MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 37/1977

Fixpunkttheorie

11.9. bis 17.9.1977

Die Tagung stand unter der Leitung von A.Dold (Heidelberg) und E.Fadell (Madison).

Die Fixpunkttheorie kann vor allem im Hinblick auf ihre Anwendungen in zwei Teilgebiete gegliedert werden, einen analytisch-geometrischen und einen algebraisch-abstrakten Bereich. Die analytisch-geometrische Fixpunkttheorie befaßt sich im wesentlichen mit der Existenz von Fixpunkten für Selbstabbildungen von Banachräumen oder allgemeiner von ANRs; Anwendung findet sie in der Geometrie, der Topologie, der Funktionalanalysis und der Theorie der Differential- und Integralgleichungen. Dagegen gewinnt die algebraisch-abstrakte Fixpunkttheorie mittels einer Indextheorie auf kompakten Liegruppen G oder auf G-Mannigfaltigkeiten Strukturaussagen z.B. für verschiedene Kohomologiealgebren; sie liefert daneben auch funktionalanalytische Resultate wie etwa in der Verzweigungstheorie.

Geometrische Probleme wurden in Vorträgen über Verallgemeinerungen des Satzes von Borsuk-Ulam (31), (32), über die Hopf-Vermutung (32) und die ebene Fixpunktvermutung (3) erörtert. Mittels verallgemeinerter Nielsen-Zahlen konnte die Deformierbarkeit von Selbstabbildungen kompakter Mannigfaltigkeiten in fixpunktfreie

Abbildungen geprüft (17) und eine untere Schranke für die Anzahl periodischer Punkte von Abbildungen auf kompakten ANRs angegeben werden (15). In einem abendlichen Seminar wurde ein Computerverfahren zur Bestimmung der Fixpunkte von Selbstabbildungen der Kreisscheibe (Satz von Brouwer) besprochen (J.Alexander). In Verallgemeinerung der klassischen Fixpunktsätze von Banach und Schauder wurde die Fixpunkteigenschaft verschiedener Teilmengen von Banachräumen für nichtexpansive (13), (19), (28), accretive (19), radialkompakte (20) und kondensierende (26) Operatoren sowie für Abbildungen mit einer kontraktiven Iterierten (22) und Abbildungen vom Frum-Ketkov-Typ (28), (30) diskutiert.

Mithilfe der asymptotischen (8), (25) und der stochastischen Fixpunkttheorie (6) wurden Differential- und Integralgleichungen behandelt, womit sich auch Referat (10) befaßte; z.T. wurde dabei eine Indextheorie für mehrwertige Abbildungen benutzt (6), (8). Fixpunktsätze für mehrwertige Abbildungen enthielten die Vorträge über m-Funktionen (18), über radialkompakte (20) und über kondensierende Abbildungen (26). Mittels der algebraischen Struktur ihrer Kohomologiegruppen konnten Grassmann-Mannigfaltigkeiten auf ihre Fixpunkteigenschaft hin untersucht werden (12). In Vortrag (11) wurden Probleme bei der Diskussion periodischer Orbite von Flüssen auf Mannigfaltigkeiten aufgezeigt. Eine Invariante aus der stabilen Homotopiegruppe π_*^s - eine Art Brouwerscher Abbildungsgrad - diente einer Erweiterung des klassischen Verzweigungssatzes von Hopf zu einer globalen Aussage (1). Auch Referat (29) entwickelte einen globalen Verzweigungssatz.

Algebraische Anwendung fand die Fixpunkttheorie bei der Bestimmung der (Ko-) Homologiealgebren gewisser Konfigurationsräume (5) und der klassifizierenden Räume BG für kompakte Liegruppen G (9). Beziehungen zwischen den Kohomologiealgebren der Fixpunktmenge eines G-Raumes und des Raumes selbst wurden für G = $\mathbb{Z}_p$, p prim, und G = S^1 erörtert (27). In Vortrag (14) wurde die Fixpunkttheorie zum Studium der Evaluationsabbildung $X^Y \longrightarrow X$ verwandt, und in Referat (2) lieferte sie eine Charakterisierung des stabilen Homotopieelementes, das einer kompakten Liegruppe zusammen mit ihrem linksinvarianten Framing durch die Pontrjagin-Thom Konstruktion zugeordnet ist.

