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# BERECHNUNG VON VERZWEIGUNGEN IN MECHANISCHEN SYSTEMEN
# 17.–23. APRIL 1988

## Exponentially small splitting of separatrices and bifurcation

For systems with a homoclinic orbit that are forced with a term having amplitude $\delta$ and frequency $\frac{1}{\epsilon}$ it is known from work by Philip Holmes, Jerrold Marsden and myself that under suitable conditions, for sufficiently small $\delta$, the separatrices split, if at all, by an amount of the form $\delta C_1 e^{-r/\epsilon}$ for constants $C_1$ and r. Moreover, if $|\delta| \leq \epsilon^p$, where p is a sufficiently large integer and $\epsilon$ sufficiently small, Melnikovs method is applicable to detect the splitting and also the transversal intersection of the separatrices implying chaos. Besides motivating and reviewing this theory we discuss an application in bifurcation theory.

Jürgen Scheurle, Hamburg

## On the computation of bifurcating manifolds in a higher singular point.

Let $x_0 \in E$ be a higher singular point for an operator $G: \mathbb{E} \to \hat{\mathbb{E}}$, with $G \in C^2(\mathbb{E})$, $G_0' = G'(x_0)$ bounded with closed range and $\dim \mathbb{N}(G_0') = m+q$, $\dim N(G_0'^{*}) = m$. In an earlier paper by Allgower-Böhmer a general theory for the numerical approximation ~~of~~ for $x_0$, $N(G_0')$, $N(G_0'^{*})$ was presented. This information is used now, to transform the classical Lyapunov-Schmidt-method for the computation of the bifurcating manifolds into the discrete counterparts. The relation of consistency and stability properties is discussed and all the necessary modifications and results for the discrete case are given. So it is possible, based on the above approximation to compute the bifurcating manifolds numerically.

Klaus Böhmer, Marburg

Surface waves in a nearly square container subjected to vertical oscillations are studied. The theoretical results are based on the analysis of a derived set of normal form equations, which represent perturbations of systems with 1:1 internal resonance and with $D_4$ symmetry. Bifurcation analysis of these equations shows that the system is capable of periodic and quasiperiodic standing as well as travelling waves. ~~The~~ Analysis also identifies parameter values at which chaotic behavior is to be expected. The theoretical results are verified with the aid of some experiments. Implications of the analysis to other physical problems are discussed. Global bifurcations of a system with 1:2 resonance are also discussed.

[illegible signature]

# Bifurcations in Stochastic Systems
## – Models, Analysis and Simulation

In practical environments ergodic perturbations are generated by wind turbulences or rough surfaces in such a way that the system parameters are superimposed with corresponding time fluctuations. The paper gives some simple examples in structural, aero or fluid dynamic problems where multiplicative fluctuating terms are involved.
The stability analysis of such non-autonomous systems is based on Lyapunov exponents and rotation numbers which substitute the eigenvalues of time-invariant linear systems. For a stochastic modelling of parameter excitations they are calculable by introducing cyclic Lyapunov coordinates and taking the expected values via orthogonal expansions. For increasing noise intensities the deterministic solution, e.g. the equilibrium position of the dynamic systems, becomes unstable and bifurcates into turbulent motions. They are bounded by cubic dissipation terms. Associated silent and noisy limit cycles are simulated by means of a Euler scheme. Normal forms are discussed.

Walter Wedig, Karlsruhe

Tutorial on
Stochastic Differential Equations.

Contents:

1.) Stochastic processes:
Stationary characteristics, white noise and Wiener process, linear time-invariant systems

2.) Ito calculus:
Stochastic differential equations, correction terms, Ito formula, diffusion equations, applications

3.) Analysis and simulation:
Taylor- and Hermite moments, generalized Hermite analysis, noise generator, cyclic coordinates

Walter Wedig, Karlsruhe

Stability of a compressible elastic rod with imperfections

Stability of a compressible elastic rod axially loaded by two concentrated forces of arbitrary intensity is studied. It is assumed that imperfections in shape and loading are present. The shape imperfections are characterized by an initial deformation of the rod axis, while the load imperfections are characterized by a small distributed force acting perpendicular to the action line of the compressive forces.

A number of solutions and their local behaviour is analyzed.

Teodor Atanacković

Parallel Algorithms for continuation of partial differential systems

Dirk Roose

We discuss how continuation procedures for partial differential equations, can be adapted to local memory parallel computers (e.g. hypercubes).

If a finite-difference discretization on a fixed grid is used, one can apply a "classical" predictor-corrector continuation procedure in which the linear systems are solved by a parallel algorithm. The problems associated with this approach are indicated.

Recently some interesting continuation procedures based on multigrid are developed. It is shown how these procedures can be parallellized. Some preliminary estimates of the efficiency of such a parallel algorithm are given.

## On the Calculation of paths of Hopf Bifurcations

Alastair Spence, Bath, U.K.

Consider a two-parameter nonlinear problem whose linearization has a double zero eigenvalue with only one ~~true~~ eigenvector. In the talk a theoretical and computational analysis of the bifurcating branches at this singular point is given using a symmetry in the system used to calculate Hopf bifurcations. The result is that standard branch-switching techniques can be used to jump on to the path of Hopf bifurcation points emanating from the singular point.

## On the Hopf bifurcation with broken O(2)-symmetry

Translation and reflection symmetries introduce the group O(2) into bifurcation problems with periodic boundary conditions. The effect on the Hopf bifurcation with O(2)-symmetry of small terms breaking the translation symmetry is investigated. Two primary branches of standing waves are found. Secondary and tertiary bifurcations involving two different types of modulated waves are analyzed in the neighborhood of secondary Takens-Bogdanov bifurcations. The effects of breaking the phaseshift (in time) symmetry is briefly considered.

Gerhard Dangelmayr, Tübingen

# Modulated Rotating Waves in O(2) Mode-Interactions

W.F. Langford, Guelph, Canada.

The interaction of steady-state and Hopf bifurcations in the presence of O(2) symmetry yields generically a secondary Hopf bifurcation, from the primary "rotating wave" branch, to a family of 2-tori. Explicit formulae for the bifurcation coefficients which determine the direction of bifurcation and stability of these tori are presented. The tori are determined by third degree terms in the normal form equations, evaluated at the origin. The flow on the torus near criticality has a small second frequency, and is topologically congugate to a linear flow, without resonances or phase locking. Existence of an additional SO(2) symmetry as found in the Taylor-Couette problem, implies that the flow is exactly linear. We have computed the bifurcation coefficients for the Taylor-Couette problem, directly from the Navier-Stokes equations, over a wide range of gap widths. These show that the 2-tori are always unstable at onset in the Taylor-Couette case. More generally, these 2-tori may manifest themselves as slowly modulated rotating waves, for example in reaction-diffusion systems or in fluid flow through an elastic hosepipe. The computations reported here may be adapted easily to other such applications.

## Splitting Iteration Technique for the Computation of the corank-2 Bifurcation Points

Mei zhen, Xi'an ; Klaus Böhmer, Marburg

A splitting iteration method is discussed here to compute the corank 2 bifurcation point and the null spaces of the corresponding derivatives of nonlinear problems. The various unknowns are divided into different groups and the iteration procedure is carried out in a block way. The iteration needs small amount of computational effort, provides much information about the bifurcation point and converges with a adjustable rate. Numerical examples are also discussed.

## Energy Measures for the Stability of Structures in Statics and Dynamics

B. Kröplin, Dortmund

When thin walled shells buckle a sequence of rapidly changing buckling patterns is passed, while the structure is moving from the prebuckling to the postbuckling range. Dynamic analysis of the phenomenon is cumbersome and the final buckling pattern depends on the in general not known damping of the structure. Static analysis can rely on equilibrium states, but has to deal with a large number of partly unstable solution paths and bifurcations. Both methods also not give estimates on the stability of the obtained solution.
In order to derive a stability estimate a perturbation strategy is tried out. It is based on accompanying eigenvalue calculation and enables to estimate the degree of stability of a solution path either in static and dynamic cases.

## Computational Methods for Bifurcation Problems with Symmetries

Bodo Werner, Hamburg

It is shown how group theoretical methods can be employed to utilize the symmetry of a bifurcation problem in numerical computations. The essential numerical point is the utilization of certain reduced instead of full systems involving appropriate subgroups of the underlying symmetry group. The group theoretical tool is an a priori knowledge of the interaction of certain subgroups at (in general) multiple steady state bifurcation points. An bifurcation graph is introduced which shows graphically this information: it edges represent possible symmetry breaking bifurcations. The main numerical aspect presented here is the efficient detection of bifurcation points. A 4-box and a 6-box Brusselator model (with dihedral symmetries) have been chosen to discuss the numerical procedure.

## O(3) symmetry breaking in variational problems

Bernold Fiedler, Heidelberg
& Konstantin Mischaikow, Michigan State University

We consider symmetry breaking bifurcations from the trivial solution $u \equiv 0$ of

$$u_t = \Delta u + \lambda f(u), \qquad f(0) = 0,\ f'(0) = 1.$$

Equivariance with respect to the orthogonal group $O(3)$ arises naturally when we consider this equation on an $O(3)$-invariant domain (ball, shell, sphere) with appropriate boundary conditions. Typically, several branches of stationary solutions with nonconjugate isotropy can bifurcate

simultaneously because, due to equivariance, high-dimensional kernels occur. We address dim = 5, 7 here. We determine the unstable dimensions associated to these solution branches, and we find heteroclinic connections between them. Our principal tool is Conley's connection matrix.

## Bifurcation analysis of a rod subjected to terminal thrust and couple.

Ernesto Buzano, Torino, Italy.

The equilibrium configurations of a rod under terminal thrust $\lambda$ and couple $\tau$ are studied.
This leads to a variational two-parameter bifurcation problem, which is studied by a uniform version of the so-called Splitting Lemma.
We prove the existence of a sequence of characteristic curves $\lambda = \Lambda_n(\tau)$, from each one of which there bifurcates a continuous surface of non-trivial equilibrium configurations. These equilibria are either supercritical or subcritical according as $\tau$ is in a neighborhood of 0 (pure compression) or in a neighborhood of a zero of $\Lambda_n(\tau)$ (pure torsion).

# Higher order predictors in continuation schemes

Klaus Ulrich, Hannover

For numerical continuation schemes variable higher order polynomial predictors are presented which allow for simultaneously monitoring step size and direction. Only first order derivatives have to be calculated, no numerical differentiation process is required to compute the additional corrector terms.
This kind of predictor process can also be viewed upon as a special reduction method using polynomial approximating subspaces.
Moreover, it allows for directly handling snap-through behaviour.
An ellipse normalization condition is suggested to automatically monitor step length and direction adjustment in the subsequent corrector process.

# Singular Perturbations and Control Theory

Dietrich Flockerzi, Würzburg

It is shown how "global" invariant manifolds for singularily perturbed system $\dot{x} = f(x,\varepsilon)$ (possessing for $\varepsilon = 0$ an invariant manifold $M_0 \subset \{x : f(x,0) = 0\}$) can be used in control theory. The applications to nonlinear control problems are directed towards (i) generating a large domain of attraction for a positive invariant set (e.g. global stabilization) by high-gain feedback and

(ii) identifying an unknown function $v(t)$, $t \in [t_0, t_e]$ in
$$\dot{x} = f(x, v(t)), \quad x(t_0) = x_0, \quad y = c^T x$$
without measuring $y(t)$.

Dietrich Flockerzi

A mathematical model of the hydrostatic skeleton and its bifurcations
Wolf-Jürgen Beyn, Konstanz

The hydrostatic skeleton is a special form of skeleton realized in many invertebrates, e.g. the leech. Basically it consists of an incompressible fluid enclosed in an elastic body wall. The shape of the body is changed by activating parts of the musculature. We present a mathematical model for the equilibria of such a system which leads to a comparatively large constrained optimization problem. Our special emphasis is on bifurcations of the equilibria for the '3D-unit worm' where the volume is taken as parameter. We show how continuation and singular point techniques ~~can~~ carry over to sparse constrained optimization problems with parameters.

Wolf-J. Beyn

Coupled Hopf- and Divergencebifurcation of pipes conveying fluid under $O(2)$-symmetry
Alois Steindl, Hans Troger.

Following an investigation by Bajaj and Sethna the bifurcations of the trivial steady state solution of an elastic pipe conveying fluid is considered. In the model damping and gravitational forces are included; in addition a rotationally symmetric elastic support with stiffness K is introduced. By fixing K and varying the fluid velocity U either a Hopf- or a Divergence bifurcation occurs. For a certain value of K an interaction of both bifurcation types takes place. Studying the equations of motion on the 6-dimensional center manifold for small variations of the critical parameter values K and U rotating waves, standing waves, stationary ~~waves~~ states and different interactions of these solution types, e.g. modulated waves, are found.

Alois Steindl

DIRECT SOLUTION OF BIFURCATION EQUATIONS.

G. MOORE – Imperial College, London

We consider the computational linear algebra problem associated with solving the extended system which characterises some particular singular behaviour. The matrix M representing the linearisation of this extended system (required for Newton's method) will generally consist of a large but structured leading principal sub-matrix A (which may be ill-posed) plus some augmented dense rows and columns. To solve such linear equations efficiently one should make use of the structure present in A while mitigating the ill-posedness. Three possibilities for doing this are:-

a) explicit deflation of M by manipulating rows/columns,
b) making use of the expected position of small pivots in the LU-decomposition of A,
c) block Gauss-elimination of M together with implicit stabilization by means of
i) implicit deflation of A ii) iterative refinement,

Gerald Moore

On the Numerical Approximation of an Invariant Curve

M. van Veldhuizen, Amsterdam

The lecture discusses several numerical algorithms for the approximation of a smooth invariant curve. Among others are mentioned the algorithms of Thoulouze-Pratt, Chan and Doedel, Kevrekides et al., Kaas-Petersen, and the author. Convergence results for the method of Kevrekides with piecewise linear interpolation are briefly discussed.
In the second part of the lecture we discuss the approximation of the rotation number. Given an approximate invariant curve, an approximate circle map is defined, an algorithm for the approximation of the rotation number is mentioned. Finally, a convergence result is briefly mentioned and discussed with respect to the absence of superconvergence.

M. van Veldhuizen.

Computation of Cusp Singularities

G. W. Reddien

A defining system for cusp points is given, allowing for underdetermined problems of arbitrary index. The approach allows the treatment of cubic turning points, winged cusps and degenerate minimizers in the same framework; the two parameters are treated symmetrically. The defining system can be solved effectively by Newton's method since explicit expressions are given for all the needed derivatives. Finally, a discretization error analysis is given for projection methods applied to the system. (Joint with A. Griewank).

Lyn Reddien

# Bifurcations in the Motion of Robots

E. Lindtner, A. Steindl, H. Troger (Wien)

The periodic motion of a simple DD-robot, i.e. of a plane double pendulum with drive moments acting at its joints is studied. The motion of the endpoint of the double pendulum is supposed to be on a circle and having constant speed $\omega_0$. For a fixed control system $\omega_0$ is increased quasistatically until the periodic solution looses stability. Calculating the Poincaré mapping and making use of Center Manifold reduction all three one parameter losses of stability which occur generically are found and analyzed. They are (i) transcritical (ii) Flip- (iii) Hopf bifurcation. The corresponding physical behavior of the robot is shown to be (i) a small shift (ii) a doubly periodic motion (iii) the motion on a torus.

H. Troger

## Chaotic motion of rail-wheel systems

G.P. Ostermeyer, Braunschweig.

Investigations in nonlinear dynamics of railway-wheel systems treat stability behavior of bogies. A typical bogie model consists on three rigid bodies, a bogie frame and two wheelsets, which are connected by viscoelastic elements. The wheelsets couples bogie and track. Main nonlinearities are to be found in geometry and contact behaviour of rail and wheel. Several authors studied the bifurcation behavior of such a modell and even found chaotic solutions. For physical reasons the rail has to be taken into account for modelling bogie-rail interaction.
Describing the rail by an infinite beam on viscoelastic foundation leads to new instability behavior in linear approximation. Classical reduction methods for the investigation of the nonlinear bogie-rail model (studies on bifurcation behaviour) failes by the existance of continuous parts of spectrum.

G.P. Ostermeyer

## Resonant forcing of nonlinear surface waves

Klaus Kirchgässner, Stuttgart

Der Einfluss von Druckwellen auf Oberflächenwellen reibungsfreier Flüssigkeitsschichten führt – im Rahmen der Eulergleichungen – auf das Studium gestörter homokliner Orbits in Funktionenräumen. Die Wirkung periodischer Druckwellen wird analysiert, ebenso wie diejenige mit endlichem Support. Im ersten Fall tritt räumliches Chaos bei großen Perioden auf, im 2. Fall gelingt eine vollständige Charakterisierung der Lösungen mit kleiner Amplitude.

K. Kirchgässner

Symbolic computation and equation on the center Manifold : application to the Couette-Taylor Problem

P. LAURE (Nice)

Reduction on the center Manifold and computation of the ~~..~~ amplitude equation is now very known. We present here ~~a~~ two ~~spe..~~ cases where it's necessary to obtain the expansion of the amplitude equation at hight order.
In the second case, we consider a degenerate Hopf bifurcation where it's necessary to compute numericaly the seventh order term.
Then we describe a method which allows us to make this computation by using symbolic system (Macsyma).

Cellular Bifurcation. Application to plate and shell buckling.

Michel Potier-Ferry (Metz)

We study bifurcation of nearly periodic solutions that appears in many physical problems: convection, plate and shell buckling... These problems are studied by multiple scale expansion. So one gets amplitude equations that are spatially modulated. In the supercritical case, the second order amplitude yields the existence of many solutions that are characterized by their wavenumber. The same amplitude equations are obtained for any "reversible" system that satisfies ~~...~~ some spectral assumptions.

# Feedback Stimulated Bifurcation

Tassilo Küpper, Hannover

Bailey and Kuszta were the first who suggested to use Bifurcation Theory for the purpose of systems identification in situations when other methods fail. Assume that an experiment has been modelled by two different dynamical systems and that standard methods (comparision of steady states, transient response) do not allow to discriminate among these models. Then a feedback procedure may be set up to force Hopf-Bifurcation such that a qualitative difference between both systems appears. Several Feedback procedures are discussed which lead to Hopf-Bifurcation, for example static and dynamic Feedback as well as Feedback with delay where the delay term is used as a parameter to force bifurcation. In addition to this qualitative criteria we propose to set up equations which can be used for the calculation of unknown quantities in the system. The equations are derived through a comparision of measurements with the asymptotic expansion of the solution.

T. Küpper

Bifurcation of homoclinic orbits and bifurcation from the essential spectrum

For a nonlinear $2^e$ ordre ODE over $\mathbb{R}$, bifurcation of solutions in terms of the $L^p(\mathbb{R})$ norm was discussed. The solutions tend to zero at $\pm\infty$, so they are associated with homoclinic orbits. The method used amounts to making an appropriate rescaling of the problem and then continuing a homoclinic orbit of the rescaled equation. Conditions for doing this can be found via bifurcation from a simple eigenvalue using the phase of the basic solution as eigenparameter. The result reduces to finding simple zeros of a Melnikov function. Related work is due to Robert Magnus

C. A. Stuart

Bifurcations in a Marangoni-Problem
R. Seydel (Würzburg)

Zone refining of cylindrical rods of silicon material is strongly affected by surface-tension-driven convection. Under some symmetry assumptions a 2-D Navier-Stokes problem is set up and solved numerically. Various branching diagrams are presented, reporting on the dependence of solutions on the Nusselt number and the Marangoni number. Based on the computational experiences, difficulties inherent to continuation are discussed. Several postulates on continuation are stated, some of which recommend to double-check computational results carefully.

R. Seydel

# Invariant Cantor sets in singularly perturbed systems

K. R. Schneider, Berlin, DDR

Consider singularly perturbed systems of the type (*) $dx/dt = f(x, y, \varepsilon, \alpha)$, $\varepsilon\, dy/dt = g(x, y, \varepsilon, \alpha)$ where $\varepsilon$ is a small parameter, $x \in R^n$, $y \in R^m$, $\alpha \in R^k$. Assume that $g = 0$ has the solution $y = \varphi(x, \alpha)$ and that the degenerated system (**) $dx/dt = f(x, \varphi(x, \alpha), 0, \alpha)$ has for $\alpha = \alpha_0$ a homoclinic orbit $\gamma$ to the equilibrium point $x = 0$ such that (**) has an invariant Cantor set near $\gamma$. In the two cases

a) rank $f_x(0,0,0,\alpha_0) = n$, $\dim(TM^s_x \cap TM^u_x) = 1$ for $x \in \gamma$ where $TM^s_x$ ($TM^u_x$) is the tangent space of the stable (unstable) manifold of $\gamma$ at $x$

b) rank $f_x(0,0,0,\alpha_0) = n-1$, $M^{cs}$ and $M^{cu}$ intersect transversally where $M^{cs}$ ($M^{cu}$) is the center-stable (center-unstable) manifold of $\gamma$.

we derive sufficient conditions that the singular perturbed system (*) has also an invariant Cantor set near $\gamma$ for $\varepsilon$ and $|\alpha - \alpha_0|$ small.

K. Schneider

# Kombinatorik geordneter Mengen 24.–30.4.88

## The bandwidth problem for distributive lattices

Let $P$ be a finite poset, and let $f: P \to \{1, \dots, |P|\}$ be a linear extension. Define

$$bw(f) = \max\{f(y) - f(x) : x \text{ is a lower neighbor of } y\}$$
$$bw(P) = \min bw(f)$$

Conjecture: If $L$ is a finite distributive lattice, then $w(L) \le bw(L) \le 3/2\, w(L)$.

Theorem: If $L$ is a distributive lattice of breadth 3, then $bw(L) \le w(L) + 1 + \sqrt{w(L) - 1}$.

Theorem: If $L$ is a distributive lattice of breadth $\le 4$, then $bw(L) \le \frac{3}{2} w(L)$

(This is joint work with F. Hergert)

Gerhard Gierz, Riverside

## Tableaux and Chains in a new partial order of $S_n$

We define a new partial order on the symmetric group $S_n$ which is a subposet of the weak order by defining $\sigma \le \tau$ if $\tau$ is gotten from $\sigma$ by a sequence of adjacent transpositions moving a left-right maximum to the left. We show that this poset has the property that every interval is a distributive lattice. We can explicitly compute the poset of join-irreducibles in the principal ideals and enumerate the chains in certain ~~are~~ cases.

Paul Edelman (Minneapolis)

## Minimal proper set families

H.-D. O. F. Gronau, Greifswald, GDR

Let $R$ be a finite set, $|R| = r$. A family $\mathcal{F} \subseteq 2^R$ is called a Sperner family if $X \not\subseteq Y$ for all $X, Y \in \mathcal{F}$. A Sperner family $\mathcal{F} = \{X_1, X_2, \dots, X_k\}$ is called <u>proper</u> if for every $x \in R$ the family $\mathcal{F}(x) = \{X - \{x\} : X \in \mathcal{F}\}$ is not a Sperner family or $|\mathcal{F}(x)| < |\mathcal{F}|$. Obviously, maximal Sperner families are proper. But what is the minimum size of a proper Sperner family on $R$? The main result in attacking this problem is the following one: Fix the size $k$ of the Sperner family $\mathcal{F}$ and ask for the maximum size $r(k)$ of $R$ such that there exists a proper Sperner family $\mathcal{F}$ on $R$.

<u>Theorem</u>:
$$r(k) = \begin{cases} 2k-2 & \text{if } 2 \leqq k \leqq 7, \\ \left\lfloor \frac{k^2}{4} \right\rfloor & \text{if } k \geqq 7. \end{cases}$$

This and related results for further proper families (e.g. $\mu$-wise intersecting Sperner families) are presented.

H.-D. Gronau

## On an ordering problem in manufacturing

A practical optimization problem that comes up in a number of flexible manufacturing systems is the following. Let a complete digraph $D_n = (V, A_n)$ on $n$ nodes and costs $c_e$ for all $e \in A_n$ be given. (The nodes correspond to machines. The costs include the costs of moving an object from one machine to another and setting up machines.) Moreover an acyclic digraph $D = (V, A)$ is given that describes precedence relations among the machines. The task is to find a hamiltonian path $H$ in $D_n$ that satisfies all precedence relations and has smallest cost. This problem is called sequential ordering problem in the flexible-manufacturing literature. We indicate that it can be viewed as an "intersection" of the asymmetric TSP and the linear ordering problem. We give several integer progr. formulations of the problem and demonstrate how the corresponding LP-relaxations can be solved in polynomial time by providing polynomial time separation algorithms for certain classes of valid inequalities. Preliminary computational experience with a cutting plane code for the sequential ordering problem is reported.

Martin Grötschel (Augsburg)

## Topology of Oriented Matroids

We outline a proof that the face lattice of an oriented matroid (as axiomatized by EDMONDS, MANDEL and FUKUDA) is the face lattice of a shellable regular CW-sphere.

For this, we use BJÖRNER's characterization of the face lattices of shellable CW-spheres, to show that every linear extension of the poset of regions (as studied by EDELMAN) introduces a recursive coatom ordering of the face lattice.

Our method leads to a new, stronger proof for the FOLKMAN-LAWRENCE Representation Theorem for oriented matroids: every oriented matroid arises from an arrangement of pseudo-hemispheres on a sphere. Moreover, such arrangements as well as their hemispheres and intersections of hemispheres (supercells) are always shellable. This sharpens Mandel's result that oriented matroids arise from constructible (hence PL-) spheres. [this is joint work with ANDERS BJÖRNER, Stockholm]

Günter M. Ziegler (Augsburg)

## Pareto extensions for spaning-tree-problems with several objectives

For a spaning-tree-problem with more than one objective the weights of the edges are n-tuples. Hence the edges give a partial order instead of the linear order. A linear extension of this partial order is called a Pareto extension if the usual algorithm like Kruskal, Prim etc produce an efficient solution. The set of efficient solution can be very large and we present bounds on its cardinality. Furthermore we study Pareto extension given by preference functions and consider the problem to find prescribed efficient solution

Dietmar Schweigert (Kaiserslautern)

## Towers of ~~po~~ Powers and Bruhat Order

Jerrold R. Griggs (I.M.A, Minneapolis & Columbia S.C. USA)

A recent paper of Brunson deals with an interesting partial order on $S_n$ which arises from comparing permutations of iterated exponentials. For $\sigma \in S_n$ and $x=(x_1,\dots,x_n)\in\mathbb{R}^n$, let $T(\sigma(x)) = T(x_{\sigma(1)},\dots,x_{\sigma(n)})$ denote the tower of iterated exponentials $x_{\sigma(1)}^{x_{\sigma(2)}^{\cdot^{\cdot^{x_{\sigma(n)}}}}}$, evaluated top-down as usual. For $\sigma,\tau\in S_n$ we have the partial ordering, called tower order, $T_n=(S_n,\le_T)$ where $\sigma\le_T\tau \Leftrightarrow T(\sigma(x))\le T(\tau(x))$ for all $x_n\ge\dots\ge x_1\ge e$. Brunson showed that $T_n$ is stronger than the dual of the well-known (strong) Bruhat order on $S_n$, and that they are identical for $n\le 4$. It was conjectured ~~(in~~ (in effect) that this true $\forall n$. However, we can provide a counterexample for $n=5$.

A closely-related poset ~~on the~~ called $A_n(c)$, is $\{a,b\}^n$ ordered for $w,w'\in\{a,b\}^n$ by $w\le_A w' \Leftrightarrow \forall b\ge a\ge c$ $T(w)\le T(w')$, where $c\ge 0$ is given. We explicitly describe $A_n(e)$. For $n=3$ we find $A_n(c)$ for all $c$ – there are 8 different posets. ~~We mean~~ It can be proven that $A_n(3.6)$ is a chain, under reverse lexicographic order, for all $n$. Stembridge has proven that for all $c\ge e$, the poset $T_n(c)$ on $S_n$ ordered as above, except $e$ is replaced by $c$ as the lower bound, can be characterized by its projections into $A_n(c)$. It follows that $\forall n$, $T_n(3.6)$ is a chain under reverse lexicographic order.

One particularly curious inequality is $b^{a^b}\ge a^{b^a}$ $\forall b\ge a\ge 0$. Still open: Is $T_n(c)$ self dual and ranked $\forall n,c$? Is $T_n(1)$ an antichain $\forall n$?

JRG

## Fibres in Ordered Sets

A fibre in an ordered set $X$ is a subset $F$ of the points of $X$ such that $|F\cap A|\ne\phi$ for all maximal antichains $A$ of $X$. This notion, dual to the more familiar idea of a cutset, was introduced by Aigner and Andreae, motivated by graph theoretic results. They conjecture that for every finite ordered set X without splitting elements there is a fibre F of size at most $|X|/2$. Rival and Lonc conjecture that such ordered sets have a subset F such that both F and $X \setminus F$ are fibres. The 2-element maximal antichains of an ordered set behave in accord with this conjecture

Dwight Duffus (Emory University, Atlanta USA)

Complexity of diagrams

Jaroslav Nešetřil (Charles University & University Bonn)

A diagram is an undirected covering graph of a poset. The class of all diagrams seems to be difficult to analyse. We further support this by giving the following:

1. Theorem For every positive $l$ there exists a graph $G_l$ with the following two properties:
   (i) $G_l$ has girth $\geq l$
   (ii) $G_l$ fails to be a Hasse diagram.

(Observe that (i) and (ii) imply $\chi(G_l) \geq l$.)
This has been proved by Nešetřil and Rödl (PAMS '78). The constructive proof is considerably more difficult. O. Pretzel solved the case $l = 6$. Recently we found a constructive proof for every $l$. Constructive examples fails to be primitively recursive.

2. Theorem For every positive $k$ there exists a graph

$G_k$ which is the Hasse diagram of poset $P_k$ such that: (i) $\dim P_k = 2$
(ii) $\chi(G_k) \geq k$.

This is due to Kříž and Nešetřil and it solves a problem due to Trotter and Nešetřil. $G_k$ fails to have a large girth and ~~(perhaps more importantly)~~ (necessarily) $P_k$ fails to be a lattice.

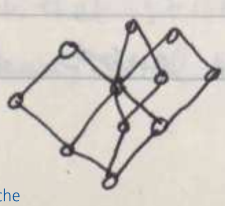
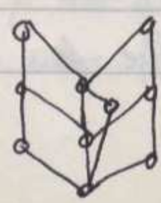
## Planar Ordered Sets    David Kelly, Univ. of Manitoba

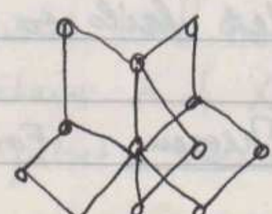
We call an ordered set $L$ a *pseudolattice* if $L \dot\cup \{0,1\}$ is a lattice. Henceforth, all ordered sets are finite. Thus, pseudolattices are defined by the implication $\{a,b\} < \{c,d\} \Rightarrow \exists x$ st $\{a,b\} \leq x \leq \{c,d\}$.

*Theorem* If a pseudolattice $L$ is a subposet of a planar ordered set, then $L$ is also planar.

In other words, nonplanar pseudolattices are obstructions to planarity. Observe that the ordered set $P = 0 < \{a,b\} < c < \{d,e\} < 1$ is planar, but the subposet $P - \{c\}$ is nonplanar. Let $\mathcal{M}$ (resp. $\mathcal{P}$, resp. $\mathcal{L}$) be the set of all minimal nonplanar meet-semilattices (resp. pseudolattices, resp. lattices). By inspecting the list $\mathcal{L}$ [Canad. J. Math. 27 (1975), 636-665], it is seen that $\mathcal{L} \subseteq \mathcal{P} \cap \mathcal{M}$. We conjecture that $\mathcal{M} \subseteq \mathcal{P}$. (In other words, if $M \in \mathcal{M}$, then $M - \{0\}$ is planar.)

Examples:

,  $\in \mathcal{M} \cap \mathcal{P}$

 $\in \mathcal{P}$

David Kelly

Removing Monotone Cycles from Graph Orientations.

Given a graph $G$ give each cycle $C$ a reference orientation. For an orientation $R$ of $G$ an edge $e$ of $C$ is forward if it is ~~directed~~ oriented by $R$ in the reference direction. Otherwise it is a backward edge. $C$ is monotone if all its edges are forward or all are backward. $C$ is $k$-good if it has at least $k$ forward edges and $k$ backward edges. $R$ is $k$-good if all cycles are $k$-good. (1-good = acyclic, 2-good = diagram orientation).

Theorem (Mosesian 1972) If $G$ has girth $\geq 4$ and an orientation in which every cycle is monotone or 2-good, then $G$ has a 2-good orientation.

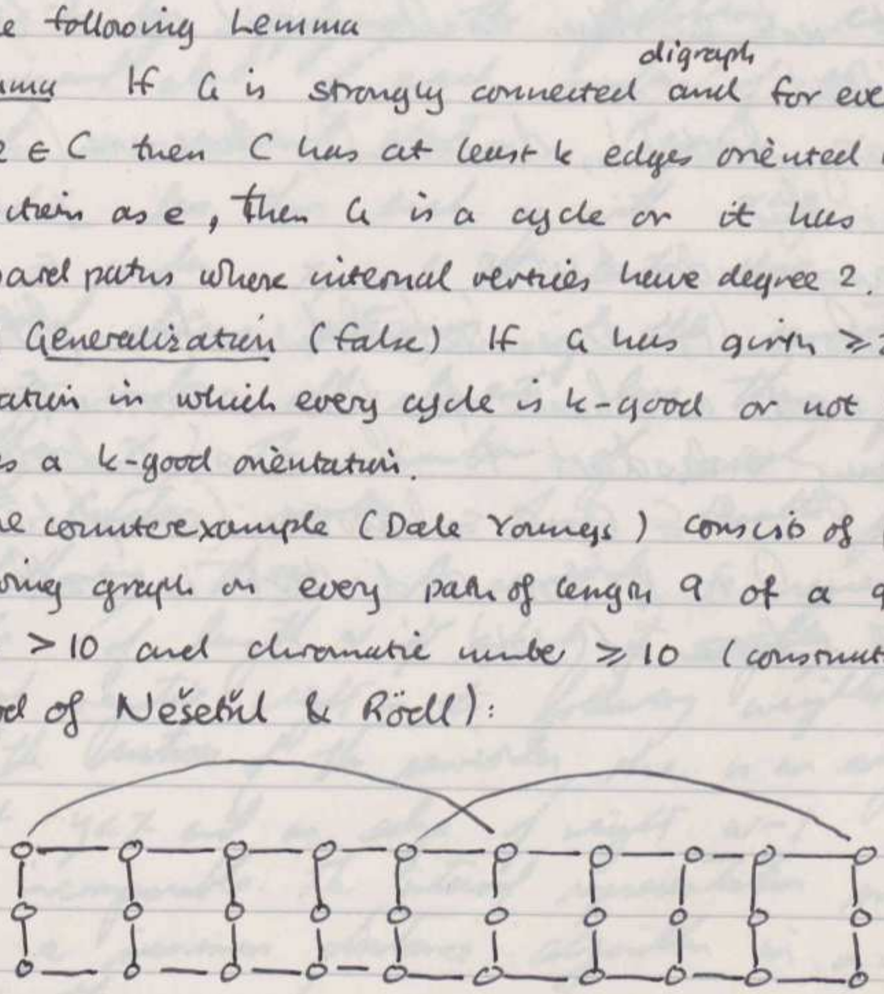
<u>Weak Generalization</u> (Pretzel) If $G$ has girth $\geq 2k$ and an orientation in which every cycle is montone or $k$-good, then $G$ has a $k$-good orientation.

The proof (which includes a new proof of the Theorem) is based on the following Lemma

<u>Lemma</u> If $G$ is strongly connected digraph and for every cycle $C$ if $e \in C$ then $C$ has at least $k$ edges oriented in the same direction as $e$, then $G$ is a cycle or it has 2 disjoint forward paths where internal vertices have degree 2.

<u>Strong Generalization</u> (false) If $G$ has girth $\geq 2k$ an an orientation in which every cycle is $k$-good or not $(k-1)$-good, then $G$ has a $k$-good orientation.

The counterexample (Dale Youngs) consists of putting the following graph on every path of length 9 of a graph of girth $>10$ and chromatic numbe $\geq 10$ (constructed, say, by the method of Nešetřil & Rödl):



O. Pretzel

Dimension Invariance of Lattice Subdivisions

Ivan Rival (Ottawa).

Although N-free ordered sets may yield to effective constructions in certain optimization problems (e.g. jump number) there is much evidence that they are as complex as all ordered sets (cf. recent work of M. Habib and R. Möhring). Indeed, every ordered set is contained in an N-free ordered set.

Lee, Liu, Nowakowski and Rival show that the dimension problem for N-free ordered sets is NP-complete, answering a recent question recently put forth by M. Habib. The proof amounts to showing that the dimension problem for N-free orders is equivalent to the dimension problem for arbitrary orders, thus confirming again that N-free orders are as complex as all.

The proof rests on these results which seem to be of independent interest.

For any lattice L, dimension(L) = dimension(subdivision(L)).

A consequence is this.

For any ordered set P,

dimension(P) = dimension(subdivision(completion(P))).

This is then used as a basis for this construction.

For any ordered set P there is an ordered set Q satisfying P ⊆ Q ⊆ subdivision(completion(P)) such that Q is N-free and |Q| is small (i.e., polynomial in terms of |P|).

# Discrete Representation Theory for Semiorders.

Kenneth Bogart (Hanover (U.S.A.))

Semiorders may be characterized as ordered sets which do not have as restrictions the sum of two two-element chains or the sum of a point and a three-element chain. It is possible to give a list of semiorders whose absence characterizes those ordered sets representable by intervals of length $n$. A semiorder is representable by intervals of length one if it has no restriction isomorphic to a sum of a point and a two element chain. The family whose absence characterizes ordered sets representable by intervals of length $n$ may be ~~characterized~~ constructed from the family for intervals of length $n-1$ by applying the following construction to each minimal element of each member of the family.

Select a minimal element $x$. Replace $x$ with 2 elements $x_1$ and $x_2$ less than each $y$ with $x<y$. Introduce a new element $z$ under $x_1$ and all elements above $x$ in the canonical linear extension of the semiorder. (That is, place $z$ under all elements less than fewer elements than $x$.) The number of such examples is the $n^{th}$ Catalan number. These results follow from the following theorem: A semiorder is representable by intervals of length $n$ iff and only if ~~weighting~~ there are no cycles of negative weight in the following weighted digraph: The vertices are the vertices of the semiorder; there is an edge of weight $-n$ from $x$ to $y$ if $y<x$ and an edge of weight $n-1$ from $x$ to $y$ if $x$ and $y$ are incomparable. The interval representation may be found by applying a minimum distance algorithm in a natural way to this graph.

# BOUNDS TO THE PAGE NUMBER OF A POSET

(Maciej M. Sysło, Inst. of Comput. Sci., Univ. of Wrocław, Poland)

The page number of a poset P is the page number of the diagram HD(P) when the elements of P can be put on the book spine in a topological order (i.e., as a linear extension). Let $pn(P)$ denote the page number of P. It is easy to show that $pn(P)$ is not a comparability invariant. We have $pn(P) = 1$ iff HD(P) is a tree. It can be shown also that $pn(P) = 2$ if HD(P) is a cycle. In these two cases, pn is the diagram invariant. Let $s(L,P)$ be the number of jumps in a linear extension L of P. We have

$$\ell(P) = \left\lceil \frac{m-n+1}{s(L,P)} \right\rceil + 1 \leq pn(P)$$

Although the $\ell(P)$ is minimized for $s(L,P)$ equal to the jump number, $pn(P)$ may not be attained for a linear extension with the maximum number of jumps. On the other hand,

$$pn(P) \leq c(P),$$

where $c(P)$ is the cover number of HD(P). Then another bound can be obtained by considering complete bipartite subgraphs in HD(P). These bounds have been used to calculate pn for some classes of posets.

We also compare this notion with that for graphs.

On Shelah's proof that the van der Waerden function is primitive recursive.

Walter Deuber Bielefeld

Ein klassischer Satz von van der Waerden besagt, dass zu $k, r \in \mathbb{N}$ eine kleinste Zahl $w(k,r)$ existiert mit der Eigenschaft, dass zu jeder Zerlegung von $\{1, \dots, w(k,r)\}$ in $r$ Klassen in mindestens einer dieser Klassen eine arithmetische Progression mit $k$ Termen sich befindet.

Während bisherige obere Schranken stets Ackermann-qualität hatten, konnte Shelah kürzlich zeigen, dass $w$ eine primitiv rekursive Funktion ist.

Some observations concerning the fixed point property for ordered sets – Aleksander Rutkowski (Warsaw, Poland)

I. Let $a, b \in P$ (an ordered set) and $x \notin P$. Define an order on $P \cup \{x\}$ in the following way:

$$p \leq q \Leftrightarrow \begin{cases} p < q & \text{if } p, q \in P \\ \text{or} & \\ (p<a \;\&\; a \leq q) \vee p<b \;\&\; b \leq q & \text{if } p = x \;\&\; q \in P \end{cases}$$

Denote $(P \cup \{x\}, \leq)$ by $P(a,b,x)$

Theorem. Assume that (i) $P$ has the FPP (fixed point property)

(ii) $\{a,b\}^*$ has the FPP &

(iii) $\{a,b\}_* \neq \emptyset$

then $P(a,b,x)$ has the FPP

[$\{a,b\}^*$ is the set of all upper bounds of $\{a,b\}$, $\{a,b\}_*$ : of all lower bounds]

Theorem. Let $Q$ be ~~the~~ a poset with FPP, (i), (ii), (iii) ~~are~~ be satisfied, and ~~one~~ $P$ or $Q$ be chaine-complet (i.e each nonempty chain has both ~~the~~ sup and ~~the~~ inf). Then $P \times Q$ has the FPP

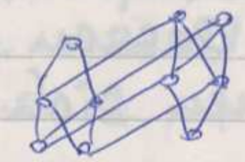
II. Let $M_P = Min_P \cup Max_P$.

Theorem. Let $P$ be a chain-complete ~~Po~~ poset and

$(\forall p \in P)(\exists x \in Min_P)(\exists y \in Max_P)\; x \le p \le y$

~~The~~ Thus, if ~~M~~$_P$ has the FPP then $M_P$ has it as well

III. Let $P$ be a poset from the following Figure:



Thm. For every positive integer $n$, $P^n$ has the FPP.

IV. ~~A normal~~ A fence $x_0 < y_0 > x_1 < y_1 > x_2 < \dots$ is normal if, for each $i$ $y_i = \sup(x_i, x_{i+1})$ and $x_i = \inf(y_{i-1}, y_i)$.

Let $P$ be a connected, chain-finite, crown-free poset. If every infinite normal fence contains an infinite subset which is up- or down-bounded then $P$ has the FPP.

On the complexity of families of sets
(Daniel Grieser, Berlin)

We consider the following problem:

Given a family $\mathcal{P} \subseteq 2^T$ of subsets of some finite set $T$, determine the complexity $c(\mathcal{P})$, which is defined as the minimal number of tests necessary to decide if an imaginary set $H \subseteq T$ is in $\mathcal{P}$ or not, a test being a question "Is $x \in H$?" for some $x \in T$.

This kind of question was first discussed by Holt and Reingold and Rosenberg 1973, in the special case where $P$ is a graph property.

Trivially $c(P) \leq t = |T|$ for all $P$, and in fact this bound is attained by almost all $P$ if $t \to \infty$.

We consider families (properties) $P$ with low complexity.
A wellknown theorem by Rivest and Vuillemin states that a property $P$ with $c(P) \leq t-k$ must be the disjoint union of intervals of length $k$.
We prove a kind of weak inversion of this:
If $P$ is the disjoint union of $r > 1$ intervals of length $\geq k$, then $c(P) \leq 2(t-k) \ln r$ (natural logarithm).
In the course of the proof we establish an interesting connection to a problem concerning edge coverings of a complete graph by bipartite graphs.

Boolean lattices, combinatorial spaces and Ramsey theory

In this talk we discuss two extensions of Hales-Jewett's theorem on combinatorial spaces with particular emphasis on the special case of Boolean lattices. The first one is a ordering version of Hales-Jewett's result, describing all natural orders on combinatorial spaces. A characterization of all these natural orders was first given by Nešetřil, Prömel, Rödl, Voigt, J. Comb Th (A), 1985. Here we present a new simplified approach. The second result we discuss is a "sparse" version of Hales-Jewett's theorem which is a joint result with B. Voigt and will appear in Trans. Amer. Math. Soc.

Hans Jürgen Prömel (Bonn)

Finite modular lattices freely generated by an ordered set

Peter Luksch (TH Darmstadt)

Free modular lattices are of central interest in lattice theory. In particular, one considers FM(P), the modular lattice freely generated by an ordered set P.

If the width of P is two, FM(P) becomes distributive and hence is isomorphic to FD(P) the free distributive lattice generated by P. In this case we state a recursive structural formula for FD(P) which can be used to obtain a reasonable line diagramm. Our basic idea is to study a decomposition by a congruence relation which has congruence classes isomorphic to a direct product $FD(Q_1) \times FD(Q_2)$ for some $Q_1, Q_2 \subsetneqq P$. Then a structural formula for FD(P) can be described which uses the knowledge of some FD(Q) for proper subsets Q of P.

For the modular lattice $FM(\underline{1}+\underline{1}+\underline{n})$ freely generated by two single elements and an n-element chain we state a recursive counting formula. This answers Problem 44 in Birkhoff (Lattice Theorie Amer. Math. Soc. (1967)) which asks one to determine $FM(\underline{1}+\underline{1}+\underline{n})$. Therefore we study subdirect products of copies of $D_2$ and $M_3$ via their scaffoldings. In this way we obtain a deeper understanding of the structure of $FM(\underline{1}+\underline{1}+\underline{n})$.

On the interval inclusion number of a partially ordered set

Douglas B. West (with Thomas Madej)

A <u>containment representation</u> of a poset P is a map f such that $x<y$ in P if and only if $f(x) \subset f(y)$. We introduce the <u>interval inclusion number</u> (or <u>interval number</u>) $i(P)$ as the smallest t such that P has a containment representation in which each $f(x)$ is the union of at most t intervals. Trivially, $i(P)=1$ if and only if $\dim P = 2$. Posets with $i(P)=2$ include the standard n-dimensional poset and all interval orders; i.e., posets of arbitrarily high dimension. In general, $i(P) \leq \lceil \dim P/2 \rceil$, with equality for Boolean algebras. For lexicographic composition, $\dim(Q)=2k+1$ and $i(P)=k$ imply $i(P[Q])=k+1$. This and $i(B_{2k})=k$ imply that testing $i(P)\leq k$ is NP-complete for fixed k. The maximum value of $i(P)$ for n-element posets remains unknown, but $i(P)=\Theta(|P|/\log|P|)$ for almost every poset. Concerning removal theorems, $i(P-x)\geq i(P)-1$ when x is a maximal or minimal element, and in general $i(P-x)\geq i(P)/2$.

Fourier analysis of a problem on finite sets

Jeff Kahn (New Brunswick) (joint w G. Kalai & N. Linial)

For $X \subseteq \{0,1\}^n$ (endowed with the usual graph structure) let $E_i$ be the set of edges having an end in each of $X, \bar{X}$, and $\alpha_i = |E_i|/2^{n-1}$. Set

$$f(n) = \min_{|X|=2^{n-1}} \max_i \{\alpha_i\}.$$

(We restrict to $|X| = 2^{n-1}$ only for simplicity.)
The problem of bounds for $f(n)$ seems first to have appeared in print in Ben-Or and Linial (see this article also for connections with computer science and game theory). Ben-Or and Linial observed that

$$\frac{1}{n} \leq f(n) \leq \frac{\log n}{n} \qquad (1)$$

and conjectured that the upper bound was close to the truth. (The lower bound follows from the well-known (easy) fact that

$$\sum \alpha_i \geq 1. \qquad (2)$$

Since (e.g.) equality in (2) requires $X = \{x : x_i = \varepsilon\}$ for some $i \in [n]$ and $\varepsilon \in \{0,1\}$ (in which case $\max \alpha_i = 1$), the lower bound in (1) seems extremely weak. Still, the best results till now were $f(n) > \frac{2-o(1)}{n}$ due to Alon, and $f(n) > \frac{3-o(1)}{n}$ due to Gerib-Grans.)
Here we settle the question (up to a constant):

THEOREM. $f(n) > c \log n / n$.

Our proof uses techniques of Fourier analysis on $\mathbb{Z}_2^n$ and has implications for (a) random walks on the cube, and (b) distributions of distances in subsets of the cube.

## Diagrams, orientation, and varieties

Hans-Jürgen Bandelt (with Ivan Rival)

A class of finite ordered sets is a diagram variety if it is closed with respect to diagram retracts (= images of order- and cover-preserving idempotent maps) and Cartesian products. An orientation variety is a diagram variety closed with respect to reorientations. For instance, the diagram variety generated by the two-element chain $\underset{\sim}{2}$ consists of all finite distributive lattices, while a finite ordered set is in the orientation variety generated by $\underset{\sim}{2}$ if and only if its covering graph is median. Thus, reorientations of such ordered sets are necessarily inversions: an inversion is a reorientation obtained by a sequence of pushdowns (sensu Pretzel). Then the orientation variety generated by a class K of finite ordered sets consists of the diagram retracts of the inversions of the Cartesian products of the reorientations of members of K. In particular, the class of all finite ordered sets for which all reorientations are inversions is an orientation variety. Such ordered sets can be described in terms of a configuration forbidden in all reorientations.

## Divisors Without Unit-Congruence Ratios

D. Kleitman (Cambridge Ma)

We address the question: how large can a collection C of divisors of a square free integer N be, if whenever $A, B \in C$, $A \mid B$ then $A \not\equiv B \bmod p$?

We show that when, for each $j$, the number of prime factors of N congruent to $j$ mod p is the same as the number congruent to $\frac{1}{j}$, and the number $\equiv -1$ is even, ~~then~~ and N has $n$ prime factors, then an upper bound is $\binom{n}{[n/2]} + \binom{n}{[\frac{n}{2}+1]}$.

This bound can be achieved when N has no prime factors $\equiv 1 \bmod p$.

## The Partial order of Equal Size Subsets,

D Kleitman (Cambridge Ma) with (F. Chung + Moulton + NJ)

Let $|S| = n$. We consider ordered pairs of subsets of S of equal size, ordered by inclusion in each component. We ~~show characterize~~ describe how to construct $\alpha$ pairs of size $k$ that are covered by fewest pairs of size $k+1$, for any appropriate $\alpha$.

Though there is no canonical order of the pairs of size $k$ such that an initial segment is the answer (for $n \geq 4$), one can construct such collections of pairs.

## Incidence Algebras

James Schmerl (Storrs, Connecticut, USA)
(w/ E. Spiegel & M. Parmenter)

Let $P$ be an arbitrary locally finite (i.e. all $[x,y]$ finite) poset and $R$ ~~an~~ a commutative ring with 1. Define the incidence algebra $I(P,R)$ to be the algebra over $R$ where
$$I(P,R) = \{ f : P\times P \to R \mid f(x,y) \neq 0 \Rightarrow x \leq y \}$$
with operations defined by
$$(f+g)(x,y) = f(x,y) + g(x,y),$$
$$(fg)(x,y) = \sum_{x \leq z \leq y} f(x,z)\, g(z,y),$$
$$(af)(x,y) = a(f(x,y)).$$
The question we consider, known as the "isomorphism problem" is the following:
Does $I(P,R) \cong I(Q,R) \Rightarrow P \cong Q$?

Theorem 1. If $P, Q$ are countable, then $I(P,R) \cong I(Q,R) \Rightarrow P \cong Q$.

A corollary to the proof is: If $R$ has $< 2^{\aleph_0}$ idempotents, then $I(P,R) \cong I(Q,R) \Rightarrow P \cong Q$.

The conclusion ~~of~~ to Theorem 1 is actually shown to be obtained for arbitrary $P$ and $Q$, but is then a little weaker: $I(P,R) \cong I(Q,R) \Rightarrow P \equiv_{\infty\omega} Q$. ($\equiv_{\infty\omega}$ is a well-known notion from model-theory).

Using concepts from Boolean-valued models in set theory we prove a converse:

Theorem 2. If $P \equiv_{\infty\omega} Q$, then there is a ring such that $I(P,R) \cong I(Q,R)$.

N.B. There exist many examples of $P, Q$ for which $P \equiv_{\infty\omega} Q$ but $P \not\cong Q$.

Chains, antichains and cut-sets for infinite posets.

N. Sauer (with A. Hajnal)

For $(P, \leq)$ a poset and $x \in P$ the set ~~x||~~ $S \subset P$ is a cut-set for $x$ in $P$ if for every maximal chain $C$ of $P$ $(C \cap \{x\} \cup S \neq \emptyset)$. $\kappa$ is the cut-set number of $P$, if $\kappa$ is the smallest cardinal such that every $x \in P$ has a cut-set $S$ with $|S| < \kappa$. $\nu$ is the chain-number of $P$, if $\nu$ is the smallest cardinal such that for every maximal chain $C$ of $P$ $|C| < \nu$ holds. A poset with chain number $\nu$ and cut-set number $\kappa$ is said to have type $(\kappa, \nu)$. If $P$ has type $(\kappa, \nu)$ the size of an antichain must be bounded in terms of $\kappa$ and $\nu$. So, $\Pi(\kappa, \nu)$ denotes the smallest cardinal such that if $A$ is an antichain of a poset of type $(\kappa, \nu)$ then $|A| < \Pi(\kappa, \nu)$. We determined $\Pi(\kappa, \nu)$ for all cardinals $\kappa, \nu$ with $\kappa + \nu \geq \aleph_0$: $\Pi(\kappa, \nu) = (\kappa^{\underline{\nu}})^+$ if $\kappa \geq \nu$ and $\kappa$ is either a successor or singular or accessible from $\nu$ and $\nu \geq 5$. $\Pi(\kappa, \nu) = (\nu^{\underline{\kappa}})^+$ if $\nu \geq \kappa$ and $\nu$ is either singular or a successor or accessible from $\kappa$ and $\kappa \geq 3$. If $\kappa < \nu$ and $\nu$ not accessible from $\kappa$ or $\nu < \kappa$ and $\kappa$ not accessible from $\nu$, so not the previous case holds, then $\Pi(\kappa, \nu) = \Pi(\nu, \kappa) = 0$. e.g.: there is no poset of this type. If $\kappa$ is weakly compact then $\Pi(\kappa, \kappa) = 0$, if $\kappa$ is a strong limit but not weakly compact then $\Pi(\kappa, \kappa) = \kappa$. Also, if $\kappa$ is ~~a limit~~ singular there is no maximal antichain of size $\kappa$ in a poset of type $(3, \nu)$ or $(4, \nu)$.

The core(s) of finite lattices. V. Duquenne, Paris.

Motivated by some practical reasons in Data Analysis in Psychology (description of Experimental Designs built on 2-permuting partition sublattices, langage for describing their statistics...; analysis of dependencies between attributes in Formal Concept Analysis...), as well as for generalyzing the celebretaded BIRKHOFF's Theorem which exhibits any distributive lattice $L$ as isomorphic to the (order-) filter lattice of the set $M(L)$ of its meet-irreducible elements, the following is proved: For a finite lattice $L$, $x \in L$ is said to be $\wedge$-essential if there exists an order filter $X \subset [x)$ with $\wedge X = x$ and $X \cup \{x\}$ a sublattice of $[x)$. Let denote by

$K_\wedge(L) := M(L) \cup \{x \in L \mid x \text{ is } \wedge\text{-essential}\}$ the $\wedge$-core of $L$. TH1: the filter lattice of the partial $\wedge$-semilattice constructed on $P \subseteq L$ is isomorphic to $L$ iff $P \supseteq K_\wedge(L)$. TH2: for a subdirect product, the $\wedge$-core is equal to the union of the factor cores. TH3: Let $L$ be modular; $x$ $\wedge$-essential iff ~~implies~~ ~~that~~ the sublattice generated by the covers of $x$ is a covering $M_n$. Some properties of the $\wedge$-core (resp. $\vee$-core) of semimodular lattices are given; and the cores ($\wedge$ and $\vee$) of a geometric lattice are characterized.

