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Non-archimedean Analysis

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This conference on non-archimedean analysis was a meeting of people who were all interested from some point of view in p-adic theory. So there have been talks on many different themes like non-archimedean function theory, functional analysis, the applications in number theory etc.. All talks have been done in English.

List of names

Amice, Y., Montrouge, F
Angermüller, G., Erlangen
Barsky, D., Paris, F
Bartenwerfer, W., Bochum
Bezivin, J.-P., Paris, F
Bosch, S., Münster
de Grande-de Kimpe, N., Brüssel, B
de Mathan, B., Talence, F
Dwork, B., Princeton, USA
Escassut, A., Talence, F
Fieseler, K., Münster
Fresnel, J., Talence, F
Frey, G., Saarbrücken
Gerritzen, L., Bochum
Güntzer, U., München
Haifawi, M., Ankara, T
Heinrich, E., Bochum
Herrlich, F., Bochum
Katz, N.M., Princeton, USA
Lütkebohmert, W., Münster
Martens, G., Erlangen
Mehlmann, F., Münster
Nastold, H.-J., Münster
Remmert, R., Münster
Robba, P., Paris, F
Schikhof, W., Nijmegen, NL
Schneider, Th., Freiburg
Springer, T.A., Utrecht, NL
van der Put, M., Groningen, NL
van Rooij, A.C.M., Nijmegen, NL

Reports of talks

DWORK, B.: Ordinary p-adic linear differential equations with analytic coefficients

(Joint work of Dwork and Robba)

Let Ω be an algebraically closed n.-a. complete valued field of characteristic 0 and residue characteristic p. For $\Delta = \{x \in \Omega, b \leq |x| < 1\}$ let W_Δ be the ring of bounded analytic functions on Δ provided with the sup-norm and W the inductive limit of the W_Δ as $b \to 1$, with the norm $|| = \sup_\Delta ||_\Delta$. Let $M \subset W$ be a subfield and $\tau : M \to W_t^1$ for some $t \in \Omega$ be an isomorphism into W_t^1 preserving differentiation and norm. (W_t^1= functions analytic and bounded in $D(t,1^-)$).

Then the extension of τ to field extensions of M in W is discussed. Furthermore comparison theorems are stated: Let $L \in M[\frac{d}{dx}]$ and L^τ be its image in $\tau(M)[\frac{d}{dx}]$.

i) The dimension of the kernel of L in the quotient field of W is bounded by the dimension of the kernel of L^τ in W_t^1.

ii) The dimension of the kernel of L in O_Δ (= ring of functions analytic in Δ, not necessarily bounded) is bounded by the dimension of the kernel of L^τ in O_t^1(= ring of functions analytic in $D(t,1^-)$).

At last the order of growth of the solutions of L is estimated. Example: M= K(X), K a subfield of Ω, $t \in \Omega$ such that $\tilde{t}$ is transcendental over $\tilde{K}$, and τ the restriction to $D(t,1^-)$.

BARSKY, D.: p-adic interpretation of Kummer's method

A new method is presented to obtain congruences between the Taylor's series coefficients of a certain class. Let p be a prime number. If a serie $\Sigma a_n \frac{X^n}{n!}$, $a_n \in C_p$ can be put in the form $\Sigma b_n (e^{cX}-1)^n$ for some constant $c \in C_p$, then $\Sigma a_n X^n = \Sigma \frac{b_n n! c^n X^n}{(1-cX),\ldots,(1-ncX)}$. If $\Sigma n! b_n T^n$ is an analytic element in Krasner's sense on the maximal ideal of C_p, then $\Sigma a_n c^{-n} X^n$ is also a p-adic analytic element in the same domain. This result is, by mean of the p-adic Mittag-Leffler Theorem, equivalent to congruences between the numbers $a_n c^{-n}$. Applications are made to Bernoulli numbers (theory of Kubota-Leopoldt), Bell numbers and to Bernoulli-Hurwitz numbers.

BARTENWERFER, W.: The first "metric" cohomology group of a smooth affinoid space

For every real number $\rho>0$ and every admissible affinoid covering $\mathfrak{U}$ of an affinoid space X $C_\rho(\mathfrak{U})$ is defined to be the complex of alternating cochains with values in the structure sheaf, which have spectral norm $<\rho$. Let $H^1_\rho(\mathfrak{U})$ be the first cohomology group of this complex and $H^1_\rho(X)$ the inductive limit of the $H^1_\rho(\mathfrak{U})$ where $\mathfrak{U}$ runs over all admissible coverings $\mathfrak{U}$ of X.