Equivarianzpunktsätze für Selbstabbildungen von G-Räumen als weitere Verallgemeinerung des Satzes von Borsuk-Ulam (5), (23) und der Lefschetzsche Fixpunktsatz für kompakte Gruppen (21) schufen eine Verbindung zur Geometrie. Der Analysis diente eine Anwendung der Fixpunkttheorie auf G-Räumen in der Verzweigungstheorie: z.B. konnten Hamilton-Systeme gewöhnlicher Differentialgleichungen behandelt werden (7).

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Vortragsauszüge

(1) J.ALEXANDER: Global bifurcation of fixed points of condensing operators

Let G_{fin}, G_{comp}, G_{cond} be spaces of finite dimensional, of compact and of condensing perturbations of the identity on some neighborhood of 0 in a real Banach space X with $g^{-1}(0) = 0$ for all $g \in G_*$. Then we have inclusions $\Omega^\infty S^\infty \hookrightarrow G_*$. Similarly one has $GL_* \hookrightarrow G_*$ where GL_* is the appropriate linear group of continuously invertible maps. In the diagram

$$\begin{array}{ccccccc} GL(\infty) & \hookrightarrow & GL_{fin} & \hookrightarrow & GL_{comp} & \hookrightarrow & GL_{cond} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \Omega^\infty S^\infty & \hookrightarrow & G_{fin} & \hookrightarrow & G_{comp} & \hookrightarrow & G_{cond} \end{array}$$

all horizontal maps are weak homotopy equivalences, and the vertical maps induce the J-homomorphism on homotopy groups.

Now let $f: \mathbb{R}^n \times X \to X$ be such that $f_\lambda(0) = f(\lambda,0) = 0$ and $f_\lambda \in G_*$ in a uniform way on compact subsets of $N - \{0\}$ where N is some neighborhood of 0 in $\mathbb{R}^n$. If S^{n-1} is a small sphere in $N \subset \mathbb{R}^n$ around 0, the map $\lambda \longmapsto f_\lambda$ for $\lambda \in S^{n-1}$ yields an element $\gamma_f \in \pi_{n-1}(G_*) \cong \pi^s_{n-1}$.

Theorem: If $\gamma_f \neq 0$, global bifurcation of zeroes of f occurs: There exists a connected set Ω_0 of zeroes of f with 1) $(0,0) \in \Omega_0$, 2) $S^{n-1} \cap \Omega_0 = \emptyset$, 3) ((exterior of S^{n-1} in $\mathbb{R}^n) \cup \{\infty\}) \cap \overline{\Omega_0} \neq \emptyset$.

The proof involves making finite dimensional approximations, putting explicit framed manifolds in the space and using the identification of framed bordism and stable homotopy and the duality theorem for generalized homology theories. To go back to infinite dimensions, one uses the continuity property of Čech cohomologies.

If f is C^1, by working with the derivative $D_x f_\lambda |_{x=0}$ in $N - \{0\}$, one receives an element $\bar{\gamma}_f \in \pi_{n-1}(GL_*) \cong \pi_{n-1}(GL(\infty))$, and one has $\gamma_f = J(\bar{\gamma}_f)$.

Applications to non-linear eigenvalue problems and Hopf bifurcation problems are discussed. Also methods of calculating γ_f are mentioned.

(2) J.C.BECKER: Compact Lie groups with their left invariant framing and the fixed point index

A compact Lie group G together with its left invariant framing L determines a stable homotopy element $[G,L] \in \pi^0(S^n)$, n = dim G. We study the problem of identifying this element. Our starting point is the relation between $[G,L]$ and the fixed point index associated with the fibre-preserving map $G \times G \longrightarrow G \times G$, $(g,\bar{g}) \longmapsto (g,g\bar{g})$. Using this relationship, we compute $[G,L]$ in several low dimensional cases and obtain general results on the possible order of $[G,L]$.

(3) H.BELL: Techniques for the plane fixed point conjecture

The plane fixed point conjecture asserts that every non-seperating plane continuum has the fixed point property for continuous maps. In this talk I intend to survey my past work on this problem.

(4) R.F.BROWN: Equations on manifolds - a survey

Fixed point and degree theory can be viewed as the study of equations of the form $f(x) = x$ and $f(x) = c$ for functions $f: X \to X$.

Motivated by problems in analysis, the main thrust of the subject has been to generalize results concerning these equations to ever broader classes of spaces and functions. However, from time to time, there have been discoveries concerning such equations that require strong hypotheses in order to give conclusions that are more precise than in the general theory. These results, many of which are motivated by questions from algebra, require at the least that X be a compact manifold. During the past few years, there has been a substantial increase in activity in this sort of fixed point theory. The lecture describes what is presently known about this subject and poses some unsolved problems.

