MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 29/1977

Scattering Theory
17.7. - 23.7.1977

Leitung: P.Werner (Stuttgart), C.H.Wilcox (Salt Lake City,USA)

This was the third meeting on Scattering Theory to be held at Oberwolfach (first two meetings: June 1971, July 1974). The last six years have seen rapid progress in this field, stimulated in part by scientific contacts which took place at these meetings.

Topics discussed this year included potential scattering (in particular long range and periodic potentials and the inverse problem), multi-particle scattering, non-linear scattering, scattering theory for electric and acoustic waves and for the linear Boltzmann transport equation.

<u>Teilnehmer</u>

Agmon, S. (Jerusalem)
Baum, C.E. (Albuquerque)
Bentosela, F. (Marseille)
Bröhl, A.P. (Bonn)
Combes, J.M. (LaGarde)
Combescure, M. (Orsay
Costabel, M. (Darmstadt)
Dermenjian, Y. (Villetaneuse)
Dolph, C.L. (Ann Arbor)
Eberlein, (Tübingen)
Eckardt,K.-J. (München)
Ginibre,J. (Orsay)
Guillot,J.C. (Villetaneuse)
Hagedorn,G.A. (New Jersey)
Hejtmanek,J. (Wien)
Herbst, I.W. (Princeton)
Jäger, W. (Heidelberg)
Kalf, H. (München)
Lavine, R. (Rochester)
Leinfelder,H. (München)
Leis, R. (Bonn)
Mäulen, J. (Stuttgart)
Meister, E. (Darmstadt)
Mohr, R. (Stuttgart)
Mourre, E. (Marseille)
Najman, B. (Zagreb)
O'Carroll,M.L.(Rio de Janeiro)
Pearson, D.B. (Hull)
Picard, R. (Bonn)
Ralston, J.V. (Los Angeles)
Saito, Y. (Osaka)
Sigal, I.M. (Zürich)
Simader, C.G. (Bayreuth)
Simon, B. (Princeton)
Streater, R.F. (London)
Thomas,L.E. (Charlottesville)
Veselic,K. (Dortmund)
Voigt,J. (München)
Walter,J. (Aachen)
Weber, Ch.(Stuttgart)
Weck, N. (Essen)
Weidmann, J. (Frankfurt)
Werner, P. (Stuttgart)
Wilcox, C.H. (Salt Lake City)
van Winter, C. (Lexington)
Wüst, R. (Berlin)
Yajima, K. (Tokyo)
Zizi, K. (Reims)

Vortragsauszüge

Shmuel Agmon: Schrödinger Operators with long Range Perturbations

Let $-\Delta+V(x,D)$ be a formally self-adjoint Schrödinger operator on $\mathbb{R}^n$ with $V=\sum_{|\alpha|\leq 2} V_\alpha(x)D^\alpha$, $V_\alpha \to o$ as $|x|\to\infty$. Let H be the self-adjoint realization of $-\Delta+V$ in L^2. We describe in this lecture some new results in the spectral and scattering theory of H in case V is long range (i.e. slowly decaying). The main results we discuss are the following.

(i) The existence in some generalized optimal sense of the boundary values $R(k^2\pm io)$, $k>o$, of the resolvent $R(z)=(H-z)^{-1}$.

(ii) The eigenfunction expansion theorem for H and its relations to the asymptitic formula:

$$R(k^2\pm io)f \sim c_n r^{-(n-1)/2} \exp(\pm i\,\psi_\pm(r;w,k))a_\pm(w;k,f) \text{ for}$$

$r=|x|\to\infty$, $w=x/|x|$. (Here $\psi_\pm$ is a certain real phase function independent of f).

(iii) The asymptotic completeness of the modefied wave operator $\tilde{W}_\pm$ of H established by means of the formula:

$$\tilde{W}_\pm = \mathcal{F}_\pm^* \mathcal{F} M(D).$$

Here $\mathcal{F}_\pm$ are generalized Fourier maps of H, $\mathcal{F}$ the ordinary Fourier map and M(D) is a unitary pseudo-differential operator.