## On the Fibonacci number of an $m \times n$ lattice

K. Engel, Rostock, GDR

Let $Z_{m,n} := \{(i,j) : 1 \le i \le m,\ 1 \le j \le n\}$ and $x_{m,n}$ be the number of subsets $A$ of $Z_{m,n}$ with the property: there are no $(i_1,j_1), (i_2,j_2) \in A$ with $|i_1 - i_2| + |j_1 - j_2| = 1$. Using linear algebraic techniques we prove several inequalities for the numbers $x_{m,n}$ and show that
$$1.503 \le \lim_{m\to\infty} \lim_{n\to\infty} x_{m,n}^{1/mn} = \lim_{n\to\infty} x_{n,n}^{1/n^2} \le 1.514.$$
We conjecture that $x_{m,2k}^2 \ge x_{m,2k-2}\, x_{m,2k+2}$ holds for all positive integers $m$ and $k$ which implies
$$\frac{x_{m,2}}{x_{m,1}} \le \frac{x_{m,4}}{x_{m,3}} \le \frac{x_{m,6}}{x_{m,5}} \le \dots \le \frac{x_{m,5}}{x_{m,4}} \le \frac{x_{m,3}}{x_{m,2}} \le \frac{x_{m,1}}{x_{m,0}}$$
for all $m$ as well as
$$\lim_{n\to\infty} x_{n,n}^{1/n^2} = 1.50304808\ldots$$

## Partial orders of interval dimension two and a channel routing problem

Rolf H. Möhring (Berlin)
[jointly with M. Habib, Brest]

It was shown by Dagan, Golumbic and Pinter (DAM, to appear) that certain VLSI channel routing problems can be modeled as the intersection of two interval orders, i.e. by partial orders P of interval dimension at most two ( $\mathrm{idim}(P) \le 2$ ). We obtain a polynomial algorithm that tests whether a partial order has $\mathrm{idim}(P) \le 2$ and, if so, finds two associated interval orders. The algorithm exploits a lower bound of $\mathrm{idim}(P)$ given by the dimension $\dim(Q)$ of the partial order $Q$ of all downsets $D(u) = \{ v \in P \mid v < u \}$ ordered by inclusion. This solves an open problem of Yannakakis (SIAM J. Alg. Discr. Meth, 1982) about the complexity of interval dimension two.

## Order dimension via Ferrers relations

K. Reuter, TH Darmstadt

A survey of my work on some problems about order dimension will be given.

1) How small can a lattice of order dimension $n$ be?
2) It is known that:
   $\max\{\dim P, \dim Q\} \le \dim P \times Q \le \dim P + \dim Q$
   Can the bounds be improved?
3) Given a convex polytope $P$. Is
   order dim (face lattice($P$)) = 1 + affine dim ($P$). ?
4) Does the removal of a critical pair in an ordered set $P$ alway decreases the dimension by at most one.

I have used the more general concept of Ferrers relations to get some new insight and partial answers to the questions raised above. The answers to 3. and 4. is "no".

# Fibres in Ordered Sets and Clique-Transversals of Graphs

T. Andreae (FU Berlin)

This is joint work with M. Schughart (Berlin) and Z. Tuza (Budapest). A <u>fibre</u> of an ordered set P is a collection F of elements of P that meets every maximal antichain. Here are two conjectures on fibres:

<u>Conjecture 1</u> (Aigner/Andreae, 1985): If P has no cutpoints, then there is a fibre F such that $|F| \leq n/2$, where $n = |P|$.

<u>Conjecture 2</u> (Lonc/Rival, 1986): If P has no cutpoints, then there is a fibre F such that the complement P−F is also a fibre.

Clearly Conj. 2 would imply Conj. 1. Lonc and Rival established Conj. 2 for some restricted classes of ~~po~~ ordered sets.–

Both conjectures have a strong graph-theoretic flavor: for a graph G without isolated vertices let $\tau_c(G)$ be the smallest number of vertices that meet every maximal clique. Consider the following two properties, where $\mathcal{G}$ is a class of graphs without isolated vertices:

($P_1$) $\tau_c(G) \leq n/2$ for all $G \in \mathcal{G}$

($P_2$) for all $G \in \mathcal{G}$ the vertices of G can be colored red and blue such that G has no monochromatic maximal clique.

(P₁) and (P₂) where observed (by M. Aigner and myself) to hold for several classes $\mathcal{G}$ of perfect graphs including triangulated graphs, cotriangulated graphs, comparability graphs, Meyniel-graphs, perfectly orderable graphs; however, it is not known whether $(P_1)$ and $(P_2)$ hold for incomparability graphs (this is exactly what the above Conjectures 1 and 2 claim).

In ~~the~~ a recent work of Schughart, Tuza and myself (1987) properties $(P_1)$, $(P_2)$ where investigated for line graphs and complements of line graphs. (The motivation for looking at line graphs and their complements results from the fact that $(P_1)$ and $(P_2)$ where known to hold for line graphs of bipartite graphs and complements of line graphs of bipartite graphs.) Our results are

1. $(P_1)$ holds for line-graphs with the exception of odd cycles,
2. $(P_1)$ and $(P_2)$ hold for all complements of line-graphs with the exception of some small graphs,
3. ~~we~~ characterize the line-graphs for which $(P_2)$ holds.

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A New Upper Bound on the Dimension of Interval Orders

Z. Füredi, V. Rödl, and T. Trotter

Let $f(n)$ be the least positive integer so that if $P$ is any interval order of length $n$, then $\dim(P) \leq f(n)$. The existence of $f(n)$ was first established by I. Rabinovitch who proved that $f(n) \leq 1 + 2\lceil \log_4 n \rceil$. This was improved (slightly) by Bogart, Rabinovitch and Trotter who showed that there exists a constant $c > 2$ and a value $n_0$ so that $f(n) < \log_c n$ when $n \geq n_0$. On the lower side, one can show $f(n) \geq \log \log n$. Using an analogy with shift graphs this can be improved to $f(n) \geq \log\log n + \frac{3}{2} \log\log\log n$. In this paper, we show $f(n) \leq c \log \log n$. It is probably true that $f(n) = (1+o(1)) \log \log n$.

A polynomial approximation algorithm for Dynamic Storage Allocation

Hal Kierstead

It is shown that the greedy algorithm for coloring interval graphs requires at most 40 times the clique size colors, in the worst case. This answers a question of Woodall (1973) and Chrobak and Slusarek (1984). It follows from independent ideas of both sets of authors that Dynamic Storage Allocation has a polynomial approximation algorithm with constant performance ratio of 80.

## Some Results On Correlation

Graham Brightwell (Cambridge)

This talk constitutes a survey of some (fairly) recent results concerning correlation in posets. For $(X,R)$ a finite poset $(R \subseteq X \times X)$, and $A \subseteq X \times X$, we define $P(A|R)$, the probability of $A$ (given $R$) to be the proportion of linear extensions $\prec$ of $R$ which respect $A$ (i.e. with $x \prec y$ whenever $(x,y) \in A$). The pair $(A,B)$ is (positively) correlated (with respect to $R$), $A \uparrow_R B$, when $P(A|R)\,P(B|R) \leq P(A \cup B|R)$. The Results in this area involve restrictions on either $R$ or $(A,B)$. Solutions are "given" to the following problems.*

1) Classify $(A,B)$ s.t. $A \uparrow_R B$ holds with respect to every poset $R$. (Winkler)

2) Classify $R$ s.t. $(x,y) \uparrow_S (u,v)$ holds for all extensions $S$ of $R$.
[This is related to the problem of classifying $(A,B)$ s.t. $A \uparrow_R B$ holds with respect to every poset $R$ on a fixed ground-set. An example is furnished by the Graham, Yao, & Yao inequality.]

3) Classify $R$ s.t. $(x,y) \uparrow_S (u,v)$ holds for all subposets $S$ of $R$.
[An example is furnished by a result of Shepp.]

## "Mechanisms and algorithms for multiple inheritance in object oriented systems"

Michel HABIB

(Brest, France)

I present a joint work with R. DUCOURNAU (INRIA) about inheritance algorithms. They are the kernel of object oriented systems. When multiple inheritance is allowed (the inheritance graph is any directed acyclic graph) then conflict may occur. We present and compare with known algorithms, our propositions for a good inheritance mechanism (i.e. satisfying some principles such as: particular-to-general, modularity...). In all these algorithms the depth-first greedy linear extensions play a great role and are very helpful. It is worth noticing that these linear extensions also called "supergreedy" were defined by O. PRETZEL during an Oberwolfach meeting in 1985.

On the skeletons of free distributive lattices / Rudolf Wille (TH Darmstadt)

The aim is to understand the structure of free distributive lattices via their skeletons. The skeleton S(L) of a finite distributive lattice L consists of all maximal Boolean intervals of L ordered by their lower (or equivalently upper) bounds; S(L) is again a lattice. To analyse the skeletons of the free bounded distributive lattices FBD(n) with n generators, methods of formal concept analysis are helpful. As key we use the basic fact that FBD(n) is isomorphic to the concept lattice $\underline{\mathfrak{B}}(B_n, B_n, \not\geq)$ and $S(FBD(n)) \cong \underline{\mathfrak{B}}(B_n, B_n, \not\succ)$ where $B_n$ is the Boolean lattice with n atoms.

<u>Theorem:</u> The maximal Boolean intervals containing n-1 of the generators generate in S(FBD(n)) a 0-1-sublattice isomorphic to FBD(n-1); if $n \leq 5$, S(FBD(n)) is the union of these n sublattices.

Corollary: $|S(FBD(5))| = 386$

The poset of closures

Gyula O.H. Katona.

Let X be a set of n elements and consider all the closures $\mathcal{L}$ on X: $\mathcal{L} : 2^X \to 2^X$,

1) $\mathcal{L}(A) \supseteq A$ 2) $A \subseteq B \Rightarrow \mathcal{L}(A) \subseteq \mathcal{L}(B)$ ~~for~~ 3) $\mathcal{L}(\mathcal{L}(A)) = \mathcal{L}(A)$

for all $A, B \subseteq X$. The ordering $\mathcal{L}_1 \le \mathcal{L}_2$ iff $\mathcal{L}_1(A) \supseteq \mathcal{L}_2(A)$ holds for all $A$, is introduced. In a paper with Burosch, Demetrovics and Sapoženko we investigated the total number of elements of the poset $P$ defined by $\le$. Also there are some asymptotic results on the number elements of the $k$th and $(2^n-2-k)$th level ($P$ is ranked). In another joint paper (Order 1987) with Burosch and Demetrovics we gave upper and lower estimates on the max and min degrees of the elements of a given rank.

After the talk, D. J. Kleitman improved our estimate on the size of $P$.

Minimal cutsets of the Boolean lattice

Zoltán Füredi (with J. R. Griggs and D. J. Kleitman)

$\mathcal{C} \subset B_n$ is a cutset if it intersects all maximal chains (i.e. chains of the form $\{L_0 = \emptyset \subsetneq L_1 \subsetneq \dots \subsetneq L_n = [n]\}$). $\mathcal{C}$ is minimal if $\mathcal{C} \setminus \{C\}$ is not a cutset for all $C \in \mathcal{C}$. E.g. a whole level of $B_n$ is a minimal cutset. However there are much larger minimal cutsets. Namely,

$$\{C \subset [n] : |C \cap \{1,2\}| = 1\}$$

has size $2^{n-1}$. It is easy to see that $c(n+1) \ge 2c(n)$, so $\lim_{n\to\infty} c(n)/2^n$ exists. Ko-Wei Lih (Taipei, Taiwan) gave a construction $c(6) \ge 33$. Here we give an almost explicit construction, proving

$$c(n) = (1 - o(1))\,2^n .$$

# Gruppen und Geometrien (1.5.88 – 7.5.88)

## Quadratic modules for finite simple groups
Gernot Stroth (FU-Berlin)

Let $G$ be a finite group and $V$ be a faithful $GF(p)G$-module. We say that $V$ is a quadratic module for $G$ iff there is some $p$-subgroup $1 \neq X$ of $G$ such that $[V,X,X]=1$. There is a well developed theory if $p$ is odd. If $p=2$, then $[V,t,t]=1$ for any involution $t \in G$. So the first interesting case is that $X$ is a fours group. Together with U. Meierfrankenfeld we studied the following situation

(*) (i) $G$ is a finite group containing a normal subgroup $H$ such that $C_G(H)=Z(H)$, $H'=H$, $|Z(H)|$ is odd and $H/Z(H)$ is simple.

(ii) There is some faithful module $V$ over $GF(2)$ and a fours group $E \leq G$ such that $[V,E,E]=1$.

We have the following result:

Theorem: If $H/Z(H)$ is a Lie-group in odd characteristic or a sporadic group, then $H$ is isomorphic to $L_2(5)$, $L_2(7)$, $L_2(9)$, $3 \cdot L_2(9)$, $U_3(3)$, ${}^2G_2(3)'$, $PSp_4(3)$, $3 \cdot U_4(3)$, $M_{12}$, $3 \cdot M_{22}$, $M_{24}$, $C_1$, $C_2$, $J_2$ or $3 \cdot Suz$

Gefördert durch DFG Deutsche Forschungsgemeinschaft

# Rank - 3 - amalgams

Andreas Böhmer, Gießen

Let $p$ be a prime, and $G$ a group generated by its finite subgroups $P_0, P_1, P_2$ satisfying

(1) $B := \bigcap_{i\in\{0,1,2\}} P_i = P_i \cap P_j$, $i \neq j \in \{0,1,2\}$, $= N_{P_i}(S)$, $S \in Syl_p(B)$

(2) $O^{p'}(P_i/O_p(P_i)) \simeq (S)L_2(p^{n_i}), (S)U_3(p^{n_i}), Sz(p^{n_i}), Ree(p^{n_i})$, $i = 0,1,2$

(3) $P_i \ntriangleright O_p(P_i) \cap O_p(P_j) \ntriangleleft P_j$ for $i = 2$ and $j = 0$ or $1$,
but $P_0 \triangleright O_p(P_0) \cap O_p(P_1) \triangleleft P_1$

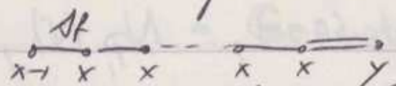
(4) $B_G := \bigcap_{g\in G} B^g = 1$

Such a group is also called a weak-BN-pair of rank 3. If $\langle P_0, P_2\rangle \ntriangleright Z := \Omega_1(Z(S)) \ntriangleleft \langle P_1, P_2\rangle$, then the local structure of $G$ is given by a theorem of Stellmacher and Timmesfeld. I investigated the situation, where $Z \triangleleft \langle P_1, P_2\rangle$, under the additional hypothesis that $\langle O^{p'}(P_1), O^{p'}(P_2)\rangle$ is „essentially" (i.e. modulo some normal subgroup) parabolically isomorphic to $(S)L_3(p^{n_1})$. It turned out, that $p^{n_1} = 2$, and the local structure of $G$ could be determined. Examples are given by the sporadic simple groups $He$ and $M_{24}$, but there may exist also examples, which have no finite analogue.

On a class of Geometries related to affine polar spaces.

Antonio Pasini Siena.

Let $\Gamma$ belong to the following diagram



(rank $n+1$)

and assume that $\Gamma$ is finite and that (IP) holds in res. Then there is a constant $\gamma$ such that, given a point $a$ and a maximal subspace $u$, ~~there is exactly one hyperpl~~ such that $a \notin u$ and $a^\perp \cap u \neq \emptyset$, there is exactly one hyperplane $w \ni a$ and $\gamma$ max. subspaces $u_1, \dots u_\gamma$ such that $u_i \supseteq w$ $(i = 1, \dots \gamma)$, $u_i \cap u$ is a hyperplane $(i = 1, \dots \gamma)$, $u_1 \cap u, \dots u_\gamma \cap u$ are pairwise parallel, $a^\perp \cap u = \bigcup_{i=1}^{\gamma} u_i \cap u$ and $u_i \cap u$ is parallel to $w$ inside $u_i$ $(i = 1, \dots \gamma)$.

It is known that $\gamma = 1$ iff $\Gamma$ is an affine polar space. I prove that $\gamma = x$ iff $\Gamma$ is either the geometry for the 2-transitive action of $x^{2n} Sp_{2n}(x)$ or one of the two geometries for the 2-transitive actions of $Sp_{2n}(x)$.

# Monoids of Coxeter type having attractors.

S. Tsaranov

Let $\Delta = \{X_i\}_{i \in I}$ ($I = \{1, 2, \dots, n\}$) be a set of subgroups in a group $G$ (not necessaryly finite) such that $X_i, X_j$ are proper subgroups in $X_{ij} = \langle X_i, X_j \rangle$. Let $F(\Delta)$ be a monoid which elements are the subsets of $G$ presented as products of subgroups of $\Delta$ and operation is the usual product of them as subsets of $G$. Moreover, let $r_s(i,j) = \underbrace{X_i X_j X_i \dots}_{s}$ and $l_s(i,j) = \underbrace{\dots X_i X_j X_i}_{s}$ be elements of $F(\Delta)$.

Define a matrix $M = M(\Delta) = (m_{ij})_{n \times n}$ as follows: $m_{ii} = 1$; for $i \neq j$ $m_{ij} = \min\{s : r_s(i,j) = X_{ij}\}$. Denote $r(i,j) = r_{m_{ij}}(i,j)$, $l(i,j) = l_{m_{ij}}(i,j)$. It is not so hard to show that if $m_{ij} = 2$ then $m_{ji} = 2$ and if $m_{ij} \neq m_{ji}$ then $\{m_{ij}, m_{ji}\} = \{2s-1, 2s\}$ for some $s > 1$. Such matrices will be called generalized Coxeter matrices (GCM). For every GCM $M$ define a Coxeter monoid $F(M)$ over $\Delta$ that satisfies the following relations

$X_i X_i = X_i$, $i \in I$

$r(i,j) = r(j,i) = l(i,j) = l(j,i)$ for all $i \neq j$

<u>Definition.</u> A word $X \in F(M)$ is called attractor if $X = X X_i = X_i X$ for any $i \in I$.

There is a natural homomorphism $\pi : F(M) \to F(\Delta)$. If $X$ is attractor for $F(M)$ then $\pi(X)$ is attractor for $F(\Delta)$ of course. The presence of attractor for $F(\Delta)$ is equivalent to finiteness of $G$ (and $F(\Delta)$). A monoid is called indecomposable if it cannot be presented as a direct product of two proper submonoids.

<u>Theorem.</u> 1) Indecomposable Coxeter monoid is finite iff $M$ is a spherical Coxeter matrix; 2) $F(M)$ has an attractor iff either $M$ is spherical or $A_n \leq M \leq A'_n$ where $A'_n$ corresponds to diagram $\circ \overset{3,4}{\text{———}} \circ \dots \circ \overset{3,4}{\text{———}} \circ$

# Some Remarks on the Cohen-Macaulay Properties for Sporadic Geometries

SATOSHI YOSHIARA Univ. of Illinois at Chicago

1. Result (joint work with Alex Ryba & Steve Smith)
Among the known sporadic geometries $\Delta$ in characteristic $p$, admitting a flag-transitive action of a group $G$ with $|G|_p \geq p^2$, it is determined the list of those $\Delta$ with projective reduced Lefschetz module $\tilde{L}(\Delta, k)$ for a field $k$ of characteristic $p$.

2. Observation Suppose $\Delta$ is one of finite GABs of dim $= 2$ associated with $\Omega_6^{\varepsilon}(p)$, $\Omega_5(p)$, (and $G_2(p)$) for an odd prime $p$, constructed by W. Kantor from Affine buildings.
Then $H_1(\Delta, \mathbb{Z})$ is a non-trivial finite $p$-group.
This implies that $\Delta$ does not have Cohen-Macaulay property over $\mathbb{Z}$, but have this property over a field $k$ except for char $k = p$.

3. Application (of 2.) Let $\Delta$ be a GAB for sporadic Suzuki group and $\Sigma$ its subgeometry on which $\Omega^-(6,3)$ acts. Then the inverse image $p^{-1}(\Sigma)$ of $\Sigma$ inside the universal 2-cover $(\tilde{\Delta}, p)$ of $\Delta$ is not connected. (It seems to me that this fact suggests the difficulty of ~~the~~ an explicit construction of $\tilde{\Delta}$)

# Subgroup structure of groups of type $E_6$.

Let $G$ be a universal group of type $E_6$ over a finite or algebraically closed field $F$. ~~If~~ Let $V$ be the 27-dimensional module for $G$ over $F$; $G$ may be regarded as the isometry group of a symmetric trilinear form $f$ on $V$. Let $\Gamma$ be the group of semilinear maps on $V$ preserving $f$. We define a class $\mathfrak{G}$ of geometric structures on $V$ and sets $S$, UNK of quasisimple subgroups of $GL(V)$ and prove:

Theorem: Let $M$ be a closed subgroup of $G$ or a subgroup of $\Gamma$ when $F$ is finite. Then either

1) $M$ stabilizes a member of $\mathfrak{G}$, or
2) $F^*(M) = L Z(M)$ with $L$ quasisimple and $C_G(L) = Z(G)$. Further one of the following holds:
   a) $L \in S$
   b) $M$ is finite, $char(F) \neq 0$, an irreducible $FL$-submodule of $V$ can be written over a proper finite subfield $F_0$ of $F$, and $N_\Gamma(M) \leq N_\Gamma(S)$ for some $S \leq G$ with $S = E_6(F_0)$
   c) $L$ is finite and in UNK

This provides a discription of maximal subgroups of groups between $G$ and $\Gamma$ when $F$ is finite and maximal closed subgroups of $G$ when $F$ is alg. closed, modulo the list UNK, where the existence and uniqueness (up to conjugation) of members of UNK is left open. UNK consists of about 15 conjugacy classes of (small) finite subgroups of $GL(V)$.

Michael Aschbacher, Pasadena, May 1988

# Combinatorial & Geometric Techniques in Modular Representation Theory

Stephen D. Smith
U. Illinois-Chicago

Part of the game is to extend, by some analogy, properties "P" of Chevalley groups & buildings to sporadic groups and their geometries.

(P) Using results of Curtis (or others) one can establish for Chevalley groups: For B a Borel subgroup, the induced module $1_B\uparrow^G$ decomposes (directly) over the orbit poset $\Delta/G$:

$$1_B\uparrow^G = \bigoplus_{\substack{J \subseteq \pi \\ \text{simple system}}} H_J$$

where $H_J$ = top homology of truncated complex $\Delta_J$ from building $\Delta$.

The goal is to prove this directly via geometry – then deduce representation theory. (Motivation: Then extend by analogy to sporadic geometries).

Sketch ① determine U-invariant cycles in $H_J$

② For each such cycle c, find $c' \in H_J$ with $c' \not\perp c$ in natural form on permutation module $1_{P_J}\uparrow^G$.

③ This shows $\downarrow H_J$ is non-degenerate. As $H_J = \left(\sum_{K \supset J} 1_{P_K}\uparrow^G\right)^{\perp}$, there is a direct decomposition $1_{P_J}\uparrow^G = H_J \oplus (\uparrow)$.

The ideas can be applied (by brute force if necessary) to sporadic geometries (The example $G = A_7$ $\Delta = \circ\!-\!\circ\!=\!\circ$ was discussed).

Stephen D. Smith
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4/5/89

# The Fifty Six Dimensional Module For Groups of Type $E_7$.

Let $\Phi = \{1, 2, \dots, 8\}$, $V$ a vectorspace of dimension 8 over a field $K$ of characteristic not two with basis $v_1, \dots, v_8$. Let $S = SL(V)$, $V^*$ a dual space of $V$ with dual basis $v_1^*, \dots, v_8^*$. Set $X = \Lambda^2(V)$, $X^* = \Lambda^2(V^*)$ and let $x_{ij} = v_i \wedge v_j$, $x_{ij}^* = v_i^* \wedge v_j^*$ be bases. $X$ and $X^*$ are dual via the pairing $X * X^* \to K$, given by $(u_1 \wedge u_2, w_1^* \wedge w_2^*) = \det(w_i^*(u_j))$. Define the alternating form $\langle\, ,\, \rangle : M \times M \to K$, where $M = X + X^*$ by $\langle a + a^*, b + b^* \rangle = (a, b^*) - (b, a^*)$ where $a, b \in X$, $a^*, b^* \in X^*$. This is $S$-invariant. Also, set $Q : M \to K$ be the quadratic form $Q(a + a^*) = (a, a^*)$. $Q^2$ is a 4-homogeneous $S$-invariant form in $K[M]$, the polynomial algebra on $M$.

The map $f : X^4 \to \Lambda^8(V) \cong K$ by $f(x_1, x_2, x_3, x_4) = x_1 \wedge x_2 \wedge x_3 \wedge x_4$ is a 4-linear map. Identify $\Lambda^8(V)$ with $K$ so that $f(x_{12}, x_{34}, x_{56}, x_{78}) = 1$. The map $h : X \to K$ by $h(x) = f(x, x, x, x)$ is 4-homogeneous. In terms of coordinates $h$ acts as follows, let $x = \sum X_{ij} x_{ij}$ (here, by convention, $X_{ij} = -X_{ji}$). Then let REP be a set of coset representatives for the centralizer of $(12)(34)(56)(78)$ in $\mathrm{Sym}(8)$.

$$h(x) = 4! \sum_{\sigma \in REP} X_{\sigma 1, \sigma 2} X_{\sigma 3, \sigma 4} X_{\sigma 5, \sigma 6} X_{\sigma 7, \sigma 8}.$$

Define $h_0(x) = \sum_{\sigma \in REP} X_{\sigma 1, \sigma 2} X_{\sigma 3, \sigma 4} X_{\sigma 5, \sigma 6} X_{\sigma 7, \sigma 8}$. This is an $S$-invariant 4-homogeneous form on $X$. Extend to $g_0$ on $M$ by $g_0(x + x^*) = g_0(x)$. In a similar way define $g_0^*$.

One more 4-homogeneous form on $M$ is defined: $X \wedge X \cong \Lambda^4(V)$ and $X^* \wedge X^* \cong \Lambda^4(V^*)$ are dual by a pairing similar to the one above; $\gamma(u_1 \wedge u_2 \wedge u_3 \wedge u_4, w_1^* \wedge w_2^* \wedge w_3^* \wedge w_4^*) = \det(w_i^*(u_j))$. Now set $c_0(x + x^*) = \gamma(x \wedge x, x^* \wedge x^*)$ and $c = \tfrac{1}{4} c_0$. This yields a 4-space, $\langle c, g_0, g_0^*, Q^2 \rangle_K$ of $S$-invariant 4-homogeneous forms.

Now, set $p_{ij} = \langle x_{ij} \rangle$, $p_{ij}^* = \langle x_{ij}^* \rangle$ and $\Omega$ the set of these 56 one-spaces. Define a graph on $\Omega$ by $p_{ij} \sim p_{kl}$ ($p_{ij}^* \sim p_{kl}^*$) iff $|\{i,j\} \cap \{k,l\}| = 1$ and $p_{ij} \sim p_{kl}^*$ iff $\{i,j\} \cap \{k,l\} = \emptyset$. The automorphism group of this graph clearly contains $\mathrm{Sym}(8)$. In fact,

<u>Thm</u> $\mathrm{Aut}(\Omega, \sim)$ is isomorphic to $\mathrm{Weyl}(E_7) \cong \mathbb{Z}_2 \times Sp(6,2)$.

Let $\sigma$ be a permutation in $\mathrm{Aut}(\Omega, \sim)$ not normalizing a $\mathrm{Sym}(8)$.

Next, let $W(S) = S_\Omega$, the set-wise stabilizer of $\Omega$ in $S$, $H(S) = \bigcap_{p \in \Omega} S_p$. Then $H(S)$ is the diagonal subgroup for the base $v_1, \dots, v_8$, $H(S) \trianglelefteq W(S)$ and $\overline{W(S)} = W(S)/H(S) \cong \mathrm{Sym}(8)$. We construct a transformation $g_\sigma \in Sp(\langle\, ,\, \rangle, M)$ so $\sigma$

preserves $\Omega$ and induces $\sigma$ on $\Omega$. It is then shown

Thm: $E = \langle S, g_0 \rangle$ leaves $C + g_0 + g_0^* - 1/4\, Q^2$ invariant

Thm: $E$ is a universal group of type $E_7(K)$.

Thm: $E$ has three orbits on one-spaces $p$ of $M$ so $\mathcal{f}(p) = 0$, where
$\mathcal{f} = C + g_0 + g_0^* - 1/4\, Q^2$.

Thm: If $\mathcal{f}(x) \neq 0 \neq \mathcal{f}(y)$, then $\langle x \rangle$ and $\langle y \rangle$ are in the same $E$-orbit iff $\mathcal{f}(x)/\mathcal{f}(y)$ is a $4^{th}$-power in $K^*$.

Cor: If $K$ is alg. closed, $E$ has four orbits on one-spaces of $M$.

Cor: If $K = GF(p^n)$, $p > 2$, then the number of orbits of $E$ on one-spaces of $M$ is 5 or 7 as $4 \nmid p-1$ or $4 \mid p-1$, respectively.

Finally,

Thm: Suppose $f(x) \neq 0$. If $-f(x)$ is a square in $K$, then $O^\infty(E_{\langle x \rangle}) \cong E_6(K)$. If $-f(x)$ is not a square, then $O^\infty(E_{\langle x \rangle}) \cong {}^2E_6(K)$. In each case $[E_{\langle x \rangle} : O^\infty(E_{\langle x \rangle})]$ is 2 or 4 depending on whether $-1$ is not, or is a square in $K$, respectively.

Bruce N. Cooperstein
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4/5/88

# SIMPLE SUBGROUPS OF SIMPLE GROUPS

Let $G$ be a finite simple group and $M$ a maximal subgroup of $G$. Then one of

1) $M$ is LOCAL ; $O_p(M) \neq 1$ for some prime $p$
2) $M$ is SEMI-SIMPLE ; $\mathrm{Aut}(M) = S_1 \times \cdots \times S_t$ $S_i$ simple non-abelian, $t \geq 2$
3) $M$ is ALMOST SIMPLE ; $\mathrm{Aut}(M)$ non-abelian simple.

Theorem 1 All local maximal subgroups of the finite simple groups are known, apart from 2-locals of Baby Monster & Monster.

Theorem 2. All semi-simple maximal subgroups of the finite simple groups are known, apart from $E_7(q)$, $E_8(q)$.

There is no corresponding Theorem 3. This is the hard problem - classifying simple subgroups of simple groups. This leads to the question: when is a simple group $X$ contained in a simple group $Y$? We focus on the question: when is a sporadic group contained in an exceptional group of Lie type? Interesting examples arise, such as $J_1 < G_2(11)$ (Janko, Coppell) $J_2 < G_2(4)$ (Wales, Suzuki) $Fi_{22} < {}^2E_6(2)$ (Fischer) $Th < E_8(3)$ (Smith, Thompson) $J_3 < E_6(4)$ (Kleidman, Aschbacher) $M_{12} < E_6(5)$ (Kleidman, Wilson). We study all other possible inclusions. This project, which is joint work with Rob Wilson, is now almost complete. The only questions left open are $M_{22} < E_7(5)$?, $HS < E_7(5)$? $Ru < E_7(25)$?

Peter Kleidman

CALTECH, PASADENA 4/5/88

## Parabolic systems of rank 3.

The main ideas of the classification of (quasi) parabolic systems of rank 3 and equivalently, locally finite classical Tit's chambersystems with discrete chamber transitive automorphism groups of rank 3 were described. The results will appear in the J. of Alg. They can not be written down here, since the theorem has to many cases. F. G. Timmesfeld, Giessen.

## BUILDINGS AT INFINITY

A building at infinity $\Delta_\infty$ of type $X_n$ is a building at infinity of some affine buildings of type $\tilde{X}_n$ (definition by Tits). E.G. Affine buildings of type $\tilde{A}_2$ (diagram) have a projective plane at infinity; affine buildings of type $\tilde{C}_2$ (diagram) have a generalised quadrangle at infinity.

THEOREM 1. The class of projective planes at infinity coincides with the class of projective planes coordinatized by some planar ternary ring with valuation (e.g. local fields)

THEOREM 2. The class of ~~projective~~ generalised quadrangles at infinity coincides with the class of generalised quadrangles coordinatised by some quadratic quaternary ring with valuation see talk of G. Hanssens, this meeting.

E.G. Several examples were constructed. We give one.

Let $K = GF(2^n)$, $\theta_i = \alpha^{2^{h_i}}$ with $(2^n - 1, 2^{1+h_1+h_2}) = 1$.

Define in $K((t))$:

$$\left(\sum a_n t^n\right)^{\theta_i} = \sum x_n^{\theta_i} t^n$$

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then $Q_1(k,a,l,a') = (k^2)^{\sigma_1} . a + a'$
$Q_2(a,k,b,k') = a^{\sigma_2} . k + k'$

defines a $T_2(0)$ (infinite). Define $k \odot_1 a = Q_1(k,a,0,0)$
$a \odot_2 k = Q_2(a,k,0,0)$

then

ax axioms of OOR with valuation for elations and dual translation g.q.

$$\begin{cases} v(k \odot_1 a) = 2v(k) + v(a) \\ v(a \odot_2 k) = v(a) + v(k) \\ \text{If } k \odot_1 a = l \odot_1 b \text{, then } v(a \odot_2 k - b \odot_2 l) = v(k-l) + v(a) + v(b) \end{cases}$$

$v$: natural valuation on $K((t))$

Residues in the corresponding building (non-classical) are $T_2(0)$'s (same definition as above restricted to $K$).

A valuation on generalised ~~quadrangle~~ polygons can be defined in such a way that:

CONJECTURE. The class of generalised polygons at infinity coincides with the class of generalised polygons with valuation

Proved for generalised $n$-gons, $n \geq 3$ and $n \neq 6$.

H. Van Maldeghem, Gent (Belgium)

Primitive groups of genus zero

If $\phi$ is a nonconstant meromorphic function on a compact connected Riemann surface $X$ of genus $g$, the monodromy group of the cover $X \xrightarrow{\phi} \mathbb{P}^1$ is called a group of genus $g$. This can be translated into a purely group-theoretic definition: a subgroup $G$ of the symmetric group $S_n$ is a group of genus $g$ if

1) $G$ is transitive (of degree $n$),

2) $G = \langle x_1, \dots, x_r \rangle$ with $x_i \neq 1$ for all $i$, and $x_1 \dots x_r = 1$, and

3) if we define $\mathrm{ind}(x_i) = n - \#\mathrm{orb}\langle x_i \rangle$ (where $\#\mathrm{orb}\langle x_i\rangle$ is the number of orbits of $\langle x_i \rangle$), then

$$\sum_1^r \mathrm{ind}(x_i) = 2(n+g-1).$$

J.G. Thompson, R. Guralnick, M. Aschbacher and others have been developing methods for studying the primitive groups of genus zero. (The assumption of primitivity here is natural, as there is a standard way in which each group of genus zero is built out of primitive groups.)

As an example of a primitive group of genus zero, take $G = S_n$ acting naturally on $\{1, \dots, n\}$, with $x_1 = (1\ 2)$, $x_2 = (1\ 3\ 4 \dots n)$, $x_3 = (1\ n\ n{-}1 \dots 2)$.

Now let $G$ be any primitive group of genus zero. Then either

(a) $G$ is affine, i.e. the socle of $G$ is $\mathbb{Z}_p^k$, $p$ prime, or

(b) the socle of $G$ is $L^k$ for some non-abelian simple group $L$.

Guralnick and Thompson have proved that in case (a), either $n \leq 2^{16}$ or $n = p$ or $p^2$ and $G'' = 1$. And in case (b) they have shown that there is a group $X$ such that $L \trianglelefteq X \leq \mathrm{Aut}\, L$, a subgroup $M$ of $X$ with $L \not\leq M$, and an element $1 \neq x \in X$ such that

$$\frac{|x^X \cap M|}{|x^X|} > \frac{1}{85} \qquad (*)$$

(i.e. $M$ contains at least $\frac{1}{85}$th of the $X$-conjugates of $x$). Writing $\mathcal{E}$ for the set of simple groups $L$ satisfying $(*)$ for some $x$ and $M$, Guralnick and Thompson make the

Conjecture There is a number $N$ such that for $q > N$, no group $G(q)$ of Lie type over $GF(q)$ lies in the set $\mathcal{E}$.

In the talk I outlined a proof, obtained jointly with J. Saxl, of
Theorem The conjecture is true for groups $G(q)$ of type $E_7$ and $E_8$.
It seems likely that the methods of the proof discussed will handle all the exceptional groups of Lie type.

M.W. Liebeck,
Imperial College, London.

On the classification of arithmetic hyperbolic reflection groups

We consider groups $W$ of isometries of $n$-dimensional hyperbolic space $H^n$ generated by reflections and such that $H^n/W$ is of finite volume. In particular (following Vinberg, Nikulin, Rummke), we are interested in arithmetic noncompact groups of that kind. That is, $W$ is commensurable to a group $O(f,\mathbb{Z})$ where $f$ is an integral quadratic form of signature $(n,1)$, and isotropic over $\mathbb{Z}$.
This means more or less that we are looking for those quadratic forms s.th. the subgroup $W(f) \trianglelefteq O(f)$ generated by all reflections preserving $f$ is of finite index. We are particularly interested in the case $n=3$.
From general results of Nikulin it follows that the list of such $f$ is finite. On the other hand, the list of candidates that come from Nikulin's proof is much to large to deal with. This problem is not only a computational one, because there is no algorithm known to us which decides for a given $f$ whether $O(f):W(f)$ is finite or infinite.
By combining an idea of Vinberg (used in the proof of the fact that $O(f):W(f)$ is always infinite

if $n \geq 30$) with methods by J. Mennicke involving the genus of a certain plane stabilizer, we hope to produce sufficiently sharp criteria that allow to prove infiniteness in each concrete case where $O(f):W(f)$ is not "obviously" finite. As an example, we have proved that for $f_p = x_0x_1 + x_2^2 + px_3^2$, $p$ prime, $p \equiv 1\ (4)$ one has $[O(f_p) : W(f_p)] < \infty$ if and only if $p = 5, 13, 17$. We have produced a list of about 50 forms s.th. the $O(f)$ are maximal, pairwise non-conjugate and $[O(f) : W(f)] < \infty$. We hope that this list will turn out to be (almost) complete. The proof of this fact will be joint work with F. Grunewald.

Rudolf Scharlau,
Univ. Bielefeld

New feasible conditions for the existence of a distance ~~[illegible]~~ regular graph.

Let $r, s, t$ be integers such that the intersection numbers of a distance regular graph satisfy the following conditions:

$$(1, a_1, b_1) = (c_{r-1}, a_{r-1}, b_{r-1}) \neq (c_r, a_r, b_r)$$

$$(c_{s-1}, a_{s-1}, b_{s-1}) \neq (c_s, a_s, b_s) = (c_{s+t}, a_{s+t}, b_{s+t}) \neq (c_{s+t+1}, a_{s+t+1}, b_{s+t+1})$$

We have the following two theorems:

Theorem 1 (A. A. Ivanov) $t < s$.

Theorem 2 (A. V. Ivanov) If $t \geq r$, then the following four conditions are satisfied:

(i) $c_r = 1$

(ii) $a_s \geq 1$

(iii) $2 \le b_s \le (b_1 - c_{s-1})/2$
(iv) $2 \le c_s \le (b_1 - b_{s+t+1})/2$

<u>Conjecture.</u> $t < r \le s$
If this conjecture is true then we have a new upper bound for the diameter $d$ of a distance regular graph of valency $k$: $d \le C_1 \cdot g \cdot k$, where

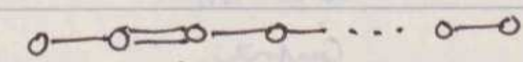
$$g = \begin{cases} 2r, & \text{if } c_2 \ge 2 \\ 2r+1 & \text{if } c_2 = 1, \end{cases}$$

and $C_1$ is a constant. The current available upper bound follows from Theorem 1: $d \le C_2 \cdot g \cdot 2^{2k}$.

A. V. Ivanov
Institute for System Studies
Moscow

Further Characterizations of Lie Incidence Systems

Suppose $\Gamma = (\mathcal{P}, \mathcal{L})$ is a weak parapolar space with the local pentagon property, such that
(1) for each non-incident point-symplecton pair $(x, S)$
$x^\perp \cap S$ is empty or contains a line.
Then $\Gamma$ is either a polar space, a metasymplectic space, a Grassman space of type $A_{n,2}$ or a polar Grassman space of type $C_{n,2}$.
Other characterizations, where the hypothesis (1) is replaced by other properties of symplecta, lead to characterizations of all residually connected geometries covered by a building with diagram



as well as homomorphic images of polar ~~space~~ Grassman spaces of type $C_{n,d}$, $d \le n-2$.

E. Shult
Kansas State U. - U. Freiberg).

## Presentations of 2-local subgroups of the Monster

Let us define $Y_{abc}$ $(a \geq b \geq c \geq 2)$ as the Coxeter group generated by a Y-shaped diagram whose arms (excluding the central node) have length a, b, c, subject to the relation that defines $3^5.O_5(3).2$ on the central $Y_{222}$. If $c=1$ other relations, that can be shown to be consequences inside larger Y-groups, are used.

Then $Y_{nn1}$ is an orthogonal group over GF(2); $Y_{n22}$ over GF(3); and $Y_{632}$ is trivial. This leaves only the cases when $a \leq 5$, $b \geq 3$, $c \geq 2$; and in these it can be shown that the values 4 and 5 for any of a, b, c lead to the same group. All cases lead to known presentations except for $Y_{433}$ $(= 2B \times 2\,?)$, $Y_{443}$ $(= 2 \times M\,?)$ and $Y_{444}$ $(= M \wr 2\,?)$. It is known that the second of these implies the last (Soicher).

One can adjoin extra generators to form a projective plane of order 3, and Soicher has shown that a single hexagon relation (extended $A_5 = 3^4S_6$) presents a group isomorphic to $Y_{555}$.

Define, for any node a, a* to be the centre of any $D_4$ for which a is an extending node. Because an extended $D_8$ generates $2^6S_8$ (follows from $Y_{432}$), this is well defined, and by further use of this extended $D_8$ we can prove that the 26 a's and a*'s corresponding to points of the projective plane generate an extraspecial group $2^{1+26}$.

Inside the $A_3$-diagram corresponding to 2 points and the line joining them, we can find an element normalizing the $2^{1+26}$. This is easy to prove. One can also determine relations among these elements. Then various subgraphs of the projective plane lead to presentations which can be shown to correspond correctly to subgroups of the $2^{1+26}.2^{24}Co_1$ normalizing the $2^{1+26}$ in M wr 2. The exceptions are $2^{1+25}.2Co_1$ (should be $2^{1+25}Co_1$) and $2^{1+26}.G$ (should be $2^{1+26}.2^{24}Co_1$) where G has the full automorphism group of the Leech Lattice as a quotient. These exceptions provide evidence that the above conjectures for $Y_{443}$ and $Y_{444}$ may be false.

One may use this method to provide elegant 26×26 matrices for the Conway group. In fact we have obtained generators for $2^{24}.Co_1$, $\mathbb{Z}^{24}.2Co_1$ and $5^{24}.2Co_1$ by this method.

Simon Norton

Cambridge

## Asymptotic properties of some GABs

Let $f = \sum_1^6 x_i^2$, $p>2$, and let $\Delta$ be the affine building for $\Omega(f, \mathbb{Q}_p)$, with diagram  if $p \equiv 3\ (4)$,  if $p \equiv 1\ (4)$. Then $G = O(f, \mathbb{Z}[\frac{1}{p}])$ is transitive on the set of vertices of type 0 or 1. For each positive integer $m>1$, $m \not\equiv 0\ (p)$, consider $G(m) = \{g \in G \mid g \equiv 1\ (m)\} \triangleleft G$, $G/G(m) = O(f, \mathbb{Z}/(m))$. This acts on the simplicial complex $\Delta/G(m)$, and is transitive on the set of vertices of type 0 or 1.

The "diameter" of $\Delta/G(m)$ can be considered in terms of either the graph of vertices of type 0 or 1, the 1-skeleton of $\Delta/G(m)$, or the chamber graph. For each of these, the diameter is at most $C \log_2 |G/G(m)|$ for some constant $C$ (proved using Kazhdan's Property (T) for $G$; an explicit estimate for $C$ is unknown).

The "geometric girth" of $\Delta/G(m)$ is the length of a shortest circuit not homotopic to 0, where the circuit can be in the simplicial complex or the chamber graph. For either definition, the geometric girth is $\geq C' \log_p m$ for a known constant $C'$; $C' = 1$ works in the case of the simplicial complex. (Here $\log_p m \simeq \frac{1}{12} \log_p |G/G(m)|$.)

Similar results hold for other GABs arising from classical affine buildings (class number 1 is required).

William M. Kantor
University of Oregon

*Fuchsian groups and Galois theory*

If $G$ is a uniformizing group for a 4-punctured sphere, there is a subgroup $H$ of $PSL(2,\mathbb{R})$ such that $G \triangleleft H$, $H/G$ being non cyclic of order 4; and $H$ is generated by involutions. There is a generator $f$ of $\mathbb{C}(G\widehat{\setminus}\mathfrak{h})$ such that $f^2+f^{-2}=f_0$ is a generator for $\mathbb{C}(H\widehat{\setminus}\mathfrak{h})$. If $\begin{pmatrix}1&1\\0&1\end{pmatrix}$ is a generator for the stabilizer of $\infty$ in $H$, $q=e^{2\pi i z}$, and $f(z)=\alpha_0+\alpha_1 q+\alpha_2 q^2+\dots$, then the following conjecture was made:

If $\alpha_0 \in \overline{\mathbb{Q}}$, then $\alpha_1^{-2}\alpha_2 \in \overline{\mathbb{Q}}$.

J. G. Thompson
University of Cambridge

DESIGNS OF STRENGTH $t$.

1. A measure $\xi$ in $\mathbb{R}^d$ is said to have strength $t$ if $\int f\,d\xi = \int f\,d\xi\circ\varphi$, for all polynomials $f$ of degree $f \le t$, and all $\varphi \in O(d)$, the orthogonal group.
2. For finite support $X$ on the unit sphere $S$, weights $w_x=1$, this amounts to spherical $t$-designs: $\operatorname{ave}_X f = \operatorname{ave}_S f$, equivalently $h(X) := \sum_{x\in X} h(x) = 0$, for $h$ harmonic homogeneous, $\partial h = 1,2,\dots,t$. Ex. $(d,n,t)=(3,12,5)$.
3. For a lattice $Y = \bigcup_{r\in R} Y_r$ (unimodular, integral, even) the condition reads $\sum_{r\in R} w(r)\,h(Y_r)=0$, where $R := \{r=(y,y) : y\in Y\}$, with smallest $0 \ne r_0 \in R \subset 2\mathbb{Z}$. By use of theta series it follows that each $Y_r$ is a spherical design of strength $\frac{1}{2}(12 r_0 - \dim) - 1$, cf. Hecke, Schoenberg, B.B. Venkov (1984). Examples: $(d,n,r_0,t)=(8,240,2,7)$: $E_8$, and $(24, 2\binom{28}{5}, 4, 11)$: Leech.
4. For finite support $Y$ on $p$ spheres, we prove the Fisher inequality $|Y| \ge \sum_{i=0}^{2p}\binom{d-1+e-i}{d-1}$.

Joint work with Neumaier (Indag. Math.) to appear
and Delsarte (Lin. Alg. Appl.)

Some remarks on the coordinatisation of generalised polygons.

Coordinatization has been carried out for projective planes, and has proved to be a valuable tool in understanding and creating such objects. We present a coordinatization theory for generalized quadrangles, that extends to generalised hexagons and 8-gons.
We use two sets $R_1$ and $R_2$, with $|R_1| = \#$ pts/line $-1$, $|R_2| = \#$ lines/pt $-1$, and two quaternary operations.
For example, for the symplectic quadrangle $W(q)$ we have

$$(a, l, a')\ I\ [k, b, k'] \Leftrightarrow \begin{aligned} a' &= ka + b \\ k' &= a^2k - 2aa' + l \end{aligned}$$

where $a, b, a' \in R_1 = GF(q)$, $k, l, k' \in R_2 = GF(q)$
$(a, l, a')$ the coordinate of a point
$[k, b, k']$ the coordinate of a line.
It appears that the more elations a GQ has, the nicer its coordinatizing structure becomes. This method might be useful to give a more elementary proof of Tits' classification of Moufang polygons. A first step in that direction is made.

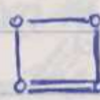
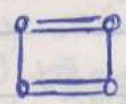
G. Hanssens
Gent (Belgium)

1-Cohomology and Ronan-Smith presheaf homology

Let $G$ be a Chevalley group over $k = \mathbb{F}_q$ and $V$ an irreducible $kG$-module. We study 1-cohomology $H^1(G, V^*)$ by looking at non-splitting $kG$-module extensions $0 \to k \to E \to V \to 0$. If either $q$ is not a prime or $V$ is fundamental $\&$, then $E$ is generated by a 1-space fixed by a Borel-subgroup (with some exceptions for $q = 2, 3$). Furthermore, if $V$ is fundamental (excluding the above exceptions), then E is a quotient of the geometrically defined module $\tilde V$ (= direct limit of the system of inclusion maps $V_B \hookrightarrow V_P$ for B (resp. P) a Borel subgroup (resp. minimal parabolic) of G, where $V_P$ is the centraliser in V of the unipotent radical of P). The module $\tilde V$ occurred first in the work of Ronan and Smith on universal presheaves. The computation of $\tilde V$ for V adjoint (not of type $C_\ell$) yields the 1-cohomology of the adjoint module.

H. Völklein (Gainesville)

Two sporadic geometries related to the Hoffman-Singelton graph.

Let $\Gamma^{(i)}$ $(i=1,2)$ be a residually connected Tits-geometry belonging resp. to the diagram $\Delta^{(1)}$ =  or $\Delta^{(2)}$ = , such that every rank 3 residue belonging to a subdiagram of type $C_3$ is the sporadic $A_7$-geometry.
An easy construction of such geometries exists. This construction makes use of the Hoffman-Singelton graph $\Lambda$ and the cocliques of size 15 in $\Lambda$. On the other hand it can be shown, that the incidences of these geometries are consequences of the structure of the rank 3 residues, which proves:

Theorem: Up to isomorphism, there exists an unique geometry $\Gamma^{(i)}$ $(i=1,2)$ which satisfies our assumptions.

Stefan Heiss (Berlin)

# A sufficient condition for an element to belong to a Sylow $p$-subgroup (a new 7-local subgroup of the monster)

Let $G$ be a finite group. An element $g$ in $G$ is called a right-engel element if there exists an integer $n$ such that $[x, \underbrace{g, \cdots, g}_{n}] = 1$ for all $x \in G$. A famous theorem of Baer states that $g$ is right-engel iff $g \in F(G)$, the Fitting subgroup of $G$. We study a generalization of this. We say that $g$ is right-engel with respect to an element $y$ in $G$ if $g$ is a right-engel element in $\langle g, y \rangle$. We look for condition on $g$ and $H \leq G$ which implies $g \in F(H)$. We notice that the condition: "$g$ is right-engel with respect to each element in $H$" is not a sufficient condition.

<u>Conjecture</u> I. Let $\pi$ = set of primes, $H$: a $\mathrm{Hall}_\pi$-subgroup and $g$: a $\pi$-element of $G$. If $g$ is right-engel with respect to each element in $H$, then $g \in F(H)$.

For a prime $p$, Baer's thm has the following version: $g \in O_p(G) \Leftrightarrow \langle g, y \rangle$ is a $p$-group $\forall\, y \in G$. This leads to the following versions of Conjecture I.

<u>Conjecture</u> II. Let $P \in Syl_p(G)$, $g$ a $p$-element. Then $g$ is right-engel with respect to each element in $P \Leftrightarrow g \in P$.

<u>Conjecture</u> III. Let $P \in Syl_p(G)$, $g$ a $p$-element. Then $\langle g, y \rangle$ is a $p$-group $\forall\, y \in P$ $\Leftrightarrow g \in P$.

<u>Prop</u>. Conjectures I, II, III are equivalent.

<u>Thm</u>. (with Völklein) Conjecture III holds for $p \geq 5$ except possibly when $p = 7$ and $G$ involves the Monster.

In treating the Monster for $p = 7$. I came across a new 7-local subgroup not appear in the list of ~~the~~ Atlas, 3 years ago. This subgroup turns to be a maximal subgroup (7-local), and the list of maximal 7-local subgroups of the Monster is then complete. (Wilson also found this subgroup independently).

Chat Y. Ho
University of Florida.

On subfield subgroups

We discussed a method of investigating the action of a group $G$ of Lie type on the set of cosets of a subgroup $S$ of the same type (possibly twisted) over a smaller field. As an illustration, we considered the case where $G$ is $F_4(q)$ with $q$ even and $S$ is either $Sz(q)$ or $F_4(q^{\frac{1}{2}})$, and the case where $G = E_8(q)$ acting on the set of cosets of $E_8(q^{\frac{1}{2}})$.

Jan Saxl
Cambridge

Suppose $G$ is a finite group acting primitively and distance transitively on a graph $\Gamma$ and $L = \mathrm{soc}\, G$ is a simple group.

Theorem If $G$ has a $B,N$ pair, then $\Gamma$ is known

theorem (joint with van Bon). If $L \cong L_n(q)$ then $\Gamma$ is either complete, a Grassmann graph or known (member of a finite list)

Theorem (joint with Liebeck & Saxl). $L \not\cong E_8(q)$ for $q \geq 4$.

theorem (joint with van Bon & Cuypers). $L \not\cong He$.

Arjeh M. Cohen Amsterdam/Utrecht

On the flocks of $Q^+(3,q)$

A complete characterization of the flocks of the hyperbolic quadric $Q^+(3,q)$ is given. As an application it follows that if $q \neq 11, 23, 59$, and $q$ is odd, there exist no maximal exterior sets of $Q^+(2n-1,q)$.

Guglielmo Lunardon
Napoli

## Component uniqueness theorems.

A heuristic discussion of "uniqueness theorems" and their role in the proof of the classification of finite simple groups was given. Let $G$ be a finite group and $\mathcal{H}$ a family of subgroups of $G$ closed under conjugation. $\mathcal{H}$ is made a graph by joining two elements of $\mathcal{H}$ iff they commute elementwise. Uniqueness theorems are concerned with the connectedness properties of $\mathcal{H}_{p^n}$, the set of all subgroups of $G$ isomorphic to the direct product of $n$ copies of $Z_p$. For instance, the celebrated Bender–Suzuki "strongly embedded subgroup" theorem determines all groups in which $\mathcal{H}_2$ is disconnected, and the "uniqueness case" theorem of Aschbacher shows that there are no simple groups in which $\mathcal{H}_p$ is disconnected for many odd primes and certain other conditions are satisfied.

If $\mathcal{H}_p$ is disconnected for some prime $p$, the normalizer $M$ of a connected component of $\mathcal{H}_p$ is called "$p$-strongly embedded" in $G$, as are all its overgroups. $M$ then has the property that for all $g \in G - M$, $M \cap M^g$ is a $p'$-group (but $M \neq G$).

The following "component" uniqueness theorem is useful at several places in the classification, for instance to detect a direct product structure in the group $G$: Let $G$ be a finite simple group all of whose proper subgroups are $K$-groups — i.e., have composition factors of known type. Let $K$ be a $p$-component of a maximal subgroup $M$, and $Q$ a Sylow $p$-subgroup of $C_M(K/O_{p'}(K))$. Assume that $m_p(K) \geq 2$, $m_p(Q) \geq 2$, and (if $p>2$) $m_p(M) \geq 4$. Then either $K \trianglelefteq M$ or $M$ is strongly $p$-embedded in $G$. (A mild additional hypothesis is required if $K$ itself has a strongly $p$-embedded subgroup.)