(5) F.R.COHEN: Structure and homology of configuration spaces; applications

Given a manifold M and a topological space X with a "nice" base point, we construct a certain space C(M,X). C(M,X) enjoys several properties:

1) If C(M,X) is connected, then it splits stably into a wedge of "j-adic constructions".

2) $H_*C(M,X)$ can be given in favorable cases.

3) $H_*C(M,X)$ gives the E_2-term of several spectral sequences, one due to Gelfand-Fuks and one due to D.W.Anderson and P.Trauber.

4) $H_*C(M,X)$ gives the homology of the space of sections $P(M)$ described by D.Mac Duft.

5) $C(\mathbb{R}^n,X)$ is weakly equivalent to $\Omega^n S^n X$ for path connected spaces X.

The construction of $C(M,X)$ is given in terms of Fadell-Neuwirth's notion of a configuration space.

(6) H.ENGL: Stochastic fixed point theorems

Let X be a separable Banach space, $(\Omega,\mathcal{A},\mu)$ a σ-finite measure space and $C: \Omega \longrightarrow 2^X$ a separable map - i.e. for all open $D \subset X$ the set $\{\omega\in\Omega / C(\omega)\cap D \neq \emptyset\}$ belongs to $\mathcal{A}$, and there exists a countable set $Z \subset X$ such that $C(\omega)\cap Z = C(\omega)$ for all $\omega\in\Omega$. T: Gr $C \longrightarrow X$ (2^X) is called a (set-valued) random operator with stochastic domain C if $\{\omega\in\Omega / x\in C(\omega)$ and $T(\omega,x)\in D$ $(T(\omega,x)\cap D \neq \emptyset\}\in\mathcal{A}$ for all $x\in X$ and all open subsets $D \subset X$. $x: \Omega \longrightarrow X$ is called a wide-sense fixed point of T if $x(\omega)\in C(\omega)$ and $x(\omega) = T(\omega,x(\omega))$ $(x(\omega)\in T(\omega,x(\omega)))$ for all $\omega\in\Omega$. A random fixed point is a measurable function $x: \Omega \longrightarrow X$ which fulfills the properties of a wide-sense fixed point for μ-almost all $\omega\in\Omega$.

Using a Chebyshef-centre method, we prove a result about the existence of a random fixed point of nonexpansive random operators. Then we prove a result which contains especially: If T: Gr $C \longrightarrow X$ (or $CB(X)$) is a continuous (set-valued) random operator with stochastic domain C which has a wide-sense fixed point, then it has a random fixed point.

Finally we give a stochastic version of the Kakutani-Bohnenblust-Karlin fixed point theorem for upper semi-continuous set-valued maps.

(7) E.FADELL: On Yang type indexes for G-spaces with applications to bifurcation theory

In bifurcation theory there are useful techniques which require the classification of invariant subspaces of a fixed free $\mathbb{Z}_2$-space by assigning to such a subspace X a non-negative integer called Index X. This index function is required to possess certain properties, e.g. monotoneity, continuity, additivity etc. The indexes of Yang developed in the 1950's have proved useful in this regard. More recently situations in bifurcation theory have been encountered where the symmetry group G is not $\mathbb{Z}_2$ and the action is not necessarily free. Accordingly, the author (with Paul Rabinowitz) has systematically developed index theories satisfying the requisite properties for paracompact G-spaces, where G is an arbitrary compact Lie group and the action is not assumed to be free. This theory is then applied to a situation involving a non-free S^1-action to obtain results on the bifurcation of time periodic solutions from an equilibrium point for Hamiltonian systems of ordinary differential equations.

(8) Chr.C.FENSKE: Asymptotic fixed point theorems for multivalued transformations

A multivalued map $f: X \longrightarrow Y$ is admissible if it is a composition of upper semicontinuous maps with compact acyclic images. If Y = X, then f is of compact attraction, if f is locally compact and has a compact attractor. We define a fixed point index for admissible maps of compact attraction on ANRs.

Let X be an ANR, $f: X \longrightarrow X$ an admissible map of compact attraction, F a component of $\mathrm{Fix}(f)$, $f(X-F) \subset X-F$, W a neighborhood of F such that $\overline{W} \cap (\mathrm{Fix}(f)-F) = \emptyset$. Assume that $f|X-F$ is of compact attraction, and let A be a compact attractor which has arbitrarily small neighborhoods V such that $\overline{f(V)}$ is a compact subset of V. Then $\mathrm{ind}(X,f,W) = \Lambda(f) - \Lambda(f|X-F)$. Hence, if $H_*(X,X-F) = 0$, then $\mathrm{ind}(X,f,W) = 0$. If there is a neighborhood U of F such that $\overline{U} \cap A = \emptyset$ and $H_*(X,X-U) = 0$, then $\mathrm{ind}(X,f,W) = 0$.

