MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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The scientists who work on interacting systems, the subject of this conference, are all mathematicians, but many of them have their training in theoretical physics or chemistry. Many of the basic ideas and methods of the subject have been developed jointly by mathematicians and physicists. In the last few years their common language has become that of probability theory. The subject is therefore one of the most active areas of probability theory. This gave our conference a special purpose: to promote and further the lively inter-change of ideas between mathematicians and physicists with common interests in certain areas of probability theory and statistical physics.

To carry out the above purpose we decided to limit the conference to those problems in statistical physics and probability theory whose mathematical formulation is already firmely established. Every one of the 6 major speakers whom we asked to give survey talks accepted our invitation, in the expectation of a strong conference. It was essential that we were able to assure the speakers from the U.S.A. of financial travel assistance if necessary. Two of the major speakers are physicists. They come from U.S.A., France, Italy, Hungary. Their hour talks set the stage for over 20 shorter lectures on special topics. These covered a lot of equilibrium theory (the theory of point processes, Gibbs states, foundations of Thermodynamics) as well as kinetic theory (stochastic time evolution of Ising and other models, deterministic evolution of infinite particle systems). Over 45 people participated, including several well known mathematicians who wanted an introduction to this area.

The hospitality and pleasant surroundings contributed to the success of the conference.

Die Tagungsleiter: H. Föllmer (Bonn)
F. Spitzer (Ithaca)

Teilnehmer

D.B.Abraham	Theoretical Chemistry Department, Oxford University
Ph.Artzner	Dépt. de Mathématique, Université de Strasbourg
F.Bertein	Lab. Calcul des Probabilités, Université Paris VI
J.Chover	Mathematics Dept., University of Wisconsin, Madison
Z.Ciesielski	Stefan Banach Center, Academy of Sciences, Warsaw
C.Cocozza	Lab. Calcul des Probabilités, Université Paris VI
D.A.Dawson	Mathematics Dept., Carlton University, Ottawa
H.Dinges	Fachbereich Mathematik, Universität Frankfurt
E.Eberlein	Statistische Abteilung, Universität Bonn
P.Ferrero	C.N.R.S., 31, cour J.Aiguier, Marseille
R.S.Ellis	Mathematics Dept., University of Massachusetts, Amherst
J.Fritz	Mathematical Institute, Academy of Sciences, Budapest
G.Gallavotti	Istituto Matematico, Università di Roma, Roma
A.Galves	Centre de Mathématiques, Ecole Polytechnique, Palaiseau
H.-O.Georgii	Inst.f.Angewandte Mathematik, Universität Heidelberg
J.Groeneveld	Instituut voor theoretische fysica, Universiteit Utrecht
Y.Higuchi	Mathematics Dept., Kyoto University
R.Holley	Mathematics Dept., University of Colorado, Boulder
K.Jacobs	Mathematisches Inst., Universität Erlangen
C.Kipnis	Centre de Mathématiques, Ecole Polytechnique, Palaiseau
K.Krickeberg	U.E.R. Mathématiques, Université René Descartes, Paris
R.Lang	Fakultät für Mathematik, Universität Bielefeld
J.Lebowitz	Physics Dept., Yeshiva University, New York
F.Ledrappier	Lab. Calcul de Probabilités, Université Paris VI
A.Martin-Löf	Mathematics Dept., Royal Inst. of Technology, Stockholm

A.Messager	C.N.R.S., 31, cour J.Aiguier, Marseille
S.Miracle-Solé	C.N.R.S., 31, cour J.Aiguier, Marseille
J.Moulin-Ollagnier	Lab. Calcul des Probabilités, Université Paris VI
M.Mürmann	Inst.f.Angewandte Mathematik, Universität Heidelberg
J.Neveu	Lab. Calcul des Probabilités, Université Paris VI
X.X.Nguyen	Fakultät für Mathematik, Universität Bielefeld
G.Papanicolaou	Courant Institute, New York
D.Pinchon	Lab. Calcul des Probabilités, Université Paris VI
C.Preston	Mathematics Dept., University of Cambridge
E.Presutti	Istituto Matematico, Università dell'Aquila, Aquila
B.Rauchenschwandtner	Math. Institut, Kepler Universität, Linz
H.Rost	Inst.f.Angewandte Mathematik, Universität Heidelberg
D.Ruelle	I.H.E.S., Bures-sur Yvette
S.Sherman	Mathematics Dept., Indiana University, Bloomington
T.Shiga	Mathematics Dept., Nara Women's University, Nara
J.L.Snell	Mathematics Dept., Dartmouth College, Hanover NH
D.W.Stroock	Mathematics Dept., University of Colorado, Boulder
W.Sullivan	Dublin Institute of Advanced Studies, Dublin
D.Szász	Mathematical Institute, Academy of Sciences, Budapest
P.Weiss	Math. Institut, Kepler Universität, Linz
W.v.Waldenfels	Inst.f.Angewandte Mathematik, Universität Heidelberg
H.Zessin	Fakultät f. Mathematik, Universität Bielefeld

