MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 20/1977

Ringe, Moduln und homologische Methoden

15.5. bis 21.5.1977

Die Tagung über "Ringe, Moduln und homologische Methoden" stand unter der Leitung von F. Kasch (München) und A. Rosenberg (Ithaca) und fand auch in diesem Jahr wieder ein breites Interesse und rege Beteiligung.

Es ist vorgesehen, daß in den kommenden Jahren Tagungen ähnlicher Art, jedoch unter wechselnden und spezielleren Titeln stattfinden werden. Dies entspricht einer Anregung des Wissenschaftlichen Beirates und einer auch sonst geübten Praxis des Oberwolfacher Instituts.

Die Tagungsleiter nehmen diesen Bericht daher zum Anlaß, allen Teilnehmern für die sich zum Teil über viele Jahre erstreckende gute Zusammenarbeit zu danken, die ihnen auch die organisatorische Tätigkeit sehr erleichtert hat. Sie wünschen künftigen Tagungen dieser Art ebenso guten Erfolg, wie ihn die diesjährige Tagung wieder verzeichnen konnte.

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Wright, D., St. Louis
Würfel, T., München
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Vortragsauszüge

V. Dlab: Structure of hereditary algebras

Some classes of hereditary algebras were discussed in terms of the properties of their module categories. In particular, an (indecomposable) finite dimensional k-algebra A is a hereditary algebra of tame representation type if and only if A is Morita equivalent to a tensor ring defined by a k-species over an Eucledian diagram or to k-algebra $\tilde{A}_n(\varepsilon,\delta)$. Here, $\tilde{A}_n(\varepsilon,\delta)$ is the k-algebra of all (upper) triangular $(n+1)\times(n+1)$ matrices (x_{ij}) with x_{ij} from a skew field F (for $i \leq j$ and $(i,j) \neq (1,n+1)$) and $x_{1\,n+1} \in M = M(\varepsilon,\delta)$, where ${}_F M = {}_F F \oplus {}_F F$ and $(a,b)f = (af+b\delta(f), b\varepsilon(f))$ with an automorphism ε and an ε-derivation δ of $F([F:k] < \infty$ and k acts centrally). Also, some functorial techniques for construction of indecomposable modules were displayed. In particular, all indecomposable $\tilde{A}_n(\varepsilon,\delta)$-modules of finite lengths were described in terms of finite sequences of integers $1,2,\dots,n+1$ (defining their composition series) and $F[z;\varepsilon,\delta]$-modules. Consequently, there is a complete explicit description of all real hereditary algebras of tame representation type and their modules of finite length. [joint work with C.M. Ringel].

J.-M. Goursaud: Anneaux de polynoms semi-hérédetaires

Let A a reduced ring, then it is well known that A[X] is semi-hereditary iff A is regular. We prove that if A is a left (or right) self-injective ring then the polynomial ring A [X] is right semi-hereditary iff A is a finite direct product of matrix rings over regular reduced self-injective rings. We give also a necessary and sufficient condition for the semi-hereditarity of $A[x,\sigma]$, where A is a product of matrix rings over fields and σ an automorphism of A. Finally, as application, we obtain that if K is a field and G a locally solv. group then KG is hereditary iff G is a finite extension of $\mathbb{Z}$ and contains no element of order p(=char K).

A. Page: Sur les anneaux héréditaires ou semi-héréditaires

1) De nouveaux exemples et contre-exemples en théorie des anneaux héréditaires ou semi-héréditaires sont mis en évidence par le résultat suivant:

Soient A et B deux anneaux, M un A-B-bimodule les conditions suivantes sont équivalentes

(i) L'anneau $R = \begin{pmatrix} A & M \\ 0 & B \end{pmatrix}$ est semi-héréditaire (resp. héréditaire) à gauche

(ii) a) A et B sont semi-héréditaires (resp. héréditaires) à gauche

b) M_B est plat

c) Pour tout idéal à gauche de type fini J de B, ${}_A(M/_{MJ})$ est un module semi-héréditaire (resp. pour tout idéal à gauche J de B, ${}_A(M/_{MJ})$ est projectif).