If X is an ANR, let $\phi = (\phi_t)_{t \geq 0}$ be a multivalued semi-flow such that for $t > 0$ each ϕ_t is locally compact and admissible, and assume that there is a compact attractor A for ϕ, i.e. for each $x \in X$ and each neighborhood U of A there is a t with $\phi_t(x) \subset U$. Let E be a closed set such that $E \subset \phi_t(E)$ for all t and such that for a suitable neighborhood U of E and for all $x \in \overline{U}-E$ there is a t with $\phi_t(x) \subset X-\overline{U}$. Then there is a neighborhood V of E and a $T > 0$ such that $\mathrm{ind}(X,\phi_t,V)$ is defined for all $t \in (0,T)$ and equals $\chi(X) - \chi(X-E)$.

(9) M.FESHBACH: A double coset theorem for the transfer

A general double coset theorem for the fixed point transfer has been proved by the author (Bull. AMS, May 1977). This theorem specializes in many cases. These formulae can be used to prove theorems about characteristic classes. In particular, Borel's theorem which says that $H^*(BG;\mathbb{Q})$ is isomorphic to $H^*(BT;\mathbb{Q})^W$ - where G is a compact Lie group, T a maximal torus and W the Weyl group - and similar results are easy corollaries. G.Brumfiel and I.Madsen have obtained some of these results by different methods. I also understand that T.tom Dieck has developed a double coset theorem.

(10) G.FOURNIER: Existence and uniqueness of the solution of a system of integral equations containing multiple convolutions

In order to prove the existence of some systems of integral equations, one considers the corresponding selfmap defined on a subspace of a product of the relevant space of functions.

First, using an Ascoli theorem, one tries to prove that this map has a compact convex attractor (for points). Since this attractor is invariant under the map, by the Leray-Schauder fixed point theorem, the map has a fixed point belonging to that compact convex subset - hence the system has a solution. Finally, if the map is locally a contraction, then the fixed point is unique. Note that the method provides a description of the compact convex set containing the fixed point; hence it often gives some property of the solution such that it is uniformly continuous. This method can be used on multiple convolutions integral equations to give the existence and uniqueness of the solutions.

(11) F.B.FULLER: Periodic orbits of flows as fixed points

Let T_t be a flow on a manifold M. The following procedure suggests itself as a way to show the existence of periodic orbits of T_t. Let X be a function space of closed curves in M, where two curves are equivalent if they differ by an orientation-preserving change of parameter. The flow T_t on M then defines a flow T_t' on X for which periodic orbits of T_t on M become stagnation points of T_t' on X.

Unfortunately no method comes to hand for showing the existence of stagnation points of $T_t^!$. If X has suitable compactness properties, then an index theory for the stagnation points of $T_t^!$ reduces to an index theory for the fixed points of the map $x \longmapsto T_t(x)$ for sufficiently small positive t. But it does not appear possible to define X with the properties needed.

To what extent are the difficulties intrinsic? An example of flows on the Möbius band shows that a satisfactory index theory for stagnation points of $T_t^!$ on X cannot assign a constant value to those stagnation points whose neighborhoods converge to them as $t \to +\infty$. This conclusion is in sharp contrast to the theorem of Leray for conventional fixed point theory that a fixed point, whose neighborhoods converge to it as the map is iterated, must have index +1.

An index theory for stagnation points applicable to the existence of periodic orbits thus cannot resemble the usual fixed point theories. At the same time, since an index theory can be constructed directly for the periodic orbits (Fuller, Amer.J.Math., January 1967), it seems likely that an applicable index theory for stagnation points exists. The talk is thus a challenge: What is this theory?

(12) H.GLOVER: <u>Selfmaps of Grassmann manifolds</u>
<u>by H.Glover and W.Homer</u>

A natural generalization to Grassmann manifolds over the real, complex and quaternion fields of the known results about the fixed point property for selfmaps of projective spaces would say:

1) $\mathbb{F}G_{p,q} \in$ FPP for $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$ if and only if pq is even and $p \neq q$,

2) $\mathbb{H}G_{p,q} \in$ FPP if and only if $p \neq q$.