Vortragsauszüge

D.B. ABRAHAM: Rigorous results in Ornstein-Zernike theory

The rigorous theory of the asymptotic behaviour of pair correlation functions $u(r,z)$ for continuum and lattice gases has been developed further by H. Kunz and the author. They considered finite range, hard-core pair potentials acting in d dimensions and used graphical techniques. The essential notions are i) analyticity of $u(r,z)$ in the fugacity z for a domain $\mathfrak{D} \ni \{0\}$ uniform in r ii) connectedness of graphs G iii) nodal classification of G. This leads to a classification of the singularities of the Fourier transform $\hat{u}(k,z)$. The nearest singularities to the real axis are simple poles in k arising from the zeros of $1-\varsigma\hat{f}(k)$ where $f(r) = \exp(-\beta\phi(r))-1$, $\phi(\cdot)$ being the pair potential and $\beta = 1/(\text{temperature})$. The asymptotic behavior of $u(r,z)$ is typically

$$u(r,z) \sim A(z,\beta)\ \exp(-k_1 r)\ \cos(k_2 r)/r^{(d-1)/2}$$

as originally suggested on heuristic grounds by Ornstein and Zernike (1917). A result was given for the variation of k_2 with β for given z . It should be noted, of course, that $\mathcal{D}$ depends on z and β . [The existence of such a domain was first established by Groeneveld and Penrose and subsequently extended by Ruelle and others.]

J. CHOVER: Majority vote model

A discrete time evolution of 2-dimensional configurations involving local majority votes, plus noise, is analyzed by percolation techniques in Z^3 . The distribution in Z^3 corresponds to a finite range potential, whose specific energy per site is analytic on the same noise parameter interval for which the evolution is nonergodic.

D.A.DAWSON: Limit theorems for quasi-multiplicative systems

This talk is concerned with the development of Feller's "diffusion approximation" to study the qualitative behavior of a stochastic population model in the context of infinite particle systems. The particles are subject to self-reproduction, immigration, death and random spatial motion as well as interaction. The diffusion approximation can be intuitively described by a formal nonlinear stochastic partial differential equation. The mathematical formulation is in terms of an appropriate "martingale problem" whose solution is a measure-valued stochastic process. The steady state random measures which arise in some special cases are described and the central limit theorem for the steady state random measure is described.

R.S.ELLIS: Limit theorems for a class of Ising models near the critical temperature

Examples are given of non-Gaussian limit theorems for block spins near the critical point. Given ϱ an even probability measure on R with suitably small tail, let $X_1,\dots,X_n$ be spin random variables with joint distribution

$$(1) \qquad P(dx_1,\dots,dx_n) = \frac{1}{Z(n)} \exp\left(\frac{\nu}{2n}\left\{\sum x_i\right\}^2 + B\sum x_i\right) \prod_1^n \varrho(dx_i) ,$$

$\nu \geq 0$, B real. Write $S_n(\nu,B) = \sum_1^n X_i$. We restrict attention to measures ϱ for which the moment generating function $\phi_\varrho(r) = \int e^{rx}\varrho(dx)$ satisfies the bound

$$(2) \qquad \phi_\varrho(r) \leq \exp(kr^2/2), \qquad k = \int x^2 \varrho(dx) , \quad \text{for all } r \text{ real},$$

together with another condition. The bound (2) holds if, for example, ϱ satisfies the GHS inequality ($(\ln \phi_\varrho)''' \leq 0$ for all $r \geq 0$), for which necessary and sufficient conditions are known; thus $\varrho = \frac{1}{2}(\delta_1 + \delta_{-1})$ and

$$(3) \qquad \varrho = \text{const} \exp(-cx^4)dx , \qquad c > 0 ,$$

are included. Formula (1) defines a mean field model which (provided ϱ satisfies (2)) exhibits spontaneous magnetization for $(\nu, B) = (\nu_c, 0)$, where $\nu_c = \int x^2 \varrho(dx)$. A typical result is the following.

