MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 49/1976

Statistical Methods

28.11. bis 4.12.1976

Als Schwerpunkte dieser Statistik-Tagung waren die Versuchsplanung (design and analysis of experiments) und die Entscheidungstheorie unter besonderer Berücksichtigung der Sequentialanalyse, gewählt worden. Damit wurden zwei wichtige Gebiete der Angewandten Mathematischen Statistik angesprochen, die in Deutschland noch dringend der Anregungen aus dem Ausland bedürfen. Die Tagung stand unter der Leitung von O. Krafft (Aachen) und N. Schmitz (Münster). Sie stieß auf außerordentlich großes Interesse; die Vielzahl von inhaltsreichen Vorträgen gab Anlaß zu anregenden Diskussionen und fruchtbaren Gesprächen, so daß die Intention, den angesprochenen Schwerpunktgebieten Impulse zu geben, durchaus erfüllt wurde. Trotz Schnee und Regen konnte eine gemeinsame Wanderung durchgeführt werden.

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Vortragsauszüge

N. BÖNNER: A sequential correlation rank test

Applying a Chernoff-Savage-technic an almost sure representation of correlation-rank-statistics T_n as a sum S_n of independent random variables is derived.

This almost sure representation yields a Wiener-process approximation for the rank-statistics T_n under a certain class $\{P_\Delta, \Delta > 0\}$ of nonparametric alternatives.

Hence it is possible to analyse the asymptotic behaviour (for $\Delta \to 0$) of the sequential rank test ϕ_Δ defined by the statistics $\Delta T_n - \frac{n\ \Delta^2}{2}$ and the stopping-bounds $a = \log((1-\beta)/\alpha)$ and $b = \log(\beta/(1-\alpha))$.

The asymptotic OC- and ASN-functions of the test are given.

I.M. CHAKRAVARTI: Design and Analysis of a Sequence of Similar Experiments

Various designs (with or without blocking) with serial balance for estimating direct effects and residual effects of different order (with or without elimination of trend and other sources of heterogeneity) have been constructed in the past by different authors - to name a few, E.J. Williams (1949, 1950), Finney and Outhwaite (1955, 1956), Sampford (1957), Federer and Atkinson (1955), Patterson (1952, 1968) and Weidman (1975).

The combinatorial problems which arise in the construction of serially balanced designs have been examined by this author in the light of the methods used in the construction and the properties of maximal period recurring sequences and DeBruijn sequences. The maximal period sequences have been widely used in recent years in coding problems connected with planetary reconnaissance.

H. DRYGAS: Hsu's theorem in variance component models

This paper deals with linear models of the kind $y = X\theta + U\varepsilon$, where ε is a random vector composed of independent random variables. Therefore $\mathrm{Cov}\,\varepsilon = \sum_{i=1}^{k} \sigma_i^2 V_i$, where $V_i = \mathrm{diag}(0,\dots,0, I_i, 0,\dots,0)$ and I_i is a unit matrix of appropiate order. Much work has been done to investigate the problem of the existence of uniformly best (invariant) quadratic unbiased estimators if ε is normally or quasi-normally distributed. It is the purpose of this paper to extend these results to the non-normal case. This extension is done in the case of best invariant quadratic estimators. A complicated matrix relation turns out to ensure optimality. But in analogy to Hsu's theorem it can be shown that this relation can be replaced by requiring it only for the diagonal elements. The obtained results still appear very complicated but it turns out that due to the diagonality of the V_i the verification of the obtained conditions is rather straightforward. This is illustrated at two examples: the balanced one-way and the balanced two-way classification model.

W.S. FEDERER: Some Unsolved Problems in Experiment Design and Linear Model Theory

Seven groups of unsolved problems were presented. These are:

Problem 1: It was shown by the method of "projecting diagonals" how to easily construct the OL(n,n-1) set for all prime numbers in a short space. Likewise a method was presented for easily constructing the OL(n,n-1) set for all prime powers in a small space; the problem here was to present a method of writing an automorphism in a simple manner for the nonmathematician.

Problem 2: It was shown how to write the OL(n = 12,5) set in an easy space-conserving manner. How does one construct the sets for n = 4t + 2 for more than the smallest prime minus one of n?

Problem 3: It was shown how to present the OL(15,3) set in a row vector of 15 three-tuples. How does one proceed in general for n odd and how does one obtain more orthogonal latin squares than the smallest prime minus one for n odd and greater than 15?

Problem 4: How does one arrange the rows of a latin square such that the resulting row x column design is as near variance-balanced as possible with the addition of each row? The method was illustrated for $n = 7, 13$ and 31. How does one find a necessary and sufficient condition that a subset of a difference set is a difference set?

Problem 5: Complete sets of orthogonal latin squares are available for all prime powers. Likewise, complete sets of F-squares of various dimensions have been obtained for all prime powers and for $n = 4t$ with two symbols. For the number three, there is only one F-square geometry. For the number 4, five F-square geometries have been found and others have been shown not to exist. For $n = 6$, not a single F-square geometry has been found but we are close to finding one. The idea of equally correlated complete sets of F-squares was presented. The problem is how to complete this theory and to obtain a simple straight-forward method of writing out complete sets of F-squares for the different geometries.

Problem 6: In a row-column array, if one knows the row and column single degree of freedom contrasts, it is a simple matter to write the single degree of freedom interaction contrasts. Now, given the interaction contrasts can one determine the row and column contrasts giving rise to this set of interaction contrasts? Also, a conjecture on sets of orthogonal contrasts was presented.

Problem 7: In optimum design theory, one could know that the response function goes through a fixed point in the X-space. How does one alter present optimum design theory to take account of this fact? It was noted that the variance of the estimated response function is zero at the known point.

