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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T A G U N G S B E R I C H T 31/1977

KATEGORIEN

31.7. bis 6.8.1977

Die Tagung stand wieder unter der Leitung der Herren J.W. Gray (Urbana,Ill.) und H. Schubert (Düsseldorf); an ihr nahmen Mathematiker aus vier Kontinenten teil. Referiert wurde unter anderem über folgende Forschungsschwerpunkte: Elementare Topoi, monoidale, geschlossene und 2-Kategorien, lokal präsentierbare und lokalisierbare Kategorien und topologische Funktoren.

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Thode, Th. (Düsseldorf)
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Vortragsauszüge

(Neben diesem Kurzbericht, in den nur Abstracts bis zu einer Länge von ca. 15 Zeilen aufgenommen werden konnten, erscheint ein informeller Tagungsbericht in der bisher üblichen Form, der allen Tagungsteilnehmern vom Mathematischen Institut der Universität Düsseldorf zugeschickt wird.)

M. BARR: $*$ - autonomous categories

Definition: An autonomous category is a symmetric monoidal category. A $*$-autonomous category has in addition, an object T such that the canonical map $A \to ((A,T),T)$ is an isomorphism for every object: This means that $A \mapsto A^* = (A,T)$ is a self duality of the category. A construction is described which gives a large class of $*$-autonomous categories. As an example, there is a closed symmetric monoidal category of topological groups which contains all locally compact groups and admits a duality extending that of Pontrjagin.

M. BARR: Logical functors between toposes

F.BORCEUX: Universal algebra in a closed category

We develop universal algebra in those complete and cocomplete closed categories $\mathbb{V}$ which satisfy the three following conditions:

(1) If A is a small $\mathbb{V}$-category with finite $\mathbb{V}$-products, $G : A \to \mathbb{V}$ is a product preserving $\mathbb{V}$-functor, $S : A^{op} \to \mathbb{V}$ is any $\mathbb{V}$ functor, and n is an integer

then $\int^A S^n A \otimes GA \simeq (\int^A SA \otimes GA)^n$.

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(2) In $\mathbb{V}$, finite products commute with exqualizers of reflective pairs

(3) in $\mathbb{V}$, finite products commute with filtered colimits.
The basic results which can be proved are
(1) existence of left adjoint for any algebraic functor
(2) completeness and cocompleteness of the algebraic categories
(2) structure-semantics adjunction
(4) characterization of algebraic categories
(5) possibility of defining a theory by a presentation.

The examples of complete and cocomplete closed categories which satisfy the conditions are numerous:

cartesian closed categories, semi-additive categories, commutatively algebraic categories, categories of semi-normed spaces, normed spaces, Banach spaces.

D. BOURN: <u>2-categories reductibles (un travail commun avec J. Penon)</u>

On part de la remarque (plus où moins connue?) que pour toute 2-categorie $\underset{\sim}{\mathbb{D}}$, $\mathrm{Cat}^{\underset{\sim}{\mathbb{D}}}$ est une 2-categorie de coalgebres sur une 2-categorie de categories internes ($\mathrm{Cat}^{\underline{\mathbb{D}}}$, ou $\underline{\mathbb{D}}$ est la categorie fermée des seuls L-morphismes de $\underset{\sim}{\mathbb{D}}$), et que ainsi ou reduit son étude à celle d'une 2-categorie de categories internes. On se propose donc d'etudier les 2-categories "reductibles" qui verifient cette proprieté.

S. BOZAPALIDES: <u>Diagonal transformations</u>

G.C.L. BRÜMMER: <u>On a local initial completion and some problems about topologicity of functors</u>

We consider a faithful functor $U : A \to X$, and show that for any U-initiality problem there is a smallest essential extension of (A,U) in which the problem is solved. These

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properties of the extension immediately yield a number of external characterizations of topologicity, in both the transportable amnestic and the absolute cases, free of all smallness assumptions. Particular interest attaches to the characterization "any full embedding of (A,U) has a left inverse over X". We consider removal of faithfulness assumptions, in particular from the characterization "(A,U) is injective with respect to full embeddings", and in this connection we consider non-transportable versions of H. Wolff's pure external characterization of topologicity.

The greater part of the talk concerns joint work with R.-E. Hoffmann.

The first author acknowledges financial support from the South African Council for Scientific and Industrial Research and the University of Cape Town.