<u>Theorem 1.</u> There exists a constant $\gamma_4 = \gamma_4(\varrho) \leq 0$ such that if $\gamma_4 < 0$ then

$$\frac{1}{n^{3/4}} S_n\left(\nu_c + \frac{b}{\sqrt{n}}, \frac{h}{n^{3/4}}\right) \xrightarrow[w]{} \text{const } e^{\gamma_4 x^4/4! + bx^2 + hx} dx$$

for all b,h real.

Since ϱ in (3) satisfies (2), Theorem 1 exhibits this measure as a fixed point. Other results include refinements of Theorem 1 (with different scalings) in the case $\gamma_4 = 0$ as well as limit theorems for the joint distribution of multi-block spins. In all cases, the limiting measure can be expressed in terms of the behavior of the free energy near the critical point. These results were obtained in joint work with Charles Newman.

J. FRITZ: <u>Non-equilibrium dynamics of infinite particle systems</u>

This talk is a survey of some existence theorems for nonequilibrium dynamics to be proved in joint papers with R.L. Dobrushin. An infinite system of Newton's equations is considered for particles interacting by a finite-range pair potential of singularity like that of an inverse power at zero. In dimensions one and two, the semigroup of motion is constructed as the weak limit of solutions to finite subsystems; uniqueness of solutions satisfying a boundary condition at infinity is also proven under some regularity conditions on the potential. The allowed set of initial configurations is characterized in terms of energy fluctuations; in the one-dimensional case the content of this condition is that the specific energy is finite, in two dimensions only logarithmic fluctuations are allowed. Several invariants of the motion are described including the order of energy fluctuations and the exponential mixing property of the distribution of initial configuration. It is shown that solutions depend continuously on initial data; an explicit bound is given for the deviation of solutions in terms of initial deviations. Analogous results hold in the presence of independent white-noise forces, where weak solutions and the Markov semigroup are constructed. A heuristic example is given to suggest that non-equilibrium solutions may die in the three-dimensional case, provided that there are no white noise forces to regularize the behaviour of solutions. In this example the hard-core condition is not needed.

H.-O. GEORGII: <u>Canonical Gibbs States</u>

Canonical Gibbs states can be shown to be the time reversible states for certain stochastic motions of interacting particles. Under rather general conditions (excluding pathologies arising, e.g., from the possibility that with positive probability only finitely many particles occur), the extremal canonical Gibbs states are just the Gibbs states with respect to arbitrary activities. In the integral representation of canonical Gibbs states the distribution of the activities come from a certain function on the configuration space which can be chosen to be tail measurable and, in many cases, actually depends only on the particle density via the appropriate thermodynamic functions.

Y. HIGUCHI: The limiting Gibbs States of two dimensional Ising model

We can show at low enough temperature, the limiting Gibbs state for arbitrary (but $N+/N- \leq \frac{1}{3}$ on the boundaries of boxes $N \times N$, $N \geq 1$) boundary condition is μ^-. And we can also show an example of boundary conditions such that $N_+/N_- \leq 1-c$ $(c \in (0, \frac{2}{5}))$ and at low enough temperature, the limiting Gibbs state for these boundary conditions is μ^+ (N_+=the number of + spins on the boundary of $N \times N$ box, N_-=the number of - spins on the same one as N_+).

R.A. HOLLEY, D.W. STROOCK: Nearest neighbor birth & death processes on R^1:

Part I: It is shown that corresponding to any reasonable choice of birth and death rates, depending only on nearest neighbors, there corresponds a unique Markov process on the space of locally finite (infinite) subsets of R^1.

