of the theory are very similar to those of the "untitled preprint" of Jacquet and Shalika for GL_n .

M. L. KAREL : Functional equations of p-adic Whittaker functions.

The talk gave a report on joint work of the author and W. Casselman.
The Fourier coefficients of Eisenstein series on a rational tube domain have Euler product expansions,and we can consider the Euler factors as functions of the weight.The main result is the existence of a functional equation for each of these functions,the local singular series.
To prove this,we observe that the local singular series are essentially p-adic Whittaker functions W attached to spherical principal series representations $PS(\chi)$ induced from a certain maximal parabolic $\mathbb{Q}$-subgroup of a reductive $\mathbb{Q}$-group G.That is,W transforms under right translation by elements of the unipotent radical N of P according to a character on N.The crucial point is to show that,up to scalar multiple,there is only one operator which intertwines $PS(\chi)$ with the space of Whittaker functions.

I. SATAKE : Symmetric domains and Jordan triple systems.

Let $\mathcal{D}$ be a symmetric domain,$p \in \mathcal{D}$,and p_∞ a "cusp" of $\mathcal{D}$.
Put $\mathfrak{g} = \mathrm{Lie}(\mathrm{Hol}(\mathcal{D}))$, $\mathfrak{k}$ (resp. $\mathfrak{v}$) = Lie(stab. of p (resp. p_∞)).
Then there exist unique elements Z and X in $\mathfrak{k}$ (resp. $\mathfrak{p} = \mathfrak{k}^\perp$) such that the standard decomposition

$$\mathfrak{g}_{\mathbb{C}} = \mathfrak{p}_+ + \mathfrak{k}_{\mathbb{C}} + \mathfrak{p}_- \quad (\text{resp. } \mathfrak{v} = \mathfrak{g}_0 + V + U)$$

is the eigenspace decomposition with respect to $\mathrm{ad}Z$ (resp. $\mathrm{ad}X$) for eigenvalues $i,0,-i$ (resp. $0,1,2$).Let V_+ be the i-eigenspace of $2\mathrm{ad}Z_0$ in $V_{\mathbb{C}}$ where $Z_0 = (\mathfrak{g}_0\text{-part of } Z)$.Then,

$$\mathcal{V}' = U_{\mathbb{C}} + V_+ \text{ with } \{z,z',z''\} = -\tfrac{1}{2}[[z,\Theta z'],z''] \quad (\Theta = \text{Cartan involution})$$

is a positive definite hermitian Jordan triple system,and e = (U-part of $2Z$) is a principal idempotent element.Moreover,

U with $uu' = \{u,e,u'\}$ is a formally real Jordan algebra (corresponding to a self-dual cone Ω) and $H(v,v') = \{v,v',e\}$ is an Ω-hermitian map defining a Siegel domain expression of $\mathcal{D}$.
In a dual expression, $2R : u \longmapsto u\,\Box\, e|V_+$ $(z\,\Box\, z' : z'' \longmapsto \{z,z',z''\})$ is a Jordan algebra representation of U on V_+ satisfying

(*) $\tau(H(v,v'),u) = \tau(v,R(u)v')$ $(v,v' \in V_+, u \in U$, τ trace form of $\tilde{V}')$
(**) $R(Hv,v'))R(u)v = R(H(v,R(u)v'))v$.

Theorem. The categories of the following four kinds of objects with suitable morphisms are equivalent :

(a) $(\mathcal{D},p,p_\infty)$, (b) $(\mathfrak{g},Z,X)$,
(c) $(\tilde{V},e)$, (d) $(U,2R)$.

(this is essentially due to Loos.)

Remark 1 A Siegel domain defined by (U,V,Ω,H) is symmetric iff
i) Ω is self-dual
ii) There exists a Jordan algebra representation 2R satisfying (*)
iii) (**) holds.
(This was obtained independently by Dorfmeister.)

Remark 2 Siegel domains satisfying only i),ii) can easily be classified.(cf. the author's paper in Nagoya Math. J.)
The above theorem can be defined to "over $\mathbb{Q}$",which gives a description of $\mathbb{Q}$-forms of $\mathfrak{g}$ with a $\mathbb{Q}$-rational cusp in terms of Jordan algebras over a totally real number field and their representations (cf. the author's paper to appear in Proc. of Takagi Sym. in Kyoto).

A. A. TERRAS : Non-analytic modular forms and applications to number theory.

We consider Eisenstein series for $GL_n(\mathfrak{O}_K)$,K any number field, but restrict ourselves to the simplest case,which is Epstein's zeta function.

