MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 35|1976

Universal Algebra

15. 8. bis 21. 8. 1976

The chairmen of this meeting were: W.Felscher (Tübingen), G.Grätzer (Winnipeg), and R.Wille (Darmstadt).

Participants

Baker, K.A. (Los Angeles)
Baldwin, J.T. (Chicago)
Banaschewski, B. (Hamilton)
Bennet, M.K. (Amherst)
Bruns, G. (Hamilton)
Burmeister, P. (Darmstadt)
Burris, S. (Waterloo)
Csâkâny, B. (Szeged)
Davey, B.A. (Bundoora)
Day, A. (Darmstadt-Thunder Bay)
Draškovičová, H. (Bratislava)
Evans, T. (Atlanta)
Felscher, W. (Tübingen)
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Ganter, B. (Darmstadt)
Gaskill, H.S. (St. John's)
Grätzer, G.A. (Winnipeg)
Gumm, H.P. (Darmstadt)
Haley, D. (Mannheim)
Hedrlin, Z. (Prag)
Heidema, J. (Johannesburg)
Herrmann, C. (Darmstadt)
Huhn, A. (Szeged)
Hule, H. (Brasilia)
Jezek, J. (Prag)
Jonsson, B. (Nashville)
Kaiser, H.K. (Wien)
Kalmbach, G. (Ulm)
Keimel, K. (Darmstadt)
Kelly, D. (Winnipeg)
Matthiessen, G. (Bremen)
Mitschke, A. (Darmstadt)
Monk, J.D. (Boulder)
Nelson, E. (Hamilton)
Nöbauer, W. (Wien)
Pixley, A.F. (Claremont)
Poguntke, W. (Darmstadt)
Quackenbush, R.B. (Winnipeg)
Rival, I. (Calgary)
Sands, B. (Winnipeg)
Schmidt, E.T. (Budapest)
Scott, D. (Oxford)
Shevrin, L.N. (Sverdlovsk)
Sichler, J. (Winnipeg)
Smirnov, D.M. (Novosibirsk)
Taimanow A. (Novosibirsk)
Urquhart A. (Mississauga)
Wenzel, G.H. (Mannheim)
Werner, H. (Darmstadt)
Wille, R. (Darmstadt)

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BAKER, K.A.: Recent results on finite equational bases for finite algebras (survey)

I. Recent example of finite algebras $\mathfrak{A}$ such that $\mathrm{Var}(\mathfrak{A})$ is not finitely definable:

1. A non-associative ring with 256 elements (Polin)
2. An algebra $\mathfrak{A} = (A, \vee)$, where A is a 4-element pseudo-ordered set (Park).

II. The congruence -distributive case:
Recent proofs of the theorem that $\mathrm{Var}(\mathfrak{A})$ is finitely definable if it is congruence-distributive and $\mathfrak{A}$ is finite. Such proofs are due to Makkai, Taylor, and Jónsson.

III. In view of Polin's example, only one direction is evident: The conjecture of Jónsson that states that if a variety contains only finitely many subdirectly irreducible members, all finite, then it must be finitely definable.

BALDWIN, J.: Malcev Conditions and the Borel Hierarchy

The paper contains a new proof ofTaylor's theorem giving a semantic characterization of classes of varieties defined by Malcev conditions. We further show the existence of $2^{\aleph_0}$ distinct Malcev conditions (answering a question of Taylor which he has also answered). Finally, we show there is an $MC_{\sigma\Delta}$ class which is not $MC_{\sigma\delta}$ answering questions of Taylor and Neumann. This last result relates descriptive set theory and universal algebra. (The work is joint with Joel Berman).

BANASCHEWSKI, B: Obstacles to finitary duality

Recently, Fajtlowicz showed that, for any quasiequational classes $\mathcal{K}$ and $\mathcal{L}$ of finitary algebras, there does not exist a dual equivlaence $\mathcal{K}^* \simeq \mathcal{L}$, provided the epimorphisms in are exactly the onto homomorphisms, generalizing an earlier result of A. Robinson's that the category of abelian groups is not dually equivalent to itself.

We say that a category $\mathcal{K}$ has finitary dual iff $\mathcal{K}^*$ is equivalent to a hereditary and finitely productive category of finitary algebras. Below, containment is understood as: full subcategory, preserving limits and underlying sets.

Proposition. A concrete category $\mathcal{K}$ fails to have finitary dual whenever it contains any one of the following:

(1) a non-trivial quasiequational class of (not necessarily finitary) algebras;

(2) a prevariety of finitary algebras with a weak injective;

(3) a prevariety of finitary algebras containing an atomic compact algebra.

BENNET, M. K.: Lattices of Convex Sets

For L any lattice , we define D(L) = {x∈L: (y∨z)∧x = (y∧x)∨(z∧x) for all y,z in L}. If V is a vector space over an ordered division ring, C a convex subset of V and L the lattices of convex subsets of C, then we call L a convexity lattice. In this case D(L) is the lattice of extreme subsets of C. We give necessary and sufficient consitions for L to be a convexity lattice in the finite dimensional case, and use this result to obtain an extrinsic characterization of the face lattice of a convex polytope. We present some further results on $D(L)$ for lattices satisfying some of the properties of convexity lattices.

BRUNS, G.: Orthomodular lattices with finitely many blocks.

Theorem: Every finitely genrated orthomodular lattice with finitely many blocks is finite.

BURMEISTER, P. Notions of validity for equations in partial algebras

Let $\mathcal{P}_\Delta$ be the class of all partial algebras of some finitary type Δ, X a countable set of variables. For $A \in \mathcal{P}_\Delta$ let $\hat{A}$ denote its free completion, and for $h: X \to A$ let $\hat{h}: \hat{X} \to \hat{A}$, $\tilde{h}: \mathrm{dom}\,\tilde{h} (\subseteq \hat{X}) \to A$ be homomorphic extensions ($\tilde{h}$ maximal while $\mathrm{dom}\,\tilde{h}$ is X-generated). In his thesis (Darmstadt, 1975) R. John subsumes the known generalizations of equational theory to $\mathcal{P}_\Delta$ to:

Def.: A binary relation $\models \subseteq \{(A,h) \mid A \in \mathcal{P}_\Delta, h: X \to A\} \times (\hat{X} \times \hat{X})$ is a notion of validity for equations in partial algebras if (I), (RR) and (TR) hold:

(I) If $(A,h) \models (p,q)$ and $p,q \in \mathrm{dom}\,\tilde{h}$, then $\tilde{h}(p) = \tilde{h}(q)$.

(RR) $\models$ is R-representable, i.e. for every A there is $R^{\models}_{A} \subseteq \hat{A} \times \hat{A}$ such that $(A,h) \models (p,q)$ if and only if $(\hat{h}(p), \hat{h}(q)) \in R^{\models}_{A}$.