In this paper we prove a part of this result, namely:

1) $\mathbb{C}G_{p,q} \in$ FPP if pq is even and $q > 2p^2 - p$,

2) $\mathbb{H}G_{p,q} \in$ FPP if $q > 2p^2 - p$.

Our method of proof is to show that every graded algebra endomorphism φ^* of $H^*(\mathbb{F}G_{p,q};\mathbb{Z})$ is determined by its action on the bottom class c_1. More specifically we prove that $\varphi^*(c_i) = k^i c_i$ for all i if $\varphi^*(c_1) = kc_1$.

(13) K.GOEBEL: Irregular sets with fixed point property for nonexpansive mappings

Generally the fixed point property for nonexpansive mappings depends very strongly on some "nice" geometrical properties of the considered spaces. The aim of this talk is to point out some singularities occuring in this field. Examples of some geometrically "bad" convex sets with fixed point property will be given. For example there exists a sequence $A_1 \supset A_2 \supset A_3 \supset \ldots$ of closed, bounded and convex sets in l^1 such that $A_1, A_3, A_5, \ldots$ have the fixed point property and $A_2, A_4, A_6, \ldots$ do not. The intersection $\bigcap_{i=1}^{\infty} A_i$ may or may not have the fixed point property according to our choice. The sequence A_i even may be Hausdorff convergent.

(14) D.H.GOTTLIEB: Fixed point theory and the evaluation map

This is a survey of how fixed point theory has led to results about the evaluation map $X^Y \longrightarrow X$ and how study of this map can lead to a fixed point result regarding liftings of actions of Lie groups to principal torus bundles.

(15) B.HALPERN: Numbers of periodic points

Consider a compact connected ANR X and a continuous map $f: X \longrightarrow X$. Set $P_n(f) = \{ x \in X \mid f^n(x) = x$ and $f^i(x) \neq x$ for $1 \leq i < n \}$. We want to find $\min \{ \mathrm{card}(P_n(g)) \mid g \simeq f \}$.

As a first step in attacking this problem we develop a theory analogous to the Nielsen fixed point theory. Numbers $D_n(f)$, $A_n(f)$, $\bar{A}_n(f)$, $S_n(f)$ and $T_n(f)$ are defined and it is shown that $\mathrm{card}(P_n(g)) \geq D_n(f)$, $A_n(f)$, ... and $D_n(g) = D_n(f)$, $A_n(g) = A_n(f)$, ... for all $g \simeq f$.

Applications are given to certain maps on tori and to all maps on the Klein bottle, and it turns out that in these cases $D_n(f) = \min \{ \mathrm{card}(P_n(g)) \mid g \simeq f \}$. The methods developed in the applications to tori give a positive answer to a question of Shub and Sullivan concerning the existence of Morse-Smale diffeomorphisms isotopic to certain diffeomorphisms on tori.

Finally, a general coincidence theory is illustrated by the following theorem: If $g: \mathbb{C}P^n \longrightarrow \mathbb{R}G_{n,m-n}$ is any map from the complex projective space $\mathbb{C}P^n$ into the Grassmann manifold of n-planes in $\mathbb{R}^m$ and if $f: \mathbb{C}P^n \longrightarrow \mathbb{R}G_{2,m-2}$ is a classifying map for the canonical line bundle over $\mathbb{C}P^n$ made real, then there exist $x_1, x_2, \ldots, x_k \in \mathbb{C}P^n$, $k \leq n+1$, such that $f(x_i) \perp g(x_{i+1})$ for $i = 1,2,\ldots,k$, where $x_{k+1} = x_1$.

(16) K.A.HARDIE: The enumeration and invariance problem for pair-homotopy classes

Let f,g be pairs in the sense of Eckmann and Hilton and let [f,g] denote the set of pair-homotopy classes of pair maps from f to g. Cofibration means closed cofibration. S denotes reduced suspension.

Theorem 1: Suppose
1) $f \simeq f' : X \longrightarrow Y$ and $g \simeq g' : E \longrightarrow B$,
2) f is a cofibration or g is a fibration,
3) f' is a cofibration or g' is a fibration.
Then $[S^n f, g] \cong [S^n f', g']$ $(n > 1)$.

If $\alpha \in [X,Y]$ contains a cofibration i or if $\beta \in [E,B]$ contains a fibration p, we may unambiguously define $\bar{\pi}_n(\alpha,\beta) = [S^n i, g] = [S^n f, p]$, where $f \in \alpha$ and $g \in \beta$.

