MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 5/1977

Mathematical Economics

30.1. bis 5.2.1977

The purpose of this first Oberwolfach meeting on Mathematical Economics was to further the active interchange between mathematicians and economists. To this end, the conference was limited to those areas of Economic Theory which are challenging both from an economic and a mathematical point of view. We scheduled 10 invited papers, all of them by leading specialists, some of which had survey character reviewing well developed domains, while others gave an exposition of current lines of research in a specific field. In addition, about 15 shorter communications on special topics were presented. Most of the talks have been concerned with problems related to General Equilibrium Analysis, such as existence, uniqueness, stability and computation of economic equilibria. Another domain was game theory in its relation to economics. A whole day was devoted to catastrophy theory and its possible applications to economics. Having discussed the first four days in the framework of Walras, on the last day Non-Walrasian Equilibrium theory was introduced. Finally we had a special evening session where "Open problems" were presented which are included in this Tagungsbericht.

It would have been impossible to gather this impressive group of specialists without the financial support of the Mathematisches Forschungsinstitut. The excellent facilities created a stimulating atmosphere which was appreciated by all the participants.

Die Tagungsleiter: H. Föllmer
W. Hildenbrand
D. Sondermann

Teilnehmer

B. Allen, Berkeley
R.J. Aumann, Jerusalem
P. Artzner, Straßburg
Y. Balasko, Paris
M.L. Balinski, Laxenburg
H. Bauer, Erlangen
T.F. Bewley, Harvard
J.M. Bismut, Paris
A. Cruceanu, Bukarest
G. Debreu, Berkeley
P. Dehez, Heverlee
E. Dierker, Bonn
H. Dierker, Bonn
W. Eichhorn, Karlsruhe
D.P. Flaschel, Berlin
H. Föllmer, Bonn
G. Fuchs, Paris
J.M. Grandmont, Paris
B. Grodal, Kopenhagen
R. Henn, Karlsruhe
U. Herkenrath, Bonn
K. Hildenbrand, Bonn
W. Hildenbrand, Bonn
W. Ibert, Bonn
K. Jacobs, Erlangen
R. John, Bonn
Y. Kannai, Rehovot
D. Kölzow, Erlangen
U. Krengel, Göttingen
J.J. Laffont, Paris
G. Mägerl, Erlangen
E. Malinvaud, Paris
A. Mas-Colell, Berkeley
W. Neuefeind, Bonn
M. Nermuth, Heverlee
C. Oddou, Marseille,
N. Reif, Hamburg
J. Rosenmüller, Karlsruhe
D. Schmeidler, Ramat-Aviv
W.J. Shafer, Detroit
D. Sondermann, Hamburg
R. Thom, Bures-sur-Yvette
M.T. Todd, Cornell University
W. Trockel, Bonn
K. Vind, Kopenhagen
C.C. v. Weizsäcker, Bonn
H. Wiesmeth, Hamburg
D. Zagier, Bonn

A. Abstracts of Invited Papers

R.J. AUMANN : Some Recent Developments in the Theory of the Shapley Value

A survey. The results include the following:

(A) Finite Games
- (i) Roth's replacement of the efficiency axiom by "Strategic Risk Neutrality" (1977).
- (ii) Myerson's characterization of the value in terms of a communication graph on the set of players (1977).
- (iii) Some "practical" applications: Littlechild's application to Airport Landing Fees (~1975); Telephone charges at Cornell (Billera + collaborator) (1977).

(B) Non-Atomic Games
- (i) The Diagonal Conjecture: Counter-example by Neyman & Tauman (1976). Counter-example by Tauman in the Reproducing Case (1977). Proof by Neyman when continuity in the variation norm is assumed (1977).
- (ii) Progress by Neyman on the existence of the asymptotic value for games $f \circ \mu$ when f has jumps or is singular continuous (1977).
- (iii) Asymptotic results for the no-side payment value of market games (Champsaur ~1975, Mas-Colell 1977).
- (iv) Guesnerie's investigation of values of syndicated economies (1977).
- (v) Rosenthal's investigation of values of public goods economies (1976).
- (vi) Dubey's investigation of non-efficient values, with independent work of Neyman.

Y. BALASKO : Regular Economies

A survey of some of the research that has followed Debreu's 197o seminal paper "Economies with a finite set of equilibria". The survey was mainly concerned with exchange economies and emphasized the following questions:

1.) structure of the equilibrium manifold
2.) structure of the set of stable equilibria
3.) singularities of the Debreu mapping
4.) global properties of the Debreu mapping.

T.F. BEWLEY : Economic Implications of the Permanent Income Hypothesis

Equilibrium theory is reinterpreted and modified in a way that, it is hoped, clarifies its significance. Existing equilibrium theory is interpreted to be a theory of the long-run behavior of an economy. It is maintained that an appropriate theory of short-run behavior is one in which consumers have a constant marginal utility of money. The constancy of this marginal utility is termed the permanent income hypothesis. This hypothesis is justified in terms of an intertemporal model of consumer behavior.

Suppose that prices and a consumer's income vary according to a stationary process and that he can save and dissave, and that his utility function is additively separable with respect to time. The solution to the consumer's finite horizon optimization problem determines a marginal utility of money in the initial period which depends on the consumer's holdings of money. This marginal utility converges to a constant as the horizon and the consumer's money holdings go to infinity.