Finally, in view of the fact that the successful assault on the finite simple groups was largely from the point of view of "semisimple" elements, whereas the main studies and understanding of the Chevalley groups are from the point of view of the building – i.e. equicharacteristic, the following question was asked: Is there a workable geometric approach to the Chevalley groups from the point of view of tori or other semisimple subgroups?

The above theorem is joint with D. Gorenstein and R. Solomon.

Richard Lyons
Rutgers University, May 1988

## Parabolic systems over GF(2)

Suppose $G$ is a group containing a minimal parabolic system $\{P_1, \dots, P_n\}$ satisfying $P_n^{+}$ and such that for each $i \in I = \{1, \dots, n\}$

$$P_i / O_2(P_i) \cong S_3 \ (\cong SL_2(2)).$$

This talk concerns those parabolic systems whose diagram $\Delta$ is of the form $\underset{n}{\circ}\!-\!\underset{n-1}{\circ}\ \cdots\ \underset{3}{\circ}\!-\!\underset{2}{\circ}\overset{\sim}{=}\underset{1}{\circ}$ ( $\underset{2}{\circ}\overset{\sim}{=}\underset{1}{\circ}$ meaning that $\langle P_1, P_2\rangle / O_2(\langle P_1, P_2\rangle) \cong \hat{S}_6$ ). Put $S = \bigcap_{i \in I} P_i$, $S_0 = \mathrm{core}_G\, S$ and $S_{123} = \mathrm{core}_{\langle P_1, P_2, P_3\rangle} S$. A proof of the following was outlined

<u>Theorem</u> If $\Delta = \underset{5}{\circ}\!-\!\underset{4}{\circ}\!-\!\underset{3}{\circ}\!-\!\underset{2}{\circ}\overset{\sim}{=}\underset{1}{\circ}$ and $|S/S_{123}| \neq 2^9$, then $|S/S_0| = 2^{46}$.

Peter Rowley UMIST, Manchester
May 1988.

# Failure-of-factorization modules for Lie-type groups in odd characteristic

A faithful $F_p[G]$-module $V$ is called a failure-of-factorization (FF-) module in characteristic $p$ for the group $G$, if there is an elementary abelian $p$-subgroup $1 \neq A \leq G$ satisfying $|A| \geq |V : C_V(A)|$.

The irreducible FF-modules for finite Lie-type groups in characteristic 2 in their natural characteristic were determined by Cooperstein, while the corresponding classification for rank-2-Lie-type groups in arbitrary (finite) characteristic follows from work of Delgado. Special cases in higher rank were treated by Thiel. I prove the following

Theorem. Let $G$ be a finite Lie-type group, rank $\geq 3$, $V$ irreducible FF-module for $G$ in the natural characteristic $p$, and let $p$ be odd. Then one of the following holds.

(1) $G$ is of type $A_n(q)$, $V$ a natural module, exterior square of this or dual to one of these

(2) $G$ is of type $B_n(q)$, $C_n(q)$, ${}^2A_n(q)$, ${}^2D_n(q)$ and $V$ is a "natural" module

(3) $G$ is of type $D_4(q)$ or $D_5(q)$ and $V$ is a spin module

(4) $G$ is of type $B_3(q)$ and $V$ is the spin module.

The proof makes heavy use of a classification theorem of quadratic modules for the corresponding Lie-type groups by Premet and Suprunenko (1981).

Thomas Meixner (Gießen)

## Distance-transitive graphs with projective subconstituents.

Let $\Gamma$ be a graph and $G$ be a distance transitive automorphism of $\Gamma$ such that for a vertex $x \in V(\Gamma)$ the permutation group induced by the stabilizer $G(x)$ on the neibourhood $\Gamma(x)$ of $x$ is a projective group:

$$PSL_n(q) \trianglelefteq G(x)^{\Gamma(x)} \leq P\Gamma L_n(q).$$

Here $PSL_n(q)$ is considered as the permutation group of degree $(q^n-1)/(q-1)$ (natural doubly transitive representations. All pairs $(\Gamma, G)$ with the abouve properties are classified. For $n \geq 3$ the list is the following:

(1) $\Gamma$ is the point-hyperplane incidence graph of the projective space $PG(n-1, q)$;

(2) $\Gamma$ is the double Grassman graph (the $q$-analog of the antipodal covering of the odd graph);

(3) $\Gamma$ is the graph of the dual polar space of type $D_n(q)$;

(4) $\Gamma$ is the hermitian forms graph over field $GF(2^2)$;

(5) $\Gamma$ is the complete graph or the double covering of the complete graph, $G$ contains $AGL(n,2)$ or $AGL(n,2) \times Z_2$.

(6) $\Gamma$ is a graph on 506 vertices related to $M_{23}$, or a graph on 330 vertices related to $M_{22}$ or a graph on 990 vertices related to $3.M_{22}$ (the nonsplit triple covering)

Alexander A. Ivanov
Institute for System Studies
Moscow, USSR
May 1988.

# Approximation und Interpolation mit Lösungen von partiellen Differentialgleichungen (8.–14.5.88)

## Approximation by solutions of elliptic equations:

I wish to report on joint work with A. Dufresnoy and W. H. Ow published in Complex Variables, 1986, vol 6, pp. 235–247. Given a function on a closed set, we wish to approximate it uniformly by solutions of a given elliptic partial differential equation.

P. M. Gauthier
Université de Montréal

## Radon-Transformation auf Polynomräumen

Für Polynome $P$ des Grades $\mu$ kann die Radontransformierte $(\mathcal{R}P)(s,t) = \omega_{r-1}(1-s^2)^{\frac{r-1}{2}}(\tilde{\mathcal{R}}P)(s,t)$, $-1 \le s \le 1$, $t \in S^{r-1}$, $r \ge 2$, „elementar" konstruiert werden, wenn man zunächst $P = \sum_{\nu=0}^{\mu} \overset{*}{P}_\nu$ in seine homogenen Bestandteile zerlegt, diese danach auf der Sphäre $S^{r-1}$ nach homogenen harmonischen Polynomen $\overset{*}{H}_{\nu\varkappa}$ vom Grade $\varkappa$ zerlegt, also von einer Darstellung

$$P(x) = \sum_{\nu=0}^{\mu} \sum_{\substack{\varkappa=0 \\ \varkappa\equiv 0(2)}}^{\nu} |x|^{\nu-\varkappa} \overset{*}{H}_{\nu\varkappa}(x), \quad x \in \mathbb{R}^r, \tag{1}$$

ausgeht.

Das Bild unter $\tilde{\mathcal{R}}$ hat dann die Darstellung

$$(\tilde{\mathcal{R}}P)(s,t) = \sum_{\nu=0}^{\mu} \sum_{\substack{\varkappa=0 \\ \varkappa\equiv 0(2)}}^{\nu} \overset{*}{H}_{\nu\varkappa}(t)\, G^r_{\nu\varkappa}(s), \tag{2}$$

wobei die $G^r_{\nu\varkappa}$ für $\varkappa = 0(1)\nu$, $\varkappa \equiv \nu(2)$, univariate Polynome vom genauen Grad $\nu$ sind, welche zu allen Polynomen $\varkappa$-ten Grades bezüglich der Gewichtsfunktion $(1-s^2)^{\frac{r-1}{2}}$ orthogonal sind und bezüglich $(\nu, \varkappa)$ eine Rekursionsgleichung erfüllen. Das Bild kann auch in der Form

$$Q(s,t) := (\tilde{\mathcal{R}}P)(s,t) = \sum_{\lambda=0}^{\mu} A_\lambda(t)\, \tilde{C}_\lambda^{\frac{r}{2}}(s), \quad A_\lambda \text{ homogen vom Grad } \lambda,$$

geschrieben werden, und die hier auftretenden rechten Seiten bilden den genauen Bildraum von $\tilde{\mathcal{R}}$ bezüglich der Polynome $\mu$-ten Grades

Die Rücktransformation von $Q(s,t)$ erfordert zunächst die Projektion von $A_\lambda$ auf die Räume der sphärisch-harmonischen Polynome vom Grade $\nu = \lambda, \lambda-2, \ldots$, was sehr aufwendig ist. Besser ist es, hier interpolatorisch vorzugehen, was nur den einmaligen Aufwand einer Inversion einer großen positiv-definiten Systemmatrix erfordert und schließlich die Bestandteile $\check{A}_{\nu\kappa}$ von $P$ liefert.

Der Übergang von (1) nach (2) wird auf dem Wege einer Mittelbildung über die Gruppe aller Rotationen mit Fixpunkt $t \in S^{r-1}$ vollzogen, die über das Haar-Integral definiert, aber auch explizit vollzogen werden kann.

M. Reimer, Universität Dortmund

## Approximation of Vector-Valued Functions

The situation to be considered is as follows. The mapping $f$ associates each element in a set $S$ with some element in a Banach space $X$. The set $S$ will be assumed to have some structure (measure-theoretic or topological) so that a Banach space of such mappings (denoted by $A(S,X)$), may be constructed. Under suitable conditions, a closed subspace $G$ of $X$ gives rise to a subspace $A(S,G)$ of $A(S,X)$. A natural question is "When does the proximinality of $A(S,G)$ in $A(S,X)$ follow from that of $G$ in $X$?" Such problems are related to "blending functions" where one half of the approximating subspace contains elements of the form

$$a_0(s) + a_1(s)t + \ldots + a_n(s)t^n$$

when $G$ is the subspace of polynomials of degree $n$. Then one approximates sections of bivariate functions $f_s(t)$ by polynomials. If $f, a_0, \ldots, a_n$ are continuous then the proximinality sought is that of $C(S, \Pi_n)$ in $C(S, C(T))$.

Will Light (Lancaster) UK

## Bernoulli Distributions and Approximation by Trigonometric Blending Functions

Jackson-Favard estimates for trigonometric approximation are related to the convolution formula $f = c_0(f) + B_r * f^{(r)}$ for periodic functions $f \in W_r^*$ (with $c_0(f)$ the mean value of $f$ and $B_r$ the r-th Bernoulli spline). We develop a similar theory for multivariate approximation using the notion of periodic distributions and the (d-dimensional) Bernoulli distribution (introduced by J. Stöckler).

A typical result along these lines is the following: If $\alpha \cdot \xi = 1$ (with fixed $0 \neq \xi \in \mathbb{Z}^d$) has an integer solution $\alpha = \alpha_\xi \in \mathbb{Z}^d$, and if $T_{n,\xi}$ denotes the periodic testfunctions of type

$$t(x) = \varphi_0(x) + \sum_{k=1}^{n} \left( \cos(k\alpha_\xi \cdot x)\, \varphi_k(x) + \sin(k\alpha_\xi \cdot x)\, \psi_k(x) \right)$$

with $\varphi_k, \psi_k$ independent of $\xi$, then the approximation constant

$$\inf\{ \|f-t\|_p ;\ t \in T_{n,\xi} \}$$

admits a Favard type estimate $\|D_\xi^r f\|_p \dfrac{K_r}{(n+1)^r}$ with $K_r$ the r-th Favard constant.

Applications of decreasing properties yield the estimates from the literature.

K. Jetter, Uni Duisburg

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Interpolation by Non-differentiable Radial Basis Functions.

Let $\mathcal{N} = \{x_1, x_2, \ldots, x_n\}$ be a prescribed set of points (called "nodes") in $\mathbb{R}^2$. We wish to interpolate data given at the nodes by a continuous function. The interpolating function is of the form $f(x) = \sum_{j=1}^{n} c_j \|x - x_j\|_1$, where the $l_1$-norm is being used. Explicitly, if $x = (s,t) \in \mathbb{R}^2$, then $\|x\|_1 = |s| + |t|$. Let $h_j(x) = \|x - x_j\|_1$ and let $\mathcal{RB}$ denote the subspace of $C(\mathbb{R}^2)$ spanned by $\{h_1, h_2, \ldots, h_n\}$. ("$\mathcal{RB}$" stands for "Radial Basis"). The functions in $\mathcal{RB}$ are continuous piecewise linear functions, whose domains of linearity are rectangles created by passing a horizontal and vertical line through each node. Our paper establishes the dimension of $\mathcal{RB}$ (usually $n$), the codimension of $\mathcal{RB}$ in the space of all piecewise linear continuous functions, and other characteristics. Conditions for solvability of the original interpolation problem are given. This is joint work with W. A. Light of Lancaster, England. E. W. Cheney

The rate of approximation by reciprocals of polynomials

Let $f \in C[-1,1]$ be nonnegative. Then we can find polynomials $p_n$ of degree not exceeding $n$ such that $\|f - 1/p_n\|_\infty \le C\,\omega_\varphi(f, \frac{1}{n})$ where $C$ is an absolute constant independent of $f$ and of $n$. The $\omega_\varphi(f,\cdot)$ is the Ditzian-Totik modulus of continuity with $\varphi(x) = \sqrt{1-x^2}$. Trying to estimate distances in the $L^p$-norm for $p < \infty$ we have a less satisfactory result. Thm: Let $f \in L^{p+1}[-1,1]$, $f \ge 0$. Then there exist polynomials $p_n$ such that $\|f - 1/p_n\|_p \le C\,\omega_\varphi(f, \frac{1}{n})_{p+1}$. This is a joint work with Ed Saff and A. Levin.

D. Leviatan Tel Aviv

## Proximinality of Tensor Product Subspaces

Let $S$, $T$, $D \subseteq S \times T$ be compact Hausdorff spaces. Let $G \subset C(S)$ and $H \subset C(T)$ be finite-dimensional subspaces of real-valued continuous functions. The question is discussed which of the spaces $W = G \otimes C(T) + C(S) \otimes H$ are proximinal in $C(D)$. It turns out that, in general, "bad functions" $f \in C(D)$ do not possess a best approximation in $W$.

M. v. Golitschek

## Splines for Solving Boundary Value Problems of Elasticity

A spline interpolation method is proposed for solving the classical displacement boundary value problems of elastostatics from discretely defined boundary displacement vectors or stress vectors. A stability theorem is developed, which is dependent on the spacing of the data on the boundary, and convergence is established for the case in which the data points become dense. A basic tool is a vectorial generalization of the addition theorem for spherical harmonics.

D. Freeden, RWTH Aachen

Mixed K-functionals: A new modulus of smoothness for Blending-type approximation

The K-functionals of J. Peetre play an important rôle in the derivation of quantitative estimates for the degree of approximation of certain approximants for univariate functions. One reason for this is the fact that they are equivalent to the standard moduli of smoothness.

In the case of "Blending-type" approximation of functions of two variables (e.g. approximation by Boolean sums of parametric extensions of univariate approximation operators or by pseudopolynomials) the so-called mixed moduli of smoothness have turned out to be appropriate devices for measuring smoothness.

In the talk at this conference we introduced "mixed K-functionals" as an analogue to the Peetre K-functionals in the context of Blending-type approximation. We stated an equivalence relation between mixed K-functionals and mixed moduli of smoothness. As applications it was shown how mixed K-functionals can be used in the method of smoothing known e.g. from the univariate case, and how they can be applied in the derivation of an optimal estimate for the degree of approximation by trigonometric pseudopolynomials.

C. Cottin, Duisburg

Approximation by harmonic functions in BMO and spectral synthesis for Hardy-Sobolev spaces.

We show that there is no obstruction to approximate planar harmonic functions in BMO. Precisely, we prove the following

Theorem. Let $X \subset \mathbb{C}$ be compact and let $f \in VMO(\mathbb{C})$ be harmonic on $\mathring{X}$. Then we can find a sequence $(f_n)_{n=1}^{\infty}$, each $f_n$ being in $VMO(\mathbb{C})$ and harmonic on a neighbourhood (depending on $n$) of $X$, such that $f_n \xrightarrow{n\to\infty} f$ in $BMO(\mathbb{C})$.

Our technique is twofold: we use Vitushkin's localization method and duality. As an application we get the following spectral synthesis result.

Theorem Let $E \subset \mathbb{R}^2$ be closed and assume $f \in I_2 H^1(\mathbb{R}^2) = \{ \log|z| * h : h \in H^1(\mathbb{R}^2) \}$ satisfies
$f = 0$ on $E$ and
$\nabla f = 0$ except for a set of zero length.
Then $\exists\ (\varphi_j)_{j=1}^{\infty}$, $\varphi_j \in C_0^{\infty}(E^c)$, such that
$\varphi_j \xrightarrow{j\to+\infty} f$ in $I_2 H^1(\mathbb{R}^2)$ (which means: $\Delta\varphi_j \longrightarrow \Delta f$ in $H^1(\mathbb{R}^2)$).

Joan Verdera
Universitat Autònoma de Barcelona,
08193 Bellaterra, Barcelona.

"Duality and shape-preserving interpolation"
Frank Deutsch, Penn State University

Let $X$ be a normed linear space and $\{K_i \mid i \in I\}$ a collection of convex sets and $K = \bigcap_i K_i$.

Theorem. If $\overline{\operatorname{con}}\, \bigcap_i (K_i - k) = \bigcap_i \overline{\operatorname{con}}\, (K_i - k)$ $\forall k \in K$, $x \in X$, and $k_0 \in K$, then the following are equivalent.

(1) $k_0$ is a best approximation to $x$ from $K$;

(2) $\exists\ x^* \in X^*$ such that $\|x^*\| = 1$, $x^*(x - k_0) = \|x - k_0\|$, and

$$x^* \in \overline{\sum_i (K_i - k_0)^{\circ}}^{\,w^*},$$

where $S^{\circ} := \{x^* \in X^* \mid x^*(x) \leq 0\ \forall x \in S\}$ denotes the dual cone of $S$.

[Here "con" means "conical hull of".]

Application: Consider $L_2 = L_2(T, \mu)$, $\{x_1, \dots, x_n\} \subset L_2$, $(d_1, \dots, d_n) \in \mathbb{R}^n$, and

$$K = \{ y \in L_2 \mid y \geq 0, \langle y, x_i \rangle = d_i \ (i = 1, 2, \dots, n) \} \neq \emptyset.$$

Then the best approximation to any $x \in L_2$ is given by

$$k_x := \left(x + \sum_1^n \alpha_i x_i\right)_+ \chi_\Omega$$

for some scalars $\alpha_i$ chosen so that the element $k_x$ satisfies

$$\langle k_x, x_j \rangle = d_j \quad (j = 1, 2, \dots, n)$$

and

$$\Omega := \{ t \in T \mid \exists\, k \in K \text{ with } k(t) > 0 \}.$$

This application contains characterization theorems (proved under more stringent conditions) established by several authors.

Approximation singulärer Lösungen partieller Differentialgleichungen in einfachen Fällen

Vorgelegt sei die Randwertaufgabe für eine Funktion $u(x) = u(x_1, \dots, x_n)$ in einem gegebenen Bereich $B$ des $\mathbb{R}^n$ mit stückweise glatten Rand $\partial B$: $Nu = r(x)$ in $B$, $Mu = s(x)$ auf $\partial B$, mit gegebenen (nichtlinearen) Operatoren $N, M$ und gegebenen Funktionen $r(x), s(x)$. Der Operator $T = (N, M)$ sei von „monotoner Art", d. h. $Tv \leq Tw$ habe $v \leq w$ in $B$ zur Folge (Ordnung punktweise auf $B \cup \partial B$ im Sinne der klassischen Ordnung reeller Zahlen, und für jede Komponente von $T$).
Dann kann man $u$ in Schranken $v \leq u \leq w$ einschliessen, sofern $Tv \leq \binom{r(x)}{s(x)} \leq Tw$ gilt. Dies wird an verschiedenen Typen von Singularitäten vorgeführt: An Ecken im $\mathbb{R}^2$ mit Eckenwinkel $\alpha$ (wobei $2\pi/\alpha$ ganzzahlig ist oder auch nicht ganzzahlig), an singulären Linien im $\mathbb{R}^3$ und an „versteckten" Singularitäten; numerische (meist im letzten Jahr gerechnete) Beispiele und noch offene Probleme werden genannt. Lothar Collatz, Hamburg

On the approximation of matrices connected with the discrete approximation of functions in two variables

An approximation of a matrix is very close to an approximation of a function in two variables. Let $S$ be a discrete point set $S = \{(x_i, y_j) : i = 1, \dots, m;\ j = 1, \dots, n\}$ and $f$ be a function in two variables. We can approximate $f(x, y)$ over $S$ by functions which can have one of the following forms:

$$\sum_{k=1}^{r} g_k(x) h_k(y) \quad , \quad \sum_{k=1}^{r} a_k f_k(x, y).$$

Then we have the following matrix problems:

$$\min_{\mathrm{rank}(Z) \le r} \| F - Z \| \quad , \quad \min_{a} \| F - \sum_{k=1}^{r} a_k F_k \|$$

with an appropriate definition of the matrices $F$ and $F_k$, and any matrix norm. We discuss some properties of these matrix problems. In particular, we give a characterization of extremal points of the unit sphere of matrices with unitarily invariant norm.

K. Ziętak (Wrocław)

open sets $D$ in $R^N$ $(N \geq 3)$ with the property that $\bar{D}$ is a closed annulus $\{x : r_1 \leq \|x\| \leq r_2\}$ are characterized by quadrature formulae involving mean values of certain harmonic functions. One such characterization is used to give a criterion for the existence of a best harmonic $L^1$ approximant to a function which is subharmonic (and satisfies some other conditions) in an annulus.

-Myron Goldstein (Tempe)

## Summen von Poisson Kern

Es sei $f(\theta)$ eine beliebige nach unten halbstetige Funktion mit Periode $2\pi$, und sei

$$P(\theta, z) = \frac{1-|z|^2}{2\pi|e^{i\theta}-z|^2}$$

der Poisson Kern. Wir suchen eine Entwicklung

$$(*) \quad f(\theta) = \sum_1^\infty c_n \, P(\theta, z_n)$$

von $f$, wo $z_n$ eine im Voraus gegebene Folge ist, und die $c_n$ nicht negative Konstanten.

Eine notwendige und hinreichende Bedingung wird für $z_n$ gegeben, so dass dies für jedes $f$ möglich sei. Hinreichend ist zum Beispiel dass jeder Punkt $e^{i\theta}$ Grenzwert von einer Unterfolge von $z_n$ in einem Stolzschen Winkel ist. Hiermit wird eine Frage von Walter Rudin beantwortet. Die Arbeit ist gemeinsam mit T. J. Lyons.

W. K. Hayman.

## Pseudohyperbolic Fourier Approximation

For $f : [0,2\pi]^2 \to \mathbb{C}$ let $(f, x_{kl})$ denote its Fourier double series coefficients. The Korobov space for $\alpha > 0$ is defined as

$$E^\alpha = \{ f : |(f, x_{kl})| = O\left(\frac{1}{|kl|^\alpha}\right), |k|, |l| \to \infty \}.$$

Examples of functions of a Korobov space are smooth functions with periodic ~~exten~~ periodic derivatives up to a certain order.

Hyperbolic Fourier partial sums $\sum_{|k \cdot l| \le n} (f, x_{kl}) \, x_{kl}$

approximate $f \in E^\alpha$ well both in $L_2$ and $L_\infty$ norm,

but the set of coefficients is difficult to organize as a data structure.
It is shown that pseudohyperbolic sums $\sum_{j=0}^{r} \sum_{|k| \leq 2^j} \sum_{|l| \leq 2^{r-j}}$ are simpler to handle and give the same asymptotic error estimates as the hyperbolic sums.

G. Baszenski.

Degree of Simultaneous Approximation by Gordon Operators

A report on investigations of the degree of approximation of bivariate functions on a rectangle by (discrete) spline blended operators was given. These are of the type ${}_xL + {}_yM - {}_xL \circ {}_yM$ and ${}_y\bar{M} \circ {}_xL + {}_x\bar{L} \circ {}_yM - {}_xL \circ {}_yM$, respectively. Our aim was to give a fuller description than is available in the literature by using mixed moduli of smoothness of higher orders. The crucial tool from the univariate case was a generalization of a theorem of Sharma and Tzimbalario on the degree of simultaneous approximation by cubic spline interpolators. The main results for the multivariate case were two theorems expressing certain permanence principles which explain how the Boolean sums and certain (discrete) blending operators inherit quantitative properties from their univariate building blocks.

Heinz H. Gonska.

Univalent harmonic mappings and approximate solutions

Let $f = u + iv$ a complex valued - univalent, orientation-preserving harmonic mapping defined on the unit disk $U$. Then $f$ can be viewed as a solution of the elliptic P.D.E. $\overline{f_{\bar z}} = a f_z$, where the dilatation $a(z) \in H(U)$ and $|a(z)| < 1$ for all $z \in U$. Since a composition $f \circ \phi$ with a conformal mapping $\phi$ remains harmonic we may assume that $f(0) = 0$ and $f_z(0) > 0$. Existence and Uniqueness of such mappings onto a given simply connected domain $\Omega$ having a prescribed dilatation $a(z)$ are discussed.

For the case when $\Omega$ is a strictly starlike domain we shall give a numerical method to construct the desired mapping.

Walter Hengartner

Uniform Harmonic Approximation with Continuous Extension to the Boundary.

Let $G$ be a domain in the complex plane $\mathbb{C}$ such that $\mathbb{C} - G$ contains a closed disk; and let $F$ be a closed subset of $G$ such that $F$ is the closure in $G$ of its interior $F^\circ$. We say $f \in C^1(F)$ if $f$ is continuous on $F$ and possesses continuous first partial derivatives in $F^\circ$ which extend continuously to $F$ as finite-valued functions. Let $G^* - F$ be connected and locally connected, $f \in C^1(F)$ be harmonic in $F^\circ$; and $E$ a subset of $\partial F \cap \partial G$ (here $G^*$ denotes the one-point compactification of $G$ and the boundaries $\partial F, \partial G$ are taken in the extended plane). Suppose there exists a sequence $\langle h_n \rangle$ of functions harmonic in $G$ such that $|f - h_n| \to 0$, $\left|\frac{\partial f}{\partial x} - \frac{\partial h_n}{\partial x}\right| \to 0$, and $\left|\frac{\partial f}{\partial y} - \frac{\partial h_n}{\partial y}\right| \to 0$ uniformly on $F$ as $n \to \infty$. We show that if $f$ extends ~~continuous~~ continuously to $F \cup E$ then each $h_n$ can be chosen to have the same property.

Myron Goldstein and Wellington H. Ow

Approximation by harmonic functions in Dirichlet and uniform norm.

In 1941, in his fundamental study of the Dirichlet problem, M.V. Keldysh characterized the compact sets $K$ in $\mathbb{R}^n$ with the property that the continuous functions on $K$ which are harmonic on int(K) can be uniformly approximated on $K$ by functions harmonic on neighborhoods of $K$. His proof was constructive and quite complicated. A proof by duality was given in 1950 by J. Deny.

In 1968 V.P. Havin studied the analogous problem for $L^2$-approximation by analytic functions on compact sets in the complex plane. He gave a necessary and sufficient condition, which is easily seen to be equivalent to the condition given by Keldysh. Havin's problem can be reformulated as an approximation problem for harmonic functions in the Dirichlet norm, and then it makes sense also in $n$ dimensions.

It is by no means obvious from the proofs why uniform approximation is possible if and only if approximation in the Dirichlet norm is possible. The talk is devoted to an effort at explaining this equivalence, by tracing its roots to H. Cartans definition of balayage by means of projections in Hilbert space.

Lars Inge Hedberg
Linköping, Sweden

PDE aspects of holographic grids

Starting with the cardinal interpolation series of the uniform sampling theorem, the elementary holograms are calculated explicitly. It is established that they give rise to the eigenfunctions of the Schwartz kernel associated with the self-adjoint hypoelliptic sub-Laplacian on the Heisenberg nilpotent Lie group. The 3-orbifolds of planar holographic grids are classified. Their existence has been established experimentally by Prof. Dr. Pál Greguss (Budapest). As a result, new identities for theta-null values are popping up. Finally, a series of applications to different fields (laser physics, neural computers, ...) are indicated.

Walter Schempp (Siegen).

Approximation by polyharmonic functions and the inverse potential problem.

1. There is an analogy between the inverse potential problem (cf. Anger G.) for high order elliptic equations and the classical moment problem. As a consequence, extremal problems for the inv. pot. prob. are dual to problems of approximation by solutions of the operators (in case of $\Delta^n$ these are the polyharmonic functions).

2. Consider homogeneous bodies which are gravíequivalent to a finite number of mass points (they are called quadrature or Zidarov domains). For them we prove characterization of the element of best $L^1$-approximation of subharmonic functions by harmonic functions, similar to the case of the ball, obtained by M. Goldstein, W. Haussmann et al.

Ognyan Kounchev (Sofia)

# H-Sets and Best Uniform Approximation by Solutions of Elliptic Differential Equations

(joint work with K. Zeller)
We consider second order elliptic partial differential operators $Lu = \sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b_i(x)\frac{\partial u}{\partial x_i}$ for $a_{ij}, b_i \in C(\mathbb{R}^n)$, and the space (for a domain $D \subset \mathbb{R}^n$) $FC(\bar{D}) := \{ w \in C^2(D) \mid Lw = 0,\ w \text{ has cont. extension to } \bar{D} \}$. Given an $f \in C(\bar{D})$ we ask for a best approximant $w^* \in FC(\bar{D})$ satisfying

$$\|f - w^*\|_{\infty,\bar{D}} \le \|f - w\|_{\infty,\bar{D}} \quad \text{for all } w \in FC(\bar{D}).$$

We give a characterization of a best approximant in terms of H-sets $H_1, H_2$ (introduced by Collatz 1965). To this end we first characterize H-sets with respect to $FC(\bar{D})$ in terms of the polynomially convex hulls $h(H_1)$ and $h(H_2)$. This leads to a result of de la Vallée Poussin type, from which the characterization of a best approximant follows. Main tool is the Runge approximation property (Browder 1962, Lax 1956). The uniqueness of a best approximant follows from the validity of the uniqueness condition in the Cauchy problem in the small (cf. eg. Browder 1962). Our results include previous investigations due to Burchard (1976), Hayman-Kershaw-Lyons (1984) and Kounchev (1985) on approximation by harmonic functions. We can admit uniformly elliptic differential operators with analytic coefficients. In the case of L-subharmonic functions f we construct the unique best approximant, and we show the monotonicity of the degree of approximation.

Werner Haußmann
(Duisburg)

# Mathematical Problems in the Kinetic Theory of Gases

[illegible]

## Orthogonality Methods for Singular Integral Equations

Singular integral equations which arise in transport problems have been solved by orthogonality methods introduced by Case and by Kuščer et al. I show that these are special cases of more general orthogonality relations for general singular integral equations with [illegible] integrals and open contours.
The orthogonality method has a number of advantages over the classical (Hilbert transform method). For example, it clarifies the mathematical structure of the solution; for example, it clarifies the mysterious independent conditions of Case and [illegible] which arise naturally as requirements for the existence of certain contour integrals, and it allows one to define solutions when zeros occur on the contour of integration and even for equations with non-integer index.

Paul F. Zweifel (Blacksburg)

## On the Cauchy Problem for the Nonlinear Full Enskog Equation in Three Dimensions

A global existence theorem is proved for the solutions to the initial value problem for the nonlinear Enskog equation in all space. It is proven that if a smooth solution for the same problem is found for the Boltzmann equation exists globally in time, then a global solution also exists for the Enskog equation, and that the distance between the two solutions tends to zero as the radius of the hard spheres also tends to zero.

Nicola Bellomo

# Mathematical Problems in the Kinetic Theory of Gases

8.–14.5.88

## Orthogonality Methods for Singular Integral Equations

Singular integral equations which arise in transport problems have been solved by orthogonality methods introduced by K.M. Case and by Kuščer, et al. I show that these are special cases of more general orthogonality relations for general singular integral equations, both on intervals and on closed contours. The orthogonality method has a number of advantages over the classical (Hilbert transform method). It is simpler; it clarifies the mathematical structure of the solutions (for example, it clarifies the mysterious "endpoint conditions" of Case as conditions which arise naturally as requirements for the existence of certain contour integrals); and it allows one to define solutions for cases that zeroes occur on the contour of integration and even for equations with non-integer index.

Paul F. Zweifel (Blacksburg)

## On the Cauchy Problem for the Nonlinear Full Enskog Equation in Three Dimensions

A global existence theorem is proven for the solutions to the initial value problem for the nonlinear Enskog equation in all space. It is proven that if a smooth solution for the same problem referred to the Boltzmann equation exists globally in time, then a global solution also exists for the Enskog Equation and that the distance between the two solutions tends to zero as the radius of the hard spheres also tends to zero.

[Signature]

Some results for the Boltzmann and Enskog equations.
The talk discusses the following three results:

i) For the spacehomogeneous B.E. exponential conv. and stability under hard forces and sufficiently high moments.

ii) the equivalence of both $L^1$ solutions and standard Young measure solutions to the spacedependent B.E. with large $L^1$ data.

iii) wellposedness and regularity for the 3D large data Enskog equation with a constant high density factor, globally in the case of bounded velocities and locally in time for unbounded velocities.

Leif Arkeryd

Kinetic Limits for stochastic particle systems

It would be nice to prove Boltzmann Eq., Euler and Navier-Stokes eqs starting from the Newton law of the motion. This is very difficult basically because the asymptotic behavior of hamiltonian systems is not very well understood. In this talk I refer on a research in progress (involving A. DeMasi, S. Caprino, E. Presutti and myself) in which the Carleman equation was derived, in the equivalent of the Boltzmann-Grad limit, from a system of interacting particles. The hydrodynamical limit is still an open problem.

Mario Pulvirenti

Mario Pulvirenti

On the Convergence of Particle Methods for Multidimensional Vlasov-Poisson Systems by H.D. Victory and K. Ganguly

For Vlasov-Poisson systems, particle methods are numerical techniques which simulate the behavior of a plasma by a large set of charged superparticles which obey the classical laws of electrostatics. The trajectories of these charged particles are then followed. We give estimates for the errors incurred for a "semi discrete" approximation to the underlying Vlasov-Poisson system, by first superimposing a rectangular grid or mesh on all of phase space and then replacing the initial continuous distribution of charges or masses by discrete charges or masses located at the center of each grid cell. Our analysis, on the one hand, generalizes that of G.-H. Cottet and P.A. Raviart (SIAM J. Numer. Anal. 21 (1984) pp. 52-76) to higher-dimensional Vlasov-Poisson systems; and, on the other the fundamental results of Ole Hald (SIAM J. Numer. Anal 16 (1979), 726-755), and of J.T. Beale and A. Majda (Math. Comp. 39 (1982), 1-52) on vortex methods for two- and three-dimensional Euler equations to particle-in-cell methods for multidimensional Vlasov-Poisson settings.

H. D. Victory Jr.

## The Initial-Value Problem for the Vlasov-Maxwell System

On $\mathbb{R}^3$ we consider the Cauchy Problem for the system

$$\partial_t f + \hat{v}\cdot\nabla_x f + (E + \hat{v}\times B)\cdot\nabla_v f = 0 \qquad (x, v \in \mathbb{R}^3,\ t > 0)$$

$$E_t = \nabla\times B - j,\quad B_t = -\nabla\times E,\quad \nabla\cdot E = \rho,\quad \nabla\cdot B = 0$$

where $\rho = \int f\,dv$, $j = \int \hat{v} f\,dv$ and $\hat{v} = v/\sqrt{1+|v|^2}$

Smooth initial values $f_0, E_0, B_0$ with compact support are prescribed, which satisfy the obvious constraints. A general sufficient condition for global classical existence is given in terms of an a priori estimate on the $v$-support of $f$. This estimate can be made when the data are "small" in an appropriate sense, and when the data are "nearly neutral".

R. Glassey

On the Existence of Smooth Solutions of the Vlasov-Maxwell Equations with Collisions

This is a report on work in progress in which R. Glassey and I are generalizing our previous work on the relativistic Vlasov-Maxwell system to allow Boltzmann-type collision terms with appropriate collision kernels. Theorem: Assuming (1) initial data in $C_c^\infty$ and (2) the a priori estimate $f_\alpha(t,x,v) \le b e^{-a|v|}$ uniformly for all species $\alpha$, all $x, v \in \mathbb{R}^3 \times \mathbb{R}^3$ and $t$ bounded, there exists a unique $C^1$ solution of the Vlasov-Maxwell-Boltzmann system for all $x, v$ and $t < \infty$. We are now working on verifying (2) in case the data are close to the relativistic Maxwellian $e^{-\sqrt{1+v^2}}$.

W. STRAUSS (Brown U.) Walter Strauss

Statistical Solutions of Boltzmann Equation and Boltzmann Hierarchy.

The understanding of the Boltzmann Hierarchy is an intermediate necessary step in the attempt to prove the validity of the Boltzmann Equation. On the other hand, BH has an intrinsic interest, because it represents the equation for the moments of the statistical solutions to the B.E. It turns out that this interpretation allows to construct solutions to the B.H. at least when solutions to the B.E. are available. It is possible to deal with the near equilibrium situation, using the individual theorem of Ukai to prove the existence and the estimates for the solution to the B.H. and the Lanford theorem to get uniqueness locally in time and then extend it globally. This approach does not work in the case of spatially homogeneous B.E. because Lanford theorem cannot be used. In this case a method based on an approximate dynamical evolution is worked out, which allows to prove uniqueness for the statistical solution of the B.E. and therefore provides existence and uniqueness for the spatially homogeneous B.H.

Esposito Boffard

## Solving the Boltzmann system for a gas mixture via the relevant conservation system.

It is first recalled that in the spatially homogeneous case the nonlinear integro-partial differential Boltzmann system for the distribution functions $f_1, f_2, \ldots, f_N$ of the $N$ gases of a mixture has been converted – on the basis of the only assumption of constant collision frequencies – into a nonlinear first-order ordinary differential system for the number densities $\rho_1, \rho_2, \ldots, \rho_N$ with $\rho_j(t) = \int_{R_3} f_j(\bar{v}, t)\, d\bar{v}$. This system – including both removal and creation effects – is then studied as a dynamical system. Can such a conversion be achieved also in the spatially inhomogeneous case? The answer is positive provided not only the assumption of constant collision frequencies, as made in the spatially homogeneous case, is maintaned, but also other assumptions are added concerning both the scattering and creation probability distributions as well as the initial data. With special delta form for the former quantities and separable form for the initial data, a quasilinear functional-hyperbolic conservation system for the number densities $\rho_j(\bar{x}, t) = \int_{R_3} f_j(\bar{x}, \bar{v}, t)\, d\bar{v}$ $(j = 1, 2, \ldots, N)$ is obtained in the spatially inhomogeneous case. This system can be studied for $N = 2$, and explicit solutions to it can be constructed by resorting also to a Lie group analysis.

Vinicio Boffi

## On Inverse Problems in Linear Kinetic Theory

Inverse problems for a class of linear kinetic equations are investigated. One wants to identify the scattering kernel of a transport equation (corresponding to the structure of a background medium) by observing the albedo-part of the solution operator for a direct (initial-) boundary-value problem.
In order to do that we derive a constructive method for solving direct half space and slab problems and prove a factorisation theorem for the solutions.
Using that we investigate stationary inverse problems w.r.t. well-posedness (e.g. reduce them to classical ill-posed problems such as integral equations of first kind).
In the time dependent case we show that a quite general inverse problem is well-posed and solve it constructively.

Klaus Dreßler

## Improved Chapman Enskog Approximation

The solutions of the Boltzmann Equation obtained by the Chapman Enskog procedure are not uniformly convergent in velocity space, because of their behaviour at large velocities. As a result the hierarchy of continuum approximations, which can be derived by the procedure – Euler, Navier Stokes, Burnett, ... – can only be asymptotically convergent.
A modified procedure is proposed, in which the parameters of the Maxwellian distribution, that is used as the lowest order approximation, are allowed to be slowly varying functions of velocity. In this way a formal solution is obtained which would be uniformly convergent.

David Butler

## The Navier-Stokes Equation in The Discrete Kinetic Theory

We investigate the Navier-Stokes equation which is formally derived from the discrete Boltzmann equation as the second order approximation of the Chapman-Enskog expansion. First, we obtain an explicit form of the Navier-Stokes equation without any particular assumptions. Next, we show that if there exists a "hydrodynamical basis" of the space of summational invariants, then our Navier-Stokes equation can be transformed into a symmetric system of hyperbolic-parabolic type. Consequently, the associated Cauchy problem is well posed on a short time interval. Finally, it is shown that the "stability condition" for the original discrete Boltzmann equation guarantees the global existence of solution of the Navier-Stokes equation.

KAWASHIMA Shuichi

(川島 秀一)

## Low Discrepancy Methods for Solving the Boltzmann Equation

Monte Carlo Methods play an important role for the numerical evaluation of the spatially inhomogeneous Boltzmann Equation. They are mostly intuitively motivated and intended to imitate the behaviours of gas particles in a reduced particle system. We investigate the mathematical structure behind one of these schemes (Nanbu's) and prove that it converges when the particle number increases to infinity. Since Monte Carlo methods are based on random numbers, fluctuation errors are high. The convergence proof indicates how to replace the stochastic game by a regular scheme (Low Discrepancy method). We show some examples in the spatially homogeneous case. Finally, we report on results we obtained for the calculation of the reentry phase of the European space shuttle Hermes.

Has Babovsky

Stohastic solution method of the B-G-K equation for diatomic molecules

Rarefied flows of monatomic gas have been calculated successfully by use of the direct simulation Monte Carlo method based on the Boltzmann equation. In simulating diatomic gas flows one has had recourse to some heuristic assumptions such as phenomenological model, together with the simulation method for monatomic gas. The result obtained by means of such a patched procedure is not a solution of any kinetic equation. Here is presented a stochastic solution method of the B-G-K type equation (Holway's model equation) for diatomic gas. The method is applied to the analysis of shock structure of diatomic gas and is shown to work well.

Kenichi Nanbu
(南部 健一)

The stationary nonlinear Boltzmann equation in unbounded domains

Half-space problems for the steady one-dimensional Boltzmann equation are considered. Two types of boundary conditions are analyzed:
specular reflection and the condition with a given distribution function of particles entering the region of interaction.
It has been made an attempt to show that these problems possess solutions without discussing the uniqueness of the solution

Andrzej Palczewski

Stationary Boltzmann Equation for a degenerate gas in a slab: boundary value problem and hydrodynamics.

To the B.E. for a gas of "vertical sticks" in the plane (exchange of $v_x$, conservation of $v_y$, cross section depending on $|v_x - v_{1x}|$), with reflected boundary conditions, a class of linear transport equations with prescribed B.C. is associated. A uniform expansion is then performed on the solution (shown to exist uniquely in a $L_\infty$ framework). Relations with an underlying Markov jump process are considered.

Rinio Riolo

On the number of collisions in Sinai's billiard in $\mathbb{R}^3$

We present a simple proof that the number of collisions in a cloud of finitely many hard spheres (which disperse in all space) is finite.

Reinhard Illner

Existence of $L^1$ solutions for the 3-d Enskog equation

The Enskog equation for dense gases is shown to have global solutions in $L^1$ for sufficiently small data. Previous proofs were either in $L^\infty$ or in lower dimensions.

Carlo Cercignani

Point approximation for collision terms occuring in semiconductor problems

The equations

$$\partial_t f + v\cdot\partial_x f + E\partial_v f = \int P(v,v')f(t,\cdot,v')dv' - C(v)f(t,\cdot,v)\,;\quad \mathrm{rot}\,E=0;\quad \mathrm{div}\,E = \int f(t,x,v)dv - n$$

are one model eq. for semiconductors in the electrostatic case. If the collision term is zero, we have the well-known Vlasov-Poisson system, for which the point approximation is well established. In order to extend this method for the semi-conductor case we consider the space-homogeneous problem restricted to 1D, which reads as

$$(1)\qquad \partial_t f(t,v) = \int_{-\infty}^{\infty} P(v,v')f(t,v')dv' - C(v)f(t,v)$$

and try to approximate $f(t,v)dv = \mu_t(v)$ by $\mu_t^N(v) = \frac{1}{N}\sum_{i=1}^{N}\delta(v-v_i(t))$.

We start with the time discretization of (1)

$$f_{n+1}(v) = (1-\Delta t C(v))f_n(v) + \Delta t\int_{-\infty}^{\infty}P(v,v')f_n(v')dv'$$

where $\Delta t$ is the time step and $f_n(v) = f(n\Delta t, v)$.

We define the approximation $\mu_0^N = \frac{1}{N}\Sigma\delta(v-v_i^0)$ by $\frac{2i-1}{2N} = \int_{-\infty}^{v_i^0} f_0(v)dv$, $i=1(1)N$; and from

$$\nu_0^N := (1-\Delta t C(v))\mu_0^N + \Delta t\int P(v,v')\mu_0^N(dv')$$

we compute $\frac{i}{N} = \int_{-\infty}^{\hat v_i}\nu_0^N(dv)$, $i=1(1)N-1$; $\hat v_0 = -\infty$, $\hat v_N = \infty$ and set $\mu_1^N = \frac{1}{N}\sum_{i=1}^{N}\delta(v-v_i^1)$; $v_i^1 = \int_{\hat v_{i-1}}^{\hat v_i} v\,\nu_0^N(dv)$, which can be continued.

This approximation converges under slightly weak conditions on P and C weakly to the solution in every finite time interval, if $\Delta t\to 0$ and $N\Delta t\to\infty$. The algorithm itself is highly vectorizable and was tested with $N=127$ for the master equation.

Since the computation of the $v_i^n$ is difficult to extend for higher dimensions we present an other method based on Lagrangian coordinates. Until now only numerical results are available, which agrees with the results quoted above

Joachim Dick

On the discrete velocity models with initial values in $L_1^+(\mathbb{R})$

We consider the Cauchy problem for a general velocity model of the Boltzmann equation in one space dimension, subjected to the additional hypothesis that the relative velocities have different from zero component along the spatial direction in which there is a variation of the densities. We show that the problem has a unique global mild solution provided that the initial data have finite entropy and stay in a weighted $L_1$ space.

Giuseppe Toscani

A Survey of One Dimensional Stationary Problems

Various methods of solving one dimensional stationary boundary value problems are examined. The eigenfunction expansion method of Case and the resolvent integration method of Larsen-Habetler are claimed to be equivalent, and have been widely employed. The diagonalization method introduced by Hangelbroek allows for functional analytic techniques not available to earlier analyses. Recent interest lies in convolution equations techniques, which allow an algebraic approach to the problem of bisemigroup construction. Typical bisemigroup perturbation results are presented, both in Hilbert and Banach space settings. These are relevant to the solution of the abstract transport equation with operator coefficients.

William Greenberg

Diffusion approximation for a free gaz with a stochastic boundary.

This is a report on a joint work with M. Babovsky and T. Platkowsky. One consider a free gaz (no collisions) in a tube (or a slab) arbitrarly long and of a thicknes of the order of $\varepsilon$ ($\varepsilon$ will go to zero). Then one shows that if the boundary of the tube create some stochastic effect, the solution behaves like the solution of a diffusion equation. The proof relies on classical analysis, and asymptotic expansion.

Cl. Bardos.

Velocity averaging techniques and their applications to kinetic theory.

by François Golse, Paris

1. (Report of joint work with P.L. Lions, B. Perthame and Sentis)

Assume that $f \equiv f(x,v) \in L^2(dx \otimes dm(v))$ where $m$ is a probability measure on $\mathbb{R}^N$ such that $\sup_{e \in S^{N-1}} m(\{v \,/\, |v.e| \leq \varepsilon\}) \leq C\varepsilon^\gamma$, $0<\gamma<2$,
and that $v.\partial_x f \in L^2(dx \otimes dm(v))$. then $\bar{f} = \int f \, dm(v) \in H_x^{\gamma/2}$, with the inequality

$$|D_x^{\gamma/2} \bar{f}|_{L^2_x} \leq C \, \|f\|_{L^2_{x,v}}^{1-\gamma/2} \, \|v.\partial_x f\|_{L^2_{x,v}}^{\gamma/2}$$

2. (Report of joint work with C. Bardos, B. Perthame and R. Sentis)

The above averaging result is used to treat the radiative transfer equation when the opacity does not depend on the frequency of photons. We prove

a. global existence of a weak solution

b. the Rosseland approximation

when the opacity $\sigma(T) \sim T^{-\alpha}$ with $\alpha < 1$ near $T = 0$.

[signature]

## On Collision Models for the Non-Linear Boltzmann Equation

For Monte Carlo simulation of rarefied gas flows one needs a suitable model for the intermolecular differential scattering cross section. Whereas for monatomic gases models are easy to construct, rotating molecules present a more difficult task. If the collisional redistribution of energy is pictured as resulting from diffusion in the space of rotational energies, a cross section can be obtained that obeys detailed balance. Via the simplest Chapman-Cowling approximations the model parameters are fitted to the known values of the viscosity and either the thermal conductivity or the volume viscosity.

Ivan Kuščer
(U. of Ljubljana, Yugoslavia)

## New Solutions and Results for the Vlasov – Poisson system

New results have been obtained in the following three directions:

1. Investigation of the "locally isotropic" solutions which are of the form

$$f(t,x,v) = \varphi\left(w(t,x) + \frac{(v-Ax)^2}{2}\right)$$

$$U(t,x) = w(t,x) + \frac{(Ax)^2}{2},$$

where $f$ is the distribution function, $U$ the Newtonian potential and $w$ (for $t=0$) a solution of the semilinear elliptic equation

$$\Delta w + \lambda = h_\varphi(w)$$

for given $\varphi: \mathbb{R} \to [0,\infty)$ and antisymmetric $3\times 3$ matrices $A$ (joint work with H. Berestycki, P. Degond, B. Perthame, to appear in Arch. Rat. Mech. Anal.)

2. Investigation of the stationary spherically symmetric solutions which correspond to the solutions of the generalized Emden-Fowler equation (in connection with numerical experiments of Hénon's with respect to the stability of these solutions) (joint work with K. Pfaffelmoser, to appear in Math. Meth. in the Appl. Sci),
3. Existence of $C^1$-stationary solutions of the relativistic VPS with compact support (to appear in the Proceedings of the Marcel-Großmann meeting on General Relativity, Perth, Australia, 1988)

J. Batt (München)

# Computational Group Theory

(15. 5. 88 - 21. 5. 88)

## Short Presentations

Define the length of a presentation $\langle X | R \rangle$ to be $|X| + \sum_{r \in R} \ell(r)$, where $\ell(r)$ is the length of $r$ as a word in $X \cup X^{-1}$.

The following results have been proved by Babai-Kantor-Luks-Pálfy:

Theorem 1. Every finite group $G$ has a presentation of length $O((\log|G|)^3)$; the exponent 3 is best possible.

Theorem 2. Every finite simple group $G$ has a presentation of length $O((\log|G|)^2)$ — and even one of length $O(\log|G|)$ if $G$ is neither an odd-dimensional unitary group nor a rank 1 group of Lie type.

Conjecture: Every finite simple group $G$ has a presentation of length $O(\log|G|)$.

William M. Kantor
University of Oregon

## Computing Modular Characters

2 short announcements - A Pritchard has been working on a low index algorithm, and S Linton on a double coset enumerator.

Conway's polynomials - used to define Brauer characters (see next page) ~~and~~

~~Lexicographically~~ order polynomials by the lexicographical order $0 < 1 < 2$ -- $< p-1$ on the field of order $p$, then $x^n - a_1 x^{n-1} + a_2 x^{n-2}$ -- are ordered on $(a_1, a_2, a_3 \quad )$

Primitive Polynomials ~~are~~ of degree $d$ and $n$ (where $d \mid n$) are consistent if $x^{(p^n-1)/(p^d-1)}$ modulo the one of degree $n$ satisfies the other.

The Conway polynomial $C(n)$ of degree $n$ characteristic $p$ is the earliest primitive polynomial consistent with all Conway polynomials of degree $d \mid n$.

We use the map $x \pmod{C_p(n)} \rightarrow e^{\frac{2\pi i}{p^n-1}}$, extended to the whole algebraic closure of $GF_p$, to define modular characters.

The meataxe system has been re-written several times, and is now available as FORTRAN programmes running on the masscomp and Sun workstation at this conference.

R.A. Parker Cambridge.

## Algebraic Combinatorics: The Use of Finite Group Actions

In that review talk the Cauchy-Frobenius lemma and Burnside's lemma were mentioned, both in the constant and weighted form. Furthermore constructions of orbit representatives were mentioned, direct ones, probabilistic and recursive ones. All that was shown how it applies in the paradigmatic case when a given action of $G$ on $X$ induces in a canonical way the action on $Y^X := \{f : X \rightarrow Y\}$. This is a situation which covers graph enumeration, for example.

A. Kerber, BT

## Subgroup presentations revisited.

Let $G = \langle g_1, \dots, g_n \mid r_1(g_i)=1, \dots, r_m(g_i)=1 \rangle$ be a finitely presented group, $U \leq G$ a subgroup of finite index. Then, in order to obtain a finite presentation of $U$, each of the following three data on $U$ suffices:

(i) a generating system $S = \{u_1(g_i), \dots, u_s(g_i)\}$, i.e. $U = \langle S \rangle$;

(ii) a "normal gen. system" $S$, i.e. $U = \overline{\langle S \rangle}$, the normal closure of $\langle S \rangle$;

(iii) a coset table of $U$ in $G$.

By the Todd-Coxeter procedure (iii) can be obtained from either (i) or (ii), and from (iii) Reidemeister's theorem allows them to write down a presentation of $U$ in terms of the Schreier generators of $U$. The large number of Schreier generators ($(n-1)(G:U)+1$) and Reidemeister relators necessitates the use of Tietze transformations in a heuristically steered attempt to eliminate Schreier generators. On the other hand, in case (i) a modified coset table can be constructed from which by an analogue of Reidemeister's theorem a presentation of $U$ in terms of the given generators can be obtained. In the talk a new third possibility in all cases is described, namely to use the techniques of the modified Todd-Coxeter method for an a priori reduction of the number of Schreier generators that enter the presentation of $U$. Times for concrete computations were given, that indicate the usefulness of the new method. The implementation of this method and of system of routines for subgroup presentations (SPAS) is due to A. Lucchini and mainly Volkmar Felsch.

Joachim Neubüser (Aachen).

Stephen Glasby (Univ. of Sydney) Algorithms for finite soluble groups

Algorithms are presented here for calculating normalizers and intersections in a finite soluble groups G. Most attention will be given to algorithms for computing the normalizer in G of Hall π-subgroups and for computing the intersection of two subgroups whose indices in G are coprime. An algorithm for conjugating one given Hall π-subgroup to another is used to construct Hall π-subgroups, and is also used ~~to~~ by the normalizer algorithm. The above algorithms may be used to construct system normalizers, Carter subgroups and Sylow bases. Details of algorithms for computing normalizers of arbitrary subgroups and for computing the intersection of two arbitrary subgroups will be described in a forthcoming joint paper with M. Slattery.

Michael Vaughan-Lee (Oxford) Collection from the left.

Following work of Leedham-Green and Soicher at QMC I have implemented an algorithm for collection from the left in CAYLEY. CAYLEY incorporates the Canberra Nilpotent Quotient Algorithm with the Havas-Nicholson algorithm for collection from the right. I have written a subroutine which can be substituted in CAYLEY for the Havas-Nicholson subroutine. A few minor modifications to other subroutines have to be made where collection from the right was assumed. Like the Havas-Nicholson algorithm, my algorithm incorporates combinatorial collection whereby $[a_j^m, a_i^n]$ is evaluated by using the formula $[a_j^m, a_i^n] = [a_j, a_i]^{mn} [a_j, a_i, a_i]^{m\binom{n}{2}} \cdots [a_j, \underbrace{a_i, \ldots, a_i}_{n}]^{m\binom{n}{n}}$

which is valid provided $2\,wt(a_j) + wt(a_i) > c$. (The group has class $c$ and generator $a_i$ is assigned weight $w$ if it lies in the $w$-th term of the lower exponent-$p$-central series of $G$.) A number of timing comparisons have been made between collection from the right and collection from the left, and time improvements (for the new algorithm) were observed of factors varying between 1·6 and 14·5 (the higher the class the better the improvement).