Part II: Necessary and sufficient conditions on the birth and death rates are found in order that such a process have a time reversible equilibrium state; and when these conditions are met the equilibrium state is a renewal measure. This resolves a question of F. Spitzer.

C. KIPNIS, C. COCOZZA: <u>Existence theorem for spin flip processes and the martingale problem</u>

In order to show the existence of a Feller process associated to a certain generator, we show that the corresponding martingale problem has a unique solution. For this purpose we exhibit a family of fundamental martingales, corresponding naturally to the behavior of the process at each site of the configuration. The uniqueness is then proved under certain assumptions on the speed coefficients by proving that any martingale orthogonal to this family (in the sense that two martingales are orthogonal when their product is also a martingale) is necessarily constant.

R. LANG: <u>Unendlich-dimensionale Wienerprozesse mit Wechselwirkung</u>

Folgendes von H. Rost stammende Modell wird untersucht: Seien Punkte $a_i \in R^\nu$ $(i=1,2,\ldots)$ gegeben, so daß in jeder kompakten Menge des R^ν nur endlich viele Punkte liegen; die a_i bewegen sich nach Diffusionen mit Varianz 1 und Drift $c_i(a) := -\frac{1}{2}\sum_{j\neq i} \operatorname{grad} \phi(a_i - a_j)$, wobei ϕ ein superstabiles Paarpotential ist (genügend glatt mit endlicher Reichweite). Ergebnis: Wenn μ ein Gibbs-Maß zum Potential ϕ ist, existiert der beschriebene Prozeß für μ-fast alle Anfangsbedingungen und ist Markov'sch. Die reversiblen Maße (hinreichende Regularität vorausgesetzt) sind kanonische Gibbs-Maße.

J.L. LEBOWITZ: <u>Time evolution and equilibrium states of harmonic and anharmonic crystals</u>

We give a full description of the equilibrium states and ergodic properties of infinite harmonic crystals (with O. Lanford).

The existence of, and approach to, stationary, Gaussian, non-equilibrium states for such systems will be discussed (with H. Spohn). Results will also be presented for the time evolution and equilibrium states of infinite anharmonic lattice systems (with O. Lanford and E. Lieb).

F. LEDRAPPIER: Local specifications, symmetric measures for a birth and death generator on a continuum

We give a differential characterization of a local specification on a continuum by a function of "local energy" satisfying a natural cocycle relation. We deduce an example of phase transition on R, i.e. a family of local specifications which admits as Gibbs measures 1) the measure of a renewal process, 2) the Dirac measure on the empty set. We show also that a birth and death generator admits a symmetric measure iff 1) the rate of growth of the population is a local energy and 2) the measure is Gibbs for that local energy.

A. MARTIN-LÖF: A limit theorem for an epidemic process with a critical phenomenon

The so called chain binomial model for an epidemic is considered. It is a discrete time Markov-chain with states (S_t, I_t), S_t = # of susceptible, I_t = # of infected individuals in the population, $t=0,1,2,\ldots$. In $(t,t+1)$ each I has probability p of meeting each S and if such a meeting occurs the S becomes infected and is counted in I_{t+1}. All I_t are well and immune at $t+1$ and do not further take part in the evolution. All encounters are independent events. Hence the transition probabilities are defined by

$$I_{t+1} \sim Bi(S_t, 1-(1-p)^{I_t}) \qquad (Bi(N,p) = \text{binomial distr.})$$

$$S_{t+1} = S_t - I_{t+1} \sim Bi(S_t, (1-p)^{I_t}) \ .$$

The initial state is called (n,m), and the process stops as soon as $I_t = 0$ at some state $(S_\infty, 0)$. We want to study the distribution of the total size of the epidemic $T = n - S_\infty$. The following limit is considered: $n \rightarrow \infty$, m finite, e.g. $q = 1-p = e^{-\lambda/n}$. In the beginning when $S_t \approx n$ large, I_t behaves approximately as a Galton-Watson process with generation distribution $P_o(\lambda)$. Hence if $\lambda < 1$ I_t is soon 0 and T has a limit distribution = that of total size of the G-W process. If $\lambda > 1$ this happens with probability p_e determined by $p_e = e^{-\lambda(1-p_e)}$, and with probability $1-p_e$ I_t soon becomes large. Then (S_t, I_t) are both large and should develop essentially deterministically as follows $(s_t = S_t/n,\ i_t = I_t/n)$:

$$s_{t+1}/s_t = e^{-\lambda i_t}, \qquad i_{t+1}/s_t = 1-e^{-\lambda i_t} \ .$$

The final state of this process is determined by

$$s_\infty/s_o = \exp(-\lambda(i_o+i_1+\ldots)) = \exp(-\lambda(m/n + 1 - s_\infty))$$

$$\therefore \quad s_\infty = \exp(-\lambda(1-s_\infty)),$$

and S_n/n ought to converge to s_∞ in probability and probably $(S_\infty - ns_\infty)/\sqrt{n}$ has a normal limit distribution. This so called threshold theorem describing the asymptotic form of the distribution of T can actually be proved. The proof does not however use the above heuristic but the trick of considering the functions $h_x(S,I) = (1-xq^{S+I})^S$ which are harmonic for all x. Hence $E_{n,m}(h_x(S_t,I_t)) = h(n,m)$ for all t and $E_{n,m}(h_x(S_\infty,0)) = h(n,m)$, so S_∞ satisfies the curious relation $E_{n,m}(1-xq^{S_\infty})^{S_\infty} = (1-xq^{n+m})^n$. This can be used to study the limit law of S_∞ and prove the threshold theorem. Work recently done in collaboration with Bengt von Bahr.

S. MIRACLE-SOLE: <u>Some remarks on the Ising model</u>

These remarks concern the Ising model with nearest neighbor ferromagnetic interactions. They arise in a common work with A. Messager on the problem of non translation invariant equilibrium states of this model. Their proof is based on some generalized Lebowitz inequalities for the correlation functions in terms of the variables $q_x = \sigma_x - \sigma_{\bar{x}}$, $s_x = \sigma_x + \sigma_{\bar{x}}$, $\chi_x = (1-\sigma_x)(1+\sigma_{\bar{x}})$ (see below).

I. The following monotonicity properties of correlations hold:

$$\langle \sigma_{00}\sigma_{mn}\rangle \geq \langle \sigma_{00}\sigma_{mn+1}\rangle \geq 0$$

$$\langle \sigma_{00}\sigma_{mn}\rangle - \langle \sigma_{00}\sigma_{mn+1}\rangle \geq \langle \sigma_{00}\sigma_{m+1n}\rangle - \langle \sigma_{00}\sigma_{m+1n+1}\rangle \geq 0$$

$$\langle \sigma_{00}\sigma_{mn}\rangle \geq \langle \sigma_{00}\sigma_{m+1n+1}\rangle \quad \text{if} \quad m \geq n$$

where σ_{mn} represents the spin of the point $(m,n) \in Z^2$. These inequalities extend to any dimension and can be generalized to higher order correlations.

II. The smoothness of the interface is proved in two dimensions for a large class of boundary conditions. Consider the system in a box $-(M+1) \leq m \leq M$, $-(N+1) \leq n \leq N$ and let ρ be a limiting Gibbs state obtained from the finite volume state in which the spins of the boundary are chosen in such a way that $\sigma_y - \sigma_{\bar{y}} \geq 0$ (x represents a point (m,n) with $n \geq 0$ and $\bar{x}$ the symmetric point $(m,-(n+1))$. Then $\rho(\sigma_x) = \rho(\sigma_{\bar{x}})$ for all x. Also $\rho(\chi_{x_1} \dots \chi_{x_n})$ is translation invariant. This result is based on the fact that the state $\rho^{\pm}$ obtained from the boundary conditions $\sigma_y = +1$ and $\sigma_{\bar{y}} = -1$ is translation invariant. This follows from Abraham's result $\rho^{\pm}(\sigma) = 0$. These results hold at all temperatures and support the conjecture that there are no non-translation invariant equilibrium states in the two dimensional case.

III. Fix in the two dimensional Ising model all spins $\sigma_{m,0} = +1$. Then the correlation functions $\rho(\sigma_{x_1} \dots \sigma_{x_n})$ corresponding to the semiinfinite system above the line $m = 0$ are unique. This follows from the smoothness of interface.