W. FIEGER: Two statistical characterizations of the normal law

L. Bondesson (Sankhya Ser. A. 36 (1974)) has characterized the normal law by the property that a Pitman estimator of a location parameter is homogeneous of first degree. In this talk it is shown that the regularity conditions needed by Bondesson can be weakened. Furthermore the following theorem is given: Let $X_1,\ldots,X_n$ be independent identically distributed random variables with distribution function $G((x-\lambda)/\sigma)$ $(\lambda \in \mathbb{R}, \sigma > 0)$ and $s(u) = \operatorname{Max}\{0, |u|^r - \alpha\}$ $(\alpha > 0, r \geq 1)$. Then there exists a translation invariant estimator $d_o(x_1,\ldots,x_n)$ which is homogeneous of first degree and differentiable at some point $(x_o,\ldots,x_o) \in \mathbb{R}^n$ such that

$$E_{\lambda,\sigma}\, s(d_o\,(X_1,\ldots,X_n) - \lambda) \leq E_{\lambda,\sigma}\, s(d\,(X_1,\ldots,X_n) - \lambda)$$

for all translation invariant estimators $d(x_1,\ldots,x_n)$ and all $\lambda \in \mathbb{R}$, $\sigma > 0$ iff $G(x)$ is a $N(0,\sigma_o^2)$ distribution function.

P. GAENSSLER: Remarks on Cramér-Rao-type inequalities and sequential estimation plans for a certain class of stochastic processes

Let $(\xi_t)_{t \geq 0}$ be a stochastic process defined on some p-space $(\Omega, \mathcal{A}, \mathbb{P})$ with $\mathbb{P} \in \{P_\theta : \theta \in \Theta\}$, Θ being a real interval. If τ is a stopping time for (ξ_t) and if $\phi = \phi_1 \circ (\tau, \xi_\tau)$ is an unbiased estimator for some function $g(\theta)$ of the unknown parameter θ, then, as one knows from Trybula (1968) and Magiera (1974), the classical Cramér-Rao inequality extends to processes (ξ_t) fulfilling certain conditions refering to the validity of Sudakov's lemma (1969). Especially for the Poisson process, the Wiener process with drift-parameter θ, the negative-binomial process and the Gamma process one obtains for τ with $E_\theta(\tau^2) < \infty$ (using Hall's generalization of Wald's equation) that

$$\mathrm{Var}_\theta\,(\phi) \geq \frac{c(\theta)\,[g'(\theta)]^2}{E_\theta(\tau)}, \text{ where } c(\theta) = \mathrm{Var}_\theta\left(\frac{\xi_t}{\sqrt{t}}\right).$$

Based on the loss function $L(\tau,\theta) = A\,(\phi_1 \circ (\tau, \xi_\tau) - g(\theta))^2 + E_\theta(\tau)$ a sequential plan as proposed by Robbins (1958) for a corresponding time-discrete situation can be extended to the present case using the fact that $\left(\frac{\xi_t}{t}\right)_{t>0}$ forms a reversed martingale.

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Robbins, H. : Sequential estimation of the mean of a normal population. In: Probability and Statistics. The Harald Cramér Volume (1956), 235-243.

Sudakov, V.N.: Measures defined by Markov times. Trudy Sem. Leningradskogo Otdelinija Mat. Inst. AN SSSR 12 (1969), 157-164

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N. GAFFKE: <u>Optimum designs in complete two-way-layouts in the irregular case</u>

There is given the following linear model (two-factor-model with interactions). $E_a X_{ijk} = a_{ij}$, $i = 1,\ldots,u$; $j = 1,\ldots,v$; $k = 1,\ldots,n_{ij}$; $a = (a_{11},\ldots,a_{uv})^T \in \mathbb{R}^{u \cdot v}$. The non-negative integers n_{ij} form a matrix N which is called a design. The random variables X_{ijk} are assumed to be uncorrelated and to have equal variance σ^2. The following problem is considered: Assuming that one is interested only in testing the hypothesis H_1, that factor 1 has mainly no influence a design N^* is to choose in an optimum way (optimality with respect to H_1). By definition the main effects of factor 1 are given by $h_i^{(1)} := a_{i.} - a_{..}$, where $a_{i.} := \frac{1}{v} \sum_j a_{ij}$, $a_{..} := \frac{1}{uv} \sum_{i,j} a_{ij}$. The hypothesis H_1 may be represented in the form $Ka = 0$ where K is a known $(u-1)\times uv$-matrix, rank $K = u-1$. Two optimality concepts are used: The uniform optimality means that the covariance-matrix of the least-square-estimator for Ka is minimized (we write $A \leq B$ for $m\times m$-matrices iff $B - A$ is nonnegative definite). Secondly the D-optimality which means that the determinant of that covariance-matrix gets a minimum. Since the values n_{ij} of a design are to be nonnegative integers both concepts lead to some discrete optimization problems. Especially in the "irregular case" that $u \cdot v \nmid n$, where n is the total number of observations, these problems cannot be solved by using standard inequalities. It is shown that for fixed values of $\sum_j n_{ij}$, $i = 1,\dots,u$, there always exist uniformly optimum designs and a complete characterization is given. In the case that only the total number of observations is prescribed all D-optimum designs are computed.

A. HEDAYAT: <u>On theory and applications of BIB designs with repeated blocks</u>

Consider BIB designs with parameters v,b,r,k and λ. Define the support of a BIB design to be the set of its distinct blocks and let the cardinality of the support be b*. If b* < b then the design is said to be a BIB design with repeated blocks. Some potential applications of such designs to experimental design and controlled sampling are given. Some necessary and sufficient conditions for the existence of these designs and some algorithmns for their construction are provided. Bounds on b* have been obtained. A necessary and sufficient condition under which a set of blocks can be the support of a BIB design are found. To cite one example, it is shown that

123, 145, 167, 178, 246, 257, 258, 347, 356, 348, 168

cannot be the support of a BIB design based on $v = 8$ and $k = 3$. A table of BIB designs with $22 \leq b^* \leq 56$ for $v = 8$ and $k = 3$ is included (distributed in the conference). A catalog of BIB designs for various values of v and k is available and the interested reader can obtain such designs with various support sizes (and sometimes many non-isomorphic ones) by writing to the author.