Two theorems are stated:

Theorem 1: For the unit polycylinder E^n one has: $H^1_\rho(E^n) = 0$ and more precisely $H^1_\rho(\mathfrak{U}) = 0$ for every rational covering $\mathfrak{U}$ of E^n.

Theorem 2: Let X be a smooth (= absolutely regular) affinoid space. Then there exists an element c in the base field,

$0<|c|\leq 1$, such that $c\cdot H^1_\rho(X)=0$ for all ρ.

For the proof of Th. 1 one uses essentially the existence of a projector, which for coverings $\mathfrak{U}$ of type 2^n will split up the sequence $0 \to T_n \to C^0(\mathfrak{U}) \to B^1(\mathfrak{U}) \to 0$ very well in a certain metric sense.

For Th. 2 then a result of Kiehl on projections for smooth spaces is needed.

LÜTKEBOHMERT, W.: Vectorbundles over non-archimedean holomorphic spaces

Let $(X=Sp(A), O_X)$ be a k-affinoid space in the sense of Tate, Kiehl etc., $E^n=\{(z_1,\ldots,z_n)\,|\,|z_i|\leq 1$ for all indices $i\}$ and $\partial E^n=\{(z_1,\ldots,z_n),|z_i|=1$ for at least one index i, $|z_i|\leq 1$ for all $i\}$.

Theorem 1: For every vectorbundle $\mathcal{F}$ on $X\times E^n\times\partial E^1$ there exists a rational covering $(U_1,\ldots,U_r)$ of X, such that for all i $\mathcal{F}|U_i\times E^n\times\partial E^1$ is the trivial bundle.

Corollary: Finitely generated projective (= f.g.p.) modules over the Tate algebra $T_n=k\langle X_1,\ldots,X_n\rangle$ are free.

More generally one can prove: Every f.g.p. module over the ring of Laurent series $L_{n,m}=T_n\langle Y_1,Y_1^{-1},\ldots,Y_m,Y_m^{-1}\rangle$ is free.

Remark: Let A be a regular complete local ring such that char (A) = char (A/m). Then every f.g.p. module over the polynomial ring $A[Z_1,\ldots,Z_n]$ is free.

In the following let the affinoid algebra A be without zero divisors. For an affinoid subdomain $U \subset X = Sp(A)$ let H_n be the Hartogs figure $H_n = (X\times\partial E^n)\cup(U\times E^n)\subset X\times E^n$.

Theorem 2: For every line bundle L on H_2 there exists a rational covering $(U_1,\ldots,U_r)$ of X, such that $\mathsf{L}_{|U_i\times\partial E^2}$ is trivial for all i.

Corollary: Let $M \subset H_n$ be a k-holomorphic set, $\dim_x M \geq \dim(X)+1$ for all $x \in M$. Then there exists a k-holomorphic set $\bar{M}\subset X\times E^n$ with $\bar{M}\cap H_n = M$ and $\dim_x \bar{M} \geq \dim(X)+1$ for all $x \in \bar{M}$.

Theorem 3: Let X= Sp(A) be smooth and the base field k algebraically closed. Then for every vectorbundle F on H_2 there exists a coherent sheaf $\bar{\mathsf{F}}$ on $X\times E^2$ with $\bar{\mathsf{F}}_{|H_2} \cong \mathsf{F}$.

For the proof of Th. 3 one needs:

Theorem 4: Let X be a Cohen-Macaulay affinoid space, $Y = \{(z_1,z_2) \in P_1\times P_1, |z_1^{-1}|\leq 1$ or $|z_2^{-1}|\leq 1\}$, $p: X\times Y \to X$ the projection. Then for every vectorbundle F on $X\times Y$ the direct image $p_*\mathsf{F}$ is a cohorent sheaf of O_X-modules.