M. BUNGE: Composite and tensor product triples (summery)

Distributive laws were introduced by Barr in connection with homological algebra and later developed for their own sake by Beck. Among the questions posed by the latter is what is the relationship between composite triples given by a distributive law and tensor product triples as studied by Freyd, Lawvere, Manes and Wraith. This paper attempts to answer this question. For this purpose, special distributive laws, called multilinear laws, are introduced. They are precisely those distributive laws for which every composite algebra is a bialgebra (or algebra for the tensor product, should the latter exist). Under further conditions on the triples involved, the two constructions for triples coincide in the presence of a multilinear law. In fact these extra conditions imply the existence of canonically defined multilinear laws for which then, both constructions agree. For example this is the

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case for triples $\mathbb{T}$, $\mathbb{S}$ where either, (i) $\mathbb{T}$ any, $\mathbb{S}$ generated by unary operations; (ii) $\mathbb{T}$ affine, $\mathbb{S}$ generated by constants and unary operations; (iii) $\mathbb{T}$ preserving finite powers, $\mathbb{S}$ of rank $\aleph_0$; (iv) $\mathbb{T}$ power preserving, $\mathbb{S}$ any. In any of these cases there exists a distributive law of $\mathbb{S}$ over $\mathbb{T}$ such that the composite triple is the tensor product $\mathbb{S} \otimes \mathbb{T}$.

Y. DIERS: Locally algebraic categories

I am working in Category Theory: algebraic, monadic, locally presentable, structured spaces categories. My last work is the study of a lot of categories which have not the usual classical universal properties: products, coproducts, coequaliser free objects, ...

For exemple, the categories of fields, local rings, linear ordered sets, metric spaces, euclidian spaces, Hilbert spaces, C^*-algebras ... . By introducing the local universal properties: localization, localizing functors, ..., we can work like in the classical cases and we prove that those categories are categories of flat presheaf on a small category, and so we prove that many of well known categories are categories of fiber of a presheaf topos, which is then classifying.

E. DUBUC: On objects of a topological nature

M. FOURMAN: Formal spaces

R.FRITSCH: Functorial properties of the classical sheaf construction

Let X and Y be topological spaces. We ask the question to which extent the construction of the sheaf S(X,Y) of germs of continuous maps from X to Y can be considered as a functor in two variables. The universal property of (S(X,Y), p:S(X,Y) → X, q:S(X,Y) → Y)

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leads to a bifunctor S':Lh x Top → Lh where Top denotes the category of topological spaces and continuous maps and Lh the subcategory of Top consisting of the local homeomorphisms. In order to extend this functor in the first variable onto Top one has also to enlarge the codomain in a span-like manner. A suitable category for this purpose seems to be the following: The topological spaces are objects, equivalence classes of diagrams $X \xleftarrow{p} T \xrightarrow{q} Y$ with p local homeomorphism and q continuous map are morphisms from X to Y; two such diagrams are said to be equivalent if the induced maps into S(X,Y) have the same image. Thus one gets functors S(-,Y) defined on Top, but there seems to be no canonical way of extending S' to a bifunctor on Top x Top.

R. GUITART: Extensions de Kan absolues

R.-E. HOFFMANN: U-legitimacy of the generalized MacNeille-completion and semi-topological funtors

"Semi-topological functors" are a special class of faithful right adjoint functors containing all topological functors and all Ens-valued monadic functors. Semi-topological functors compose. They lift limits. Colimits are (generally) not lifted, but if they exist in the codomain, they exist in the domain (without being preserved). The main structural insight is obtained by: A functor V is semi-topological iff it is reflective in its MacNeille-completion category. This is, in this case, U-legitimate.The U-smallness condition for V, however, does not carry over to the MN-completion (H.Herrlich, Dec.77). In this connection it is noted that results of L.Kučera and A. Pultr answer the problem when a MacNeille completion of a faithful functor satisfies the U-s.c. for base category Ens (trivial generalization: category with a bicategory structure).

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Details (including further references): Note on semi-topological functors, Math.Z.(1978?)