Remark: This is a joint work with Walter Foody.

S. HOLM: Asymptotic properties of SPR type tests

The asymptotic optimality properties of SPR tests of a composite hypothesis against a composite alternative will be discussed for the one-parameter case with special reference to exponential classes. This includes asymptotic power and asymptotic expected sample size considerations. For the multiparameter case I concentrate on a class of tests of 'asymptotic SPR type' including the Bartlett and Cox tests. The asymptotic power and asymptotic expected sample size will be obtained and the possibility of steering their asymptotic power will be demonstrated.

P. W.M. JOHN: Factorial Experiments in Incomplete Block Designs

When a 2^n factorial design is carried out in an incomplete block design it is desirable that the estimates of the various effects should be mutually independent (orthogonal). A necessary and sufficient condition for this is that the corresponding contrasts should be a set of eigenvectors of the matrix $\underset{\sim}{N}\,\underset{\sim}{N}'$, where $\underset{\sim}{N}$ is the incidence matrix of the incomplete block design.

If a factorial experiment has two factors A and B with a and b levels respectively, it has factorial structure in the sense of J.A. John and T.M.F. Smith, J. Royal Statist. Soc., $\underset{\sim}{B}$ 1972, if, and only if, the A,B and AB subspaces are each spanned by mutually orthogonal subsets of eigenvectors of $\underset{\sim}{N}\,\underset{\sim}{N}'$. Some examples will be given and some consequences of these results will be discussed.

W. KLONECKI: Best unbiased estimation, a coordinate-free approach

Linear models from the coordinate-free point of view are considered. Let $\mathcal{K}$ stand for an Euclidean space endowed with inner product $(.,.)$. Necessary and sufficient conditions for the existence of a best estimator in an arbitrary subspace $\hat{G}_o$ of estimators of the form (A,Z), $A \in \mathcal{K}$, are given. Z stands for an $\mathcal{K}$-valued random vector, and the expected value and the covariance of Z are assumed to exist. The concept of the Gauss Markov estimator (GME) of EZ is extended so that the existence of the extended GME is equivalent to the existence of a best estimator in $\hat{G}_o$ for each parametric function $E(A,Z)$, where $(A,Z) \in \hat{G}_o$. The developed theory is exemplified by considering a random vector Z with a covariance operator of the same structure as the covariance operator YY', where Y stands for an $\mathcal{R}^n$-valued normal random vector.

E. KÖHLER: On t-designs

My talk was concerned with the following theorem:
Let $B_p = (B_{\cdot p}, \underline{B}_p)$ be a graph ($B_{\cdot p}$ = set of vertices, $\underline{B}_p$ = set of edges) with

$$B_{\cdot p} := \{\bar{\alpha} = \{\alpha, \frac{1}{\alpha}, \frac{-\alpha}{\alpha+1}, -\alpha - 1, -\frac{1}{\alpha} - 1, \frac{\alpha}{\alpha+1} - 1\} \mid \alpha \in GF(p) \setminus \{0,1,-1,-2,\frac{p-1}{2}\}\}$$

and $\{\bar{\alpha}, \bar{\beta}\} \in \underline{B}_p$ iff there exist $a \in \bar{\alpha}$ and $b \in \bar{\beta}$ such that $a = b+1$ or $a = b-1$. Now suppose that p is prime and that $p \equiv 77 \mod(120)$ holds. Then the following assertion is true:

If there is no bridge in B_p then a cyclic (3,4,2p)-Steiner quadrupel system exists.

V. KUROTSCHKA: Neuere Ergebnisse in der Theorie der optimalen statistischen Experimente

Für komplexe Experimente mit qualitativen Einflußfaktoren existiert eine umfangreiche Literatur, die noch laufend durch viele einzelne Ergebnisse bereichert wird (vgl. meine Beiträge dazu bei früheren Tagungen hier und die Beiträge der Herren Gaffke, Schaefer, Sonnemann während dieser Tagung). Das gleiche gilt auch für komplexe Experimente mit rein quantitativen Einflußfaktoren (vgl. dazu z.B. den Vortrag von Herrn H.P. Wynn während dieser Tagung). Für allgemeine lineare Modelle mit sowohl qualitativen als auch quantitativen Einflußfaktoren sind außer einigen allgemeinen Tatsachen (Harville, 1975) bisher keine konkreten Ergebnisse veröffentlicht worden. Die ersten konkreten Ergebnisse dazu, die mein Student J. Köster mit mir in den letzten beiden Jahren in Göttingen erzielt hat, stellt den ersten Teil der "Neueren Ergebnisse ......" dar, den zweiten Teil unsere Ergebnisse über optimale Versuchsplanung von multivariaten Experimenten, für die ebenfalls nur erste Ansätze in der Literatur zu finden sind.

Aus Zeitgründen wurde die Darstellung dieser Ergebnisse zugunsten einer Darstellung der allgemeinen Problemstellungen in der Versuchsplanung auf Wunsch einiger Tagungsteilnehmer zurückgestellt, um die Einordnung der verschiedenen Probleme, die während dieser Tagung diskutiert wurden, zu ermöglichen und somit ein besseres Verständnis für die Einzelergebnisse innerhalb der allgemeinen Gesichtspunkte zu erleichtern.

V. MAMMITZSCH: Die Konvexität der Risikobereiche im Fall atomloser Hypothesen

Vorgegeben seien W.-Maße $P_1,\dots,P_m$ auf (M,F), ferner eine aufsteigende Familie von Unter-σ-Algebren F_t über M, sowie Schadensbereiche $S(x,t) \subset R^m$ für $0 \leq t < \infty$ bzw. $S(x,\infty) = \{(\infty,\dots,\infty)\}$; $x \in M$. Dann besteht ein Sequenztest (T,s) aus einer für alle $i = 1,\dots,m$ P_i-fast endlichen Stoppzeit der Familie (F_t) sowie einer Schadensfunktion s bezüglich T, d.h. einer Abbildung $M \to \bar{R}^m$ derart, daß s F_T-meßbar ist und für alle $x \in M$ gilt $s(x) \in S(x,T(x))$.