Dr. Carl E.Baum: The Singularity Expansion Method in Electromagnetic Theory: A Recent Development of Practical Importance in Electrical Engineering which Raises Important Mathematical Questions

Beginning from empirical observations of the transient response of complex electromagnetic scatterers it was noted that damped sinusoids were important parts of the transient-response waveforms. This led to the concept of expanding the solution to the scattering (and antenna) problem(s) in terms of the singularities of the solution in the s plane; this is the basic concept of the singularity expansion method (SEM). It has been further observed by analytic examples and numerical results that the pole terms are adequate, at least in some cases, to describe the complete response. An important aspect of the pole terms is the efficient way they characterize the response of the scatterer as a function of the various physical parameters of the problem by separating the dependences on the various parameters. This subject appears to be one in which there can be fruitful collaboration between the mathematicians and the engineers.

F. Bentosela: Periodic Media: the Effective Mass Approximation

A common method in the solid state physics to calculate the isolated eigenvalues of the quantum mechanical operator $H = -\frac{\Delta}{2m} + V + Q$ ($V(x)$ is a periodic potential, $Q(x)$ a potential which goes to zero as $|x| \to +\infty$, m the electron mass) in a real interval on the resolvent set of $H_B = -\frac{\Delta}{2m} + V$ is to replace this operator by a simpler one: $-\frac{\Delta}{2m^*} + Q$; where m^*, called the effective mass is related to the energy of H_B.
We give a mathematical justification of this approximation by constructing a family of operators, derived from H, the resolvent of which converge to the resolvent of $-\frac{\Delta}{2m^*} + Q$.
We give also some corrections to the effective mass approximation by expressing the eigenvalue of H in the form of asymptotic series.

L.C. Dolph: A New Challenge to Scattering Theory - the Singularity Expansion Method [SEM]

The use of complex singularities, first introduced by Lord Kelvin [J.J.Thomson, Proc. London Math. Soc. 15, 1884, p.197], in a discussion of the free modes of oscillations of the electromagnetic field around a perfectly conducting sphere has led to similar treatments for other obstacles. [c.f., e.g. Tesche, F.M., IEEE Trans. on Antennas and Propagation Vol. Ap-21, 1973, p. 53 - this article also contains a description of the SEM method]. The poles or "resonances" in the SEM method are determined by numerical methods, and their physical reality is often in doubt. Here the results so far obtained toward the resolutions of the many difficulties of the SEM method, {as obtained in collaboration with C.T.Tai of the University of Michigan (EM)} will be summarized.

J. Ginibre: Existence of Solutions and Scattering Theory for a Class of non linear Schrödinger Equations
(Joint work with G.Velo)

We study a class of non linear Schrödinger equations of the form

$$i \frac{du}{dt} = (-\Delta + m)u + f(u) \qquad (1)$$

in $\mathbb{R}^n$, where m is a real constant and f a complex valued non linear function. Under suitable assumptions on f, we prove the existence of global solutions of the Cauchy problem for an integral version of the equation, in two steps: we prove the existence of solutions in a small time interval by a contraction method, and we extend the solutions to all times by the use of the conservation laws of the L^2-norm and of the energy. We then study the scattering theory for the pair of equations (1) and (2) below:

$$i \frac{du}{dt} = (-\Delta + m)u. \qquad (2)$$

We prove the existence of the wave operators by solving the initial value problem with infinite initial time, again by a contraction method. Finally, we prove asymptotic completeness for a class of repulsive interactions by the use of an approximate conservation law, which we call pseudoconformal. The previous program is implement in a variety of theories that differ by the choice of the basic space and of the assumptions on f. We present (1) a general theory that holds for any $n \geq 2$, with basic space $H^1(\mathbb{R}^n)$, but with some topological complications. (2) Special theories for n=2 and 3, with basic space $H^1(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n)$, free of the previous complications. (3) A simple theory for n=1, with basic space $H^1(\mathbb{R})$.
The assumptions made on f cover the case of a single power $f(u) = \lambda |u|^{p-1} u$ with various restrictions on λ and p. We require (1) lower bounds on p, which range between 1 and 2 for the Cauchy problem at finite initial time, and between $1+2/n$ and $1+4/n$ for the Cauchy problem at infinite initial time, depending on the particular version of the theory. (2) upper bounds on p, namely $p<(n+2)/(n-2)$ if $\lambda > 0$ (for $n \geq 3$) only), and $p < 1+4/n$ if $\lambda < 0$. (3) for asymptotic completeness only) $\lambda \geq 0$ and $p > 1 + 4/n$.

J.C.Guillot: Wave Propagation and Scattering in Stratified Media

Recent work of C.H.Wilcox and myself is discussed.
One studies the selfadjoint operator $A = -c^2(y)\Delta_{x,y}$ in the Hilbert space $H = L^2(c^{-2}(y)dydx)$ where $x = (x_1,x_2)\in\mathbb{R}^2$, $y\in\mathbb{R}$, $c(y) = c_0$ for $y<0$, $c(y) = c_1$ for $0<y<h$, $c(y) = c_2$ for $y>h$. We prove that A is an absolutely continuous operator whose spectrum is $[0,\infty)$. We also give a generalized eigenfunction expansion. Finally we prove existence and completeness of the usual wave operators for local perturbations of the function $c(y)$. The physical interpretation of the solutions of the wave equation associated with A is given.

George A.Hagedorn: Resolvent Formulas for 3 and 4 Body Schrödinger Operators

Let H be the Schrödinger Hamiltonian for a system of three or four particles with the center of mass motion removed. Formulas are derive for the resolvent $(z-H)^{-1}$, which may be used to prove asymptotic completeness for a class of dilation analytic two-body potentials when the space dimension is greater than or equal to three.

J.Hejtmanek: Recent Results on the Scattering Theory of the Linear Boltzmann Operator

The first part of this lecture is a review on results of the scattering theory of the neutron transport equation in $L^1(R^3\times R^3)$:

$$\frac{\partial n}{\partial t} = -\, v\,\mathrm{grad}_x n - h(x,v)n + \int_{R^3} k(x,v',v)n(x,v',t)dv'$$

$$n(\cdot,\cdot,t)\in L^1(R^3\times R^3)$$

$$n(x,v,o) = n_o(x,v) \qquad \text{(initial condition)} \quad .$$

The wave operator Ω^+ and $\tilde{\Omega}^-$ have the following properties:
1) $\tilde{\Omega}^-$ exists $\Rightarrow$ The transport system is subcritial. 2) Ω^+ exists $\Leftrightarrow$ The transport system is subcritical. Sufficient conditions for the existence of $\tilde{\Omega}^-$ (Simon, Voigt, Protopopescu) are discussed.

The second part is a complete description of the spectrum of the collision free neutron transport operator T in $L^1(R^3\times R^3)$ and then in $L^p(R^3\times R^3)$ $1<p\leq\infty$.

Ira W. Herbst: Topics in the Study of Schrödinger Operators with a Constant Magnetic Field

This talk is a report of some recent work done in collaboration with J.Avron and B.Simon. I will cover three disjoint topics:

I. Correlation Inequalities and Properties of Ground States. Correlation inequalities from the theory of spin systems will be used to prove that the ground state of hydrogen in a magnetic field has $\vec{L}\cdot\vec{B}=0$.

II. Enhanced Binding in Magnetic Fields. Since a constant magnetic field binds a particle in the plane perpendicular to $\vec{B}$, it effectively reduces a three dimensional problem to a one dimensional one where binding is easier. Thus it will be shown that He^- has a bound state in an arbitrarily weak magnetic field (although detailed calculations indicate this is not the case if $\vec{B}=0$).

III. Separation of the Center of Mass in N-Body Systems. Translation symmetry is realized in general by a projective representation of the translation group. Only if the total charge is zero is there a real representation. In this case one can introduce a quasi-momentum. The consequences for the HVZ theorem are explained.

Willi Jäger: On the absolute Continuity of the Spectrum of Schrödinger Operators with long Range Potentials.

Consider a domain G in $\mathbb{R}$, $\mathbb{R}^n \setminus G$ bounded, $p \in C^0(\overline{G},\mathbb{R})$; assume that $p_\infty = \lim_{|x|\to\infty} p(x)$ exists in $\hat{\mathbb{R}}$. We ask for conditions on p such that the selfadjoint extensions H of the operator $L = \Delta + p$, defined on $D(L) = C_0^2(G,\mathbb{C})$, has absolute continuous spectrum in $]p_\infty,\infty[$. There are many results in the case of ordinary differential operators, for instance proved by Tischmarsh, Neumark, Walter and Rejto. This joint work with P.Rejto gives a generalization to the case of partial differential operators using the method of constructing approximate potentials p_λ with help of approximate solutions θ_λ of the equation

$$i\vartheta_r - \vartheta^2 + \lambda - p = 0 \quad .$$

Under certain assumptions on Θ_λ one can control the limit absorption using weighted L^2-norms and the radiation condition

$$\int |u_r - i\Theta^\lambda u|^2 r^{-1} dx \quad (r=|x|).$$

Choosing different approximations Θ_λ one gets different classes of potentials p such that absolute continuity follows. The class of long range potentials treated by Lavine, Ikebe and Saito is a subclass. Example of an exploding potential which can be handled:

$$p(x) > -|x| + O(|x|^{-1/2-\varepsilon}) \quad \text{for} \quad |x| \to \infty .$$

Richard Lavine: The Scattering Cross Section.

The differential cross section for scattering of a quantum mechanical particle with energy operator $H = -\Delta + V(x)$ is traditionally expressed in terms of $|t(k,k')|^2$ where t is the kerned of T = S-I, where S is the scattering operator. Although the differential scattering cross section can be measured for the long range Coulomb potential, T is known to possess reasonable kernel only for potentials decaying faster than r^{-2} at ∞. We give a generalized expression for the differential cross section as a measure on $S^2 \times S^2 - \{(w,w');\ w \in S^2\}$ in terms of the asymptotic in- and outmomenta $P^{\pm}$, which are known to exist for long range potentials, and prove that this measure is finite on products of disjoint subsets of S^2 for a class of potential V which can decay as slowly as $r^{-1/2-\varepsilon}$ at ∞ .

Rolf Leis: Zur Theorie elektromagnetischer Wellen in anisotropen inhomogenen Medien

The purpose of this survey lecture is the development of a theory of boundary- and initial value problems for Maxwell's equations. We start by introducing the general time dependent equations for inhomogeneous anisotropic media. Intending to give a Hilbertspace theory several notations regarding solutions in the weak sense are needed. We then treat the time harmonic equations. The damped case can very easily be handled. The undamped case is more difficult. First we deal with boundary and eigenvalue problems for bounded domains. Afterwards exterior boundary value problems are solved using

the method of limiting absorption. In order to apply this method a-priori estimates have to be derived, and for proving unicity the principle of unique continuation has to be verified.
These preparations being done the general time dependent case (initia value problems) can be handled. The spectral theorem for the Maxwell operator has to be formulated first. Initial value problems for bounded domains can be solved afterwards and some aspects of a genera theory of wave propagation will be given.

Michael Lovis O'Carroll: On the Inverse Scattering Problem for the Schrödinger, Klein-Gordon and Dirac equations in dimension $n>1$

We treat the inverse scattering problem associated with the Schroedinger, Klein-Gordon and Dirac equations in dimension $n>1$ with a linear potential term and with minimal coupling to a static electromagnetic field. For a specified class of potentials by considering certain high energy, fixed momentum transfer limits of the scattering amplitude we recover the potentials, show uniqueness at the inverse function and continuity and differentiability properties of the inverse.

D.B. Pearson: Pathological Spectral Properties of Schrödinger Hamiltonans

Given a sequence $\{f_n(k,y)\}$ of functions periodic in y with period c and satisfying, in addition to regularity conditions,

$$\text{(i)} \quad \bar{f}(k) = \frac{1}{c}\int_0^c f_n(k,y)dy = 1 , \qquad \text{(ii)} \sum_{n=1}^{\infty} -m_n(k) = \infty ,$$

where $m_n(k) = \frac{1}{c}\int_0^c \log f_n(k,y)dy \quad (f_n \geq \text{const.} > 0)$,

the measure μ defined on intervals Σ by

$$\mu(\Sigma) = \lim_{n\to\infty} \int_{\Sigma} \prod_{i=1}^{n} f_i(k,N_i k)dk$$ may be extended (for $\{N_i\}$ increasing sufficiently rapidly) to a singular continuous measure on Borel subsets of $\mathbb{R}$.

This result is applied to potential scattering to generate potentials $V(r)$ such that $H = -\Delta + + V$ has singular continuous spectrum. Typicall these potentials consist of a sequence of "bumps" of height g_n separated by intervals N_n (with $\sum g_n^2 = \infty$),but potentials giving rise to these spectral properties also arise from perturbations of both periodic and short-range (singular) potentials.

Rainer Picard: Zur Existenz des Wellenoperators bei Anfangsrandwertaufgaben vom Maxwell-Typ.

Es wird das folgende Anfangsrandwertproblem betrachtet:

$$(\mathcal{M} + i\frac{\partial}{\partial t})\begin{bmatrix} E(t) \\ H(t) \end{bmatrix} := i\begin{bmatrix} \varepsilon^{-1}\operatorname{div} H(t) + \frac{\partial}{\partial t}E(t) \\ \mu^{-1}\operatorname{rot} E(t) + \frac{\partial}{\partial t}H(t) \end{bmatrix} = o$$

in einem Außengebiet $G \subset \mathbb{R}^d$;

$$E(t) = o \quad \text{auf } \partial G,\ t \in [o,\infty);$$
$$E(o) = E_0,\ H(o) = H_0 .$$

Hierbei ist $E(t)$ q-Form, $H(t)$ (q+1)-Form, E_0, H_0 entsprechende Anfangsdaten, rot die Cartan-Ableitung, div die Co-Ableitung, ε,μ sind lineare Abbildungen von q-Formen bzw. (q+1)-Formen in sich. Für den Operator $\mathcal{M}$ wird im Rahmen der Hilbertraumtheorie die Existenz und Vollständigkeit der Wellenoperatoren $W_{\pm}$ in Bezug auf ein passendes Anfangswertproblem im $\mathbb{R}^d$ nachgewiesen.

Yoshimi Saito: Spectral Representations for the Schrödinger Operators $-\Delta + Q(y)$ with long-range Potentials $Q(y) = O(|y|^{-\varepsilon}), \varepsilon > 0$

Let us consider the Schrödinger operator

$$S = -\Delta + Q(y) \qquad (y \in \mathbb{R}^N)$$

with a long-range potential $Q(y) = O(|y|^{-\varepsilon})$, $\varepsilon > 0$, as $|y| \to \infty$. The purpose of this report is to develop a spectral representation theory for S. First we shall construct the eigenoperator $\tilde{\eta}^{\pm}(r,k)$, $r \in [o,\infty)$, $k \in \mathbb{R} - \{0\}$, which is a bounded operator on $L_2(S^{N-1})$ for fixed r and k. Further, $\varphi_\alpha^{\pm}(y,k) = \tilde{\eta}^{\pm}(|y|,k)\alpha$ ($\alpha \in L_2(S^{N-1})$) satisfies $(S-k^2)\varphi_\alpha^{\pm} = 0$. By the use of the eigenoperator the generalized Fourier operators $\mathcal{F}^{\pm}$ can be defined. The main tools are the limiting absorption principle for S and the asymptotic behavior of the solutions v of the equation $(S-k^2)v = f$. For the details see the papers to appear in Osaka J.Math 14(1977).

Israel M.Sigal: Scattering Theory for Many Quantum Systems with

We consider a n-body quantum system with local, two-body, dilation analytic potentials which, essentially, have the L_2-local behavior and fall off at infinity faster than $|x|^{-2}$. In the formulations of the results we use the notions of quasibound state and resonance which we don't define here. If the potentials satisfy the conditions described above then the set of all $g \in \mathbb{R}^{\frac{n(n-1)}{2}}$ such that $H(g) = H_0 + \sum_{i<j} g_{ij}v_{ij}$ has no quasibound states is nondense in $\mathbb{R}^{\frac{n(n-1)}{2}}$.

If in addition, no subsystem has quasibound states then (a) number of bound states of our system is finite and resonances have no real accumulation points, (b) the set of channel wave operators is complete. The proofs of these statements are based on the spectral and scattering theories for the Combes-Balslev family $H(\theta)$. The channel wave operator in the scattering theory for $H(\theta)$ are defined as follows

$$\Omega^{\pm}_{a,m}(\theta) = \operatorname*{s-lim}_{t\to\pm\infty} e^{iH(\theta)e^{2\vartheta}t}\, e^{-iH_{a,m}(\theta)e^{2\vartheta}t}, \quad H_{a,m}(\theta) = \lambda^{a,m}\mathbb{1} + T_a e^{-2\vartheta}$$

where $\lambda^{a,m}$ is the internal energy and T_a the operator of kinetic energy in the channel (a,m).

C.G.Simader: Essential self-adjointness of Schrödinger operators bounded from below

We consider Schrödinger operators

$$\text{(H.1)}\begin{cases} Lu := -\Delta u + qu \quad \text{on } D(L) := C_0^\infty(\mathbb{R}^N), \text{ where } q \in L^2_{loc}(\mathbb{R}^N) \\ \text{and assume them to be bounded from below; without loss of} \\ \text{generality we assume } (Lu,u) \geq \|u\|^2 \text{ for } u \in D(L). \end{cases}$$

We split the potential: $q = q_1 + q_2$, $q_1 \in L^2_{loc}(\mathbb{R}^N)$, where $q_1 \geq 0$ and $q_2 \leq 0$. Then we ask for conditions on q_2 guaranteeing together with (H.1) essential self-adjointness of L. For q_2 we assume:

(H.2) For $k \in \mathbb{N}$ let $q_k(x) := q_2(x)$ if $|x| \leq k$ and $q_k(x) := 0$ otherwise. Then we assume:
i) $L_k u := -\Delta u + q_1 u + q_k u$, $D(L_k) := C_0^\infty(\mathbb{R}^N)$ is ess. s.a.
ii) For every $k \in \mathbb{N}$ there exists $0 \leq a_k < 1$ and $C_k \geq 0$ such that $|(q_k u, u)| \leq a_k \|\nabla u\|^2 + C_k \|u\|^2$ for $u \in C_0^\infty(\mathbb{R}^N)$.

(H.2) is satisfied if e.g. $q_2 \in Q_{\alpha,loc}$ or q_2 is (locally) "p-canonical" or q_2 satisfies Kato's (Israel J.Math. 13(1972)) mild Stummel-type condition. We prove the following

<u>Theorem.</u> Assume (H.1) and (H.2). Then L is essentially self-adjoint.

This theorem generalizes Wienholtz's famous result (1958) and those of Stetkaer-Hansen (1966) and J.Walter (1969). Its elementary proof follows from Wienholtz's wonderful "cut-off"-idea (with some little, but essential changes in the arguments used) and is in our situation based on the following lemma, replacing Wienholtz's use of Weyl's lemma:

<u>Lemma.</u> Let $g \in L^2(\mathbb{R}^N)$ and assume that there is a $k \in \mathbb{N}$ and a constant $C \geq 0$ such that

$$|(g, L_k u)| \leq C(\|u\| + \|\nabla u\|) \text{ for } u \in C_0^\infty(R^N)$$

where L_k is defined as in (H.2). Then, $g \in H^1(R^N)$.
The proof of the lemma is very easy.

Barry Simon: <u>Geometric Methods in Multi-Particle Systems</u>

Ideas in scattering and spectral theory which emphasize the geometric separation in configuration space are presented. Applications include the HVZ theorem, completeness of scattering in the two-cluster energy range, and conditions for the finiteness of the number of bound states.

Lawrence Thomas: Planchered Theorem for Ground State Representation of the Heisenberg Chain

In its ground state representation, the infinite spin 1/2 Heisenberg Chain provides a model for spin wave scattering that entails many features of the quantum mechanical N-body problem. A complete eigenfunction expansion for the Hamiltonian of the chain in this representation will be described, for all numbers of spin waves scattering will be discussed.

Jürgen Voigt: Scattering Theory for the Linear Boltzmann Equation; a "Uniformly Absorbing" Result.