M. R. Vaughan-Lee

COMPUTING IN GROUPS OF EXPONENT FOUR

The study of groups of exponent four continues to throw up interesting computational challenges. Some additions and improvements to the nilpotent quotient algorithm were described which should allow the determination of the order of the 5-generator (relatively) free group B of exponent four with reasonable resources. These improvements are based primarily on a more careful analysis of the structure of an appropriate system of linear equations over GF(2). From the work so far it follows that an upper bound for the order of B is $2^{2728}$ and it seems likely this is the order of B.

M. F. Newman

VERTICES AND SOURCES

Let $F$ be a (finite) field of characteristic $p$, $G$ a finite group (such that $p$ divides $|G|$) and let $M$ be an indecomposable $FG$-module. A vertex is a subgroup $P$ of $G$ of smallest possible order which has an indecomposable $FP$-module $N$ (a source) such that $M$ is a direct summand of the induced module $N\uparrow^G$.

We present methods that automatically determine vertex and source for an indec. $FG$-module $M$. In particular, we describe how to compute the ring of $FG$-endomorphisms of a module $M$ and how to prove the indecomposability of $M$ or to find an explicit decomposition

The methods have been implemented as part of the CAYLEY system and have been used to compute a number of examples.

Gerhard Schneider
(Essen)

MOC: a modular character system – theoretical background

MOC is a computer system for dealing with modular characters. It was developped in joint work with R. Parker and K. Lux. Some theory behind this system is described in my talk.
Certain bases for the rings of generalized Brauer characters and projective characters are introduced. Firstly, basic sets of Brauer characters resp. projective characters are defined and secondly, in duality to these, bases of projective atoms and Brauer atoms. The problem of finding all possible decomposition matrices for a finite group, which are consistent with a given set of information, is reduced to the following problem. Let $A, B, U$ be integral

matrices, such that $U$ is square of determinant 1 and has non-negative entries. Find all square matrices $U_1, U_2$ with $U = U_1 \cdot U_2$ such that all of $U_1, U_2$, $A \cdot U_1$ and $B \cdot U_2$ have non-negative entries.

Gerhard Hiß
(Aachen)

Simplifying group presentations

A system has been developed which contains many small primitive functions which can be combined to give Todd-Coxeter, Reidemeister-Schreier, Tietze transformation facilities etc.

Two specific ideas which come from the flexibility of the system allowing easy experimentation are discussed. The first of these look at extra relations which can be obtained from a modified Todd-Coxeter algorithm. Relations of type A come from the early closing of rows. Relations B come from coincidences between cosets yielding relations between coset representatives.

In using Tietze transformations it is possible to direct the process towards getting relations of a particular type by defining a weighted length on substrings which come from weighting the generators. Giving a generator low weight compared with others improves to chances of the order of the generator being produced as one of the relations

Edmund F. Robertson
(St Andrews)

PRESENTATIONS FOR SIMPLE GROUPS

In a recent paper with E.F. Robertson (St Andrews) and P.D. Williams (San Bernardino) we give presentations for the groups $PSL(2,p^n)$, $p$ prime, which show that the deficiency of these groups is bounded below. For the groups $SL(2,2^n)$ the best general result still contains one extra relation. However for $n \leq 6$ efficient presentations have been obtained by using a computer. Deficiency $-1$ presentations for direct products of $SL(2,2^{n_i})$ for coprime $n_i$ are also given.

New efficient presentations have also been obtained for certain groups $PSL(2,p^n)$, $p$ odd; in particular $PSL(2,3^4)$, $PSL(2,5^3)$, $PSL(2,11^2)$, $PSL(2,13^2)$ and $PSL(2,19^2)$. Further we considered the groups $PSL(2,p) \times PSL(2,p)$, giving a 2-generator 6-relation presentation for these groups. Finally, based on computer evidence of efficient presentations obtained for $p = 5, 7, 11, 13$ and $17$ we conjectured an efficient (2-generator 4-relation) presentation for the groups $PSL(2,p) \times PSL(2,p)$.

Colin M. Campbell (St Andrews)

Algorithms for the determination of finite p-groups.

The groups of order 256 have been determined by computer. The algorithms used in the determination are extensions of the p-group generation algorithm described by Newman 1977. The basic algorithm will be reviewed briefly and the extensions described in some detail. Implementation & performance details will be provided together with a summary of results.

E. A. O'Brien (Australian National University, Canberra)

Algorithms for finite soluble groups and permutation groups.

This talk was a report on recent discussions with Charles Leedham-Green and Leonard Soicher aimed at developing algorithms for groups which could be implemented quickly in a high level language like CAYLEY. We concentrated on the problem of finding the kernel of a group homomorphism. We developed three algorithms: Let $G = \langle X \rangle$ and $H$ be groups and $\phi : X \to H$ a map.

Algorithm 1 assumes that $G$ and $H$ are permutation groups for which bases are known. It tests whether $\phi$ determines a homomorphism $\phi : G \to H$, and if so finds the kernel.

Algorithm 2 assumes that $G$ and $H$ are soluble groups with power commutator presentations and that $\phi : G \to H$ is a homomorphism. It returns power commutator presentations for the kernel and image.

Algorithm 3 is nondeterministic. It assumes that $G$ and $H$ are permutation groups or soluble groups, that $|G|$ is known and that $\phi : G \to H$ is a homomorphism. It returns the kernel and image with a high probability.

Cheryl Praeger (Western Australia)

The Knuth-Bendix Procedure and Coset Enumeration

The Knuth-Bendix procedure for strings is outlined. Two examples are presented. In these examples, the Knuth-Bendix procedure is able to provide more information than coset enumeration. A family of orderings on free monoids is defined. The orderings have proven useful in computations with polycyclic groups. Four implementation issues associated with the Knuth-Bendix procedure are discussed: rewriting strategy, indexing the rules, overlap strategy, and provision for length increasing rules.

Charles Sims
Rutgers

Finite varieties and a finitely presented group

It is known that there exists a sequence $w_1, w_2, \ldots$ of words in two variables with the following property:
The finite group $G$ is soluble if and only if $w_k(G) = 1$ for all but finitely many values of $k$.
We present some explicit sequences in four variables and discuss a question whose answer would yield a satisfactory series involving two variables. This leads to the groups $G(a,b) = \langle X, Y \mid X = [X,_a Y], Y = [Y,_b X] \rangle$ that have $SL(2,q)$ as quotients for various values of $q$, $a$ and $b$. For example, $G(5,5)$ maps onto $SL(2,5)$. Nothing seems to be known about $G(2,3)$, however adjoining the extra relations $X^r = 1$, $Y^s = 1$ where $r$ and $s$ are coprime causes the group to collapse in many cases.

Rolf Brandl
WÜRZBURG

Power-series groups

Iain York (research student) has made use of REDUCE (Nottingham) and CAYLEY (Manchester) packages to find invariants of "power-series groups"

$G_n(p) = \{$integer polynomials under substitution$\} / (x^{n+1}, p)$, a group of order $p^{n-1}$. As a result, several conjectures have been formulated and some of them have been proved. For example, the class and exponent of $G_n(p)$ are now known explicitly in all cases.

Dogdson
NOTTINGHAM

Construction of Representations of Hecke Algebras

We describe a computational technique called Condensation which turns representations of a finite group into representations of a related Hecke algebra. Under certain circumstances condensation sets up a Morita equivalence between modules of the group algebra and the very much smaller modules for the Hecke algebra; this allows us to use condensation to obtain otherwise inaccessible information about the group. We describe the use of the condensation programs in the calculation of the 2-modular characters of $G_2(3)$.

Alexander Ryba,
Ann Arbor.

# A Generalization of the Alternating Groups

The class of groups

$$Y(m,n) = \{ a_i \ (1 \le i \le m) \mid \underset{(1\le i \le m)}{a_i^n = 1}, \ \underset{(1 \le i < j \le m,\ 1 \le k \le \frac{n}{2})}{(a_i^k a_j^k)^2 = 1} \}$$

was introduced in the J. of Alg. vol. 75 (1982).

Extensive computational investigation done with J. Neubüser in Aachen during Jan - March 1987, and independently by Robertson + Campbell in St. Andrews confirm a conjecture that this series is formed by orthogonal + symplectic groups defined over certain fields of characteristic two of finite order.

The class $Y(m,n)$ is related to

$y(m,n) = \{ \Sigma_m, a \mid \Sigma_m$ symmetric gp generated by the transposition $\tau_{12}, \tau_{23}, \ldots, \tau_{m-1,m}$,

$a^n = (a\tau_{12})^n = 1$,

$[\tau_{12}^{a^i}, \tau_{12}] = 1 \quad (1 \le i \le [\frac{n}{2}]) \quad , \quad (a\tau_{i,i+1})^2 = 1 \quad (2 \le i \le m-1)\}$

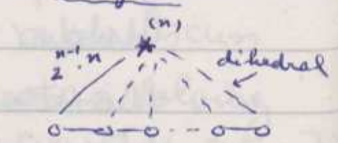
For instance, $Y(m,n) \cong y(m,n)$ for $n$ odd.

The first difficult case of the problem is $y(3,n)$.

We identify $y(3,\infty) = \{ u, v, a \mid u^2 = v^2 = (uv)^3 = 1$

$[v^{a^i}, v] = 1 \ \forall i, \quad a^u = a^{-1} \}$

Diagram




as being $SL(2, \mathbb{Z}_2[t, t^{-1}]) . \langle \begin{pmatrix} t^{1/2} & 0 \\ 0 & t^{-1/2} \end{pmatrix} \rangle$. The proof uses Nagao's theorem on the structure of $SL(2, \mathbb{Z}_2[t])$ as a free product with amalgamation.

Said Sidki

University of Brasília

# Finite subgroups of $E_8(\mathbb{C})$

The question of which finite nonabelian simple subgroups occur in $E_8(\mathbb{C})$ has been completely solved (work joint with David B. Wales). In the talk the construction of a subgroup isomorphic to $L(2,19)$ has been treated as an example. It was found by solving a system of 19 equations (having 42 monomials each) in 8 variables. From work joint with Robert L. Griess, the finite nonabelian simple subgroups of $E_8(\mathbb{C})$ are known up to a few questions, involving $L(2,31)$, $L(2,32)$, $L(2,61)$, $U(3,8)$. It has been discussed how the $L(2,61)$ case could be solved by finding a solution to a set of 58 equations in 8 variables. Such a solution has not yet been found.

## Lie: a software package for computations with Lie group representations and Weyl groups.

In the work described above, use has been made of C programs for computing traces and centralizer types of elements of finite order in $E_8(\mathbb{C})$. Ron Sommeling and I have taken these programs as ingredients for a package as described in the title. Presently it includes routines to compute degrees and multiplicities of highest weight modules of semisimple Lie groups, traces and centralizer types of elements of finite order, tensor product decompositions, the orbit of a vector under the Weyl group, a reduced expression of a Weyl group element.

Arjeh M. Cohen, CWI, Amsterdam.

# A soluble quotient algorithm

The algorithm can compute the biggest finite soluble factor group of a finitely presented group in case it exists. The basic idea, which is not restricted to finite or soluble groups, is as follows: Deciding whether an epimorphism $\varepsilon$ of a finitely presented group $G$ onto a group $H$ can be lifted to an epimorphism of $G$ onto ~~epimorphism~~ an extension $\tilde{H}$ of an (irreducible) $H$-module by $H$ leads to a system of linear equations. In the situation of finite soluble images the construction of the relevant modules and extensions is also largely a matter of linear algebra. Finding the relevant new prime divisors for $|\tilde{H}|$ can be approached by using the rational representations of $H$.

W. Plesken

Aachen

The abelian groups determined by (i) the point-hyperplane incidence matrix A and (ii) the complement of A for a finite projective geometry derived from $V_n(p)$, where $p$ is prime and $n>1$, are described.

R. J. List

Collection

Given a finite soluble group G described by a power-conjugate presentation, the elements of G can be multiplied by a collection process. We give strong experimental evidence that collection from the left (always collecting the leftmost minimal non-normal subword) is vastly more efficient than collection from the right, which had been the most commonly used method of collection.

We also describe the "deep thought" method of multiplying elements of a polycyclic nilpotent group. The technique involves preprocessing to determine a multiplication formula. (joint work with C. Leedham-Green)

Leonard Soicher
QMC, London

P.S. The first implementation of collection from the left for soluble groups was coded at the meeting, and in a group of composition length 53, performed about 100 times faster than the existing implementation.

# Fast Wedderburn transforms

Let $G$ be a finite group. Then, according to Wedderburn's Theorem, the complex group algebra $\mathbb{C}G$ is isomorphic to a suitable algebra $\bigoplus_{i\le h} \mathbb{C}^{d_i\times d_i}$ of block diagonal matrices. Every isomorphism $W: \mathbb{C}G \to \oplus\, \mathbb{C}^{d_i\times d_i}$ will be called a Wedderburn transform for $\mathbb{C}G$. With respect to natural $\mathbb{C}$-bases, $W$ can be viewed as a $|G|$-square matrix. The *linear complexity* $L_s(A)$ of a matrix $A \in \mathbb{C}^{r\times t}$ is the minimal number of arithmetic operations sufficient to compute $A\cdot x$, for an arbitrary $x \in \mathbb{C}^{t\times 1}$. Since for non-abelian groups $G$ the Wedderburn transforms are not uniquely determined by $G$, we define the *linear complexity of the group* $G$ by $L_s(G) := \min\{L_s(W) \mid W \text{ a Wedderburn transform for } \mathbb{C}G\}$. Trivially, $|G| \le L_s(G) < 2\cdot|G|^2$. For cyclic groups $G$, the FFT algorithms show that $L_s(G) = O(|G|\log|G|)$. By results of Atkinson ('77) and Karpovsky ('77) this last result extends to finite abelian groups. Recently, Beth ('84) showed that for finite soluble groups $L_s(G) = O(|G|^{3/2})$. Using Wedderburn transform adapted to a tower of subgroups of $G$, Beth's result can (with slightly greater "constants") be generalised to arbitrary finite groups. As a second result, we show that for symmetric groups $L_s(S_n) = o(|S_n|\cdot\log^3|S_n|)$. The final result (jointly with U. Baum and T. Beth) states that every $p$-group with an abelian normal subgroup of index $\le p^2$ has a fast Wedderburn transform. E.g., $L_s(G) \le \frac{3}{2}|G|\cdot\log|G|$, for all groups of order 64. These fast algorithms can be applied to the design of suboptimal Wiener filters, as described in the work by Trachtenberg and Karpovsky.

M. Clausen
Karlsruhe

# Parallel Computation in Permutation Groups

A fundamental issue in parallel computation is the determination of which problems with 'efficient' sequential solutions have 'substantially' faster solutions on a parallel machine. We look at this question for permutation group problems, focussing on the problems known to have polynomial-time solutions. Following a current paradigm, parallel efficiency will be established by inclusion in the class NC, roughly, the class of problems solvable in 'superfast' time $O(\log^{c} n)$ using a 'feasible' number $O(n^{c_2})$ processors. Recent results of Babai, Luks, and Seress show that basic problems are in NC. (Assuming, as usual, that $G \leq S_n$ is specified by generating permutations) these include: finding the order of $G$, testing membership in $G$, finding a composition series, finding a presentation $G = \langle X \mid R \rangle$, finding pointwise stabilizers of subsets. Two striking observations:
① All these problems seem to demand structural information about the group, and timing arguments appeal to the classification of finite simple groups; ② The techniques have led to an order of magnitude improvement in the computational complexity of sequential methods and all the above problems are now solvable in time $O(n^4 \log^c n)$ (vs. $O(n^5)$ for usual good approaches to finding $|G|$, etc).

Eugene M. Luks
Eugene, Oregon

## MOC: A modular character system, some algorithms.

The MOC 2/3 - system is a collection of programs, which try to determine the Brauer character table of a finite group. It was developped in Aachen and Cambridge by R. Parker, G. Hiß and myself. The problems solved by using the programs are the following:
The Brauer trees of the sporadic groups and their covering groups (400 trees, of which 375 trees are determined up to algebraic conjugacy) $2\,Sp_6(2)$ for the primes 3, 5, $2J_2.2$ for $p=$ all primes, ${}^3D_4(2)$ $p=3,7$, $2\,G_2(4)$ $p=3,5$, He for $p=7$ by A. Ryba.

In order to calculate the Brauer character table of a finite group, we proceed as follows:
The program PST determines the restriction of the ordinary characters to the $p$-regular classes, the distribution of the ordinary characters into the blocks and a basis for the Brauer characters modulo a large prime. Using the programs TEN, SYM and inducing up from subgroups we are able to generate a large set $\mathcal{P}$ of projectives and a large set $\mathcal{B}$ of Brauer characters. Using GETBAS we find ~~a~~ $\mathbb{Z}$-basis of genuine characters for $\mathcal{P}$ and $\mathcal{B}$. The program PIMTEST checks whether we have found any proj. indecomposable characters by decomposing the projectives into projective atoms. Finally RDC tries to subtract the projective indecomposables found sofar from other projective characters and thereby it improves ~~on~~ our $\mathbb{Z}$-basis. These methods can be iterated and lead in general to several (ideally one) possibilities for the Brauer character table, which can now be attacked by more sophisticated methods.

K. Lux
(Aachen)

# Knuth-Bendix Algorithm and Dehn Algorithm

Let $G = \langle \mathcal{G}, \mathcal{E} \rangle$ be a finite group presentation. In the Knuth-Bendix algorithm, we restrict the computation of critical pairs to the "normal" ones, i.e. those induced by the standard complete set of rules for the equational theory of groups: if $R: a_1 \cdots a_n \to b_1 \cdots b_m$ is a rule, $a_i$, $b_j$ in $\mathcal{G}$, then these pairs are $(a_2 \cdots a_n, a_1^{-1} b_1 \cdots b_m)$, $(a_1 \cdots a_{n-1}, b_1 \cdots b_m a_n^{-1})$ and $(a_n^{-1} \cdots a_1^{-1}, b_m^{-1} \cdots b_1^{-1})$. Then the K.B. algorithm halts on the presentation $G$, given a reduction ordering which is non-length increasing. Call $\mathcal{R}$ the resulting set of rules. We now ask: what are the minimal conditions that imply the solvability of the word problem for $G$ by $\mathcal{R}$. Answer: the usual condition $C'(1/4)$ of small cancellation theory plus some new condition: the non-existence of some diagrams (closed ladders) in the Cayley graph of $G$. So this is yet another proof of the fundamental result of small cancellation theory. It sharpens the usual ones in the following way: 1) the usual metric conditions imply the new ones, 2) the group $G = \langle A, B, C \mid ABC = CBA \rangle$ becomes a small cancellation group under these new conditions, 3) we have structural hints on the Cayley graph of small cancellation groups.

# An application of SOGOS

If groups $G$, $G_1$ are given, and $\sigma: L(G) \to L(G_1)$ is an isomorphism of their subgroup lattices, it is well known that the image $N^\sigma$ of a normal subgroup $N$ of $G$ needs not be normal in $G_1$. In the critical case where $G/N$ is cyclic, $N^\sigma$ is core-free in $G_1$, and both $G$ and $G_1$ are finite $p$-groups, then $N$ is abelian if $p \neq 2$. An example (Busetto - Stonehewer) shows that if $p=2$ $N$ may be non abelian.

A. Lucchini + myself, using SOGOS to do part of the checking, found one more example:

$$G = \langle a, h, k \mid a^{2^7} = h^{2^4} = k^{2^4} = 1,\ h^k = h^9,\ h^a = h^{-1},\ k^a = hk^{-1} \rangle$$

$$G_1 = \langle a_1, h_1, k_1 \mid a_1^{2^7} = h_1^{2^4} = k_1^{2^4} = 1,\ h_1^{k_1} = h_1^5,\ h_1^{a_1} = a_1^{-8} h_1^{-1},\ k_1^{a_1} = h_1^{-1} k_1^{-1} \rangle$$

where $N = \langle h, k \rangle$ has $|N'| = 2$, $N \trianglelefteq G$, $N^\sigma = \langle h_1, k_1 \rangle$ is core-free in $G_1$.

The computations ran on a VAX/VMS; CPU time approximately $2^h$.

Federico Menegazzo
(Padova)

# Computing with infinite finitely presented groups

Certain infinite groups defined by finite presentations that arise naturally from geometry and topology (the von Dyck groups, for example) can be shown to have very regular properties, in a precise sense. This means that a normal form can be found for the group elements (which will usually consist of the shortest words for the elements), and efficient algorithms exist for putting arbitrary words into normal form. These algorithms involve computations using finite state automata, and they are expected to have applications to the underlying geometrical or topological structure. Practical methods for constructing these automata were discussed. Methods that have been attempted to date include Todd-Coxeter Coset Enumeration and Knuth-Bendix reduction.

Derek Holt,
Warwick

# Computation in Permutation Groups
# Using Labelled Branchings

Labelled branchings are data structures for implicitly representing the coset representatives for the point stabilizer sequence of a permutation group using $O(n^2)$ space. This data structure was invented by M. Jerrum who gave the first $O(n^5)$ time $O(n^2)$ space algorithm for finding a strong generating set for a permutation group. We have used labelled branchings to give a new base change algorithm which runs in $O(n^3)$ time. This improves by two orders of magnitude the previous algorithm (joint with C. Brown and P. Purdom). In addition, if $B$ is a complete labelled branching for $G$, then we can construct a presentation for $G$ using at most $n-1$ generators and $(n-1)^2$ relations. This leads to a method for a strong generating test which runs in $O(n^4)$ time (joint with C. Brown and G. Cooperman). This test has been used to give a substantial speed up of Jerrum's original algorithm

Larry Finkelstein
Boston, MA

Constructing machines for automatic groups.

Basically a group is automatic if a finite state automaton can be used to recognise a well structured normal form for its elements (see

"Derek Holt", previous page. For many automatic groups (eg groups of hyperbolic isometries) such a normal form is provided by the reduced words which are shortest and lexicographically least according to an ordering of the generators.

In this talk David Epstein's algorithm for the construction of such a machine was outlined. The machine is constructed in terms of a finite set of word differences which in turn are derived from a set of Knuth Bendix rules and associated long strings of reduced words. All these are collected within a partial Cayley graph in a construction weighted heavily towards those areas of the graph leading to a most rapid increase of the set of word differences.

Sarah Rees

Warwick

Presentations of groups acting discontinuously on $\mathbb{H}^3$

Call $\Gamma \subset SL_2\mathbb{C}$ <u>algorithmic</u> when $\Gamma = \langle X_1, \cdots, X_m \rangle$ where we have two algorithms,

AD(1) to solve the word problem in $\Gamma$ on $\vec{X}$,

AD(2) to compute the entries of each $X_k \in \vec{X}$ to any desired accuracy.

Jørgensen's Inequality: $|\mathrm{tr}(X)^2 - 4| + |\mathrm{tr}\, XYX^{-1}Y^{-1}|^{0} < 1 \Rightarrow$ either $\langle X, Y \rangle$ is not discrete in $SL_2\mathbb{C}$ or is elementary (contains no free subgroup of rank 2),

gives an effectively computable criterion for $\Gamma$ not to be discrete. Poincaré's theorem on Fundamental Polyhedra gives an effectively computable criterion for $\Gamma$ to be discrete, and a search procedure to set up a fundamental domain $\mathcal{D} \subset \mathbb{H}^3$ for the action of $\Gamma$ is simultaneously an efficient search for elements $X, Y$ generating a nondiscrete subgroup of $\Gamma$. When $\Gamma$ is discrete and geometrically finite a good search for $\mathcal{D}$ will eventually finish with the correct domain, and when $\Gamma$ is not discrete Jørgensen's Inequality will eventually show that $\Gamma$ is not discrete. If $\Gamma$ is discrete and geometrically infinite we would compute forever.

I have a file of Fortran subroutines, PNCRE =: $\mathcal{P}$, which search for a Ford fundamental domain $\mathcal{D} \subset \mathbb{H}^3$ for a given $\Gamma$, using either the half-space model $\mathcal{U}^3$ of $\mathbb{H}^3$ or the ball model $\mathbb{B}^3$. The output of $\mathcal{P}$ is a decision: discrete / not discrete, and if $\Gamma$ is discrete & geometrically finite a presentation for $\Gamma$ on $\vec{T}$, the side pairing transformations of $\mathcal{D}$, together with the expressions writing $\vec{T}$ on the original generators $\vec{X}$. If a graph plotter is available it can be used to draw a diagram of $\mathcal{D}$ when $\mathcal{U}^3$ was used. There is a manual for the use of $\mathcal{P}$, the system is available from me or other sources, and I would be ready to respond to reports of errors in the system.

Robert F Riley
Binghamton New York

COMPUTING IN P-GROUPS

A number of remarks about computing in groups of prime power order were made.

(i) An important source of examples comes from linear groups over local fields. We have programs to compute in such groups, including a program for arithmetic in local fields of characteristic 0 written by a PhD student C. Murgatroyd.

(ii) The dramatic improvement in our collection algorithm over the traditional method has a theoretical explanation. Theory and practice show that the time required for our method increases exponentially in the derived length rather than the class of the group.

(iii) Our symbolic method of multiplying elements of a p-groups, called 'deep thought', as an alternative to collection, allows us to perform calculations very rapidly that are completely out of range for collection. The method requires further development, and we would welcome collaboration.

This is joint work with L. Soicher.

C.R. Leedham-Green. Queen Mary College, London.

Procedures and algorithms of computational Group Theory had recent dramatic applications in the area of Finite Geometries and Combinatorial Designs. These structures usually have groups of automorphisms from which the geometries can be reconstructed. We can divide these algorithms into 3 broad categories. The first category is closest to computational group theory and is related to the construction of fused incidence matrices such as intersection matrices, tactical decompositions and $A_{t,k}(G)$ related to a group action $G|\Omega$. The second category is largely number theoretic and deals with solving large knapsac problems $AX = B$ where $A$ is non-negative integral, $B$ integral, and $X$ subject to $0 \le X \le M$. Several algorithms are used for these problems including branch & bound, ALGOR, $L^3$-based, BUCKETS (time-space trade off). The third category relates to solving isomorphism and automorphism of designs and graphs problems. Algorithms in the above categories have been developed and reside in LIB111 at the University of Nebraska - Lincoln.

Spyros S. Magliveras

Lincoln, NE.

## Supercomputer Applications

Some applications of supercomputers to group theory were considered. Specific examples were a nilpotent quotient algorithm for Lie rings, Cayley, and integer matrix diagonalization.

The nilpotent quotient algorithm for Lie rings (as distinct to that for groups) is particularly well suited for vectorization of the Lie product operation (whereas collection in groups is much more difficult). Some results on Lie algebras related to Burnside groups were presented.

The integer matrix diagonalization algorithm is ideal for vectorization. Vectorization ratios exceeding 90% are readily achieved. Use of a vector supercomputer effectively cuts the calculation time, which is polynomial in the matrix rank, by reducing the polynomial degree by 1.

George Havas (Canberra)

## Braid orbits on classes of generators of finite groups

In the first section of the talk a report was given on the realizations of finite simple groups as Galois groups of regular field extensions over $\mathbb{Q}^{ab}(t)$ and $\mathbb{Q}(t)$.

In the second section the braid orbit theorems were applied to prove, that the groups $PSL_2(5^2)$, $PSL_2(7^2)$ and $M_{24}$ are Galois groups of regular field extensions over $\mathbb{Q}(t)$. Therefore by Hilbert's irreducibility theorem there exist infinitely many Galois extensions over $\mathbb{Q}$ with these groups as Galois groups.

B. H. Matzat (TU Berlin)

The problem of Galois groups.

Given $f \in \mathbb{Q}[x]$ find $G = \mathrm{Gal}_{\mathbb{Q}} f$. Lower bounds for $G$ from shapes of elements obtained from mod $p_i$ factor degrees $p_i \nmid \operatorname{disc} f$. Čebotarev density theorem is impractical to use even with G.R.H. Upper bounds are obtained from proving that $I(\alpha_1, \ldots \alpha_n) \in \mathbb{Z}$ where $I$ is an invariant of $G$. p-adic methods (Darmon) have been used for this. Symmetric fn. theory is used to construct polynomials with zeros $\{\alpha_{i_1}\alpha_{i_2}\cdots\alpha_{i_r}\}$ & $\{\alpha_{i_1}+\alpha_{i_2}+\cdots+\alpha_{i_r}\}$ & $\{k_1\alpha_{i_1}+k_2\alpha_{i_2}+\cdots k_r\alpha_{i_r}\}$ $k_i \in \mathbb{Z}$ pairwise distinct. The factorization of these polynomials give orbit data on the action of $G$ on $r$-sets & $r$-sequences. Unramified primes would give further data. A program for computing Galois gps for polynomials are degree $\leq 7$ is incorporated in MAPLE (written by Ron Sommeling).

John McKay
Concordia U., Montréal

meeting on:

name of speaker: V. Zaychenko

subject of talk: Computations in algebras of invariant relations

duration of talk: 25 min

short summary (not more than 15 lines): Computer algorithms are designed for the study of invariant relations algebras. Given a group $(G, W)$, the orbits of action of $G$ on $W^k$ are represented by the $x$-paths in a tree $T_G$, constructed by the algorithms. The complexity of sub-algebras study is $O(2^{\mathrm{rank}\, G})$ for $V$-rings and it can't be reduced. Applications for the study of distance-regular graphs, Hamming association schemes, transitive extentions ~~of~~ of permutation groups are given.

## The Knuth-Bendix Procedure

The K-B procedure attempts to change an arbitrary finite presentation for a group G into a confluent presentation. Given a confluent presentation one can determine the cardinality of G, and one can solve the word problem in G. There is also a heuristic procedure for the membership problem, but in general this problem is unsolvable even for confluent presentations. A slight modification of the K-B procedure produces a procedure for enumerating the cosets in G of a finitely generated subgroup H. This procedure can work even when the K B procedure applied to G itself fails to terminate.

R Gilman
Hoboken

### Computing conjugacy classes of elements in finite soluble groups.

In a joint paper with R. Laue and U. Schoenwaelder in the proceedings of the 1982 LMS Durham Symposium on computational group theory two general principles for orbit calculations were proposed: the use of homomorphisms for G-sets and the use of the fact that the orbit of a normal subgroup $N \triangleleft G$ is a block for G. An algorithm for the determination of the conjugacy classes of soluble groups based on these principles was implemented in the SOGOS system in 1986 by M. Mecky and in CAYLEY 1987 by M. Slattery. However in this case further improvements are possible: Let $N$ be a minimal normal subgroup of the finite soluble group $G$, let $g_1N, \ldots, g_rN$ be representatives of the classes of $G/N$ and $C_i/N := C_{G/N}(g_iN)$. Then by the general algorithm one has to find the orbits of $C_i$ on $g_iN$ under conjugation, and for each representative $g_{ij}$ its stabilizer $\mathrm{Stab}_{C_i}(g_{ij})$. This can be done very efficiently by the following observation (cf. Pahlings/Plesken, J.f.d.r.u.a.M. 380 (1987) 178–195): via the mapping $g_in \to n$ of $gN \to N$ the operation of $C_i$ on $g_iN$ by conjugation ($g_in \to (g_in)^c$) can be replaced by the "affine" action of $C_i$ on $N$ given by $\alpha_c : n \to n^c[g_i, c]$. The elements $m \in N$ act by "translation" ($\alpha_m : n \to nm[g_i, m]$), hence it suffices to consider the action of $C_i$ on $N/[g_i, N]$. This reduction, together with more suitable data structures allows to compute classes much more efficiently. Test examples: For an iterated semidirect product of extraspecial groups of order $2^{11} \cdot 3^{13}$ computing time (on a MC 5400) dropped from 2496 sec to 96 sec; the 52 195 classes of $(S_4 \, wr \, S_3) \, wr \, S_3$ (order $2^{31} \cdot 3^{13}$) were found in $6^h 38^{min}$.

Joachim Neubüser (Aachen).

Normalizers and Intersections in Solvable Groups

Using traditional orbit stabilizer techniques, one can compute normalizers and intersections in a finite solvable group. Well-known orbit reduction tricks reduce the amount of work by working down a normal series in G with elementary abelian factors. Further speed ups are achieved for these algorithms by using S. Glasby's ideas (developed for normalizers of Hall subgroups and intersection of subgroups with coprime indices) when appropriate. In this way, orbit calculations are replaced by integer or linear algebra, thus permitting reasonable computation in some situations with very large orbits.

Michael C. Slattery
Milwaukee, Wisconsin

Galois Theory and Computing Subfields.

Let $f(X) \in \mathbb{Z}[X]$ be a monic irreducible polynomial and $\Omega := \{\alpha = \alpha_1, \dots \alpha_n\}$ its set of roots. Consider $G := \mathrm{Gal}(f)$ as a permutation group on $\Omega$. Then $\mathbb{Q}(\alpha)$ is the fixed field of the point stabilizer $G_\alpha$ and so

$\mathbb{Q}(\alpha)$ has a subfield $F$ of index $d \Leftrightarrow G$ has a block of imprimitivity of size $d$.

If $\Delta = \{\alpha_1, \dots, \alpha_d\}$ is such a block, then 'generically' $F = \mathbb{Q}(\delta)$ where $\delta := \alpha_1 \cdots \alpha_d$.

The theorem of Frobenius-Chebotarev shows that if $p \nmid \mathrm{disc}(f)$ then the degrees $n_1, \dots n_r$ of the irreducible factors of $f(X) \bmod p$ imply that $G$ contains a permutation of type $(n_1, \cdots, n_r)$. Using this technique we can often obtain evidence that $\mathbb{Q}(\alpha)$ has a subfield of index $d < n$ (in the case $G$ is imprimitive). In my talk I outlined a method of computing the roots of $f(X)$ in an extension field of $\mathbb{Q}_p$ (for some prime $p \nmid \mathrm{disc}(f)$), determining a small list of $d$-subsets of these roots which might form a block, and deciding whether or not the corresponding $\delta$ does generate a subfield of index $d$.

John D. Dixon (Carleton University, Ottawa)

Dirichlet Series Associated with Groups

If $G$ is a f.g. group we define $\zeta_G(s) = \sum_{H \leq_f G} |G:H|^{-s}$, so, for example, $\zeta_{\mathbb{Z}}(s) = \zeta(s)$, the Riemann zeta function. If $p$ is a prime $\zeta_G^p(s) = \sum_{H \leq_f G, |G:H| \text{ a } p\text{-power}} |G:H|^{-s}$. If $G$ is also torsion-free nilpotent then $\zeta_G(s) = \prod_p \zeta_G^p(s)$.
For example, $\zeta_{\mathbb{Z}^n}(s) = \prod_{i=0}^{n-1} \zeta(s-i)$. Methods were explained for computing the zeta function of such a group. In particular, it was noted that for relatively small such groups, the formula for $\zeta_G(s)$ can be complex – in examples so far calculated the formula was commonly a quotient of products of translates of $\zeta(s)$, perhaps with one very large irreducible polynomial* factor in $p$ & $p^{-s}$ (one computes one prime at a time).
The computer algebra system REDUCE was used to perform algebraic manipulations and factorizations.
The purpose of this investigation is to study $s_G(x) = \sum_{H \leq_f G, |G:H| < x} 1$, which can be bounded using the Tauberian theorem if $\zeta_G(s)$ is sufficiently well understood. The formulas are of interest in their own right. Contributors to this work include D. Segal (All Souls, Oxford), F. Grunewald (Bonn) and D. Grenham, a p.g. student of Segal

Geoff Smith:
(G.C. Smith, Bath University, UK)

* so far always reciprocal in $p$ & $p^{-s}$

GAP

Groups and Programming (GAP) is a new system for computational group theory, currently developed in Aachen. It addresses the problems that it takes so much effort to write new programs. It trys to support a programmer in the task of writing a program in three ways.

- A language specially designed with group theory in mind. This generally PASCAL like language provides as data types permutations, finite field element, vectors and matrices.
- A programming environment, which is an interpreter for the language GAP and provides the ability to examine bugs, fix them and restart a computation...
- A library containing already written functions, i.e. an a Schreier-Sims algorithm, polynomial functions...
  It is important to remark that all the algorithms are written in GAP and thus are available to the user who can easily modify them.

The system is distributed from Aachen without costs. Mark Schönert Aachen

Cayley Version 4

The group theory system Cayley includes a high level programming language that has been in use for a number of years. Experience thereby gained has led to the design of a new language. This language is designed around the standard algebraic notions of algebraic structure, set, sequence and mapping. Of particular interest is the use of a set constructor which enables the user to describe sets by listing predicates. The types of the language include algebraic and combinatorial domains (groups, rings, fields, modules, finite geometries, linear codes, designs and graphs).

John Cannon

# KONSTRUKTIVE ALGEBRAISCHE ZAHLENTHEORIE
(22. Mai – 28. Mai 1988)

## Zeros of the Riemann zeta function

A new algorithm, invented by A. Schönhage and the speaker, makes it possible to compute large sets of zeros of the zeta function much faster than was possible with the classical algorithms based on the Euler-Maclaurin and Riemann-Siegel formulas. Asymptotically, the new algorithm ought to make it possible to verify the Riemann Hypothesis in $n^{1+o(1)}$ operations, as opposed to $n^{2+o(1)}$ operations for the E-M method and $n^{\frac{3}{2}+o(1)}$ for the R-S method. The new algorithm was implemented recently and it turns out to be very fast in practice as well as in theory. It has computed almost 79 million zeros in the neighborhood of zero # $10^{20}$ as well as several other large sets of zeros. These zeros all satisfy the Riemann Hypothesis and provide evidence in favor of other conjectures that link the zeros to eigenvalues of random hermitian matrices.

Andrew Odlyzko
AT&T Bell Laboratories
Murray Hill, NJ, USA

E

Algorithms for computing class numbers of imaginary quadratic fields.

Let $d>0$, $d \equiv 3 \pmod 4$, and let $C(-d)$ denote the Gauss class group with cardinality $h(-d)$. Previously the best known algorithm for computing $h(-d)$ was due to Shanks, and uses $O(d^{1/5+\varepsilon})$ operations under the assumption of the extended Riemann hypothesis for $L(s,\chi)$, when $\chi(\cdot) = \left(\frac{-d}{\cdot}\right)$. A new probabilistic algorithm is described for computing $h(-d)$, whose expected running time is $O(L^c)$, where $L = \exp(\sqrt{\log d \log\log d})$. (A.K. Lenstra and C.P. Schnorr have suggested that $c = 1+o(1)$ should be possible). The idea of the algorithm is to combine an approximation to $h(-d)$ provided by Dirichlet's class number formula with a method for generating random relations on a set of generators for $C(-d)$. The generation of random relations is closely related to an integer factoring algorithm of M. Seysen. In addition, an algorithm for computing discrete logarithms in $C(-d)$ can be described, again with expected running time $O(L^c)$. Both algorithms have some significance for a cryptographic key distribution scheme proposed by J. Buchmann and H. Williams. In addition, it can be proved using these methods that the problem of computing $h(-d)$ and the structure of $C(-d)$ belongs to the complexity class NP. This answers a question posed by E. Bach, G. Miller, and J. Shallit.

Kevin McCurley

Los Angeles, USA

# Efficient, Perfect Random Number Generators.

joint work with S. Micali (M.I.T)

A random number generator is an efficient algorithm that transforms short random seeds into long pseudo-random strings. The concept of <u>perfect</u> random number generator has been introduced by Blum, Micali (1982) and A. Yao (1982). A random number generator is perfect if it passes all polynomial time statistical tests, i.e. the distribution of output sequences cannot be distinguished from the uniform distribution of sequences of the same length.

We extend and accelerate the RSA-generator in various ways. Let $N = p \cdot q$ be product of two large random primes $p$ and $q$ and let $d$ be a natural number that is relatively prime to $\varphi(N)$. We conjecture that the following distributions are indistinguishable by polynomial time statistical tests

- the distribution of $x^d \pmod N$ for random $x \in [1, N^{2/d}]$
- the uniform distribution on $[1, N]$.

This hypothesis is closely related to the security of the RSA-scheme. By the hypothesis we obtain an improved RSA-random number generator that is almost as efficient as the linear congruential generator.

We describe a method that transforms every perfect random number generator into one that can be accelerated by parallel evaluation. Using $m$ parallel processors we can speed the generation of pseudo-random bits by a factor $m$.

C. P. Schnorr
Universität Frankfurt

Generalization of Schoof's algorithm to Abelian varieties and applications

We describe a generalization to Abelian varieties over finite fields of Schoof's algorithm for elliptic curves. The algorithm computes the characteristic polynomial of the Frobenius endomorphism of the Abelian variety A over $\mathbb{F}_p$ in time $O_\Delta((\log p)^\Delta)$ where $\Delta$ depends only on the form of the equations defining A. The method, generalizing that of Schoof, is to use the machinery developed by Weil to prove the Riemann hypothesis for curves and Abelian varieties. As applications we show how to count the rational points on the reductions mod p of a fixed curve in time polynomial in log p, and we show that, for a fixed prime l, we can compute the l-th roots of unity mod p, when they exist, in time polynomial in log p.

Jonathan Pila
Stanford University.

Factoring into Sparse Polynomials

A new algorithm for factoring multivariate polynomials over a field of characteristic 0 is introduced. The algorithm takes as input an "oracle black box" that allows to evaluate the polynomial at an arbitrary point. By probing this box it returns a program that allows to evaluate the irreducible factors of the polynomial. The program fixes once and for all the enumeration and associates of these factors. It operates in a quadratic number of probes of the input box in terms of the total degree of the polynomial.

If one wants to obtain the sparse representation of one of the factors one can apply algorithms by Ben-Or & Tiwari or Zippel to the output program. We show how this scheme is useful to check conjectures on factorization properties of determinants of Moufang loop tables or how to factor the resultant of a system of some of polynomial equations. These examples constitute the largest polynomials in number of terms (≈ 300 000 000) factored by computer today.

Erich Kaltofen
Rensselaer Polytechnic Institute

Representation of one by binary cubic forms with positive discriminant.

We computed the solutions of the diophantine equations

$x^3 - cxy^2 + dy^3 = 1$ $\quad 0<c\le 30\,;\ 46\le c\le 50$

$x^3 + x^2y - cxy^2 + dy^3 = 1$ $\quad 0<c\le 20\,;\ c=50$

$x^3 - ax^2y - bxy^2 + y^3 = 1$ $\quad 1\le a\le 60\,;\ 0\le b\le a$

with $|y| \le 10^{41}$ under the condition that the discriminant $D_f$ of the polynomials are positive.

Summarizing the observations we conjecture the following connections between cubic forms $f(x,y)$ with $D_f > 0$ and the number of solutions $N_f$ of the diophantine equation

$$f(x,y) = 1.$$

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

| | $N_f$ | |
|---|---|---|
| $f$ is not equivalent to a reversible form | 0 | |
| | 1 | |
| | 2 | $f$ is equivalent to a reversible form |
| | 3 | |
| | 4 | |
| | 5 | |
| | 6 | $D_f = 81, 148, 257, 361$ |
| | 7 | none |
| | 8 | |
| | 9 | $D_f = 49$. |

Similar connection were proved by Delone (1930) and Nagell (1928) for cubic forms with negative discriminant

Attila Pethő
Kossuth Lajos University, Debrecen

On Computations over Finite fields

Denote by $F_k$ the field of $p^{p^k}$ elements, $p$ a prime and suppose $F_0 \subset F_1 \subset F_2 \subset \cdots$ Put $\tilde{F} = \cup F_k$. The transformation $S: x \to x^p - x$ maps $\tilde{F}$ onto itself. Let $W_m$ be the kernel of $S^m$. When $m = p^k$, $W_m = F_k$ and, in any case, $W_m$ is an $m$-dimensional linear space. We obtain analogues of the Fast Fourier Transform and related algorithms, with $W_m$ playing the role of the roots of unity in the ordinary FFT. The complexity of evaluating a polynomial of degree $N < p^m$ on $W_m$ is $N(\log N)^{1 + \log_p\left(\frac{p+1}{2}\right)}$.

David G. Cantor
UCLA – Los Angeles, CA

# Recognizing Primes in Random Polynomial Time

A random polynomial time algorithm for recognizing the set of primes is presented. The techniques used are from arithmetic algebraic geometry, algebraic number theory and analytic number theory. The proof of the efficiency of the algorithm involves the classification and counting of the curves of genus 2 and their Jacobians over finite fields. The notion of good Weil number is introduced. It is proved that (1) for any good Weil number $\pi$ for a prime $p$, there exists an $\mathbb{F}_p$-principally polarized Abelian variety $A$ associated with $\pi$, and with endomorphism ring $R = \mathbb{Z}[\pi, \bar{\pi}]$, (2) let

$\vartheta = \{ R\text{-ideal } I :$ $I$ is prime to $p$ and the conductor of $R$, and
$I\bar{I} = \alpha R$ for some real $\alpha \}$

$\forall\, I \in \vartheta$, $\exists$ $\mathbb{F}_p$-ppav $A_I$ with ring $R$ and $\mathbb{F}_p$ isogenous to $A$.
$\forall\, I, J \in \vartheta$, $A_I$ is $\mathbb{F}_p$-isomorphic to $A_J$ iff $I$ is $R$-isomorphic to $J$. It is proved that any 2-dim. $\mathbb{F}_p$-ppav associated with a good Weil number $\pi$ is the canonically polarized Jacobian of an $\mathbb{F}_p$-curve of genus 2. It then follows that the number of $\mathbb{F}_p$-isom classes of $\mathbb{F}_p$-curves of genus 2 whose Jacobian is assoc. with a good Weil number $\pi$ is at least the number of $R$-isom classes in $\vartheta$. It is then proved that the latter is at least $p^{1.5}/\log^c p$ for some constant $c$, for most good Weil numbers.

Leonard M. Adleman
Ming-Deh Huang
U. S. C.

## Necessary conditions for the existence of a relative power basis in Algebraic Number Fields.

We use local theory of integers to find N.C. to have $Z_E = Z_F[\theta]$ for the rings of integers in a galois extension $E/F$. The general observation is the following: If $H$ is any cyclic subgroup of $G$, then, for all generator $\sigma$ of $H$, the ideal $(\theta-\theta^{\sigma})Z_E = \mathfrak{D}_H$ is independent of $\sigma$ and is known by upper ramification theory (it is a part of a relative different); then the quotients $N_{E/F}\left(\frac{\theta-\theta^{\sigma}}{\theta-\theta^{\sigma'}}\right)$, $\sigma \neq \sigma'$ generating $H$, are units in $Z_F$ satisfying some strong conditions. We deduce many examples where there is no power basis: the less technical one is for instance: Let $E/\mathbb{Q}$ be abelian of degree $n$ prime to 2 and 3; there there exists only finitely many such extensions ($n$ fixed) with a power basis (M.-N. G.).

Marie-Nicole & Georges Gras
Besançon.

## SMALL DISCRIMINANTS FOR A GIVEN PERMUTATION GROUP.

Let $n$ be a positive integer. One asks to construct extensions $E/\mathbb{Q}$ with Galois closure $F$ such that (i) $Gal(F/\mathbb{Q})$ is isomorphic to a given transitive group of degree $n$, (ii) the permutation group $G$ is afforded by $E/\mathbb{Q}$, and (iii) the infinite Frobenius is a prescribed conjugacy class of order 1 or 2 in $G$. We recalled some results on permutation groups, including 2-dimensional invariants, then discussed various methods of construction (geometry of numbers, class field and Kummer theory, embedding problems), and at last gave examples for degrees $\leq 8$. In particular new results by A.-M. Bergé, M. Olivier and myself were given, which provide a table up to discriminant $5 \cdot 10^7$ of totally real quartic fields containing a quadratic field.

Jacques MARTINET
Bordeaux I

## SIMATH, ein Computer-Algebra-System

SIMATH, i.e. SImix MATHematics, is a computer algebra system developed at Saarbrücken on a Siemens PC MX-2.
We give the basic ideas of the system and an overview of the features of SIMATH:

* developed for applications in constructive number theory
* open System, the sources will be available
* higher level number theory algorithms
* written in "C"
* library of functions for use in "C"-programs
* dialogue system SIMCALC, i.e. SIMath CALCulator, for interactive problem solving.

In the near future SIMATH will be available also on other computers such as SUN, Appollo and VAX.

Markus M. Reichert
Saarbrücken

## SUR LA CONSTRUCTION EXPLICITE DES EXTENSIONS RELATIVES

On décrit une méthode générale qui permet de construire explicitement toutes les extensions relatives $k/k'$ où $k$ est un corps de nombres de degré et signature fixées et dont le discriminant est, en valeur absolue, plus petit qu'une constante donnée.
Cette méthode semble bien adaptée pour le calcul de tables de corps de nombres.

F. Diaz y Diaz
Orsay

A New Bound for the first Case of Fermat's Last Theorem

We present an improvement to Gunderson's function, which gives a bound for the exponent in a possible counterexample to the first case of Fermat's "Last Theorem" assuming that the generalized Wieferich criterion is valid for the first $n$ prime bases. The new function increases beyond $n = 29$, unlike Gunderson's. The first case of Fermat's "Last Theorem" has been proved for all exponents up to 156 442 236 847 241 729.

Samuel S Wagstaff, Jr
Purdue University

Counting points on elliptic curves over finite fields

In 1987 AOL Atkin devised a practical algorithm to count the number of points on an elliptic curve $Y^2 = X^3 + AX + B$ modulo a prime $p$. His algorithm is based on computations with the $l$-torsion points of the curve and on calculations on the modular curves $X_0(l)$ for small primes $l$. It seems that Atkin can count the points on elliptic curves over $\mathbb{F}_p$ where $p$ is a prime upto 50 decimal digits.

René Schoof

Heuristics on class groups of number fields.
(joint work with J. Martinet)

We generalise the C–Lenstra heuristics to arbitrary extensions $L/K_0$ of number fields, galois or not, and with arbitrary base field $K_0$. One consequence, which is surprising but in accordance with the tables is that quartic fields of type $S_4$ have a density strictly less than 1 ~~among~~ all quartics, contrary to what is believed to be true for $S_n$ in general.

Henri Cohen
Talence

Polylogarithms and Special Values of Zeta Functions

The dilogarithm function, defined for $|z|<1$ by $Li_2(z)=\sum_{n=1}^{\infty}\frac{z^n}{n^2}$ and for $z\notin[-1,\infty)$ by analytic continuation, has many surprising properties and often occurs in unexpected connections. For instance, it can be evaluated in closed form for 8 values of $z$, and one of these $\left(Li_2\left(\frac{3-\sqrt5}{2}\right)=\frac{\pi^2}{15}-\log^2\left(\frac{1+\sqrt5}{2}\right)\right)$ played a role in analyzing the bizarre claim by Ramanujan that the continued fractions $1-\cfrac{q^x}{1+\cfrac{q^2}{1-\cfrac{q^3x}{1+\cfrac{q^4}{1-\ddots}}}}$ and $\cfrac{q}{x+\cfrac{q^4}{x+\cfrac{q^8}{x+\cfrac{q^{12}}{x+\ddots}}}}$ $(0<q,x<1)$ are "nearly equal."

The modified function $D(z)=\mathrm{Im}\left[Li_2(z)+\log|z|\log(1-z)\right]$ (Bloch-Wigner function) extends (real-) analytically to all of $\mathbb{C}\smallsetminus\{0,1\}$ and has even nicer properties than $Li_2$.

<u>Theorem.</u> The value at $s=2$ of the Dedekind zeta-function of an arbitrary number field $F$ equals $\frac{\pi^{2(n-r_2)}}{\sqrt{|\mathrm{disc}(F)|}}$ times a rational linear combination of products $D(x^{(1)})\dots D(x^{(r_2)})$ with $x\in F$. (Here $n=[F:\mathbb{Q}]$ $=r_1+2r_2$ as usual and $x^{(1)},\overline{x^{(1)}},\dots,x^{(r_2)},\overline{x^{(r_2)}}$ are the images of $x$ under the non-real embeddings $F\hookrightarrow\mathbb{C}$.) For example,

$$\zeta_{\mathbb{Q}(\sqrt{-7})}(2) = \frac{4\pi^2}{21\sqrt{7}}\left[2D\left(\frac{1+\sqrt{-7}}{2}\right) + D\left(\frac{-1+\sqrt{-7}}{4}\right)\right].$$

The theorem is proved using either algebraic K-theory or (in a slightly weaker form) the interpretation of $D(z)$ as the volume of an ideal hyperbolic tetrahedron whose four vertices have cross-ratio $z$ and the relation of $\zeta_F(2)$ to the volume of a hyperbolic 3-manifold. We conjecture a similar formula for $\zeta_F(m)$ for all integers $m \geq 3$, with $D(z)$ replaced by the Ramakrishnan function $D_m$ (a modification of the polylogarithm $Li_m(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^m}$) and $r_2$ replaced by $r_1 + r_2$ if $m$ is odd. Thus we should have

$$\zeta_{\mathbb{Q}(\sqrt{5})}(3) = \frac{32}{25\sqrt{5}} D_3(1)\left[D_3\left(\frac{\sqrt{5}-1}{2}\right) - D_3\left(\frac{1-\sqrt{5}}{2}\right) + \tfrac{1}{3}D_3(2-\sqrt{5}) - \tfrac{1}{3}D_3(\sqrt{5}-2)\right]$$

with $D_3(x) = Li_3(x) - \log|x|\, Li_2(x) - \frac{1}{2}\log^2|x| \log(1-x) + \frac{1}{12}\log^3|x| \quad (0<x<1)$.

## Principal factors in pure cubic fields.

(joint work with K. D. Mayer)

Let $K = \mathbb{Q}(\sqrt[3]{D})$, an integral $\alpha \in K$ is called a principal factor (p.f.), if $\alpha$ is primitive and $(\alpha)$ consists only of totally ramified primes. Various congruence conditions which are necessary for the existence of a p.f. and the connection with class numbers are discussed. If $K$ has p.f. then at most 2 of its primitive principal factors are minima in the geometric sense. Criteria for a p.f. to be a minimum are given and a statistics for $D \leq 15000$ is presented.

F. Halter-Koch, Graz

## Some remarks concerning the computation of the class number of a real quadratic field

Let $D \in \mathbb{Z}^{>0}$ be square free and let $K = \mathbb{Q}(\sqrt{D})$ be the quadratic field formed by adjoining $\sqrt{D}$ to the rationals $\mathbb{Q}$. We describe several different methods of computing the class number $h$ of $K$. The best of these methods which are unconditional will determine $h$ in $O(D^{1/2+\epsilon})$ elementary operations, whereas the best known complexity result for computing $h$ conditional on the truth of the Riemann Hypothesis for $\zeta_K$ is $O(D^{1/5+\epsilon})$. We further discuss some large scale computations that have been carried out by utilizing these techniques and possible generalizations to arbitrary algebraic number fields. Finally, it is pointed out that under suitable Riemann Hypotheses the problem of evaluating $h$ and the regulator of $K$ is in complexity class NP.

Hugh Williams, Winnipeg

In my talk I presented the number theory package developped in Düsseldorf. There are more than 200 subroutines written in standard FORTRAN 77. The main algebraic topics implemented until now are: integral bases, algebraic integer arithmetic, ideal arithmetic units (independent and fundamental), norm equations and class groups.

N. Schulte Düsseldorf

Neue Resultate aus der konstruktiven Galoistheorie

Unter Verwendung der bekannten Rationalitätskriterien für Galoiserweiterungen (siehe z.B. L.N.M. 1284) konnten neuerdings die Gruppen $PSp_4(p)$ für $p \equiv \pm 2 \bmod 5$ $(p \neq 2)$ von R. Dentzer (Berlin), die Gruppen $PSU_3(p)$ für $p \equiv -1 \bmod 4$ von R. Nauheim (Karlsruhe) und G. Malle (Berlin), die Gruppen $F_4(p)$ für $p \equiv \pm 2, \pm 6 \bmod 13$ $(p \geq 19)$ von G. Malle sowie die sporadischen Gruppen $J_3, J_4, Mc, Ru, Ly$ von H. Pahlings (Aachen) als Galoisgruppen regulärer Körpererweiterungen über $\mathbb{Q}(t)$ nachgewiesen werden. Experimente mit den neuen Zopfbahnenkriterien führten ferner erstmalig zu Darstellungen der Gruppen $PSL_2(p^2)$ für $p=5$ und $p=7$ sowie der Mathieugruppe $M_{24}$ als Galoisgruppen regulärer Körpererweiterungen über $\mathbb{Q}(t)$.

B. H. Matzat (TU Berlin)

Improvements in Primality Testing.

Reporting on joint work with M.P. van der Hulst, it was shown how theoretical simplifications of the Cohen-Lenstra version of the primality test devised by Adleman, Pomerance and Rumely lead to practical improvements in the algorithm, currently being implemented in Berkeley/Amsterdam. In particular it is now possible to incorporate Lucas-Lehmer type tests (utilizing known factors of $n^u-1$, for small values of $u$ - as done by Williams) into the general purpose Jacobisum primality test. Other improvements include the possibility to work on smaller rings than in the Cohen-Lenstra version and the possibility of combining the necessary (but expensive) powerings of Jacobisums for several different characters.

Wieb Bosma (Berkeley/Amsterdam)

# An Overview of Computational Group Theory

The purpose of this talk was to introduce the audience to some of the algorithms that have been developed for computations with groups. Over the past twenty years quite powerful algorithms have been developed for working in most of the important branches of group theory: permutation groups, p-groups, soluble groups, matrix groups over finite fields, representation theory and cohomology of groups. The talk mainly concerned itself with examining work on permutation group algorithms. After introducing the important concepts of base and strong generating set (BSGS) it was observed that, generally, permutation group algorithms fall into three classes: (1) those that directly depend upon the ability to compute a BSGS (e.g. stabilizer of a sequence, normal closure); (2) those that utilize a backtrack search over base images (e.g. set stabilizer, centralizer) and (3) those that employ homomorphism methods (e.g. Sylow p-subgroup). It was observed that we now have techniques which enable us to obtain BSGSs for groups of degree up to 50,000. Finally attention was drawn to the important role played by probabilistic algorithms.

John Cannon

Some Polynomials Associated with Pollard's "Rho" Method

Define polynomials $f_i$, $i=0,1,\dots$ by $f_0 = x$, $f_i = f_{i-1}^2 + y$. We show that $f_i - f_j$ factors in $\mathbb{Z}[x,y]$ into absolutely irreducible polynomials. By associating a unique $p_{ij}$ (a factor of $f_i - f_j$) with each pair $i<j$ we find that for fixed $k$, $p \to \infty$

$$\Pr[\exists \text{ distinct } i,j < k \text{ with } f_i(x,y) \equiv f_j(x,y) \bmod p]$$
$$= \binom{k}{2}/p + O(1/p^{3/2})$$

when $x$ and $y$ are chosen at random from $\mathbb{Z}/p\mathbb{Z}$. If $p$ is the smallest prime divisor of a composite number $n$, then the heuristic assumption that $p_{ij} = 0$ is a "random curve" implies that the least $k$ for which $\exists i<k$ $\gcd(f_{2i+1} - f_i, n) \neq 1, n$ has expected value $\sqrt{\pi/2} \cdot \sqrt{p}$; this was found by Pollard using a different heuristic argument.

Eric Bach
Madison, WI USA

On the construction of large amicable numbers

The talk reports on the discovery of 526-digits pair of amicable numbers made by H. Wiethaus (Dortmund). The idea behind the construction is a new type of Thabit-rules (following W. Borho (1972)). It provides sufficient conditions for two numbers of the type

$$m_1 = g \cdot p^n \cdot \prod_1^k r_i \cdot (h_1 p^n - 1), \quad m_2 = g \cdot p^n \cdot c\,(h_2 \cdot p^n - 1)$$

to be amicable.

E. Becker (Dortmund)

# ALGEB – A COMPUTER ALGEBRA LANGUAGE

THE ALGEB LANGUAGE IS AN ALGOL DERIVATIVE, DESIGNED SPECIFICALLY TO FACILITATE THE EXPRESSION OF THE ZASSENHAUS ROUND 4 MAXIMAL ORDER ALGORITHM. IT IS GENERALLY APPLICABLE TO COMPUTATIONS IN ALGEBRA AND ALGEBRAIC NUMBER THEORY; IT IS PARTICULARLY WELL-SUITED FOR COMPUTING IN FINITE-DIMENSIONAL $\mathbb{Q}_p$-ALGEBRAS.
ALGEB HAS NOW HAD THREE IMPLEMENTATIONS:

1977: PDP-11
1986: VAX/VMS (NATIVE MODE; VIRTUAL MEMORY)
1988: IBM-PC

THE VAX AND IBM-PC VERSIONS ARE AVAILABLE AT NO COST FROM THE AUTHOR.

DAVID FORD
CONCORDIA UNIVERSITY
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Unramified extensions of fields containg many roots of unity.

This talk reported on the following theorem (and generalizations): Suppose $L = K(\zeta_n)$ and all the prime divisors of $n$ split completely in $K$. Then the ray class field of $K$ with conductor $n\cdot\infty_1\ldots\infty_s$ is an unramified extension of $K(\zeta_n)$

GARY CORNELL

## The PARI library

The PARI library, designed by ~~H. Cohen C. Batut, M. Olivier~~ C. Batut, H. Cohen and M. Olivier in Bordeaux, and D. Bernardi in Paris, is a package running on machines with a 68020 processor (presently SUN/3 and Macintosh II). It consists in I. Core (more than 6000 lines of assembly language) implementing the basic operations on unlimited integers and floating point numbers with arbitrary precision. II A library, written in C, which give access to these basic types: integers modulo another one, fractionnal numbers (reduced or not), p-adic, complex, quadratic numbers, polynomials, power series, vectors, matrices, polynomials modulo another, rational functions (reduced or not). The last types are recursive. A few fundamental arithmetic functions and many (real) transcendental ones are implemented. We plan to add more, and also p-adic transc. functions. One can use the library from a C or pascal program. One can also use a so-called "Super-Calculator" to use interactively the package.

Dominique Bernardi.

## Computing Galois groups

For details see page 132. Is $Gal_{\mathbb{Q}} f_n(x)$ the Sylow 2-group of the symmetric group $S_{2^n}$ where $f_0 = x$ $f_n = f_{n-1}^2 + 1$ ? True for $n \leq 5 \times 10^7$ (Cremona & Odoni, Exeter U.).

John McKay.

### Hecke operations on ternary quadratic forms:

The action of the Hecke algebra on forms of weight 2 on $\Gamma_0(N)$ has considerable importance, and has been computed in many ways — one can consider the algebra as acting on the modular forms, on the one-dimensional homology of $X_0(N) = \widehat{H/\Gamma_0(N)}$, or (in the manner of Oesterlé & Mestre) on the free module on the set of supersingular elliptic curves of characteristic $N$. This last representation is simple, natural, and very rapid to calculate — but of course only works when $N$ is prime. In the talk, it was suggested that we should consider a Hecke action on the free module on the set of reduced ternary quadratic forms of determinant $2N$; ~~this suffo~~ this seems to be essentially the same, but unfortunately it only gives the subspace not fixed by the involution $W_N$ (it may perhaps ~~more appropriately~~, better be viewed as equivalent to the action of the operators $T_{p^2}$ on forms of weight $3/2$).

B J Birch.

Congruent numbers and elliptic curves.

A one parameter family of elliptic curves each of positive rank and its application to the congruent number problem is discussed.

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Algorithms in algebraic number theory and their complexity

We discuss the algorithms for computing maximal order, unit- and class group of an algebraic number field F implemented in the Düsseldorf library for computational algebraic number theory.
We mention the Round 2 algorithm of Ford and Zassenhaus, its analysis by Hendrik Lenstra which shows that it is polynomial time if and only if largest square factors of integers can be computed in polynomial time.
We describe methods of Pohst, Zassenhaus and the author for unit computation and we mention that a system of units can be computed in time $O(RD^{\varepsilon})$, R being the regulator and D the discriminant of F. We also discuss the infrastructure idea of Shanks and its

generalization to arbitrary fields by the author. Finally we study the recent work of Lenstra, Pohst and the author on the analysis of the class group algorithm.

Johannes Buchmann

<u>On non commutative arithmetics I</u>

The constructive treatment of $\mathbb{Z}$-orders $\Lambda$ with simple central quotient algebra $A$ is discussed. The first task is the embedding of the commutative order $\Lambda_0 = C(A) \cap \Lambda$ into the maximal order $\mathfrak{O}(\mathbb{Z}, C(A))$ of the center $A_0 = C(A)$ of $A$. It suffices to deal with the case that $\Lambda_0 = \mathbb{Z}[t]/f(t)\mathbb{Z}[t] = \Lambda(f/\mathbb{Z}) = \sum_{i=0}^{n-1} \mathbb{Z}\xi^i$ $(\xi = t/f(t)\mathbb{Z}[t])$ is the equation order of the monic separable polynomial $f(t) = t^n + a_1 t^{n-1} + \dots + a_n$ over $\mathbb{Z}$. Round 5 of the maximal order programme is motivated by the desire suggested by the work of the Saarbrücken group (Zimmer, Böffgen et al.), D. Ford, Lenstra and Buchmann to postpone factorizations as long as possible. As a result the new core algorithm produces an overorder $\Lambda_1$ of $\Lambda_0$ that is pseudo-Eisenstein over separable. The maximal order is obtained after suitable factorizations and square testing of $d(f)$.

The second task is the establishment of an efficient calculus in $A$. The <u>new matrix calculus</u> (based on Galois theory and a crossed product construction) attaches to each element $a$ of $A$ (within given precision bounds) an el-t of $\mathbb{N}$ as denominator and $m$ rational integers or vectors as indices ($\dim A/C(A) = m^2$). It produces algorithms for finding the indices of $a \pm b$, $ab$ $(a, b \in A)$, needing $O(m^2)$ steps for each operation.

Hans Zassenhaus

# ORDERS AND THEIR APPLICATIONS

— in memory of Irving Reiner —

(29.5. – 4.6. 1988)

## Representation Rings

Given a finite group $G$ and an algebraically closed field $k$ one constructs a commutative ring $a(kG)$, whose additive group is generated by symbols $[V]$, one for each isomorphism class of (left, finitely-generated) $kG$-modules $V$; subject to defining relations $[V] = [V'] + [V'']$ whenever $V, V', V''$ are $kG$-modules such that $V \cong V' \oplus V''$. Multiplication in $a(kG)$ is given by $[V][V'] = [V \otimes V']$ ($V \otimes V' := V \otimes_k V'$, $G$ acts 'diagonally' $g(v \otimes v') = gv \otimes gv'$). One defines similarly a 'representation ring' $a(RG)$ formed of $RG$-lattices $V$, where $R$ is a suitable local coefficient domain.

Early in the study of these rings arose the question: does $a(kG)$ [or $a(RG)$] contain (non-trivial) nilpotent elements? When a Sylow $p$-subgroup $G_p$ of $G$ is cyclic ($p = \text{char.} k$) there are no such elements in $a(kG)$, but during 1965–1973 Irving Reiner & his pupils produced numerous examples of nilpotent elements in $a(RG)$ & $a(kG)$, for $G$ satisfying various conditions. From this work (particularly that of Reiner's student J. Zemanek) it is known that (non-zero) nilpotent elements exist in $a(kG)$ in following cases: (1) $p$ odd & $G_p$ not cyclic, and (2) $p$ even & $G_2$ has subgp $D_8$, $Q_8$, or $C_4 \times C_2$. The question seems to be still open in case ($p = 2$) $G = C_2 \times C_2 \times C_2$.

30.5.88

J. A. Green
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The Mathematics of Irving Reiner

The published works of Professor Reiner (1924-1986) appeared between 1943 and 1987; there were around one hundred including books and survey articles. We survey his work on number theory, integral representation theory, classical groups, algebraic K-theory and analytic noncommutative number theory.

William H. Gustafson
Lubbock, Texas

The Representation Ring of a Group of Prime Order

We compute the indecomposable $\mathbb{Z}G$-lattices, where $|G|$ is prime, using a method that avoids most matrix manipulations. The invariants determining the isomorphism class of a lattice are determined. From this, the structure of the representation ring $a(\mathbb{Z}G)$ is easily computed.

William H. Gustafson
Lubbock, Texas

Uniqueness of Presentations of Module

If $\Lambda$ is a ring and $f: P \twoheadrightarrow U$ a presentation of the finitely generated $\Lambda$-module by a finitely generated $\Lambda$-module $P$ (which is projective) we say $U$ is uniquely presented by $P$ if any other presentation $g: P \twoheadrightarrow U$ is equivalent to $f: P \twoheadrightarrow U$. As a first step in studying the collection of equivalence classes of presentations over orders, we consider the case $\Lambda$ a maximal order. We prove:

<u>Theorem</u> (Guralnick, Levy, 1988) If $f: P \twoheadrightarrow U$ is a presentation over a maximal order $\Lambda$ and either the (uniform) rank of ker is $\geq 2$ or $\Lambda$ satisfies the Eichler condition (if $\Lambda$ is a $\mathbb{Z}$-order), then P uniquely represents U.

Odenthal has extended this to hereditary orders.
This can be translated to a statement about equivalence classes of matrices and yields a solution to Nakayama's problem.

<u>Theorem</u> (Guralnick, Levy, Odenthal) If $\Lambda$ is a (noncommutative) PID, A, B $m \times n$ matrices over $\Lambda$, then for rank $A \geq 2$ A and B are equivalent $\Leftrightarrow$ coker $A \cong$ coker B.
If $\Lambda$ is a $\mathbb{Z}$-order, then the rank 2 condition can be dropped $\Leftrightarrow$ $\Lambda$ satisfies the Eichler condition.

Robert M. Guralnick
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## On orders and multiple pullbacks

To every semisimple order $\Lambda$ there is associated a canonical overorder $\tilde{\Lambda}$ which can be characterized as the unique minimal overorder of $\Lambda$ which is a multiple pullback. Some basic properties of $\tilde{\Lambda}$ are established; the question is treated when $\Lambda = \tilde{\Lambda}$.

Ernst Kleinert
Universität zu Köln

Group rings of p-groups over fields of characteristic p

In connection with the modular isomorphism problem, the following question is of interest: Let $V$ be a set of words, $G$ a finite p-group and $\Delta G$ the augmentation ideal of $\mathbb{F}_p G$. Under what conditions does $V(1+\Delta G) \cap G = V(G)$ hold? This question will be answered for various sets $V$ of words – among others those sets $V_i$, which determine the modular dimension subgroup series – by using the following Thm: For $\operatorname{gr} G := \bigoplus \mathfrak{n}_n(G)/\mathfrak{n}_{n+1}(G)$, $\mathfrak{n}_n(G)$ the n-th modular dimension subgroup of $G$ and analogously $\operatorname{gr}(1+\Delta G)$, one has $\operatorname{gr}(1+\Delta G) \cong \operatorname{gr} G \oplus C$, where $C$ is a Lie-p-algebraideal. This Thm. is of interest in its own right, since it shows that an object closely related to $G$ admits always a complement in an object close to the group of normalized units of $\mathbb{F}_p G$.

I. R. Roll

Tuscaloosa, Ala, USA

Brauer Invariance II

Four years ago the speaker (jointly with Sudarshan K. Sehgal, Surinder K. Sehgal) defined Brauer invariance of a finite group $G$ as the property that every automorphism of $\mathbb{Z}G$ can be composed from an automorphism of $G$ and an inner automorphism of $\mathbb{Q}G$. It must be distinguished from Higman invariance of $G$ demanding that $\mathbb{Z}G = \mathbb{Z}H$ implies $G \simeq H$. They reported on partial results about the Brauer invariance of solvable groups with all Sylow subgroups elementary abelian. Now it is proved that all groups $G$ with only abelian Sylow subgroups are Brauer-Higman invariant.

More generally, if $G = A \rtimes B$, $A$ is the abelian p-Sylow subgroup of $G$, $B$ is Brauer-Higman invariant then $G$ is Brauer-Higman invariant. After reductions that were previously explained, the proof is reduced to the task of showing that every automorphism $\alpha$ of the group ring $RG$ over $R = \frac{\mathbb{Z}}{\mathbb{Z} \setminus p\mathbb{Z}}$ that merely permutes the $G$-conjugacy class sums and that leaves $B$ elementwise fixed, can be modified by a suitable automorphism of $G$ over $B$ to an inner automorphism of $\mathbb{Z}G$.

A decisive tool is the "epidemics-lemma" stating that $\alpha$ is inner if $\alpha$ fixes the $G$-conjugacy class sums for the elements of a system of $G$-invariant direct components of $A$.

Hans Zassenhaus
Columbus, Ohio, USA

Some new invariants for blocks

Let $G$ be a finite group, $\mathbb{Z}_p$ the $p$-adic integers. In 1986, Scott asked whether the defect group $\delta(B)$ of a $p$-block $B$ of $\mathbb{Z}_pG$ is determined up to conjugation and "normalisation" by the block, independently of the group $G$; weakening this, Alperin asked whether at least the isomorphism type of $\delta(B)$ is determined by $B$.
Using some new cohomological invariants, we can give a contribution to this question even for more general coefficient rings $A$ such as complete discrete valuation rings with residue field of characteristic $p$, or even fields of characteristic $p$. For certain classes of $p$-groups $D$, including in particular abelian $p$-groups, the invariants for $AD$ and for blocks $B$ of $AG$ with $D$ as a defect group coincide. In the abelian case, the invariants for $AD$ determine the isomorphism type of $D$. Thus, if we know in advance that the defect group is abelian, we can determine its isomorphism type from the block.
The invariants are also used to give an improvement of Green's lower bound on the $p$-part in the rank of an $AG$-module.

Christine Bessenrodt - Timmerscheidt
Universität Essen

# Construction of units of integral group rings of finite nilpotent groups I, II

J. Ritter and S. K. Sehgal

In this series of two talks, a set of generators for $\mathcal{U}\mathbb{Z}G$, the group of units of the integral group ring $\mathbb{Z}G$ for nilpotent groups $G$, were presented upto a finite index. An outline of the proof was given. To state the theorem, some notation is required.

Write $|G| = n$, $\varphi(n) = m$. Let $a \in G$, $o(a) = d$. Choose $(i, d) = 1$. Then

$$u = (1+a+\cdots+a^{i-1})^m + \frac{1-i^m}{d}\hat{a}, \quad \hat{a} = 1+a+\cdots+a^{d-1}$$

is a unit as can be seen by projecting to the Wedderburn Components of $\mathbb{Q}\langle a\rangle$. The units above obtained by varying $a \in G$ and $i$ relatively prime to $d$, we shall call the <u>Bass Cyclic units</u> of $\mathbb{Z}G$. We denote by $B_1$ the group generated by them. A theorem of H. Bass says that if $G$ is abelian then $(\mathcal{U}\mathbb{Z}G : B_1) < \infty$.

The Bass cyclic units are not enough to generate $\mathcal{U}\mathbb{Z}G$, upto finite index, if $G$ is non abelian as can be seen by taking $G = S_3$. We introduce new units.

For $a, b \in G$, $u_{a,b} = 1 + (a-1)b\hat{a}$ is a unit with $u_{a,b}^{-1} = 1-(a-1)b\hat{a}$. We call the units $u_{a,b}$ obtained by varying $a, b \in G$, the <u>Bicyclic units</u> of $\mathbb{Z}G$ and denote by $B_2$ the group generated by them. Further, let $B = \langle B_1, B_2\rangle$. Then our result is

<u>Theorem</u> Let $G$ be a nilpotent group such that

$$\mathbb{Q}G = \sum^{\oplus}(K_i)_{n_i \times n_i}, \quad K_i \text{ fields}.$$

Further, suppose that if $n_i = 2$ then $K_i \neq \mathbb{Q}$ or $\mathbb{Q}(i)$. Then $(\mathcal{U}\mathbb{Z}G : B) < \infty$.

The proof uses the Congruence subgroup theorems of Bass-Milnor-Serre, Serre and Vasserstein. Clearly, the theorem applies to all nilpotent groups of odd order. The restriction on the Sylow 2-subgroup of $G$ is genuine as seen by the following:

<u>Example</u>. Let $G = \langle a^4 = 1 = b^4,\ a^b = a^{-1}\rangle$. Then

$$(\mathcal{U}\mathbb{Z}G : B) = \infty.$$

We must look for more units in this case.

J. Ritter
University of Augsburg

S K Sehgal
Edmonton/Alberta Canada

<u>On a conjecture of Zassenhaus on finite group rings</u>

Zassenhaus had conjectured that, whenever $\mathbb{Z}G = \mathbb{Z}H$ as rings with augmentation, for finite groups $G$ and $H$, then $G$ is conjugate to $H$ by a unit of $\mathbb{Q}G$. The author, in collaboration with Klaus Roggenkamp, has found many positive answers to this conjecture, including all $G$ with a normal $p$-subgroup containing its centralizer. However, we believe we have now found a counterexample to the general conjecture. The group in question has order $2^6 \cdot 3^2 \cdot 5$ and contains an abelian normal subgroup $C_2 \times C_2 \times C_3 \times C_3 \times C_5$ with quotient $C_2 \times C_2 \times C_2 \times C_2$. The central (class preserving) automorphisms of its quotients are plentiful and ill-behaved. This enables eventually the construction of unusual central automorphisms of quotient orders of $\mathbb{Z}_\pi G$ in the semilocal case ($\pi = \{2, 3, 5\}$). Passage to the global situation is facillitated by specifically looking at units giving rise to class group obstructions.

Details in the semilocal case have now been thoroughly checked, and it is anticipated to complete similar checking of the global case in the very near future.

Leonard Scott
Charlottesville

<u>Some Auslander orders of finite lattice type</u>

Let $R$ be a complete Dedekind domain, and let $\Lambda$ be a connected $R$-order of finite lattice type. Let $\mathcal{A}(\Lambda)$ be the Auslander order of $\Lambda$. If $\mathcal{A}(\Lambda)$ is again of finite lattice type, then denote by $\mathcal{A}^2(\Lambda)$ the Auslander order of $\mathcal{A}(\Lambda)$, etc. We give some answers to the following questions of M. Auslander:

Q.1: When exists $\mathcal{A}^i(\Lambda)$ for all $i \in \mathbb{N}$?

Q.2: When is $\mathcal{A}(\Lambda)$ again of finite lattice type?

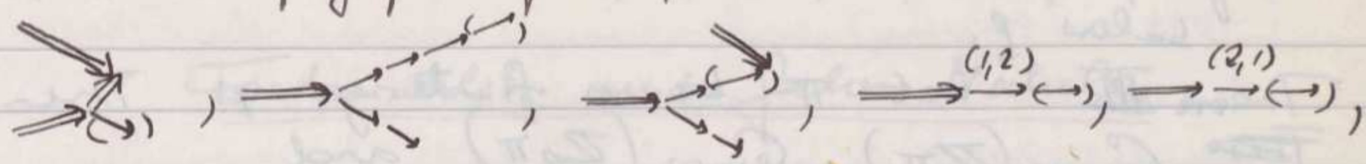
The fact that Q.1 has a positive answer for an artin algebra $A$ if and only if $A$ is semisimple gives the following answer to

Q. 1 essentially due to C. Munroe:

Theorem 1: For $\Lambda$ all $A^i(\Lambda)$ exist iff $\Lambda$ is a Bäckström order with associated graph $G(\Lambda)$ a disjoint union of Dynkin diagrams of types $A_2, A_3, B_2$ and $C_2$.

To Q. 2 we have answers in special situations:

Theorem 2: If $\Lambda$ is generalized Bäckström with associated graph $G(\Lambda)$, then $A(\Lambda)$ is of finite lattice type iff $G(\Lambda)$ is a disjoint union of graphs of the form:



where "$\Longrightarrow$" stands for "$\to \to \cdots \to$", and the arrow in brackets possibly can be omitted.

The knowledge of the Auslander Reiten quivers of the local orders of finite lattice type allows it also to answer Q. 2 for $\Lambda$ local.

Alfred Wiedemann

Stuttgart

Some finiteness results in the higher K-theory of orders and group-rings

Let $R$ be the ring of integers in a number field $F$, $\Lambda$ any $R$-order in a semi-simple algebra $\Sigma$. It is well-known that $K_0(\Lambda)$, $K_1(\Lambda)$, $G_0(\Lambda)$, $G_1(\Lambda)$ are finitely generated Abelian groups and that $SK_0(\Lambda)$, $SK_1(\Lambda)$, $SG_0(\Lambda)$, $SG_1(\Lambda)$ are finite groups. Questions about such finiteness results for higher K-groups have been open for some time. In this lecture, we answer these questions positively as follows:

Theorem I. For all $n \geq 1$:

(i) $K_n(\Lambda)$ is a finitely generated Abelian group

(ii) $SK_n(\Lambda)$ is a finite group.

(iii) If $\mathfrak{p}$ is a prime ideal of $R$, $\hat{\Lambda}_{\mathfrak{p}}$ the completion of $\Lambda$ at $\mathfrak{p}$, then $SK_1(\hat{\Lambda}_{\mathfrak{p}})$ is a finite group.

We also have similar results for $G_n$

Theorem II. ~~Let~~ $\forall n \geq 1$:

(i) $G_n(\Lambda)$ is a finitely generated Abelian group

(ii) $SG_{2n}(\Lambda) = SG_{2n}(\Lambda_{\mathfrak{p}}) = SG_{2n}(\hat{\Lambda}_{\mathfrak{p}}) = 0$.

$SG_{2n-1}(\Lambda)$ is finite, $SG_{2n-1}(\hat{\Lambda}_{\mathfrak{p}})$ $SG_{2n-1}(\Lambda_{\mathfrak{p}})$ are finite groups of order relatively prime to the prime $p$ lying below $\mathfrak{p}$.

Theorem III. Let $\pi$ be a finite group. Then $G_{4n+3}(\mathbb{Z}\pi)$, $G_{4n+3}(\mathbb{Z}_p\pi)$ and $K_{4n+3}(\mathbb{Z}\pi)$ are finite groups.

Remi Kuku

Ibadan

## Resolutions of periodic lattices.

Let $A$ be a periodic lattice over $\mathbb{Z}G$ ($G$ a finite group). This means $\mathrm{Ext}^{n+q}_{\mathbb{Z}G}(A,-)$ is naturally equivalent to $\mathrm{Ext}^{n}_{\mathbb{Z}G}(A,-)$ for all $n \geq 1$. The minimum such $q$ is the projective period of $A$.

1) $A$ has ~~proj~~ period $q$ iff $A$ has a projective resolution of period $q$; $A$ also has a periodic free resolution of period some multiple of the proj. period.

What is the relation between the free and projective periods?

2) Two minimal projective resolutions with the same rank sequences are in the same genus as augmented complexes over $A$.

This fails for minimal free resolutions and, moreover, a periodic $A$ need not have a periodic minimal free resolution. However, if $\mathbb{Z}G$ allows cancellation, all minimal free resolutions lie in one genus; and if $\mathbb{Z}G$ is not a summand of the periodic $A$, then $A$ has a periodic minimal free resolution.

3) For given $A$, we may define a sequence $\sigma_n(A)$ of invariants of $A$: these are elements in various factor groups of the projective class group of $\mathbb{Z}G$. They generalise the Swan obstruction for $\mathbb{Z}$ of projective period $q$: $\sigma_{q-1}(\mathbb{Z})=0$ iff $q$ is also a free period. We show that if $A$ has projective period $q$, then $q$ is essentially also a free period iff $\sigma_{q-1}(A)=0$.

Karl Gruenberg,
QMC, London.

## Factorisability and Galois Modules

$\Gamma$ a finite group, $A$ an Abelian group:

$B_\Gamma$ = Burnside ring, $R_\Gamma(\mathbb{Q})$ rational character ring.

A homomorphism $B_\Gamma \to A$ is factorisable if it factors through $B_\Gamma \to R_\Gamma(\mathbb{Q})$. Arithmetic invariants are factorisable maps, e.g. Dedekind Zeta.

This is used to gain information on Galois modules (additive, and multiplicative). It leads to a new equivalence relation $X \wedge Y$ on $\mathbb{Z}\Gamma$-lattices weaker than belonging to the same genus. Ex.

If $\Gamma = \mathrm{Gal}(N/K)$ (number fields), $\mathfrak{o}_N$, $\mathfrak{o}_K$ the rings of integers, then $\mathfrak{o}_N \wedge \mathfrak{o}_K\Gamma$.

A. Fröhlich

Cohen-Macaulay approximations.

Let $R$ be a complete local Cohen-Macaulay ring which is a finitely generated module over a regular local ring. Then for each $R$-module $C$ there exists a unique (up to isomorphism) exact sequence, called a Cohen Macaulay approximation of $C$, $0 \to Y_C \to X_C \to C \to 0$ having the following properties: a) $X_C$ is Cohen-Macaulay, $Y_C$ has finite injective dimension & $X_C$ has no indecomposable summands contained in $Y_C$. The lecture was mainly devoted to showing various consequences of the existence and properties of these sequences including a way of computing the multiplicity of hypersurfaces.
(Joint work with Buchweitz)

M. Auslander
Brandeis University

<u>Cohen Macaulayness of invariant modules</u>

If $R$ is a regular domain (char. zero) and $G$ is a reductive algebraic group acting on $R$ then the famous Hochster Roberts states that $R^G$ is Cohen Macaulay. However if $M$ is a free $M$ module with $G$ action then $M^G$ is not necessarily C-M. In the talk we give a criterion under which this holds. An application for $SL_2(k)$ is given. In particular we recover L. Lebruyn's result that the trace ring of generic 2x2 matrices is C.M.

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# Integral Group Rings of Groups of Square-free Order

For finite group $G$, the integral group ring $\mathbb{Z}G$ is of finite representation type if and only if $G$ is of cube-free order with cyclic Sylow subgroups. For such groups we seek a usable description of the indecomposable $\mathbb{Z}G$-lattices.

In the case where $G$ is of square-free order, we have such a description of the genera of indecomposable $\mathbb{Z}G$-lattices. As an example of how we might apply this description, we have the following theorem.

Theorem: If $G$ is of square-free order, then $\mathbb{Z}G$ has the property that every indecomposable left $\mathbb{Z}G$-lattice is isomorphic to a left ideal of $\mathbb{Z}G$ if and only if $G$ has one of the following forms:

(1) abelian
(2) dihedral
(3) $P \rtimes H$, where $|P|$ is prime and $H$ acts faithfully on $P$ by conjugation.

Lee Klingler
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## Some tame curve singularities

Let $C$ be an affine-algebraic complex curve, with singular point $0 \in C$. Let $\Lambda = \hat{\mathcal{O}}_{C,0}$ be the complete local ring of $(C,0)$, and denote by latt $\Lambda$ the category of $\Lambda$-lattices. We consider the problem of characterizing those curve singularities $\Lambda$ for which latt $\Lambda$ is tame, and the problem of ~~characterizing~~ giving a complete classification for latt $\Lambda$ in case $\Lambda$ is tame. These questions are answered for a special class $\mathfrak{C}$ of curve singularities, namely for those which have 4 branches and whose conductor contains the radical squared of the normalization.

Theorem. $\mathcal{C}$ consists of $16 + 1\cdot\infty$ analytical isomorphism classes of curve singularities. Among these, 10 are wild and $6 + 1\cdot\infty$ are tame. Among the tame ones, 3 are domestic, the infinite family is non-domestic of finite growth (tubular of tubular type $(2,2,2,2)$), and 3 have infinite growth.

Ernst Dieterich
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Zürich

Hall subgroups, isomorphic integral group rings and a question of R. Brauer

Let $G$ be a finite group, and $\mathbb{Z}G$ be its integral group ring.

Theorem 1. (jt. work with R. Sandling) $\mathbb{Z}G$ determines hamiltonian Hall subgroups of $G$ up to isomorphism.

Theorem 2. The character table of $G$ determines abelian Sylow-subgroups up to isomorphism.

The proof of both theorems is based on the earlier result that $\mathbb{Z}G$ determines the chief series of $G$ (joint paper with R. Lyons and R. Sandling). Theorem 2 answers an old question of R. Brauer (Reps of fin. groups, Lectures on modern math., Vol I., pp. 133-175, problem 12, 1963) positively. The chief series result holds even with respect to character tables. All results are proved making use of the classification of the finite simple groups. ∎

Wolfgang Kimmerle
Universität Stuttgart

Rationality problem for the $\mathbb{R}$-subspace problem.

We want to study rationality of the quotient varieties.

$$X_{\underline{\underline{a}}} = \prod_{i=1}^{r} \mathrm{Grass}(a_i, a_0)^{ss} / SL(a_0)$$

in the case that one has stable points.
A special case of a joint result with A.H. Schofield asserts that the functionfield is stably equivalent to the rational invariants of $n \times n$ matrices where $n = \gcd(a_0, a_1, \dots, a_r)$. In this way we obtain (1): $Br(\tilde{X}_{\underline{a}}) = 1$ ; (2) $X_{\underline{a}}$ is retract rational if $n$ is squarefree and (3) $X_{\underline{a}}$ is stably rational if $n \leq 4$.

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## On Stickelberger modules for group rings.

For a finite group $G$, a $\mathbb{Q}$-bilinear map $\langle , \rangle : \mathbb{Q}R_G \times \mathbb{Q}G \longrightarrow \mathbb{Q}$ (where $\mathbb{Q}R_G$ is the $\mathbb{Q}$-span of the virtual character ring $R_G$) is defined as follows. For $s \in G$ and a character $\chi$ of degree one, $\langle \chi, s \rangle$ is defined by $0 \leq \langle \chi, s \rangle < 1$ and $\chi(s) = e^{2\pi i \langle \chi, s \rangle}$. For an arbitrary character $\chi$, $\mathrm{res}^G_{\langle s \rangle} \chi$ is a sum of degree-one-characters of $\langle s \rangle$ and we put $\langle \chi, s \rangle = \langle \mathrm{res}^G_{\langle s \rangle} \chi, s \rangle$. A Stickelberger map $\Theta_G : R_G \longrightarrow Z(\mathbb{Q}G)$ (center of $\mathbb{Q}G$) is defined by $\Theta_G(\chi) = \sum_{s \in G} \langle \chi, s \rangle s$, and a Stickelberger module

$S_G = \mathbb{Z}G \cap \Theta_G(R_G)$. Among numerous relations between $S_G$ and the class group $Cl(\mathbb{Z}G)$, we can show $[c(\mathbb{Z}G)^- : S_G^-] = |Cl(\mathbb{Z}G)^-|$ when $G$ is abelian of type $(p^n, \dots, p^n)$ or non-abelian of order $p^3$ ($p$ an odd prime). (The "minus" parts are with respect to the canonical involution $s \longmapsto s^{-1}$ of $G$.)

Leon McCulloh
Urbana, Illinois

# Non-uniqueness of Presentations of Modules

(This is joint work with R. Guralnick, and is a continuation of the work described on p.p. 159-160)

~~the~~ Let $f: P \twoheadrightarrow U$ be a presentation of a $\Lambda$-module, where $\Lambda$ is either ~~a~~ a global order or the coordinate ring of an affine curve. What can be said when $\Lambda$ is not a maximal order? There can be many non-equivalent presentations of $U$ by $P$.

Theorem 1 If $\Lambda$ is commutative or satisfies an Eichler condition, then the set $\mathrm{pres}(P, U)$ becomes an abelian group if we define the sum $[s]$ of the equivalence classes $[g_1]$, $[g_2]$ of presentations $g_i: P \twoheadrightarrow U$ by $[s] = [g_1] + [g_2] \Leftrightarrow f \oplus s \sim g_1 \oplus g_2$ ($\sim$ meaning "equivalent to").

In the absence of the Eichler condition, a "stable" version of Theorem 1 holds.

Theorem 2 $(\exists\, n = n(\Lambda))$ ($\forall$ presentations $f: P \twoheadrightarrow U$ of $\Lambda$-modules) $|\mathrm{pres}(P, U)| < n$. [If $\Lambda$ is a global order.]

On the other hand, if $\Lambda$ is a geometric order, $\mathrm{pres}(P, U)$ can be infinite, but:

Theorem 3 If $\Lambda$ is commutative and $U$ has finite length, then $\mathrm{pres}(P, U)$ is a (possibly infinite) torsion group of finite exponent.

In other words, $(\exists\, n$ such that$)$ if $f, g: P \twoheadrightarrow U$, then $f^n \sim g^n$

Example 4 To show that the group $\mathrm{pres}(P, U)$ can be quite large, even when Theorem 1 forces it to be finite, we show: Let $G$ be a ~~p~~ group of prime order $p \geq 3$. Then the set of numbers that can occur as $|\mathrm{Pres}(P, U)|$, when $P$ is free and $U$ is finite, is $\{$all divisors of $(p-1)/2\}$.

Lawrence Levy

Madison, Wisconsin USA

## Torsion units in $\mathbb{Z}G$ via permutation lattices

Given finite groups $H, G$ the goal is to classify the group homomorphisms $\varphi: H \longrightarrow U_1(\mathbb{Z}G)$, with $U_1$ the augmentation 1 units, up to conjugation by units of $\mathbb{Z}G$. The 'double action' construction associates to $\varphi$ a lattice $M(\varphi)$ for the group $H \times G$, which classifies $\varphi$. In the spirit of integral representation theory it is then natural to place homomorphisms $\varphi', \varphi$ in the same genus precisely when they are conjugate by $p$-adic units for all primes $p$ and to emphasize the tentative

Genus Conjecture Every $\varphi$ has a group homomorphism $\sigma: H \to G$ in its genus.

This sharper version of a conjecture of Zassenhaus holds when $G$ is a $p$-group and yields very complete information in that case. More generally it is perhaps too optimistic but is nevertheless suggestive as a model for the goal.

Al Weiss (Edmonton, Alberta, Canada)

## Lattices with a condition on the exponent of $\hat{\mathrm{Ext}}^0_{RG}(M,M) = \underline{\mathrm{Hom}}(M,M)$

Let $R$ be a complete discrete valuation ring with maximal ideal $\pi R$, $G$ finite, $M, N$ $RG$-lattices. Let $\underline{\mathrm{Hom}}(M,N)$ be homomorphisms modulo projectives. For $\alpha \in \underline{\mathrm{Hom}}_{RG}(M,N)$ let $\exp\alpha = \pi^a$ if $\pi^a\alpha$ factors through a projective and $\pi^{a-1}\alpha$ does not. Let $\exp M = \exp I_M$.

For $M$ with $\exp M = \pi^a$ and almost split sequence $0 \to \Omega M \xrightarrow{\alpha} E \xrightarrow{\beta} M \to 0$, the following are equivalent:

1. $E = \Omega\left(\frac{M}{\pi^{a-1}M}\right)$, 2. $\mathrm{socle}\, \underline{\mathrm{Hom}}(M,M) = \pi^{a-1}\underline{\mathrm{Hom}}(M,M)$ 3. $\exp\alpha < \exp M$ 4. $\exp\beta < \exp M$ 5. $\exp E < \exp M$ 6. If $\gamma: N \to M$ is not a split epi then $\exp\gamma < \exp M$. 7. If $\gamma: M \to N$ is not a split mono then $\exp\gamma < \exp M$.

The condition is conserved under Green correspondence and undertaking sources, for absolutely indecomposable lattices. Jacques Thévenaz has shown that the absolutely indecomposable lattices that satisfy this condition are the Knörr lattices.

Joint work with Jon Carlson A. Jones

São Paulo.

Let $R$ be a Dedekind ring with quotient field $K$ and let $A$ be a central simple $K$-algebra. Let $\Lambda$ be an $R$-order in $A$ and let $S$ be a maximal commutative suborder of $\Lambda$ such that $KS = L$ is separable over $K$. Let $\Lambda_1 = \Lambda, \Lambda_2, \dots, \Lambda_t$ represent the isomorphism classes of orders in the genus of $\Lambda$. Then

$$\sum_{i=1}^{t} H(\Lambda_i)\, e_{\Lambda_i^*}(S,\Lambda_i) = h(S)\, e_{U(\Lambda)}(S,\Lambda)$$

where $H(\Lambda_i)$ is the two-sided class number of $\Lambda_i$, $h(S)$ is the class number of $S$, $e_{\Lambda_i^*}(S,\Lambda_i)$ is the number of equivalence classes of optimal embeddings $S \to \Lambda_i$ modulo the action of $\Lambda_i^*$, and $e_{U(\Lambda)}(S,\Lambda)$ is the number of local optimal embeddings $\varphi = (\varphi_{\underline{p}}) : \varphi_{\underline{p}} : S_{\underline{p}} \to \Lambda_{\underline{p}}$, $\underline{p} \in \operatorname{Spec} R$, modulo the action of $U(\Lambda) = \prod \Lambda_{\underline{p}}^*$ (the actions by conjugation). The above formula generalizes the result of Eichler in case of quaternion algebras and hereditary orders (and Vigneras for Eichler orders). It is a special case of a combinatorial result on transitive actions of groups on pairs of sets and relations invariant with respect to these actions. A special case is also a similar result for lattices with tensor structure.

Julius Brzezinski, Göteborg, Sweden.

Let $D_\infty$ be the infinite dihedral group, $U_1(RD_\infty)$ – the group of normalized units of the group ring with coefficients in $R$.

We prove:

1) if $H$ is a finite subgroup in $U_1(\mathbb{Z}D_\infty)$ then $H \simeq C_2$

2) $U_1(\mathbb{F}_2 D_\infty) \cong K \rtimes D_\infty$, $K \cong \mathop{*}_{j\in\mathbb{Z}} \bigoplus_{i\in\mathbb{N}} C_2$.

3) The groups $U_1(\mathbb{F}_2 D_\infty)$ and $U_1(\mathbb{Z}D_\infty)$ are not finitely generated.

Z. Marciniak, Warsaw, Poland.

## On the Number of Solutions of $x^{p^k}=a$ in a $p$-group

This talk concerns a small application of the representation theory of groups to the enumeration theory in $p$-groups. For $p$ a prime divisor of $|G|$ ($G$ a finite group), and an element $a \in G$, let $N_a = \#\{x \in G : x^p = a\}$. The following two theorems are classical: (A) If $G$ is a $p$-group, $G \neq$ cyclic and $p > 2$, then $p^2 \mid N_1$; (B) If $G$ is a $p$-group, $G \neq$ metacyclic and $p > 3$, then $p^3 \mid N_1$. Here, (A) is due to Miller-Blichfeldt-Dickson (1919) and Kulakoff (1931), while (B) is due to Huppert and Berkovich (1967).

In this talk, a representation theoretic proof is offered for the following theorem: <u>Let $G$ be a finite group, and $H \triangleleft G$ be an elementary abelian group of order $p^r$. Suppose $a \in G$ is a $p$-element in the quotient group $G/C_G(H)$. Then $\#\{x \in G : x^{p^k} = a\}$ is divisible by</u> $p^{\min\{r,\, p^k - 1\}}$. (Here $k$ is a fixed integer $\geq 1$.)

The proof of this theorem relies on the knowledge of the indecomposable representations of cyclic $p$-groups over a field of characteristic $p$. By applying the theorem to $p$-groups, one gets the following refinements of the classical results stated in the first paragraph above: (A′) If $G$ is as in (A) above, then $p^2 \mid N_a$ $\forall\, a \in G$, and (B′) If $G$ is as in (B) above, then $p^3 \mid N_a$ $\forall\, a \in G$.

Tsit-Yuen Lam
T. Y. Lam
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## Orders of finite global dimension

Let $D$ be a DVR and let $\Lambda$ be an order in $M_n(D)$, $\mathrm{gldim}\,\Lambda < \infty$. This talk is concerned with the problem of finding an upper bound on $\mathrm{gldim}\,\Lambda$. A survey of what is known about the problem is given. The diagrammatic techniques of Wiedemann and Roggenkamp are discussed. Related Artin algebras $\Lambda/I$ and orders $e\Lambda e$ of finite global dimension are described.

Ellen Kirkman
Wake Forest University
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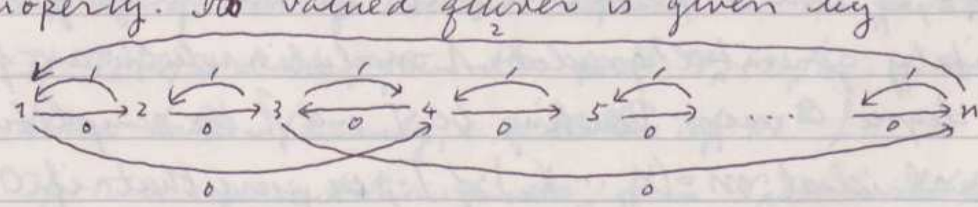
## Tiled orders of finite global dimension

We introduce a projective link between maximal ideals of an arbitrary ring with identity, with respect to which an idealizer preserves being of finite global dimension. Let $D$ be a local Dedekind domain with the quotient ring $K$. When $2 \leq n \leq 5$, every tiled $D$-order of finite global dimension in $(K)_n$ is obtained by iterating the idealizers w.r.t. projective links from a hereditary order. If $n \geq 6$ then there exists a tiled $D$-order in $(K)_n$ which does not have the above property. Its valued quiver is given by



$(n \geq 6)$.

This is also a counterexample to Tarsy's conjecture. Using the above result, a list of the representatives of isomorphism classes of tiled $D$-orders of finite global dimension in $(K)_n$ is obtained where $n = 4, 5$.

Hisaaki Fujita
University of Tsukuba
Tsukuba-Shi Ibaraki 305 Japan

Tame and wild generalized Bäckström orders and socle categories:

We describe the class of generalized Bäckström orders by using their closed relation to hereditary algebras. The basic tool is a representation equivalence found by Ringel and Roggenkamp 1979 ff: for each gen. B-order (defined by: $\exists$ hereditary order $\Gamma$ with $\mathrm{rad}\,\Gamma \subset \Lambda \subset \Gamma$ and the radical of each projective $\Lambda$-lattice has as direct summands only $\Gamma$-lattices or projective $\Lambda$-lattices) the category of lattices is representation equivalent to the category of f.g. modules with projective socle over some hereditary artinian algebra.
For these orders we give a classification theorem (finite, tame or wild repr.-type depends on a graph assigned to such an order), also we give a complete description of the Auslander-Reiten quivers in the tame case and some structure properties of the Auslander-Reiten-quiver in the wild case; all this is done by considering socle categories (= categories of socle proj. modules over hereditary algebras) and then using the results of Ringel and Roggenkamp.
Our main tool for describing the Auslander-Reiten-quivers ~~at~~ is the graph-theoretical notion of reducibility which we characterize by the existence of a preprojective but not projective simple module over the hereditary algebra.

Steffen König

Universität Stuttgart, D-7000 Stuttgart

Graded rings and their completions

Let $k$ be a field and $T = k[X_1, \cdots, X_n]$ be $\mathbb{Z}$-graded ring with $\deg X_i > 0$, and $j: T \to \Lambda$ a $\mathbb{Z}$-graded $T$-algebra, such that $\Lambda$ is a finitely generated free $T$-module and $\mathrm{gl.dim}\,\Lambda_p = \dim T_p$ when $p$ is a prime ideal in $T$ which is not maximal. $\Lambda$ is said to be of finite representation type if there is only a finite number, up to shift, of indecomposable objects in the category $CM(\mathrm{gr}\,\Lambda)_0$ of finitely generated $\mathbb{Z}$-graded $\Lambda$-modules which are free $T$-modules, with degree 0 maps. Denoting be $\hat{T}$ and $\hat{\Lambda}$ the completions at the maximal ideal $m = (X_1, \cdots, X_n)$ of $T$, we prove that if $0 \to A \to B \to C \to 0$ is an almost split sequence in $CM(\mathrm{gr}\,\Lambda)_0$, then $0 \to \hat{A} \to \hat{B} \to \hat{C} \to 0$ is almost split in $CM(\hat{\Lambda})$. ~~And~~ $\Lambda$ is of finite representation type if and only if $\hat{\Lambda}$ is.

If the group $\mathbb{Z}$ is replaced by an abelian group $G$ (such that $T_0 = k$ and each $T_i$ is finite dimensional), we can only conclude that $0 \to \hat{A} \to \hat{B} \to \hat{C} \to 0$ is a direct sum of almost split sequences (unless $G$ is torsionfree). But the result on finite representation type remains true.

This talk was based on joint work with M. Auslander, and for the last generalization we profited from conversations with M. Van den Bergh during the conference.

Idun Reiten, University of Trondheim, AVH

## A Stability Theorem for Representation-finite Orders

Let $\Lambda$ be an order over a complete d.v.r. $R$ with quotient field $K$, $A = K\Lambda$, and $S$ a simple $A$-module. The set $\mathcal{V}_\Lambda(S)$ of $\Lambda$-representations $I$ with $KI = S$ is a lattice w.r.t. $+$ and $\cap$ on which the unit group $G = D^*$ of the skewfield $D = \mathrm{End}_A S$ operates from the right-hand side. If $\Delta$ is the (unique) maximal order in $D$, we have an exact sequence $G_0 \rightarrowtail G \xrightarrow{v} \mathbb{Z}$, where $G_0 = \Delta^*$, $G = D^*$, and $v$ is the discrete valuation on $D$. If $\Gamma$ is a minimal hereditary overorder of $\Lambda$, define for $i \in \mathbb{Z}$, $I \in \mathcal{V}_\Lambda(S)$,

$$\chi_i^\Gamma(I) := l_\Lambda\left(\left((I \cap J_i) + J_{i+1}\right)/J_{i+1}\right) \in \mathbb{N},$$

where $\mathcal{V}_\Gamma = \{ \ldots \supsetneqq J_i \supsetneqq J_{i+1} \supsetneqq \ldots \}$. Then we have a monotonous map

$$\chi^\Gamma : \mathcal{V}_\Lambda(S) \longrightarrow \mathbb{N}^{\mathbb{Z}}$$

which is constant on the $G_0$-orbits of $\mathcal{V}_\Lambda(S)$. We call $\mathcal{V}_\Lambda(S)$ <u>stable</u> if the implication $\chi^\Gamma(I) = \chi^\Gamma(I') \implies I, I'$ belong to the same $G_0$-orbit holds for arbitrary $I, I' \in \mathcal{V}_\Lambda(S)$.

Thm. 1: Definition of stability does not depend on the particular choice of $\Gamma$

Thm. 2: If $\mathcal{V}_\Lambda(S)$ is stable, then the poset $\mathcal{V}_\Lambda(S)/G_0$ of $G_0$-orbits is a distributive lattice (namely, a sublattice of $\mathbb{N}^{\mathbb{Z}}$)

Thm. 3: If $\Lambda$ is representation-finite, then $\mathcal{V}_\Lambda(S)$ is stable for any simple $A$-module $S$.

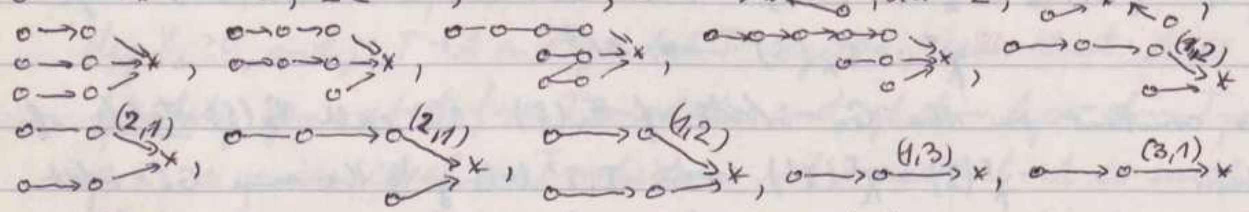
For $\Lambda = D$ (skewfield) the converse of Thm. 3 holds.

Wolfgang Rump, Eichstätt

# Right peak rings, valued posets and indecomposable socle projective modules

A semiperfect ring $R$ with $1$ is a right peak ring if $\mathrm{soc}(R_R)$ is essential in $R$ and it is of the form $\mathrm{soc}(R_R) \simeq P_*^t$, $t<\infty$, where $P_*$ is indecomposable projective module. Our main interest is to classify indecomposable f.g. socle projective right $R$-modules. We call $R$ sp-representation-finite if the category $\mathrm{mod}_{sp}(R)$ of finitely generated socle projective right $R$-modules has only finitely many isoclasses of indecomposables. To any right peak ring $R$ one can associate an order $\Lambda$ in a simple algebra $C$ and a representation equivalence $\mathrm{latt}(\Lambda) \to \mathrm{mod}_{sp}(R)/[P_*]$.

If $R$ is artinian, schurian (i.e. $eRe$ is a division ring for any primitive idempotent $e\in R$) PI-ring we associate a valued poset $(I_R^*, d)$. We prove that an indecomposable artinian, schurian right peak PI-ring $R$ is sp-representation-finite iff $(I_R^*, d)$ does not contain the following peak subposets: $\circ \xrightarrow{(d,d')} *$, $dd' \geq 3$, $\circ \xrightarrow{(d,d')} * \xleftarrow{(e,e')} \circ$, $2 \leq dd'$, $ee' \leq 3$, $\begin{tikzcd}[row sep=tiny] & & \circ \arrow[dl] \\ \circ \arrow[r, "{(d,d')}"] & * & \\ & & \circ \arrow[ul] \end{tikzcd}$, $dd'=2$, $\begin{tikzcd}[row sep=tiny] \circ \arrow[dr] & & \circ \arrow[dl] \\ & * & \\ \circ \arrow[ur] & & \circ \arrow[ul] \end{tikzcd}$,



Moreover a list of 30 right peak rings $R_i$, $i \leq 30$, and a list of 82 indecomposable socle projective $R_i$-modules $X_j$, $j \leq 82$ such that if $R$ is schurian sp-representation-finite and $X$ is indecomposable in $\mathrm{mod}_{sp}(R)$ then there are $i, j$ and a tensor type-functor $T: \mathrm{mod}_{sp}(R_i) \to \mathrm{mod}_{sp}(R)$ such that $X \simeq T(X_j)$, $X_j \in \mathrm{mod}_{sp}(R_i)$.

Daniel Simson
Nicholas Copernicus University
Toruń, Poland

## The isomorphism problem: Defect groups, the $Z^*$ theorem, and philosophical remarks

This second talk at this conference is also joint work with Klaus Roggenkamp. I discussed briefly the ingredients of our theorem for finite groups $G$ with a normal $p$-subgroup containing its centralizer. This theorem asserts that, if $\mathbb{Z}G \cong \mathbb{Z}H$ as augmented $\mathbb{Z}$-algebras, $H$ a second finite group, then $H$ is conjugate to $G$ by a unit of $\mathbb{Z}_pG$. This gives a positive answer to the Zassenhaus conjecture and the isomorphism problem in this case. (The Zassenhaus conjecture appears to be false in general; see previous talk)

The ingredients of the proof include a Green correspondence theory for automorphisms of blocks stabilizing a defect group, a study of Coleman's theory of normalizers in ring unit groups of $p$-subgroups of $G$, and Weiss's new results on permutation modules for $p$-groups.

I also discussed an application of these permutation module methods to give a positive answer to a version of the conjugacy problem I posed, (cf. C. Bessenrodt's talk at this conference) for defect groups (coming from different groups) in blocks, in the case of cyclic T.I. set Sylow $p$-subgroups and the principal block. I assumed the defect groups were $D$, $\alpha(D)$ for $\alpha$ an augmentation preserving automorphism of the principal block, though this hypothesis may be removable.

I mentioned that the general defect group conjugacy problem, but just for the case of the principal block, implies the $Z^*$ theorem (finite group theory), through a reduction of G. Robinson ($p>3$).

There was also time for some brief philosophical remarks about the pathway provided by the group ring and its associated unit groups between the theory of arithmetic groups and modular representation theory, as well as perhaps other aspects of finite group theory.

Leonard Scott
The University of Virginia
Charlottesville

Critical simply connected algebras with proj. socle.

$k$ an alg. closed field, and $S$ a fin. dim. $k$-algebra with proj. socle. All such algebras occur in the representation-theory of lattices over $R$-orders ($R = k[[X]]$ p.e). Denote the cat. of socle proj. $S$-modules by $\mathcal{F}(A)$. We should decide wether one given algebra $S$ is of finite or infinite repr. type with respect to the cat. $\mathcal{F}(S)$. Ringel and Roggenkamp defined reduction of $S$ which does not change the repr. type with respect of $\mathcal{F}$ for hereditary $S$. For these algebras $S$ is $\mathcal{F}$repr. finite if $S$ can not be further reduced and the quiver of $S$ is a Dynkin diagramm. For simply connected algebras we define strong-reduction of $S$ by removing Objects $x$ in the corresponding categorie $\mathcal{S}$ which are maximal or minimal in $\mathcal{S} \setminus \operatorname{Soc} \mathcal{S}$ or which are minimal in $S$ and $\dim_k (\mathcal{S}(x,-)) = 2$

For simply connected s.p. algebras we have a list of ca 300 $\mathcal{F}$-critical $k$-algebras, such that a given s.c. sp algebra $S$ is of $\mathcal{F}$-infinite type iff it can be strong-reduced to one of them or to an Algebra with extended Dynkindiagramm $\tilde{A}_n$ as quiver.

Thomas Weichert
Universität Stuttgart.

# ALGEBRAISCHE K-THEORIE

5. 6. — 11. 6. 1988

## On $K_2(O_F)$

Since a long time, we have known the structure of the K-groups $K_1(O_F)$ for rings $O_F$ of integers of every number field $F$ (Dirichlet). Since recently, we also know the structure of the K-groups $K_3(O_F)$ for every number field $F$ (Merkurjev, Suslin). As of today, the information about the structure of the finite abelian K-groups $K_2(O_F)$ is still limited.

We proposed the study of the structure of $K_2(O_F)$ modulo the knowledge of the structure of related S-class groups, and exhibited 4-rank formulas for $K_2(O_F)$. This led to a characterization of all number fields $F$ with a wild kernel (Hilbert kernel) of odd order, and the determination of infinite families of number fields $F$ for which the structure of the 2-primary subgroup of $K_2(O_F)$ can be determined.

Jürgen Hurrelbrink, LSU

## Hilbert's Satz 90 in Milnor K-Theory

For a quadratic extension $L = F(\sqrt{a})$, Char $F \neq 2$, Hilbert's Satz 90 states that the following sequence is exact:

$$K_n^M L \xrightarrow{1-\sigma} K_n^M L \xrightarrow{N_{L|F}} K_n^M F$$

Here $K_n^M$ denotes Milnor K-Theory and $\sigma$ is the generator of $Gal(L|F)$. Hilbert's Satz 90 for quadratic extension is proved for $n \leq 4$. The method of proof is to use specialization arguments relating Hilbert's Satz 90 to certain homology groups of the localization sequence in Milnor K-Theory for quadrics defined by Pfister forms. In computing these groups one is led to consider the complex

$$\bigoplus_{v \in X_{(1)}} K_{n+1}^M k(v) \xrightarrow{d} \bigoplus_{v \in X_{(0)}} K_n^M k(v) \xrightarrow{N} K_n F$$

for (projective) quadrics $X$ (where $d$ is given by the tame symbol and $N = \sum_v N_{k(v)|F}$). The exactness of this complex is proved for $n \leq 1$ if the form defining $X$ is of type $\varphi \oplus c\varphi' \oplus \langle d \rangle$, where $\varphi = \varphi' \oplus \varphi''$ is a Pfister-form and for $n = 2$, $\dim X \leq 2$ (which leads to a proof for Hilbert's Satz 90 for $n \leq 3$, $n = 4$ respectively) and for $n = 3$, $\dim X = 1$.

Markus Rost, Regensburg

# THE STRUCTURE OF CLASSICAL GROUPS BELOW THE STABLE RANGE AND NONABELIAN K-THEORY

Let A denote an associative ring which is finite over a commutative ring with 1. Let $G_n(A)$, $n \geq 3$, denote a classical group over A, i.e. either $G_n(A) = GL_n(A)$ or $G_n(A)$ is the automorphism group of a nonsingular form of Witt index $\geq n$. Let $E_n(A)$ denote the elementary subgroup of $G_n(A)$. Algebraic K-theory treats the groups $G(A) = \varinjlim_n G_n(A)$ and via stability theory, one can apply K-theory to obtain information about certain subquotients of $G_n(A)$, for example $G_n(A)/E_n(A)$, providing $n > sr(A) =$ stable range of A. Until recently, almost nothing was known about $G_n(A)/E_n(A)$ when $n \leq sr(A)$, one reason being that there is no K-theory for these groups. The following results close these gaps.

THEOREM A. There is a filtration $G_n^{-1} = G_n \supset G_n^0 \supset \cdots G_n^i \supset \cdots E_n$, functorial in A, satisfying:

(1) $G_n^i(A) \trianglelefteq G_n(A)$.

(2) If A is commutative and $G_n = GL_n$ then $G_n^0(A) = SL_n(A)$.

(3) $G_n^{-1}(A) / G_n^0(A)$ is abelian.

(4) $G_n^0(A) \supset G_n^1(A) \supset \cdots \supset G_n^i(A) \supset \cdots$ is a descending central series.

THEOREM B. If $sr(A)$ is finite then $G_n^i(A) = E_n(A)$ whenever $i \geq sr(A)$.

Theorem B says that $G_n^0(A)/E_n(A)$ is nilpotent of class $\leq sr(A)$. This result can be improved to the following: If $z \in \mathbb{Z}$, let $[z] = z$ if $z \geq 0$, and $0$ if $z \leq 0$.

THEOREM C. If $sr(A)$ is finite then $G_n^0(A)/E_n(A)$ is nilpotent of class $\leq 1 + [sr(A) + 2 - n]$.

The results above are proved by introducing 'nonabelian K-theory'. For each functor $G_n^i$ above an algebraic K-theory with K-theory groups $K_j G_n^i$ $(j \geq 1)$ is defined such that $K_1 G_n^i(A) = G_n^i(A)/E_n(A)$.

Whereas, $K_j$ for $j \geq 2$ is always abelian, $K_1$ is not necessarily abelian, hence the rubric 'nonabelian K-theory.' The main theorems are deduced with the help of certain exact Mayer-Vietoris sequences for the K-theory above, in particular the M.-V. sequence associated to a localization - completion square.

Anthony Bak, Bielefeld

## Structure of gauge groups

Let $G = G(\mathbb{R})$ be a simple Lie group. E. Cartan and van der Waerden proved that $G(\mathbb{R})^0/\text{center}$ is simple as abstract group. Let $A$ be a ring of continious functions $X \to \mathbb{R}$ on a topological space $X$. Assume that $A \supset \mathbb{R}$ and $GL_1 A$ is open in $A$. We define $G(A)$ as a subgroup in the group of continious maps $X \to G$. When $X = S^1$, these groups are known as loop group. In general, they appear in mathematical physics as gauge groups. Assume that $G$ is of classial type or splits (e.g. $G$ is compex) (this condition probably is not necessary) and that there are $N$ roots of 1 for elements of $A$ or $A[i]$ close to 1 (where $N$ is a certain number depending on $G$). Then a subgroup $H$ of $G(A)$ is normalized by $G(A)^0$ iff $G(B)^0 \subset H \subset G(B)$ for an ideal $B$ of $A$.

When $X = \{\text{point}\}$, this is the Cartan-van-der Waerden result. When $X = S^1$, the maximal normal subgroups of $G(A)^0$ were described by de la Harpe and (for some $G$) Segal-Prestly (~~the correspo~~ they are $G(B)$ with maximal ideals $B$ of $A$).

L. VASERSTEIN
Penn State University.

## Traces and Fixed Points

The main point of the talk was to give a particular description of Dennis' trace map from the K-theory $K(A)$ of a ring $A$ to the Hochschild homology $H(A)$. The description is as follows:

Define $K(A)$ by the Waldhausen method, so $K(A) = \Omega|BiS.\mathcal{C}|$ where

$\mathcal{C}$ = category of $A$-modules (finitely generated proj.)

$S_k\mathcal{C}$ = category of filtered objects in $\mathcal{C}$
$$0 = P_0 \subset P_1 \subset \cdots \subset P_k = P$$

$iS_k\mathcal{C}$ = category with these same objects, but only isomorphisms.

$B$ = nerve

Then Dennis' map can be described as the composition

$$(*)\qquad \Omega|BiS.\mathcal{C}| \xrightarrow[\alpha]{} \Omega|\Lambda iS.\mathcal{C}| \xrightarrow[\beta]{} \Omega|\Lambda S.\mathcal{C}| \xrightarrow[\gamma]{} \Omega|HS.\mathcal{C}| \xleftarrow{\ \simeq\ } H(A)$$

Here $\Lambda$ is "cyclic nerve" (whereas a $p$-simplex of $B\mathcal{C}$ is a diagram $P_0 \xrightarrow{f_0} \cdots \xrightarrow{f_{p-1}} P_p$ in $\mathcal{C}$, a $p$-simplex of $\Lambda\mathcal{C}$ is a diagram

$$P_0 \xrightarrow{f_0} P_1 \xrightarrow{f_1} \cdots \qquad P_p \xrightarrow{f_p} P_0$$

.)

The map $\alpha$ ~~extends~~ ~~is~~ is the based on the fact that $B\mathcal{C} \hookrightarrow \Lambda\mathcal{C}$ when every arrow in $\mathcal{C}$ is invertible

$$\underset{\substack{\cap \\ B}}{(f_0, \dots f_{p-1})} \longmapsto \underset{\substack{\cap \\ \Lambda}}{(f_0, \dots f_p)}\,, \qquad f_p \circ f_{p-1} \circ \dots \circ f_0 = 1$$

The map $\beta$ forgets the requirement that maps are invertible. The map $j$ takes products of Hom-sets to tensor products of Hom-groups. Its target is defined like its source except that in the forming cyclic nerves a $p$-simplex is an element of

$$\bigoplus_{P_0, \dots P_p} \mathrm{Hom}(P_0, P_1) \otimes \dots \otimes \mathrm{Hom}(P_{p-1}, P_p) \otimes \mathrm{Hom}(P_p, P_0)$$

rather than

$$\coprod_{P_0, \dots P_p} \mathrm{Hom}(P_0, P_1) \times \dots \dots \times \mathrm{Hom}(P_p, P_0)$$

The inclusion $H(A) \to \Omega\Sigma H(A) \to \Omega\Sigma HS_\bullet\mathcal{C}$

$$\to \Omega|HS.\mathcal{C}|$$

(analogous to inclusion ~~of the units of~~ $BGL_1(A) \to K(A)$) is an equivalence, by a theorem of Randy McCarthy.

($H(A)$ here is the "tensor product cyclic nerve" of the one-object category $A$; it is isomorphic to the usual model for cyclic homology.)

One point of the construction is that the circle group acts ~~equivariantly~~ on the diagram $(*)$ because cyclic nerves are cyclic objects in the sense of Connes.

The intermediate terms can be identified as follows:

(1) $\Omega|\Lambda S.\, i\mathcal{C}| = \Omega|B.\, i\mathrm{Aut}_{\mathcal{C}}|$, the K-theory of A-modules-with-automorphism.

(2) $\Omega|\Lambda S.\, \mathcal{C}|$ seems to be equivalent to the ~~category~~ K-theory of A-modules-with-endomorphism, minus $K(A)$, that is

$$K(\mathrm{End}_{\mathcal{C}}| = \Omega|B.\, i\,\mathrm{End}_{\mathcal{C}}| \simeq K(A) \times \Omega|\Lambda S.\, \mathcal{C}|$$

(The idea of proving (2) only came up after the talk, in response to a question of Thomason. With a little help from Grayson it ~~now~~ looks like it can be proved)

Thomas Goodwillie
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## Algebraic vector bundles over real algebraic varieties and applications.

J. BOCHNAK (AMSTERDAM).

Let $X$ be an affine nonsingular, compact connected real algebraic variety and let $\mathcal{R}(X)$ be the ring of regular functions from $X$ into $\mathbb{R}$.
The groups $\operatorname{Pic}(\mathcal{R}(X))$, $\operatorname{Pic}(\mathcal{R}(X)\otimes_{\mathbb{R}}\mathbb{C})$, $K_0(\mathcal{R}(X))$, $K_0(\mathcal{R}(X)\otimes_{\mathbb{R}}\mathbb{C})$ contain precious informations about the geometry and topology of $X$. Each of these groups is a subgroup (in a natural way) of the corresponding group of the ring $C(X)$ of continuous functions from $X$ into $\mathbb{R}$ (embedding is induced by the inclusion map $\mathcal{R}(X) \hookrightarrow C(X)$).
$\operatorname{Pic}(\mathcal{R}(X))$ is naturally isomorphic to a subgroup $H^1_{alg}(X,\mathbb{Z}/2)$ of $H^1(X,\mathbb{Z}/2)$, where $H^1_{alg}(X,\mathbb{Z}/2)$ is the image of

$$H^{alg}_{n-1}(X,\mathbb{Z}/2) = \{\text{homology classes in } H_{n-1}(X) \text{ represented by algebraic hypersurfaces of } X\}$$

by the Poincaré duality isomorphism $H_{n-1} \to H^1$ ; $n = \dim X$.

<u>Theorem</u>. Let $M$ be a compact connected $C^\infty$ manifold of dimension $\geq 3$, and let $G$ be a subgroup of $\operatorname{Pic}(C(M))$, containing the first Stiefel-Whitney class of $M$. Then there is an algebraic model $X$ of $M$ and a diffeomorphism $\varphi: X \to M$ such that $\varphi^*(G) = \operatorname{Pic}(\mathcal{R}(X))$. $\square$
(here $\varphi^*: \operatorname{Pic}(C(M)) \to \operatorname{Pic}(C(X))$ is the isomorphism induced by $\varphi$).
<u>Remark</u>. A slightly weaker version of this theorem is valid also for surfaces.
<u>Corollary</u>. For each compact connected $C^\infty$ manifold $M$, orientable of $\dim \geq 2$, there exist an algebraic model $X$ of $M$ with $\mathcal{R}(X)$ factorial. $\square$

### $K_0(\mathcal{R}(X))$ of real affine surfaces and 3-folds.

Define the following invariants of a nonsingular real algebraic surface $X$.

$$\beta(X) = \dim_{\mathbb{Z}/2} H^1_{alg}(X,\mathbb{Z}/2)$$
$$\delta(X) = \dim_{\mathbb{Z}/2} \{v \in H^1_{alg}(X,\mathbb{Z}/2) \mid v \cup v = 0\}.$$

<u>Theorem</u> (i) Let $X$ be a compact connected affine real algebraic surface. Then

$$K_0(\mathcal{R}(X)) = \mathbb{Z} \oplus (\mathbb{Z}/4)^{\beta(X)-\delta(X)} \oplus (\mathbb{Z}/2)^{\beta(X)+1-2(\beta(X)-\delta(X))}$$

(ii) As $X$ runs through all algebraic models of a compact connected smooth surface $M$ of genus $g$, the groups $K_0(\mathcal{R}(X))$ take (up to isomorphism) precisely $q(M)$ values, where

$$q(M) = \begin{cases} 2g+1 & \text{if } M \text{ orientable} \\ g & \text{if } M \text{ nonorientable, } g \text{ odd} \\ 2g-2 & \text{if } M \text{ nonorientable, } g \text{ even.} \end{cases}$$

□

(Remark. Similar results holds true for algebraic 3-folds).

<u>Theorem</u>. Let $M \subset \mathbb{RP}^k$ be a $C^\infty$ compact hypersurface. Then there exists a diffeomorphism $h: \mathbb{RP}^k \to \mathbb{RP}^k$ (which can be chosen arbitrarily close to the identity), such that:

(i) $X = h(M)$ is an algebraic nonsingular subset of $\mathbb{RP}^k$.

(ii) $\tilde{K}_0(\mathcal{R}(X))$ and $\tilde{K}_0(\mathcal{R}(X)\otimes_{\mathbb{R}}\mathbb{C})$ are finite groups.

(iii) If $H^{even}(M,\mathbb{Z})$ is torsion free, then $\tilde{K}_0(\mathcal{R}(X)\otimes\mathbb{C}) = 0$

(iii) If $M$ is orientable, and $\dim M = k-1$ is <u>even</u>, then each regular mapping $X \to S^{k-1}$ (standard $k-1$ sphere) is homotopic to a constant. □

There are many applications of these and similar results to the study of the structure of the set of regular mappings ($\mathcal{R}(X, S^k)$) from affine real algebraic varieties $X$ into $S^k$ (= the standard sphere). A sample of results:

<u>Theorem.</u> Given a compact connected $C^\infty$ surface $M$, the following conditions are equivalent:

(i) For <u>each</u> algebraic model $X$ of $M$, the set $\mathcal{R}(X, S^2)$ is dense in $C^\infty(X, S^2)$ (= set of $C^\infty$ mappings from $X$ into $S^2$ equipped with the $C^\infty$ topology).

(ii) $M$ is nonorientable of odd genus.

□

Remark. In particular one gets an algebraic model $X$ of the Klein bottle (with $\mathcal{R}(X, S^2)$ NOT dense in $C^\infty(X, S^2)$) by constructing a model with $\widetilde{K}_0(\mathcal{R}(X)\otimes\mathbb{C}) = 0$.

Theorem. Let $\Sigma^2_k$ be a Fermat sphere i.e.

$$\Sigma^2_k = \{(x,y,z)\in\mathbb{R}^3 \mid x^{2k}+y^{2k}+z^{2k}=1\}$$

Then $\mathcal{R}(\Sigma^2_k, S^2)$ is dense in $C^\infty(\Sigma^2_k, S^2)$. □

Remark. The Fermat spheres are quite exceptional, since for „most" algebraic surfaces $X$ in $\mathbb{R}^3$, the set $\mathcal{R}(X, S^2)$ contains only mappings homotopic to a constant!

Theorem. Given a compact connected orientable $C^\infty$ manifold $M$, $\dim M = 4$, the following conditions are equivalent:

(i) There exists an algebraic model $X$ of $M$ such that each regular map $X \to S^4$ is homotopic to a constant.

(ii) The signature of $M$ is $0$.

Theorem. Let $C$ be a nonsingular complex projective curve, and let $C_{\mathbb{R}}$ be the underlying real algebraic variety. Then $\mathcal{R}(C_{\mathbb{R}}, S^2)$ is dense in $C^\infty(C_{\mathbb{R}}, S^2)$.

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J. BOCHNAK (AMSTERDAM)

Connections between $|K_2O_F|$ for real quadratic fields F and class numbers of appropriate imaginary quadratic fields

I gave some connections between the order of the group $K_2O_F$ for real quadratic fields F and class numbers of appropriate imaginary quadratic fields. I applied an old series (see the paper of M. Lerch in Acta Mathematica, 1905). From the obtained formulas we got some congruences for $|K_2O_F|$ modulo powers of 2. These congruences are more general and modulo larger powers of 2 than ones of Gras (see Manuscripta Math. 57 (1987), 373-415). We got the exact divisibilities of $|K_2O_F|$ by powers of 2 from them. They answer questions (conjectures) of Candiotti (Acta Arithm., to appear).

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# Some Remarks on $H^1(X, \underline{K}_2)$ of Curves

Let $X$ be a smooth, projective, geometrically connected curve over a number field $k$ and set

$$V(X) =: \mathrm{Ker}(H^1(X, \underline{K}_2) \xrightarrow{N} k^*).$$

A conjecture of Bloch and a more general conjecture of Vaserstein say that $V(X)$ should be a torsion group. Let now $\bar{k}$ be an algebraic closure of $k$ and $\bar{X} = X \times_k \bar{k}$. Then one can easily show that $V(X)$ is torsion if and only if

$$V(\bar{X})^{Gal(\bar{k}/k)} = 0.$$

In this lecture I stated and outlined the proof of the following

Theorem: Let $X$ be as above with $X(k) \neq \emptyset$. Then the natural map

$$V(X) \longrightarrow V(\bar{X})^{Gal(\bar{k}/k)}$$

is surjective.

Since $V(\bar{X})^{Gal(\bar{k}/k)}$ is uniquely divisible, the theorem states that either $V(X)$ is a torsion group or it is quite large.

The proof of the theorem uses results of Saito to prove the corresponding local statement and then a recent theorem of Jannsen to pass from the local to the global.

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# Operations in cyclic homology of commutative algebras.

Jean-Louis LODAY.

The notion of descents for a permutation $\sigma \in S_n$ permits us to define the Eulerian partition of $S_n$ : $S_n = S_{n,1} \cup \dots \cup S_{n,n}$. The elements $\ell_n^k = (-1)^{k-1} \sum_{\sigma \in S_{n,k}} \mathrm{sgn}(\sigma)\, \sigma$ of the group algebra $K[S_n]$ have very nice properties. They lead to $\lambda_n^k = \sum_{i=0}^{k-1} (-1)^i \binom{n+i}{i} \ell_n^{k-i}$.

Let $S_n$ act on the left on $A \otimes A^{\otimes n}$ where $A$ is a commutative $K$-algebra. Denote by $b$ the Hochschild boundary and by $B$ the map defined by Connes.

PROP. $b\,\ell_n^k = (\ell_{n-1}^k + \ell_{n-1}^{k-1})\,b$ and $\ell_n^k B = B\,(k\,\ell_n^k + (n-k+1)\ell_{n-1}^{k-1})$.

COR. $b\,\lambda_n^k = \lambda_{n-1}^k\, b$ and $\lambda_n^k B = B\, k\, \lambda_{n-1}^k$.

Therefore these $\lambda^k$ maps permit us to endow Hochschild homology and cyclic homology with a special $\lambda$-ring structure.

In the rational case it implies a natural splitting :

$HH_n = HH_n^{(1)} \oplus \dots \oplus HH_n^{(n)}$ and $HC_n = HC_n^{(1)} \oplus \dots \oplus HC_n^{(n)}$, with $HH_n^{(n)} = \Omega^n$, $HC_n^{(n)} = \Omega^n / d\Omega^{n-1}$ and $HH_n^{(1)} = \mathrm{Harr}_n = HC_n^{(1)}$ ($n \geq 3$ for this last equality) where $\mathrm{Harr}_n$ is Harrison homology.

All these properties are valid for any functor $\underline{\mathrm{Fin}} \to$ (K-modules) where $\underline{\mathrm{Fin}}$ is the category of finite sets. In fact, the relations in PROP and COR above may be seen as relations in the universal ring $\mathcal{L} = K[\underline{\mathrm{Fin}}]$.

Ref. J-L. LODAY, Partition eulérienne et opérations en homologie cyclique, Cptes Rend. Acad. Sci. Paris (1988).

# $SK_1$ of punctuated Spec of 2-dimensional local rings

Shuji Saito (University of Tokyo)

Let $A$ be a 2-dimensional normal local domain

Let $F = A/m_A$ its residue field, $K = Q(A)$ its quotient field, $P$ the set of all prime ideals of height 1 in $A$ and put

$$X = Spec(A) - \{m_A\}$$

Let

$$SK_1(X) \overset{def}{=} Ker(K_1(X) \longrightarrow A^\times)$$

By the localization theory on $X$ we know

$$SK_1(X) \simeq Coker(K_2(K) \xrightarrow{\partial} \bigoplus_{p \in P} \kappa(p)^\times)$$

where $\partial$ is given by tame symbols. The localization sequence

$$K_2(K) \to \bigoplus_{p\in P} \kappa(p)^\times \longrightarrow \mathbb{Z}$$

gives rise to

$$\delta : SK_1(X) \longrightarrow \mathbb{Z}$$

and we put

$$SK_1(X)^0 = Ker(\delta)$$

Bloch proves

<u>Th</u> If $A$ is regular, $\delta$ is an isomorphism

In this talk I give the following theorem which treat $SK_1(X)$ in general case but assuming $F$ is <u>finite</u>

Th. Assume that $F$ is finite.

(1) $SK_1(X)^0$ is torsion

(2) Let $D(X) \subset SK_1(X)$ be the maximal divisible subgp. Then $SK_1(X)^0/D(X)$ is finite.

(3) There exist a canonical isomorphism

$$SK_1(X)^0/D(X) \simeq \mathrm{Gal}(\hat{K}^{ur}/\hat{K})_{tor}.$$

Here $\hat{K}$ is the quotient field of the completion $\hat{A}$ of $A$. $\hat{K}^{ur}$ is the maximal abel extension of $\hat{K}$ which is unramified over any $\mathfrak{p} \in P$.

We conjecture $D(X) = 0$. Concerning this we have

Prop. Assume that $A$ has rational singularity. Then the prime-to-$ch(F)$ part of $D(X)$ is trivial.

As a corollary of thm and Prop. we get

Cor. Let $B$ be a 2-dimensional regular local ring with finite residue field $F$. Let $G$ be a finite group acting on $B$ such that

(1) for any $\sigma \in G - \{id\}$

$$\mathrm{length}_B\, B/I_\sigma < \infty$$

where $I_\sigma = \langle b^\sigma - b \mid b \in B \rangle$.

(2) any $\sigma \in G$ acts trivially on $F$.

Put $A = B^G$ which is a 2-dimensional normal local ring. Then we have

$$SK_1(X)^0 \simeq G^{ab} \oplus (p\text{-primary torsion divisible group}).$$

$(p = ch(F))$.

# Higher Algebraic K-theory of schemes and of derived categories.

Robert W Thomason and Thomas F. Trobaugh(†)

Let $X$ be a quasiseparated quasicompact scheme. Recall from SGA 6 Grothendieck's ~~[illegible]~~ notion of a perfect complex on $X$. This is a complex of $\mathcal{O}_X$-modules which is locally quasi-isomorphic to a bounded complex of algebraic vector bundles. Using quasi-isomorphisms as the weak equivalences, this is a category with cofibrations and weak equivalences in the sense of Waldhausen. His work then defines a K-theory spectrum $K(X)$. When $X$ has an ample family of line bundles, for example when $X$ is quasiprojective over an affine or is regular noetherian, then this $K(X)$ is homotopy equivalent to Quillen's $K(X)$.

<u>Key</u> <u>Lemma</u>: Let $U$ be a quasicompact open in $X$. A perfect complex $F^\cdot$ on $U$ is the restriction of some perfect complex on $X$ up to quasi-isomorphism iff the class $[F^\cdot] \in K_0(U)$ is in the image of $K_0(X)$.

Using this, and techniques of Waldhausen K-theory, we prove:

(Bass Fundamental Thm)

<u>Thm 1</u>: There is a functorial spectrum $K^B(X)$ such that

a) $K^B_n(X) = K_n(X)$ for all integers $n \geq 0$

b) there is an exact sequence for all $n \in \mathbb{Z}$

$$0 \to K^B_n(X) \to K^B_n(X \otimes_{\mathbb{Z}} \mathbb{Z}[T]) \oplus K^B_n(X \otimes_{\mathbb{Z}} \mathbb{Z}[T^{-1}]) \to K^B_n(X \otimes_{\mathbb{Z}} \mathbb{Z}[T,T^{-1}]) \xrightarrow{\partial} K^B_{n-1}(X) \to 0$$

with $\partial$ naturally split by multiplication by $T \in K_1(\mathbb{Z}[T,T^{-1}])$

(Quillen Projective Space Thm)

<u>Thm 2</u>: If $\mathcal{E}$ is a rank $r$ vector bundle over $X$, there is a homotopy equivalence

$$K^B_\cdot(\mathbb{P}\mathcal{E}_X) \simeq \prod_{1}^{r} K^B(X).$$

For $Y \subseteq X$ closed, define $K(X \text{ on } Y)$ as the K-theory of the category of those perfect complexes on $X$ which are acyclic on $X - Y$. There is a $K^B(X \text{ on } Y)$ satisfying the analog of the "Bass fundamental theorem", Thm 1.

(Localization)

Thm 3: For $U \subseteq X$ a quasicompact open, there is a homotopy fibre sequence

$$K^B(X \text{ on } X-U) \longrightarrow K^B(X) \longrightarrow K^B(U)$$

Hence there is a long exact sequence

$$\cdots \to K^B_n(X \text{ on } X-U) \to K^B_n(X) \to K^B_n(U) \xrightarrow{\partial} K^B_{n-1}(X \text{ on } X-U) \to \cdots$$

(Excision)

Thm 4: If $i: Y \to X$ is a finitely presented closed immersion and $f: X' \to X$ is a map such that

1) $\mathcal{O}_{X',y'}$ is flat over $\mathcal{O}_{X,y}$ if $f(y') = y \in Y$
2) $f$ induces an isomorphism $f^{-1}(Y) \xrightarrow{\cong} Y$

then $f^*: K^B(X \text{ on } Y) \xrightarrow{\sim} K^B(X' \text{ on } Y')$ is a homotopy equivalence.

(Mayer-Vietoris)

Thm 5: If $U$ and $V$ are quasicompact opens in $X$, there is a homotopy cartesian Mayer-Vietoris square

$$\begin{array}{ccc} K^B(U \cup V) & \longrightarrow & K^B(U) \\ \downarrow & & \downarrow \\ K^B(V) & \longrightarrow & K^B(U \cap V) \end{array}$$

(Brown-Gersten)

Thm 6: If $X$ is noetherian of finite Krull dimension, there is cohomological descent for the Zariski and Nisnevich topologies

$$K^B(X) \xrightarrow{\sim} \mathbb{H}^{\cdot}_{Zar}(X; K^B)$$

$$K^B(X) \xrightarrow{\sim} \mathbb{H}^{\cdot}_{Nis}(X; K^B)$$

hence spectral sequences $H^p_{Zar}(X; \tilde{K}^B_q) \Rightarrow K^B_{q-p}(X)$

The Nisnevich descent part of Thm 6 allows one to remove the hypothesis that $X$ is regular in my old theorem that

$$K/\ell^\nu(X)[\beta^{-1}] \simeq K^{TOP}/\ell^\nu(X)$$

# Generalized Trace Map for K-Theory of Spaces, and Applications

Crichton Ogle

A conjecture due to T. Goodwillie asserts that
$\bar{A}(\Sigma X) \simeq \tilde{D}(|X|) \simeq \prod_{q\geq 1} \tilde{D}_q(|X|)$, $\tilde{D}_q(|X|) \overset{\text{def.}}{=} \Omega^\infty\Sigma^\infty(\Sigma(E\mathbb{Z}/q_+ \wedge_{\mathbb{Z}/q} |X|^{[q]}))$,
where $A(Z)$ denotes the Waldhausen K-Theory of the space = simplicial set $Z$, $\bar{A}(Z) = $ hofibre$(A(Z) \to A(*))$. A proof of this conjecture has been announced by ~~Z~~ G. Carlsson, R. Cohen, T. Goodwillie + W.-C. Hsiang [CCGH] and independently by myself. Both previous proofs are incorrect. We correct this.

We follow the techniques used by Waldhausen in his proof of the splitting $A(Y) \simeq Wh^{Diff}(|Y|) \times \Omega^\infty\Sigma^\infty(|Y|_+)$, and the outline of the proof of Goodwillie's conjecture given in [CCGH] in showing

<u>Thm 1</u> There exists a trace map $\bar{Tr}_X(Y)$ natural in $X$ and $Y$, ($X$ a connected simplicial set, $X$ and $Y$ basepointed):

$$\bar{Tr}_X(Y) : \varinjlim_n \Omega^n \text{fibre}(\bar{A}(\Sigma(X \vee \Sigma^n Y)) \to \bar{A}(\Sigma X))$$
$$\to \Omega^\infty\Sigma^\infty(\Sigma(\bigvee_{q\geq 1} |X|^{[q-1]} \wedge |Y|)) \simeq \prod_{q\geq 1} \Omega^\infty\Sigma^\infty(\Sigma(|X|^{[q-1]} \wedge |Y|)).$$

The decomposition on the right decomposes $\bar{Tr}_X(Y)$ as $\prod_{q\geq 1} \bar{Tr}_X(Y)_q$.

There exist maps $\tilde{\rho}_q : \tilde{D}_q(|X|) \to A(\Sigma X)$ as constructed in [CCGH] ~~and~~ and [O]. These constructions, as well as the entire proof of the above Theorem, admit and require a precise simplicial formulation. This we do. We then get

<u>Thm 2</u> $$(\bar{Tr})_X(Y)_q \circ (D_1\tilde{\rho}_p)_X(Y) \simeq \begin{cases} * & \text{if } p \neq q \\ (-1)^{q-1} & \text{if } p = q \end{cases}$$

this homotopy is natural in $X$ and $Y$. Here $(D_1\tilde{\rho}_p)_X(Y)$ denotes the 1st derivative of the map $\rho_p$ at $X$, evaluated at $Y$ in the sense of Goodwillie. It now follows from the fundamental results of Goodwillie and Waldhausen, who have computed $(D_1\bar{A}\Sigma)_X(Y)$ that

<u>Cor. 3</u> $\bar{A}(\Sigma X) \simeq \tilde{D}(|X|)$ by a homotopy natural in $X$.

Naturality of Pic, $SK_0$ and $SK_1$

This talk reports on joint work with C.A. Weibel. Transfer maps are constructed for $SK_0$ and $SK_1$. From these it follows that if $A = \bigoplus_{i\geq 0} A_i$ is a graded commutative ring with $A_+ = \bigoplus_{i>0} A_i$ and $A_0 = R$ then $SK_0(A, A_+)$, $SK_1(A, A_+)$, $\mathrm{Pic}(A, A_+)$, $NSK_0(R)$, $NSK_1(R)$, $N\mathrm{Pic}(R)$ are all modules over the ring $W(R)$ of Witt vectors over $R$. Various consequences of these module structures are discussed. In particular we consider the case where $A = \bigoplus_{i\geq 0} A_i$ is reduced, graded and finitely generated as an algebra over the field $A_0 = k$. Let $B = \bigoplus_{i\geq 0} B_i$ be the seminormalization of $A$, $GW(B) = \{f = 1 + b_1 t + \cdots \in W(B) \mid b_i \in B_i\}$. There is an injection $\gamma : \mathrm{Pic}(A) \to GW(B)/GW(A)$ of $W(k)$-modules. If $A_n = B_n$ for $n >> 0$ then $\gamma$ is an isomorphism. If $\mathrm{char}(k) = 0$, composing $\gamma$ with the ghost map gives an isomorphism of $k$-modules $\mathrm{Pic}(A) \longrightarrow B/A$.

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Is The KABI Conjecture True?

Sue Geller

(This is joint work with Chuck Weibel)

KABI CONJECTURE: Let $A$ and $B$ be rings, $I$ an ideal of $A$, and $f : A \to B$ such that $f(I)$ is an ideal of $I$ and $I \cong f(I)$. Then for all $n \geq 1$

$$K_n(A, B, I) \otimes \mathbb{Q} \cong HC_{n-1}(A, B, I) \otimes \mathbb{Q}.$$

Previously, the conjecture was known to be true for

a) $n = 1$ (Geller - Weibel)

b) $I$ nilpotent (Goodwillie)

c) $B = A/J$ (Ogle - Weibel)

also, it is sufficient to prove that $K_n(A, B, I) \simeq HC_{n-1}(A, B, I)$ for $\mathbb{Q}$-algebras $A \subseteq B$ with $I$ an ideal of both rings.

In this talk, for $\mathbb{Q} \subseteq A \subseteq B$ and $I$ an ideal of both rings, triple relative groups $K_n(A, B, I, J)$, $J$ an ideal of $A$, were defined, a module structure over the ring of Witt vectors $W(\mathbb{Q})$ was discussed and the following results were announced with some proofs given.

For $\mathbb{Q} \subseteq A \subseteq B$ and $I$ an ideal of both $A+B$

1) KABI Conjecture $\Leftrightarrow NK_n(A, B, I) \simeq NHC_{n-1}(A, B, I) \quad \forall n \geq 1$

2) KABI Conjecture $\Leftrightarrow K_n(A[t], B[t], I[t], t^k) \hookrightarrow HC_{n-1}(A[t], B[t], I[t], t^k) \quad \forall n \geq 1, k$

3) KABI Conjecture $\Leftrightarrow$ the weight $s$ summand of $K_n(A[t], B[t], t^k I[t])$ is zero for $s < k$ and $\forall n \geq 1$ (hence, if the weight $s$ summand of $K_n(A[t], t^k I[t]) = 0$ for $n \geq 2$, then the KABI conjecture is true).

4) $K_2(A, B, I) \to HC_1(A, B, I)$ is onto.
Hence, for $A, B, I$ as in the conjecture $K_2(A, B, I) \otimes \mathbb{Q} \to HC_1(A, B, I) \otimes \mathbb{Q}$ is onto.

# Higher K-theory of orders and integral group rings

This talk gives an exposition of the speaker's recent results on the higher K-theory of orders and group-rings. First solutions were given to recent questions on finite generation of $K_n$, $G_n$ of orders as well as finiteness of $SK_n$ and $SG_n$ of orders as follows. More precisely we prove the following results

(I) Let $R$ be the ring of integers in a number field, $F$, $\Lambda$ any $R$-order in a semi-simple $F$-algebra $\Sigma$, $\mathfrak{p}$ any prime ideal of $R$, then for all $n \geq 1$,

(i) $K_n(\Lambda)$ is a finitely generated Abelian group

(ii) $K_n(\Lambda) \to K_n(\Gamma)$ is an isomorphism mod torsion if $\Gamma$ is the maximal $R$-order containing $\Lambda$.

(iii) $SK_n(\Lambda)$ is a finite group.

(iv) $SK_n(\hat{\Lambda}_{\mathfrak{p}})$ is finite where $\hat{\Lambda}_{\mathfrak{p}}$ is the completion of $\Lambda$ at $\mathfrak{p}$

(II) Let $R$, $\Lambda$, $F$, $\Sigma$ be as in (I). Then $\forall n \geq 1$

(i) $G_n(\Lambda)$ is a finitely generated Abelian group

(ii) $G_{2n-1}(\Lambda_{\mathfrak{p}})$ is a finitely generated Abelian group

(iii) $SG_{2n}(\Lambda_{\mathfrak{p}}) = SG_{2n}(\Lambda) = SG_{2n}(\hat{\Lambda}_{\mathfrak{p}}) = 0$

(iv) $SG_{2n-1}(\Lambda)$ is finite; $SG_{2n-1}(\Lambda_{\mathfrak{p}})$, $SG_{2n-1}(\hat{\Lambda}_{\mathfrak{p}})$ are finite groups of order relatively prime to the prime $p$ lying below $\mathfrak{p}$.

We also have the following results on Cartan maps: For all $n \geq 1$

III (i) If $k$ is a field of characteristic $p$ and $\pi$ any finite group, then $K_{2n}(k\pi)$ is a finite $p$-group and $\mathrm{Ker}(K_{2n-1}(k\pi) \to G_{2n-1}(k\pi))$ is the Sylow $p$-subgroup of $K_{2n-1}(k\pi)$

(ii) $K_n(\Lambda) \to G_n(\Lambda)$ induces a surjection $SK_n(\Lambda) \to SG_n(\Lambda)$

(iii) $G_{4n+3}(\mathbb{Z}\pi)$, $K_{4n+3}(\mathbb{Z}\pi)$, $G_{4n+3}(\mathbb{Z}_p\pi)$ are finite groups

Finally we show that induction theory can be used to reduce the study of K-theory of integral group-rings of finite groups to the study of the K-theory of group-rings over the p-hyperelementary subgroups of $\pi$.

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## Whitehead groups of finite groups

This talk was a summary of current knowledge of the groups $K_1(ZG)$ and $Wh(G)$ for finite groups $G$. By results of Bass, they are finitely generated, and their ranks are known. Also, by a theorem of Wall, the torsion subgroup of $Wh(G)$ is precisely the group

$$SK_1(ZG) = Ker[K_1(ZG) \to K_1(QG)] = Ker[nr : K_1(ZG) \longrightarrow Z(QG)^*].$$

Localization sequences are needed to make systematic computations of the $SK_1(ZG)$. One way to see these is to consider the relative K-theory exact sequence

$$K_2(Z/n[G]) \longrightarrow SK_1(ZG, nZG) \longrightarrow SK_1(ZG) \longrightarrow K_1(Z/n[G]).$$

Upon taking the inverse limit over all $n$, this gives an exact sequence

$$\Pi_p K_2^c(\hat{Z}_p G) \longrightarrow \varprojlim_n SK_1(ZG, nZG) \longrightarrow SK_1(ZG) \xrightarrow{\ell} \prod_p SK_1(\hat{Z}_p G) \to 1.$$

For any $Z$-order $\mathfrak{O}$ in a finite dimensional semisimple $Q$-algebra $A$, $\varprojlim SK_1(\mathfrak{O}, n\mathfrak{O})$ vanishes iff the congruence subgroup problem holds for $\mathfrak{O}$; i.e., iff any subgroup of finite index in $SL_r(\mathfrak{O})$ $(r \geq 3)$ contains some congruence subgroup $SL_r(\mathfrak{O}, n\mathfrak{O})$. The group

$$C(A) = \varprojlim_n SK_1(\mathfrak{O}, n\mathfrak{O}) \cong Coker[K_2(A) \longrightarrow \bigoplus_p K_2^c(\hat{A}_p)]$$

is independent of $\Omega$; and in many cases — including the case $A = \mathbb{Q}G$ — has been completely described in work of Bass, Milnor, Serre; Bak, Rehmann, Prasad, Raghunathan, and others.

The $SK_1(\mathbb{Z}G)$ are thus described by 2 exact sequences

$$1 \to Cl_1(\mathbb{Z}G) \longrightarrow SK_1(\mathbb{Z}G) \xrightarrow{\ell} \prod_p SK_1(\hat{\mathbb{Z}}_pG) \to 1 \qquad (Cl_1(\mathbb{Z}G) := \mathrm{Ker}(\ell))$$

and $\prod_p K_2^c(\hat{\mathbb{Z}}_pG) \longrightarrow C(\mathbb{Q}G) \longrightarrow Cl_1(\mathbb{Z}G) \to 1.$

The $SK_1(\hat{\mathbb{Z}}_pG)$ can be described precisely, for any finite $G$, in terms of the functor $H_2(-)$ applied to subquotients of $G$. The map $\ell$ is naturally split in odd torsion. Formulas for the odd torsion in $Cl_1(\mathbb{Z}G)$ are known. For example, if $G$ is a $p$-group for odd $p$, if $\mathbb{Q}G \cong \prod_{i=1}^k A_i$, $A_i \cong M_{r_i}(F_i)$ has irreducible representation $V_i$, and $F_i = \mathbb{Q}(\mu_i)$ where $\mu_i$ is a group of $p$-power roots of unity, then:

$$Cl_1(\mathbb{Z}G) \cong \left[\prod_{i=1}^k \mu_i\right] \Big/ \langle \psi(g\otimes h) : g, h \in G,\ gh = hg \rangle \quad \text{where}$$

$$\psi(g\otimes h) = \left(\det_{F_i}(g, V_i^h)\right)_{i=1}^k. \qquad (V_i^h = \{x \in V_i \mid hx = x\})$$

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## Bivariant Chern character.

The Chern character (also called generalized Dennis trace map) ch: $K_*(A) \longrightarrow HC_*^-(A)$ from algebraic K-theory to negative cyclic homology can be extended to a bivariant Chern character ch: $K^*(A,B) \longrightarrow HC^*(A,B)$ from a suitably defined bivariant algebraic K-theory to a bivariant version of cyclic cohomology. Both bivariant theories are covariant in B and contravariant in A. One recovers the usual Chern character when $A = \mathbb{Z}$. As an immediate consequence of the multiplicativity of the bivariant Chern character, two Morita-equivalent algebras have ~~the~~ isomorphic (bivariant) cyclic (co)homology groups.

The bivariant K-groups are obtained from the exact category of A-B-bimodules which are finitely generated projective over B.

The bivariant cyclic cohomology groups have the following properties

i) (Product) There exists a graded product

$$HC^*(A_1, B_1 \otimes C) \otimes HC^*(C \otimes A_2, B_2) \longrightarrow HC^*(A_1 \otimes A_2, B_1 \otimes B_2)$$

ii) (Bivariant Connes exact couple) There exists an exact couple

$$\begin{tikzcd} HC^*(A,B) \arrow[rr, "S"] & & HC^*(A,B) \arrow[dl, "I"] \\ & HH^*(A,B) \arrow[ul, "B"] & \end{tikzcd}$$

where $\deg(S) = 2$, $\deg(I) = 0$, $\deg(B) = -1$ and $HH^*(A,B)$ is a bivariant version of Hochschild homology.

iii) For any extension of algebras $0 \to I \to R \to S \to 0$ such that I is H-unital in the sense of Wodzicki, one has the exact triangles

$$\begin{tikzcd} HC^*(A,I) \arrow[rr] & & HC^*(A,R) \arrow[dl] \\ & HC^*(A,S) \arrow[ul] & \end{tikzcd} \quad \text{and} \quad \begin{tikzcd} HC^*(I,A) \arrow[dr] & & HC^*(R,A) \arrow[ll] \\ & HC^*(S,A) \arrow[ur] & \end{tikzcd}$$

iv) If SA is the suspension of the algebra A, one has the following isomorphisms

$$HC^n(A,B) = HC^0(A, S^nB) \quad \text{and} \quad HC^{-n}(A,B) = HC^0(S^nA, B) \quad (n \geq 0).$$

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# Absolute stable rank and Witt cancellation for noncommutative rings

In a ring $A$, a list $a_0, \dots, a_n$ "can be shortened" if there are $t_i \in A$ with $a_0 + t_0 a_n, \dots, a_{n-1} + t_{n-1} a_n$ lying in exactly those maximal left ideals containing $a_0, \dots, a_n$; if every such list in $A$ can be shortened, we say $A$ has <u>absolute</u> <u>stable</u> <u>rank</u> $\mathrm{asr}(A) \leq n$. This condition is designed to imply transitive action of $\mathcal{U}(q)$ on all nonsingular vectors $v$ (in a $(\Lambda, \varepsilon, \alpha)$-quadratic space $(M,q)$) of equal length. By a standard argument it implies $(M,q)$ is cancellative when $q$ has Witt index $\geq \mathrm{asr}(A) + 2$ (or $\mathrm{asr}(A) + 1$ provided the involution $\alpha$ on $A$ is trivial). In general $\mathrm{asr}(A) \geq \mathrm{sr}(A)$ = the stable rank of $A$. By a recent theorem of J.T. Stafford, $\mathrm{asr}(A) \leq \mathrm{Kdim}(A/\mathrm{rad}\,A) + 1$, where $\mathrm{Kdim}(A)$ is the Krull dimension of a left noetherian ring. So Witt cancellation (for sufficiently large index) applies to quadratic spaces over $\mathbb{Z}G$ when $G$ is polycyclic by finite.

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## G-theory of integral group rings.

Let $G$ be a finite group, and consider $G_*(\mathbb{Z}G)$ (or more generally, $G_*(RG)$, for a Noetherian ring $R$). We first deduce a Lenstra-type decomposition for $G$ nilpotent.

Prop.: Let $G$ be finite nilpotent, and write $\mathbb{Q}G \cong \prod_\rho \mathbb{Q}(\rho)$, where $\rho$ ranges over irreducible rational representations and $\mathbb{Q}(\rho)$ is simple; let $\mathbb{Z}(\rho)$ be a maximal $\mathbb{Z}$-order in $\mathbb{Q}(\rho)$, $\mathbb{Z}\langle\rho\rangle = \mathbb{Z}(\rho)[\frac{1}{|\rho|}]$, where $|\rho| = [G : \ker\rho]$. Then $G_*(\mathbb{Z}G) \cong \bigoplus_\rho G_*(\mathbb{Z}\langle\rho\rangle)$.

I. Hambleton, L. Taylor, and B. Williams prove this result independently, and they conjecture a general answer:

Conjecture (HTW): Let $G$ be a finite group, and write $\mathbb{Q}G \cong \prod_\rho M_{n_\rho}(D_\rho)$, $D_\rho = \mathrm{End}_{\mathbb{Q}G}(V_\rho)$ the division algebra associated to the irreducible rational representation $\rho : G \to GL(V_\rho)$. Let $k_\rho = |\ker G \xrightarrow{\rho} GL(V_\rho)|$, $\ell_\rho$ the degree of any the irreducible constituents of $\mathbb{C}\otimes_{\mathbb{Q}} V_\rho$, $w_\rho = \frac{|G|}{k_\rho \ell_\rho}$, $\mathcal{D}_\rho$ a maximal $\mathbb{Z}$-order in $D_\rho$. Then $G_*(\mathbb{Z}G) \cong \bigoplus_\rho G_*(\mathcal{D}_\rho[1/w_\rho])$.

Prop.: The HTW conjecture holds for dihedral extensions of finite abelian groups.

Prop.: The HTW conjecture holds for $|G|$ square-free.

The proofs use Lenstra-type techniques; one defines the Lenstra functor, a self homotopy equivalence of $BQ\underline{\underline{M}}(\mathfrak{M})$, $\mathfrak{M}$ a $\mathbb{Z}$-order in $\mathbb{Q}G$ containing $\mathbb{Z}G$; this induces a map of the homotopy fibre sequence $\underline{\underline{M}}^{tor}(\mathbb{Z}G) \to \underline{\underline{M}}(\mathbb{Z}G) \to \underline{\underline{M}}(\mathbb{Q}G)$ to the sequence $\underline{\underline{M}}^{tor}(\mathfrak{A}) \to \underline{\underline{M}}(\mathfrak{A}) \to \underline{\underline{M}}(\mathbb{Q}G)$, where $\mathfrak{A}$ is a ring whose $G_*$ is the desired answer.

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# Trivializing Milnor's K-theory

Let $F$ be a field. The talk defined two series of groups $\hat{K}_n(F)$, $\tilde{K}_n(F)$, "lifting" the Milnor K-groups $K_n^M(F)$. $\hat{K}_n(F)$ (resp. $\tilde{K}_n(F)$) is defined as $\mathbb{G}_m^{\otimes n}$ (resp. $\Lambda^n(\mathbb{G}_m)$) in the category of Mackey functors. So, loosely speaking, $\hat{K}_n(F)$ is defined by generators $\mathrm{cor}_{E/F}(x_1\otimes\ldots\otimes x_n)$, $[E:F]<+\infty$, $x_i\in E^*$, with relations given by the projection formula. Same thing for $\tilde{K}_n(F)$ with $x_1\wedge\ldots\wedge x_n$. There are surjective homomorphisms:

$$\hat{K}_n(F) \twoheadrightarrow \tilde{K}_n(F) \twoheadrightarrow K_n^M(F),$$

and $\mathrm{Ker}(\hat{K}_n(F)\to K_n^M(F))$ and $\mathrm{Ker}(\tilde{K}_n(F)\to K_n^M(F))$ are <u>divisible</u>.

Thus the Milnor-Kato conjecture may be phrased as follows: the natural maps $\hat{K}_n(F)/m \to H^n(F,\mathbb{Z}/m(n))$ (resp. $\tilde{K}_n(F)/m \to H^n(F,\mathbb{Z}/m(n))$) are isomorphisms.

? <u>Conjecture 1</u>. There are canonical isomorphisms:

$$H^{n-1}(F,\mathbb{Q}/\mathbb{Z}(n)) \xrightarrow{\sim} \hat{K}_n(F)_{tors}$$
$$H^{n-1}(F,\mathbb{Q}/\mathbb{Z}(n)) \xrightarrow{\sim} \tilde{K}_n(F)_{tors}.$$

I am able to construct such maps for $n=2,3$ (at least, away from 2-torsion: for 2-torsion I have to assume that $\mathrm{Gal}(F(\mu_{2^\infty})/F)$ is torsion free).

Assume that $F$ is perfect; define $\mathbb{Z}(1)$ as $\mathbb{G}_m[-1]$ (as a complex of $\mathrm{Gal}(\bar{F}/F)$-modules) and $\hat{\mathbb{Z}}(n)$ as $\mathbb{Z}(1)^{\hat{\otimes} n}$ (in the corresponding derived category). Set $\hat{K}'_n(F) = \mathbb{H}^n(F,\hat{\mathbb{Z}}(n))$. Then cup-product induces a homomorphism

$$\hat{K}_n^{\bullet}(F) \xrightarrow{\alpha} \hat{K}'_n(F),$$

and

<u>Conjecture 2</u> $\alpha$ is an isomorphism.

The link between conjectures 1 and 2 is the following (easy) theorem.

<u>Theorem 1</u>. a) There is a canonical isomorphism

$$H^{n-1}(F, \mathbb{Q}/\mathbb{Z}(n)) \xrightarrow{\sim} \hat{K}'_n(F)_{tors}.$$

b) There is a canonical injection

$$\hat{K}'_n(F)/m \hookrightarrow H^n(F, \mathbb{Z}/m(n)).$$

If the Galois symbol in degree $n$ is surjective, this injection is an isomorphism.

It is easy to see that $\operatorname{Ker}\alpha$ and $\operatorname{Coker}\alpha$ are torsion. On the other hand, there is the following result:

<u>Theorem 2</u>. a) $\alpha$ is surjective iff the Galois symbol in degree $n$ is surjective.

b) Assume $n = 2$ or $3$. Then the restriction of $\alpha$ to $\hat{K}_n(F)_{tors}$ is split surjective, with divisible kernel.

Bruno Kahn Paris 7.

## Grothendieck - Riemann - Roch for general schemes

Let $S$ be a base scheme, Noetherian and of finite Krull dimension, separated. Let $\ell$ be a prime number, fixed once for all so that 1) $\ell^{-1} \in \mathcal{O}_S$ 2) ~~all~~ all residue fields of $S$ have bounded uniformlly, $\ell$-étale cohomological dimension. e.g. $\mathbb{Q}$ if $\ell \neq 2$, $\mathbb{C}$, $\mathbb{F}_{alg}$, $\mathbb{Z}[\frac{1}{\ell}]$, .... .
Schemes we consider are essentially of finite type over $S$.

Theorem There exists a topological $G$-theory spectrum $G^t(X)$ so that

1) Atiyah-Hirzebruch s.s. $H^*(X_{et}; i^! \hat{\mathbb{Z}}_\ell(*)) \Rightarrow G^t_*(X)$, $i : X \to S$, the str. morph.
2) Grothendieck - Riemann - Roch : When $f : X \to Y$ is proper morph.,

$$\begin{array}{ccc} G^{alg}(X) & \xrightarrow{\tau_X} & G^t(X) \\ f_* \downarrow & & \downarrow f^t_* \\ G^{alg}(Y) & \xrightarrow[\tau_Y]{} & G^t(Y) \end{array}$$

where $G^{alg}(X)$ is the spectrum associated to coherent sheaves on $X$, $f_*$ is induced by an alternative sum of higher direct image sheaves.

And it induces the Hirzebruch - Riemann - Roch formula, the main theorem of Baum - Fulton - MacPherson and its generalization to higher K-theory, the theorem of Gillet.

The proof and the construction is **based** on the facts that 1) $f_*$ can be localized with respect to the étale topology on $Y$. 2) $K^{alg}(\ )^\wedge_\ell$ is locally constant on the étale topology.