Constancy of the marginal utility of money determines a consumer's demand as a function of prices. This function is called a short-run demand function. When utility functions are strictly concave, the aggregate demand function determined by the short-run demand functions is such that the Tâtonnement price adjustment process is globally stable. This justifies the assumption that markets always clear. This equilibrium is termed short-run equilibrium. Short-run equilibria should be thought of as fluctuating randomly. Because of the constancy of the marginal utility of money, the time path of short-run equilibria gives a Pareto optimal allocation of goods over time. This optimality is achieved without Arrow-Debreu markets for contingent claims.

It is also shown that if the marginal utilities of money are thought of as adjusting slowly over time in response to consumer's average net expenditures, the resulting adjustment process may be unstable. This instability corresponds to the instability that may appear in the Tâtonnement price adjustment process when demand functions are defined in the usual way.

G. DEBREU : The Core of a Large Economy

A survey of some of the research of the last sixteen years on the relationship between two concepts of equilibrium for an economy E (a) the set $W(E)$ of the competitive allocations of E and (b) the core $C(E)$ of E, and specifically on the asymptotic equality of $W(E)$ and of $C(E)$ for economies with a large number of nearly insignificant agents. Three theorems were emphasized:

1.) Aumann's theorem on the equality of $W(E)$ and of $C(E)$ for an atomless economy
2.) W. Hildenbrand's theorem on the behavior of $C(E_n)$ for a competitive sequence $\{E_n\}$ of economies without assumptions of convexity or differentiability on preferences
3.) Grodal's theorem on the rate of convergence to zero of the distance between $C(E_n)$ and $W(E_n)$ for a competitive sequence of economies with convex, differentiable preferences.

J.M. GRANDMONT : The Logic of the Fix Price Method

This paper surveys some recent models which have used the fixed price method in order to study problems which were exclusively in the realm of macroeconomics (unemployment, inflation, etc.). In these models, short-run adjustments take place through quantity rationing instead of price movements. A crucial assumption common to these models is that, when there is a disequilibrium on some market, the short side of the market always imposes his views. For instance, in the case of an excess demand, all the sellers realize their plans, while some buyers are constrained to buy less than intended.

This survey included a precise presentation of some models currently used, in a general equilibrium framework. In particular, the equilibrium concepts of Drèze and of Benassy were discussed in detail and compared.

The efficiency properties of these equilibrium concepts were investigated. It was shown that the resulting allocation of commodities was efficient if one restricted the recontracting process to take place market by market. This result emphasizes the role of money in the exchange process and thus makes precise some ideas which have been put forward for a long time by macro-economists in order to explain unemployment.

Lastly,an analysis of this equilibrium concept by means of the theory of games was presented. It was shown that this equilibrium concept would be justified by exhibiting a bargaining process which made the equilibrium allocation the only stable outcome in large economies.

E. MALINVAUD : <u>The Theory of Fixed Price Equilibria and the Macroeconomic Analysis of Unemployment</u>

The theory may start from the following notions concerning only the aggregate situation of each market: state of the market, (aggregate) purchase and sale, effective demand and effective supply. These characteristics depend on both the prevailing price vector and a vector of parameters defining the nature and extent of rationing. A fixed price equilibrium is defined by two conditions:

(i) net trade is zero on each market

(ii) the state of each market agrees with the inequality between purchase/sale and demand/supply on this market.

In such a framework two adjustment processes can be defined: a quantity adjustment process operating with fixed prices (stability then requires that the roots of a particular matrix have negative real parts), a price adjustment process along which prices are revised as a function of excess demands and fixed price equilibria are simultaneously adjusted.

This theory is appropriate for a full discussion of the policies intended to reduce unemployment. A macroeconomic simple model with three commodities (good,labor,money) permits the examination of fixed price equilibria corresponding to three main types of market states: Keynesian unemployment where supplies of good and labor are rationed, Inflation where demands of good and labor are rationed, Classical unemployment where supply of labor but demand of good are rationed. Stability of each type of equilibrium follows from three conditions that can safely be assumed to hold. Comparative statics can then be dealt with.

If the stability conditions are met, there is a neighbourhood of "the" Walrasian equilibrium (stationary point of the price adjustment process) in which the fixed price equilibrium exists for any price vector and is unique.

Stability of the price adjustment process can then be studied. But more important questions are involved for the macroeconomics of medium term dynamics, which is now a real challenge.

A. MAS-COLELL : On the Smoothing of Demand by Aggregation: a Survey

Work of the last 4 or 5 years on regularizing effects of demand aggregation was surveyed. Three smoothing problems were posed: uniqueness of mean maximizers, continuity of aggregate demand, differentiability (C^1) of aggregate demand. Two problems were emphasized:

1.) find sets of sufficient conditions
2.) try to establish denseness results in contexts where stability of the smoothing property is guaranteed.

The survey included:

a.) the distribution approach of Aumann and Hildenbrand and its convexifying effects
b.) genericity (of continuity) and denseness (of smoothness) of aggregate demand in the distribution approach (Araujo, Neuefeind,...)
c.) the parameterized approach of Sondermann
d.) theorems on the uniqueness of maximizers (Sondermann, Araujo,...) in the parametric approach
e.) theorems on the continuity of mean demand in the parametric approach (Sondermann)
f.) difficulties with smoothness.

