MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 27/ 1977

Measure Theory - Applications to Stochastic Analysis

July 3 - 9, 1977

This Conference was organized by G. Kallianpur (Minneapolis, U.S.A. - Calcutta, India) and D. Kölzow (Erlangen).
Participants: 41, Countries represented: 11, Talks held: 28.
To the knowledge of the referees, this Conference has been the first focussing on the applications of Measure Theory to Stochastic Analysis.

Participants

L. Arnold, Bremen
V.E. Beneš, Murray Hill (U.S.A.)
E. Bertsch, Berlin
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D. Dawson, Ottawa (Canada)
T.E. Duncan, Lawrence (U.S.A.)
N. El Karoui, Paris (France)
P. Gänßler, Bochum
M. Gattinger, Erlangen
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M. Hitsuda, Nagoya (Japan)
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Th. Kailath, Stanford (U.S.A.)

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W. Kliemann, Bremen

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M. Metivier, Palaiseau (France)

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F. Österreicher, Salzburg (Austria)

H. Oodaira, Yokohama (Japan)

P. Ressel, Freiburg

R.W. Rishel, Lexington (U.S.A.)

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W. Wedig, Karlsruhe

A. Weron, Wrozlaw (Poland)

V. Wihstutz, Bremen

J.C. Willems, Groningen (Netherlands)

M. Yor, Paris (France)

Abstracts

MARTINGALES, STOCHASTIC INTEGRALS

N. EL KAROUI : Temps d'arrêt optimaux en théorie générale des processus

L'objet de ce travail est de caractériser les temps d'arrêt "optimaux", c'est à dire qui maximisent $E(Y_T)$ où T parcourt la classe des temps d'arrêt associé à un espace filtré $(\Omega, \mathfrak{F}, F_t, P)$, et où Y est un processus optionnel positif.
L'outil essentiel de ce travail est le gain optimal conditionnel défini par Z_T = P-ess sup $E(Y_S/F_T)$, qui est la plus petite surmartingale qui majore Y.
Lorsque Y est làd làg, on montre qu'on peut trouver trois temps d'arrêt, liés étroitement au début de l'ensemble $\{ Y = Z$ ou $Y^- = Z^- \}$ tels que $\sup_T E(Y_T) = E(Z_0) = E(Y^-_{D^-} + Y^+_{D^+} + Y_D)$, et que si Y satisfait à

$$\lim_{T_n \downarrow T} E(Y_{T_n}) \leqslant E(Y_T) \quad \text{et} \quad \lim_{T_n \uparrow T} E(Z_{T_n}) \leqslant E(Z)$$

, il existe toujours un temps d'arrêt optimal.
Nous remarquons ensuite qu'en fait, ce problème d'optimalité admet toujours une solution dans un espace des formes linéaires convenablement défini et que l'on sait caractériser la plus petite et la plus grande forme optimale.

M. METIVIER : Stochastic integration with respect to Hilbert valued martingales, representation theorems and infinite dimensional filtering

We describe a very general stochastic integral with respect to Hilbert valued martingales, as it was introduced by M. Metivier and G. Pistone in Z. Wahrscheinlichkeitstheorie und Verw. Gebiete 33 (1975), 1-18 .
This integral seems well suited to the study of stochastic partial differential equations and the filtering of related systems.
The main feature of this integral, beside its isometry property, is that it makes possible the integration of a wide class of processes, the values of which may be unbounded operators. It is shown that we have to pay this price to get convenient representation theorems.
As an illustration we indicate how can be developped martingale argu-

ments in the infinite dimensional filtering problem, as they were developped for example by Th. Kailath, A.V. Balakrishnan, M. Fujisaki, G. Kallianpur and H. Kunita in the finite dimensional setting.

M. YOR : On the representation of martingales as stochastic integrals

Let $(\Omega, \mathcal{F}^o)$ be a measurable space, with a right-continuous filtration $(\mathcal{F}^o_t)$, and X a $(\mathcal{F}^o_t)$ adapted, right-continuous, real process. Note $\mathcal{M} = \{$ P probability measure on $(\Omega, \mathcal{F}^o) \mid$ X is a $(P, \mathcal{F}^o_t)$ local martingale$\}$.

The main result I gave is :

Theorem : Let $P \in \mathcal{M}$. Then, (1) and (2) are equivalent:

(1) P is an extremal point of $\mathcal{M}$

(2) Every (local) martingale Z (for P) can be represented as: $Z_t = c + \int_0^t H_s \, dX_s$, where $c \in \mathbb{R}$, and H is a predictable process such that $(\int_0^t H_s^2 \, d[X,X]_s)^{1/2}$ is locally integrable.