J. ISBELL: Some concrete duals

A. JOYAL: The meaning of the Barr representation theorem

P.J. JOHNSTONE: A condition inequivalent to De Morgan's law

The following conditions are equivalent in any topos:

a) De Morgan's law: $\neg\varphi \vee \neg\psi = \neg(\varphi \wedge \psi)$.

b) the law $\neg\varphi \vee \neg\neg\varphi$ = true.

c) $1 \overset{\text{false}}{\rightarrowtail} \Omega$ has a complement.

d) Every $\neg\neg$-closed subobject has a complement.

e) Every $\neg\neg$-separated object is decidable (i.e. has complemented diagonal).

f) $\Omega_{\neg\neg}$ is decidable.

g) $\binom{\text{true}}{\text{false}} : 1 \amalg 1\ (=2) \longrightarrow \Omega$ is an isomorphism.

h) 2 is an internally complete lattice.

i) 2 is injective.

j) 2 is a $\neg\neg$-sheaf.

k) $\Omega_{\neg\neg}$ is a sublattice of Ω.

The following (equivalent) conditions are strictly weaker than the above:

l) If Y is decidable, so is Y^X for any X.

m) 2^X is decidable for any X.

n) 2 is a "naively complete" lattice, i.e. the diagonal map $2 \longrightarrow 2^X$ has right and left adjoints for any X.

A counterexample to (n) $\Rightarrow$ (h) is provided by the topos of right M-sets, where M is the three-element monoid $\{1,a,b;a^2=ab=a,b^2=ba=b\}$.

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R. JOHNSTONE: On a topological topos

G.M.KELLY: Strict and pseudo adjoints for algebraic 2-functors

Categories with equational structure are the algebras for a 2-monad on the 2-category Cat; and more generally we have to consider the algebras for a 2-monad D = (D,i,m) on a 2-category K. In practice we find that the appropriate concepts at this level are not, as in ordinary universal algebra, the "strict" ones, but the "to within a coherent isomorphism" ones. In fact it does no harm to suppose that the D-algebras are algebras in the strict sense, for the relaxed algebras for D are the strict algebras for some other 2-monad D'. However we must accept that the appropriate morphisms for these algebras are not the strict ones; for example, if D is such that its algebras are categories-with-limits, the natural morphisms to consider are those which preserve these limits, not strictly, but within isomorphism. Since the category of algebras with these relaxed morphisms is not the Eilenberg-Moore category, new problems arise when we consider algebraic functors: the classical proof that these have left adjoints no longer applies. In fact we don't expect them to have strict left adjoints, but relaxed ones. We show that they do have these under reasonable conditions, and we give a counter-example to a plausible suggestion about their nature.

A. KOCK: Synthetic differential geometry

We argue that such a geometry (which in particular involves no limit processes) is possible, but not if metaphysical (in contrast to dialectical) logic is presupposed.

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We give the basic axiom for the geometric line A: it is a commutative ring; and if $D \subseteq A$ denotes the set of elements of square zero, every map $f: D \to A$ is uniquely of form

$$f(x) = a + b \cdot x, \qquad \forall x \in D;$$

the b occuring here can be used to define the derivative f'. Models for this axiom exist in certain toposes, but not in the category of sets; for, the metaphysical disjunction used in the following description of an $f: D \to A$

$$f(x) = \begin{cases} 1 & \text{if } x=d \\ 0 & \text{else} \end{cases} \qquad \text{(d a fixed non-zero element in D)}$$

leads to absurdity. (This argument is due to Schanuel.)

Lit.: Kock, Math. Scand. 40 (1977), 183-193

M. LATCH: A homotopy equivalence between the categories of simplicial sets and small categories

Since the Milnor geometric realization [Milnor, Ann.of Math., 65 (1975), 357-362] $|\_|: K \to Top$, preserves products, it induces a functor $|\_|: K/\tilde{} \longrightarrow Top/\tilde{}$ between the homotopy categories of simplicial sets and spaces. However, this induced functor is not full; i.e., it is possible for the realization of a simplicial map to be a homotopy equivalence (HE) in Top, and for no simplicial homotopy inverse to exist. Such simplicial maps are called weak homotopy equivalences (WHE) in K. The localization $(K/\sim)[\Sigma^{-1}]$ wrt the collection Σ of WHE's in K is K', the homotopic category of K.

In [Latch, JPAA,9 (1977), 221-237], similar definitions were given for homotopy and homotopic categories of Cat, the category

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of small categories; and the nerve functor N:Cat → K was shown to induce an equivalence between corresponding homotopic categories.