(TR) $\models$ is T-representable, i.e. for every $(p,q) \in \hat{X} \times \hat{X}$ there is a set $\underset{\sim}{T}^{\models}_{pq}$ of X-generated relative subalgebras of $\hat{X}$ such that $(A,h) \models (p,q)$ if and only if for all $T \in \underset{\sim}{T}^{\models}_{pq} : T \subseteq \mathrm{dom}\,\tilde{h} \Rightarrow \tilde{h}(p) = \tilde{h}(q)$.

The Galois-correspondences are described and applications are given.

BURRIS, S.: New foundations for sheaves over Boolean spaces.

Bounded Boolean powers are generalized in a natural way to "Boolean subdirect products" (which are equivalent to sheaf constructions over Boolean spaces), and we survey parts of a forthcoming paper (with H. Werner) "Sheaf constructions and their elementary properties" plus some results on bounded Boolean powers. In the joint paper we are primarily concerned with positive decidability results and the existence of model companions (the sheaf representations for discriminator varieties due to Bulman-Fleming, Keimel & Werner play a major role); and for the bounded Boolean powers the emphasis is on B-separating algebras.

CSÁKÁNY, B.: Malcev-type theorems on congruences and subalgebras.

A subset and an equivalence relation on the same set are said to be connected if the set is a class of the equivalence. Properties of algebras, based on the notion of connectedness of subalgebras and congruences are investigated. Especially, Malcev-type theorems are proven for the following properties:

1. No subalgebra can be a class of two different congruences.
2. Two distinct subalgebras cannot be classes of the same congruence.
3. No proper subalgebra can be a congruence class.

DAVEY, B.A.: Weak injectivity and Boolean Powers

For a set $\underset{\sim}{A}$ of algebras, a formula $\alpha(x,y)$ is a simplicity formula if it is a $\exists\forall$ conjunct of equations and for each

$A \in \mathfrak{A}$, $\{\Theta(a,b) \mid \alpha(a,b)\} = \{\Delta, \nabla\}$. THR$^{\underline{M}}$ (B.A. Davey, H. Werner). Let $\mathbb{K}$ be an equational class and let $\mathfrak{A}$ be a finite set of finite algebras in $\mathbb{K}$ and assume that $Si(\mathbb{K}) \subseteq S(\mathfrak{A})$. If there is a simplicity formula for $\mathfrak{A}$ and no finite product of members of $\mathfrak{A}$ has skew congruences, then T.F.A.E.:

(i) I is a (weak) injective in $\mathbb{K}$;

(ii) $I \cong A_o[B_o] \times \ldots \times A_n[B_n]$ where for all $j \leq n$, $A_j \in H(\mathfrak{A}) \cap Si(\mathbb{K})$, A_j is (weak) injective in $\mathbb{K}$, and B_j is a complete Boolean algebra.

This result can be applied to the case where $\mathfrak{A}$ consists of simple algebras (in particular, quasiprimals, planar squags, or planar sloops) or subdirectly irreducible lattices, p-algebras, double p-algebras, Heyting algebras, or Brouwerian algebras.

DAY, A.: A construction in lattice theory.

Let L be a lattice, $I = [u,v]$ a closed interval in L. We define $L[I] = (L \setminus I) \cup (I \times 2)$. $L[I]$ is a lattice with the order relation defined by: $x \leq y$ iff one of the following holds

(a) $x, y \in L \setminus I$ and $x \leq y$ in L

(b) $x = (p,i), y \in L \setminus I$ and $p \leq y$ in L

(c) $x \in L \setminus I$, $y = (q,j)$ and $x \leq q$ in L

(d) $x = (p,i)$, $y = (q,j)$ and $p \leq q$ in L and $i \leq j$ in 2

This construction can be used to prove several results in lattice theory

1) Every projective lattice satisfies (W)

2) Every free lattice satisfies (W)

3) Finitely generated free lattices are weakly atomic

(i.e. every proper quotient contains a prime quotient).

4) Every Mal'cev prevariety of lattices idempotent under Mal'cev multiplication contains all free lattices

5) The only Mal'cev idempotent varieties of lattices are $\mathcal{L}(x=y)$ and $\mathcal{L}$.

DRASKOVICOVA, H.: Some representation problems for lattices

B.M. Schein [Algebra Universalis 2/2 (1972), 177-178] proved that any lattice can be represented by orders of some set A and asked whether in the case of finite lattices a similar representation exists with A finite. B. Sivák characterized the class of finite lattices having this property. Another question is the following. Let L be a closed sublattice of the lattice of all equivalence relations on a set A, containing the identity and $A\times A$. Does there exist an algebra on A having L as the congruence lattice! The author gave a positive answer in the case L is completely distributive (or dually Brouwerian). A.F. Pixley [ibid. 179-196] proved: If L is a distributive lattices of pairwise permutable equivalence relations on a finite set A, then there is a function $f:A^3 \to A$, compatible with L and satisfying $f(x,x,y) = y = f(y,x,x) = f(y,x,y)$. I. Korec showed that an analogous assertion holds in the case A is countable. This yields a partial answer to a question of A.F. Pixley.

FRIED, E.: Connectednesses and Disconnectednesses in general.

The aim is to give a general frame for known theories like Radical-Semisimple Theory (Kurosh-Amitsur), Torsion-Torsionfree Theory (Dvinsky), Connectednesses-Disconnectednesses Theory for topological spaces (Archangelski-Wiegand), Connectednesses- Disconnectednesses Theory for graphs (Fried-Wiegand). The general theory was developped by Fried and Wiegand. To each element S of a given subcategory of all sets we assign a set $P(S)$, where P is a special covariant functor and we choose some special subset R of $P(S)$. We build up a category whose objects are of the form $A(S,R)(R \subseteq P(S))$. $Af:A(S_1,R_1) \to A(S_2,R_2)$ is a mapping in this category iff $f:S_1 \to S_2$ and $Pf(R_1) \subseteq R_2$. One can get an inner characterization of Connectednesses and Disconnectednesses in the sense written down in the paper of Fried-Wiegand similar to that given in this paper and there is a characterization of pairs in Torsion-Torsionfree sense. The general theory is applied to the ones listed above but it has other applications, too.

GANTER, B.: Combinatorial designs and algebras.

Results from the theory of block-designs and their generalizations can be used for the investigation and construction of (finite) algebras. Especially R.M Wilsons existence theorem has interesting and very general consequences. These methods are particulary helpful when the class of algebras under consideration is idempotent and can be described by axioms of the form

$$\Phi = \forall_{x,y} \exists_{z_1,\dots,z_n} \psi,$$

where ψ contains no quantifiers and Φ contains no free variables. Generalizations to other structures than algebras are possible.

GASKILL, H.: Distributive lattices which generate finite sublattices of free lattices.