The proof was based on a theorem due to R.G. Douglas (1964), which I recall briefly: if $(A, \mathcal{O}\!\iota, \mu)$ is a probability space, and F a set of real functions $f : (A, \mathcal{O}\!\iota) \longrightarrow (\mathbb{R}, \mathcal{B}(\mathbb{R}))$, such that $1 \in F$, then, if $\mathcal{N}_\mu = \{\nu$ probability measure on $(A, \mathcal{O}\!\iota) \mid$ for all $f \in F$, $\int f d\nu = \int f d\mu \}$ - the integrals are obviously supposed to be defined (e.g. $F \subset L^1(\mu)$, and, if $\nu \in \mathcal{N}_\mu$, $F \subset L^1(\nu)$) - μ is extremal in $\mathcal{N}_\mu$ iff F is total in $L^1(\mathcal{O}\!\iota, \mu)$.

To apply Douglas theorem, I need the following stability proposition, for stochastic integrals:

Proposition : Let (V_t) be a local martingale, and $Y^n = \int_0^\cdot h^n_s \, dV_s$ - h^n and h are predictable processes - a sequence of uniformly integrable martingales. Suppose $Y^n_\infty \xrightarrow[\sigma(L^1, L^\infty)]{} Y_\infty$. Then,

$Y_t = E[Y_\infty | \mathcal{F}_t]$ is also a stochastic integral:

$Y_t = \int_0^t h_s \, dV_s$.

STOCHASTIC FILTERING AND CONTROL

R.S. BUCY : A priori bounds for the cubic sensor

The Ornstein-Uhlenbeck process is considered as the signal and is observed via the cubic sensor corrupted by white noise. Upper and lower bounds for the error variance in estimation of the signal are given as explicit functions of the generalized signal to noise ratio. In particular asymptotic behavior of the upper bound is conjectured to be the signal to noise ratio to the minus three half power so that both upper and lower bounds agree asymptotically.

M.H.A. DAVIS : On a non-linear semigroup of stochastic control

Suppose $\{x_t\}$ is a controlled Markov process on a state space S; let $Q_t\phi(x)$ be the maximal reward for a control problem of duration t with terminal pay off function $\phi \in C(S)$, starting at $x_o = x$. One formulation of Bellman's principle of optimality states that Q_t is a semigroup, i.e. $Q_{t+s}\phi(x) = Q_t(Q_s\phi)(x)$. I described some recent work of M. Nisio in which this semigroup (acting on C(S)) is constructed directly by considering approximating sequences of piecewise-constant controls. I then indicated the possibility of treating in a similar way problems with noisy observations, i.e. where control has to be based on observations $\{y_t\}$ of the form $dy_t = h(x_t)dt + dw_t$ ($h \in C(S)$ and $\{w_t\}$ is Brownian, independent of x_t). Denoting $\pi_t(f) = E\{f(x_t) \mid Y_t\}$ for $f \in C(S)$, $Y_t = \sigma\{y_s , s \leq t\}$, the Fujisaki/ Kallianpur/ Kunita filtering formula states that $\pi_t(f)$ satisfies

$$\pi_t(f) - \pi_o(f) = \int_o^t \pi_s(Af)ds + \int_o^t (\pi_s(hf) - \pi_s(h)\pi_s(f))dI_s \ , \text{ where}$$

$dI_t = dy_t - \pi_t(h)dt$ is the "innovations process" and A the infinitesimal generator of x_t. This means that π_t is actually a Marcov process on the state space M(S) (= set of probability measures on S). Giving M(S) the weak topology, π_t has continuous paths and is Feller, i.e. the corresponding semigroup P_t maps C(M(S)) into itself. Thus one can formulate the partially observable control problem in terms of constructing a semigroup $\tilde{Q}_t$ on C(M(S)), related to $\{\pi_t\}$ in the same way that Nisio's semigroup Q_t relates to the original process $\{x_t\}$.

T.E. DUNCAN : Applications of differential geometry to stochastic filtering and control

Since many physical systems evolve in a smooth manifold and not in a linear space it is natural to study problems of stochastic filtering and control in this geometric setting. Brownian motion, as well as some other stochastic processes, has an inherent formal differential geometric interpretation and its natural setting is a Riemannian manifold. Some notions from differential geometry such as parallelism of vectors along a curve will be used to formulate and solve some stochastic filtering and control problems that are described by stochastic differential equations. Both continuous and discontinuous processes will be considered. Some differential geometric interpretations of the techniques used in many stochastic filtering and control problems in Euclidean spaces will be made as well as the generalization of these techniques to processes in manifolds.