Comparable "fibred" definitions are given in [Latch, "A fibred homotopy equivalence and homology theories for Cat," to appear JPAA] and N:(Cat↓𝔹) → (K↓N𝔹) is shown to induce a weak fibred homotopy equivalence of categories.

H.LINDNER: A characterization of Mackey-functors

The category of Mackey-functors from a category C, satisfying suitable assumptions, to a category D, is characterized as the category of finite-product-preserving-functors from SpC, the category of "spans" in C, to D. This characterization permits to apply all results on categories of functors preserving a given class of limits to the case of Mackey-functors.

F.E.J. LINTON: Finiteness & Decidability

Theorem (Acuna). The full subcategory $\mathbb{E}_{dKf}$ of decidable K-finite objects of a topos $\mathbb{E}$ is a boolean topos, closed under finite limits, finite coproducts, and exponentiation; the inclusion is a logical iff $\mathbb{E}$ is Boolean; and in $\mathbb{E}_{dKf}$ there holds the internal axiom of choice: $[(A \to B) \text{ epi}] \Rightarrow \forall C\ [(A^C \to B^C) \text{ epi}]$

The core of the proof (details in the Durham proceedings) is the

Lemma. For X decidable and K-finite, 2^X and K(X) coincide, live in $\mathbb{E}_{dKf}$, and serve there as power object P(X) for X.

Remarks. 1. One must not expect all K-finites to form a topos (though they do when $\mathbb{E}$ is Boolean and all objects are decidable): in

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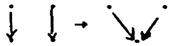
Sets^2, an epic  is not only epic among the K-finites, it is also monic there (though not in Sets^2), so those K-finites, having a non-iso bimorphism, are no topos.2. For $\mathbb{E}$ with NNO, $\mathbb{E}_{dKf}$ coincides with the familiar topos $\mathbb{E}_{lcf}$ of locally constant finite objects (= objects locally isomorphic to a finite cardinal).

J. MESEGUER: Order-continuous algebras

Ch.MULVEY: Hahn-Banach in a category of sheaves

Banaschewski has proved that for Banach sheaves on a topological space X the Hahn-Banach theorem fails for all but the most trivial spaces when functionals are taken to have values in the Banach sheaf of bounded continuous real functions on X. However, the category of Banach sheaves may be shown to be equivalent to the category of Banach spaces in the category of sheaves on X. For Banach spaces in a category of sheaves the Hahn-Banach theorem may be proved intuitionistically provided that functionals are taken into the extended reals $^*\mathbb{R}$ which are the order-completion of the rationals.

D. SCHUMACHER: The preservation of smallness

L.SCHUMACHER: On the interpretation of cohomology groups by extensions (summary)

Let $\underline{E}$ be a category with finite limits and $\mathfrak{I} \subset \underline{E}$ a projective class in the sense of Tierney-Vogel. Then for $X \in \underline{E}$ and an

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abelian group object $\Pi \in$ $\underline{E}$ cohomology groups $H^n(X,\Pi)$ of X with coefficients in Π are defined. Let $\underline{E}$ be exact in the sense of Barr. Under weak additional assumptions on $\underline{E}$ and $\mathfrak{T}$ we give a Yoneda-like interpretation of $H^n(X,\Pi)$ in terms of n-fold simplicial extensions of X by Π rel. $\mathfrak{T}$. As consequences we obtain interpretations of the corresponding

I. André cohomology for models $\underline{M} \subset \underline{E}$ if in addition 1) $\underline{E}$ has coproducts of models and 2) M is a small regular generating subcategory closed under α -coproducts and consists of α-presentable objects of $\underline{E}$ (α a regular cardinal).

II. cotriple cohomology for a cotriple $\mathbb{G}$ on $\underline{E}$ if in addition $\mathbb{G}$ is of 'descent' type.

Note that for cotriple cohomology in contrast to Duskin's work on $K(\Pi,n)$-torsors we do not refer to an underlying object functor and therefore do not use tripleability conditions. For this reason we can obtain results in model induced cotriple situations where often no suitable underlying object functor can be found, e.g. in I. above.