Theorem 2: There is an exact sequence $(n > 1)$
$\ldots \longrightarrow \bar{\pi}_n(C_\alpha, F_\beta) \longrightarrow \bar{\pi}_n(Y,E) \longrightarrow \bar{\pi}_n(\alpha,\beta) \longrightarrow \bar{\pi}_{n-1}(C_\alpha, F_\beta) \longrightarrow \ldots,$
where C_α is the cofibre of α and F_β is the fibre of β.

Sample application: Let $\eta \in \bar{\pi}_3(S^2)$, $\nu \in \bar{\pi}_7(S^4)$ denote the Hopf classes. Then $\bar{\pi}_n(\eta,\nu)$ is an extension of $\bar{\pi}_{n+2}(S^7)$ by $\bar{\pi}_{n-1}(C_\eta, S^3)$ $(n > 1)$.

(17) S.Y.HUSSEINI: Relating Nielsen numbers, obstruction classes and generalized Lefschetz numbers

Suppose that $f: M \longrightarrow M$ is a map of a compact manifold M into itself. Then, in the simply connected case, f is deformable to a fixed point free map if and only if the Lefschetz number $\Lambda(f) = 0$. But this need not be so if M is not simply connected.

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Therefore, we

1) define an obstruction o(f) for so deforming f and then express this obstruction in terms of the indices of essential Nielsen classes so that o(f) = 0 if and only if the Nielsen number N(f) is 0,

2) define a generalized Lefschetz number $\Lambda_{\pi}(f)$, when $\pi = \pi_1(M)$ is finite, and express o(f) in terms of $\Lambda_{\pi}(f)$, and

3) express $\Lambda_{\pi}(f)$ in terms of local indices so that one relates $\Lambda_{\pi}(f)$ directly to N(f) with π being finite.

(18) R.JERRARD: Multiple-valued functions applied to fixed points of continuous functions

Certain multiple-valued functions called m-functions are useful in describing the behavior of fixed points and in solving coincidence problems. These are finite-valued functions, with each point in the graph weighted by a value taken from a ring. There is an additivity condition which requires that the sum of the weights is conserved locally in the range space as the domain variable changes. If $f_t: X \to X$ describes a homotopy, then the multiple-valued function $\phi: I \to X$ with $\phi(t)$ equal to the set of fixed points of f_t and with weights equal to their indices, can be an m-function. One can also define homology (and homotopy) groups using the m-functions themselves, and can prove fixed point theorems for m-functions.

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(19) W.A.KIRK: Fixed point theorems for nonexpansive mappings in Banach spaces

If X is a Banach space and D a subset of X, then a mapping T of D into X is called nonexpansive if $\|T(x) - T(y)\| \leq \|x - y\|$ for all x and y in D. Recent developments in fixed point theory for nonexpansive mappings will be discussed, including an exposition of the fundamental role this theory plays in the study of accretive operators, a class of operators important in the theory of nonlinear semigroups. The talk will also include an announcement of new results for non-expansive mappings defined on unbounded convex sets.

(20) R.J.KNILL: Fixed points of radially compact maps on Hilbert spaces

Let $F: E \to 2^E$ be a compact convex set-valued upper semicontinuous function on a Hilbert space E. Let $B \subset E$ be the closed unit ball and let $r: E \to B$ be radial projection. F is said to be radially compact if $rF(E)$ has compact closure.

Suppose that F is radially compact and that F(A) is relatively compact for every bounded subset A of E. Suppose further that there is a positive distance d and a real number t such that $|t| < 1$, and that there are finitely many points $x_1, x_2, \ldots, x_n$ in E such that for every point x of E there is an index i, $1 \leq i \leq n$, such that at least one of the following three conditions holds:

1) $\|x - x_i\| < d$.
2) For every $y \in F(x)$, $\|y - x_i\| < d$.
3) For every $y \in F(x)$, $(y - x_i, x - x_i) < t\,\|y - x_i\|\,\|x - x_i\|$.

Then for some x_0 in E, x_0 is a point of $F(x_0)$. Furthermore for any such x_0 there is an index i with $1 \leq i \leq n$, such that $\|x_0 - x_i\| < d$.

This extends a theorem of Van De Vel for $E = \mathbb{R}^n$.

(21) R.J.KNILL: <u>The Lefschetz fixed point theorem for compact groups</u>

It is shown that every compact group G is a Q-simplicial space where Q is any field of characteristic zero. As a consequence it follows that G satisfies the Lefschetz fixed point theorem for maps $f: G \to G$ such that the induced homology homomorphism with coefficients in Q has a finite dimensional image.