J. MOULIN-OLLAGNIER: Mesures quasi-invariantes et mécanique statistique

Les mesures de Gibbs sur $\mathcal{P}(S)$, S dénombrable, pour un potentiel regulier au sens de Preston, sont quasi invariantes pour le F-cocycle correspondant: $\{V_\Lambda\}_{\Lambda \in \mathcal{P}_f(S)}$, c'est à dire verifient: $\forall \Lambda \in \mathcal{P}_f(S) \forall f \in \mathcal{C}(X)$ $\mu[f \circ \tau_\Lambda] = \mu[f e^{V_\Lambda}]$. On generalise aux groupes abeliens les resultats de Krieger sur les caractérisations des cocycles qui admettent des mesures quasi-invariantes. Un contre-exemple dans le cas resoluble est fourni par un modele élémentaire de mécanique statistique.

J. NEVEU: A survey of stationary point processes

Cette conférence présente quelques uns des résultats principaux obtenus dans l'étude des processus ponctuels ou des mesures aléatoires positives définies sur un espace localement compact à base dénombrable.

a) théorèmes d'existence et de convergence en loi utilisant les fonctionnelles de Laplace;

b) mesures de Palm des processus ponctuels stationnaires (définition de Mecke; relations avec certaines probabilités conditionnelles; images $A_S \cdot P$ pour des points aléatoires S du processus; changement de processus ponctuels et de mesures de Palm; etc. ...).

Une représentation plus complète doit paraître dans les Lecture Notes à la suite d'un cours fait par l'auteur à Saint-Flour cet été 1976.

D. PINCHON: Décroissance de l'energie libre dans les processus de spin-flip

Le formalisme des cocycles permet de montrer dans un cadre très général les resultats de Holley sur la décroissance de l'energie libre. La démonstration s'en trouve raccourcie par l'utilisation d'un lemme technique permettant en un certain sens de controler les conditions frontières. L'introduction de la notion d'espace moyennable est le cadre abstrait adaptée à l'etablissement du lemme. Cette notion recouvre non seulement les groupes Z^d habituels mais de nombreux réseaux auxquels on ne peut pas naturellement associer des groupes.

C. PRESTON: <u>More on canonical Gibbs states</u>

One of the results stated in Georgii's summary, viz. that each extreme canonical Gibbs state is some kind of grand canonical Gibbs state (with some activity) can be proved in a fairly abstract setting, namely that of a consistent set of probability kernels with respect to a decreasing family of σ-fields (as introduced by Hans Föllmer), though, as above, conditions must be imposed in order to avoid the possible pathologies; also the method deals with the microcanonical case (provided regularity assumptions are made); similar results have been obtained by Goldstein, Aizerman and Lebowitz.

E. PRESUTTI: <u>Uniqueness of DLR states for infinite spin systems by use of the Dobrushin-Vasershtein technique</u>

In this summary I present some of the results obtained with M.Cassandro, E.Olivieri, A.Pellegrinotti, B.Tirozzi. I consider a lattice Z^ν and for $x \in Z^\nu$ the random variable $S_x \in R$: $X = \{S: Z^\nu \to R,\ S = \{S_x, x \in Z^\nu\}\}$. By S_Λ I denote $S|_{x\in\Lambda}$, $\Lambda \subset Z^\nu$. Statistical Mechanics is introduced by giving a "free" distribution $\mu(dS_x)$ for each spin and a pair interaction between them: we assume the pair interaction of the form $-J_{xy}S_xS_y$, $x \neq y$, $x,y \in Z^\nu$ with

(i) J_{xy} (space) translationally invariant

(ii) $\sum_{y \neq x} |J_{xy}| < \infty$

We assume that $\mu(dS_x)$ is a Borel regular measure on R, the same for each $x \in Z^\nu$, and we include in μ the self interaction of the spins and finally we assume that

(iii) the system characterized by $\mu(dS_x)$ and J_{xy} is superstable in the sense of [1,2].

We define for any $\Lambda \subset Z^\nu$ bounded

$$q(dS_\Lambda | S_{\Lambda^c}) = Z(S_{\Lambda^c})^{-1} \prod_{x\in\Lambda} \mu(dS_x) \exp[-\beta U(S_\Lambda) - \beta W(S_\Lambda | S_{\Lambda^c})]$$

$$U(S_\Lambda) = -\frac{1}{2} \sum_{\substack{x,y\in\Lambda \\ x\neq y}} J_{xy} S_x S_y \; ; \qquad W(S_\Lambda | S_{\Lambda^c}) = -\sum_{x\in\Lambda, y\notin\Lambda} \sum J_{xy} S_x S_y \; .$$

We say that the regular Borel probability measure ν on X satisfies the DLR equations if its conditional probabilities are given by $q(dS_\Lambda | S_{\Lambda^c})$.

Thm 1 [2]. Let (i,ii,iii) hold then card $\mathcal{E} \neq 0$ with

$$\mathcal{E} = \{\nu \text{ satisfy DLR} \mid \exists c(\nu) : \int \nu(dS) |S_x| \leq c(\nu) \; \forall x \in Z^\nu\}$$

In [2] it is also proven that the physically interesting states are in $\mathcal{E}$. We study uniqueness in $\mathcal{E}$, and we use the following

Thm 2 (Dobrushin [3]). Let (i,ii,iii) hold, let r_{xy}, $x\neq y, x,y \in Z^\nu$ such that

$$\sup_{x\in Z^\nu} \sum_{y\neq x} r_{xy} < 1$$

$$R[q(dS_x | S^{(1)}_{\{x\}^c}), q(dS_x | S^{(2)}_{\{x\}^c})] \leq \sum_{y\neq x} r_{xy} |S^{(1)}_y - S^{(2)}_y| \quad \forall S^{(1)}_y, S^{(2)}_y$$

where R is the Vasershtein distance [3] defined as follows. Let λ, σ be two regular probability measures on R, $\hat{P}(dx,dy)$ is a joint representation for λ and σ if it is a regular probability measure on R^2 with projections on the two axes λ and σ. Then

$$R(\lambda,\sigma) = \inf_{\hat{P}} \int \hat{P}(dx,dy) |x-y| \quad \text{(over all joint representations)}$$

To apply Thm 2 we need

Thm 3. Let λ and σ be two regular probability measures on R with finite first moments then

$$R(\lambda,\sigma) = \int_{-\infty}^{+\infty} dx \left| \int_{-\infty}^{x} [\lambda(dy) - \sigma(dy)] \right|$$

As a consequence we obtain

Thm 4. Let (i,ii,iii) hold. Then card $\mathcal{E} = 1$ if

$$\sum_{y \neq x} \beta |J_{xy}| \sup_{t \in R} \left\{ \int p(dS_x | t) [S_x - \bar{S}_x(t)]^2 \right\} < 1$$

where $p(dS_x|t) = q(dS_x|S_{\{x\}})$, $t = \sum_{y \neq x} \beta J_{xy} S_y$, $\bar{S}_x(t) = \int p(dS_x|t) S_x$

Remark. The above condition is essentially a mean field theory estimate: it proves that the mean field theory gives an upper bound for the critical temperature.

[1] D.Ruelle. Superstable interactions in statistical mechanics. Commun.Math.Phys.18,127-159 (1970). Superstable interactions for unbounded spin systems. In press.

[2] J.L.Lebowitz,E.Presutti. Statistical mechanics for unbounded spin systems. In press.

[3] R.L.Dobrushin. Prescribing a system of random variables by conditional distributions. Th.Prob.Appl.XV,458 (1970)

D. RUELLE: Statistical Mechanics on Axiom A Basic Sets.

The space of differentiable dynamical systems satisfying Smale's Axiom A (in particular Anosov diffeomorphisms or flows) has a local product structure. This permits the application of the formalism of equilibrium statistical mechanics, and in particular the definition of Gibbs states. Counting periodic orbits with the weights suggested by statistical mechanics, one is led to a natural definition of zeta functions. In particular the geodesic flow on a manifold of constant negative curvature yields Selberg's zeta function. The meromorphy of the latter is a natural consequence of the transfer matrix formalism of statistical mechanics.