2) Si A est un anneau à identité polynomiale vérifiant la condition maximale sur les annulateurs à gauche d'éléments, le quotient de A par son radical premier est un anneau de Goldie. Si de plus A est semi-héréditaire à gauche (ou à droite), il est semi-héréditaire.

L. A. Bokut: New results on ring theory in Novosibirsk

Some new result on ring theory of Novosibirsk's ring theorists.

H. H. Brungs: Unique factorization in rings with right a.c.c.$_1$

Let R be an integral domain that satisfies the maximum condition for principal right ideals. We show that every nonzero non-unit in R can be factored in generalized atoms. If in addition the sets $V(a) = \{bR;\ bR \ni a\}$ are modular lattices for every $a \neq 0$ in R, a uniqueness theorem is obtained. These results can be applied and sharpened in case R is a weak Bezout domain.

F. L. Sandomierski: Localization in Maximal Orders

A Noetherian prime ring R is a maximal order in its classical (simple, Artinian) quotient ring Q, provided whenever S is an order in Q with $R \subset S$ and $aSb \subset R$ for units $a,b \in Q$, then $R=S$.

Throughout R is a prime Noetherian maximal bounded order.

Theorem 1. If $\mathrm{glb}(R) \leq 2$ (Global Homological Dimension = glb), then R has enough clans (where a clan is a Goldie localizable semiprime ideal).

Theorem 2. If $\mathrm{glb}(R) \leq 2$ and P is a height one prime of R, then R/P is hereditary (Dedekind) if and only if $P \not\subset MS$ for each maximal ideal M and clan S of maximal ideals with $P \subset S \subset M$ (and maximal ideals above P commute module P).

Theorem 3. If $\mathrm{glb}(R) \leq 2$, P height one prime with R/P hereditary, then the following are equivalent.

1. $PM \neq MP$
2. $P \subset M^2$
3. $P \subset M^n$, $n = 1,2,\ldots$

An example of Ramras shows that the situation of Theorem 3 can occur even when R is a k-order with k regular local 2-dimensional ring.

L. Lesieur: Noetherian Conditions in polynomial Ore Rings $A[X,\sigma,\delta]$.

Let A be a ring with unit 1, σ an injective endomomorphism of A, δ a σ-derivation in A: $\delta(a+b) = \delta(a) + \delta(b)$, $\delta(ab) = \sigma(a)\cdot\delta b + \delta a \cdot b$. The Ore polynomial ring $A[X,\sigma,\delta]$ is defined by the usual left operations on $A[X]$, together with the right multiplication: $Xa = \sigma(a)X + \delta a$. It provides a lot of interesting examples to illustrate various more or less recent theories in non commutative Algebra.

My topic is to give sufficient conditions on A and σ in order to obtain a left noetherian ring $A[X,\sigma,\delta]$; those conditions are also necessary in the case $\delta = 0$. They are valid, in particular, when A is artinian simple or semi-simple or "quasi-frobeniusien".

G. Renault: Sur les anneaux de polynomes de Ore

Soit A un anneau unitaire de radical de Jacobson R qui vérifie la condition suivante:

* Si P est un A-module projectif tel que P^n soit isomorphe à A^n, alors P est isomorphe à A.

Alors un anneau de polynômes de Ore $M_n(A)[X,\sigma,\delta]$ est isomorphe à un anneau de matrices $M_n(A[X',\sigma',\delta'])$. (C'est toujours le cas si A/R est auto-injectif à droite et fini).

Si A est artinien à gauche, l'anneau $A[X,\sigma]$ est noethérien à gauche si, et seulement si on a $A\sigma(R) = R$.

G. Cauchon: On The Krull Dimension Of The Skew Polynomial Rings $A[X,\sigma,\delta]$.

A ring A is said to be left bounded if every essential left ideal in A contains a nonzero two-sided ideal. We say that A is left fully bounded if every prime factor of A is left bounded. We define in the same way the notion of right fully bounded ring.

I just expect to prove the following result:

Consider a ring A which is noetherian on both sides and fully bounded on both sides. Consider also an automorphism σ of A and a σ-derivation δ of A. Suppose that the skêw polynomial ring $R = A[X,\sigma,\delta]$ is left fully bounded.

Then, if the krull-dimension of A is finite, we have

$$\text{Kdim } R = \text{Kdim } A + 1.$$

(Kdim R is the krull dimension of R, that is the maximal length of the chains of prime ideals of R. Kdim A is defined in the same way).

L. Márki: On linearly compact rings

This is a report on some results of P. N. Anh.

Linearly compact rings (modules) are generalizations of artinian rings (modules). An important subclass of linearly compact rings is that of inverse limits of artinian modules; these rings are said to be L-compact. The results to be reported characterize semisimple linearly compact rings by means of discrete modules over them, describe the additive structure of L-compact modules, further characterize those L-compact rings in which a) every ideal (left ideal) is closed or b) every non-zero ideal (left ideal) is open.

M. Auslander: <u>An observation about finitely presented modules</u>

S. Jøndrup: <u>Projective modules</u>

Let P be a projective module over a ring A. We consider the following problem. Assume $P = P_o + Jac(A) P$, P_o finitely generated $P_o \subset P$ is $P = P_o$?
If the ring A is a P.I. ring, we show that the answer is yes. An application of this result to P.I. rings is given. Next we consider some problems concerning left hereditary P.I. rings. Among other results we prove: If A is left hereditary and finitely generated as a module over its center, then the center is hereditary.

G. Michler: <u>Blöcke mit abelschen Defektgruppen</u>

Zur Bestimmung der Struktur der unzerlegbaren, projektiven Moduln eines Blocks B einer endlichen Gruppe G ist das Auffinden der Vertices vxM der einfachen B-Moduln M ein wichtiger Schritt. Ist G p-auflösbar und D eine Defektgruppe von B mit Zentrum Z(D), so gilt:
(1) $Z(D) \leq_G vxM \leq D$, (2) $Z(D) = {}_G vxM$ genau dann, wenn D abelsch ist. Dies wurde in einer mit W. Hamernik gemeinsamen Arbeit gezeigt. Für beliebige Gruppen G erhielten P. Landrock und der Vortragende den folgenden Satz: Ist G eine endliche Gruppe mit abelscher 2-Sylowgruppe D, so ist D ein Vertex eines jeden einfachen Moduls des 2-Hauptblocks B_o von G. Zum Beweis wurde ein Überblick über die Struktur der 2-Hauptblöcke der endlichen einfachen Gruppen G mit abelschen 2-Sylowgruppen gegeben, wobei besonders auf eine mit P. Landrock gemeinsame Arbeit über die Blockstruktur der Janko-Gruppe J_1 eingegangen wurde.

B. J. Müller: Noncommutative Localization and Invariant Theory

Most examples of noetherian PI-rings are similar to the following: a matrix ring $R = \begin{pmatrix} A & X \\ Y & B \end{pmatrix}$, where A and B are subrings of a commutative noetherian domain C, and X and Y are ideals of C whose product XY is contained in A and B. If A and B are the iverse images of the invariant subring of $\bar{C} = C/\sqrt{XY}$ under the action of two finite groups G_1 and G_2 of automorphisms, then the classically localizable semiprime ideals of R correspond to the $G_1 * G_2$-stable semiprime ideals of $\bar{C}$. In particular if $G_1 * G_2$ is Zariski-dense in an affine algebraic group G acting rationally on an affine ring $\bar{C}$, then they correspond to the Zariski-closed G-stable subsets of the affine variety max-spec($\bar{C}$); hence the classification of the classically localizable semiprime ideals of R amounts essentially to the classification of the G-orbits in an affine variety, ie. to the central problem of invariant theory. This observation suggests many interesting concrete examples.

B. Zimmermann-Huisgen: Decomposability of direct products of modules

Necessary and sufficient conditions are given for a direct product of modules (over an associative ring with 1) to be projective or a direct sum of submodules not exceeding a prescribed cardinality lity. It is shown that these properties of a direct product are closely related to a certain chain condition on the factors which, on the other hand, is known to be characteristic of algebraically compact direct sums.
The main theorem contains Chase's Theorem [Trans.Amer.Math. Soc. 97, Thm.3.1], the Faith-Walker-Theorem on decompositions of injective modules [J. Algebra 5, Thm. B] and results on direct products of abelian groups which extend those of Mishina and Łoś [Math. Publ. Debrecen 3] . Applications to v.N. regular rings are given. Moreover, a class of examples exhibited of those rings for which all projective modules are algebraically compact.

J. Cozzens: <u>Representations of 2-dimensional semiperfect orders</u>

Let Λ be a bounded, semiperfect order $\subset D_n$, D a division ring, with gl b Λ = 2. Then if $\Lambda \subsetneqq \Gamma$, Γ a 2-dimensional (global dimension), maximal, quasilocal order with Γ_Λ f.g. projective, then one can show that if Λ is tiled, ie. Λ contains n orthogonal idempotents. In particular, Λ is localizable in the sense of Silver with each localization quasilocal and maximal. Call a sequence of distinct integers $\underset{n}{k} = (k_1,\dots,k_e)$ with $k_1=1$, $k_l=n$ and k_i a divisor of k_{i+1} $\forall i \leq k-1$ a divisor sequence for n. Let d be an arbitrary maximal, local oder in D and $p_1,\dots,p_m$ prime ideals of d. For a positive integer n with divisor sequence $\underset{m}{k} = (k_1,\dots,k_{m+1})$, we inductively define a class of integral formes in d_n, denoted $\Delta(n,\underset{m}{k};p_1,\dots,p_m)$, as follows:

$$\Delta(n,\underset{m}{k};p_1) = \begin{pmatrix} d & \cdots & d \\ & \ddots & \vdots \\ p_1 & & d \end{pmatrix} ;$$

assuming that $\Delta(n',\underset{n}{k'};p_1,\dots,p_{m-1})$ has been defines for all $n',\underset{m}{k'}$, we define

$$\Delta(n,\underset{m}{k};p_1,\dots,p_m) = \begin{pmatrix} \Delta_m & \cdots & \Delta_m \\ & \ddots & \vdots \\ p_m\Delta_m & & \Delta_m \end{pmatrix} \quad \text{where}$$

$\Delta_m = \Delta(k_m,\underset{m}{k'};\ p_1,\dots,p_{m-1})$ and $\underset{m}{k'} = (k_1,\dots,k_m)$.

<u>Theorem:</u> If Λ is as above, then there exists a 2-dimensional local order $d \subset D$, an integer $m \leq n$ (m = # of maximal ideals of Λ), a divisor sequence $\underset{m}{k}$ for m and distinct height primes $p_1,\dots,p_l$ of d such that Λ is Morita equivalent to $\Delta(m,\underset{m}{k};p_1,\dots,p_l)$. In particular, if Λ is reduced, $\Lambda \sim \Delta(n,\underset{m}{k};p_1,\dots,p_l)$.

<u>M.-P. Malliavin:</u> <u>Chain conditions on prime ideals of universal algebras of nilpotent Lie algebras</u>

Let k be a field of characteristic zero, G a Lie nilpotent algebra of finite dimension over k and $A = U(G)$ the universal algebra of G. Then 1) for every prime ideal P of A and every maximal ideal M of A such that $P \subset M$, there exists a saturated chain of prime ideals, from (0) to M through P, of length equal to the height of M. 2) if P and Q are prime ideals of A if M is a maximal ideal of A which is minimal with respect to: $P + Q \subset M$ then height of M is less or equal to the sum of the heights of P and Q.

W. Shelter: <u>Affine PI Rings</u>

Some results on the spectrum of a prime affine PI ring R, its centre Z, and the ring $R[T]$ formed by adjoining all coefficients of the characteristic polynomials of elements of R. We show the equichain condition for chains of primes in R (conjectured by Procesi). We show that Spec R $\longrightarrow$ Spec Z is surjective for dim R = 2 but not for 3 . Use is made of integral extensions. Another result used is that if C is the evaluation of a central polynomial, then $C^m R[T] \subset R$ for some m. We give an example to show that spec $R[T] \simeq$ Spec R does not imply R is noetherian, but do give a positive result in this direction.

M. Boratyński: <u>Ideals of polynomial rings which are locally complete intersections</u>

My lecture will be dedicated to the proof of the following result: Let R be a twodimensional regular ring / commutative and noetherian / . If $p \subset R[X]$ is a prime ideal with ht p = 2 which is locally a complete intersection then the number of generators of p is ≤ 3. This bound can not be improved.

H. Lenzing: <u>On isomorphisms of endomorphism rings</u>

Let P_R, Q_S be generators, E and F (resp. E_o and F_o) their rings of endomorphisms (of finite rank). A ring isomorphism $\varphi: E \rightarrow F$ is induced by a category equivalence if there is an equivalence Φ: Mod-R $\longrightarrow$ Mod-S s.t.

$\Phi(P) = Q$ and φ is the map $u \longrightarrow \Phi(u)$.

Prop. 1. $\varphi : E \xrightarrow{\cong} F$ is induced by a category equivalence if and only if $\varphi(E_o) = F_o$.

Prop. 2. Any ring isomorphism $\psi : E_o \longrightarrow F_o$ is induced by a category equivalence and extends uniquely to a ring isomorphism $\varphi : E \longrightarrow F$.

Prop. 3. If P_R, Q_S are projective modules of infinite rank. Then any isomorphism $\varphi : E \longrightarrow F$ is induced by a category equivalence and $\varphi(E_o) = F_o$.

F. Van Oystaeyen: a) Graded Rings and Modules of Quotients, oder b) Zariski Central Rings

a) The category of left graded R-modules, R-gmod for some graded ring R, is a Grothendieck category but it is not a full subcategory of R-mod. We relate localization in R-mod and R-gmod. Graded kernel functors are being associated to homogenuous prime ideals and multiplicative sets. The left (graded-) Ore-condition is being studied.

b) A Zariski central ring is a ring R such that $I^n \subset R\,(I \cap C)$ for some $n \in \mathbb{N}$, where C is the center of R. It is proved that for such a ring:

   1° Symmetric localization at prime ideals is central.
   2° Symmetric localization at primes has property (T).
   3° Under symmetric localization at primes ideals of R extend to (two-sided) ideals of the ring of quotients.

   Any two symmetric kernel functors are compatible. Certain rings of twisted power series are Zariski central but not an Azumaya algebra.

D. S. Passman: The Jacobson radical of a group ring of a locally solvable group

The object of this work is to describe the Jacobson radical $JK[G]$ of the group algebra $K[G]$ of a locally solvable group. If K is a field of characteristic 0, then it is known that $JK[G] = 0$. Thus we assume that char $K = p > 0$. The description of JK[G] is then given in terms of its controller subgroup C(JK[G]). Indeed it is shown that this subgroup is equal to $S = \mathcal{S}(\Lambda^P(G))$. Furthermore we then have $JK[G] = JK[S]\cdot K[G]$ and the structure of JK[S] is easily described in terms of the radicals of group rings of certain finite groups involved in S. Thus a complete description of JK[G] is obtained.

K. Sugano: On projective Separable extensions

We have introduced the notion of H-separable extensions, which is a kind of generalizations of Azumaya algebras. A ring Λ is an H-separable extension of a subring Γ, if $\Lambda\otimes_\Gamma\Lambda$ is isomorphic to a direct summand of a finite direct sum $\Lambda\oplus\cdots\oplus\Lambda$ of copies of Λ as Λ-Λ-module. We study now in what case an H-separable extension Λ of Γ is Γ-projective. Especially if Γ is Γ-Γ-direct summand of Λ, Λ is Γ-f.g. projective. In case an H-separable extension Λ of Γ is left Γ-progenerator, there exists a one to one correspondece between the class of right ideals of Γ and the class of right Λ and left $V_\Lambda(\Gamma)$-submodules of Λ.

J. Osterburg: Fixed Rings and Skew Group Rings

Let R be a ring and G a finite group of ring automorphisms of R. By R^G, we mean the fixed subring of R. R_sG, the skew group ring is a free R module with basis G such that $rg=r^g g$. We discuss some properties that pass between R^G, R and R_sG. For example, if R is semiprime and R has no |G| torsion, then R is left Goldie if and only if R^G is left Goldie if and only if R_sG is left Goldie. Replacing Goldie with Noetherian, perfect or R has Krull dimension yields many of the implications. The affect of |G| torsion will also be discussed.

B. Fein: Ulm Invariants of the Brauer Group of a Field

Let DB(K) denote the maximal divisible subgroup of the Brauer group of a field K and let $B(K)=DB(K)\oplus RB(K)$. For p prime, let $RB(K)_p$ denote the p-primary component of RB(K). If $RB(K)_p$ is countable, then it is characterized by its Ulm invariants as defined in Kaplansky, Infinite Abelian Groups. We let $U_p(\beta,B(K))$ denote the β-th Ulm invariant of $RB(K)_p$ and we let $l_p(B(K))$ denote the Ulm lenght of $RB(K)_p$. In joint work with M. Schacher, these invariants are studied for the case when K is finitely generated over its prime field. A typical result is that if K has characteristic q and K is finitely generated over its prime field F with the transcendence degree of $K/F > 0$ if $q=0$ and >1 if $q>0$, then $l_p(B(K)) \geq \omega 2$ for $p \neq q$ and $U_p(\beta,B(K)) = \omega$ for all but finitely many choices of p and finite ordinals β for $q=0$.

R. B. Warfield: Noncommutative Prüfer Rings

A Prüfer prime ring is a prime left and right Goldie ring for which all finitely generated one-sided ideals are progenerators. Semilocal Prüfer prime rings are Bezont rings. Bounded semilocal Prüfer prime rings of Krull dimension one are elementary divisor rings, and this is used to show that over a Prüfer algebra of Krull dimension one (i.e. a Prüfer prime ring which is a finite algebra over a commutative ring of Krull dimension one), every torsion f.p. module is a direct sum of cyclic modules. Generalizations are obtained for a number of other results about maximal orders.

C. Lanski: Chain Conditions in Rings with Involution

Let R be a ring with involution $*$, $S = \{r \in R / r^* = r\}$, and S' the subring generated by S. If R is a semi-prime ring and S' is Goldie, Noetherian, Artinian, or has Krull dimension, the same holds for R. When R is Goldie, S' is Goldie and when R is Noetherian and contains 1/2, S' is Noetherian and has the same Krull dimension as R.

G. Krause: Quotient rings of noetherian rings

It is proved that for a right noetherian ring R and an arbitrary ring S the deviation of a bimodule ${}_SM_R$ is at most equal to the classical Krull dimension of R, provided ${}_SM$ is noetherian and M_R has Krull dimension. This is useful in proving that two-sided noetherian right fully bounded rings are right ideal invariant, and that a noetherian right ideal invariant ring with the right Macaulay condition, i. e. $K\text{-dim}(A_R) = K\text{-dim}(R_R)$ for each right ideal A, has an artinian classical quotient ring. These results arise from recent investigations by Lenagan, Warfield, Stafford, and Cauchon.

E. Taft: The continuous dual of a polynomial algebra

Let F[x] be the algebra of polynomials in one variable over a field F, $F[x]^*$ its linear dual considered as infinite sequences of scalars. The continuous dual $F[x]^o$ consists of those sequences vanishing on cofinite ideals, i. e., vanishing on non-zero ideals. For such an ideal generated by $x^n - a_{n-1}x^{n-1} - \dots - a_1x - a_o$, the sequences vanishing on it are those whose m-th entry, for all $m \geq n$, are a fixed linear combination of the previous n entries. This space $F[x]^o$ of recursive sequences has the structure of a coalgebra. Given an f in $F[x]^o$, of minimal recursive degree n, it generates a subcoalgebra of dimension n. This subcoalgebra has a basis consisting of $f, Df, D^2f, \dots, D^{n-1}f$, where D is the difference operator shifting each entry of a sequence one position to the left. We present an algorithm for diagonalising f in terms of this basis. It involves inverting the symmetric matrix whose n rows are the n-tuples formed from f starting with its first n entries. Also, given a recursive relation, we discuss bases for its solution space. If F is algebraically closed of characteristic 0, the theory resembles that of ordinary linear homogeneous differential equations. At characteristic $p > 0$, certain ideas involving divided power sequences intervene to cause difficulty.

F. Kasch (München)

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