It was suggested that the smoothness (C^1) problem can be made more tractable, perhaps, with simple kinds of consumption sets (e.g. $X = K \times (0,\infty), \# K < \infty$) and settling for smoothing less than C^1 (but not much less, e.g. C^1 on an open set of full measure).

W.J. SHAFER : <u>Recent Results on the Existence of Competitive Equilibrium</u>

This lecture surveys some recent results in proving existence of equilibrium in an Arrow-Debreu type model with weakened hypotheses on individual preferences. Specifically, it is shown, that the hypotheses of transitivity and completeness of preference relations are unnecessary for existence (a result due to A. Mas-Colell). It is then demonstrated that with these weakened assumptions on preferences, one can include externalities of a very general form into the model and still retain existence of equilibrium. Finally, it is demonstrated that in this same context the hypothesis of free disposal can be eliminated, and that the technique of quasi-equilibrium can also be applied.

R. THOM : <u>Catastrophe Theory and Some Applications to Economics</u>

We start first by defining catastrophe theory as a theory of automata where the input determines the output up to a finite choice: input and output are points of Euclidian spaces $\mathbb{R}^n$, $\mathbb{R}^p$ resp. and the automaton is defined by a correspondence graph Γ which is (in general) an embedded n-dimensional manifold in the product space $\mathbb{R}^n \times \mathbb{R}^p$. Where such a manifold Γ is transversal to the fibers $\pi : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$, the output is a smooth function of the input. "Catastrophes" do occur at those points $q \in \mathbb{R}^p$ which are singular values of the projecting map $\pi : \Gamma \to \mathbb{R}^n$ (control or input space). Catastrophe theory deals with cases where these singular values are of "generic type", that is topologically structurally stable w.r. to a slight perturbation of the embedding Γ. Moreover it is supposed that for any $q \in \mathbb{R}^p$ the corresponding possible outputs are given by the equilibria of a fast dynamic in the fiber $\pi^{-1}(c)$. Elementary catastrophe theory deals with the case where this internal fast dynamics is a gradient dynamics $\dot{X} = -\mathrm{grad}_x V(x,u)$, $x \in \mathcal{O}$, $u \in \mathcal{J}$. In that case, equilibria are minima of the potential $V(x,u)$. For any $u \in \mathcal{J}$, the choice of the dominating regime is one of the minima of $V(x,u)$. To choose this minimizing point, we may apply Maxwell's rule: the dominating regime is the point where V reaches its absolute minimum.

This convention justifies considering in any function space $C^\infty(M,\mathbb{R})$ of smooth functions on a compact manifold M the subset Mxw (Maxwell set) with the following definition: any function f in the complementary $C^\infty(M,\mathbb{R}) \setminus$ Mxw reaches its absolute minimum at only one simple minimizing point. The Maxwell set is closed with no interior point. Moreover it admits a stratification Mxw = $\cup X_i$, union of manifolds of increasing codimension. The codimension one stratum corresponds to a double minimum, and there are two types of strata of codimension two. The one associated to the triple edge , and the one associated with a minimum of local type $V = x_0^4 + \Sigma x_i^2$, whose local model in the Maxwell set is the free edge. The notion of transversal intersection of a stratified set is then defined.

In the economic case, we deal with a map of the product A × P, A continuum of agents (preferences + endowment), P prices, to a function space $C^\infty(R^n,\mathbb{R})$ of utility functions u (defined on the budget hyperplane for every point $a \in A$). Let us call $\mu(a,p)$ one of the maximizing points of u for $(a,p) \in A \times P$. We study integrals of type

$$I(p) = \int_A \mu(a,p)\,da$$

from the point of view of smoothness.

Using Hironaka's desingularization theorem one may prove the following: if $S \subset \mathbb{R}^n$ is a stratified space (semi-analytic for custom), f a map of $\mathbb{R}^m$ into $\mathbb{R}^n$ which is transversal onto S, if X is a stratum of S of codimension k, then $f^{-1}(X)$ is a stratum of $f^{-1}(S)$ of dimension m - k. Let ω be a (m - k) differential form on $\mathbb{R}^n$. Then the integral

$$I(f) = \int_{f^{-1}(X)} f^*(\omega)$$

is defined if f is proper on S.

If f varies in such a way as to stay transversal to S and proper on S, then I(f) is a smooth function of the map f.

This theorem then stems from the fact that at any point of the boundary of the stratum X, there exists a local desingularizing model with a manifold with corners.

But in case of the integral I(p) a new difficulty arises due the fact that $\mu(a,p)$ is not a smooth function of a in the neighbourhood of singularity strata. This difficulty may be taken care of by the use of Mather's theory of singularities of functions,at least for the "good" dimensions, when dim P is less than seven. In such a case, there are only finite models up to equivalence, and we may construct a flow leaving the singularity isomorphic. Starting from codim 7 (double cusp $V = x_2^4 + x_2^4$) the presence of modules introduces a new difficulty - which, quite likely, may be disposed of by using a stratumwise differentiable controlled flow.