S. SHERMAN: Correlation inequalities for the classical Heisenberg ferromagnet

Let $N := \{1,\dots,n\}$ so $2^N \supset \binom{N}{2} \supset \mathcal{E} \supset \mathcal{A}, \mathcal{B} \ni E, F$, where $\binom{N}{2}$:= the collection of 2 element subsets of N. Let $\sigma_1,\dots,\sigma_n$, $\tau_1,\dots,\tau_n$ be independent random vectors each uniformly distributed on $S^{m-1} := \{z \in R^m : \|z\| = 1\}$. For $E = \{i,j\}$ let $\sigma_E := \sigma_i \sigma_j$ and $\sigma_{\mathcal{A}} := \prod_{E \in \mathcal{A}} \sigma_E$. τ_E and $\tau_{\mathcal{A}}$ are defined similarly.

MAIN THEOREM. $(\forall m,n \in N)(\forall \mathcal{E})(\forall \pm)\ \langle \prod_{E \in \mathcal{E}} (\sigma_E \pm \tau_E) \rangle_o \geq 0$.

Consider the following Hamiltonian

$$H(\sigma) = -\sum_{\mathcal{A}} J_{\mathcal{A}} \sigma_{\mathcal{A}} \quad , \quad (\forall \mathcal{A})\ J_{\mathcal{A}} \geq 0 .$$

The Gibbs distribution yields correlations $\langle \sigma_{\mathcal{A}} \rangle = \dfrac{\langle \sigma_{\mathcal{A}} \exp -H(\sigma) \rangle_o}{\langle \exp -H(\sigma) \rangle_o}$.

MONOTONICITY THEOREM. $(\forall \mathcal{A};\mathcal{B})\ \langle \sigma_{\mathcal{A}} \rangle \nearrow_{J_{\mathcal{B}}}$.

Both theorems generalize to the case where N is replaced by $\bar{N} := N \cup \{0\}$, σ_o is a random vector in R^m independent of $\sigma_1,\dots,\sigma_n,\tau_1,\dots,\tau_n$, and where the common distribution of $\sigma_1,\dots,\tau_n$ is radial.

T. SHIGA: Remarks on Gibbs states, canonical Gibbs states and Markovian time evolutions

Gibbs states and canonical Gibbs states are characterized as quasi-invariant measures of some families of transformations on configuration spaces. By this fact we can give a simple proof of the identification theorem of extremal canonical Gibbs states and extremal Gibbs states. Moreover this technique enables to show the equivalence and singularity relations for two distinct Gibbs states. Finally we remark on limiting distributions of certain Markovian time evolutions.

W.G.SULLIVAN: Some remarks on Markov interaction processes

We consider the problem of defining a Markov interaction process in terms of a parametrized family of Markov transition functions together with stochastic boundary conditions as a generalized specification. From this point of view we deduce the existence-uniqueness theorem of Dobrushin&Liggett including the Griffeath&Gray generalization, by using a perturbation argument. For attractive processes in the compact case we deduce existence and uniqueness when the + and - stochastic boundary conditions give the same limit.

H. ZESSIN/ NGUYEN X.X.: Integral and differential characterizations of the Gibbs process

We consider general Gibbs processes, i.e. point processes in a locally compact second countable Hausdorff topological space where the conditional probabilities in bounded volumes are given by the grand canonical Gibbs distribution. For a general class of interactions U we characterize Gibbs processes P by means of the following integral equation:

$$\int_M \int_X h(x,\mu)\,\mu(dx)P(d\mu) = \int_X \int_M h(x,\mu+\varepsilon_x)\exp E(x|\mu)P(d\mu)\,\varrho(dx)$$

for all $h \in H$, where H is the set of measurable non-negative functions on $X \times M$, and M denotes the space of "configurations" in X. $E(x|\mu)$ is the "energy of x given the configuration μ" and ϱ is some measure in X. In the case $U \equiv 0$ (the case of the Poisson point process) this equation reduces to an integral equation due to Mecke. We also show the equivalence of this equation to the equilibrium equations due to Lanford/Ruelle. Some easy corollaries show that this new integral equation is an effective tool in analyzing Gibbs processes. In particular we get a differential characterization of Gibbs processes in terms of their Palm measures which generalizes a recent result of Georgii. In the case $U \equiv 0$ this reduces to characterizations of the Poisson process via Palm measures due to Mecke, Jagers, Sliwujak and Ambartzumian. Finally we point out some relations of this integral equation to the Kirkwood-Salzburg equations.

H.Föllmer (Bonn)