F. GRAEF : Optimal filtering of infinite-dimensional stationary signals

To analyse the pulse amplitude modulation (PAM) of time-discrete signals in frequency domain, operator-valued measures must be used. Signals will be represented in time-domain by stationary sequences of Hilbert-Schmidt operators, modulation is interpreted as subordination of such sequences, and filters will be represented by elements of spaces of type $\mathcal{S}_X(\mathcal{H},\mathcal{K})$ introduced by V. Mandrekar and H. Salehi.
The construction of optimal filters for PAM demands the minimization of a nonlinear functional on such spaces. Some propositions regarding the structure of minimizing elements are derived which allow to reduce the optimization problem to one in $L^1(0,\infty)$, which can be solved explicitely.

R.W. RISHEL : Filtering and control of jump processes

The generalization of filtering formulas of Rudemo to conditionally Markov jump processes provide a mean of generalizing a recent separation principle for jump processes of Segal . A synthesis of the minimum principle for jump processes can be interpreted as a "method of characteristics" for solving the partial differential equation of the seperation principle. A solution of the partial differential equation of the separation principle leads to a solution of the dynamic programming equations. Thus these three different optimality conditions can be compared and interpreted.

STOCHASTIC EQUATIONS

V.E. BENEŠ : Realizing a weak solution on a probability space

Let $T : (X,\mathfrak{X}) \longrightarrow (Y,\mathcal{Y})$ be a Borel application, ν a given probability measure on $\mathcal{Y}$, and μ a weak solution of the stochastic equation $\mu T^{-1} = \nu$. With $(\Omega,\mathcal{F},P)$ a probability space, and $w : \Omega \longrightarrow Y$ a random variable with $Pw^{-1} = \nu$, it is of interest to know when there is a random variable $x : \Omega \longrightarrow X$ such that $Px^{-1} = \mu$ and $Tx = w$ a.s. (P). We give a necessary and sufficient condition for the existence of such a "realization" x : There is a random variable $f : \Omega \longrightarrow \mathbb{R}$, a measure isomorphism h, and a decomposition (mod P) $\Omega = E_0 \cup E_1 \cup E_2 \cup \ldots$, with E_0 conditionally $w^{-1}\mathcal{Y}$-atomless in $w^{-1}\mathcal{Y} \vee f^{-1}\mathcal{R}$, and E_n, $n > 0$, disjoint conditional $w^{-1}\mathcal{Y}$-atoms in $w^{-1}\mathcal{Y} \vee f^{-1}\mathcal{R}$, such that (i) $h : (\Omega, w^{-1}\mathcal{Y} \vee f^{-1}\mathcal{R}) \longleftrightarrow (\mu,\mathfrak{X})$, (ii) $hw^{-1}B = T^{-1}B$ for $B \in \mathcal{Y}$, (iii) under $(P|E_0)/P(E_0)$, $f|E_0$ is uniform on $[0,1]$ and independent of $E_0 \cap w^{-1}\mathcal{Y}$, (iv) $f = n + 1/2$ on E_n, $n > 0$.

D. DAWSON : A class of measure-valued Markov processes

A measure-valued Markov process is a Markov process whose state space is the space of Borel measures on $\mathbb{R}^d$. Hence at each epoch the system is described by a random measure. Such processes arise naturally as models of spatially distributed populations in population biology, chemical kinetics, etc. An interesting class of mesure-valued Markov processes are obtained as solutions of appropriate martingale problems in the sense of Strood and Varadhan. In this context we describe a number of specific examples. Of these a basic one is the multiplicative process which we describe in detail. Examples are also given of martingale problems which have no solution. We also discuss the problems of determining the thermodynamic limit of the system and the limiting behavior of the system for large values of the time parameter.

K. SATO : Uniqueness for some diffusion processes and convergence of genetical Marcov chains

A new class of degenerate diffusion processes is introduced. Let $\mathbb{K} = \{(x_1,\ldots,x_{d-1}) \in \mathbb{R}^{d-1} ; x_p \geq 0 \text{ for } p=1,\ldots,d\}$, a (d-1)-dimensional

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simplex, where $x_d = 1 - \sum_{p=1}^{d-1} x_p$, and let

$$L = \frac{1}{2} \sum_{p,q=1}^{d-1} a_{pq}(x) \frac{\partial^2}{\partial x_p \partial x_q} + \sum_{p=1}^{d-1} b_p(x) \frac{\partial}{\partial x_p} , \quad x \in \mathbb{K}$$

$$a_{pq}(x) = x_p x_q \Big(\sum_{r=1}^{d} \beta_r(x) x_r - \beta_p(x) - \beta_q(x) \Big) + \delta_{pq} \beta_p(x) x_p ,$$

$$b_p(x) = x_p \sum_{r=1}^{d} \gamma_r(x) (\delta_{pr} - x_r), \qquad \delta_{pq} = \begin{cases} 1 : p=q \\ 0 : p \neq q \end{cases} .$$