M.T. TODD : Fixed-Point Algorithms for Computing Economic Equilibria

We describe recent fixed-point algorithms and their application to the computation of economic equilibria. The algorithms of Scarf, Kuhn, Merrill and Eaves are presented. While all these algorithms are based on a common combinational argument, the latter two can be viewed as tracing fixed points as a simple affine function is deformed into (possibly an approximation of) the function of interest. The deformation is performed using piecewise linear approximations based on triangulations. While the algorithms are capable of approximating fixed points of upper semi-continuous point-to-set mappings, they can be implemented so as to achieve quadratic convergence in the case of smooth functions and compare favorably with discrete Newton methods.

Abstracts of Contributed Papers

P. ARTZNER : On Some Search Models

The following Markov chain is introduced: the set of states is $S = \{s | s : \{1,2,..,n\} \to \mathbb{N}, \sum_i s_i = \Delta\}$, Δ a given integer, $m = [\frac{\Delta}{n}]$.

The one-step transitions allowed (i.e.(strictly) positive transition probabilities) are the following: $s \to s'$ if and only if there exist $i,j \in \{1,..,n\}$, $i - j = \pm 1 \pmod n$ and $s(i) > s(j)$; then $s'(i) = s(i) - 1$, $s'(j) = s(j) + 1$, $s'(k) = s(k)$ for $k \neq i,j$ (moreover if $\Delta = m \cdot n$, the constant function $\frac{\Delta}{n}$ is absorbing by definition). It is proven that the chain has only one ergodic class namely $F = \{s | \Sigma s_i = \Delta, m \leq s(i) \leq m + 1, i = 1,..,n\}$.

Another search model is sketched where a "form behavior" is introduced.

E. DIERKER : Mean Demand in Case of Non-Convex Preferences

In general the demand of an individual agent cannot be described by a function but only by a correspondence. If one has a measure suitably spread out on a large set of agents' characteristics one may hope that aggregation yields a C^1 mean demand function. Since the Lebesgue measure is not available to describe the distribution of preferences one is led to aggregate first over the wealth keeping preferences fixed. This way one obtains the result that demand becomes C^1 except on a closed null set of prices that depends on the preference considered. Integrating over preferences then yields, under suitable assumptions, a C^1 aggregate demand function.

W. EICHHORN : Ray-Homothetic Correspondences

Let the correspondence

(1) $P : \mathbb{R}^n_+ \to$ power set of $\mathbb{R}^m_+$

be given. A generalization of the homogeneous correspondences (1) satisfying

(2) $P(\lambda x) = \lambda^r P(x)$ for all $(\lambda,x) \in \mathbb{R}_{++} \times \mathbb{R}^n_+$

are the so-called ray-homothetic correspondences (1) which satisfy the functional equation

(3) $P(\lambda x) = \psi(\lambda,x)P(x)$ for all $(\lambda,x) \in \mathbb{R}_{++} \times \mathbb{R}^n_+$

where $\psi : \mathbb{R}_{++} \times \mathbb{R}^n_+ \to \mathbb{R}_+$, $\psi(1,x) = \psi(\lambda,o) = 1$.

Consider the problem(s)

(4) $\max \{pu \mid u \in P(\lambda x^*)\}$ for every fixed $\lambda \in \mathbb{R}_{++}$,

where p is the vector of the prices of the goods produced, u is the vector of the quantities of these goods, and x^* is a given input vector.

Theorem 1: Let P be ray-homothetic and let u^* be a solution of (4) for $\lambda = 1$. Then $\psi(\lambda,x^*)u^*$ is a solution of (4) for every λ. ("Linear expansion path property" of P satisfying (3)).

Theorem 2: Let P satisfy (3) and let the inverse $P^{-1} = :L$ satisfy

(5) $L(\mu u) = \chi(\mu,u)L(u)$ for all $(\mu,u) \in \mathbb{R}_{++} \times \mathbb{R}^m_+$,

where $\chi : \mathbb{R}_{++} \times \mathbb{R}^m_+ \to \mathbb{R}_+$, $\chi(1,u) = \chi(\mu,o) = 1$.

Let both $\lambda \to \psi(\lambda,x)$ and $\mu \to \chi(\mu,u)$ be strictly increasing from 0 to $+\infty$. Then free disposability of inputs (i.e., $x \leq x^*$ implies $P(x) \subseteq P(x^*)$) implies the homogeneity (2) of P on $\mathbb{R}^{n+1}_{++}$, whereas weak disposability of inputs (i.e., $P(x) \subseteq P(\lambda x)$ for all $\lambda \in [1,\infty[$, $x \in \mathbb{R}^n_+$) implies the so-called semi-homogeneity of P:

(6) $P(\lambda x) = \lambda^{r(x/|x|)}P(x)$ on $\mathbb{R}^{n+1}_{++}$.

Proofs of the above theorems are due to R.W.Shephard, R.Färe, and W.Eichhorn (yet unpublished).

G. FUCHS : <u>Dynamics of Expectations in Temporary Equilibrium</u>

In temporary general equilibrium theory the action of the agents depends on their forecasts of what the future will be. These forecasts themselves are values of expectation functions defined on some set of attainable information. If one considers that expectation function should not be taken as exogenous data, the problem arises of their formation and revision along time.