Der Risikovektor $r(T,s)$ bestehe aus den Komponenten $\int s\, dP_i$, $i = 1,\dots,m$. Sind die P_i atomlos auf F_t für alle $t > 0$, so sind die Mengen aller eigentlich existierenden Risikovektoren folgender Familien von Tests konvex:

a) alle Tests mit positiver Stoppzeit;

b) alle zur Zeit k abgebrochenen Tests mit positiver Stoppzeit;

c) alle Tests mit einer festen positiven Stoppzeit T_o.

J.A. MÜHLBAUER: Progressively measurable families of correspondences and sequential tests

Given a monoton increasing family of σ-fields $(\mathbb{K}_t)_{t\in[0,+\infty)}$ over a set M, and a family $(\Gamma_t)_{t\in[0,+\infty)}$ of correspondences from M to a topological vector space X, a subspace X' of the topological dual of X, we call $(\Gamma_t)_{t\in[0,+\infty)}$ progressively (R_i)-measurable iff for all $x'\in X'$ $(g(x',\Gamma_t(\cdot)))_{t\in[0,+\infty)}$ is progressively measurable where $g(x',\Gamma_t(m)) := \inf\{x'x : x\in\Gamma_t(m)\}$. If X furnished with $\sigma(X',X)$ is a Hausdorff space and if X' is furnished with $\tau_{X'}$ which is metric and separable such that X is isomorphic to the topological dual of $(X',\tau_{X'})$ and if furthermore $\Gamma_t(m)$ is closed and convex for all $(m,t)\in M\times[0,+\infty)$ and satisfies the following conditions

(a) there exists $x'(m,t)$ such that $g(\cdot,\Gamma_t(m))$ is finite and continuous at $x'(m,t)$,

(b) for each $x'\in X'$ and for every $(m,t)\in M\times[0,+\infty)$ it holds that $g(x',\Gamma_t(m)) > -\infty$, then for any progressively measurable process $(u'_t)_{t\in[0,+\infty)}$ from M to X' there is a progressively measurable process $(\gamma_t)_{t\in[0,+\infty)}$ from M to X such that $\gamma_t(m) \in \Gamma_t(m)$ for all $(m,t) \in M\times[0,+\infty)$ and $u'_t(m) \cdot \gamma_t(m) = g(u'_t(m), \Gamma_t(m))$.

This theorem is used to prove the existence of Bayes tests in the model mentioned above. Certainly this is done under some additional assumptions.

D.W. MÜLLER: On the power of one-sided tests

The aim is to develop an asymptotic theory for tests of a single hypothesis vs. a nonparametric alternative. For probability measures P, Q let $\alpha_{P,Q}(A) = \int_A \sqrt{dP\,dQ}$ be the "affinity measure",

$\gamma^A = \alpha_{P,Q}(A)\ (P(A)\,Q(A))^{-\frac{1}{2}}$ the "conditional affinity".

If $P = N(0,1)$ then $\chi(Q) = (\int x\, d\,\alpha_{P,Q})\ (2\,(1 - \|\alpha_{P,Q}\|))^{-\frac{1}{2}}$ is a measure of one-sidedness of Q. Let $\mathcal{E}_n = (\mathbb{R}, P \in \mathcal{P}_n)$ be a sequence of experiments. Assumptions:

(1) $\inf\{\gamma^n_n;\ n, \mathcal{P}_n\} > 0$; (2) On $\mathcal{P}_n$ there is a metric d_n such that (a) the ε-entropy (d_n) of $\mathcal{P}_n$ is bounded uniformly (n), (b) for every $\varepsilon > 0$ there is $\delta > 0$ such that for all n $d_n(Q',Q'') < \delta$ implies $\|\alpha_n(Q',Q'')\|^n > 1-\varepsilon$; (3) for every

$\varepsilon > 0$ there is $\delta > 0$ such that $P(A) > 1 - \delta$ implies $|(\gamma_n^A)^n - (\gamma_n)^n| < \varepsilon$ for all n; (4) $\chi(Q) \geq \beta$ $(Q \in \mathcal{P}_n)$.

Then: for every $\varepsilon > 0$ there exist m_o, n_o such that $\mathcal{E}_n^n$ can be approximated (up to ε) by some $(N(\mu, I_m) : \mu \in \Theta)$ such that $m \leq m_o$ and $\cos \sphericalangle (e,\mu) \geq \beta$ $(\mu \in \Theta)$ (some direction e).

The asymptotic form of the likelihood ratio test is discussed.

J. OLKIN: Admissibility and minimax results for some multivariate distributions

The multivariate distributions considered are (i) the multinomial distribution, (ii) independent binomial distributions, (iii) the Wishart distribution. The results for (i) and (ii) are joint with M. Sobel; those for (iii) are joint with J. Selliah.

The specific results are the following. Suppose $(X_o, X_1, \ldots, X_k)$ has a multinomial distribution with parameter $\theta_o, \theta_1, \ldots, \theta_k$, $\sum_o^k X_i = n$, $\sum_o^k \theta_i = 1$. Then $(X_1, \ldots, X_k)/n$, $X_o = n - \sum_1^k X_i$, is admissible and minimax for the loss function $L(\delta,\theta) = (\delta - \theta)\, \Sigma^{-1} (\delta - \theta)'$, where $\Sigma = (\sigma_{ij})$, $\sigma_{ii} = \theta_i(1 - \theta_i)$, $\sigma_{ij} = -\theta_i \theta_j$, $i \neq j$, $i,j = 1,\ldots,k$. The method of proof depends on a generalized version of the Cramér-Rao inequality as used by Hodges and Lehmann. A key feature in the proof is the proof of a differential inequality as a consequence of the divergence theorem. A parallel argument yields the result for k independent binomial distributions $B(n_j, \theta_j)$, $j = 1,\dots,k$ with loss function