The projection formula $f_*(x \cap f^* y) = (f_* x) \cap y$ is formulated as the comm. diagram of spectra

$$\begin{array}{ccccc} G^{alg}(X) \otimes K^{alg}(Y) & \xrightarrow{1 \otimes f^*} & G^{alg}(X) \otimes K^{alg}(X) & \xrightarrow{\cap} & G^{alg}(X) \\ f_* \otimes 1 \downarrow & & & & \downarrow \\ G^{alg}(Y) \otimes K^{alg}(Y) & & \xrightarrow[\cap]{\hspace{6em}} & & G^{alg}(Y) \end{array}$$

The facts gives us

$$\begin{tikzcd}[column sep=small]
\mathbb{H}(X_{et}; \underset{\sim}{G}^{alg}/\ell^\nu) \overset{\mathbb{L}}{\otimes}_{K^t(S)^\wedge_\ell} K^t(Y)^\wedge_\ell \arrow[r, "1\otimes f^*"] \arrow[d, "f_*\otimes 1"'] & \mathbb{H}(X_{et}; \underset{\sim}{G}^{alg}/\ell^\nu) \overset{\mathbb{L}}{\otimes}_{K^t(S)^\wedge_\ell} K^t(X)^\wedge_\ell \arrow[r, "\cap"] & \mathbb{H}(X_{et}; \underset{\sim}{G}^{alg}/\ell^\nu) \arrow[d, "f_*"] \\
\mathbb{H}(Y_{et}; \underset{\sim}{G}^{alg}/\ell^\nu) \overset{\mathbb{L}}{\otimes}_{K^t(S)^\wedge_\ell} K^t(Y)^\wedge_\ell \arrow[rr, "\cap"'] & & \mathbb{H}(Y_{et}; \underset{\sim}{G}^{alg}/\ell^\nu)
\end{tikzcd}$$

When $X$ and $Y$ are proper over $\tilde{S}$, compose the Gysin mapping to $S$

$$\begin{tikzcd}
\mathbb{H}(X_{et}; \underset{\sim}{G}^{alg}/\ell^\nu) \arrow[dr] \arrow[dd, "f_*"] & \\
& G^{alg}/\ell^\nu(S)_K \\
\mathbb{H}(Y_{et}; \underset{\sim}{G}^{alg}/\ell^\nu) \arrow[ur] &
\end{tikzcd}$$

and taking the adjunction as $K^t(S)^\wedge_\ell$-module, we get the theorem 2).

To prove the theorem 1), we look at the Postnikov filtation on them.

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# Acyclic groups

Acyclic groups are those groups whose homology (trivial $\mathbb{Z}$ coefficients) is that of the trivial group. This survey attempts to indicate the importance of acyclic groups and examine their group-theoretic structure.

## Examples

Acyclic groups are to be found in work of G Higman (1951), McLain (1954), Baumslag & Gruenberg (1967), Epstein (1968), J Mather (1971), Wagoner (1972), Kan & Thurston (1976), Baumslag, Dyer & Heller (1980), de la Harpe & McDuff (1983), and elsewhere. Many examples have few normal subgroups.

## Ubiquity results

For a group extension $N \rightarrowtail G \twoheadrightarrow Q$ with $Q$ acting trivially on $H_*N$,

(i) $N$ acyclic $\Leftrightarrow H_*G \xrightarrow{\cong} H_*Q$ (ii) $Q$ acyclic $\Leftrightarrow H_*N \xrightarrow{\cong} H_*G$.

[K & T 1976]: $\forall$ group $G$, $G \trianglelefteq D \trianglelefteq$ acyclic.

This prompts the study of normal-in-acyclic groups, e.g. abelian groups [BD & H 1980, B 1983], $GLR$ ($R$ ring) [W 1972].

## Group structure $\Rightarrow$ acyclicity

Techniques used to prove acyclicity include Mayer-Vietoris sequences, preservation of dirlim by homology, and binate structure: $G = \cup G_n$ where $G_1 \leq G_2 \leq \dots$ and $\forall n\ \exists \varphi_n : G_n \to G_{n+1}$ and $a_{n+1} \in G_{n+1}$ s.t. $\forall g \in G_n \quad g = [\varphi_n(g), a_{n+1}]$. Binate groups are acyclic [B, to appear in Proc. Singapore Group Theory Conf., de Gruyter].

## Acyclicity $\Rightarrow$ group structure

T: Any f.d. complex representation of an acyclic group restricts trivially to all finite subgroups.

C1: Finite normal-in-acyclic groups are abelian.

C2: A (non-central) normal subgroup $N$ of a torsion-generated acyclic group has $N/N''$ f.g. iff $N$ is (infinite perfect)-by-f.g. abelian. (Possible example $GLR \triangleleft GLCR$ $\because$ $GLR$ is $ER$-by-$K_1R$.)

C3: If perfect $N \triangleleft$ torsion-gen'd acyclic $A$ and $\mathrm{Out}N$ has a series with factors residually finite and/or hypoabelian and/or torsion-free, then $A \cong N \times A/N$, so $N$ also tors. gen'd acyclic.

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# Cyclic and Hochschild Homologies of an Exact Category

$k$ - a comm. ring

For $\mathcal{C}$ a small $k$-linear category, we define the cyclic nerve of $\mathcal{C}$, $CN(\mathcal{C})$ to be the cyclic $k$-module:

$$CN_n(\mathcal{C}) = \bigoplus_{c_0, \dots, c_n} \mathrm{Hom}(c_1, c_0) \otimes_k \cdots \otimes_k \mathrm{Hom}(c_0, c_n)$$

i.e. $c_0 \xleftarrow{a_0} c_1 \xleftarrow{a_1} c_2 \leftarrow \cdots \leftarrow c_n$ with $a_n : c_0 \to c_n$, $(a_0 \otimes \cdots \otimes a_n)$.

Face and degeneracy operators are like those of Hochschild homology.

Thm: If $A$ is a unital $k$-algebra, and $\mathcal{P}_A$ = cat. of f.g. projective modules, then $A^{\natural} \simeq CN(\mathcal{P}_A)$ [by def. retract].

For $\mathcal{M}$ an exact category, which is also $k$-linear, we can form $CN.S.\mathcal{M}$, where $S.\mathcal{M}$ is Waldhausen simplicial category for a cofibered category.

Def: $HH_*(\mathcal{M}) = HH_{*+1}(CN.\,S.\mathcal{M})$

$HC_*(\mathcal{M}) = HC_{*+1}(CN.\,S.\mathcal{M})$.

Thm: $CN.\,S_m\mathcal{P}_A \simeq CN.\,\mathcal{P}_A^m$

Cor: The map $CN.\mathcal{P}_A \longrightarrow \Omega\, CN.\,S.\,\mathcal{P}_A$ is a homotopy equivalence.

Cor: We have trace map (by Goodwillie earlier)

$$\begin{array}{ccccc} \Omega N i S.\mathcal{P}_A & \longrightarrow & \Omega CN S.\mathcal{P}_A & \xleftarrow{\ \sim\ } & CN.\mathcal{P}_A \\ \wr\mid & & & & \wr\mid \\ \Omega N.Q\mathcal{P}_A & & & & CN.A \end{array}$$

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# Motivic Cohomology

It would be highly desirable to have an algebraic cohomology theory bearing the same relation to algebraic K-theory as ordinary singular cohomology bears to topological K-theory. This theory should also have serious applications to the study of special values of zeta-functions and to arithmetic duality theorems.

Such a theory should be the hypercohomology (in the étale and Zariski sites) of a complex of sheaves $\mathbb{Z}(r)$ $(r = 0, 1, 2, \dots)$ on a noetherian regular scheme $X$ satisfying (at least) the following properties

(0) $\mathbb{Z}(0) = \mathbb{Z}$ $\quad \mathbb{Z}(1) = \mathbb{G}_m[-1]$

(1) For $r \geq 1$, $\mathbb{Z}(r)$ is acyclic outside of $[1, r]$

(2) There is a product pairing $\mathbb{Z}(r) \overset{\mathbb{L}}{\otimes} \mathbb{Z}(s) \to \mathbb{Z}(r+s)$

(3) a) If $n$ is invertible on $X$, there is a distinguished triangle in the étale site

$$\mathbb{Z}(r) \xrightarrow{n} \mathbb{Z}(r) \to \mathbb{Z}/n\mathbb{Z}(r) \to \mathbb{Z}(r)[1]$$

b) If $X$ has characteristic $p$, there is a distinguished triangle in the étale site

$$\mathbb{Z}(r) \xrightarrow{p^m} \mathbb{Z}(r) \to \nu_m(r)[-r] \to \mathbb{Z}(r)[1]$$

(4) If $\alpha$ maps the étale site to the Zariski site,

$$\alpha^* \mathbb{Z}(r)_{zar} = \mathbb{Z}(r)_{ét}, \quad t_{\leq r} R\alpha_* \mathbb{Z}(r)_{ét} = t_{\leq r+1} R\alpha_* \mathbb{Z}(r)_{et} = \mathbb{Z}(r)_{zar}.$$

In particular, $R^{r+1}\alpha_* \mathbb{Z}(r) = 0$ (Hilbert Theorem 90)

(5) $R^r \alpha_* \mathbb{Z}(r) = \underline{\underline{K}}^M_{r, zar}$

(6) The homology sheaves $\mathcal{H}^i(\mathbb{Z}(r))$ should be isomorphic to the sheaves $gr^\gamma_r \underline{\underline{K}}_{2r-i}(\mathcal{O}_X)$, up to $p$-torsion for primes $p < r$.

For $r = 2$, we have constructed a cohomology theory satisfying all of these properties, with the exception that we do not know, for property (6) that $gr^\gamma_2 \underline{\underline{K}}_{4-i}(\mathcal{O}_X) = 0$ for $i \leq 0$.

A possible candidate for a motivic cohomology complex in the case of a field $F$ is the following:

Let the $i$-th term of the complex $\mathbb{Z}(r)$ $(0 \le i \le r)$, be

$$\varinjlim_{V,S} K_i^{1,M}(V-S;\ I_1, I_2, \ldots I_r)$$

where $V$ runs over all reduced $i$-dimensional subschemes of $\mathbb{A}_F^r$ whose intersection with all faces of the hypercube $X_i(X_i-1) = 0$, $i = 1, \ldots r$ is proper. $S$ runs over all finite subsets of $V$ whose intersection with the $(r-i)$-skeleton of the hypercube is empty, and $I_j$ is the ideal defined by $X_j(X_j-1)$. $K_i^{1,M}$ here denotes multirelative Milnor $K'$-theory.

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# Relative Chow Groups

S. Landsburg

Let $Y \subset X$ be a closed inclusion of regular schemes of finite type over a field. (Regularity can be relaxed in much of what follows) We want to define a relative Chow Theory $Ch^P(X, Y)$.

To see what this theory should look like, consider the usual absolute Chow theory $Ch^P(X)$. We have

$$gr^P K_0(X) \xleftarrow[\sim]{} Ch^P(X) = Z^P(X)/\sim \; = H^P(X, \underline{\underline{K}}_P) = E_2^{P,-P}(X)$$

iso up to torsion.

where $Z^P$ is cycles, $\sim$ is rational equivalence, $\underline{\underline{K}}_P$ is sheafified K-theory, and $E_2^{P,-P}(X)$ is from the Quillen spectral sequence.

Here are the relative analogues of some of these objects:

① Let $\tilde{Z}^P(X)$ be free abelian on cycles meeting $Y$ properly. Then $Z^P(X,Y)$ is defined by $0 \to Z^P(X,Y) \to \tilde{Z}^P(X) \to Z^P(Y)$.

② $\mathcal{K}_P(X)$ is the complex $\underline{\underline{K}}_P(X) \longrightarrow i_* \underline{\underline{K}}_P(Y)$.

③ We get a spectral sequence for relative K-theory by taking fibers vertically in the diagram

$$\begin{array}{ccccc}
F^{m+1} & \longrightarrow & F^m & \longrightarrow & F^{m/m+1} \\
\downarrow & & \downarrow & & \downarrow \\
K\mathcal{M}^{m+1}(X) & \longrightarrow & K\mathcal{M}^m(X) & \longrightarrow & K\mathcal{M}^{m/m+1}(X) \\
\downarrow & & \downarrow & & \downarrow \\
K\mathcal{M}^{m+1}(Y) & \longrightarrow & K\mathcal{M}^m(Y) & \longrightarrow & K\mathcal{M}^{m/m+1}(Y)
\end{array}$$

Here $\mathcal{M}^m(X)$ is the category of $X$-modules of cod $\geq m$. The spectral sequence is $E_1^{pq} = \pi_{-p-q}(F^{p/(p+1)}) \Longrightarrow K_{-p-q}(X)$.

The construction of the spectral sequence leads immediately to maps

$$\begin{array}{ccc}
E_2^{P,-P} & \longrightarrow & H^P(X, \mathcal{K}_P) \\
\downarrow & & \\
Z^P(X,Y)/\sim & &
\end{array}$$

for appropriate $\sim$.

We also get a cycle map: $Z^p(X,Y)/\sim \longrightarrow \mathcal{H}^p(X,\mathcal{K}_p)$ directly by noticing that for $z \in Z^p(X,Y)$, $\mathcal{H}^p_z(X,\mathcal{K}_p)$ is free abelian on the components of $z$.

Before defining $Ch^m(X,Y)$, we ~~recall~~ generalize some of this to higher Chow groups. There is a map from Bloch's higher Chow complex to the Gersten-Quillen complex induced by

$$Z^m(X,n) \longrightarrow \coprod_{x \in X^{m-n}} K_n k(x) \quad \text{via} \quad z \longmapsto \left(p_* z, \left\{\frac{Nt_1}{\Sigma Nt_i - 1}, \ldots, \frac{Nt_n}{\Sigma Nt_i - 1}\right\}\right)$$

where $N = \mathrm{Norm}_{z/p_* z}$ and $\{\ \}$ is the Steinberg symbol.

This gives $Ch^m(X,n) \longrightarrow H^{m-n}(X, \underline{K}_m)$; this is iso for $n \leq 1$.

Now define $Ch^m(X,Y,n) = \pi_n(\mathrm{Cone}(Z^m(X,\cdot) \to Z^m(Y,\cdot))[-1])$.

(To define the map, first replace $Z^m(X,\cdot)$ by the quasi-isomorphic complex consisting of things that restrict properly to $Y$.)

Define $Ch^m(X,Y) = Ch^m(X,Y,0)$. Then we get a Bloch Formula

$$Ch^m(X,Y) \xrightarrow{\ \approx\ } \mathcal{H}^m(X,\mathcal{K}_m).$$

Finally, to get a cycle map, note that an element of $Ch^m(X,Y)$ is represented by a cycle $z$ on $X$ with a choice of trivialization of $z|_Y$. This gives data consisting of compatible cycles on two copies of $X$ and one of $Y \times \mathbb{A}^1$ (namely $z^+$, $z^-$ and the trivialization). Under favorable circumstances, these can be "patched" to give a class in $K_0(X \amalg_{Y\times 0} (Y \times \mathbb{A}^1) \amalg_{Y \times 1} X) \simeq K_0(X,Y) \oplus K_0(X)$.

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# GRAPHENTHEORIE

## 12.-18. Juni 1988

### Some Results on Well-covered Graphs

M. D. Plummer

The maximum independent set problem for graphs is well-known to be NP-complete. But suppose one has a graph in which the greedy algorithm for an independent set always yields a maximum indep. set. In other words, every maximal indep. set is maximum. We call such graphs well-covered (w-c).

The structure of w-c graphs is not completely understood. In this talk we first consider the case of cubic graphs (i.e., regular of degree 3). In joint work with Stephen Campbell, we present complete characterizations of 1-(but not 2-), 2-(but not 3-) and 3-connected cubic w-c graphs, the last of these only for the planar case. The first two families are infinite, but the third contains but four members.

We also discuss another approach taken by Denbow, Hartnell and Nowakowski who have recently characterized all w-c graphs of girth at least 5.

## Decomposition of graphs on surfaces

A. Schrijver

We discuss the following two theorems

Theorem. Let $G=(V,E)$ be a graph embedded on a compact surface $S$. Let $C_1,\dots,C_k$ be closed curves on $S$. Then: there exist pairwise disjoint simple circuits $\tilde{C}_1,\dots,\tilde{C}_k$ in $G$ where $\tilde{C}_i$ is freely homotopic to $C_i$ (for $i=1,\dots,k$), if and only if:

(i) there exist pairwise disjoint simple closed curves $\tilde{C}_1,\dots,\tilde{C}_k$ on $S$ so that $\tilde{C}_i$ is freely homotopic to $C_i$ (for $i=1,\dots,k$);

(ii) for each closed curve $D$ on $S$: $cr(G,D) \geq \sum_{i=1}^{k} \mathrm{mincr}(C_i;D)$;

(iii) for each "doubly odd" closed curve $D$ on $S$: $cr(G,D) > \sum_{i=1}^{k} \mathrm{mincr}(C_i;D)$.

Theorem. Let $G=(V,E)$ be an eulerian graph on the Klein bottle. Then the maximum number of pairwise edge-disjoint orientation-reversing circuits in $G$ is equal to the minimum number of edges intersecting all orientation-reversing circuits.

The first theorem is motivated by results of N. Robertson and P.D. Seymour on graph minors, and by VLSI-design algorithmics. The second theorem has applications to multi-commodity flows.

## Graphs not containing certain subgraphs

H.J. Prömel (Bonn)

For a finite graph $K$ let $Forb(K)$ denote the class of all finite graphs which do not contain $K$ as a (weak) subgraph. In the present talk we give a complete characterization of all those graphs $K$ with chromatic number at least 3 which have the property that almost all graphs in $\mathrm{Forb}(K)$ are bipartite. This extends earlier results of Erdős, Kleitman and Rothschild (1976) showing that almost all triangle-free graphs are bipartite and of Lamken and Rothschild (1985) showing that almost all graphs in $\mathrm{Forb}(C_{2k+3})$ are bipartite for every odd cycle $C_{2k+3}$.

## Hamilton Surfaces

Nora Hartsfield (Santa Cruz; Bellingham)

The two-dimensional analog of a Hamilton cycle in a graph is a genus embedding of the graph, composed of polygons. In approximately 1960, Ringel showed that the set of squares in the n-dimensional cube graph can be partitioned into classes so that each class forms a genus embedding of the graph after appropriate edge identifications have been made. In 1987, Hartsfield, B. Jackson, and G. Ringel showed that the set of squares in $K_{2n,2n}$ can be partitioned into classes $C_{i,j}$ such that $C_{i,j} \cup C_{k,l}$ is a Hamilton surface if and only if $(k-i, 2n-1)=1$ and $(l-j, 2n-1)=1$.

A finite undirected graph $\mathcal{G}=(V,E)$ without loops and multiple edges is said to be almost embeddable in $\mathcal{F}$ - where $\mathcal{F}$ is an arbitrary orientable surface

$\mathcal{F}_p$ ($p \in \mathbb{N}_0$) or an arbitrary non-orientable surface $\mathcal{F}_{\underline{q}}$ ($q \in \mathbb{N}$) or the spindle-surface $\mathcal{S}_1$ – if $\mathcal{G}$ is not embeddable in $\mathcal{F}$ and for each vertex $v \in V$ the graph $\mathcal{G} - v$ is embeddable in $\mathcal{F}$. In case of $\mathcal{F} = \mathcal{F}_0 \triangleq$ sphere Klaus Wagner gave a characterization of the set $\Delta(\mathcal{F}_0) = \{\mathcal{G} \mid \mathcal{G}$ is almost embeddable in ~~$\Delta(\mathcal{F}_0)$~~ $\mathcal{F}\}$ about 25 years ago. By means of the spindle-surface $\mathcal{S}_1$ and the proper partial ordering relation $\geq^{e}_{03}$ we obtain two (4) other characterizations of $\Delta(\mathcal{F}_0)$ which are shorter and more elegant.

Theorem 1: $\Delta(\mathcal{F}_0) = \bigcup_{i=1}^{4} \Gamma_i$, where $\Gamma_1 = \{\mathcal{G} \in \Delta(\mathcal{F}_0) \mid \mathcal{G} \notin S(K_5)\}$, $\Gamma_2 = \{F_2, F_4, F_5\} \cup \{\mathcal{G} \in \Delta(\mathcal{F}_0) \mid \mathcal{G}$ is Möbius graph$\}$, $\Gamma_3 = \{\mathcal{G} \in \Delta(\mathcal{F}_0) \mid \mathcal{G}$ is $\mathcal{S}_1$-graph$\}$, $\Gamma_4 = \Delta(\mathcal{F}_0) - (\bigcup_{i=1}^{3} \Gamma_i)$.

Theorem 2: $M^{e}_{03}(\Delta(\mathcal{F}_0)) = \bigcup_{i=1}^{4} \overline{\Gamma}_i$, where $\overline{\Gamma}_1 = \{K_5\}$, $\overline{\Gamma}_2 = \{F_1, F_2, K_{3,3}, K_{3,3} \cup \{a_1, a_2\}, K_{3,3} \cup \{a_1, a_2\} \cup \{a_1, a_3\}, \mathcal{G}_2\}$, $\overline{\Gamma}_3 = \{K_2 + C_n \mid n \geq 4\}$, $\overline{\Gamma}_4 = \{F_3, 1 * P_3\}$.

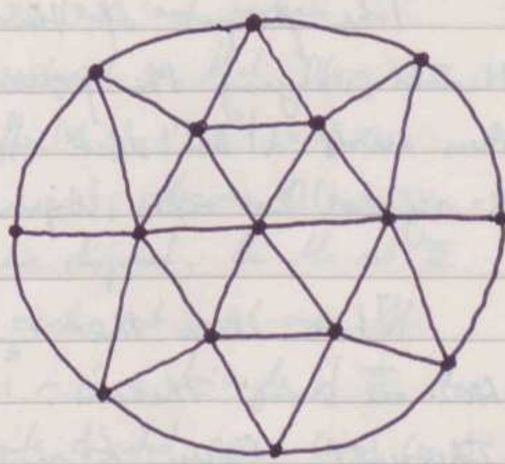
Rainer Bodendiek, Kiel

## Clean Triangulations

Gerhard Ringel Santa Cruz Californien

In a triangulation T of a surface S each face is a triangle. If also each triangle is a face then T is called <u>clean</u>.

If the number of triangles in T is minimal for a given S, T is called minimal. The picture is a minimal clean triangulation of the projective plane.



Denote by $\tau(S)$ the number of triangles in a minimal clean triangulation of S. Let $S_g$ be the orientable surface of genus g. We can prove that $\tau(S_1) = 24$ and that $\lim_{g\to\infty} \frac{\tau(S_g)}{g} = 4$.

This was joint work with Nora Hartsfield.

## Applications of Connectivity

The unique capabilities of fiber optic technology have made it necessary to implement new communication networks. One of the most important practical problems in this area is the design of minimum-cost survivable networks. This problem leads to interesting new connectivity concepts in graph theory. We show in this talk how "survivability" can be phrased in terms of connectivity parameters, we formulate integer programming models of the corresponding optimization problems, and present optimum solutions of some real-world problems. This talk is based on joint work with Clyde Monma and Mechthild Stoer.

Martin Grötschel (Augsburg)

## Decomposing a complete bipartite graph into copies of a k-regular graph

The purpose of this communication is to present some theorems varying the following theme: "For every natural number k there exists a smallest natural number $c_k$ such that every k-regular 2n-order bipartite graph B decomposes $K_{c_k n, c_k n}$."

It is clear that $c_k$, if it exists, is a multiple of k. Here are some theorems.

Theorem 1: Every k-regular bipartite graph on 2n vertices decomposes $K_{(k^2)!n,\,(k^2)!n}$.

Theorem 2: Let $G$ be a 3-regular bipartite graph on $2n$ vertices without any component a Heawood graph. Then $6G \mid K_{6n,6n}$. Moreover, if $n$ is even $G \mid K_{3n,3n}$ and if every vertex belongs to a 4-cycle $3G \mid K_{3n,3n}$

Roland Häggkvist

Cubic Combinatorial Maps

In recent years, there has been much interest in studying imbeddings of graphs in surfaces from a combinatorial point of view. For example, such imbeddings have been modelled by means of cubic combinatorial maps, i.e, cubic graphs with a proper edge colouring in three colours. We discuss the Jordan curve theorem in this context.

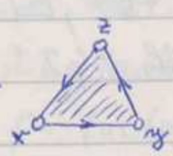
Charles Little

Groups and Graphs

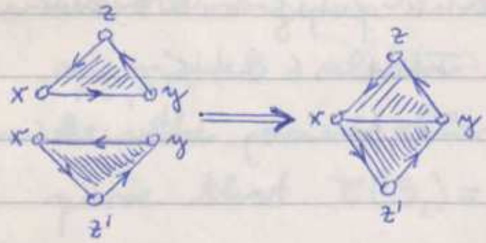
Let $G$ be a finite group.

A triple $(x, y, z)$ of elements $x, y, z \in G$ is said to be regular if $x \neq y \neq z \neq x$ and $xyz = e$. With $xyz = e$ also $yzx = e$ and $zxy = e$. If $(x, y, z)$ is a regular triple then $(y, x, z')$ with $z' = (yx)^{-1}$ is regular too.

To each regular triple $(x, y, z)$ an oriented triangle $D(x, y, z)$ with arcs $(x,y), (y,z), (z,x)$ and vertices

$(x, y, z) \Longrightarrow$ 

$x, y, z$ is assigned so that $D(x, y, z) \equiv D(x', y', z')$ iff $(x', y', z') \in \{(x,y,z), (y,z,x), (z,x,y)\}$; otherwise $D(x,y,z)$, $D(x' y', z')$ are disjoint. In the set $\Sigma$ of all so obtained triangles an arc $(x,y)$ occurs at most once. If $(x,y)$ is in some triangle of $\Sigma$ then $(y,x)$ is in a triangle of $\Sigma$ too. In the sense of combinatorial topology opposite directed arcs $(x,y), (y,x)$




are identified so that an edge $[x, y]$ is obtained. Identifying step by step all opposite directed edges triangulations of oriented surfaces are obtained. Thus to each group $G$ a set $S$ of triangulations of oriented surfaces is assigned.

The general problem is to find the interconnections between the (group-theoretical) properties of $G$ and the (topological, graph-theoretical) properties of $S$.

The main result presented to the conference was a characterization of a large class of triangulations of oriented surfaces corresponding to some group.

Heinz-Jürgen Voss

Transformationen Eulerscher Linien

(Transformations of Eulerian Trails, Трансформации Эйлеровских линий)

Using the concept of $\kappa$-transformations A. Kotzig showed in the 60-ies that any two eulerian trails of a connected eulerian graph $G$ can be transformed into each other by a sequence of $\kappa$-transformations. However, this concept does not suffice if one considers the set of all eulerian trails $G$ satisfying certain restrictions. For example, if $G$ is plane, then one may consider the set of all nonintersecting eulerian trails; if $G$ is arbitrary and satisfies $\delta(G) \geq 4$, one may consider all eulerian trails compatible with a given system of transitions. In such cases one has to introduce additional transformations. These are: 1) $\kappa$-detachments $\kappa'$: This operation transforms an eulerian trail into two subtrails $T', T''$ such that $E(T') \cap E(T'') = \emptyset$, $E(T') \cup E(T'') = E(G)$.

2) $\kappa$-absorption $\kappa''$: Here, two subtrails $T', T''$ as above are being transformed into an eulerian trail of $G$.

3) Denote $\kappa^* = \kappa''\kappa'$.

In fact, in the case of nonintersecting/compatible eulerian trails, $\kappa^*$-transformations are the appropriate tool to transform any two eulerian trails of the respective type into each other.

Herbert Fleischner

# Functions on Graph Languages Generated by Edge Replacement Systems

This is work done in collaboration with A. Habel and H.-J. Kreowski, Bremen.

A graph grammar is a finite system for the generation of graphs, the generated set of graphs is called a language. We have studied a specific type of grammars, namely hyperedge replacement systems, which include edge replacement systems.

Given a graph function f into the natural numbers the problem is the following: Decide whether for given graph grammar GG f is unbounded on the language of GG! This decision problem can be solved for edge replacement systems, if g is compatible with the derivation process in a specific way, involving addition and maximum taking only. Examples of such functions are the number of edges, the maximum degree and the maximum path length.

Walter Vogler

## f-factors of countable graphs

Let $G=(V,E)$ be a graph and $f: V \to E$ be a function such that $0 < f(x) \le d_G(x)$ for each $x \in V$. A subgraph $F=(V^*, E^*)$ of $G$ is said to be an f-factor if $d_F(x) \le f(x)$ for all $x \in V^*$. An f-factor $F=(V^*,E^*)$ is called perfect if $V^*=V$ and $d_F(x) = f(x)$ for each $x \in V$. Let $s$ be a fixed vertex and $F$ be a fixed f-factor of $G$. A vertex $v$ is called an outer vertex if there is an $F$-alternating trail from $s$ to $v$ starting with an edge of $E-F$ and ending with an edge of $F$. We show that a countable graph $G$ has no perfect $f$-factor if and only if there exist an $f$-factor $F$ of $G$ an unsaturated vertex $s$, a set $O$ of outer vertices with $s \in O$ and a set $L(v)$ of edges incident with $v$ for each $v \in O$ such that

(i) $E(F) \cap L(v) = \emptyset$ for each $v \in O$,

(ii) $|L(v)| = f(v) - d_F(v)$ for each $v \in O - \{s\}$ and $|L(s)| = f(s) - d_F(s) - 1$,

(iii) there is no $f$-augmenting trail $(v_i : i < k \leq \omega)$ starting at $s$ such that $\{v_{2i}, v_{2i+1}\} \notin L(v_{2i})$ for each $i$ with $2i+1 < k$.

K. Steffens

Perfect Graphs With Additional Min-Max Properties

A system $L$ of linear inequalities in the variables $x$ is called totally dual integral (TDI) if for every linear function $cx$ such that $c$ is all integers, the dual of the linear program : maximize $\{cx : x$ satisfies $L\}$ has an integral optimum solution or no optimum solution. A system $L$ is called box TDI if $L$ together with any inequalities $l \leq x \leq u$ is TDI. It is a corollary of work of Fulkerson and Lovász that: where $A$ is a 0-1 matrix with the 1-columns of any row not a proper subset of the 1-columns of any other row, and with no all-0 column, the system $L(G) = \{x : Ax \leq \underline{1}, x \geq \underline{0}\}$ is TDI if and only if $A$ is the matrix of maximal cliques (rows) versus nodes (columns) of a perfect graph.

We will describe a class of graphs in a graph-theoretic way, and characterize them as the as the graphs G for which L(G) is box TDI. We thus call these graphs box perfect. We also describe some classes of box perfect graphs.

Kathie Cameron

Let $\chi^C$ denote the incidence vector of a simple cycle C in an undirected graph G. The cycle cone C(G) is the cone generated by all these vectors and the cycle polytope P(G) is their convex hull. Seymour [1979] gave a linear system sufficient to define C(G) for general graphs. For any $\hat{x} \in C(G)$, let

$$\lambda_{\hat{x}} = \min\{\textstyle\sum \lambda_C : \hat{x} = \sum(\lambda_C \chi^C : C \text{ a cycle in } G),\ \lambda_C \geq 0\}$$

$$\mu_{\hat{x}} = \max\{ \text{———————————————} \}.$$

We let $L(G) = \{x \in C(G) : \lambda_x \leq 1\}$, $U(G) = \{x \in C(G) : \mu_x \geq 1\}$. Then $C(G) = L(G) \cup U(G)$ and $P(G) = L(G) \cap U(G)$.

For the case of Halin graphs, we describe min max relations for $\lambda_x, \mu_x$ which enable us to give explicit linear formulations of L(G), U(G) and hence P(G).

This is joint work with Collette Coullard, of Purdue university

Bill Pulleyblank

16-6-88

On Well-Quasi-Ordering Infinite Graphs

Robertson & Seymour proved that given an infinite sequence $G_1, G_2, \ldots$ of finite graphs there are indices $i, j$ such that $i<j$ and $G_i$ is isomorphic to a minor of $G_j$. We are interested in extending this result to infinite graphs. The infinite analogy is false in general, but holds for example if $G_1$ is finite and planar.

Robin Thomas
Prague, Czechoslovakia & Columbus, Ohio

On Seymour's self - minor conjecture

Paul Seymour conjectured that every infinite graph is isomorphic to a proper minor of itself. A counter-example to this conjecture, presented in the talk, is based on the counter-example to the Wagner conjecture about well-quasi-ordering of infinite graphs due to Robin Thomas. The validity of the conjecture for graphs with an isolated planar end has been shown and the implications, if Seymour's conjecture in the still open countable case is true, have been discussed.

Bogdan Oporowski
Columbus, Ohio, USA

Cai Ning's solution of the extremal problem for diameter 2 over the 3 element alphabet.

In his dissertation (Bielefeld 88) Cai proved:

In $\{-1,0,1\}^n$ a set of diameter $\le 2r$ (taxi metric $d(x,y) = \sum |x_i - y_i|$ has at most as many elements as the unit ball around 0. $(n \ge 2r+1)$

W. Deuber Bielefeld

Absolute retracts in graph theory

Both absolute retracts of reflexive graphs and absolute retracts of $n$-chromatic graphs $(n \ge 2)$ admit characterizations involving Helly type conditions, leading to polynomial time recognition procedures; for the bipartite case see Discrete Appl. Math. 16 (1987) 191–215, and for a survey see Mathematical Systems in Economics 110 (Athenäum Verlag, 1988).

This is joint work with E. Pesch, A. Dählmann & H. Schütte, resp.

H.-J. Bandelt, Bielefeld

Upper bounds of the chromatic number of graphs via clique number with restrictions to graphs' structure.

It's a small review of results on coloring of graphs from family $\mathcal{G}_i$ $(i=1,2,3,4)$ with given clique number or girth, where $\mathcal{G}_1 = \{G \mid \Delta(G) \le k\}$, $\mathcal{G}_2 = \{G \mid G$ is $k$-degenerate$\}$, $\mathcal{G}_3 = \{G \mid G \notin \mathcal{G}_2 \ \& \ \forall e \in E(G) \ \ G \setminus e \in \mathcal{G}_2\}$, $\mathcal{G}_4 = \{G \mid G \notin \mathcal{G}_2 \ \& \ \forall v \in V(G) \ \ G \setminus v \in \mathcal{G}_2\}$. The main result is a description of $\{G \in \mathcal{G}_3 \mid \chi(G) > k\}$.

A.V. Kostochka, Novosibirsk

# ON SUM OF CIRCUITS OF GRAPHS

Let $G = (V,E)$ be an undirected graph and $w: E \to \mathbb{Z}_+$ a non-negative integral vector on the edges such that $\sum(w(e) : e$ incident to $v)$ is even for every node $v \in V$. Assume furthermore that there are no 5 pairwise disjoint edges of $w$-value bigger than 1.

THEOREM It is possible to assign non-negative integers $z(C)$ to the circuits of $G$ so that $w = \sum(z(C) : C$ a circuit$)$ if and only if $w(e) \le \frac{1}{2} w(B)$ holds for every cut $B$ of $G$ and for every edge $e \in B$.

The Petersen graph (when $w$ is defined to be 2 on a specified perfect matching and 1 on the other 10 edges) shows that the theorem does not hold if 5 is replaced by 6 in the assumptions.

András FRANK
Budapest, Eötvös University

## THE TREE GAME AND THE ARBORESCENCE GAME

The Tree Game on a graph is a variant of the Shannon Switching Game solved by A. Lehman in 1964 in terms of matroids. The Arborescence Game is a directed version of the Tree Game.

In the Arborescence Game, two players, Black and White, plays alternately edges of a connected undirected graph $G$ with a distinguished vertex $x_0$. A move of Black resp. White consists of deleting resp. directing an unplayed edge. White wins if he forms a spanning arborescence of $G$ rooted at $x_0$.

We characterize winning positions in the case when $G$ is a union of two edge-disjoint spanning trees. A general strategy follows. (joint work with Y.O. Hamidoune)

Michel LAS VERGNAS.
C.N.R.S., PARIS

CLUMPS, MINIMAL ASYMMETRIC GRAPHS, AND INVOLUTIONS

A graph G is minimal asymmetric [minimal bilaterally asymmetric] if it has no non-trivial automorphism [no involution] but every proper non-trivial induced subgraph of G does. A useful parameter for classifying such graphs is the induced length, i.e. the length of a longest induced path. Denote by $\mathcal{A}_n$ the class of all minimal asymmetric graphs of induced length $n$; similarly $\mathcal{B}_n$ for bilateral symmetry. With J. Nešetřil we conjecture that there are only finitely many minimal asymmetric graphs, and that these are also the only minimal bilaterally asymmetric graphs. In fact, we believe there are only 18 such graphs (9 complementary pairs). We can prove that $\mathcal{A}_n = \mathcal{B}_n = \emptyset$ for $n \geqq 6$, $\mathcal{A}_5 = \mathcal{B}_5$ consists of two graphs, $\mathcal{A}_4 = \mathcal{B}_4$ of seven. That $\mathcal{A}_n = \mathcal{B}_n = \emptyset$ for $n = 1, 2$ is trivial. The only open case is $n = 3$. These results follow from the following considerably stronger theorem dealing with the structure of graphs which contain no minimal asymmetric subgraphs.

THEOREM. Let G be a finite graph of induced length $\geqq 4$, and suppose that G has no induced minimal asymmetric subgraph (actually, none of a list of 13 minimal asymmetric graphs). Then G contains a non-trivial clump (homogeneous set), or G has an involution.

Gert SABIDUSSI
Université de Montréal

## Girth and Face-Width of Embedded Graphs

Let $G$ be a graph embedded on an orientable surface. The face-width of the embedding is the minimum value $|C \cap G|$ taken over all noncontractible cycles $C$ in the surface. The face-width measures how densely the graph embeds on the surface; an embedding with large face-width represents a surface well. Robertson and Vitray conjectured that if the face-width $> 10^{10}$ then the embedding was a minimum genus embedding for $G$. We present counterexamples to this conjecture. Specifically, we construct a graph with two embedding in different orientable surface, each of face-width $> 10^{10}$. An essential ingredient is the construction of an embedded graph where both the graph and its dual are of large girth.

Dan Archdeacon (Burlington)

## Critically connected digraphs

A digraph is called critically connected, if it is connected, but the deletion of any vertex destroys the connectivity. It is proved that every critically connected, finite digraph has two vertices of outdegree one.

W. Mader (Hannover)

Distance-Regular Digraphs with Q-polynomial property

Let $(X, \{R_i\}_{0 \le i \le d})$ be a commutative nonsymmetric association scheme. Let $A_0, A_1, \cdots, A_d$ and $E_0, E_1, \cdots, E_d$ be the adjacency matrices and the primitive idempotents of the Bose-Mesner algebra over $\mathbb{C}$. Assume the association scheme is of (P and Q)-polynomial, i.e., there exist polynomials $v_i(x)$ and $v_i^*(x)$ of degree $i$ $(i = 0, 1, \cdots, d)$ such that $A_i = v_i(A_1)$ w.r.t. the ordinary multiplication and $nE_i = v_i^*(nE_1)$ $(n = |X|)$ w.r.t. the Hadamard product (entrywise product). Then it is shown that the association scheme is self-dual, i.e., $v_i(x) = v_i^*(x)$ for all $i$. This result is obtained by D. Leonard, independently.

The P-polynomial property is equivalent to the distance-regularity of the graph $(X, R_1)$. Notice that $v_i(x)$, $v_i^*(x)$ of symmetric (P and Q)-polynomial association schemes are Askey-Wilson polynomials (Leonard Theorem). We expect that $v_i(x)$, $v_i^*(x)$ of nonsymmetric (P and Q)-polynomial association schemes are a kind of Askey-Wilson polynomials with weight $w(x)$, $x \in \mathbb{C}$.

Tatsuro Ito (Joetsu)

Covering the Vertices of a Graph with Cycles

Our main result is: Let $G$ be a 2-connected graph with $n$ vertices and $k$ an integer such that $n > k \ge 1$. If the minimum degree of a vertex of $G$ is greater than or equal to $n/k+1$, then there exist $k$ cycles in $G$ which cover all the vertices of $G$.

We conjecture: Let $G$ be a graph and $k$ a positive integer. If the maximum size of an independent set of vertices in $G$ is less than or equal to $k$ times the vertex connectivity of $G$, then there exist $k$ cycles which cover all the vertices of $G$. Where $k=1$, this is a theorem of Erdös and Chvátal.

Y. Egawa 江川 (東京) Tokyo

# On the number of distinct induced subgraphs of a graph

Let $i(G)$ denote the number of distinct subgraphs of a graph $G$.

$G = \langle V, E \rangle$ is $\ell$-canonical if there is a partition

$\bigcup_{i<\ell} A_i = V$ such that for $x, x' \in A_i$, $y, y' \in A_j$

$\{x, y\} \in A_i \Leftrightarrow \{x', y'\} \in A_j$.

$G$ is $\ell, m$-almost canonical if there is a canonical graph $G_0 = \langle V, E_0 \rangle$ such that the symmetric difference $G \Delta G_0$ has only components of size at most $m$.

<u>Theorem</u> Assume $k \geq 1$ and $i(G_n) = o(n^{k+1})$ for a sequence $G_n = \langle V_n, E_n \rangle$ of graphs with $|V_n| = n$. Then there are $W_n \subset V_n$, $|W_n| = o(n)$ and $\ell_n, m_n$ such that

$\ell_n + m_n \leq k+1$ and $G[V_n \setminus W_n]$ is $\ell_n, m_n$-almost canonical.

This is a joint work with Paul Erdős.

András Hajnal

## On transitive graphs with polynomial growth

Results of Gromov and Trofimov imply that transitive, connected, locally finite infinite graphs of polynomial growth are closely related to Cayley graphs of virtually nilpotent groups. This suggests that the automorphism groups of such graphs retain some of the properties of

nilpotent groups.
A survey of results and open problems in this area is presented.

Wilfried IMRICH
Montanuniversität Leoben

End-faithful spanning trees in infinite graphs

Let $G$ be an infinite connected graph. Two rays $P, Q \subset G$ are end-equivalent if there exists a ray $R \subset G$ which meets both $P$ and $Q$ infinitely often. Let $\mathcal{E}(G)$ denote the set of the corresponding equivalence classes, the ends of $G$. If $T$ is a spanning tree of $G$, and if $P, Q$ are end-eq. rays in $T$, then clearly $P$ and $Q$ are also end-eq. in $G$. We thus have a natural map $\eta : \mathcal{E}(T) \to \mathcal{E}(G)$ mapping each end of $T$ to the end of $G$ containing it. In general $\eta$ need be neither 1-1 nor onto; if it is both, then $T$ is called end-faithful. The following question was raised by Halin in 1964:

Problem. Does every connected graph have an end-faithful spanning tree?

Halin settled this question in the affirmative for countable graphs $G$. We do the same for any $G$ not containing any subdivided infinite complete graph as a subgraph. The general problem remains open.

Reinhard Diestel, Cambridge

Paths and cycles in $k$-edge-connected graphs.

For a graph $G=(V(G), E(G))$, $\lambda(G)$ denotes the edge-connectivity of $G$. We call a graph $G$ weakly $k$-linked, if for every $k$ pairs of vertices $(s_i, t_i)$, there are edge-disjoint paths $P_1, \cdots, P_k$ such that $P_i$ joins $s_i$ and $t_i$ $(1 \le i \le k)$. Let

$$g(k) := \min\{n \mid \text{if } \lambda(G) \ge n, \text{ then } G \text{ is weakly } k\text{-linked}\}.$$

It is known that

$$g(2)=g(3)=3, \quad g(4)=5 \quad \text{and} \quad k \le g(k) \le 2k-3 \quad (k \ge 5).$$

Our results are

Theorem 1. If $\lambda(G) \ge 2k$ $(k \ge 2)$ and $f_1, f_2 \in E(G)$, then there is a cycle $C$ such that $\{f_1, f_2\} \subset E(C)$ and $\lambda(G-E(C)) \ge 2k-2$.

Theorem 2. $g(5) \le 6$, $g(6) \le 8$, $g(7) \le 9$.

$$g(3k) \le 4k \quad \text{and} \quad g(3k+1) \le g(3k+2) \le 4k+2 \quad (k \ge 2)$$

Haruko Okamura 岡村治子
(OSAKA)

Matchings, monotone path systems and some selected applications

Eine Reihe graphentheoretischer Probleme hängt eng mit linearer Algebra zusammen: so können z.B. die Anzahl der Gerüste beliebiger Graphen und die Anzahl der Linearfaktoren gewisser (insbesondere: ebener) Graphen durch die Determinante einer Matrix ausgedrückt werden, die

sich in einfacher Weise aus der Adjazenzmatrix des Graphen gewinnen läßt (Satz von KIRCHHOFF/TUTTE bzw. Sätze von KASTELEYN und LITTLE). Der Verfasser gibt eine graphentheoretische Methode zur Reduktion von linearen Gleichungssystemen und Determinanten an und benutzt diese zur Bestimmung der Anzahl der Linearfaktoren in Ausschnitten aus ebenen Gittergraphen. Die entspringenden Algorithmen – besonders diejenigen, die sich auf monotone Wege stützen – erweisen sich als sehr effizient. –
Die Resultate haben Anwendungen in der Chemie der aromatischen Kohlenwasserstoffe (Bindungsordnung) und in der Physik der Kristalloberflächen (Dimer-Problem).
(Teilweise gemeinsam mit K. Al-Khnaifes.)

Horst Sachs (Ilmenau)

## Entropy splitting and perfect graphs

The entropy of a graph G can be defined combinatorially as

$$H(p,G) = \min_{a} \left[ - \sum_{i \in V} p_i \log a_i \right]$$

where p is a probability distribution on V and a ranges through the vertex packing polytope of G. In a joint work with Csiszár, Körner, Marton and Simonyi we prove that

$$H(p,G) + H(p,\overline{G}) \geq H(p) = -\sum_{i \in V} p_i \log p_i$$

where equality holds if and only if the graph is perfect. This yields a rather strong covering property of perfect graphs by cliques and independent sets.

László Lovász (Budapest)

An Algorithm Related to Hadwiger's Graph Colouring Conjecture
Neil Robertson (Reporting on joint work with Paul Seymour.)

We have developed an algorithm, which in polynomial-time for fixed $k$ accepts a finite graph $G$ and either $k$-colors $G$ or exhibits a non-$k$-colorable minor $H$ of $G$. This is based on an excluded minor theorem which states that if the complete graph $K_{k+1}$ is not a minor of $G$, then one of four alternatives must hold: (1) for a given function $f$, $G$ has tree-width $\leq f(k)$, (2) $G$ has a vertex $x$ of valency $\leq k$, (3) $G$ has a so-called "one-sided clique separation", or (4) for some $X \subseteq V(G)$, $|X| \leq k-4$, the graph $G-X$ is planar. It is known that graphs with tree-width bounded have linear algorithms for computing chromatic number and making optimal colorings. Also planar graphs may be efficiently colored in 4-colours. The algorithm for Hadwiger colourings cycles through (2) and (3) reducing the size of graphs considered, taking minors in both cases, leaving a recipe for $k$-coloring if the pieces formed are all $k$-colorable. Condition (4) gives a $k$-coloring; and condition (1) gives a $k$-coloring or locates a non-$k$-colorable minor. By finiteness the algorithm terminates in a desired way. The fact that all non-$k$-colorable minors have bounded tree-width means their size can be bounded and hence they can be effectively determined. If Hadwiger's conjecture is true, the only non-$k$-colorable minor is the obvious one, $K_{k+1}$. itself.

Neil Robertson, Ohio State University
Columbus, Ohio 43210

Complex Graphs With a Large Girth

We state the following results:

① For every N, there exists a graph G and two linear orderings $\leq_1$, $\leq_2$ of $V(G)$ such that: 1) $\chi(G) \geq N$ 2) there is no edge $\{x, y\}$ and $z$ such that $z$ lies between $x$ and $y$ in both orderings $\leq_1$ and $\leq_2$.

② For every graph G, there exists a graph H such that:
1) girth H = girth G
2) for every partition $E(H) = E_1 \cup E_2$ a copy of G is contained in either $E_1$ or $E_2$.

Several relevant results are considered.

Jarik Nešetřil
Charles Univ., Prague

A Binary Search Problems for Graphs

We consider a search problem which generalises the group testing problems studied in papers of Chang/Hwang and Chang/Hwang/Lin. In its general form for arbitrary graphs, this problem was proposed by Aigner. Let $G = (V, E)$ be a finite simple graph, and let $e^* \in E$ be an unknown edge. In order to find $e^*$ we choose a sequence of test-~~sets~~ sets $A \subseteq V$ where after every test we are told whether or not $e^*$ is an edge of the subgraph induced by A. Find the minimum number $c(G)$ of tests required. We relate $c(G)$ to the coloring number $k(G)$. (The coloring number was introduced by Erdős and Hajnal in ~~the~~ about 1965.) $k(G)$ is the smallest number $k$ such that there exists an ordering $x_1, \ldots, x_n$ of the vertices of G such that $x_i$ has at most ~~k~~ k-1 neighbors among $x_1, \dots, x_{i-1}$ ($i = 1, \dots, n$).)

Thomas Andreae
FU Berlin

# VARIATIONSRECHNUNG
# 19.-25. JUNI 1988

## Line Singularities in Liquid Crystals – Robert M. Hardt

Here we discuss results concerning a model for nematic liquid crystals that admits the possibility of 1 dimensional singularities. The standard static model for a liquid crystal involves a unit vectorfield n defined on a spatial region $\Omega$. This n can be thought of as a statistical average of direction vectors of liquid crystal molecules and an order parameter $s \in [0,1]$. Critical points for the Oseen-Frank energy $\int_\Omega W(n, \nabla n)\,dx$ may admit point singularities and energy minimizers have been studied by Hardt, Kinderlehrer, and F.H. Lin (Comm. Math. Physics 105 (1986) 547-570) However their result that the singular set has dimension less than one rules out line singularities, as observed in experiments. C. Dafermos, following practices of physicists, formulated a 2-phase model in 1969. Ericksen suggested in 1986 that a more tractable model was possible using s as well as n as a variable. J. Maddocks found some planar critical points for Ericksen's functional. In joint work with F.H. Lin we show that weak solutions $(n,s)$ are Hölder continuous and exhibit specific examples where point and line singularities occur as $s^{-1}\{0\}$. This work is related to harmonic maps into cones and the notion of frequency.

## Curvature estimates and existence ~~of~~ for minimal surfaces – Rugang Ye

Consider stable minimal surfaces (of dimension 2). The following two themes are known: 1) (strong) curvature estimates imply theorems of Berstein type. 2) Berstein theorems imply curvature estimates. Here we add one more theme: 3) Curvature estimates (or Berstein theorems) for stable immersed minimal surfaces imply existence of embedded minimal surfaces. The real content of this theme is ~~an~~ a parametric approach to ~~an~~ (the existence problem for) embedded minimal surfaces. The main ingredients of this approach are: 1) Douglas' theorem which asserts the existence of minimal surfaces of a given topological type (satisfying some boundary condition) under the so-called Douglas' condition, 2) Immersion theorems which garantee the immersed character of the minimal surfaces provided by Douglas' theorem, 3) a classical cut-paste argument which leads to the Douglas conditions in case the above immersed surfaces are not embedded, thereby providing minimal surfaces of higher topology, 4) curv. estimates. We apply this approach to minimal surfaces with free boundary. The immersion theory is delicate, ~~one has to combine~~ ~~since~~ non-immersed surfaces lead to new Douglas conditions. A general existence theorem for emb. minimal surf. with free boundary is proved.

## An existence result for quasilinear elliptic equations

[Giuseppe BUTTAZZO
Univ. of Ferrara]

Quasilinear elliptic equations of the form $(*) \begin{cases} -\mathrm{div}\,(a(x,u)\,Du) = f(x) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$ are considered. Here $\Omega$ is a bounded open subset of $\mathbb{R}^n$, $f \in H^{-1}(\Omega)$, and $a(x,s)$ is an $n \times n$ matrix satisfying the usual ellipticity and boundedness conditions $\begin{cases} \lambda |z|^2 \leq \langle a(x,s) z, z \rangle \\ |a(x,s)| \leq c \end{cases}$ for every $x \in \Omega$, $s \in \mathbb{R}$, $z \in \mathbb{R}^n$. The existence for problem $(*)$ is studied when $a(x,s)$ does not necessarily satisfy the Carathéodory continuity condition. An example of non-existence for problem $(*)$ is shown, with $a(x,s)$ highly discontinuous in $(x,s)$.

# Global existence and partial regularity for the heat flow for harmonic maps

Given two compact manifolds $\mathcal{M}$ and $\mathcal{N}$, $\partial\mathcal{M} = \emptyset = \partial\mathcal{N}$, $\mathcal{N} \subset \mathbb{R}^n$ consider the evolution problem for harmonic maps from $\mathcal{M}$ into $\mathcal{N}$

(1) $$\partial_t u - \Delta_{\mathcal{M}} u = -\Gamma_{\mathcal{N}}(u)(\nabla u, \nabla u)_{\mathcal{M}} \in (T_u \mathcal{N})^{\perp} \subset \mathbb{R}^n$$

with initial data

(2) $$u\big|_{t=0} = u_0 : \mathcal{M} \to \mathcal{N}.$$

Here, $\Delta_{\mathcal{M}}$ is the Laplace-Beltrami operator on $\mathcal{M}$ and $\Gamma_{\mathcal{N}}(u)(\nabla u, \nabla u)_{\mathcal{M}}$ is a term involving the Christoffel symbols of the induced metric on $\mathcal{N}$, growing quadratically in $\nabla u$ and orthogonal to $T_{u(x,t)}\mathcal{N}$, $(x,t) \in \mathcal{M} \times \mathbb{R}_+$.

The following results extend the classical Eells-Sampson result to arbitrary target manifolds $\mathcal{N}$ as above:

<u>Theorem 1</u> (Struwe, '85): If $\dim \mathcal{M} = 2$, then for any $u_0 \in H^{1,2}(\mathcal{M}, \mathcal{N})$ there exists a global weak solution $u$ of (1), (2) with finite energy and which is regular with exception of at most finitely many points $(\bar{x}, \bar{t})$ where non-constant harmonic maps $\bar{u} : S^2 \cong \overline{\mathbb{R}^2} \to \mathcal{N}$ separate. $u$ is unique in this class. Finally, as $t \to \infty$ suitably, $u(t)$ converges to a regular harmonic map $u_\infty : \mathcal{M} \to \mathcal{N}$, weakly in $H^{1,2}(\mathcal{M}, \mathcal{N})$.

<u>Theorem 2</u> (Chen-Struwe, '88): If $\dim \mathcal{M} > 2$, then for any $u_0 \in H^{1,2}(\mathcal{M}, \mathcal{N})$ (1), (2) admits a global weak solution $u$, converging to a weakly harmonic map $u_\infty : \mathcal{M} \to \mathcal{N}$ as $t \to \infty$ suitably. $u$ and $u_\infty$ are regular off closed singular sets of co-dimension $\geq 2$ (in the parabolic metric).

Theorem 2 is based on a combination of a monotonicity formula for (1), cp. Struwe, '88, with a penalty approach to (1), cp. Chen, '88.

Michael Struwe, 22.6.88

## Harmonic maps between spheres and ellipsoids

[ANDREA RATTO (Un. of WARWICK)]

I would like to present the results of a paper by James Eells and me (to appear in Publ. MATH. I.H.E.S.): in particular, we proved that classical homotopy groups of spheres (such as $\pi_n(S^n)$, $n \in \mathbb{N}$, $\pi_3(S^2)$) can be represented by a harmonic map provided that suitable ellipsoidal metrics are introduced.
I will also discuss the following problem, which is closely related to the above results: how does the choice of the metric influences the existence of certain harmonic maps?
The proof of the results is based on the study of an associated 1-dimensional variational problem, according to a new method recently introduced by Ding.

## Surfaces of Gauß curvature 1 and arbitrary genus

We look at a new type of Plateau problem: For a given curve $\Gamma \subset \mathbb{R}^3$ and fixed genus $g$ we look for a 2-surface $\mathcal{S} \subset \mathbb{R}^3$ with boundary $\Gamma$ and genus $g$, and with Gauß curvature $K(\mathcal{S}) \equiv 1$.
If $\Gamma$ is sufficiently close to a smooth curve $\Gamma_0 \subset S^2$ which 'winds around twice' and $\Gamma$ is in a submanifold of curves of finite codimension, then we can even fix the branch points of $\mathcal{S}$ and solve the problem.
The Gauß curvature 1 is defined at the branch points even as the spherical image infinitesimally. But the result is not expected against the theorem of Gauß-Bonnet, since the integrals of the curvature and along the boundary $\partial\mathcal{S}$ are positive, and the Euler characteristic is very negative. We have therefore to assume that the branch points are carrying a negative mass, even if the spherical image is positive.