In such a context a state of an economy is a point in the infinite dimensional space describing both prices of commodities and the individual expectation function of each agent. The problem then is to define a reasonable dynamics on the space of states. For that one can add to the standard market clearing conditions rules of evolution for expectations. The rules investigated here are of the error learning type. According to the hypothesis made on the information structure of the model one can define in this way several dynamics. Three of them are studied here corresponding to extreme situations where the stationary states correspond to only one or to any expectation function. In each case results are given for asymptotic stability of stationary states, so one can discuss the problem of formation of expectations and price convergence, separately or together, according to the circulation of information.

J.M. GRANDMONT : <u>Intermediate Preferences and the Majority Rule</u>

The Majority Rule has been shown to be transitive when the preferences of the agents in the society satisfy some restrictions, e.g. Single Peakedness (Block, Arrow, Sen). The spirit of these restrictions is to assume that some preferences are not present in the society under consideration.

In this work, one does not exclude a priori some preorderings, but rather one focuses the attention upon the shape of the distribution of preferences. More precisely, one considers a family of preferences $(R_a)_{a \in A}$ indexed by a vector a belonging to an open convex subset A of an Euclidean space E^n. This family displays the following regularity property. For any pair of parameters a' and a", and any point a in the open segment (a',a"), the preference R_a is "between" the preferences $R_{a'}$, and $R_{a''}$, in the sense that its graph is contained in the union of the graph of $R_{a'}$ and of $R_{a''}$, and contains their union. It is then shown that if the distribution of preferences is nicely distributed around some point a* of the parameter space, the majority rule coincides with R_{a*}. It is also shown the usual theorem on the transitivity of the majority rule when preferences are single peaked can be obtained as a particular case of this analysis, and corresponds essentially to a one dimensional family (n = 1).

U. HERKENRATH : Generalized Random Systems with Complete Connections and their Applicability

A generalized random system with complete connections (GRSCC) and its associated processes are defined. Especially the associated Markov-process of the GRSCC's is studied. This is done by making continuity assumptions on the underlying transition probabilities of the system. Convergence properties for the associated Markov-process imply similar properties for the associated event process. Some possibilities of applications of the theory are indicated.

Y. KANNAI : Non Concave Utility Functions and Pareto Sets

This is a report on work done jointly with R. Mantel. It is proved that there exists a pure exchange economy with 3 agents and 2 commodities with strictly convex and strongly monotone preferences so that for no choice of utility functions is the disposable hull of the utility possibility set (alias the Pareto set) convex.

J.J. LAFFONT : Taxing Price Makers

The conventional wisdom in optimal taxation theory is that a monopolist producing a single commodity in an otherwise competitive economy can be "controlled" with appropriate linear indirect taxation of the commodity he produces. In particular such a taxation scheme is believed to be able to restore Pareto efficiency.

In this paper we study in a general equilibrium framework the characterization of optimal taxes and we make in particular the following points:

The use of perceived demand functions in optimal taxation theory is shown to be informationally inconsistent. The use of true demand functions faces the difficulty of the general non concavity of profit functions. The main consequence of this non concavity is the general impossibility of decentralizing some first best Pareto optima,and therefore the need to settle for second best Pareto optima at which production efficiency might not even be desirable We construct an economy for which no Pareto optimum can be decentralized through an appropriate linear tax scheme. When lump sum taxes are not available, it is not possible to restore (even for non critical cases) first best Pareto efficiency unless taxation of profits is allowed. Finally we argue that there is a large number of cases where optimal taxation puts the monopolist at a critical (and therefore very unstable) production level.

A. MAS-COLELL : Blocking at Random and the Core Equivalence Theorem

Consider an exchange economy $E : I \to P \times P$ where I is a finite indexing set, $P = \mathbb{R}^{\ell}_{++}$ and P is the space of C^2, strictly monotone, convex preference relations satisfying the boundary condition: $\{y : y \succsim x\}$ is $\mathbb{R}^{\ell}$-closed for all $x \in P$ (see Econometrica, 1972, Debreu:"Smooth preferences" for all this); P has the topology of C^2 uniform convergence on compacta of the functions $g_{\succsim} : P \to S$ defined by : $g_{\succsim}(x) = Du(x)/\|Du(x)\|$, where u is a C^2 utility function for $\succsim$. Let $\underset{\sim}{x}$ be a Pareto optimum, then it has shadow prices $p(\underset{\sim}{x})$. Define the competitiveness gap of $\underset{\sim}{x}$ as

$$d_2(\underset{\sim}{x}) = [\frac{1}{\#I} \cdot \sum_{i \in I} (p(\underset{\sim}{x})(\underset{\sim}{x}(i)-\omega(i))^2]^{1/2}.$$

Let

$$\tau(E,\underset{\sim}{x}) = \frac{1}{2^{\#I}-1} \#\{C \subset I : C \text{ blocks } \underset{\sim}{x}\}$$

(for the notion of blocking see Hildenbrand:"Core and Equilibria of a Large Economy", Princeton 1974).