Let $\Gamma_p = \{ x \in \mathbb{K} ; x_p = 0 \}$ for $p=1,\dots,d$. Under the assumption that $\beta_p > 0$ on $\mathbb{K} \setminus \Gamma_p$ and β_p , γ_p continuous on $\mathbb{K}$, locally Lipschitz on $\mathbb{K} \setminus \Gamma_p$, existence and uniqueness (in the sense of martingale problem) of a diffusion process on $\mathbb{K}$ associated with L are proved. This class of diffusions includes many processes that appear in diffusion approximation in population genetics. A sequence of genetic Markov chains is introduced and its convergence to the above diffusion with β_p and γ_p constant (depending on p) is proved. This justifies and generalizes J.H. Gillespie's heuristic one-dimensional argument.

GAUSSIAN PROCESSES

M. HITSUDA : Some topics on representation of Gaussian processes

Let $X = (X(t);\ 0 \leq t \leq 1)$ be a N-ple Markov Gaussian process which is canonically represented in the form $X(t) = \sum_{i=1}^{N} F_i(t) B_i(t) =$
$= [F_1(t),\dots,F_N(t)]^t [B_1(t),\dots,B_N(t)]$, $0 \leq t \leq 1$.

Theorem : If a N-ple Markov Gaussian process $Y(t) = (Y(t);\ 0 \leq t \leq 1)$ is of multiplicity N, $Y(t)$ has the canonical representation

$$Y(t) = [F_1(t),\dots,F_N(t)] \int_0^t [f_{ij}(u)][f_{ij}(t)]^{-1}\, {}^t[dB_1(u),\dots,dB_N(u)] ,$$

where $\mathbf{f}(t) = [f_{ij}(t)]$ is a non-singular matrix for each t, and the components have the Radon-Nikodym derivatives satisfying

$$\int_0^1 \int_0^t |\mathbf{f}(u)^{-1} \mathbf{f}(t)'|^2 du dt < \infty \quad \Big(|A|^2 = \sum_{i,j=1}^{N} a_{ij}^2 \ \text{ for } A = [a_{ij}] \Big).$$

G. KALLIANPUR : Gaussian processes of two parameters

Work done jointly with N. Etemadi and C. Bromley is described.
Let $X_{\underline{t}}$, $\underline{t} = (t_1,t_2) \in [0,1]^2$ be the 2-parameter Wiener process and let $Y_{\underline{t}}$ be a zero mean Gaussian process equivalent to $X_{\underline{t}}$ (in the sense of mutual absolute continuity of measures). If $Y_{\underline{t}}$ has a representation of the form

(✱) $(I + K)X_{\underline{t}}$ where K is a Hilbert-Schmidt, Volterra operator on the Hilbert space $L(X) \equiv$ linear span $\{X_{\underline{t}}\ ,\ \underline{t} \in [0,1]^2\}$ such that $KL(X;\underline{t}) \subseteq L(X;\underline{t})$ $\forall\, \underline{t}$, then $Y_{\underline{t}}$ has a non-anticipative representation which is shown to be the same as the canonical representation of the type considered by Tjøstheim. In general, however, a representation of the type (✱) is not available but a more general representation holds which depends on a four-fold factorization of the operator S relative to the commuting chains Π_1 and Π_2 . Here S is the positive, invertible, self-adjoint operator whose existence is guaranteed by the equivalence of $(Y_{\underline{t}})$ and $(X_{\underline{t}})$, $\Pi_i = \{ P^{\varkappa}_{t_i}\ ,\ 0 \leq t_i \leq 1 \}$, and $P^{\varkappa}_{t_1}$ is the orthogonal projection onto the linear subspace $\vee\{X_{\underline{u}}\ ,\ 0 \leq u_1 \leq t_1\ ,\ 0 \leq u_2 \leq t_2\}$ ($P^{\varkappa}_{t_2}$ is similarly defined).

A consequence is the fact that not every Gaussian process of two parameters which is equivalent to the 2-parameter Wiener process has a Tjøstheim canonical representation or a non-anticipative representation.

H. OODAIRA : Note on Freidlin-Wentzell type estimates for stochastic processes

Let C be the space of real continuous functions on [0,1] with the sup norm $\|.\|_\infty$, and let $\{\mu_n\}$ be a sequence of probability measures on C converging weakly to a Gaussian measure with mean 0 and covariance kernel Γ. Let H denote the reproducing kernel Hilbert space with kernel Γ whose norm is described by $\|.\|_H$. Then, under certain conditions, the following estimates (Freidlin-Wentzell type estimates) are obtained. If $0 < \alpha(n) \uparrow \infty$ as $n \longrightarrow \infty$, then, for any $\phi \in H$, δ, $h > 0$,

$$\mu_n\{\|\tfrac{x}{\alpha} - \phi\|_\infty < \delta\} \geq \exp[-(\alpha^2/2)(\|\phi\|^2_H + h)] \quad \text{and} \quad \mu_n\{d(\tfrac{x}{\alpha}, K_r) > \delta\} \leq$$

$\leq \exp(-(\alpha^2/2)(r^2-h))$ for all sufficiently large n, where K_r is the closed ball of radius r in H and d is the distance in C. This is a generalization of recent results of J. Gärtner. Some examples of Gaussian processes, partial sums of independent identically distributed random variables and empirical distribution functions are given.