$$\sum_1^k n_i(\delta_i - \theta_i)^2 / \theta_i(1-\theta_i).$$

For the multivariate normal distribution with known mean, we wish to estimate various functions of the covariance matrix Σ. The particular functions considered are estimating a single covariance σ_{ij}, estimating a linear combination of the covariances $\operatorname{tr} A\Sigma$ (for A positive semidefinite of rank r and for A indefinite), estimating the generalized variance, and for estimating Σ. A typical is the following. Suppose we wish to estimate Σ with loss function $\operatorname{tr}(\Sigma^{-1}\delta(V) - I)^2$, where V is the sample cross-product matrix. Write $V = TT'$, where T is a lower triangular matrix. The estimator $\delta(V) = T D_a T'$, where $D_a = \operatorname{diag}(a_1,\dots,a_k)$ and the a_i are chosen to minimize the risk, is admissible.

D. PLACHKY: Optimal tests and extreme points: A note on the generalized fundamental lemma of Neyman and Pearson

With the help of a characterization of the extreme points of the tests at a given level for a finite hypothesis against a simple alternative it is shown, that there exists an optimal test in the sense of Neyman and Pearson, which has the well-known

0-1-structure and is a primitive function on the "domain of randomization". The number of constants on the "domain of randomization" is less than or equal to the cardinality of the finite null hypothesis.

F. PUKELSHEIM: Linear models and convex programs: nonnegative estimation of variance components

One of the unsolved problems in linear model theory is the possibility of negative estimates $Y'\hat{A}Y$ for the variance components σ_n^2 in the general linear model $Y \sim (\sum_{\pi=1}^{p} b_\pi x_\pi, \sum_{n=1}^{k} \sigma_n^2 V_n)$ where the decomposing vectors x_π and the decomposing dispersion matrices V_n are known while the coefficients b_π and σ_n^2 are to be estimated.

Here we propose a solution to this problem and give two constructive representations of the "best nonnegative estimate" $Y'A^*Y$. The first characterization of A^* is based on the "best defective estimate" $Y'\hat{A}Y$ by appropriately correcting its negative part $\hat{A}_-$. The second characterization of A^* is built on "negativity eliminating projectors" Q which are so defined that in the Q-reduced model (the one that is generated by QY) the best defective estimate is not only automatically nonnegative but, even more, equal to the best nonnegative estimate $Y'A^*Y$ in the original model.

Since the above setting shifts the problem from linearity (linear space of symmetric matrices) to convexity (convex cone of nonnegative definit matrices), our main tool is Fenchel's duality theorem for convex programs. The dual program may also be interpreted within linear model theory and determines, in fact, an interval $[Q_*, Q^*]$ of negativity eliminating projectors Q.

G. ROTHE: A functional limit theorem for quadratic rank statistics

Let X_n, $n \in \mathbb{N}$, be a sequence of i.i.d. random variables with continuous distribution function; let $h \in \mathcal{L}_2(\mathbb{R}^2, \lambda_o^2)$ be symmetric, where λ_o denotes the Lebesgue-measure on $]0,1]$.
Furthermore, for $N \in \mathbb{N}$ let

$$\mathcal{F}_N = \sigma(\{]\tfrac{n-1}{N}, \tfrac{n}{N}], 1 \leq n \leq N\}), \quad h_N = E(h \mid \mathcal{F}_N^2)$$

and let R be the rank vector of $(X_1, \ldots, X_N)$.
For $t \in [0,1]$ define

$$S_N(t) = \begin{cases} \sum_{i,j=1}^{m} h_N\left(\frac{R_i}{N}, \frac{R_j}{N}\right), & \text{if } t = \frac{m}{N} \\ \text{linearly interpolated}, & \text{otherwise} \end{cases}$$

and $T_N = \frac{1}{N} \cdot (S_N - E\,S_N)$.

Then the following theorem holds.

<u>Theorem:</u> If $\int_0^1 h(x,y)\,dy = O\,[\lambda_o]$ then

$$T_N \xrightarrow{\mathcal{D}} \sum_{k \in \mathbb{N}} \gamma_k \,[B_k^2(t) - t(1-t)] \quad ,$$

where B_k, $k \in \mathbb{N}$, are independent Brownian bridges and (γ_k) is a sequence of real numbers depending on h only.

S. SCHACH: <u>Nonparametric tests for randomized block experiments</u>

Kannemann (1976) proposed the following nonparametric test for the comparison of p treatments in N blocks: Let R_{ij} be the rank of the j^{th} observation in the i^{th} block. Define D_{kj} as number of blocks $i = 1,\ldots,N$ with $R_{ij} = k$. Reject the hypothesis of no treatment effects if $K_N = \frac{p}{N} \sum_{k,j=1}^{p} (D_{kj} - \frac{N}{p})^2$ is too large. The asymptotic distribution $(N \to \infty)$ of K_N is derived under the hypothesis as well as under contiguous alternatives. Using Bahadur's concept of approximate asymptotic efficiency it is shown that the test is more efficient than the Friedman or the P a p test. Extensions to more general experimental designs are indicated.

M. SCHAEFER: On the existence of completely symmetric binary designs

Considered is the usual one-way elimination of heterogeneities with fixed effects in the linear model. For testing the hypothesis that all treatment effects are equal it is tried - at least in a special case - to characterize those situations or more precisely those parameters n (total number of observations), u (number of blocks), v (number of treatments) for which the estimation for the D-functional given by Kiefer is sharp. In those situations D-optimum designs are called completely symmetric. It is wellknown that completely symmetric designs do not exist in general. But in the following special case which seems to be quite general it is possible to give a condition which is necessary and sufficient for the existence of completely symmetric binary designs:

Theorem: Let $k = \left[\frac{n-u}{v-1}\right]$, $r = n - u - k(v-1) > 0$. If $v - 1$ is prime then the condition $v(v-2) - r(v-1) \leq k \leq u + r - v$ is necessary and sufficient for the existence of completely symmetric designs.

M. SCHÄL: On dynamic programming and statistical decision theory

A statistical decision model can be described by a triplet (Θ,Δ,R) where Θ is the set of possible states of nature, Δ is the set of all decision rules available to the statistician, and $R: \Theta \times \Delta \to [0,\infty]$ is the risk function. For such a model the following properties play a fundamental part: (i) Δ is compact with respect to the coarsest topology such that $R(\theta,\cdot)$ is lower semi-continuous for all $\theta \in \Theta$. (ii) Δ and R are convex in the following sense:

$$\forall\ \delta_1,\delta_2,\ 0<\alpha<1\ \exists\ \delta_o:\ R(\cdot,\delta_o) \leq \alpha\ R(\cdot,\delta_1) + (1-\alpha)\ R(\cdot,\delta_2)$$

It is know that these assumptions imply the following statements: (I) Existence of optimal decision rules: Bayes solutions, minimax solutions or more general: optimal decision rules in the sense of Bierlein, Bunke and Menges. (II) Minimax-theorem (Sion) (III) Complete class theorems (LeCam).

The purpose is to give sufficient conditions for properties (i) and (ii) in a statistical decision model including as special cases the decision model of the theory of dynamic programming (Markov decision theory) as well as the usual sequential and fixed-sample-size statistical decision model.

E. SEIDEN: A problem in repeated measurement design

The motivation for the research described here came from a medical problem. An orthopedic surgeon was concerned with statistical evaluation of resultant laxity which is caused by each of the possible combinations of ligamenteous injuries of a knee. Since the experimental units were human joints it was desired that the number of them be as small as possible. The design problem in general can be formulated as follows. Given that the problem is to investigate all possible combinations of n treatments in a repeated measurement design it is clear that at least $\binom{n}{(n-1)/2}$ experimental units will be required when n is odd. It is shown that this minimum number of experimental units will in fact suffice. The crucial part of the design problem is how to proceed from $\binom{n}{(n-1)/2}$ layout to $\binom{n}{(n+1)/2}$ layout. Another utilization of this type of design could occur when one has already carried out a design using an unreduced BIB of the type $\binom{n}{(n-1)/2}$ and would like to enlarge it to an unreduced BIB of type $\binom{n}{(n+1)/2}$ adding one plot to each block. For n = 5 it is shown that there are exactly three non isomorphic ways of assigning the treatments to the 10 experimental units. Non isomorphic here means that no permutation of the treatments will take one onto the other. It is shown that for the whole experiment there are 336 balanced non isomorphic designs in the case n = 5. Some results regarding the whole experiment for n > 5 are also discussed.

E. SONNEMANN: <u>Optimality and existence of complete latin squares</u>

Consider the following linear model with residual effects λ_k of first order, $X_{ij} = \rho_i + \kappa_j + \lambda_{d(i,j-1)} + \tau_{d(i,j)} + e_{ij}$, where the design $d : \{1,\dots,r\} \times \{1,\dots,c\} \to \{1,\dots,t\}$ is surjective with $d(i,0) := d(i,1)$. If one is interested in testing $H_0 : \tau_1 = \ldots = \tau_t$ or in estimating $\tau_k - \tau_t$, $k = 1,\ldots,t-1$, one knows that in the regular case (i.e. $t|r\cap c$) d is D-optimum iff the matrix $(d(i,j))$ forms a row-complete latin rectangle (i.e. that $(d(i,j))$ is balanced w.r.t. rows and columns, and w.r.t. pairs of neighbours within rows).

Specializing the concept of sequenceable groups to that of generating Hamiltonian paths in the complete digraph and generalizing this into another direction, it is possible to show the existence of row-complete latin squares of order 9×9 and 15×15 which do not base on a group and which cannot be completed.

E. SPJØTVOLL: <u>Ordering ordered parameters</u>

A stepwise multiple comparison method similar to the Newman-Keuls method is proposed for the case where it is a priori known that the parameters are ordered. It is shown how the significance levels at the various steps can be chosen to keep the probability of at least one false rejection less than a given upper bound.

The method can be used for both continuous and discrete distributions since it is essientially based upon tests ordering any two of the parameters.

J. SRIVASTAVA: Some combinatorial design problems in search linear models

Consider the model $E\underset{\sim}{y} = A_1 \underset{\sim}{\xi}_1 + A_2 \underset{\sim}{\xi}_2$, $V(\underset{\sim}{y}) = \sigma^2 I_N$, where $\underset{\sim}{y}$ $(N\times 1)$ is a vector of observations, $A_i(N\times v_i)$, $i = 1,2$, are known matrices, $\underset{\sim}{\xi}_1$ is a vector of unknown parameters, and so is $\underset{\sim}{\xi}_2$, except that it is known that atmost k elements of $\underset{\sim}{\xi}_2$ are non-zero, where k is a known or unknown positive integer. Usually, in applications k is much smaller than v_2. In this talk a survey of results in this area is given along with some new result. The advantage of search linear models over ordinary models is pointed out. As an example, an application to industrial psychology is demonstrated. An introduction to factorial designs is given, and the relevance of search models then is pointed out. The impact of the concept of search models on optimal designs is discussed, and a theorem on A,D-optimality presented. The property P_t of matrices (no set of t columns linearly dependent) is discussed in the context of factorial designs and coding theory. It is shown that the same property arises in search models. This is discussed specially for the case when $v_1 = 0$. For this case, when $\underset{\sim}{y}$ corresponds to observations in a factorial design, a necessary condition that A_2 has property P_{2^n} is presented. This last result, whose proof is sketched to give a flavor of the subject, is new.