<u>Theorem</u>: For every compact $E \subset P \times P$, compact $K \subset P$ and $0 < \varepsilon \leq \frac{1}{2}$ there is $H > 0$ such that for every economy $E : I \to E$ and allocation $\underset{\sim}{x} : I \to K$ the following holds: if $\tau(E,\underset{\sim}{x}) \leq \frac{1}{2} - \varepsilon$ then $d_2(\underset{\sim}{x}) \leq \frac{H}{[\#I]}$.

M. NERMUTH : <u>On the Continuity of the Choice Correspondence</u>

Let X be a locally compact Hausdorff space, M a nonempty compact subset of X, and r a <u>preference relation</u> on X, i.e. a reflexive negatively transitive relation with closed graph in $X \times X$. The <u>choice set</u> $a(M,r)$ of maximal elements in M with respect to r is called <u>solution</u> of the choice <u>problem</u> (M,r), where M is called the <u>constraint set</u>. It is shown that the most general (i.e.coarsest) topologies such that $a(M,r)$ depends upper-hemi-continuously on both M and r are the following: the Vietoris topology T_V on the space of constraints and the "compact-open" topology T_K on the space of preferences. Moreover, if $f(u)$ denotes the preference relation associated with a continuous utility function u on X, then the coarsest topology on the space of utility functions which makes the mapping f continuous is the topology of uniform convergence on compacta, modulo strictly monotone increasing transformations (when the range space is endowed with the topology T_K).

As an application one can show, for example, that "initial segments" of optimal T-period consumption-investment programs are " insensitive" with respect to the length of the planning horizon, a kind of turnpike theorem, but without convexity assumptions on technology or utility.

W. NEUEFEIND : <u>Boundary Behavior of Supply: a Continuity Property of the Maximizing Correspondence</u>
Report on a joint paper with Ph.Artzner, University of Strasbourg

The normal cone correspondence, which assigns to each point of a convex set the cone of normal vectors to the set at this point is well known to be necessarily lower hemi-continuous. Sufficient conditions for weaker approximability property of some points in the graph of this correspondence are given. The result is applied to study the boundary behavior of supply functions of producers, who maximize profit on a strictly convex production set.

J. ROSENMÜLLER : <u>Values of Non-Side Payment Games and Transfer of Utility</u>

Let $\Omega = \{1,\dots,n\}$ be the "set of players" and $\underline{B} = P(\Omega)$ the "coalitions". Consider the set $\mathbb{V}$ of "nice" mappings $V : \underline{B} \to P(\mathbb{R}^n)$ (describing the utility prospects of coalitions). Also, let $\mathbb{W}$ be the set of mappings $v : \underline{B} \to \mathbb{R}^+ (v(\emptyset) = 0)$. τ is a map $\mathbb{V} \to \mathbb{W}$ via $V \to v^V$, $v^V(S) \subset \max\{\sum_S x_i \mid x \in V(S)\}$. A value is a mapping $\chi : \mathbb{V} \to \mathbb{R}^n$ that commutes with permutations and affine transformations of utility such that $\chi(V)$ is Pareto optimal in $V(\Omega)$. A rescaling is a mapping $L : \mathbb{V} \to \{$linear transformations of utility$\}$, $L(L(V)V) = \mathrm{id}$. Next, a value χ is generated by L if $\chi(V) = (L(V))^{-1}\ \Phi\ (v^{L(V)V})$ (Φ the side payment Shapley value) or $(L(V)V = L^*V)$ for short $\chi \circ L^* = \Phi \circ \tau \circ L^*$. It can be seen that a proper version of Shapley's side value as well as Harsanyi's solution may be summarized under this concept. Also, value-equilibrium-equivalence theorems (in the utility space) may be established via such a mechanism since the competitive equilibrium obeys a similar law as the value: it is obtained from a "primitive = side payment" concept via rescaling and applying the "side payment operator".

D. SCHMEIDLER : Outcome Functions Guaranteeing Existence and Optimality of Nash Equilibria
(written jointly with L. Hurwicz)

An outcome function f maps, by definition, a nonempty Cartesian product $S_1 \times S_2 \times \ldots \times S_n$ of finite sets into a nonempty finite set A. We refer to A as the set of outcomes and S_i as the set of strategies of person i, with $N = \{1,2,\ldots,n\}$ denoting the set of persons. The set of orders on A is denoted by Σ. The elements of Σ^n will be called profiles. An outcome function f and a profile $R = (R_1,\ldots,R_n)$ in Σ^n define an n-person ordinal game in strategic form. An outcome function is said to be acceptable for a set of profiles Σ^n if for every profile there is a Nash Equilibrium and every Nash Equilibrium is Pareto optimal.

Theorem 1 states that for two persons a function that is acceptable is dictatorial.

For three and more persons there are classes of nondictatorial outcome functions. Existence of symmetric across persons outcome functions is shown for some values of n and $\# A$.

One of the aims of this model is to get insight to the problem of existence of mechanisms for allocation of public and private goods which are efficient and incentive compatible.