HOMOGENEOUS CHAOS AND MULTIPLE WIENER INTEGRALS

S. CAMBANIS : Stochastic and multiple Wiener integrals for Gaussian processes

Multiple Wiener integrals and stochastic integrals are defined for general Gaussian processes, extending the related notions for the Wiener process. It is shown that every L_2-functional of a Gaussian process admits an adapted stochastic integral representation and an orthogonal series expansion in terms of multiple Wiener integrals. Some results of Wiener's theory of nonlinear noise are generalized to noises other than white. Also the stochastic differential rule is given.

T. HIDA : White noise and Lévy's functional analysis
(Communicated by M. Hitsuda)

Let $(L^2) \equiv L^2(\mathcal{S}^*, \mu)$, where μ is the measure of white noise, $C(\xi) = \exp[-\|\xi\|^2/2] = \int_{\mathcal{S}^*} \exp(i\langle\xi, x\rangle)\mu(dx)$, $\xi \in \mathcal{S}$. Let $\mathcal{F}$ be the Hilbert space with reproducing kernel $C(\xi - \eta)$. As the decomposition of $(L^2) = \sum_{n=0}^{\infty} \oplus \mathcal{H}_n$, we get a decomposition of $\mathcal{F} = \sum_{n=0}^{\infty} \oplus \mathcal{F}_n$, $\mathcal{F}_n$ is the Hilbert space with kernel $C_n(\xi, \eta) = (n!)^{-1} C(\xi)(\xi, \eta)^n C(\eta)$. We get an isomorphism $\mathcal{H}_n \simeq \mathcal{F}_n \simeq \sqrt{n!}\, \widehat{L^2(R^n)}$. This isomorphism can be extended to the "n-ple generalized Brownian functional" as in the form:

$$\begin{array}{ccccc}
\sqrt{n!}\, \widehat{H^{(n+1)/2}}(R^n) & \hookrightarrow & \sqrt{n!}\, \widehat{L^2(R^n)} & \hookrightarrow & \sqrt{n!}\, \widehat{H^{-(n+1)/2}}(R^n) \\
\updownarrow & & \updownarrow & & \updownarrow \\
\mathcal{F}_n^{(n)} & \hookrightarrow & \mathcal{F}_n & \hookrightarrow & \mathcal{F}_n^{(-n)} \\
\updownarrow & & \updownarrow & & \updownarrow \\
\mathcal{H}_n^{(n)} & \hookrightarrow & \mathcal{H}_n & \hookrightarrow & \mathcal{H}_n^{(-n)}
\end{array}$$

where $\widehat{H^{(n+1)/2}}(R^n)$ is the symmetrized Sobolev space of order $(n+1)/2$.

The members of $(L^2)^- = \sum_{n=0}^{\infty} \oplus \mathcal{H}_n^{(-n)}$ should be called the "generalized functionals". These functionals are useful to define the derivations of the form $\frac{\partial}{\partial x(t)}$, $\frac{\partial^2}{\partial x(t)^2}$, ... etc. ($\int \frac{\partial^2}{\partial x(t)^2}$ dt is the Lévy Laplacian).

W. SŁOWIKOWSKI : <u>Ito's integration obtained by the second quantization of the Wiener integration</u>

To a Hilbert space H we assign the commutative Wick algebra $\Gamma_w H$ which is an algebra containing H as a linear subspace, $\Gamma_w H$ is provided with a scalar product in such a way that it coincides on H with the original scalar product, that every linear contraction extends uniquely to a contractive morphism of $\Gamma_w H$ and, finally, that the class of the linear span of n-fold products of elements of H is complete.
We give examples of Wick algebras essentially due to Wiener, Bargmann and Fock. We derive the existence of the multiple Wiener integral and, finally, we build the Ito algebra which is a Wick algebra which incorporates time and non-anticipation into the algebra structure and gives rise to the Ito stochastic integral.