(22) J.MATKOWSKI: <u>Fixed point theorems for mappings with a contractive iterate at a point</u>

Let (X,d) be a complete metric space, $T: X \to X$, and let $\alpha: [0,\infty)^5 \to [0,\infty)$ be nondecreasing with respect to each variable. Suppose that for the function $\gamma(t) = \alpha(t,t,t,2t,2t)$ the sequence of iterates γ^n tends to 0 in $[0,\infty)$ and that $\lim_{t\to\infty}(t - \gamma(t)) = \infty$. Furthermore, suppose that for each $x \in X$ there exists a positive integer $n = n(x)$ such that for all $y \in X$, $d(T^n x, T^n y) \leq \alpha(d(x,T^n x), d(x,T^n y), d(x,y), d(T^n x,y), d(T^n y,y))$. Under these assumptions, T has a unique fixed point.

This theorem generalizes an earlier result of V.M. Sehgal and some recent results of L.Khazanchi and K.Iseki.

(23) M.NAKAOKA: Equivariant point theorems

The Borsuk-Ulam type theorems due to Conner-Floyd, Munkholm, Fenn and Lusk, as well as an equivariant point theorem of Milnor, will be generalized and proved by a unified method. The proofs are patterned on the cohomological proof of the Lefschetz coincidence theorem and based on analysis of the image in $H_G^*(M^p;\mathbb{Z}_p)$ or in $H_G^*(M^p,dM;\mathbb{Z}_p)$ of the equivariant fundamental cohomology class $\hat{U}\in H_G^*(M^p,M^p-\Delta M;\mathbb{Z}_p)$ of a G-manifold M, where $G=\{1, T, \ldots, T^{p-1}\}$ is a cyclic group of prime order p and $\Delta: M\longrightarrow M^p$ is the map $x\longmapsto(x,Tx,\ldots,T^{p-1}x)$.

(24) J.PAK: On the Jiang spaces

Let $f: X\longrightarrow X$ be a continuous map from a compact metric ANR X into itself. Let M(X) be the space of all continuous maps from X into itself with the compact open topology. The evaluation map α at $x_o\in X$ is defined by $\alpha(f)=f(x_o)$ for each $f\in M(X)$. If the induced homomorphism $\alpha_*: \pi_1(M(X),id)\longrightarrow\pi_1(X,x_o)$ is onto, then we call X a Jiang space.

I will try to enlarge the category of Jiang spaces so far we have known.

(25) H.-O.PEITGEN: Leray endomorphisms and asymptotic fixed point theory

Fundamental studies of R.D.Nussbaum concerning the existence of periodic solutions of autonomous functional differential equations have recently underlined the usefulness of asymptotic fixed point theory. The central notion in Nussbaum's considerations is the one of an ejective (attractive) fixed point. Using basic properties of Leray endomorphisms and the generalized Lefschetz number due to J.Leray, we have

1) a fixed point index characterization for ejective (E) and attractive (A) sets for sufficiently compact maps on an ANR X:
   $\mathrm{ind}(X,f,E) = \Lambda_f(X,X-E)$, $\mathrm{ind}(X,f,A) = \Lambda_f^{\check{H}^*}(A)$,
2) a complete classification of ejective fixed points on infinite and finite dimensional manifolds:
   $\mathrm{ind}(M,f,x) = 0$ (∞-dim), $\mathrm{ind}(M,f,x) = (-1)^n \deg_x f$ (n-dim),
3) a fixed point principle for operators leaving invariant a wedge in a Banach space - i.e. expansion (compression) properties are imposed for high iterates of an operator rather than for the operator itself (here 1) is crucial),
4) an application of 3) to the problem of finding a continuum of periodic solutions of $\dot{x}(t) = -\lambda f(x(t-1))$.

(26) W.V.PETRYSHYN: Existence of nonzero fixed points for noncompact mappings

Suppose X is a real Banach space and K is either a wedge or a cone in X. Let D_1 and D_2 be open bounded neighborhoods of $0 \in X$ with $\overline{D}_1 \subset D_2$. It is shown that if either $T: D_2 \cap K \to K$ is condensing (single- or multi-valued) or I - T is A-proper, then under certain additional conditions the map T has nonzero fixed points in $(D_2 - \overline{D}_1) \cap K$. New, as well as some known earlier and recent results are deduced as special cases of two general theorems.