D. ZAGIER : Indices of Inequality

Given an income distribution f(x) (i.e. $f(x) \geq 0$, $\int_0^\infty f(x)dx = 1$, $f(x)dx$ = fraction of the population with income in $[x,x+dx]$, one can define various indices I(f) which measure the inequality of the distribution; reasonable requirements are that $I \geq 0$, with $I = 0 \Leftrightarrow$ everybody has the same income, and that I satisfies the "Pigou-Dalton condition" (a transfer of wealth from a richer to a poorer person decreases I). Several indices used are of the form

$$(*) \qquad I = \int_0^\infty \psi\left(\frac{x}{\mu}\right) f(x)\,dx, \qquad \text{where } \psi(1) = 0,\ \psi''(x) \geq 0$$

(here $\mu = \int_0^\infty x f(x)\,dx$ = average income), i.e. I is the social utility, where individual utility depends only the ratio of the individual's to the mean income. Examples are the Theil index ($\psi(x) = x\log x$) and relative variance ($\psi(x) = (x-1)^2$), both of which have the property of being <u>decomposable</u>: i.e. if the population consists of several components (e.g. men and women, or various races), then $I = I_o + \sum_{k=1}^{K} a_k I_k$ where I_k is the index of the k^{th} component and $I_o, a_1, \ldots, a_K$ depend only on the sizes and mean incomes of the various components but not on the individual distributions.

<u>Theorem 1</u>: All indices decomposable in this sense are of the form (*), with $\psi(x) = Cx^\alpha (C > 0, \alpha \in R)$.

This is a negative result since the indices (*) are unsatisfactory (they depend only on comparing individuals with the average, not with each other). For the <u>Gini coefficient</u>

$$G = \frac{1}{2\mu} \int_0^\infty \int_0^\infty f(x) f(x')\ |x - x'|\ dx\ dx',$$

which does not suffer this defect but is (by Theorem 1) not decomposable, we prove:

<u>Theorem 2</u>: The Gini coefficient of a population of size n and total income $X = n\mu$ consisting of two components (of size n_i, total income X_i, Gini coefficient G_i, so $n=n_1+n_2, X=X_1+X_2$) satisfies

$$\max\ \left(\left|\frac{n_1}{n}\frac{X_2}{X} - \frac{n_2}{n}\frac{X_1}{X}\right| + \frac{n_1}{n}\frac{X_1}{X} G_1 + \frac{n_2}{n}\frac{X_2}{X} G_2\ ,\right.$$

$$1 - \frac{1}{nX}\left(\left(n_1 X_1 (1-G_1)\right)^{1/2} + \left(n_2 X_2 (1-G_2)\right)^{1/2}\right)^2 \Big) \leq$$

$$\leq G \leq \left|\frac{n_1}{n}\frac{X_2}{X} - \frac{n_2}{n}\frac{X_1}{X}\right| + \frac{n_1}{n}\frac{X_1}{X} G_1 + \frac{n_2}{n}\frac{X_2}{X} G_2 +$$

$$+ 2 \min \left(\frac{n_1}{n}\frac{X_2}{X}, \frac{n_2}{n}\frac{X_1}{X}\right)(G_1+G_2-G_1G_2).$$

Both the upper and lower bounds are best possible.

As a corollary, one deduces a kind of convexity property: $G \geq \frac{n_1}{n} G_1 + \frac{n_2}{n} G_2$ (and also $G \geq \frac{X_1}{X} G_1 + \frac{X_2}{X} G_2$), which is equivalent to the inequality (valid for any $x_1,\dots,x_n,y_1,\dots,y_m \geq 0$)

$$2 \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{j=1}^{m} y_j\right) \sum_{i=1}^{n} \sum_{j=1}^{m} |x_i - y_j| \geq$$

$$\geq \left(\sum_{i=1}^{n} x_i\right)^2 \sum_{j=1}^{m} \sum_{j'=1}^{m} |y_j - y_{j'}| +$$

$$+ \left(\sum_{j=1}^{m} y_j\right)^2 \sum_{i=1}^{n} \sum_{i'=1}^{n} |x_i - x_{i'}|$$

The upper bound in Theorem 2, which is much harder to prove, uses the following result

<u>Theorem 3</u>: Let $f,g : [0,\infty) \to [0,1]$ be monotone decreasing functions with $\int_0^\infty f(t)dt, \int_0^\infty g(t)dt < \infty$. Then

$$\int_0^\infty f(t)g(t)dt \geq \left(\int_0^\infty f(t)^2 dt\right)\left(\int_0^\infty g(t)^2 dt\right)/\max\left(\int_0^\infty f(t)dt, \int_0^\infty g(t)dt\right).$$

This inequality, which is also best possible, can be considered as a converse to Cauchy's inequality.

Session on Open Problems

R.J.AUMANN

1. Let A be a set on the sphere S^n, $n \geq 2$, such that every hyperplane through the center of the sphere cuts A into half w.r.t. the natural measure on S^n. Then is A centrally symmetric a.e., i.e. is the antipodal of a.e. point in A also in A ? *)

   Background: The hypothesis is equivalent to the aggregation condition of Grandmont.

2. Let λ be Lebesgue measure on [0,1], and define

$$v(S) = \begin{cases} 1 & \text{if } \lambda(S) > \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

   Does v have an asymptotic value ?

   Background: v is a model for a non-atomic majority game. The game v is a member of the space bv'NA, on wich a value is known to exist.