The existence proof depends essentially on an explicit calculation of the commutator between the singular term of the metric at a branch point and all the terms of the Gauß curvature operator in polar coordinates of $\mathbb{R}^3$. The Taylor development of the Gauß curvature at $\{r=1\}$ has a leading term being the trace of the curvatures and the leading term "mean curvature" behaves nicely.

Reinhold Böhme 22.6.88

## Existence of closed convex hypersurfaces with prescribed Gauss Curvature

Let $f: \mathbb{R}^{n+1} \to [0,\infty)$ be a continuous function. When $f$ is the Gauss curvature of some closed convex hypersurface in $\mathbb{R}^{n+1}$? A possible approach is to look for a solution among graphs of functions over $S^n$. Then the problem can be formulated as a problem of solvability of a special equation of Monge-Ampère type on $S^n$. For a smooth ~~da~~ $f$ it is possible to formulate conditions for solvability of this equation (V. Oliker, Comm. on PDE, 1984). We consider now a different approach via variational methods. Namely we construct a functional for which the above mentioned equation is the Euler-Lagrange equation in ~~sa~~ a certain weak sense. It is shown that this functional admits the first variation and the solution to the "Gauss curvature" problem can be found as minimizers of the functional. The assumptions on the data in this approach are geometrically natural and simple. ~~For example~~ In particular, $f$ is required to be only nonnegative and continuous + a condition assuring existence of $C^0$-bounds. The corresponding results appeared in Trans. AMS, 86.

Vladimir Oliker, 23. 6. 88

On the Expansion of Convex Hypersurfaces by Symmetric Functions of Their Principal Radii of Curvature

Let $M_0$ be a smooth closed uniformly convex hypersurface in $\mathbb{R}^{n+1}$ given by an embedding $X_0 : S^n \to \mathbb{R}^{n+1}$, and consider the initial value problem

$$\frac{\partial X}{\partial t}(x,t) = k(x,t)\nu(x,t),$$

$$X(\cdot,0) = X_0,$$

where $\nu(\cdot,t)$ is the outer unit normal vectorfield to the hypersurfaces $M_t$ parametrized by $X(\cdot,t)$ and $k(\cdot,t) > 0$ is some curvature function of $M_t$. Under some reasonable conditions it can be shown that a solution of this problem exists for all time, for each $t \geq 0$, $X(\cdot,t)$ is a parametrization of a smooth closed uniformly convex hypersurface $M_t \subset \mathbb{R}^{n+1}$, and $M_t$ becomes round as $t \to \infty$. This problem can be reduced to the initial value problem

$$\frac{\partial h}{\partial t} = F(\nabla_{ij} h + \delta_{ij} h) \quad \text{on } S^n \times [0,\infty)$$

$$h(\cdot,0) = h_0 \quad , \quad [\nabla_{ij} h + \delta_{ij} h] > 0$$

where $h$ is the support fuction of the hypersurface represented by $X(\cdot,t)$, and $F$ is a function defined by the curvature function $k$. This problem has a unique smooth solution for all time, and after a suitable rescaling $h$ converges to a constant $h^*$ as $t \to \infty$. The corresponding assertions for the first problem follow from this.

A similar result concerning the expansion of convex hypersurfaces has recently been proved by Gerhard Huisken using a different method.

John Urbas 23/6/88

<u>Mean Curvature Evolution of Entire Graphs and a New Bernstein Type Result</u>

The following represents joint work with G. Huisken, Canberra.
Mean curvature evolution of hypersurfaces in Euclidean space has attracted considerable interest over the last years. However, only compact surfaces have been studied so far. We present methods suitable to deal with the evolution of entire graphs. An interesting feature is that linearly growing initial graphs become asymptotically selfsimilar during the evolution.

We furthermore apply our techniques to establish new curvature estimates for mean curvature graphs. Apart from providing a natural interior curvature bound for capillary surfaces, they lead to a new Bernstein type result for minimal graphs. We show in fact, that any entire solution of the minimal surface equation satisfying

$$|Du(x)| = o(|x|)$$

has to be an affine function.

Klaus Ecker 23/6/88

REGULARITY OF MINIMIZERS OF INTEGRALS OF THE CALCULUS OF VARIATIONS WITH NON STANDARD GROWTH CONDITIONS · P. Marcellini (Firenze)

Let $f \in C^2(\mathbb{R}^n)$ be a function satisfying the following properties:

(i) $$m \sum_{j=1}^{n} |\xi_j|^{q_j} \leq f(\xi) \leq M \left(1 + \sum_{j=1}^{n} |\xi_j|^{q_j}\right)$$

(ii) $$m |\lambda|^2 \leq \sum_{ij=1}^{n} f_{\xi_i \xi_j}(\xi)\, \lambda_i \lambda_j \leq M \left(1 + \sum_{j=1}^{n} |\xi_j|^{q_j - 2}\right) |\lambda|^2$$

for some positive constants $m, M$, for every $\xi, \lambda \in \mathbb{R}^n$ $(n \geq 2)$ and for some exponents $q_j$ such that

$$2 \leq q_j < \frac{2n}{n-2}, \quad \forall j = 1, 2, \dots, n.$$

Let $\Omega$ be a bounded open set of $\mathbb{R}^n$. Then, it can be proved that every minimizer $u$ of the integral

$$F(u) = \int_\Omega f(Du)\,dx$$

in the Sobolev class

$$\left\{ v \in H^{1,2}(\Omega) : \ v_{x_j} \in L^{q_j}(\Omega),\ \forall j = 1, 2, \dots, n \right\}$$

has the gradient $Du$ locally bounded in $\Omega$.

It follows that, if $f \in C^{k,\alpha}(\mathbb{R}^n)$ for some $k \geq 2$, then $u \in C^{k,\alpha}_{loc}(\Omega)$.

Paolo Marcellini 23. 6. 88

Calculus of Variations for Elastic Crystals. M. Chipot (Metz)

The energy density of an elastic crystal is a function $W$ satisfying

$$W(QFH) = W(F) \quad \forall F \in M^+,\ \forall Q \in O_3^+,\ \forall H \in H$$

$M^+$ is the set of matrices with positive determinant,

$O_3^+ = \{ Q \mid Q^T Q = Id,\ \det Q = 1 \}$,

$H = L\, GL^+(\mathbb{Z}^3) L^{-1}$, $GL^+(\mathbb{Z}^3)$ is the set of matrices with integer entries with determinant 1, $L$ is the basis of the lattice of the crystal.

Under certain conditions we prove

$$\operatorname*{Inf}_{A_\Omega(\varphi)} \int_\Omega \phi^{**}(\det \nabla v)\,dx = \operatorname*{Inf}_{A_\Omega(\varphi)} \int_\Omega W(\nabla v)\,dx$$

where $A_\Omega(\varphi) = \{ u : \Omega \to \mathbb{R}^3 \mid u \in (W^{1,\infty}(\Omega))^3,\ u = \varphi \text{ on } \partial\Omega,\ \det \nabla u > 0 \text{ a.e} \}$ and $\phi^{**}$ is the convex minorant of the subenergy defined by J. Ericksen as

$$\phi(t) = \operatorname*{Inf}_{\det A = t} W(A)$$

This result is part of a joint work with D. Kinderlehrer.

# The motion of convex hypersurfaces along symmetric curvature functions

Let $F_0 : M^n \longrightarrow \mathbb{R}^{n+1}$ be the smooth embedding of a closed, uniformly convex hypersurface in Euclidean space. We consider the evolution equation

$$(*) \begin{cases} \frac{d}{dt} F(p,t) = \left(f^{-1} \cdot \nu\right)(p,t) & \\ F(p,0) = F_0(p) \, , & p \in M^n \end{cases}$$

where $\nu$ is the exterior unit normal to the hypersurface and $f$ is a smooth, positive and symmetric function of the principal curvatures on $M^n$. We give natural structure conditions for $f$ which ensure the existence of a longtime solution to $(*)$, which becomes more and more round as it expands. We also show that the same structure conditions imply an analoguous behaviour for contracting hypersurfaces moving in direction $-\nu$ with speed $f$.

A similar result was recently obtained by John Urbas using different techniques involving the support function of the hypersurfaces.

Gerhard Huisken

# Regularity of harmonic mappings at a free boundary

This is a report on joint work with Franz Duzaar at Düsseldorf. We consider the following situation: $M^m$ is a Riemannian manifold of dimension $m$ ($m \geq 3$ is the case of interest) called the parameter domain and $\Sigma \neq \emptyset$, the free boundary of $M$, is an open subset of $\partial M$; the target manifold $N^n$ is a Riemannian manifold which is isometrically embedded as a closed submanifold of some $\mathbb{R}^{n+k}$ and the supporting manifold for the free boundary values is a closed submanifold $S^s$ of $N$. There are no restrictions on the dimensions $n, k, s$. The Sobolev space $H^{1,2}(M,N)$ consists of mappings $v: M \to \mathbb{R}^{n+k}$ in the usual linear Sobolev space $H^{1,2}(M,\mathbb{R}^{n+k})$ such that $v(x) \in N$ for almost all $x \in M$. For such mappings the energy $E(v) = \int_M |Dv|_M^2 \, d\,vol_M$ is defined where the norm $|Dv|_M^2 = \operatorname{trace}(Dv)^* Dv$ is taken with respect to the Riemannian metric on $M$. We say that $u \in H^{1,2}(M,N)$ is energy minimizing (locally on $M \cup \Sigma$) with respect to the free boundary condition $u(\Sigma) \subset S$ (to be understood in the sense of the trace of $u$ on $\Sigma$ a.e.) if $E(u) \leq E(v)$ for all $v \in H^{1,2}(M,N)$ such that $v(\Sigma) \subset S$ and $v = u$ outside some (sufficiently small) compact subset of $M \cup \Sigma$.

Existence of energy minimizing maps is easily obtained with the direct method of the calculus of variations by minimizing $E$ in a class $\mathcal{F} = \{v \in H^{1,2}(M,N) : v(\Sigma) \subset S,\ v|_\Gamma = g\}$ where $\Gamma \subset \partial M \setminus \Sigma$ has positive $(m-1)$-measure and $g: \Gamma \to N$ is prescribed. (One then calls $v|_\Gamma = g$ an additional fixed boundary condition; of course, $\mathcal{F}$ should be nonempty.) In the interesting case $\Sigma = \partial M$, i.e. only a free boundary condition is prescribed, the minimizers of $E$ on $\{H^{1,2}(M,N) \ni v : v(\Sigma) \subset S\}$ will be constant mappings $u: M \cup \Sigma \to S$, however, and the problem of finding nontrivial mappings which are locally energy minimizing on $M \cup \Sigma$ remains.

Regularity of energy minimizing mappings $u$ in the interior $M$ and at a fixed boundary $\Gamma$ has been studied by Schoen & Uhlenbeck in the general situation described above. We are able to extend their results to the free boundary situation and obtain the optimal estimate for the size of the singular set of $u$ in $\Sigma$.

Theorem A  Suppose $u \in H^{1,2}(M,N)$ is locally on $M \cup \Sigma$ energy minimizing with respect to the free boundary condition $u(\Sigma) \subset S$ and $u$ is bounded. Then the Hausdorff $(m-2)$-measure of the singular set in the free boundary $\Sigma \cap \mathrm{Sing}(u)$ vanishes. On the regular set $\Sigma \cap \mathrm{Reg}(u)$, $u$ is as smooth up to the boundary $\Sigma$ as the data $M, \Sigma, N, S$ allow, $u$ satisfies the differential equation for harmonic mappings from $M$ into $N$ and the natural boundary condition $\partial_{\nu(x)} u(x) \perp \mathrm{Tan}_{u(x)} S$ for $x \in \Sigma \cap \mathrm{Reg}(u)$.

If $u$ is unbounded we have the same result provided we assume some global curvature bounds on $N \subset \mathbb{R}^{n+k}$ and $S \subset N$. The natural boundary condition corresponds, in suitable local coordinates on $N$, to $n-s$ conditions of Dirichlet type $u_i = 0$ and $s$ conditions of Neumann type $\partial_\nu u_j = 0$ on $\Sigma$. The major difficulty in the proof of theorem A is that one cannot localize the problem in the target manifold as long as one does not know continuity of $u$ nor can one use reflection across $S$ in $N$ since the image $u(M)$ need not be bounded away from the focal set of $S$ in $N$. We overcome this problem by combining the methods of Schoen & Uhlenbeck with a novel "partial reflection construction" to prove continuity of $u$ on a set of full $(m-2)$-measure in $M \cup \Sigma$. We then can use reflection methods to prove higher regularity of $u$ on $\mathrm{Reg}\,u$ up to $\Sigma \cap \mathrm{Reg}\,u$ as in the work of Gulliver & Jost. We also can reduce the dimension of the singular set to obtain the optimal

Theorem B  If $u(\Sigma)$ is bounded then Hausdorff-dim$(\Sigma \cap \mathrm{Sing}(u)) \leq m-3$.
(And $\Sigma \cap \mathrm{Sing}(u)$ is discrete in $\mathrm{Sing}(u)$ in case $m=3$)
One can further reduce to $\dim(\Sigma \cap \mathrm{Sing}(u)) \leq m-\ell-1$ if one knows that all "blow-up-tangent-maps" in dimensions $\leq \ell$ are trivial.

Examples show that singularities at the free boundary can be caused by the geometry or by the topology of the data. We can construct a domain $M \subset \mathbb{R}^3$ with 2 boundary components $\Gamma, \Sigma$ and Dirichlet data $g : \Gamma \to \mathbb{R}^3$ such that the minimizer $u$ in $\{ v \in H^{1,2}(M, \mathbb{R}^3) : v_{|\Gamma} = g,\ v(\Sigma) \subset S \}$ for $S = S^1 \times \{0\}$ or $S = S^1 \times \mathbb{R}$ must have isolated singularities on $\Sigma$. ($u$ is a classical harmonic function!). If $M = [0,1] \times T^{m-1}$, $\Gamma = \{0\} \times T^{m-1}$, $\Sigma = \{1\} \times T^{m-1}$, $N = T^n$ ($T^\ell$ flat tori, $m \geq 3$, $n \geq m-1$) then we can find $S \subset N$, $g : \Gamma \to N$ such that $C^0(M \cup \Gamma \cup \Sigma, N)$ contains no admissible map but $\mathcal{F}$ above is nonempty, hence minimizers $u \in \mathcal{F}$ exist and have singularities (only) on $\Sigma$.

Klaus Steffen

## On the Holomorphic and Geodesic Convexity of Dirichlet's Energy on Teichmüller's Moduli Space

A. J. TROMBA

Let $T(M)$ be Teichmüller's moduli space of a surface $M$ of fixed genus $p$, $p>1$. Let $g_0$ be a fixed metric of Gauss curvature $-1$, and $g$ an arbitrary metric with the same curvature. Let $s(g)$ be the unique harmonic map from $(M,g)$ to $(M,g_0)$ homotopic to the identity and let $E(g)$ be its Dirichlet energy. Then $E(g)$ can be considered as a map on Teichmüller's moduli space. Now Teichmüller's moduli space carries a natural complex structure and a natural metric, called the Weil-Petersson metric. With respect to the complex structure we have the result:

Theorem A. $E : T(M) \to R$ is proper and holomorphically convex, i.e. w.r.t. the natural complex structure

$$\frac{\partial^2 E}{\partial z \partial \bar z} > 0$$

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Let $\sigma(t)$ be any W.P. geodesic then we have

Theorem B. $E$ is convex along W.P. geodesics, i.e.

$$\frac{d^2}{dt^2}\left(E(\sigma(t))\right) > 0$$

These convexity properties yield a short proof of Nielsen's famous conjecture on the existence of fixed points for the action of ~~the~~ subgroups of finite order of the surface modular group.

A. Tromba 24/6/88

Regularity of viscosity solutions of second order, nonlinear elliptic equations. (Neil Trudinger)

We are concerned with the regularity of weak solutions in the viscosity sense of Crandall and Lions (or equivalently in the sense of the classical Perron process) of second order, elliptic equations of the general form,

$$F[u] = F(x, u, Du, D^2u) = 0 .$$

Here, $F \in C^0(\Gamma)$, $\Gamma = \Omega \times \mathbb{R} \times \mathbb{R}^n \times \mathcal{S}^n$, $\Omega$ is a domain in $\mathbb{R}^n$, $\mathcal{S}^n$ denotes the linear space of real, $n \times n$, symmetric matrices.

For uniformly elliptic operators, satisfying natural continuity and structure conditions, we proved that continuous viscosity solutions are continuously differentiable with Hölder continuous derivatives [1] [2] and moreover are twice differentiable almost everywhere [3]. The techniques involve semi-concave approximation (as introduced by Jensen for comparison principles), introduction of new variables and the Krylov-Safonov estimates, (particularly the weak Harnack inequality) for linear equations. The second result depends on an idea of Nadirashvili - the backwards use of the Aleksandrov maximum principle. The $C^{1,\alpha}$ regularity was also obtained independently by Caffarelli. Further regularity is an open problem except when $F$ is concave (or convex) with respect to $D^2u$ or $n = 2$.

References: [1] Hölder estimates for fully nonlinear elliptic eqns. Proc. Roy. Soc. Edinburgh. 108A, 1988, 57-65.

[2] On regularity and existence of viscosity solutions of nonlinear 2nd order elliptic eqns. to appear in Volume dedicated to De Giorgi's 60th birthday.

[3] Still in preparation.

# SURFACES OF MINIMAL AREA ENCLOSING BODIES IN $\mathbb{R}^3$.

Roberta MUSINA - SISSA Trieste -
(with Gianni MANCINI - Bologna)

We are interested in the problem of finding a closed (namely: $S^2$-type) surface having minimal area among all surfaces which are parametrized by $S^2$ and which "enclose" a given connected body ~~in~~ $\Omega$ in $\mathbb{R}^3$.

We prove the existence of such a surface for every regular obstacle $\Omega$, by showing that there exists a harmonic map from $S^2$ into $\mathbb{R}^3 \setminus \Omega$, which is not homotopic to a constant.

In case of unconnected obstacles, we give a sufficient condition for the existence of such a surface. We also study the Plateau's Problem for disk-type minimal surfaces with obstructions, and we prove that a suitable "Douglas criterion" is a sufficient condition for existence.

Roberta Musina Cassano
24.6.88

# ASYMPTOTIC BEHAVIOUR OF MINIMAL SURFACES WITH OBSTACLES

Gianni DAL MASO (SISSA Trieste)
(with M. Carriero, A. Leaci, E. Pascali)

The asymptotic behaviour, as $h \to +\infty$, of the solutions of obstacle problems of the form

$$(P_h) \quad \min_{u \geq \psi_h} \left[ \int_\Omega \sqrt{1+|Du|^2} + \int_{\partial\Omega} |u-\varphi| \, d\mathcal{H}^{n-1} \right]$$

can be sometimes described in terms of a limit problem

$$(P) \quad \min_u \left[ \int_\Omega \sqrt{1+|Du|^2} + \int_{\partial\Omega} |u-\varphi| \, d\mathcal{H}^{n-1} + G(u) \right]$$

which satisfies the following conditions:

(1) the minimum values of $(P_h)$ converge to the minimum value of $(P)$ as $h \to +\infty$;

(2) if, for every $h \in \mathbb{N}$, $u_h$ is a minimum point of $(P_h)$ in $BV(\Omega)$, then, up to a subsequence, $(u_h)$ converges in $L^1(\Omega)$ to a minimum point $u$ of problem $(P)$.

Let $\varphi \in L^1(\partial\Omega)$ and let $(\psi_h)$ be an arbitrary sequence of obstacles (i.e. functions from $\Omega$ to $\overline{\mathbb{R}}$) which satisfies the following compatibility condition: there exists $w \in W^{1,1}(\Omega)$ such that $w = \varphi$ on $\partial\Omega$ and $w \geq \psi_h$ in $\Omega$ for every $h \in \mathbb{N}$. Then there exists a subsequence $(\psi_{h_k})$ of $(\psi_h)$ for which the corresponding sequence of problems $(P_{h_k})$ admits a limit problem $(P)$ in the sense considered in (1) and (2). The functional $G$ which appears in $(P)$ does not depend on $\varphi$ and can be represented in the form

$$G(u) = \int_\Omega g(x, \bar{u}(x))\, d\mu(x),$$

where

(a) $g: \Omega \times \mathbb{R} \to [0, +\infty]$ is a Borel function, with $g(x, \cdot)$ convex, non-increasing, and lower semicontinuous on $\mathbb{R}$ for every $x \in \Omega$;

(b) $\mu$ is a non-negative Borel measure on $\Omega$, absolutely continuous with respect to $\mathcal{H}^{n-1}$;

(c) $\bar{u}(x)$ denotes the approximate upper limit of $u$ at the point $x$.

Gianni Dal Maso
24.6.88

# Geometry of level sets of entire solutions of semilinear elliptic equations

Luciano Modica (Pisa)

Consider a smooth real function $F$ and let $u$ be a smooth solution on the whole of $\mathbb{R}^n$ of the equation $\Delta u = F'(u)$. Assume that $F$ is non-negative and $u$ is uniformly bounded on $\mathbb{R}^n$. Note that our equation is the Euler-Lagrange equation of the following non-negative energy integral:

$$\mathcal{E}(v;\Omega) = \int_\Omega \left(\frac{1}{2}|\nabla v|^2 + F(v)\right)dx.$$

Theorem. Suppose $n < 8$. If $u$ is locally minimizing energy, i.e.

$$\mathcal{E}(u;A) \leq \mathcal{E}(u+\varphi;A) \quad \forall A \subset\subset \mathbb{R}^n, \forall \varphi \in C_0^1(A),$$

then all level sets of $u$ are parallel hyperplanes.

Luciano Modica

24.6.88

Minimal surfaces with a free boundary on a polyhedron

Jürgen Jost

Thm.: Let $\Sigma$ be a convex polyhedron in $E^3$. Then there exists an embedded minimal disk $M$ meeting $\Sigma$ orthogonally along its boundary. $M$ is nontrivial in the sense that it is not contained in a face of $\Sigma$, nor does it contain an edge of $\Sigma$ in its boundary.

The proof uses approximation of $\Sigma$ by smooth surfaces where previous results of the author are available, barrier constructions utilizing catenoids, a blow-up technique and a regularity theorem of the author.

Mathematical foundations of string theory

Jürgen Jost

In this series of lectures, a mathematical approach to the quantization of Plateau's problem is described, the physical motivation is discussed, and the necessary mathematical tools from Riemannian geometry, global analysis, nonlinear elliptic PDE, Riemann surfaces, Teichmüller theory, and algebraic geometry are presented.

Jürgen Jost
24.6.88

# Variational Convergence of Minimal Submanifolds to a Singular Variety

Robert Gulliver, Minneapolis

Suppose a Lipschitz Riemannian manifold $M_h$ is represented by a mapping $\Phi_h : \Omega_h \longrightarrow M_h$, where $\Omega_h = \Omega \setminus E_h \subset \mathbb{R}^n$, and $\Phi_h$ is locally bi-Lipschitz and one-to-one except on $\partial E_h$, where it is two-to-one: $\Phi_h(x) = \Phi_h(Tx)$ for $x \in \partial E_h$. Here $T : \Omega \longrightarrow \Omega$ is a Lipschitz involution and corresponds to an isometry of $M_h$. The Dirichlet integral on $M_h$ is represented in $\Omega$ by the functional

$$D_h(u) = \int_{\Omega_h} g_h^{ij} \partial_i u \, \partial_j u \sqrt{\det(g_{ij}^h)}\, dx + \frac{1}{4}\int_{\bar\Omega} (u(x) - u(Tx))^2 \, d\nu_h .$$

Here $\nu_h : \mathcal{B}(\bar\Omega) \to [0,\infty]$ is the measure defined by $\nu_h(A) := +\infty$ if $\operatorname{cap}(A \cap \partial E_h) > 0$, and zero otherwise. The second term has the effect of enforcing the periodicity condition $u(x) = u(Tx)$ for $x \in \partial E_h$, which implies that $u$ is equivalent to an $H^1$-function on $M_h$. Let $b \in L^\infty(\Omega)$ be the weak-$L^\infty$ limit of the volume functions $\sqrt{\det(g_{ij}^h)}$ as $h \to \infty$ (after passing to a subsequence). Then we may consider $D_h$ as defined on $L^2(\Omega, b)$ by defining $D_h(u) = +\infty$ if $u \notin H^1(\Omega)$.

Theorem (Dal Maso – Mosco – G.) Suppose that (1) The domains $\Omega_h$ are uniformly strongly connected in $\Omega$, that is, for some extensions $\Pi_h : H^1(\Omega_h) \to H^1(\Omega)$ there holds $\|\Pi_h u\|_{H^1(\Omega)} \le c_0 \|u\|_{H^1(\Omega_h)}$; (2) $T(E_h) = E_h$; and (3) $\lambda |\xi|^2 \le g_h^{ij}(x)\, \xi_i \xi_j \le \Lambda |\xi|^2$, $x \in \Omega_h$. Then for some subsequence $D_{h'} \xrightarrow{\Gamma} D$, where

$$D(u) = \int_\Omega a^{ij}(x)\, \partial_i u \, \partial_j u \; b(x)\, dx + \frac{1}{4}\int_{\bar\Omega} (u(x) - u(Tx))^2 \, d\nu ,$$

for some $a^{ij} \in L^\infty(\Omega)$ satisfying (3) with $\lambda$ replaced by $\lambda c_0^{-2}$ and for some positive Borel measure $\nu$.

The notion of $\Gamma$-convergence requires that if $u_h \xrightarrow{L^2} u$ then $D(u) \le \liminf_{h\to\infty} D_h(u_h)$, and for every $u \in L^2(\Omega, b)$ there are $v_h \in L^2$ with $v_h \xrightarrow{L^2} u$ and $\lim_{h\to\infty} D_h(v_h) = D(u)$. With somewhat stronger hypotheses, we show that the spectrum of the Laplace operator (with Dirichlet conditions on the boundary of a fixed compact set) on $M_h$ converges to the spectrum of $D$ with the weight function $b$. Examples show that $\det(a^{ij})$ need not equal $b^{-1}$.

We give an explicit example in the convergence of Scherk's "second" surface to its tangent cone at $\infty$. Consider

$$M_h = \{(x,y,z) \in \mathbb{R}^3 : \sin(hz) = \sinh(hx)\sinh(hy)\}$$

with its isometric involution $T(x,y,z) := (y,x,z)$. As $h \to \infty$, $M_h \to M$ in the weak-varifold and Hausdorff senses, where $M$ is the union of the $(x,z)$-plane and the $(y,z)$-plane. Let $M$ be represented isometrically by two copies of $\mathbb{R}^2$: $\Omega = \mathbb{R}^2 \cup \mathbb{R}^2$ where one component $\mathbb{R}^2$ is mapped to $M \cap \{x \geq y\}$ and the other to $M \cap \{x \leq y\}$. Each point of $M_h$ is required to correspond to the nearest point on $M$, which allows one to define $E_h \subset \Omega$ and $\Phi_h : \Omega_h = \Omega \setminus E_h \longrightarrow M_h$ which is locally bi-Lipschitz. In this case, it can be shown that the $\Gamma$-limit of $D_h$ is the Euclidean Dirichlet integral on $\Omega$, plus the penalty term with $\nu(A) = \infty$ if $A$ meets the $z$-axis in either component in a set of positive measure, and $0$ otherwise. We also construct examples of the same (unbounded) topological type, a sequence of minimal surfaces bounded by four lines parallel to the $z$-axis which converge as varifolds to a doubly-covered plane, and with limit measure $\nu = c\,\mathcal{H}^1(z\text{-axis})$ for $c < \infty$, including the possibility $c = 0$.

20. June 1988

# PROBABILITY IN BANACH SPACES

26. Juni – 2. Juli 1988

## Measures of dependence involving B-valued random variables

This talk concerned measures of dependence between pairs of σ-fields in a probability space, with special emphasis on certain measures of dependence involving B-valued random variables. One particular measure of dependence can be made equivalent to either that for "strong mixing" or that for "absolute regularity", depending on appropriate choices of a Banach space B, but it seems to be an open question whether these are the only two possible equivalence classes. A similar open question was also posed for another closely related measure of dependence.

Richard C. Bradley
Indiana University
27. June 1988

## Rearrangements of sequences of random variables and exponential inequalities.

Bernard HEINKEL, Strasbourg

Exponential bounds are studied for $P(\|X_1+\dots+X_n\|>t)$ where $(X_1,\dots,X_n)$ denotes a sequence of independent random variables with values in a real separable Banach space $(B,\|.\|)$. In our results the usual boundedness assumptions on $\|X_1\|,\dots,\|X_n\|$, are replaced by hypotheses on the weak $l_p$ norm of the sequence $(\|X_1\|,\dots,\|X_n\|)$.

28. 6. 1988

## A remark on Gaussian isoperimetry and logarithmic Sobolev inequalities

Michel LEDOUX, Strasbourg

We use isoperimetric inequality to show that a function on $\mathbb{R}^n$ whose gradient is in $L^1$ of the canonical Gaussian measure belongs to the Orlicz space $L^1(\mathrm{Log}\,L)^{1/2}$ of this measure. This complements the logarithmic Sobolev inequality of L. Gross.

## Nonlinear functionals of empirical measures and the bootstrap

Let $(X,\mathcal{A},P)$ be a probability space and $\mathfrak{F}$ a class of functions on $X$ such that the central limit theorem for empirical measures $\nu_n=\sqrt{n}(P_n-P)\xrightarrow[\mathcal{L}]{}G_P$ holds in $\ell^\infty(\mathfrak{F})$ for the sup norm $\|\cdot\|_{\mathfrak{F}}$. Let $T$ be a Functional on a class $\mathcal{P}$ of laws on $(X,\mathcal{A})$ which is

Fréchet differentiable for $\|\cdot\|_{\mathcal{F}}$, so that

$$T(Q)-T(P) = \int f_P \, d(Q-P) + o(\|Q-P\|_{\mathcal{F}}), \quad P, Q \in \mathcal{P},$$

where $f_P \in \mathcal{F}$ for all $P \in \mathcal{P}$. Then $\sqrt{n}(T(P_n)-T(P))$ is asymptotically normal. This is extended to suitable equi-$C^1$ classes of functionals and to a bootstrap form.

R. M. Dudley, MIT, Cambridge, Mass.

Rates of convergence in the central limit theorem in the space $D[0,1]$

The estimate of the rate of convergence in the CLT for i.i.d. summands with values in separable metric space $D[0,1]$ is obtained. We consider the convergence on balls (with respect to sup norm) and under rather natural conditions we get non-uniform (with respect to radius of ball) estimate of order $n^{-1/6}$. As a corollary we get the estimate for convergence for weighted empirical process.

V. Paulauskas, Vilnius university, Vilnius USSR

Gaussian measure of translated balls

Werner Linde, Jena

Let $E$ be a Banach space and let $\mu$ be a Gaussian measure on $E$. Then we define a function $F: (0,\infty)\times E \to \mathbb{R}$ by $F(s,z) := \mu\{x\in E;\ \|x-z\| < s\}$. Normally this function is studied as function of $s>0$ ($z\in E$ fixed). We prove that $z \to F(s,z)$, $s>0$ fixed, is Gateaux differentiable at every point $z$ belonging to the support of $\mu$.

29.6.88

Necessary conditions for the bootstrap of the mean

We show that if a very mild form of the bootstrap of the mean holds a.s. then $EX^2 < \infty$, and that if it holds in probability, then $X$ is in the domain of attraction of a normal law. In particular this shows that some current results in the literature can not be improved. Join work with J. Zinn.

Evarist Giné, College Station TX
28-VI-88

The Asymptotic Distribution of Magnitude-Winsorized Sums

For $X, X_1, X_2, \ldots$, iid, arrange $\{X_1, \ldots, X_n\}$ in descending order of magnitude, denoting the results $|X_1^{(n)}| \geq |X_2^{(n)}| \geq \cdots \geq |X_n^{(n)}|$. Take integers $0 \leq r_n \leq n$ with $r_n \to \infty$ but $r_n/n \to 0$. Put $\hat{b}_n = |X^{(n)}_{r_n+1}|$, and then

$$S_n(r_n) = \sum_{j=r_n+1}^{n} X_j^{(n)} + \sum_{j=1}^{r_n} \hat{b}_n \operatorname{sgn}(X_j^{(n)}) = \sum_{j=1}^{n} (|X_j| \wedge \hat{b}_n) \operatorname{sgn}(X_j)$$

$$V_n^2 = \sum_{j=1}^{n} (X_j^2 \wedge \hat{b}_n^2).$$

If $\mathcal{L}(X)$ is symmetric and nondegenerate, then $\mathcal{L}(S_n(r_n)/V_n) \to N(0,1)$. We use this result to study the asymptotic distribution of $S_n(r_n)/c_n$, for suitable constants $c_n$. A universal law (à la Doeblin) is constructed having all the allowable subsequential limit laws for $\{S_n(r_n)/c_n\}$. This work was joint with M. Hahn & J. Kuelbs.

Daniel Ch. Weiner, Boston University
27 June 1988

Rates of convergence in the CLT in $\mathbb{R}^k$ via Stein's Method

Let $X_1, \ldots, X_n$ denote i.i.d. random vectors taking values in $\mathbb{R}^k$ with mean zero, identity covariance and finite third absolute moment, say $\beta_3$.

Using solutions of the Ornstein-Uhlenbeck diffusion equation as a substitute for Stein's equation in one dimension the error in the CLT for a shift and scale invariant class of sets a Berry-Esseen estimate can be proved by induction on n. Assuming that the Gaussian probability of the $\varepsilon$-boundary of sets of this class is uniformly bounded by $\varepsilon\Delta$ the error in the CLT over this class of sets is bounded by $(5.4 + 23\,\Delta\sqrt{k})\,\beta_3\, n^{-1/2}$. This method can be applied similar as Bergström's method to prove rates of convergence in Banach spaces. Further application are to exchangeable r.v. and statistics of indep. r.v. with normal distribution under minimal moment conditions on the remainder of Hajek's projection like e.g. multivariate rank statistics and von Mises statistics.

Friedrich Götze Fakultät für Mathematik
Universität Bielefeld, 4800 Bielefeld 1

A LAW OF THE ITERATED LOGARITHM

Let $T: X \to X$ be a pointwise dual ergodic, measure preserving transformation on the infinite ($\sigma$-finite) measure space $(X, \mathcal{F}, m)$. Denote by $\hat{T}$ its dual operator on $L^1(m)$ and assume that $\frac{1}{n^\alpha h(n)} \sum_{k \le n} \hat{T}^k f \to \int f\,dm$ $(\forall f \in L^1(m)^+)$, where $0<\alpha<1$ and $h$ is slowly varying. Define recursively $\Lambda(0,t) \equiv 1$ $(t \ge 0)$ and

$$\Lambda(p+1,t) = \frac{\Gamma(1+\alpha(p+1))}{\Gamma(\alpha)\Gamma(1+\alpha p)} \int_0^1 u^{\alpha-1}(1-u)^{\alpha p} \frac{h(ut)}{h(t)} \left(\frac{h((1-u)t)}{h(t)}\right)^p \Lambda(p,(1-u)t)\,du$$

and set $p^* = p^*(n) = \left[\frac{1}{1-\alpha} L_2 n\right]$ $(L_2 = \log\log)$, $\Lambda(n) = \Lambda(p^*, n)^{1/p^*}$.

The following results are announced:

1) $f \in L^1(m)^+$ then

$$(*) \quad \limsup_{n\to\infty} \frac{1}{n^\alpha h(n) (L_2 n)^{1-\alpha} \Lambda(n)} \sum_{k\le n} f\circ T^k \le \frac{\Gamma(1+\alpha)}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \int f\,dm \quad \text{a.e.}$$

2) Assume that $T$ admits a Darling-Kac set $A$ for which the return time process is uniformly mixing. Then for $f \in L^1(m)^+$ equality in $(*)$ holds.

3) Let $m(A)=1$, $\beta' < 1 < \beta$. There exist constants $M$, $C(p,n)$, $C'(p,n) \sim \Lambda(p,n)$ such that for all $n, p \ge 1$

$$\int_A \Big(\sum_{k\le n} 1_A \circ T^k\Big)^p dm \begin{cases} \le C(p,n)\, \beta^p M \exp\{M \frac{p^{\alpha+1}}{n^\alpha h(n)}\} \\ \ge C'(p,n)\, \beta'^p \end{cases} \frac{p!\, \Gamma(1+\alpha)^p}{\Gamma(1+\alpha p)} h(n)^p n^{\alpha p}$$

where "$\ge$" holds if $A$ is a Darling-Kac set.

The results are obtained jointly with Jon Aaronson.

Manfred Denker

Institut f. Mathem. Stochastik, Lotzestr. 13, 3400 Göttingen, FRG

Bootstrapping General Empirical Measures

The almost sure central limit theorem for the bootstrap of empirical measures is characterized by the central limit theorem for the empirical measure and the finiteness of the second moment of the envelope function. Joint with E. Giné.

Joel Zinn

Texas A and M Univ.

## Sudakov's minoration for Rademacher Processes

Consider a subset $T$ of $\mathbb{R}^n$; Let $(\varepsilon_i)_{i\le n}$ be a Bernoulli sequence, and set

$$r(T) = E \sup_{t\in T} \left| \sum_{i\le n} \varepsilon_i t_i \right|$$

For a set $D \subset \mathbb{R}^n$, denote by $N(T,D)$ the minimum number of translates of $D$ needed to cover $T$. We prove the existence of a universal constant $K$ such that for $\eta > 0$

$$\eta \sqrt{\log N(T, K r(T) B_1 + \eta B_2)} \le K r(T)$$

where $B_2$ is the euclidean ball, $B_1 = \{ (t_i)_{i\le n} ;\ \sum |t_i| \le 1 \}$ and $A+B = \{ t = u+v ;\ u\in A,\ v\in B \}$.

M. Talagrand.

## Some Aspects of The Bootstrap

Let $Z, X_1, X_2, \ldots$ be iid; we study also $Z_1^*, X_{11}^*, Z_2^*, X_{21}^*, X_{22}^*, \ldots$ and assume that given $\{X_1, \ldots, X_n\}$, $\{X_{ij}^* : ij \le n-1\}$, and $\{Z_i^* : i \le n-1\}$, The random variables $Z_n^*, X_{n1}^*, \ldots, X_{nn}^*$ are iid $P_n = \sum_1^n \delta_{X_i}$. Write $P^*\{\cdot\}$ for $P\{\cdot \mid X_1, \ldots, X_n\}$. For $0<\alpha<1$ define $t_\alpha$, $t_\alpha^*$ by $P(\frac{\bar X_n - Z}{\sigma} \le t_\alpha) = \alpha$, $P^*\{\frac{\bar X^* - Z^*}{S_n} \le t_\alpha^*\} = \alpha + O_p(n^{-1})$, where $\sigma = \sqrt{\operatorname{Var} Z}$, $S_n = (n^{-1}\sum_1^n (X_i - \bar X)^2)^{1/2}$. Then under some regularity $t_\alpha - t_\alpha^* = O_P(n^{-\frac12})$. A plausibility argument was given that notwithstanding, $P(\frac{\bar X_n - Z}{\sigma} \le t_\alpha^*) - \alpha = O_p(n^{-1})$. At present, rigorous arguments prove only that The cited difference is $o_p(n^{-\alpha})\ \forall \alpha < 1$. These results on prediction intervals are in contrast to Those for confidence intervals insofar as $t_\alpha - t_\alpha^*$ is concerned. This work is joint with Chongen Bai and Peter Bickel.

In addition, mention was made of an almost sure $\sqrt{\frac{\log n}{n}}$ rate of convergence for The supremum of differences between true and bootstrapped coverage probabilities in a prediction problem (for random coefficient trigonometric polynomials), and finally an open problem concerning Vapnik–Chervonenkis classes was given.

Richard Olshen

The Concentration of Partial Sums in Small Intervals: Improvements on Berry-Esseen

This talk is based on joint work with M.J. Klass. Let $X, X_1, X_2, \ldots$ be i.i.d. mean zero random variables with $P(-a \le X \le b) = 1$ for some $a, b > 0$. Let $S_n = \sum_1^n X_i$ and let $I$ be a closed interval. If $|I| \ge b+a$ and $I$ is not too far into the tails of the distribution, then

$$P(S_n \in I) \propto \left( \frac{|I|}{\sqrt{\operatorname{Var} S_n}} \wedge 1 \right).$$

The result is optimal in two senses: it can fail if either $|I| < b+a$ or if $I$ is located too far into the tails. Explicit conditions specify how far is too far. The proof is derived from first principles modulo one application of the Berry-Esseen Theorem, which could in fact be circumvented.

Marjorie D. Hahn
Tufts University, Medford, MA, USA

An inequality on two dimensional Gaussian random variables

Let $(X,Y)$ be two dimensional Gaussian r.v.'s. with mean vector 0, cov. matrix $\begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix}$. Set $\varepsilon = \sqrt{E[(X-Y)^2]} = \sqrt{2(1-r)}$. Assume that $r \ge 0$. Then $\forall x > 0$, $\forall y > 0$

$$P(X \ge x + \varepsilon y, Y \le x) \le 2e^{-y^2/2} \int_x^\infty e^{-u^2/2} du / \sqrt{2\pi}$$

$$\text{or} \quad \le \varepsilon e^{-y^2/2 - x^2/2}.$$

One application is given.

N. Kôno (Kyoto Univ)
河野敬雄

Un modèle presque sûr pour la convergence en loi (X. Fernique, Strasbourg)

Dans les années 50, Skorohod prouvait que toute suite de mesures de probabilités sur un espace polonais convergeant étroitement est la suite des lois de certaine suite de variables aléatoires convergeant presque sûrement. L'exposé a été consacré à l'énoncé et à la démonstration du théorème suivant qui étend et précise le résultat de Skorohod:

Soit E un espace polonais, il existe un espace d'épreuves $(\Omega, \mathcal{A}, \mathbb{P})$ et pour toute probabilité $\mu$ sur E une variable aléatoire $X(\mu)$ sur $\Omega$ à valeurs dans E ainsi qu'une partie négligeable $N(\mu)$ de $\Omega$ telles que:

(1) $X(\mu)$ ait pour loi $\mu$,

(2) Pour tout filtre $\Phi$ sur l'ensemble $M(E)$ des probabilités sur E convergeant étroitement vers une probabilité $\mu$ et pour tout $\omega$ n'appartenant pas à l'ensemble $N(\mu)$, le filtre image $X(\Phi)(\omega)$ converge vers $X(\mu)(\omega)$.

Fernique

The empirical process of long-range dependent observations

Let $X_i$, $i \geq 1$, be a stationary Gaussian sequence with $EX_i = 0$, $\operatorname{Var} X_i = 1$ and $r(k) = EX_j X_{j+k} = k^{-D} L(k)$ for $0 < D < 1$ and a slowly varying function $L(x)$. Let $G: \mathbb{R} \to \mathbb{R}$ be measurable. We study the e.d.f. of $Y_i = G(X_i)$, i.e. $F_n(s) = \frac{1}{n} \sum_{j=1}^{n} 1_{\{Y_j \leq s\}}$. Define $J_q(s) = \int (1_{\{x \leq s\}} - F(s)) H_q(x) \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx$ where $H_q(x) = e^{x^2/2} \frac{d^q}{dx^q} e^{-x^2/2}$ is the $q$-th Hermite polynomial. Let $m = \inf\{q : J_q(s) \neq 0$ for some $s\}$.

Theorem: Assume $D < 1/m$. Then

$$\sup_{s \in \mathbb{R}} \sup_{0 \leq t \leq 1} \left(n^{2-mD} L^m(n)\right)^{-1/2} \left| [nt] \left(F_{[nt]}(s) - F(s)\right) - \frac{J_m(s)}{m!} \sum_{j=1}^{[nt]} H_m(X_j) \right| \xrightarrow[\text{in prob.}]{} 0$$

As a corollary of this and results of Taqqu, Dobrushin/Major we obtain the weak convergence of the empirical process $(n^{2-mD} L^m(n))^{-1/2} \cdot [nt] (F_{[nt]}(s) - F(s))$ in $D(\mathbb{R} \times [0,1])$ to $\frac{J_m(s)}{m!} Z_m(t)$ where $Z_m(t)$ is an $m$-th order Hermite process.

Herold Dehling, Groningen, The Netherlands

Low and High Density Approximation of the Reaction - Diffusion Equation

It is shown that a nonlinear reaction-diffusion equation can be approximated by stochastic space-time fields with local interaction (low density) and by fields with ~~global~~ (medium range interaction (high density).

Peter Kotelenez, University of Utrecht, The Netherlands

Embedding and approximating vector-valued martingales

Results of Morrow and Philipp (TAMS, 1982) suggest that the canonical process to embed $\mathbb{R}^d$-valued martingales is an $\mathbb{R}^d$-valued Gaussian process $\{G(C), C \in \mathcal{C}\}$ (where $\mathcal{C}$ is the collection of all positive semidefinite $d \times d$ matrices) with the following properties:

(i) $G(0) = 0$

(ii) $G(C) = N(0, C)$

(iii) $G(C_1), G(C_1+C_2) - G(C_1), \ldots, G(C_1 + \ldots + C_n) - G(C_1 + \ldots + C_{n-1})$ are independent for $C_1, \ldots, C_n \in \mathcal{C}, n \geq 1$.

A simple argument shows that for $d > 1$ such processes do not exist.

Other possibilities to obtain strong approximation theorems for vector-valued martingales and counterexamples to some natural conjectures are also discussed.

Walter Philipp, Univ. of Illinois, Urbana, IL

Self-normalized laws of the iterated logarithm

Using suitable self-normalizations for partial sums of i.i.d. random variables, a law of the iterated logarithm, which generalizes the classical LIL, is proved for all distributions in the Feller class. A special case of these results applies to any distribution in the domain of attraction of some stable law. This work is joint with Phil Griffin.

James Kuelbs, University of Wisconsin, Madison, WI.

Stochastic iterations for linear problems in a Banach space

For recursive estimates in linear filtering and prediction theory, problems of convergence and rate of convergence appear which can be reduced to corresponding problems with limit 0 for a sequence $(X_n)$ of random elements in a real separable Banach space $B$ (especially $C([0,1]^2)$ and Hilbert space) iteratively defined by $X_{n+1} = X_n - a_n(A_n X_n - V_n)$ with $a_n \in [0,1)$, $a_n \to 0$, $\sum a_n = \infty$. Here $A_n$, $V_n$ are $L(B)$- and $B$-valued random variables, resp., with a.s. convergence of weighted or arithmetic means of the $A_n$'s to $A \in L(B)$ which satisfies a certain spectral condition. A.s. convergence of $X_n$ (investigated jointly with L. Zsidó) and in the case $a_n = 1/n$ rates of convergence (functional central limit theorem and loglog invariance principle) are obtained from corresponding assumptions on weighted and arithmetic means of the $V_n$'s, resp., under weak additional assumptions.

Harro Walk, Universität Stuttgart

Series representations of i.d. random vectors with applications to 0-1 laws.

A general form of LePage-type series representations for infinitely divisible (i.d.) random vectors without Gaussian components ~~are~~ given and some specjal cases are discussed. As an application of such representations it is shown that the zero-one laws for i.d. measures (Jannsen (1984), LNM1064) follow directly from basic zero-one laws (Hewitt-Savage, Borel-Cantelli lemma) and from a generalized version of a theorem of P. Lévy.

Jan Rosinski, Univ. of Tennessee, Knoxville, TN.

Statistical mechanics on graphs

Random tree-type partitions for finite sets are used as a model of a chemical polymerization process when ring formation is forbidden. The study rigorously establishes theoretically the existence of three stages of polymerization and of a critical point dependent upon the ratio of association and dissociation rates. Distributions on Banach spaces ~~related to~~ arising in the study are also analyzed. (joint work with Pittel and Mann)

Wojbor A. Woyczynski, Case Western Reserve University, Cleveland, Ohio

Continuity properties of diffusion semigroups in Hilbert space.

Let $(P_t)$ ~~be~~ the semigroup of transition probabilities of a diffusion process in a real separable Hilbert space, the diffusion process given as the solution of a Stochastic

differential equation.
We consider $P_t$ acting as ~~a~~ linear operators $P_t: V \to V$ for different spaces $V$ of continuous functions on $H$ and derive continuity properties of these operators from bounds on the growth of the diffusion and drift coefficient of the underlying diffusion process.

Gottlieb Leha, University of Passau, FRG.

Rates for the CLT via ideal metrics

Let $(B, \|\cdot\|)$ be a seperable Banach space and $\mathcal{X} = \mathcal{X}(B)$ the vector space of all random variables defined on a probability space and taking values in $B$. It is shown that new ideal metrics for $\mathcal{X}$ may be used to obtain refined rates of convergence of normalized sums to a stable limit law. The rates are expressed in terms of a variety of uniform metrics on $\mathcal{X}$. In the $B$-space setting, the rates hold w.r.t. the total variation metric and in the Euclidean space setting the rates hold w.r.t. uniform metrics between density and characteristic functions. The main result provides a sharp order estimate of the rate of convergence in local limit theorems w.r.t. the uniform distance between densities. The method is based on the theory of probability metrics, especially those of convolution type.

Joseph E. Yukich, Lehigh University, Bethlehem, Pa.

## Characterization of the Cluster Set of the LIL Sequence in Banach Space

Let $S_n = X_1 + \dots + X_n$, where $X_1, X_2, \dots$ are iid Banach-space-valued random variables with weak mean 0 and weak second moments. Let K be the unit ball of the reproducing kernel Hilbert space associated to the covariance of $X_1$. We show that the cluster set (set of limit points) of $\{S_n/(2n\log\log n)^{1/2}\}$ either is empty or has the form $\alpha K$, where $0 \le \alpha \le 1$. A series condition is given which determines the value of $\alpha$. For each such $\alpha$ there exist examples in which $\alpha K$ is the cluster set.

Kenneth S. Alexander, University of Southern California, Los Angeles.

## The decomposition theorem for functions satisfying the law of large numbers

Suppose that B is a Banach space with the Radon-Nikodym property. Then $f \in LLN(\mu, B)$ if and only if there exists $f_1 \in L^1(\mu, B)$ (Bochner $\mu$-integrable) and $f_2 \in L^1_P(\mu, B)$ (Pettis $\mu$-integrable), with $\| f_2 \|_{GC} = 0$ ($\| \; \|_{GC}$ - the Glivenko-Cantelli norm) such that $f = f_1 + f_2$. Moreover, if $f \in LLN(\mu, B)$ such a decomposition is unique. The neccessary condition holds if $\| \; \|_{GC}$ is substituted by the Pettis norm $\| \; \|_P$, but then the sufficiency fails. Namely, there is a example of a function that is Pettis $\mu$-integrable, has the Pettis norm 0, but it does not satisfy the strong law of large numbers.

Vladimir Dobric, Lehigh University, Bethlehem, Pennsylvania

A law of the iterated logarithm for trimmed sums

Let $X_1, X_2, \ldots$ be a sequence of non-negative independent and identically distributed random variables with common distribution function $F$ in the domain of attraction of a non-normal stable law. We discuss the law of the iterated logarithm behavior of trimmed sums of the form $\sum_{i=1}^{n-r_n} X_{i,n}$, where $X_{1,n} \leq \ldots \leq X_{n,n}$ are the order statistics of $X_1, \ldots, X_n$ for $n \geq 1$ and where $(r_n)_{n\geq 1}$ is a sequence of integers with $r_n \to \infty$ and $r_n/n \to 0$ as $n \to \infty$.

Erich Haeusler, University of Munich

Relationship between Gaussian Processes and the local time of Markov Processes

Dynkin's Isomorphism theorem gives a relationship between Gaussian processes and the local time of a killed tied down right Markov process with symmetric transition probability density. The theorem shows that if the ~~local the~~ Gaussian process is continuous so is the local time. In fact if the Gaussian process is continuous the local time satisfies the central limit theorem in the space of continuous functions. This result is joint with R. Adler & J. Zinn.

Michael B. Marcus The City College of CUNY

## Uniform Convergence of Martingales

Let $\{X_n(t), \mathcal{F}_n \mid n \geq 1\}$ be a martingale for each $t \in T$, and let $\{X(t) \mid t \in T\}$ be a Bochner measurable stochastic process (i.e. $X(\cdot,\omega)$ takes values in a $\|\cdot\|_T$ separable subset of $\mathbb{R}^T$), such that $X_n(t) \to X(t)$ a.s $\forall t$. Now suppose that $T$ is a separable topological space and

(i) $\sup_n E \sup_{t \in S} |X_n(t)| < \infty \quad \forall S$ countable $\subseteq T$

(ii) $X(\cdot,\omega)$ is continuous for a.s. $\omega$

Let $\Delta_n(\omega) = \sup_{t\in T} \{ \inf_{U \text{ nghb of } t} \{ \sup_{s \in U} |X_n(t) - X_n(s)| \} \}$, and suppose that $\Delta_n \to 0$ a.s., then $X_n(t) \rightrightarrows X(t)$ uniformly in $t$ a.s. Moreover if $T$ is hereditarily separable, then this holds even if we drop condition (ii)

J. Hoffmann-Jørgensen   Mat. Inst. Århus Universitet

## Large deviation result for a class of Markov chains

Let $\{X_n^{(N)}\}_{n\geq 0}$ be an array of stationary Markov chains in $\mathbb{R}^d$. Suppose that with scaling $t = n\beta$, $\beta \to 0$, the chain $(X_n)$ resembles a diffusion that solves a stochastic differential equation of Wentzell-Freidlin type. That is, the diffusion is a small random perturbation of a dynamical system. The time it takes the chain to escape a neighborhood of a stable fixed point of a dynamical system in discrete time is evaluated along an exponential scale as roughly the same amount of time it takes the corresponding diffusion to leave this neighborhood. The Markov chains are motivated by models of population genetics.

Gregory J. Morrow, University of Colorado, Colorado Springs

Representations of Banach space valued martingales as stochastic integrals

If $M = (M_{(t)})_{t \geq 0}$ is a real-valued, continuous local martingale whose quadratic variation is absolutely continuous relative to Lebesgue measure, then by a theorem of Doob, $M(t)$ is the stochastic integral of a certain function relative to a Brownian motion (on a possibly extended probability space). This result is also well-known in the $\mathbb{R}^d$-case. The classical method of proof is restricted to the case that $M$ takes values in a Hilbert space. For continuous, local martingales with values in a real, separable Banach space, we give a complete different proof of Doob's theorem. One application is a uniqueness theorem for the so-called martingale problem (in the sense of Stroock and Varadhan) on Banach spaces.

Egbert Dettweiler, University of Tübingen

Strong invariance principles and stability results for sums of Banach-valued random variables

We present a strong approximation theorem for sums of i.i.d. d-dimensional r.v.'s with possibly infinite second moments. Using this result, one obtains strong invariance principles for Banach-valued r.v.'s in the domain of attraction of a Gaussian law generalizing the known strong invariance principles for r.v.'s satisfying CLT. These new strong invariance principles immediately imply compact as well as functional laws of the iterated logarithm. We also mention a related stability result for sums of i.i.d. B-valued r.v.'s.

Uwe Einmahl, Universität Köln

Strong approximation of continuous time stochastic processes

Let $(X^n(t))_{t\geq 0}$, $(Y^n(t))_{t\geq 0}$ be two sequences of stochastic processes. We study sufficient conditions under which an almost sure approximation

$$\| X^n(t) - Y^n(t) \|_{t_n} \ll \varepsilon_n$$

holds, where $(\varepsilon_n)_{n\geq 1}$ is a given approximation order, $(t_n)_{n\geq 1}$ a nondecreasing sequence of numbers and $\|\cdot\|_t$ means the supremum norm on the interval $[0,t]$.
Two different approaches are discussed. The first one is based on a Berkes-Philipp type theorem, the second one uses measurable selections.

Ernst Eberlein, Universität Freiburg