OPERATOR VALUED MEASURES AND INFINITE DIMENSIONAL PROCESSES

W. HACKENBROCH : <u>Some properties of operator measures related to prediction theory</u>

By means of spectral representation, prediction theory of stationary operator sequences leads to the study of "invariant subspaces" of an abstract space $L^2(\Phi;H)$ with H a complex Hilbert space and Φ a weakly σ-additive measure on the circle group taking values in the positive bounded linear operators on H. As is well-known, the crucial properties of (simple and double) invariance depend in a delicate manner on the 0-sets of Φ as related to the 0-sets of Lebesgue measure. For general Φ, problems arise from the lack of finite variation properties of Φ, of "pointwise structure" of $L^2(\Phi;H)$ and of a Radon-Nikodym derivative of Φ with respect to Lebesgue measure. The talk gives i) Some general characterizations of 0-sets for operator measures, ii) Necessary conditions for the existence of Radon-Nikodym derivatives, iii) Equivalent formulations of the problem of decomposing self-adjoint operator measures into positive parts.

R. JAJTE : Spectral and semi-spectral Gleason measures

Some vector-valued measures defined on a lattice of all orthogonal projectors acting in a Hilbert space are considered. In particular: Hilbert space-valued orthogonally scattered measures, projector-valued and positive operator-valued measures.

The problems concerning the structure, convergence and extension in tensor products are taken into consideration.

H.H. KUO : Differential calculus for measures on Banach spaces

The study of differential calculus for measures on Banach spaces is motivated by Hodge theory and distribution theory on such spaces. In the case of infinite dimensional Banach spaces there is no natural way to regard bounded measurable functions as distributions because the Lebesgue measure does not exist. Thus one cannot expect to represent nice distributions, e.g. harmonic distributions, by smooth functions. However, finite Borel measures can be regarded as distributions in a natural way. To be able to represent nice distributions by smooth measures one needs to develop differential calculus for measures. In this expository lecture we give a brief survey on some topics such as the chain rule, Weyl's lemma, Kolmogorov's forward equation and prove a new result for differential operators associated with differentiable measures with logarithmic derivatives.

V. MANDREKAR : On subordination of decomposable processes

Let $(T,\mathcal{B})$ be a standard Borel space and $(\Omega,\mathcal{F},P)$ be a complete probability space. A decomposable process is a map on $\mathcal{B} \longrightarrow L^0(\Omega,\mathcal{F},P)$ such that $X_{A_1},\dots,X_{A_n}$ are independent for A_i disjoint and countably additive. We study in general the problem of subordination of X with respect to a Gaussian process X', which is also decomposable. We show using $L_{2,F}$ introduced by the author and H. Salehi that X is a stochastic integral with respect to X' except for a shift by arbitrary signed measure. In the above problem we assume X_A is H-valued for H real separable. We show that the problem cannot be generalized to a real separable Banach space preserving main result of J. Feldman.

P. MASANI : The frequency response function as a Radon-Nikodym derivative

Theorem 1 : Let (i) $\mathcal{A}$ be a σ-algebra over a space Λ and μ be a σ-finite countably additive measure on $\mathcal{A}$ to $[0,\infty]$; (ii) W and H be Hilbert spaces over $\mathbb{F}$ ($\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$) and W be separable; (iii) $T(.)$ be a W-to-H c.a.q.i. measure on the δ-ring $\mathcal{A}_\mu = \{A : A \in \mathcal{A}$ and $\mu(A) < \infty\}$ with control measure $\mu(.)I_W$, i.e. $\forall$ A, $B \in \mathcal{A}_\mu$, $T(B)^* T(A) = \mu(A \cap B) I_W$ (Bull. AMS, 1970, p.449); (iv) $\forall B \in \mathcal{A}$, $\mathcal{M}_T(B) \underset{d}{=} \sigma\{T(A)(W) : A \in \mathcal{A}_\mu$ and $A \subseteq B\}$ and $Q_T(B)$ be the projection on $\mathcal{M}_T(\Lambda)$ onto $\mathcal{M}_T(B)$; (v) R be a continuous linear operator on H to H which commutes with $Q_T(B)$, $\forall B \in \mathcal{A}$. Then $R = \Sigma_T \cdot \mathcal{M}_\phi \cdot \Sigma_T^*$, where Σ_T is the isometry on $L_{2,\mu} \underset{d}{=} L_2(\Lambda, \mathcal{A}, \mu; W)$ onto $\mathcal{M}_T(\Lambda) \subseteq H$ given by $\Sigma_T(\phi) \underset{d}{=} \int_\Lambda T(d\lambda)\phi(\lambda)$, $\phi \in L_{2,\mu}$, and $\phi(.)$ is a function on Λ to $CL(W,W) \underset{d}{=}$ the class of all continuous linear operators on W to W such that $\forall w \in W$, $\phi(.)w$ is $\mathcal{A}$-Borel (W) - measurable, and $\forall f \in L_{2,\mu}$

$$\{\mathcal{M}_\phi(f)\}(\lambda) = \phi(\lambda)\{f(\lambda)\}\ , \text{ a.e. } \mu .$$

We prove this theorem by showing that the set-function $L_R(.)$ on $\mathcal{A}_\mu$ given by $L_R(A) \underset{d}{=} T(A)^* RT(A)$, $A \in \mathcal{A}_\mu$, is a c.a. measure on $\mathcal{A}_\mu$ to $CL(W,W)$, which satisfies the hypothesis of the Dunford-Pettis theorem (TAMS(47),1940, p.323). Hence it has a Radon-Nikodym derivative $\phi(.)$ on Λ with values to $CL(W,W)$. This $\phi(.)$ does the job.