The notion of sublattice of a free lattice generated by a finite distributive lattice is described. A complete characterization of those finite distributive lattices which generate finite sublattice of free lattices is given. Results are given which completely elucidate the structure of these distributive lattices.

GUMM, H.P.: Algebras in permutable varieties.

We use a geometric approach investigating algebras in permutable varieties, i.e. algebras having a polynomial p satisfying $p(xxy) = y$ and $p(xyy) = x$.
If $\mathfrak{A}$ is an algebra such that in the congruence lattice of $\mathfrak{A}^2$ the projection congruences have a common complement then in a natural way $\mathfrak{A}$ can be endowed with an abelian group structure $\mathcal{G}$, such that every fundamental operation on $\mathfrak{A}$ is a homomorphism with respect to $x-y+z$. This gives a characterization of abelian groups in a permutable variety, moreover we obtain:
Characterizations of simple algebras in permutable varieties, characterizations of hamiltonian loops (improving a result of Evans), characterizations of paraprimal algebras which are not quasiprimal and various characterizations of hamiltonian varieties of universal algebras.

GRÄTZER, G.: Congruence Schemes.

A congruence Scheme S is a finite sequence of polynomials $p_o, p_1, \ldots p_n$ and a function $f:\{0,\ldots,n-1\} \to \{0,1\}$. Write $S(a_o,a_1,b_o,b_1)$ for element a_o,a_1,b_o,b_1 of an algebra $\mathfrak{A}$ if $b_o = p_o(a_{f(0)},c_1,c_2,\ldots.)$, $p_i(a_{1-f(i)},c_1,c_2,\ldots) = p_{i+1}(a_{f(i)},c_1,c_2,\ldots)$, $p_n(a_{1-f(n)},c_1,c_2,\ldots) = b_1$ for suitable $c_1,c_2,\ldots \epsilon A$. We write $\mathfrak{A} \models S$ if $(c \equiv d\Theta(a,b) = S(a,b;c,d))$ for $a,b,c,d \epsilon A$. The lecture surveys three papers. The first one by E. Fried, G.Grätzer, and R.W. Quackenbush examines the consequences of assuming $\mathfrak{A} \models S$ for all $\mathfrak{A} \epsilon \underset{\sim}{K}$, where $\underset{\sim}{K}$ is an equational class. The second and third paper are by J. Bermann and G. Grätzer and they deal with constructions of equational classes $\underset{\sim}{K}$ such that $\mathfrak{A} \models S$ for all $\mathfrak{A} \epsilon \underset{\sim}{K}$. Sample result: let S contain no constant. Then there is such an equational class iff each p_i is at least binary. (All the results can be found in the Notices Amer. Math. Soc.)

HEDRLIN, Z.: Statistics in algebraic and relational structures.

A frequence table of a structure was defined and its connections with some statistical questions established.

Having a structure (with defined notion of subobject and isomorphism) we define: the frequency of an object O in the object P with respect to this structure is the number of subobjects of P isomorphic with O. This can be considered like a table.

Theorems about frequency tables of functions, equivalences and graphs were shown.

HEIDEMA, J.: Algebra $\vee$ Topology = Structure Theory
< Category Theory.

'Structure Theory" is the theory of sets with (finitary and infinitary) relations on them. By generalizing notions from algebra and topology to structures, rather than directly to categories, one can i.a.

(1) solve the problem that B.H. Neumann put as follows: ".....I know of no honest and simple way of defining homomorphisms and congruences on relational algebras so that they do what we want them to do", ("Special Topics in Algebra: Universal Algebra," Courant Inst., 1962). "What we want them to do" would for instance be to make the homomorphism theorem true in general (and not just for full homomorphisms);

(2) describe an explicit construction of the free structure (in a certain class, generated by ....), which in the case of algebras yields the free algebra, in the case of Tychonoff-spaces yields the Stone-Čech compactification, etc.;

(3) describe an explicit construction of the tensor product (of a set of structure in a certain class).

HERRMANN, C.: On congruence varieties of mudules.

Recent results of G. Czêdli and G. Hutchinson are presented. Let R be a ring with 1, L(R) the class of all lattices of submodules of R-modules. We outline a method which associates te each lattice identity α a system Σ_α of linear equations with integer coefficients such that α is valid in L(R) iff Σ_α is solvable over R. Conversely we associate with each Σ an identity χ_Σ such that χ_Σ is valid in a modular lattice, iff this is true for substitutions of elements forming a frame of suitable order.

Moreover, $\chi_{\Sigma\alpha}$ is equivalent to α. By use of this and the classical Frobenius theorem for systems of linear congruences we conclude, that the solvability of Σ in R is equivalent to a finite set of sentences $\delta_{p,k} \equiv \exists x(p^k \cdot 1 \cdot x = p^{k-1} \cdot x)$ with p a prime. Then, the congruence variety HL(R) is in two ways related to a map $f = f_R$, $f: P \cup \{\infty\} \to \mathbb{N} \cup \{\infty\}$: firstly HL(R) is defined by the $\delta_{p,f(p)}$ $(p \in P)$ and secondly it is generated by the lattices $L(\mathbb{Z}_p^n{}_{f(p)-1})$ $(p \in P, n < \infty)$ and $L(\mathbb{Q}^n{}_{\mathbb{Q}})$ $(n < \infty)$ in case that $f(\infty) = 1$. In particular, there are uncountable many congruence varieties of modules. If f_R is recursive then the validity of equations in L(R) can be decided.

HUHN, A.P.: <u>Some New Results Concerning n-distributive Lattices.</u>

The notion of an n-distributive lattice was introduced to characterize dimension-like properties of modular lattices. Recently it turned out that there are natural examples of n-distributive lattices even in the non-modular case. <u>Example:</u> The lattice all convex subsets of the n-dimensional Euclidean space is (n+1)-distributive, as well as dually (n+1)-distributive. The finite n-distributive lattices can be characterized as the subalgebra lattices of universal algebras of rank $\le n$ (a universal algebra A has rank $\le n$ if whenever an element $a \in A$ lies in the subalgebra $\langle a_\gamma \rangle_{\gamma \in \Gamma}$, then there are $\gamma_1, \gamma_2, \ldots, \gamma_n \in \Gamma$ such that $a \in \langle a_{\gamma i} \rangle_{i=1}^{n}$). n-distributivity has been applied also to characterize certain properties of graphs and matroids by L. Lovâsz and the author.

HULE, H.: Relations between the Amalgamation Property and systems <u>of algebraic equations.</u>

It is known that in a variety $\mathcal{V}$ the Amalgamation Property implies the following property concerning systems of algebraic equations: If A is a sublagebra of $B \in \mathcal{V}$ and if a system over A is solvable (in $\mathcal{V}$) then it is also solvable as a system of equations over B. We can show that also the converse is true.