I. Fourier expansions (involving K-Bessel functions) lead to a formula for hR (h=class number,R=regulator).
Ref. Acta Arit. (3 papers) and Durham L-function Symposium.
Notation : $[K:\mathbb{Q}]=m, x \longmapsto x^{(j)}$ conjugations of K, j=1..r real, j=r+1..r+r' complex, $e_j = \begin{cases} 1 & ,j=1...r \\ 2 & ,j=r+1...r+r' \end{cases}$, δ = different,
d = |discriminant|,w = #{roots of 1},$\zeta_K(s)$ Dedekind zeta function

$A = 2^{-r'}\pi^{-\frac{m}{2}}\sqrt{d}$, $\Lambda_K(s) = A^s\Gamma(\frac{s}{2})^r\Gamma(s)^{r'}\cdot\zeta_K(s)$, $\sigma_s(\mathfrak{a}) = \sum_{\mathfrak{b}|\mathfrak{a}} N\mathfrak{b}^s$,

$K_s(z) = \int_0^\infty \frac{1}{2}t^{s-1}\exp(-\frac{1}{2}z(t+\frac{1}{t}))\,dt$,$M_s(z) = K_s(z) + 2z\frac{d}{dz}K_s(z)$,

$T_s(u) = M_{\frac{e_1 s}{2}}(2\pi e_1|u^{(1)}|)\prod_{j=2}^{r+r'} K_{\frac{e_j s}{2}}(2\pi e_j|u^{(j)}|)$.

<u>Theorem.</u> $(2-s)\Lambda_K(s-1)+s\Lambda_K(s) = -2^{r+r'}d^{\frac{s-1}{2}}\sum_{0\neq u\in\delta^{-1}}|Nu|^{\frac{s-1}{2}}\sigma_{1-s}(u\delta)T_{s-1}(u)$.

<u>Cor.</u> K totally real $\Rightarrow$

$hR = 4(2\pi)^{-m}d\zeta_K(2)-2^{3-m}d^{\frac{1}{2}}\pi\sum_{0\neq u\in\delta^{-1}}|u^{(1)}|\sigma_{-1}(u\delta)\exp(-2\pi(|u^{(1)}|+\dots+|u^{(m)}|))$

The proof makes use of Fourier expansions of the Epstein zeta functions $Z_n^{\mathfrak{a}}(P,s)$.

II. Here K=$\mathbb{Q}$, $Z_n^{\mathbb{Z}}(P,s) = Z(P,s)$ (P positive def. n×n), $m_P = \min_{x\in\mathbb{Z}^n\setminus\{0\}} P[x]$,
$\Lambda_n(p,s) = \pi^{-s}\Gamma(s)Z_n(P,s)$,and considering an expansion of these functions involving incomplete gamma functions one obtains
<u>Theorem.</u> $u\in(0,1)$,$P>0$ with $|P|=1$, m_P or $m_{P^{-1}} \leq \frac{nu}{2\pi e}$ $\Rightarrow$
$Z_n(P,\frac{nu}{2}) > 0$ if n large (dependent on u),and then the Riemann hypothesis is false for $Z_n(p,s)$.
The theorem,together with an expression for $\Lambda_K(s)$ proved by Hecke (Werke p.198) leads to a new proof of a result on the minima of quadratic forms.

L.-CH. TSAO : <u>Saturated subgroups of modular groups.</u>

Let Γ be an arithmetic subgroup of the holomorphic automorphism group of a rational tube domain,$F(\Gamma)$ be the field of automorphic functions,and $E(\Gamma)$ be the subfield of $F(\Gamma)$ generated by Eisenstein series.For Siegel's upper half planes,Hermitian upper half planes and exceptional tube domains,one can determine the degree $[F(\Gamma):E(\Gamma)]$ for principal congruence subgroups Γ. The talk also answered (sometimes partially) the following more general question: In the set of all arithmetic subgroups of a given modular group (of one of the above three types of tube domains), partially ordered by inclusion, does there exist a cofinal family of saturated subgroups ? (An arithmetic subgroup Γ is saturated iff $E(\Gamma) = F(\Gamma)$.)

A. NOBS : Exceptional representations of $GL_2(\mathbb{Z}_2)$.

Let K be a non-archimedean locally-compact field with residue field F_q , of characteristic p , and $\bar{K}$ be a closure of K . Langlands conjectured that there exists a one-to-one correspondence between the representations of degree two of $Gal(\bar{K}/K)$ and the admissible representations of $GL_2(K)$. This conjecture was proved by Jacquet and Langlands (Springer Lecture Note 114) for $p \neq 2$. It is still a conjecture if $p = 2$. In the special case $K = \mathbb{Q}_2$, A. Weil (Exercises dyadiques) gave a complete description of all irreducible representations of degree 2 of $Gal(\bar{K}/K)$ $(K = \mathbb{Q}_2)$. (He has also described the exceptional representations, i.e., those representations which are not induced from a character of a subgroup of index 2 in $Gal(\bar{\mathbb{Q}}_2/\mathbb{Q}_2)$.) As an approach to the classification of the admissible representations of $GL_2(\mathbb{Q}_2)$, the author gave a complete description of all irreducible representations of $GL_2(\mathbb{Z}_2)$. If one decomposes the Weil representations attached to binary quadratic forms, one finds all but 58 of the primitive representations of $GL_2(\mathbb{Z}_2)$. These 58 exceptional representations were constructed by the use of tensor products.