ROBBA, P.: <u>Schwarz's lemma and approximation lemma</u>

Let K be a n.-a. valued complete field, locally compact, $q = \# \tilde{K}$. Let $\lambda>1$ such that $|K^*| = \langle\lambda\rangle$ and $\|a\| = \max\{|a_i|\}$ for $a \in K^d$. We assume that f is an analytic function in $B(0,R^+)$, $f = \Sigma a_n X^n$. For r<R let $|f|_r = \sup\{|a_n| r^{|n|}\}$, then we have $\sup\{|f(x)|, \|x\|\leq r\} \leq |f|_r$. Let $\Gamma\subset B(0,r^+)$, r<R, $r \in |K|$, be a finite set,

$h = \# \Gamma$ and $\delta = \inf\{\|\gamma-\gamma'\|, \gamma\neq\gamma', \gamma,\gamma'\in\Gamma\}$, s the integer such that $\frac{r}{\delta} = \lambda^{s-1}$. Necessarily we have $h\leq q^{sd}$; in the case of equality we say that Γ is well distributed (W.D.) in $B(0,r^+)$.

We know that $|f|_r \leq |f|_R$. We want to improve that estimate when it is known that f is zero in Γ (resp. small).

Theorem 1: (Schwarz's lemma) If f has multiplicity $\geq m(\gamma)$ in $\gamma \in \Gamma$, then $|f|_r \leq (\frac{r}{R})^N |f|_R$ with $N = \frac{1}{q^{(d-1)s}} \sum_{\gamma \in \Gamma} m(\gamma)$.

Theorem 2: Let $\varepsilon = \sup\{|D^n f(\gamma)|, |n| \leq k-1, \gamma \in \Gamma\}$, then $|f|_r \leq \max\{(\frac{r}{R})^N |f|_R, C\varepsilon(\frac{r}{\delta})^{N-1}\}$ with $N = \frac{hk}{q^{(d-1)s}}$, $C = \sup_{|n| \leq k-1} \{\frac{\delta^{|n|}}{n!}\}$ ($C = 1$ if $\delta \leq p^{-1/p-1}$, $p = \mathrm{char}\,(\tilde{K})$).

Theorem 3: Assume that Γ is W.D. and let ε be as in Th. 2. Then $|f|_r \leq \max\{(\frac{r}{R})^{hq^s} |f|_R, C\varepsilon\lambda^{\sigma}\}$ with $\sigma = (q^s - q/q-1)k - s - 1$ and $\sup_{\|x\| \leq r}\{|f(x)|\} \leq \max\{(\frac{r}{R})^{hq^s} |f|_R, C\varepsilon\}$.

The results are used to prove properties of p-adic transcendance, diophantine approximation etc..

KATZ, N. M.: Some applications of p-adic measures

Let K be a p-adic field, O_K its integers. Suppose given a 1-parameter formal group G over O_K of finite height h and a parameter X, such that the coordinate ring A(G) of G is $O_K[\![X]\!]$. Let D be the unique translation-invariant derivation of A(G) into A(G) with DX(0) = 1. Given a function $f \in A(G)$ one can form the sequence $c(n) := D^n f(0)$ of numbers in O_K.

Then in the cases h = 1, 2 estimates are given for the divisibility of c(n) as a function of n like $c(n) \equiv 0 \bmod p^{*(n)}$ and congruences between the various $c(n)/p^{*(n)}$ are proved for variable n.

The principal applications of this are to the p-adic interpolation of Bernoulli numbers and Hurwitz numbers.

DE MATHAN, B.: <u>p-adic Fourier analysis</u>

Let K be a complete extension of Q_p, which contains roots of unity of all orders, and G be an abelian compact totally disconnected group, $\tilde{G}$ the group of all continuous characters $G \to U$, U the group of roots of unity of K. Denote by $L^1(\tilde{G})$ the algebra of all functions $f:\tilde{G} \to K$, which tend to zero at infinity, with the convolution as product $f*g(\gamma)=\sum_{\delta\in\tilde{G}} f(\delta)g(\gamma\delta^{-1})$ and normed by $\|f\|= \sup|f(x)|$. Now let $\hat{f}$ be following continuous function on G: $\hat{f}(x)=\Sigma f(\gamma)\gamma(x)$; $F:L^1(\tilde{G}) \to C(G,K)$, $f \to \hat{f}$ is an algebra homomorphism. It is known by Schikhof that the maximal ideals of $L^1(\tilde{G})$ are the $\underline{m}_x:=\{f|\hat{f}(x)=0\}$, $x \in G$. So the kernel of F is the intersection of all maximal ideals and it is shown that this equals the closure of the nilradical. For the further study of F it is sufficient to look at pro-p-groups G. Such groups are always of the form $H \oplus Z_p^I$, where H is the smallest closed subgroup such that G/H has no elements of finite order other than 0.

Theorem: F is injective if and only if G= H. F is surjective if and only if H is finite.

FRESNEL, J.: <u>Topological tensor product of valued fields</u>

Let k,L,M be subfields of C_p,L and M linearly disjoint extensions of k. If $\widehat{LM}$ is the closure of the compositum LM in C_p, there is a conjecture: The canonical map f: $L\hat{\otimes}_k M \to \widehat{LM}$ is surjective and the quotient $L\hat{\otimes}_k M/\ker(f)$ isometrically isomorphic to $\widehat{LM}$. Now it is sufficient to consider the case, where L and M are algebraic over k.

Proposition: $L\hat{\otimes}_k M$ is a local ring with maximal ideal ker(f).

In the next let $K_\infty = \bigcup_n K_n, K_n = Q_p(\sqrt[p^n]{1})$.

Proposition: The tensornorm and the absolute value on $L\otimes_k K_\infty \tilde{\to} LK_\infty$ are equivalent if and only if the different $D_{L/K_1} \neq (0)$.

Theorem: In the case $D_{L/K_1} = (0)$ f is not injective but surjective. Moreover $L\hat{\otimes}_k K_\infty/\ker(f)$ is isometrically isomorphic to $\widehat{LK_\infty}$. This theorem is a consequence of the surjectivity of the Fourier transform in the case $G = Z_p$.

FREY, G.: <u>Some application of tori to number theory</u>

Let K be a p-adic field, E an elliptic curve defined over K with absolute invariant j and Hasse-invariant γ. If v(j)<0 and γ trivial then the theory of Tate implies that there is an element $q \in K^*$ with $j = \frac{1}{q} + \sum_{i=1}^{s} a_i q^i$, $a_i \in Z$, and with $E(K) \cong K^*/\langle q\rangle$, i.e. E can be viewed as an analytic torus.

Applications: Let K_o be a number field, then the Tate theory together with Nêron's reduction theory helps to get information about the torsion points of elliptic curves defined over K_o, so the following deep result could be proved: If $K_o = Q$, then $|E(K)_t| \leq 12$.

ii) As Fund (E) = Z we conclude: any elliptic curve over any field with complex multiplication has an absolutely algebraic invariant j, that is an integer with respect to all real n.-a. valuations of K_o.

iii) Using the isogeny theorem for E we describe the Galois group of the maximal unramified abelian extension of the function field F. Using function theory (Θ-functions) and the generalized Jacobian we describe moreover the maximal abelian extension of F unramified outside a finite set S of places of F in a very explicit manner; by going to the limit we get the Galois group of the maximal abelian extension of F.

This theory can be generalized to curves C of higher genus with split degenerated reduction in the sense of Mumford (i.e. the Jacobian of C is a torus again) by using techniques developped by Gerritzen, Mumford, Manin, Drinfeld.

ESCASSUT, A.: The ultrametric spectral theory

Let K be an algebraically closed complete ultrametric field and let A be a commutative unitary Banach algebra. In A procee-dings of holomorphic functional calculus are defined, using a class of Banach algebras H(D,P), which extends the class of Krasner's algebras.

The three principal seminorms $|\cdot|_{sa}, |\cdot|_s, |\cdot|_{si}$ are defined, they satisfy $|\cdot|_{sa} \leq |\cdot|_s \leq |\cdot|_{si}$. Then the properties $|\cdot|_{sa} = |\cdot|_s$ and $|\cdot|_s = |\cdot|_{si}$ are compared.

Let $x \in A$ and $s(x)$ be the spectrum of x. If the number of the infraconnected components of $s(x)$ is finite, one can define some idempotents u of A associated to everyone.

If K owns the strongly valued property, the maximal spectrum of A is in a one-one-correspondance with the set of the multiplicative seminorms of A whose kernel is a maximal ideal. Then one has harmonic synthesis results which can be compared to the complex analysis results.

GERRITZEN, L.: p-adic automorphic forms

Let Γ be a subgroup of $SL_2(k)$, k groundfield with a n.-a. complete valuation which is assumed to be algebraically closed. We consider Γ to be acting on $P_1(k) = k \cup \{\infty\}$. Γ is called a Schottky group if all elements of $\Gamma \neq id$ are hyperbolic.

Then there is an unbounded Stein-domain X of $P_1(k)$ such that the quotient space $S = X/_\Gamma$ is a compact analytic manifold and a projective curve the genus of which is the rank of Γ.

Theorem 1: Any meromorphic function f on X has a product decomposition

$$f(z) = const. z^r \cdot \prod_{i=1}^{\infty} \frac{z-a_i}{z-b_i}, \quad a_i, b_i \in k,$$

and where $\lim_i |a_i - b_i| = 0$.

Let $c \in G = Hom(\Gamma, k^*)$ and f a meromorphic function on X. f is called automorphic form of degree c, if $f(\gamma(z)) = c(\gamma) f(z)$ for all $\gamma \in \Gamma$. Let $\Theta(a,b;z) = \prod_{\gamma \in \Gamma} \frac{z-\gamma(a)}{z-\gamma(b)}$, where $a, b \in X$.

Then $\Theta(a,b;z)$ is an automorphic form of some degree.

Theorem 2: If f is an automorphic form, we have a decomposition $f(z) = \prod_{i=1}^{r} \Theta(a_i, b_i; z)$. Th. 2 allows to show that the Jacobian variety of S is an analytic torus: $J_S = G/L$ where L is some lattice in the algebraic torus G.

Theorem 3: Let N be the normalizer of Γ in $PGL_2(k)$. Then $N/_\Gamma$ is the automorphism group of S.

The proofs for these theorems can be given by elementary function theoretic methods. An example with N isomorphic to the classical modular group $SL_2(Z)/_{(\pm 1)}$ has been given.

BOSCH, S.: <u>On the reduction of rigid analytic spaces</u>

Let X be a (reduced) rigid analytic space (in the sense of Kiehl) over an algebraically closed field k and assume that X admits a formal covering $\mathfrak{U}$. Then dependent on $\mathfrak{U}$ one associates to X the "reduction" $\tilde{X}$ which is a scheme of locally finite type over the residue field $\tilde{k}$. For $x \in X$ denote by $X_+(x)$ the fibre at x with respect to the projection $X \to \tilde{X}, x \to \tilde{x}$.

Proposition: Let $X = Sp(A)$ be affinoid. Then the following is equivalent:

i) X is non singular in x.

ii) There exists functions $f_1, \ldots, f_d \in \underline{m}_x \cap \mathring{A}$, $d := \dim_x X$, such that the morphism $X \to B_d = Sp(T_d)$ which is defined by the homomorphism $T_d = k<X_1, \ldots, X_d> \to A, X_i \to f_i, i=1, \ldots, d$, induces an isomorphism $X_+(x) \xrightarrow{\sim} B_d^+(0)$.

Theorem: Let X be separated and quasi-compact. Then $\dim_k H^q(X,O_X) \leq \dim_{\tilde{k}} H^q(\tilde{X},O_{\tilde{X}})$ for all q.

Corollary 1: i) X is affinoid, if $\tilde{X}$ is affine.

ii) A formal morphism $\phi: X \to Y$ is finite if and only if $\tilde{\phi}: \tilde{X} \to \tilde{Y}$ is finite.

Corollary 2: $\left.\begin{array}{l} H^o(\tilde{X},O_{\tilde{X}}) = \tilde{k} \\ H^q(\tilde{X},O_{\tilde{X}}) = 0, q>2 \end{array}\right\} \Rightarrow \begin{array}{l} H^o(X,O_X) = k \\ H^q(X,O_X) = 0, q>2 \\ \dim_k H^1(X,O_X) \\ = \dim_{\tilde{k}} H^1(\tilde{X},O_{\tilde{X}}). \end{array}$

Cor. 1 is true without any special assumption on k and X,Y. Cor. 2 applies in particular to the case where X is a complete curve. In case $\tilde{X}$ is an elliptic curve, it follows that X is an elliptic curve with good reduction.

BEZIVIN, J.-P.: <u>Interpolation of bounded analytic functions</u>

Let K be a complete ultrametric algebraically closed field and D be the open unit disk. Let $B = \{\sum_{n=0}^{\infty} a_n X^n, \sup_n |a_n| < \infty\}$, the space of bounded analytic functions on D, x_n be a sequence of points in D, $x_n \neq x_m$ for $n \neq m$ and b_n some sequence of elements of K.

Proposition 1: There exists a function $f \in B$ with $f(x_n) = b_n$ if and only if the sequence $b_n^{(n)}$ is bounded:

$$b_n^{(n)} = \Sigma \frac{b_k}{\pi_{kn}} \quad \text{where } \pi_{kn} = \prod_{\substack{j \leq n \\ j \neq k}} (x_k - x_j).$$

As a corollary of this proposition one gets the results of van der Put on interpolation sequences. Furthermore, a question of van der Put is answered:

Proposition 2: There exists in B closed, not maximal, prime ideals, which are stable under differentiation.