Next we let $H \underset{d}{=} L_{2,\mu}$ and $\forall A \in \mathcal{A}_\mu$, $T(A)w = \mathcal{M}_\chi(A)(w) \underset{d}{=} w\chi_A(.)$, we easily deduce (since $\Sigma_T = I_{L_{2,\mu}}$ and $Q_T(A) = \mathcal{M}_{\chi_A}$) :

Theorem 2 : Let R be a continuous linear operator on $L_{2,\mu}$ to $L_{2,\mu}$, which commutes with multiplication by indicator functions χ_B, $B \in \mathcal{A}$. Then R is the multiplication operator $\mathcal{M}_\phi$. Here

$$\phi = \frac{dL_R}{d\mu}, \text{ where } L_R(A) = \mathcal{M}_\chi(A)^* R \mathcal{M}_\chi(A),\ A \in \mathcal{A}_\mu .$$

Next we let $\Lambda = \mathbb{R}$, $\mathcal{A}$ = Borel $(\mathbb{R})$, μ = Lebesgue measure, let R be a time-invariant linear filter with signals in $L_2(\mathbb{R})$ and $\hat{R} = VRV^*$, where V is the Fourier-Plancherel transform on $L_2(\mathbb{R})$. Then $\hat{R}$ commutes with multiplication by the indicator functions χ_B, $B \in$ Borel $(\mathbb{R})$. Hence by Theorem 2, $\hat{R} = \mathcal{M}_\phi$, where $\phi = \dfrac{dL_{\hat{R}}}{d\mu}$.

Thus the frequency-response function Φ of the filter is the Radon-Nikodym derivative of an operator-valued measure.
The first part of Theorem 2 is essentially due to Foures and Segal (TAMS 98 (1955),385-). The Radon-Nikodym approach is due to the writer, cf. "Vector and operator-valued measures" , by D.H. Tucker and H.B. Maynard, Acad. Press 1973, pp. 217-232 .

A. WERON : Operator valued measures related to multivariate stochastic processes

We consider some problems arising in connection with the linear prediction of multivariate stochastic processes. We present the "operator model" for second order processes with values in Banach spaces.
For stationary processes and additively correlated processes the spectral representations are obtained. The measures arising here taking values in the space of linear continuous operators $L(B,H)$, (where B is a Banach space and H is a Hilbert space) and $L(B,B^*)$.
As an application we discuss some aspects of the dilation theory. We prove that $L(B,B^*)$-valued positive definite function K over the group G may be dilated to the unitary representation D_g in $L(H_X)$, where H_X is a time domain of a stationary process $(X_g)_{g\in G}$, where $\forall g$ $X_g \in L(B,H)$.

STABILITY

L. ARNOLD : On the stability of stochastic linear differential equations

Let $\dot{x}_t = A_t x_t$, where A_t is "real noise" (i.e. a matrix-valued stationary, sometimes ergodic and Markov stochastic process). What are conditions for $x_t \longrightarrow 0$ $(t \to \infty)$ a.s. ? Put $w_t = x_t/|x_t|$. We get
$|x_t| = |x_0| \exp(\int_0^t q(A_s,w_s)ds)$, where $q(A,w) = w'(\frac{A+A'}{2})w$. Thus, the growth of x_t depends on the ergodic behavior of the pair $z_t = (A_t,w_t)$. If we assume that A_t is a stationary and ergodic solution of the Ito equation

$$\left.\begin{array}{l} dA_t = F(A_t)dt + G(A_t)dw_t \text{ and if we add the differential equation for } w_t \\ dw_t = (A_t - q(A_t,w_t)I)w_t \text{ ,} \end{array}\right\} (*)$$

the investigation of the stability of the original d.e. reduces to finding stationary solutions of (*). The example $\ddot{y} + f_t y = 0$ is treated in detail.

In this case, ($\divideontimes$) has stationary distributions, such that

$\frac{1}{t}\int_0^t q(A_s, w_s)ds \longrightarrow R$, where R is a constant. For $f_t > 0$ this constant is

$$R = \int_{x=0}^{\infty}\int_{\alpha=0}^{2\pi} \frac{(1-x)\sin 2\alpha}{x\cos^2\alpha + \sin^2\alpha}\, d\mu,$$ where μ is the (uniquely existing) invariant measure of ($\divideontimes$).