(27) V.PUPPE: Cohomology of fixed point sets and deformation of algebras

Under certain conditions the cohomology algebra (with appropriate coefficients) of the fixed point set of a G-space ($G = \mathbb{Z}_p$, p prime, or $G = S^1$) can be considered a deformation - in a purely algebraic sense - of the cohomology algebra of the space itself.

On one hand this gives restrictions on the isomorphism type of algebras that can occur as the cohomology algebra of the fixed point set of a G-space if the cohomology algebra of the space itself is given. On the other hand it leads to the definition of obstructions for the cohomology algebra of the fixed point set to be isomorphic (as an ungraded algebra) to the cohomology algebra of the space itself.

(28) J.REINERMANN: Some topological aspects in the fixed point theory of nonexpansive mappings

A mapping $f: X \longrightarrow E$ of a nonempty subset X of a Banach space E into E is said to be nonexpansive if $\|f(x) - f(y)\| \leq \|x - y\|$ for all x and y in X. In general, weak compactness and convexity of X and smoothness properties of E are assumed to be fulfilled in order to get $Fix(f) \neq \emptyset$.

We have shown, however, that some of the most important results in this area remain true if "convexity of X" is replaced by "starshapeness of X". Now we intend to announce some further results if X is assumed to be a (contractible) finite union of closed bounded convex subsets of E or if X is merely weakly compact. Some remarks on the topological structure of Fix(f) are added.

(29) V.STALLBOHM: Coincidence index and global bifurcation theorems without multiplicity assumptions

Using the coincidence degree for couples (L,N) as developed by Mawhin-Hetzer, we compute the coincidence index of an isolated zero of L - N. Special interest is devoted to the case that the linear part of L - N has a nontrivial kernel. We assume that L is a Fredholm operator of index 0 and N is a k-set contraction satisfying some growth conditions. As an application we obtain a global bifurcation theorem for equations of the type $Lx - B(\lambda)x - R(x,\lambda) = 0$ without multiplicity assumptions.

(30) R.SCHÖNEBERG: Fixed point principles for mappings of Frum-Ketkov-type via a generalization of the fixed point index

Let E be a Banach space, $X \subset E$ and $f: X \longrightarrow E$. f is said to be of Frum-Ketkov-type if f is continuous and if there exist a nonempty compact subset K of E and a constant $m < 1$ such that $d(f(x),K) \leq md(x,K)$ for all $x \in X$, where $d(y,K)$ denotes the distance from a point y to K.

In this talk we present some new fixed point theorems for mappings of Frum-Ketkov-type which satisfy certain boundary conditions. Essential to our work is an appropriate generalization of the classical fixed point index.

(31) H.STEINLEIN: Generalized Borsuk-Ulam theorems and asymptotic fixed point theory

Let $f: X \to X$ be a continuous map from a closed convex subset of a normed space into itself such that $f^k(X)$ is relatively compact for some $k \geq 2$. We ask if f has a fixed point.

Let $h: G \to G$ be a free $\mathbb{Z}_p$-action (p prime) on a Hausdorff space G. Define $C(G,h) = \{ H \subset G$ / H is the disjoint union of closed subsets H_i, $i = 0, 1, \ldots, p-1$, such that $h^i(H_o) = H_i$ for all $i \}$ and $g(G,h) = \min \{ card(C)$ / $C \subset C(G,h)$ and C covers $G \}$ (Švarc 1961/62).

Assume $Fix(f) = \emptyset$. For prime numbers p we define $\varphi(p) = g(Fix(f^p),f)$. One knows that $\varphi(p) \gtrsim 2p/k$. Hence one would get a contradiction if one could show $\varphi(p) = o(p)$. Generalizations of the Borsuk-Ulam theorem due to Švarc and of the Borsuk-Ljusternik-Schnirelman theorem show that $\varphi(p) = O(p)$. There might be a chance to obtain a better version of the Borsuk-Ljusternik-Schnirelman theorem which would yield the desired o(p) estimate.

(32) F.WILLE: New results in the theory of degree of mappings

In 1946, H.Hopf gave the following problem: Let $f: S^n \to S^n$ be a continuous mapping on the n-sphere and let $\alpha \in (0,\pi]$ be a real number such that $f(x) \neq f(y)$ whenever $\sphericalangle(x,y) = \alpha$. Conjecture: The degree of f is not zero.

It will be proved that the conjecture is true for all even positive integers n. In the case of $\alpha = \pi/2$, the conjecture holds for all positive integers n. These results may be transformed into fixed point theorems by usual methods. In the case of $\alpha = \pi$ we have the well known theorem of Borsuk.

H.Ulrich (Heidelberg)