3. Let A denote a set of agents, $(\Omega, \mathcal{B}, \pi)$ a probability space (interpreted as the states of the world). For each agent a, let $u_a : R_+^\ell \times \Omega \longrightarrow \mathbb{R}$ be a's Neumann-Morgenstern utility function, $e_a : \Omega \longrightarrow R_+^\ell$ his endowment, and $\mathcal{C}_a$ a sub-σ-field of $\mathcal{B}$, which is interpreted as a's information. It is assumed that e_a is $\mathcal{C}_a$-measurable, but u_a need not be. Define an equilibrium to be a pair

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*) Remark added by R.J. Aumann: Thom gave a beautiful solution of my Problem No.1 in case the set A has a smooth boundary. However it is not clear whether the general case follows.

of functions (p,f), which map Ω into the price simplex and into the set of allocations (which may vary as ω varies) respectively, with the following property: If $\mathcal{P}$ is the sub-σ-field of $\mathcal{B}$ generated by p, then for each a in A and each ν in Ω , the conditional expected utility $E\left(u_a(x,\omega) \mid \mathcal{C}_a \vee \mathcal{P}\right)$, evaluated at ν, is maximized over the budget set

$$\left\{ x \in R_+^\ell : p(\nu) \cdot x \le p(\nu) \cdot e(\nu) \right\} \quad \text{when} \quad x = f_a(\nu).$$

<u>Problem</u>: Find sufficient conditions of a "general" nature for existence of equilibrium.

<u>Background</u>: The situation is one in which agents start out with different information, and can deduce additional information from the equilibrium prices. Green, Grossman, Kreps, and perhaps others have found simple examples in which there is no equilibrium. Radner has some (as yet unpublished) results on the subject, including a generic existence result. With sufficient "noise" in the system, or with sufficient diversity among the traders, it should be possible to prove existence. Presumably a diversity result would require a continuum of agents.

Y. KANNAI

1. Let X be a compact metric space, let P be the space of complete continuous preference orders. Construct a continuous function $F : X \times P \to \mathbb{R}$ such that $F(x,\succsim)$ is a utility function for $\succsim$ for all $\succsim \in P$. (Note that A. Mas-Colell proved the existence of F).

2. Let X be an unbounded convex subset of $\mathbb{R}^n$. Let $\succsim$ be a complete, convex, continuous preference order on X. Clarify the role of the "points at infinity" for the possibility of (i) constructing concave utility functions for $\succsim$, and if (i) is impossible, (ii) "good" approximation of $\succsim$ by orders having concave utility function. (i) is due to Fenchel, 1956.

3. Find conditions for the non-emptiness of cores (or at least ε-cores) for games with infinitely many players and no side payments.

4. Pareto sets: Given X and $f : X \to Y$, determine the boundary of f(X). Conversley, given X and $P \subset Y$, find $f : X \to Y$ with $f(X) = P$. This should be answered in various categories (topological, differential, monotone and convex).

D. Sondermann

1. Let A and P be finite-dimensional smooth manifolds, N a submanifold in A x P. Let μ be a measure on A with compact support which is smooth, i.e., with local C^1 densities. Consider a real-valued C^1 function f defined on $(A \times P) \setminus N$. Find control conditions for f on a (tubular) neighborhood of N, such that, for a.e. $p \in P$, the integral

$$I(p) = \int_A f(a,p)\, \mu(da)$$

is C^1.

2. Let A,P,X be finite-dimensional smooth manifolds, Y a closed submanifold in X. Let $f: A \times P \to X$ be a C^2 map and consider the induced map $\pi: M_f \to P$, where $M_f := f^{-1}(Y)$ and π is the restriction of the projection $A \times P \to P$ to M_f.

   a) Is the following conjecture correct: If dim P=1, then there exists an open and dense set $\mathcal{U} \subset C^2(A \times P, X)$ such that, for any $f \in \mathcal{U}$, the map $\pi: M_f \to P$ has only non-degenerate critical points?

   b) What can be expected generically for dim P >1 ?

M. TODD

Let Δ = set of triangles in $\mathbb{R}^2$.
A policy is a function $f : \Delta \to \mathbb{R}^2$ such that $f(\sigma) \in \sigma$, $\sigma \in \Delta$.
A f-son of σ, with vertices y^1, y^2, y^3 is one of the triangles with vertices $y^i, y^j, f(\sigma)$, $i \neq j$, (1,2 or 3 f-sons of each triangle) $\Rightarrow$ kth f-descendants.

The worst-case rate of f is $\alpha(f)$ defined as

$$\underline{\lim}_{k\to\infty} \left(\min \left\{\frac{-\log \operatorname{diam} \sigma}{k} \mid \sigma \text{ a kth f-descendant of } \sigma_o\right\}\right)$$

with σ_o a given triangle.

| Find $\sup_f \alpha(f)$. |

$(1/2 \leq \sup_f \alpha(f) \leq 1/2 \log 3)$.

Let λ be Lebesgue measure on $\mathbb{R}^2$. The expected rate of $f, \beta(f)$, is

$$\underline{\lim}_{k\to\infty} \left(\Sigma \left\{\frac{-\lambda(\sigma) \log \operatorname{diam} \sigma}{k} \mid \sigma \text{ a kth f-descendant of } \sigma_o\right\}\right).$$

| Find $\sup_f \beta(f)$. |

$(1/2 \leq \sup_f \beta(f) \leq 1/2 \log 3)$.

D. Sondermann (Hamburg)