W. THOLEN: Semitopological functors and adjoint liftings

Zur Verallgemeinerung topologischer Funktoren werden zwei Wege beschritten: Anstelle initialer Liftungen werden semiinitiale Faktorisierungen von Kegeln betrachtet, und anstelle finaler Liftungen werden semifinale Erweiterungen von Cokegeln (verallg. "proclusions" i.S. von Wyler) untersucht. Beide Konzepte erweisen sich als gleichwertig (verallg. "Dualitätssatz") und führen zum Begriff des semitopologischen Funktors. Topologische Funktoren, monadische Funktoren über Ens, volle reflexive und volle bicorefl. Einbettungen sowie Komposita semitopol. Funktoren sind semitopol. Semitopol. Funktoren sind treu, rechtsadjungiert, gestatten die Liftung der Existenz von Limites und Colimites sowie die Liftung

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der Existenz von Links- und Rechtsadjungierten (verallg. "Taut Lift Thm."). Etwas spezieller als semitopol.Funktoren sind die orthogonalen M-Funktoren, die jedoch den Vorteil haben, sich stets voll und reflexiv in topol. Kategorien einbetten zu lassen.

F. ULMER: Bialgebras

H. WEBERPALS: On the heredity of limits to free colimit completions

For a non-empty class Δ of small categories let $L(\Delta)$ be the class of those small categories $\underline{X}$ such that $\underline{X}^{op}$-limits commute with Δ-colimits in Set. Following Gabriel/Ulmer, we consider the Yoneda-embedding $Y : \underline{C} \longrightarrow K_\Delta(\underline{C})$ of a category $\underline{C}$ with small hom-sets into the free Δ-colimit completion for a regular class Δ.

Lemma. Let $G : \underline{X} \longrightarrow K_\Delta(\underline{C})$ be a functor on a small category $\underline{X}$. If $\underline{C}$ has limits of type $\underline{X}$ and if there is a final functor $\underline{D} \longrightarrow [\underline{X},Y]/G$ into the comma-category with $\underline{D} \in \Delta$, then G has a limit in $K_\Delta(\underline{C})$.

Theorem. Let Δ be regular and let the category  be contained in $L(\Delta)$.

a) If $\underline{C}$ has products of type X for some set X, then so does $K_\Delta(\underline{C})$.

b) If $\underline{C}$ has equalizers, then so does $K_\Delta(\underline{C})$.

M.B. WISCHNEWSKY*: Every topological category is convenient for Gelfand and Pontrjagin

We generalize our work on Gelfand dualities in cartesian closed topological categories on categories which are only monoidal closed. Using heavily enriched category theory we show

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*this is a joint work with H.E.PORST.

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that under very mild conditions on the base category function algebra functor and spectral space functor exist forming a pair of adjoint functors and establishing a duality between function algebras and spectral spaces. Using recent results in connection with semitopological functors we show that every (E,M)-topological category is endowed with at least one convenient monoidal structure admitting a generalized Gelfand as well as Pontrjagin duality. So it turns out that there is no need of a cartesian closed structure on a topological category in order to study generalized Gelfand or Pontrjagin dualities.

R.J. WOOD: <u>Coalgebras for closed cotriples</u>

The underlying category 2-functor from monoidal categories to C A T creates the construction of coalgebras. If $\underline{V}$ is monoidal closed and $\mathfrak{G} = (G, \varepsilon, \delta)$ is a monoidal cotriple on $\underline{V}$, then $\underline{V}_{\mathfrak{G}}$ is closed precisely when $\underline{V}_{\mathfrak{G}}$ has certain equalizers.

<u>Proposition:</u> For $\underline{V}$ and $\mathfrak{G}$ as above, $\underline{V}_{\mathfrak{G}}$ is closed and $\underline{V}_{\mathfrak{G}} \to \underline{V}$ preserves the internal hom if and only if for every $(A,\alpha) \in \underline{V}_{\mathfrak{G}}$ and every $X \in \underline{V}$ $[AX]G \xrightarrow{\hat{G}} [AG, XG] \xrightarrow{[\alpha,1]} [A, XG]$ is an isomorphism. In case $\underline{V}$ is symmetric the above condition just says that G, regarded as a $\underline{V}_{\mathfrak{G}}$-functor via $\underline{V} \to \underline{V}_{\mathfrak{G}}$, preserves cotensors. If $\mathfrak{G}$ is an "admissable" cotriple in the sense of Keigher (Closed Categories of Coalgebras, to appear in Communications in Algebra), the above condition may be easily verified.

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