Similary, a stronger form of the Amalgamation Property implies that a solvable system with at most one solution in any extension of A in $\mathcal{V}$ has a solution in A itself. This latter condition turns out to be equivalent to a "strong symmetric" form of the Amalgamation Property. The question remains open if this one implies the "strong" Amalgamation Property.

JEŽEK, J.: <u>Recent Results of Prague's Universal Algebraists.</u>

J. Ježek, T. Kepka and P. Němec were interested in the last year in the following questions:

1. <u>Distributive groupoids.</u> Some theorems on the structure of commutative distributive groupoids and distributive division groupoids were proved. The main problem in the theory of distributive groupoids is: Does every distributive groupoid satisfy $xy \cdot zx = xz \cdot yx$? This problem is yet open; in several special cases the positive answer was obtained.
2. <u>Varieties of quasigroups.</u> Varieties of medial quasigroups and quasigroup varieties defined by short balanced identities were studied.

3. EDZ-varieties.

4. Some properties of varieties of groupoids (mainly extensivity and epimorphisms onto).

JONSSON, B.: Lattice varieties covering the smallest non-modular variety.

There are sixteen varieties of lattices that are known to cover $\mathcal{N}$, the variety generated by the five-element non-modular lattice N. Fifteen of these are generated by finite subdirectly irreducible lattices $L_1, L_2, \ldots, L_{15}$, and the sixteenth is jointly generated by N and the diamond M_3. It is shown that every variety $\mathcal{V}$ of lattices that properly contains $\mathcal{N}$ includes one of the lattices $M_3, L_1, L_2, \ldots, L_{15}$. Of these sixteen lattices, the first six fail to be semidistributive, and it is shown that every variety $\mathcal{V}$ of lattices in which the semidistributive law fails contains one of these six. These two theorems generalize, respectively, and earlier result by I.Rival and a result by B.A. Davey, W. Poguntke and I. Rival, where $\mathcal{V}$ was assumed to be generated by a lattice in which every chainis finite.

KAISER, H.K. New results on complete universal algebras.

Polynomially complete algebras are algebras having the following property: Every function on A with values in A is a polynomial (=algebraic)function. One can characterize them in the following way: A finite universal algebra A is polynomially complete iff the following conditions hold:

(i) A is simple,

(ii) there is an element $0 \in A$ and $p, s \in P_2(A)$ such that

$p(x,x) = 0$ for all $x \in A$, $p(x,a) := p_a(x)$ is bijective for all $a \in A$ and $s(x,0) = s(0,x) = x$ for all $x \in A$.

(iii) There is an element $q \in P_n(A)$, $n>1$ such that q is not constant and takes the value 0 if any $n-1$ of its arguments are 0.

Applications of this theorem to the variety of groups, rings, loops, L-algebras, Boolean algebras and nearrings are discussed. A local version on this theorem has been given.

KEIMEL, K.: <u>A lemma on primes in continuous lattices.</u>

The following lemma is the starting point of this report on considerations due to G. Gierz and the speaker: Let L be a continuous lattice (in the sense of D. Scott). Let K be a subset of L which is compact in the Scott topology. If p is a prime element of L such that $\inf(K) \leq p$, then $x \leq p$ for some element x in K. We illustrate that this lemma can be viewed as a common background to results in very different areas as, for example, Jónsson's lemma, the "converse" of the theorem of Krein-Nilman on exteme points in compact convex sets, the theorem of Gelfand and Kolmogoroff that the closed prime ideals of the ring $C(X)$ of real valued continuous functions on a compact Hausdorff space are all of the form P_x, where P_x is the set of all f in $C(X)$ vanishing in the point x of X.

KELLY, D.: <u>Free products of lattices with infinitary operations.</u>

(A report on joint work with G. Grätzer and A. Hajnal). Let $\mathfrak{m}$ be an infinite regular cardinal. A lattice L is <u>$\mathfrak{m}$-complete</u> (or L is an <u>$\mathfrak{m}$-lattice</u>) iff for any nonempty set $S \subseteq L$ with with $|S| < \mathfrak{m}$, the join and meet exists in L. We prove a normal form theorem for any $\mathfrak{m}$-lattice $\mathfrak{m}$-generated by a subset which generalizes a result of B. Jónsson (Canad. J. Math. 14 (1962) 476-481). The structure theorem for the free $\mathfrak{m}$-product of $\mathfrak{m}$-lattices is similar to the finitary ($\mathfrak{m} = \aleph_0$) case. The free $\mathfrak{m}$-product preserves $(W_{\mathfrak{m}})$ and $(SD_v^{\mathfrak{n}})$ for $\mathfrak{n} < \mathfrak{m}$; the latter result uses the normal form theorem. Similary to the finitary case, the theory of $\mathcal{E}$-reduced free $\mathfrak{m}$-products is developed and applied similarly as in the finitary case. We use results of P. Erdös and R. Radó (J. London Math. Soc. 35 (1960), 85-9o) To determine the maximum size of chains in free $\mathfrak{m}$-lattices and free $\mathfrak{m}$-products.

MATTHIESSEN,G.: <u>Birkhoff Type Theorems of Heterogeneous Algebras.</u>

In the model theory of heterogeneous algebras not only the variables which occur in a formula (equation or quasi-equation) may play a role but also variables which don't occur. Thus we get the notions of equational classes and finitary equational classes, where in finitary equational classes we consider only finite (heterogeneous) variable sets. These notions are equivalent if and only if the set H of phyla is finite. The same holds for quasi-equational classes and finitary quasi-equational classes. For all these notions semantical descriptions are given.

MITSCHKE, A.: <u>Sums of directed systems of algebras.</u>

Sums of directed systems of algebras (without nullary operations and of finite type) have been investigated by J. Płonka in various papers. There is a connection between the decomposability of an algebra in a sum of a directed system and the existence of a binary function - called P-function - of the algebra that satisfies certain identities. This yields that the class $\Sigma(\mathfrak{K})$ of all sums of directed systems of an equational class $\mathfrak{K}$ is equational if $\mathfrak{K}$ is idempotent or has a binary term t satisfying $t(x,y) = x$. If one has a binary term t on $\mathfrak{K}$ satisfying $t(x,y) = t(x,x)$ - which is the only nontrivial remaining case - then $\Sigma(\mathfrak{K})$ need not be an equational class. Conditions are given which make $\Sigma(\mathfrak{K})$ equational by means of a generalized P-function and the free algebras of $\Sigma(\mathfrak{K})$ are described by finitely generated free algebras of a subclass of $\mathfrak{K}$. Examples are given for binary algebras.

MONK, J.D.: Some conjectures about Boolean Algebras.

We formulate 15 or so conjectures about B.A.'s and indicate some background on each. Examples:

1. If $\mathfrak{m}^{\aleph_0} = \mathfrak{m}$, then there are exactly $2^{\mathfrak{m}}$ isomorphism types of complete BA's of power $\mathfrak{m}$.
5. Any infinite complete BA $\mathfrak{A}$ has $2^{|A|}$ subalgebras.
7. $|$Ideals of $\mathfrak{A}|$ is a power of 2.
8. Every infinite complete BA $\mathfrak{A}$ has a free subalgebra of power $|A|$.
9. There is an uncountable BA $\mathfrak{A}$ such that every set of pairwise incomparable elements in $\mathfrak{A}$ is countable.
14. For every $\mathfrak{m} > \aleph_0$ there are exactly $2^{\mathfrak{m}}$ isomorphism types of rigid BA's of power $\mathfrak{m}$.

NELSON, E.: Power simplicity and filtered products of congruences.

An algebra A is power simple iff every congruence on a power of A is a filter congruence; this concept arises quite naturally in the study of certain algebras of continuous functions, as follows: For a topological space X and an algebra A, each filter in $\mathfrak{B}X$ (the Boolean algebra of open-closed subsets of X) induces a congruence on $C(X,A)$ (the algebra of all functions $X \to A$ continuous with respect to the discrete topology on A) consisting of all pairs (f,g) which coincide on a set in the filter. This produces a complete lattice embedding of the filter lattice of $\mathfrak{B}X$ into the congruence lattice of $C(X,A)$ which is in addition an isomorphism for all X iff A is power simple. Examples of power simple algebras are all fields, all quasiprimal algebras, and all finite simple algebras in a congruence distributive equational class.

For congruences Θ_i on A_i $(i \in I)$ and a filter $\mathfrak{f}$ on I, the filtered product $\Pi_{\mathfrak{f}}\Theta_i$ is the congruence on ΠA_i constisting of all $(f,g$ with $\{i \mid f(i)\Theta_i g(i)\} \in \mathfrak{f}$. In any congruence distributive equational class, if there is a finite bound on the cardinalities of the A_i then every congruence on ΠA_i is the intersection of filtered products of congruences on the A_i. On the other hand, if every congruence on every power of A is a filtered sum of congruences on A then A is power simple.

NÖBAUER, W.: Algebras of functions and polynomial functions.

Let A be a universal algebra and k a natural number. The full function algebra $F_k(A)$ on A is the set of all functions $f:A^k \to A$ together with the operations which are obtained from the operations of A in the usual way, and with the composition of functions. First there are given some results on the congruence lattice $\mathcal{L}$ of $F_k(A)$: If $k>1$, then $F_k(A)$ is always simple. If $k = 1$, then $F_k(A)$ for the most algebras which occur in nature, is either simple or has order 3. Then the concept of a "general subalgebra" of $F_k(A)$ is introduced. Such subalgebras either can be obtained by generating elements, or by properties of invariance. Some well known and some new examples of general subalgebras are listed and some results on these general subalgebras are mentioned.

QUACKENBUSH, R.W.: On Murskii's theorem.

A report on Murskii's theorem, that almost all finite algebras are quasiprimal, is given.

RIVAL, I.: Planar sublattices of free lattice.

A lattice L is semidistributive if, for every $a,b,c \in L$, $a \vee b = a \vee c$ implies $a \vee b = a \vee (b \wedge c)$ and $a \wedge b = a \wedge c$ implies $a \wedge b = a \wedge (b \vee c)$. Furthermore, L satisfies whitman's condition (W) if, for every $a,b,c,d \in L$, $a \wedge b \leq c \vee d$ implies either $a \leq c \vee d$ or $b \leq c \vee d$ or $a \wedge b \leq c$ or $a \wedge b \leq d$.

Theorem. A finite planar lattice is embeddable in a free lattice if and only if it is semidistributive and satisfies (W).

In addition, there is a list $\mathcal{L}$ of finite lattices subject to the following condition:

Theorem. A finite lattice is a planar sublattice of a free lattice if and only if it contains no sublattice isomorphic to a lattice in $\mathcal{L}$. Moreover, $\mathcal{L}$ is the minimum such list.

SANDS, B. Two Lattice Constructions

For lattices L and M, $\mathrm{Hom}(L,M)$ denotes the set of homomorphisms of L into M, with the pointwise partial order. A lattice L is catalytic if $\mathrm{Hom}(L,M)$ is a lattice for all lattices M.

Theorem (T.Kucera and B. Sands). Let L be a lattice.

i) If L is a retract of $CF(P)$ for some partially ordered set P, then L is catalytic.

ii) If L is catalytic then L is semidistributive and satisfies (W).

A construction used in this theorem can be modified to prove that if a lattice is sharply transferable it satisfies SD.

Secondly, we construct lattices L and M such that L is finite and satisfies (W), there is a bounded homomorphism f of M onto L, but there is no embedding g of L into M such that $gf = id_L$.

SHEVRIN, L.N.: On Lattices and Groupoids of Varieties.

The lecture gives a survey of results obtained in Sverdlovsk during the last two-three years by participiants of a seminar guided by the lecturer. A part of investigations in this seminar is devoted to the theory of varieties and prevarieties of some algebras (mainly semigroups, inverse semigroups, rings, lattices). The lecture concerns only two branches of this investigations which are mentioned in the title. The following themas are considered. On lattices of subvarieties:

1) The covering condition, 2) Irreducible bases of identities,
3) Some aspects of a structure of the lattice of subvarieties.

On the groupoid of subvarieties:

1) Identities on a product of two varieties,
2) The groupoid of varieties of idempotent semigroups,
3) Some subgroupoids of the groupoid of prevarieties of lattices.

Some open problems are raised.

SICHLER, J.: Endomorphisms of finite lattices.

Let E_n denote the class of monoids isomorphic to $End_{01}(L)$ for a lattice L of finite length n.

Proposition 1: (M. Adams, J. Sichler) $E_n \subsetneqq E_{n+1}$ for all integers $n \geq 1$

Proposition 2: (M. Adams, J. Sichler) There is a countable monoid M such that $M \cong End_{01}(L)$ implies that L has an infinite chain.

Theorem (M. Adams, J. Sichler) Every finite monoid is isomorphic to $End_{01}(L)$ of a finite lattice L.

<u>Proposition 3:</u> (M. Adams, J. Sichler) E_3 includes all left-cancellative monoids, in particular, E_3 includes all groups.

SMIRNOW, D.M.: <u>On regular varieties.</u>

Let V be a regular variety (cf. [1]) and let $F_r(V)$ denote the V-free algebra of rank $r \geq 1$. If $A = \mathrm{Aut}(F_1(V))$, then

$$\mathrm{Aut}(F_r(V)) \cong \mathrm{Sym}(r) \lambda A^r .$$

For every group G, having the positiv generating set of cardinality s, there exists a regular variety V of unary Ω-algebras such that

$$\mathrm{Aut}(F_1(V)) \cong G, \ |\Omega| = s, \ |F_1(V)| = |G| .$$

The dual of the lattice $\mathrm{Con}(P)$ of all congruence relations of every semigroup P is isomorphically embeddable in the the lattice of varieties of unary algebras of some similarity type.

References:

[1] B. Jónsson and E. Nelson, Relatively free products in regular varieties, Algebra univers., 1974, 4, N1, 14-19.

TAIMANOV, A.D.: <u>Über topologisierbare Algebren.</u>

Wenn $K = \{\mathfrak{A};\ \mathfrak{A}$ ist unendliche Algebra vom Typ $\sigma\}$ dann $\mathrm{Top}(K) \overset{df}{=} \{\mathfrak{A};\ \mathfrak{A} \in K;\ \mathfrak{A}$ läßt eine nicht-diskrete Hausdorffsche Topologie zu$\}$. Sei Kg die Klasse der unendlichen Gruppen

1. $\exists \mathfrak{A},\ \mathfrak{A} \in Kg \setminus \mathrm{Top}(Kg)$, card $\mathfrak{A} = \aleph_0$ (S. Shelah, 1976)
2. $\mathrm{Top}(Kg)$ ist abgeschlossen unter Ultraprodukten.
3. Card $Z(\mathfrak{A}) \geq \aleph_0 \Rightarrow \mathfrak{A} \in \mathrm{Top}(Kg)$
4. Wenn $Z(\mathfrak{A}) = \{e\} \& \forall X$ endlich $\leq \mathfrak{A}$ $Z(X) \neq \{e\}$ dann $\mathfrak{A} \in \mathrm{Top}(Kg)$.

5. Wenn Card $\mathfrak{A} = m > \aleph_0$ und $\exists B, \exists f$ [card $B < m$, $f: \mathfrak{A} \to$ Aut (B)],

   dann $\mathfrak{A} \in \mathrm{Top}(Kg)$.

   Folgerung: Für beliebige Algebren $\mathfrak{A}$ mit Card Aut $(\mathfrak{A}) >$ card $\mathfrak{A}$

   folgt Aut$(\mathfrak{A}) \in \mathrm{Top}(Kg)$

   (Taimanov 1975, Sib. Math. J. 1976)

Sei K_r die Klasse aller unendlichen Ringe

1. (Card $\mathfrak{A} = \aleph_0$) v ($\mathfrak{A}$ ist kommutativ und assoziativ)

   $\Rightarrow \mathfrak{A} \in \mathrm{Top}\ (Kr)$.

2. $\exists \mathfrak{A}\ (\mathfrak{A} \notin \mathrm{Top}(Kr)$ & card $\mathfrak{A} \geq 2^{2^{\aleph_0}}$

   (Arnautow W. 1973)

URQUHART, A.: <u>Topological representation of lattices.</u>

A topological representation theory is given which generalizes to arbitrary bounded lattices the corresponding theory for distributive lattices due to Stone and Priestley. The space of prime ideals is generalized to the space of pairs of filters and ideals, each of which is maximal in the family of filters (ideals) disjoint from the other. The space of a lattice is a compact space with two quasi-orders defined on it. The lattice itself is represented as a lattice of continuous partial functions into the two element discrete space. Many results of the classical representation theory generalize in a natural way. Duals of filters, ideals epimorphisms and congruences are discussed.

WERNER, H.: <u>Algebras having a ternary discriminator.</u>

A discriminator variety is a variety $\mathfrak{A}$ having a ternary polynomial such that $A \in \mathfrak{A}$ is subdirectly irreducible iff $A \models t(x,x,z) = z\ \&\ (x \neq y \to t(x,y,z) = x)$. Examples are

Boolean algebras, relatively complemented distributive lattices, n-valued Post algebras, n-valued Łukasiewicz - algebras, monadic algebras, cylindric algebras of dimension n, $x^n = x$-rings, biregular rings, Baer*-rings, strongly regular rings, complementary semigroups of rank n.

Theorem Each residually small discriminator variety has a decidable elementary theory and its countable members have a decidable theory with quantification over congruences.

This result is obtained by a sheaf representation method similar to the one developed by S. Comer to prove the decidability of the theory of a residually finite variety of monadic algebras.

WILLE, R.: Order Polynomial Complete Lattices.

A lattice L is called order polynomial complete if every order-preserving map of L into itself is an algebraic function of L.

Theorem: A lattice L is order polynomial complete if and only if L is finite and the identity and the constant zero are the only v-preserving maps $\delta : L \to L$ with $\delta x \leq x$ for all $x \in L$.

Corollary: An order polynomial complete lattice is simple.

Theorem: Let L be a finite lattice whose greatest element is the join of atoms. Then L is order polynomial complete if and only if L is simple.

Theorem: Let L be a finite modular lattice. Then L is order polynomial complete if and only if L is an irreducible projective geometry.

Problems

1. Nelson: Is the unbounded Boolean power $A[B]$ a subalgebra of a power of A ? (Solved negative by K. Keimel, Darmstadt)

2. Quackenbush: Let A be finite, Var(A) congruence permutable with definable principal congruences and only finitely many critical algebras. Is A finitely based ? (Conj.: Yes)

3. Quackenbush: Let $\mathcal{V}$ be a locally finite discriminator variety such that the lattice of subvarieties is $\omega+1$. When is $\mathrm{Th}(\mathcal{V})$ decidable ? (negative for $\mathcal{V}$ = monadic algebras, RUBIN)

4. Felscher: $\mathcal{R}$ class of commutative rings. Is there a recursive procedure to generate the operation of equational closure?

5. Felscher: Can one derive the axiom of choice from the statement that every complete Boolean algebra is injective ?

6. Burris: For Boolean algebras we have only $\aleph_0$-many types and decidable theory, how is the situation for $x^4 = x$-rings?

7. Burris: In the type (1,1) there are $2^{\aleph_0}$ equationally complete varieties e.g. $fx = gx = x$ or $fx = fy \;\&\; gx = x$ or $fx = x \;\&\; gx = gy$ or $fx = fy \;\&\; gx = gy$. Find some others, preferably finitely based ones.

8. Bacsich: Is there a decidable variety $\mathcal{V}$ such that every $A \in \mathcal{V}$ can be embedded into a simple $S \in \mathcal{V}$?

9. Sands: Which varieties $\mathcal{V}$ of algebras have the following property: "For all finite $A \in \mathcal{V}$, A is subdirectly irreducible iff $\forall B \in \mathcal{V}\ (A \hookrightarrow B^2 \Rightarrow A \hookrightarrow B)$" ? (Varieties of lattices have this property, are there others having finite members ?)

10. Davey: What can be said about endo-primal algebras (= all funktions preserving all endomorphisms are polynomials) ?

11. Davey: Is there a variety with filtral congruences which is not congruence-distributive ?

12. Werner: Let $A \in \Gamma^e(\mathfrak{A})$. Is there a first-order way to describe the fact that $A \simeq \prod_{i \in I} A_i[B_i]^*$ $(A_i \in \mathfrak{A})$? (positive if $\mathfrak{A}$ is a finite set of finite algebras).

13. Werner: For which classes $\mathfrak{A}$ of algebras does "$\forall A, B \in \mathfrak{A}$ $A \times B$ has no skew congruence" imply "$\forall A_1, \ldots, A_n \in \mathfrak{A}$ $A_1 \times \ldots \times A_n$ has no skew congruence" ?

14. Werner: If no finite product of algebras in $\mathfrak{A}$ has skew congruences, does is follow that $A_1^{I_1} \times \ldots \times A_n^{I_n}$ has no skew congruence ? (true for $A_1 \ldots A_n$ finite)

15. Werner: $\mathcal{V}$ discriminator variety. Can $A \in \mathcal{V}$ carry two different compact topologies ? (Answered in the negative by D. Haley, Mannheim).

16. Fried, Grätzer, Quackenbush: Let $\mathcal{V}$ be an equational class, S a congruence scheme. $\underline{A \models S}$ means that S defines principal congruences on A. Assume $A \models S$ for all $A \in \mathcal{V}$. Is $\mathcal{V}$ congruence modular ?

17. Berman, Grätzer: For congruence schemes S and T we can define $S \leq T$ in 3 ways:

$S \leq T \Leftrightarrow (S(a,b,c,d) \Rightarrow T(a,b,c,d))$

$S \leq T \Leftrightarrow (A \models S \Rightarrow A \models T)$ $\quad$ A algebra

$S \leq T \Leftrightarrow (\forall A \in \mathcal{V}\ A \models S \Longrightarrow \forall A \in \mathcal{V}\ A \models T)$ $\quad$ $\mathcal{V}$ variety

Characterize $\leq$ in these three senses.

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18. Bermann, Grätzer: Define the <u>rank</u> of a congruence scheme in the obvious way. For which congruence schemes S is there an equational class $\mathcal{V}$ such that $\forall A \in \mathcal{V}\ A \models S$ and $\forall A \in \mathcal{V}\ A \models T$ does not hold for any congruence scheme T of smaller rank.

19. Berman, Grätzer: Let S be a congruence scheme containing constants. When does $\forall A \in \mathcal{V}\ A \models S$ hold for some nontrivial variety $\mathcal{V}$?

2o. Berman, Grätzer: For a congruence scheme S, when is there an equational class $\mathcal{V}$ with operations that actually occur in S such that $\forall A \in \mathcal{V}\ A \models S$?

21. Shevrin: Has every $A \in \mathcal{V}$ a finite equational base ?
    a) $\mathcal{V}$ = inverse semigroups (type $\cdot, {}^{-1}$) (<u>Conj.:</u> No)
    b) $\mathcal{V}$ = Cliffordian semigroups (=union of groups) (<u>Conj.:</u> Yes)

22. Shevrin: A group G is <u>extremal</u> if it is the extension of an abelian group having minimal condition for subgroups by a finite group. Does the subgroup-lattice of G have finite dimension for extremal G ?.

23. Shevrin: What can be said about a semigroup S for which the subsemigroup-lattice $\mathcal{L}(S)$ satisfies some nontrivial identity ? In particular is an infinite cyclic semigroup such ?

24. Smirnov: Does the free lattice FL(3) have a finite basis for its quasi-identities (=universal Horn-theory) ?

25. Smirnov: Does every a) finite group, b) free group, c) finite lattice have an independent basis for its quasi-identities ?

26. Ježek: Does $x \cdot yz = xy \cdot xz$ & $xy \cdot z = xz \cdot yz$ imply $xy \cdot zx = xz \cdot yx$? (Answered positive by A. Mitschke, Darmstadt)

27. Ganter: Let $\mathfrak{A}_n^2$ denote the variety of the same type as groups which is defined by all 2-variable indentities that hold in abelian groups of exponent n. What is spec $\mathfrak{A}_n^2$?

28. Knoebel: Let $(A,\cdot)$ be a finite groupoid, $|A|\geq 3$, such that $\cdot$ is onto and depends on both arguments. Is there a unary operation u on A such that $\langle A,\cdot,u\rangle$ is primal ? For which groupoids is the answer yes, for which ones no ?

29. Kaiser: A is <u>k-affine complete</u> if each k-ary operation on A, which preserves Con(A), is a polynomial-function (=algebraic function) on A. Does 2-affine completeness imply k-affine completeness for every $k \in \mathbb{N}$?

3o. Kaiser: A is <u>k-locally complete</u> if each k-ary operation on A, which on each finite subset of A^k coincides with some polynomial function on A, is a polynomial function on A. Find all semigroups which are k-locally complete for every $k \in \mathbb{N}$.

31. Csakany: Characterize semigroups of all translations (=unary algebraic functions) of algebras having essential at least binary operations.

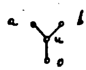
32. Haley: For an algebra A let $P_A(x)$ be the set of all positive formulas in one free variable x with constants from A. Let T_A be the topology on A determined by the solution sets of formulas in $P_A(x)$, taken as a basis for the closed sets. The compactness of T_A, which in general is only T_1-separating and not compatible with the structure of A, nonetheless characterizes A being equationally compact. If A is a compact topological algebra, are then the fundamental operations continuous with respect to T_A ? Or at least in the class of unital rings ?

3. Pixley: Let A be a quasi-primal algebra. We know that most important properties of the variety of Boolean algebras carry over to Var(A). For Boolean algebras we also have e.g. very simple equational bases and a normal representation theorem for algebraic functions. Can one find "canonical" equational bases for quasi-primal algebras? (see McKenzie, JSL <u>40</u>, 2, 1975)

4. Pixley: Is there a normal representation theorem for algebraic functions in members of Var(A), A quasi-primal? (solved for primal algebras by Foster)

5. Pixley: Can one find some sort of generalized complementation in the members of Var (A), A quasi-primal?

6. Nöbauer: For a finite algebra A there is a procedure to determine whether or not a function $f:A \to A$ is an algebraic function by listing all algebraic functions. Find a shorter procedure.

7. Fried, Padmanabhan, Park: On the poset 

we define

$$x\cdot y := \begin{cases} 0 \text{ if } & \{x,y\} = \{a,b\} \\ \min\{x,y\} & \text{otherwise} \end{cases}$$

a) Is $(\{a,b,u,0\},\cdot)$ finitely based ?

b) What is the number of essentially n-ary polynomials ?

8. Jónsson: Find a procedure for determining whether a finite structure (algebra) A is cancelable in the class of finite structures of the same type (i.e. $\forall B\, \forall C \quad A\times B \cong A\times C \Rightarrow B \cong C$) .

9. Jónsson: Is it true for every variety $\mathcal{V}$ of algebras that if the class $\mathcal{V}_{FSI}$ of all finitely subdirectly irreducible algebras of $\mathcal{V}$ is strictly elementary, then $\mathcal{V}$ is finitely based ?

<u>Note 1:</u> An algebra A is said to be finitely subdirectly irreducible if the intersection of two nontrivial congruences is always nontrivial.

Note 2: The answer is affirmative for congruence-distributive varieties $\mathcal{V}$. Without congruence-distributivity the following holds:
If $\mathcal{V}_{FSI}$ is strictly elementary and $\mathcal{V} \leq \mathcal{K}$, $\mathcal{K}$ and $\mathcal{K}_{FSI}$ strictly elementary, then $\mathcal{V}$ is finitely based.

4o. Jónsson: Find a construction Q such that

(1) For a finite set $\mathcal{K}$ of finite algebras $Q(\mathcal{K})$ is strictly elementary,

(2) If in addition $\mathcal{K}$ is contained in a congruence distributive variety then $Q(\mathcal{K})$ is a variety.

Illustration: A possible candidate was $Q = P_S HS$, but McKenzie showed that it violated (1).

41. Baldwin: Characterize $\aleph_1(\aleph_0)$-categorical structures in your favorite variety. Is there an $\aleph_0$-categorical projective plane?

42. Bennett: For a lattice L define the distributive center of L
$\mathcal{D}(L):=\{x \in L \mid (a \vee b) \wedge x = (a \wedge x) \vee (b \wedge x) \ \forall a,b \in L\}$.

a) Which properties of $\mathcal{D}(L)$ can be deduced from properties of L ?

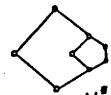
b) Which finite distributive lattices are distributive centers of compactly atomistic lattices ?

43. Rival, Sands: For a finite distributive lattice L let $L^* := \{L/\Theta \mid \Theta$ atom of $\mathrm{Con}(L)\}$. Note that L^* may contain two or more isomorphic lattices. Does $L^* "=" M^*$ imply $L \cong M$ for finite distributive lattices L,M?
An equivalent formulation is the following: is every finite partially ordered set characterized by the collection $\{P-\{x\} \mid x \in P\}$?

44. Herrmann: Does there exist an $n \geq 5$ such that there are uncountably many isomorphism types of subdirectly irreducible arguesian lattices with n generators ?

45. Day: For which finite modular lattices does there exist a finite lattice L satisfying Whitman's condition (W) and having M as a homomorphic image ?

46. Day: Let $\mathcal{V}$ be a variety which is not congruence modular.
   a) Does there exist an $A \in \mathcal{V}$ such that ConA contains a sublattice isomorphic to N_5^* ?



N_5^*

   b) Let L be a bounded lattice (in the sense of R. McKenzie) that is embeddable into ConA for some $A \in \mathcal{V}$. For a closed interval I of L, does there always exist a $B \in \mathcal{V}$ such that L[I] can be embedded into ConB?

47. E.T. Schmidt: Let L be a finite lattice, and let a/b be a prime quotient of L. Putting a bounded distributive lattice D between a and b, one gets a partial lattice L(aDb). A finite subdirectly irreducible modular lattice L has the property (*) if for each prime quotient a/b and each distributive D the free modular lattice generated by L(aDb) is a subdirect power of L. Is (*) equivalent to being a splitting modular lattice ?

48. Wille: Does 1-affine order completeness imply n-affine order completeness for lattices ?

49. Pixley, Wille: Is there a reasonable concept of order-quasi-primal lattices?

50. Fried: In the lattice of subvarieties of the variety of all weakly associative lattices, five covers of the variety of distributive lattices are known, namely besides the four trivial ones the variety generated by all w.a.l. having the unique bound property. Is there another cover generated by a finite algebra ?

51. Huhn: a) Is the variety of modular 2-distributive lattices generated by its finite members ?
   b) Is the variety of modular lattices generated by all its members which are n-distributive for some n ?

52. Monk: In the following fifteen conjectures, all Boolean algebras and cardinals are assumed to be infinite.

(1) Let $m^{\aleph_0} = m$. Then there are exactly 2^m isomorphism types of complete Boolean algebras of power m.

A Boolean algebra $\mathfrak{A}$ is called <u>weakly σ-complete</u> if for any $X, Y \subseteq \mathfrak{A}$ with $X \leq Y$, there is an $a \in \mathfrak{A}$ with $X \leq a \leq Y$.

(2) Let $m^{\aleph_0} = m$. Then there are exactly 2^m isomorphism types of weakly σ-complete Boolean algebras of power m.

(3) $\mathfrak{A}$ is weakly σ-complete if and only if $\mathfrak{A}$ is a homomorphic image of a complete Boolean algebra.

A set $X \subseteq \mathfrak{A}$ is <u>irredundant</u> if for any $x \in X$, x is not in the subalgebra generated by $X - \{x\}$.

(4) $\mathfrak{A}$ has an irredundant subset $X \subseteq \mathfrak{A}$ with $|X| = |\mathfrak{A}|$.

(5) $\mathfrak{A}$ has $2^{|\mathfrak{A}|}$ many subalgebras.

Let $n(\mathfrak{A})$ be the cardinality of the set of ideals of $\mathfrak{A}$.

(6) $n(\mathfrak{A})^{\aleph_0} = n(\mathfrak{A})$.

(7) $n(\mathfrak{A}) = 2^m$ for some m.

(8) Every complete Boolean algebra $\mathfrak{A}$ has a free subalgebra of power $|\mathfrak{A}|$.

(9) There is an uncountable Boolean algebra $\mathfrak{A}$ such that every family of pairwise incomparable elements in $\mathfrak{A}$ is countable.

(10) $\mathfrak{A}$ has a well-founded generating set.

For any Boolean algebra $\mathfrak{A}$, let $t\mathfrak{A}$ be the smallest cardinal m such that $\mathfrak{A}$ is the union of a strictly increasing sequence of subalgebras of type m.

(11) $t\mathfrak{A} = \aleph_0$ or $\aleph_1$.

(12) $t\mathfrak{A} = \aleph_1$ if and only if $\mathfrak{A}$ is weakly σ-complete.

(13) If $|\mathrm{Aut}(\mathfrak{A})| = \aleph_0$, then $|\mathfrak{A}| \geq 2^{\aleph_0}$

(14) If $m > \aleph_0$, then there are 2^m isomorphism types of rigid Boolean algebras of power m.

(15) If $m^{\aleph_0} = m$, then there are 2^m isomorphism types of rigid complete Boolean algebras of power m.

H.P.Gumm (Darmstadt)