T. YAMAZAKI : On the zeta function of a strongly non-degenerate form.

Let f be a homogeneous polynomial of degree m in n variables with coefficients in an algebraic number field. To any such form f , we can attach a zeta function by an adelic integral. If f is strongly non-degenerate, it follows from the Weil conjecture proved by Deligne that this integral converges absolutely in a half plane. Applying Igusa's Poisson formula the author showed that this zeta function satisfies a functional equation. He also determined the residues at $s = 0$ and $s = 1$.

P. GÉRARDIN : Local representations.

I. Let Θ be a größencharacter of a Galois extension E of degree n of a global field K ; then for each place v of K, Θ defines a character Θ_v of E_v^* where $E_v = E \otimes K_v$ .If each component E_w of E_v is tamely ramified,then Θ_v defines a representation of $GL_n(K_v)$ when $\Theta_v | E_w^*$ is in general position with respect to the Galois group.It is expected that the L function and the ϵ factor corresponding to that representation are those defined by Langlands relatively to Θ_v,and,if the hypothesis is satisfied for all v,that $\otimes \gamma_{\theta_v}$ occurs in the space of cusp-forms of $GL_n(K_{\mathbb{A}})$.

II. More generally,we can associate to each character Θ_v of a maximal torus T_v of a reductive group G_v over a local field K_v a representation γ_v^Θ of G_v when T_v splits in a tamely ramified extension of K_v and Θ_v satisfies certain regularity conditions.It is expected that $\otimes \gamma_v^\Theta$ occurs in the space of automorphic forms of $G(K_{\mathbb{A}})$ if Θ is a "regular" character of $T(K_{\mathbb{A}})/T(K)$.

R. BERNDT : Kählers Modulformen eines arithmetischen Körpers.

Let K be a field,finitely generated and of transcendence degree n over $\mathbb{Q}$,k the largest number field contained in K ,$[k:\mathbb{Q}] = g$, and $\mathfrak{O}$ the ring of integers of k.By means of the following notions we associate to K two functions $\Psi_K(s)$ and $\Phi_K(s)$:
Let S denote rank one valuation rings of K and Ω_S their rings of differentials,[Sd S] the image of Ω_S in $\Omega_K = \Omega_S \otimes K$.
We define $D(K) := \bigcap [Sd\ S]$,the intersection taken over all S of finite type over $\mathbb{Z}$ .This is a graduated module,$D_\mu(K)$ being a finitely generated $\mathbb{Z}$-module of rank gp_μ (p_μ = μ-genus of K).
Let V be a complete regular model of K of finite type over k and X an associated scheme.Then $X \otimes \operatorname{Spec} \mathbb{C} = R = \bigcup_{\nu=1}^{g} R_\nu$ with n-dimensional compact complex-analytic manifolds R_ν .
Any differential $\omega \in D(K)$ induces on R a holomorphic differential which is denoted by ω again.

I. For n=g=1 there exists an unique Riemann surface R of genus p_1 associated to K .Therefore we can define

$$\psi_K(s) = \sum_{j,\gamma} \frac{1}{\left|\int_\gamma \alpha_j\right|^s} \qquad \text{for} \quad \mathrm{Re}\ s > p+1 \ ,$$

where the summation is done over all inequivalent systems of retrosections ,and α_j $(j = 1...p_1)$ runs through any system of generators for $D_1(K)$.

This definition can be extended for g≥1 ,but not to the case n>1.

II. For n≥1 we can attach to K

$$\Phi_K(s) = \sum_{\alpha \in D_n(K)} \frac{1}{\left|\int_R \alpha \wedge \bar{\alpha}\right|^{\frac{s}{2}}} \qquad \text{for} \quad \mathrm{Re}\ s > p_n g$$

and a 2n differential

$$\sum_{\alpha \in D_n(K)} \frac{\alpha \wedge \bar{\alpha}}{\left|\int_R \alpha \wedge \bar{\alpha}\right|^{\frac{s}{2}+1}}$$

on R which in certain cases can be used to define a Kähler metric.

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