HAIFAWI, M.: On Non-Standard aspects in certain n.-a. normed spaces

Let K be a n.-a. complete valued field and E a n.-a. normed space over K. Let $^*K$ and $^*E$ be enlargements of K and E respectively (in the sense of Robinson). E is considered as a subpace of $^*E$. Define $^*E_{fin} := \{x \in {}^*E, {}^*\|x\| \leq r$ for certain $r \in \mathbb{R}\}$ and $^*E_{inf} := \{x \in {}^*E, {}^*\|x\| < r$ for all $r \in \mathbb{R}\}$ and $^*\tilde{E} := {}^*E_{fin}/{}^*E_{inf}$. $^*\tilde{E}$ is a normed space with the usual inf. norm. Non-Standard proofs of the following theorems are given:

Theorem 1: (Hahn-Banach) E has the extension property if and only if E is spherically complete.

Theorem 2: $^*\tilde{E}$ is spherically complete.

Theorem 3: For every normed space E there is a spherical completion E_s of E contained in $^*\tilde{E}$.

Def.: An element $x \in E$ is said to be a best approximate of $y \in E_{fin}$ if $^*\|x-y\| = \inf\{{}^*\|y-e\|, e \in E\}$.

Theorem 4: E is spherically complete iff every $x \in E_{fin}$ has a best approximate in E.

Theorem 5: E has the orthogonal complement property iff there is at least one $x \in E_{fin}$ orthogonal to E. The non-standard techniques developped can be used to handle non-standard proofs for a variety of interesting results in non-archimedean theory.

VAN DER PUT, M.: Cohomology of constant sheaves

For a holomorphic space X it is tried to calculate the cohomology groups $H^i(X,F)$ where F is a constant sheaf with respect to the Grothendieck topology on X. Following results are stated:

1.) $H^i(X,F)=0$ for $i>\dim(X)$.

2.) Let X be a hyperelliptic curve of genus g. Then $0\leq\dim H^1(X,F)\leq g$.

The extreme cases $\dim H^1(X,F)=0$ and $\dim H^1(X,F)=g$ seem to occur when X has a good reduction resp. the reduction of X consists of projective lines.

Following methods are used: A good class of sheaves is introduced, namely constructible sheaves. For those sheaves cohomological dimension of spaces can be computed. Moreover, constructible sheaves satisfy a "base change" theorem.

DE GRANDE-DE KIMPE, N.: Structure theorems for locally K-convex spaces

Let K be a n.-a. valued field with a dense valuation under which it is spherically complete. A characterization is given for all the subspaces (locally K-convex) of c_o^I with the product topology, for some power I. ($c_o = \{(a_n) \mid a_n \in K, \lim_n a_n = 0\}$

with $\|(a_n)\| = \sup_n |a_n|$). Every Schwartz space and every space which has an orthogonal basis, is a subspace of c_o^I. Let S_o (resp. S, S_ω) denote the class of all subspaces of c_o^I (resp. of all Schwartz spaces, of all locally convex spaces with the property that all the operators to c_o are compact). Then $S_o \cap S_\omega = S$. Other characterizations of the elements of S_ω are given.

VAN ROOIJ, A. C. M.: Open questions on Banach spaces

Let K be a complete n.-a. valued field. If E is a Banach space over K and if D is a closed linear subspace of E, a "complement" of D is a closed linear subspace F of E, such that $D \cap F = 0$, $D + F = E$. A Banach space E is said to have the "complementation property" (CP), if every closed linear subspace of E has a complement. Question: Which Banach spaces have the CP? If the value group is discrete, every Banach space does. (In the proof one uses the fact that on every Banach space E one can define a norm $\|\cdot\|_o$, equivalent to the given one, such that $\|E\|_o = |K|$). Question: Is the discreteness of the valuation crucial for this?

Henceforth assume the valuation to be dense.

Let E be a Banach space. If E has a base, then every closed subspace of countable type has a complement. (Question: Is the converse true?) On the other hand, c_o^N has no complement in b^N. There exists a continuous linear surjection $c_o^K \to b^N$. It follows that c_o^K does not have the CP. However, if a Banach space E has the CP, then it is a quotient space of c_o^K. Thus, spaces with the CP cannot be too large.

(A Banach space over R or C has the CP if and only if it is linearly homeomorphic to a Hilbert space!)

K. Fieseler (Münster)