We consider the linear Boltzmann equation

$$\frac{\partial f}{\partial t}(t,x,v) = -v\cdot \mathrm{grad}_x f(t,x,v) + \int k(x,v',v) f(t,x,v')dv' - s_a(x,v)f(t,x,v)$$

in $L_1(\mathbb{R}^n \times \mathbb{R}^n)$. Let $(W_o(t); t\in\mathbb{R})$ be the strongly continuous group of free motion (i.e., $k=o$, $s_a=o$), and let $(W(t);\ t\geq o)$ denote the s.c. semigroup describing the perturbed motion. Let $\sup_{t\geq o}\|W(t)\|<\infty$. We motivate and prove:

If there is a convex compact set $D\subset\mathbb{R}^n$ such that $k(x,v',v)=o, s_a(x,v)=o$ for all $x\in\mathbb{R}^n\setminus D$, and if there exist $\beta>o, \alpha>o$ such that

$s_a(x,v) - \int k(x,v,v')dv' \geq \alpha$ for all $x\in D, |v|<\beta$,

then the wave operator

$$W_+(T_o,T) := s-\lim_{t\to\infty} W_o(-t)W(t)$$

exists.

Christian Weber: A Compactness Theorem for Maxwell's Equations

The talk deals with the following compactness statement: Let $G\subseteq R_3$ be a bounded domain and $\{\vec{F}_n\}\subseteq L_2(G)^3$ such that $\|\vec{F}_n\|\leq C$, $\|\mathrm{curl}\ \vec{F}_n\|\leq C$, $\|\mathrm{div}(\beta\vec{F}_n)\|\leq C$, and either $\vec{n}\times\vec{F}_n|_{\partial G}=o$ or $\vec{n}\cdot(\beta\vec{F}_n)|_{\partial G}=o$: then $\{\vec{F}_n\}$ has a $L_2(G)^3$-convergent subsequence. (Norms refer to $L_2(G)$ or $L_2(G)^3$, derivatives are understood in the sense of distributions, $\beta=\beta(x)$ should be tensor-valued, real, symmetric, bounded, and uniformly positively definite, and the boundary conditions are meant in a generalized sense). This compactness statement has several application to Maxwell's equations (if β = dielectricity constant or β = permeability).

The talk gives an approach to this theorem (assuming that G has the restricted cone property) which is different from a previous one given by N.Weck (1972) who assumed that G is a "cone domain" (a property distinct from the "restricted cone property").

C.H.Wilcox: <u>Sonar Echo Analysis</u>

Pulse mode sonar operation is analyzed under the assumption that the scattering object Γ lies in the far fields of both the transmitter and the receiver. It is shown that, in this approximation, the sonar signal is a plane wave $s(x\cdot\theta_0 - t)$ near Γ, where θ_0 is a unit vector directed from the transmitter toward Γ, and similarly the echo is a plane wave $e(x\cdot\theta - t)$ near the receiver, where θ is a unit vector directed from Γ toward the receiver. Moreover, if Γ is stationary with respect to the sonar system then it is shown that

$$e(\tau) = \mathrm{Re}\left\{ \int_0^\infty e^{i\tau\omega}\, \hat{s}(\;)\, T_+(\omega\theta, \omega\theta_0)\, d\omega \right\}$$

where $\hat{s}(\omega)$ is the Fourier transform of $s(\tau)$ and $T_+(\omega\theta, \omega\theta_0)$ is the scattering amplitude in the direction θ due to the scattering by Γ of a time-harmonic plane wave with frequency ω and propagation direction θ_0. A generalization of this relation is derived for moving scatterers.

Clasine van Winter: Semigroups Generated by Analytic Multiparticle Hamiltonians

The Hamiltonian of a multiparticle system with analytic interactions is continued analytically in the dynamical variables. A study is made of the resolvent $R(\lambda)$ of the continued operator H. Under suitable assumptions on the two-body interactions, there is an operator Z such that $ZR(\lambda)Z$ remains bounded if λ tends to the continuous spectrum of H. This suffices to show that H generates bounded semigroups in one-to-one correspondence with the scattering channels. The semigroups are used to construct wave operators, which are related to projections onto invariant subspaces of H. The wave operators can be continued analytically and appear to tend to limits as the dynamical variables return to real values. Preliminary results indicate a relation to physics, but the precise interpretation is not yet understood.

Kenji Yajima: An Abstract Stationary Approach to Three-body Scattering

The completeness of wave operators for non-relativistic quantum mechanical three-body systems is discussed. The abstract stationary method will be used.

P. Werner (Stuttgart)
C.H.Wilcox (Salt Lake City)