W. STADJE: Asymptotische Effizienz der Sequentialquotiententests im Vergleich mit dem besten nicht-sequentiellen Test

Gegeben ein W-Raum $(\mathfrak{X}, \mathfrak{A}, P)$; weitere W-Maße $Q, Q_1, Q_2, \ldots$; sind $\alpha, \beta \in (0, \frac{1}{2})$, so kann man folgende Zahlen definieren:

$$K(\alpha,\beta,P,Q) := \inf \{m \mid \exists\, \phi : \mathfrak{X}^m \to [0,1] \text{ mb. mit } \int \phi \, dP^m \leq \alpha,\ \int (1-\phi)\, dQ^n \leq \beta\},$$

$$S(\alpha,\beta,P,Q) := \inf \{\max(E_P(\tau), E_Q(\tau)) \mid (\phi,\tau) \text{ sequentieller Test mit Fehler } 1. \text{ Art bzw. } 2. \text{ Art} \leq \alpha \text{ b.z.w. } \leq \beta\} .$$

Zwei Probleme werden behandelt:

1. $Q \to P$ in einem zu präzisierenden Sinne;
2. P, Q feste W-Maße, (i) α fest, $\beta \to 0$, (ii) $\alpha, \beta \to 0$.

Zu 1.: $\|P - Q_k\| \to 0$ ist äquivalent mit $K(P,Q_k) \to \infty$ sowie mit $S(P,Q_k) \to \infty$ für irgendein festes Paar (α,β). Damit $\frac{K(P,Q_k)}{S(P,Q_k)}$ konvergiert, hat man "$Q_k \to P$" so zu definieren:

Sei $P = f\, d\nu$, $Q_k = f_k\, d\nu$, ν ein W-Maß.

μ_k, σ_k^2, c_k^3 b.z.w. μ_k', $\sigma_k'^2$, $c_k'^3$ seien die Erwartung, die Varianz, das absolute dritte Moment von $\log \frac{f_k}{f}(X_1)$ unter P b.z.w. Q_k.

Dann:

$$\text{(i)}\ \mu_k \to 0, \qquad \text{(ii)}\ \frac{\mu_k}{\sigma_k^2} \to \lambda, \quad \frac{\mu_k'}{\sigma_k'^2} \to \lambda', \quad \frac{\sigma_k'^2}{\sigma_k^2} \to a$$

mit $-\infty < \lambda < 0 < \lambda' < \infty$, $a > 0$

$$\text{(iii)}\ \left(\frac{c_k}{\sigma_k}\right)_{k \geq 1}, \quad \left(\frac{c_k'}{\sigma_k'}\right)_{k \geq 1} \text{ sind beschränkt.}$$

Es gilt nun: Aus (i) - (iii) folgt:

$$\frac{E_P(\tau_k)}{K(P,Q_k)} \to \frac{1}{\lambda} \left(\frac{\lambda - \lambda' a}{2\alpha + \sqrt{a}\, 2\beta}\right)^2 \left(\alpha \log \frac{1-\beta}{\alpha} + (1-\alpha) \log \frac{\beta}{1-\alpha}\right) \quad \text{und}$$

$$\frac{E_{Q_k}(\tau_k)}{K(P,Q_k)} \to \frac{1}{\lambda'} \left(\frac{\lambda - \lambda' a^{-1}}{2\alpha + \sqrt{a^{-1}}\, 2\beta}\right)^2 \left(\beta \log \frac{\beta}{1-\alpha} + (1-\beta) \log \frac{1-\beta}{\alpha}\right),$$

((ϕ_k, τ_k) sei dabei der SQT von P gegen Q_k mit Fehlern α, β)

Dies ist eine Erweiterung und Verbesserung einer Formel von Paulsen (1947), die sich für $\lambda = -\frac{1}{2}$, $\lambda' = \frac{1}{2}$, $a = 1$ ergibt.

(Beweis dieses Satzes durch Anwendung von Invarianzprinzipien in $C[0,\infty)$).

<u>Zu 2.:</u> (i) $\dfrac{E_P(\tau_k)}{K(\alpha,\beta_k)} \to 1 - \alpha$, $\dfrac{E_Q(\tau_k)}{K(\alpha,\beta_k)} \to 0$

((ϕ_k, τ_k) SQT für P gegen Q mit Fehlern $\alpha, \beta_k \to 0 (k \to \infty)$).

(ii) Hier ergeben sich durch eine leichte Verallgemeinerung einer Arbeit von Berk (1973; 15) vernünftige Grenzwerte.

W. URFER: <u>Berechnung der Maximum-Likelihood-Schätzer für die Parameter eines Varianzanalyse-Modells</u>

Gegeben sei das lineare Modell $y = X\beta + \sum_{i=1}^{e} V_i v_i$ mit bekannter $(n\times m)$-Matrix X, unbekanntem $\beta \in \mathbb{R}^m$, bekannten $(n\times k_i)$-Matrizen V_i und Zufallsvektoren $v_i \sim N_{k_i}(0, \sigma_i^2 I_{k_i})$ für

$i = 1,\dots,\ell-1$. Ferner sei $V_\ell = I_n$, $v_\ell \sim N_n(0,\sigma_\ell^2\, I_n)$ und $E(v_i\, v_j') = 0$ für $i \neq j$. Zu schätzen sind β und σ_i^2 für $i = 1,\dots,\ell$.

Bezeichnet man mit $Z := (V_1,\dots,V_{\ell-1})$, $v := (v_1',\dots,v_{\ell-1}')'$,

$$D := \sum_{i=1}^{\ell-1} \oplus \ \sigma_i^2\, I_{k_i} \quad \text{und} \quad (Z'Z + \sigma_\ell^2\, D^{-1})^{-1} = \begin{pmatrix} \Gamma_{1,1} & \cdots & \Gamma_{1,\ell-1} \\ \vdots & & \vdots \\ \Gamma_{\ell-1,1} & \cdots & \Gamma_{\ell-1,\ell-1} \end{pmatrix}$$

sind ferner β^* und v^* Lösungen von

$$\begin{pmatrix} X'X & X'Z \\ Z'X & Z'Z + \sigma_\ell^2\, D^{-1} \end{pmatrix} \begin{pmatrix} \beta^* \\ v^* \end{pmatrix} = \begin{pmatrix} X'y \\ Z'y \end{pmatrix}$$

, dann führt folgendes iterative Verfahren zur Lösung der ML-Gleichungen bei positiver Startlösung zu positiven Iterationen:

$$(\sigma_i^2)_{k+1} = \frac{1}{k_i}\,\{(v_i^{*\prime})_k\, (v_i^*)_k + (\sigma_\ell^2)_k\, \mathrm{tr}(\Gamma_{ii})_k\} \quad \text{für } i = 1,\dots,\ell-1$$

$$(\sigma_\ell^2)_{k+1} = \frac{1}{n}\,\{y'y - (\beta^{*\prime})_k\, X'y - (v^{*\prime})_k\, Z'y\} .$$

H. WITTING: Nonparametric alternatives and asymptotic theory of sequential rank tests

Using the concept of a nonparametric alternative, which is discussed at the beginning, a sequential linear rank test (S L R T) is given, which is optimal in the sense that asymptotically for near alternatives it is equivalent to the S P R T , i.e. it has asymptotically the same OC- and ASN-function. The theory follows the lines of the asymptotic theory of non-sequential linear rank tests, introduced by Hajek (AMS 33, 1962) and extended by Behnen (AMS 42 + 43, 1971 + 1972), but is technically more involved, since it uses invariance principles for linear rank statistics. These were proved for the case of testing symmetry w.r.t. O by M. Müller-Funk (1976) and for the case of testing independence by N. Bönner (1976), using certain almost sure representations of linear rank statistics by sums of independent random variables (of the subsequent paper of N. Bönner). The relation to the sequential rank test of Sen-Ghosh (Ann. Stat. 2, 1974) is mentioned.

H.P. WYNN: A review of optimum design algorithmns

Since the introduction of optimum design algorithmns (Wynn, 1970, 1972; Federov, 1972) there has been a successful search for algorithmns for more general optimality criteria (Federov & Madynkov, 1972; Gribanek & Kortanek, 1975; Whittle, 1973; Atwood, 1976 a,b). The problem is to minimize a convex function $\Phi(\cdot)$ on the moment space $\mathfrak{M} = (M / M = \{ \int f_i(x)\, f_j(x)\, \xi(dx) \})$ of a design problem (Kiefer & Wolfowitz notation). A special feature of design is that $\Phi(M)$ may be equal to ∞ (e.g. $\Phi = -\log\det(M)$ D-optimality). The main theorems are of two kinds:

(I) optimum direction $(\min_{M} \nabla \Phi(M_n, M))$, arbitrary step length, $M_{n+1} = (1-\alpha_n)\, M_n + \alpha_n \bar{M}_n$, $\sum \alpha_n = \infty$, $\alpha_n \to 0$.

(II) optimum direction, optimum step length (Atwood, 1974 + 1976). A new condition on Φ, $\|\nabla \Phi\| < c_k \|\nabla \Phi\|^2$ for $\Phi > k$ in the direction $\bar{M}_n - M_n$ (optimum) extends the proofs for D-optimality (Wynn, 1970; Tsay, 1976) to other "non-singular criteria": trace (M^{-1}), trace $(A M^{-1})$, trace (M^{-P}).

New work of Wu based on the normalized gradient $[\nabla \Phi(M_n, M)]\ (\|M - M_n\|)^{-1}$ lead to finite dimensional (finite support) algorithmns of the projected gradient constrained steepest descent conjugate gradient type. The main feature of these new algorithmns is that they do not move in a vertex direction (I,II) but in some more general direction. This leads to faster algorithmns but the line search at each stage is harder.

W.R. VAN ZWET: A proof of a conjecture of Kakutani

Define random variables $X_1, X_2, \ldots$ as follows. X_1 has a uniform distribution on $(0,1)$; given $X_1, \ldots, X_{N-1}$, the random variable X_N has a uniform distribution on the largest of the N subintervals into which $X_1, \ldots, X_{N-1}$ sub-divides $(0,1)$.

Let F_N be the empirical distribution function of $X_1,\ldots,X_N$.

Is it true that $\limsup_{N\to\infty} \sup_{x\in(0,1)} |F_N(x) - x| = 0$ with probability 1?

This question, which gave rise to much discussion at the meeting on "Stochastics" at Oberwolfach earlier this year, is answered in the affirmative.

W.R. VAN ZWET: <u>A Cramér-Rao type inequality for adaptive estimators</u>

Let $\mathcal{F}$ be the class of densities f on $\mathbb{R}^1$ that are positive and symmetric about zero, possess a positive and finite Fisher information $I_f = \int (f'/f)^2 \, f \, d\lambda$ and satisfy:

there exists $\varepsilon \to 0$ such that for all θ with $|\theta| \leq \varepsilon$

$$\left| \frac{f(x-\theta) - f(x)}{\theta\, f(x)} \right| \leq h(x) \text{ holds with } \int h^2 \, f \, d\lambda < \infty .$$

<u>Theorem:</u> $X_1,\ldots,X_N$ are i.i.d. with density $f(\cdot-\theta)$ and $f \in \mathcal{F}$.

$T = \tau(X_1,\ldots,X_N)$ is a location invariant unbiased estimator of θ.

Then there exists $R(x) = \rho(X_1,\ldots,X_N; x)$ where ρ depends only on τ and <u>not</u> on f or θ, such that

$$\sigma^2_{f(\cdot-\theta)}(T) \geq \frac{1}{N I_f} + \frac{1}{N} E_f \left\{ \int \left(R(x) + \frac{1}{I_f} \frac{f'(x)}{f(x)} \right)^2 f(x) \, dx \right\} .$$

The implications of this result - which was obtained in collaboration with C. Klaassen - for adaptive estimators are discussed.

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