W. WEDIG : On the integration of sequences of moments equations in the stability theory of stochastic systems

Linear stochastic systems with coloured noise coefficients generated from white noise by a filter equation lead to an infinite set of moments equations. In order to avoid such sequences a linear transformation of the state vector is introduced by use of Ito's calculus, where the associated transformation matrix is defined as a second order eigenvalue problem on the entire range of the filter's process.

Under the condition that the transformation matrix is non-singular and bounded with probability one the eigenvalue equation is integrated by means of Hermite polynomials resulting in a recursion formula for the determination of the associated coefficient matrices the convergence of which can be proofed. The evaluation of the mean square stability condition is carried out in case of a single degree of freedom system and a scalar first order filter equation.

MISCELLANIES

TH. KAILATH : Classifications of operators by their complexity of inversion

Many physical problems lead ultimately to the solution of linear equations, say $Ra = m$, where R and m are N×N and N×1 matrices determined by the physical problem. To solve such equations generally takes $O(N^3)$ operations (multiplications and additions), which may be burdensome for large N. Therefore it is desirable to see if R has any special structure that might help to reduce the number of computations. A frequently made assumption is that we have an underlying stationarity (or shift-invariance or homogeneity) property that allows us to take R to be Toeplitz, i.e. of the form $R = [\lambda_{i-j}]$. This is nice because it is known that such Toeplitz equations can now be solved with $O(N^2)$ operations instead of $O(N^3)$. In many cases

however R might rather be the product of Toeplitz matrices, or the inverse of a Toeplitz matrix, or the non Toeplitz covariance matrix of an asymptotically stationary process, or other non Toeplitz matrices growing out of operations on Toeplitz matrices. According to present theory, these matrices being non Toeplitz would seem to require $O(N^3)$ operations for their inversion; however one feels that this is too high and that such non Toeplitz but certainly nonarbitrary equations should be soluble with a number of operations between $O(N^2)$ and $O(N^3)$. We shall show that with any N×N matrix we can associate an integer α, $1 \leq \alpha \leq N$, that roughly speaking gives a measure of how non-Toeplitz the matrix is; moreover the associated linear equations can be solved with $O(N^2\alpha)$ operations. We present several results on this method of classifying matrices and also integral operators (i.e. continuous kernels).
These results, which grew out of studies on the structure of the nonlinear Riccati - and Chandrasekhar - type differential equations that arise in linear filtering theory, were done jointly with B. Friedlander, S. Kung, L. Ljung, B. Lèvy and M. Morf.

F. ÖSTERREICHER : <u>On the construction of least favourable pairs of distributions</u>

A composite testing problem (P, Q) can, in the case when P and Q are described by 2-alternating capacities, be reduced to a testing problem of single hypotheses (P^*, Q^*) (Huber and Strassen, AS 1973). (P^*, Q^*) is called a least favourable pair.
This paper presents a principle of constructing (P^*, Q^*), which is based on the use of the corresponding risk sets and which works for the case when P and Q are given in terms of some neighbourhoods. Both in the ε - contamination model and the total variation model, which were already solved by Huber (AMS 1965), the solution is derived straight forward. But also Prokhorov-type -neighbourhoods are treated successfully, at least under the condition of monotone likelihood ratio. This condition, however, may be weakened.

P. RESSEL : <u>The continuity of $(\mu, \nu) \longmapsto \mu \otimes \nu$</u>

For a given arbitrary (not necessarily Hausdorff) space denote by $\mathcal{P}_\tau(X)$ the space of all τ-smooth probability measures. We prove that given $\mu \in \mathcal{P}_\tau(X)$,

$\nu \in \mathcal{P}_\tau(Y)$ there exists an uniquely determined measure $\mu \hat{\otimes} \nu \in \mathcal{P}_\tau(X \times Y)$ extending $\mu \otimes \nu$. The mapping $(\mu, \nu) \longmapsto \mu \hat{\otimes} \nu$ is continuous and $\mu \hat{\otimes} \nu$ is Radon iff μ and ν are (X and Y then being Hausdorf). Extensions to infinite products are given, too.

J.C. WILLEMS : Representations of dynamical systems

In this talk definitions of input/output dynamical systems, systems in state space form, and recursive "white noise" representations were defined and their relationship discussed. The problem of finding, for a given continuous time stationary zero mean Gaussian $\mathbb{R}^n$-valued process y, a continuous time stationary zero mean Gaussian Markov $\mathbb{R}^n$- valued process x, and a matrix C such that $y(t) = Cx(t)$ was discussed and all y-measurable solutions x were presented.

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft