MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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## TOPOLOGIE

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Die Tagung stand unter der Leitung von T. tom Dieck (Göttingen), K. Lamotke (Köln) und C.B. Thomas (London).

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## Vortragsauszüge

### H. ABELS: Cocompactness of proper transformation groups

The following question is discussed: Given a locally compact topological group $G$ and a locally compact, locally connected, connected, paracompact topological space $X$. Is it true that for any two proper actions of $G$ on $X$ both orbit spaces are compact or both are non-compact? The answer is no in general. It is conjectured that the answer is yes if $\dim_k H_c^*(X;k) < \infty$ for some finite field $k$. The conjecture is true in the following cases: (1) $X$ a differentiable manifold possibly with boundary, $G$ a Lie group acting effectively and differentiably. (2) $G$ a closed subgroup of a Lie group having a finite number of connected components, $X$ arbitrary. (3) $G$ a Lie group, the action is p-free, $p \neq \mathrm{char}(k)$, i.e. all isotropy subgroups of $G$ are finite of order prime to $p$; e.g. $G$ is discrete and has no p-torsion.

### J. BERRICK: Detecting sections in K-theory

We study obstructions in $K_G$-theory to the existence of a section of the sphere bundle $SE$ of an n-dimensional complex $G$-vector bundle $E$ over a compact $G$-space $B$, where $G$ is a compact Lie group. In $K_G(B)$ there exist the following related obstructions:

$$\lambda_{-1}(E) = 0 \Leftrightarrow \lambda^j(E-\underline{1}) = 0 \ \forall j \geq n \Rightarrow k \mid \rho^k(E) \ \forall k \geq 1.$$

By considering the Gysin sequence of $E$,

**Theorem 1:** $K^*_G(SE) \cong K^*_G(B \times S^{2n-1})$ as $K^*_G(B)$-algebras.

Now restrict to $G = \langle 1 \rangle$ . Over the rationals $\exists$ an isomorphism as in Thm. 1 respecting $\lambda^k$-operations. Examination of the classifying space $\lambda_{-1}(E) = 0$ reveals the existence of secondary obstruction classes $v_i \in K^1(B)$ , $i = 1,2,\ldots,n-1$ .

<u>Theorem 2</u>: If $v_i = 0 \ \forall \ i$ , then $\exists$ an isomorphism as in Thm. 1 respecting $\lambda^k$-operations.

An application to the non-immersibility of lens spaces is presented.

M. C. CRABB: <u>$\mathbb{Z}/2$-equivariant homotopy: the Euler class, the Kahn - Priddy theorem</u>

Suppose that $\xi$ , $\xi'$ are real vector bundles over a compact space $X$ . Let $L$ be the non-trivial 1-dimensional real representation of $\mathbb{Z}/2$. If the sphere-bundles $S(\xi)$ and $S(\xi')$ are stably fibre-homotopy equivalent, then $S(L \otimes \xi)$ and $S(L \otimes \xi')$ are $\mathbb{Z}/2$-equivariantly stably fibre-homotopy equivalent. The proof is the commutative diagram:

$$\begin{array}{ccc} KO(X) & \xrightarrow{S^2} & KO_{\mathbb{Z}/2}(X) \\ \downarrow & & \downarrow \\ J(X) & \xrightarrow{S^2} & J_{\mathbb{Z}/2}(X) \end{array}$$

where $S^2$ is the sum operation which takes the vector (or sphere) bundle $\xi$ (or $S(\xi)$ ) to the sum $\xi \oplus \xi$ (or fibre join $S(\xi) * S(\xi)$) with $\mathbb{Z}/2$ acting by switching the factors ( $\xi \oplus \xi$ with this involution is just $\xi \oplus L \otimes \xi$ ) .

This basic idea leads to consideration of power operations in stable cohomotopy and a geometric definition of the generalized Hopf invariant (giving by duality, where appropriate, recent interpretations by R. Wood, U. Koschorke and B. Sanderson). We note that G. Segal's proof of the Kahn-Priddy theorem now takes a very simple form. Several points are emphasized: the split exact sequence $0 \to \{X^+; RP^{\infty\,+}\} \to {}_{\mathbb{Z}/2}\pi^0_S(X) \to \pi^0_S(X) \to 0$ ($X^+ = X \cup$ base point, ${}_{\mathbb{Z}/2}\pi^*_S$ = $\mathbb{Z}/2$-equiv. stable cohomotopy) ;

the rôle of duality; the importance of defining relative classes; the $\mathbb{Z}/2$ -equiv. Euler class / characteristic of a vector bundle with the antipodal involution.

J. DUPONT: Characteristic classes of flat bundles

Let $G = GL(n,\mathbb{C})$, let $G_d$ be the underlying discrete group and $\iota : G_d \to G$ the natural map. Simons and Cheeger have constructed classes $s_k \in H^{2k-1}(BG_d, \mathbb{C}/\mathbb{Z})$ such that $\beta(s_k) = \iota^*(c_k)$, where $\beta$ is the Bockstein homomorphism and $c_k \in H^{2k}(BG,\mathbb{Z})$ are the Chern classes. We give a simple direct construction of cochains representing the classes $s_k$ in the Eilenberg-Mac Lane group cohomology of $G_d$ with coefficients in $\mathbb{C}/\mathbb{Z}$. Secondly we show how the construction of similar classes of Simons and Cheeger for any real Lie group $G$ is a direct consequence of the Chern-Weil theory for the universal $G$-bundle in the framework of simplicial De Rham cohomology.

S. GITLER: Vector bundles over a stable complex

Conditions both necessary and sufficient on geometric dimension of vector bundles over a suspension space are analyzed. If the complex $X$ of dimension $n$ is stable, we conjecture that a single obstruction in $J_*(DX \wedge P^n_N)$ is necessary and sufficient for $gd(\alpha) \leq N$ if $2N > n$. This is joint work with K.Y. Lam and M. Mahowald.

H.A. HAMM: Zur Homologie Steinscher Räume

Es handelt sich um den folgenden Satz: Jeder n-dimensionale Steinsche Raum hat den Homotopietyp eines n-dimensionalen CW-Komplexes. Im Fall einer Mannigfaltigkeit wurde dies von

Andreotti - Frankel mit Morsethorie bewiesen. Im allgemeinen Fall wird nun eine verallgemeinerte Morsetheorie benutzt. Über das oben angegebene Resultat hinaus läßt sich noch zeigen: Ist X ein Steinscher Raum, Y eine analytische Teilmenge, X-Y n-dimensional, so entsteht X aus Y bis auf Homotopieäquivalenz durch Anheften von Zellen der Dimension $\leq$ n .

H.M. HASTINGS: Čech and Steenrod homotopy theory

Inverse systems of spaces (pro-spaces) occur in many settings, for example, shape theory, proper homotopy theory, localizations and completions, and ètale cohomology. The basic algebraic topology of pro-spaces is developed and the first two settings above are discussed.

Theorem: Let C be any of Top, simplicial sets, simplicial groups, simplicial spectra. Then pro-C admits a natural closed model structure in the sense of D.G. Quillen.

Corollary: There is a homotopy inverse limit holim: Ho (pro-C) $\rightarrow$ Ho (C) adjoint to the inclusion Ho (C) $\hookrightarrow$ Ho(pro-C). There are comparison theorems relating Ho (tow-C) (tow denotes towers) and tow-Ho (C) $\subset$ pro-Ho (C) , the category studied by Artin and Mazur. This relation extends the relation between Čech and Steenrod homology.

Typical applications: Reproof of Ross's results on vanishing of $\lim^s$ , inadequacy of M-L condition, construction of generalized Steenrod homology theories, strong Chapman complement theorem (categorical) , classification of open principal fibrations. (Joint work with D.A. Edwards.)

I. M. JAMES: Induced automorphisms of homotopy groups

Let points of the Stiefel manifold $V_{n,k}$ be represented by k x n matrices, in the usual way, and let $\lambda$ , $\mu$ be the involutions given by changing the sign of a row, column, respectively. Then

$$\lambda_* = 1 - u_* S_* \Delta ,$$

where

$$\pi_r(V_{n,k}) \xrightarrow{\Delta} \pi_{r-1}(S^{n-k-1}) \xrightarrow{S_*} \pi_r(S^{n-k}) \xrightarrow{u_*} \pi_r(V_{n,k}) .$$

Also

$$\lambda_* \mu_* = 1 - \Delta S_* p_* ,$$

where

$$\pi_r(V_{n,k}) \xrightarrow{p_*} \pi_r(S^{n-1}) \xrightarrow{S_*} \pi_{r+1}(S^n) \xrightarrow{\Delta} \pi_r(V_{n,k}) .$$

These relations facilitate the calculation of $\lambda_*$ and $\mu_*$, at least for low values of $k$, and the information obtained applies to the index theory of multiple vactor fields with finitely many singularities. The case of complex Stiefel manifolds under complex conjugation will also be discussed.

JOHANNSON: <u>Homotopy equivalences of bounded 3-manifolds</u>

Let $M$ be a compact 3-manifold (pl, orientable, irreducible). Suppose $\partial M$ is incompressible. Let $V$ be a submanifold of $M$. Then $V$ is called the characteristic submanifold of $M$ if the following holds:

1) Every component $S$ of $V$ admits either a fibration $p : S \to F$ as an I-bundle over a surface such that $(\partial S - p^{-1}\partial F)^- = \partial S \cap \partial M$ or a Seifert fibration with fibre projection $p : S \to F$ such that $p^{-1}p(\partial S \cap \partial M) = \partial S \cap \partial M$.

2) Every component of $(\partial V - \partial M)^-$ is an essential annulus or torus in $M$.

3) If $W$ is a non-empty submanifold of $M$ whose components are components of $\overline{M-V}$, then $V \cup W$ is not a submanifold with 1) and 2).

4) Every submanifold with 1) and 2) can be deformed into $V$, using a proper isotopy.

<u>Theorem</u>: Every homotopy equivalence $f : M_1 \to M_2$ ($M_1, M_2$ 3-manifolds as above) can be deformed so that afterwards

$f \mid \overline{M_1 - V_1} : \overline{M_1 - V_1} \longrightarrow \overline{M_2 - V_2}$ is a homeomorphism and

$f \mid V_1 : V_1 \longrightarrow V_2$ is a homotopy equivalence,

where $V_j$ is the characteristic submanifold of $M_j$, $j = 1,2$ .

U. KOSCHORKE: Selfintersections and higher Hopf invariants

I. James, P. May and others have constructed combinatorial models for iterated loop spaces $\Omega^m S^m X$ , $0 \leq m \leq \infty$ . If $X$ is the Thom complex of a bundle $\xi$ , we can interprete these models in a natural way as "Thom spaces for immersions". Hence bordism of smooth embedding into $M \times \mathbb{R}^m$ ($M$ a manifold), equipped with a description of the normal bundle as a pullback of $\xi \oplus \mathbb{R}^m$ , turns out to be isomorphic to the corresponding bordism of embeddings which project to immersions into $M$ . An analysis of transverse k-tuple points of the resulting immersions leads to bordism invariants which translate into homotopy operations $\Theta^k$, $k = 1,2,\ldots$ . As special cases we deduce the generalized and higher Hopf invariants of James, the Hopf ladder of Boardman-Steer as well as the (un)stable cohomotopy operations of G. Segal.

M. KRECK: Isotopy classes of diffeomorphisms

Consider orientation preserving diffeomorphisms on closed oriented diff. manifolds. Let $\pi_o(\mathrm{Diff}\ (M))$ (resp. $\tilde{\pi}_o(\mathrm{Diff}(M))$) be the group of isotopy (resp. pseudo-isotopy) classes of diffeomorphisms. A general method was introduced for the complete computation of $\pi_o(\mathrm{Diff}\ (M))$.

Theorem: Let $M$ be a $(k-1)$-connected $2k$-manifold.

a) For $k \equiv 5$ (8) there is an exact sequence

$0 \to \Theta_{2k+1} \to \pi_o(\mathrm{Diff}\ (M)) \to \mathrm{Aut}\ H_k(M,o)$ , $\Theta_{2k+1}$

the group of homotopy spheres.

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b) For $k \equiv 3\ (4)$, $k \neq 3$, there are exact sequences

$$0 \to \text{Kernel} \to \pi_o\,(\text{Diff}\,(M)) \to \text{Aut}\,(H_k(M), o) \quad \text{and}$$

$$0 \to \Theta_{2k+1} \to \text{Kernel} \to H_k(M)\ .$$

c) For simply connected 4-manifolds there is an injection

$$\tilde{\pi}_o\,(\text{Diff}\,(M)) \longrightarrow \text{Aut}\,(H_k(M), o)\ .$$

Using a result of Wall it turns out that for $M$ of the form $M' \# S^2 \times S^2 \# S^2 \times S^2$ this map is an isomorphism.

P. LÖFFLER: Some remarks on smooth involutions on homotopy spheres

Let $M$ be a $\mathbb{Z}_2$-manifold. We call $M$ $(n,k)$-framed if we have $t(M) \oplus \mathbb{R} \times M \cong M \times (\mathbb{R}^{n,k} \oplus \mathbb{R}^s)$ where $\mathbb{R}^{n,k}$ is the $\mathbb{R}^{n+k}$ with a nontrivial $\mathbb{Z}_2$-action on the first $n$ coordinates ($\mathbb{R}^s$ no action).
Now a well-known lemma asserts that if $(\Sigma^n,T)$ is a smooth involution on a homotopy sphere then it is $(n+1,-1)$-framed. Using this lemma and some well-known results of surgery and equivariant homotopy theory we can show:

1) Every element of the bordism group of $(n,-1)$-framed $\mathbb{Z}_2$-manifolds has a representative which is a homotopy sphere with a free involution.

2) If $\Sigma_d$ denotes the $(4k+1)$-dimensional Brieskorn-sphere associated to the polynomical $z_o^d + z_1^2 + \ldots + z_{2k+1}^2 = \varepsilon$, d odd, and free involution $(z_o, z_1, \ldots, z_{2k+1}) \mapsto (z_o, -z_1, \ldots, -z_{2k+1})$ then $\Sigma_d$ is diffeomorphic to $\Sigma_{d'}$, up to an action of $L_2(\mathbb{Z}_2,+)$ iff $d \equiv d' \mod 2^{2k+2}$.

3) Call a free involution on a homotopy sphere a standard involution if it admits a framing such that this element

bounds in this bordism setting. Our approach gives a fairly good description of the standard involutions in dimensions $\not\equiv 3$ (4). But using a recent result of Snaith we can show, that there exist nonstandard involutions on $S^{4k+1}$ (the standard sphere) for almost all $k$ .

S. MARDEŠIĆ: Approximate fibrations and pro-fibrations

In joint work with T.B. Rushing we introduce the notion of a shape fibration. A map $p : E \to B$ between metric compacta is a shape fibration provided one can find ANR-sequences $\underline{E} = (E_i, q_{ij})$ , $\underline{B} = (B_i, r_{ij})$ and maps $p_i : E_i \to B_i$ such that $p_i q_{ij} = r_{ij} p_j$, $p_i q_i = r_i p$ and $\underline{p} = (p_i) = \underline{E} \to \underline{B}$ has the following approximate homotopy lifting property (AHLP) : $(\forall i)$ $(\forall \varepsilon > o)$ $(\exists j \geq i)$ $(\exists \delta > o)$ whenever one has maps $h_j : X \to E_j$, $H_j : X \times I \to B_j$ with dist $(p_j h_j, H_{jo}) < \delta$ , then there is a map $\tilde{H}_i : X \times I \to E_i$ such that dist $(\tilde{H}_{io}, q_{ij}\, h_j) < \varepsilon$ , dist $(p_i\, \tilde{H}_i, r_{ij}\, H_j) < \varepsilon$ . If this happens for one expansion $\underline{p}$ of $p$ , it also happens for any other expansion $\underline{p}'$ of $p$ .
If $E, B$ are ANR's , then shape fibrations $p : E \to B$ coincide with approximate fibrations of Coram and Duvall.
Among the results obrained is an exact sequence for homotopy pro-groups. Every cell-like map $p : E \to B$ is a shape fibration provided dim $E < \infty$ , dim $B < \infty$ .

A. RANICKI: Equivariant S-duality

Let $\pi$ be a (discrete) group. Given pointed $\pi$-spaces $X, Y$ let $[X,Y]_\pi$ be the pointed set of pointed $\pi$-homotopy classes of $\pi$-maps $f : X \to Y$ , and define the abelian group of $S\pi$-maps

$$\{X,Y\}_\pi = \varinjlim_{p} [\Sigma^p X, \Sigma^p Y]_\pi$$

with $\Sigma X = X \wedge S^1$ the reduced suspension. Also, define a pointed space

$$X \wedge_\pi Y = (X \wedge Y)/\pi \ .$$

Given a finite connected subcomplex $X \subset S^N$ with fundamental group $\pi_1(X) = \pi$ and closed regular neighbourhood $E$ define the map $\alpha$ by $\alpha : S^N \xrightarrow{\text{collapse}} S^N/\overline{S^N - E} = E/\partial E = (\tilde{E}/\partial\tilde{E})/_\pi \xrightarrow{\text{diagonal}}$ $\to (\tilde{E}\times\tilde{E}/\tilde{E}\times\partial\tilde{E})/_\pi = \tilde{E}_+ \wedge_\pi \tilde{E}/\partial\tilde{E} \simeq \tilde{X}_+ \wedge_\pi \tilde{E}/\partial\tilde{E}$ with $\tilde{E} \simeq \tilde{X}$ the universal cover of $E \simeq X$ and $\tilde{X}_+ = \tilde{X} \cup \{\text{point}\}$ . Then $\alpha$ is an "S$\pi$-duality" between $\tilde{X}_+$ and $\tilde{E}/\partial\tilde{E}$ in the sense that for any pointed $\pi$-space $Y$ the slant products

$$\alpha\backslash - \ : \ \{\tilde{X}_+, Y\}_\pi \to \{S^N, Y \wedge_\pi \tilde{E}/\partial\tilde{E}\} \ ,$$

$$\alpha\backslash - \ : \ \{\tilde{E}/\partial\tilde{E}, Y\}_\pi \to \{S^n, \tilde{X}_+ \wedge_\pi Y\}$$

are both isomorphisms. In particular, if $X$ is a Poincaré complex (e.g. a manifold) this establishes an S$\pi$-duality between $\tilde{X}_+$ and the Thom space $\tilde{E}/\partial\tilde{E} = T(\nu_{\tilde{X}})$ of the pullback $\nu_{\tilde{X}}$ to $\tilde{X}$ of the normal spherical fibration $\nu_X$ of $X \subset S^N$ , generalizing the traditional Milnor-Spanier-Atiyah-Spivak S-duality between $X_+$ and $T(\nu_X)$ .

S. A. ROBERTSON: Topological exact fillings

The concept of exact filling was introduced about fifteen years ago by Robertson, and was studied in the context of vector bundles by Robertson and Schwarzenberger over several years. Recently, I have found a simple way of formulating the basic ideas in purely categorical language. The result is that the range of examples and problems is now very wide indeed.

The most interesting cases seem to occur in the topological category, where there is a close connexion with problems in fibre spaces. The talk attempted to explain this connexion, and included a description of how many familiar objects in topology (for example the family of maximal tori in a connected compact Lie group) may be presented as exact fillings.

R. RUBINSZTEIN: Some remarks on the cohomotopy of infinite real projective space

Let $\mu : \hat{A}(\mathbb{Z}/2) \to \omega^0(\mathbb{R}P^\infty)$ be the well-known homomorphism, where $A(\mathbb{Z}/2)$ is the Burnside ring of $\mathbb{Z}/2$ and $\omega^*(-)$ denotes ordinary cohomotopy theory. Let $\bar{\varepsilon} : \hat{A}(\mathbb{Z}/2) \to \mathbb{Z}$ be the augmentation. Consider $B = \bar{\varepsilon}^{-1}(1) \subset \hat{A}(\mathbb{Z}/2)$. One has $\omega^0(\mathbb{R}P^\infty) = [\mathbb{R}P^\infty, \Omega^\infty S^\infty] \supset [\mathbb{R}P^\infty, SG]$, where $SG = \Omega^\infty S^\infty_{(1)}$. Then $\mu(B) \subset [\mathbb{R}P^\infty, SG]$.
There is a map $i : SG \to G/PL$. It induces the transformation

$$i_* : [\mathbb{R}P^\infty, SG] \to [\mathbb{R}P^\infty, G/PL] .$$

Consider the composition

$$\beta : B \xrightarrow{i_* \circ \mu} [\mathbb{R}P^\infty, G/PL] .$$

The right hand side has a cohomological description.
We describe the map $\beta$. In particular, for $b \in B$, the value $\beta(b)$ depends only on "mod 8 reduction of b" (in some sense) and, consequently, $\beta$ is not an injection.
In the case of odd primes the situation is different.

P. SCOTT: Ends of group pairs

Let $e(G)$ denote the number of ends of a group $G$. When $G$ is finitely generated, one can interpret $e(G)$ geometrically. A famous result of Stallings says: If $G$ is a finitely generated group, then $e(G) \geqslant 2$ if and only if $G$ splits over some finite subgroup.

I have generalized this result in the following way. There is a natural definition, due to Houghton, of the number of ends, $e(G,C)$, of pair of groups $(G,C)$ where $C$ is a subgroup of $G$. If $G$ is finitely generated, $X$ is a finite CW complex, $\tilde{X}$ a regular covering of $X$ with group $G$ and $\tilde{X}/C$ is the quotient of $\tilde{X}$ by the action of $C$, then the number of ends of $\tilde{X}/C$ equals $e(G,C)$.

We say that $G$ is $C$-residually finite if given $g \in G-C$, there is a subgruoup $G_1$ of finite index in $G$ such that $G_1 \supset C$ but $g \notin G_1$. I have proved

Theorem: If $G$ and $C$ are f.g. groups and $G$ is $C$-residually finite, then $e(G,C) \geqslant 2$ if and only if $G$ has a subgroup $G_1$ of finite index which contains $C$ and splits over $C$.

W. SINGHOF: Generalized higher order cohomology operations induced by the diagonal mapping

A method is developed for the calculation of the category of a space, where we put cat $X \leqslant n$ if $X$ can be covered by $n$ open subsets each of which is contractible in $X$.

Let $\Psi$ be a generalized stable $n$-th order cohomology operation, $X$ a finite CW-complex with base point $*$, $T^nX = \{(x_1,\ldots,x_n) \in X^n \mid \exists\, i : x_i = *\}$, $d : (X,*) \to (X,*)^n = (X^n, T^nX)$ the diagonal map. If there exist cohomology classes $u_1,\ldots,u_n$ of $(X,*)$ such that $\Psi_d(u_1 \times \ldots \times u_n)$ is defined and does not contain $0$, then cat $X \geqslant n+1$.

This is applied to sphere bundles over spheres: If $S^q \to E \to S^m$ is such a bundle, $m,q > 1$, $m < 4q - 2$, then cat $E$ is expressed in terms of the element of $\pi_{m-1}(SO(q+1))$ classifying the bundle, answering for this range of dimensions a problem of T. Ganea.

J. VRABEC: Knotting a k-connected $M^m$ in $\mathbb{R}^{2m-k}$

Let $M$ be a k-connected closed PL m-manifold, $0 \leq k \leq m-3$, and let $q=2m-k$. It is known that $M$ embeds in $\mathbb{R}^q$ and that any two embeddings of $M$ in $R^n$ are isotopic if $n > q$. A natural problem is, therefore, to describe the set of isotopy classes of embeddings of $M$ in $\mathbb{R}^q$. Denote this set by $I$. It turns out that we have to distinguish four cases, which we name 00,0E,N0,NE; the first letter of the name denotes orientability type of $M$ (0 =orientable, N=nonorientable), and the second letter denotes parity of m-k (0=odd, E=even).

The set $I$ is described as follows. There exists a (geometrically defined) bijective map $I \to \mathfrak{X}_{k+1}(M)$, where $\mathfrak{X}_{k+1}(M) = H_{k+1}(M;\mathbb{Z})$ in case 00, $\mathfrak{X}_{k+1}(M) = H_{k+1}(M;\mathbb{Z}_2)$ in cases 0E and N0, and $\mathfrak{X}_{k+1}(M) = \mathfrak{X}_1(M) = H_1(M,x_o;\mathbb{Z}^t)/2\cdot\mathrm{im}[H_1(M;\mathbb{Z}^t) \to H_1(M,x_o;Z^t)]$ in case NE ; here $x_o \in M$ is any point, and $\mathbb{Z}^t$ denotes the twisted integer coefficients.

Only a part of this is new. J.F.P. Hudson obtained in 1969, by a different construction, a description of I for most cases. But he published only some of his proofs, and one of the results claimed was false.

W. Singhof (Bonn)

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