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Asymptotic Methods of Statistics

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Die Tagung "Asymptotic Methods of Statistics" stand unter der Leitung von I. Ibragimov (Leningrad), J. Pfanzagl (Köln) und H. Witting (Freiburg). Leider mußte Herr Ibragimov seine Teilnahme kurzfristig absagen.

Wie in den vergangenen Jahren waren wieder zahlreiche ausländische Gäste anwesend, so daß viele interessante Diskussionen geführt werden konnten. Die thematischen Schwerpunkte der Tagung lassen sich durch folgende Stichworte charakterisieren:
(1) Asymptotische Entwicklungen, (2) "large deviations", (3) Asymptotische Methoden der Sequentialanalyse.

In weiteren Vorträgen wurden u.a. einige spezielle Aspekte der Robustheit sowie nicht-parametrische Problemstellungen untersucht. Zum Abschluß wurden verschiedene Beziehungen zwischen grundlegenden Prinzipien der asymptotischen Statistik erörtert.Die Tagung hat zu den oben genannten, wichtigen und aktuellen Themenkreisen -die im Ausland zum Teil auf breiterer Basis als in Deutschland bearbeitet werden- zahlreiche Anregungen für weiteres wissenschaftliches Arbeiten gegeben. Darüber hinaus konnte eine Reihe wertvoller persönlicher Kontakte geknüpft werden.

<u>Teilnehmer</u>

R. Ahmad, Glasgow
M. Akahira, Tokyo
W. Albers, Enschede
P. Ba'rtfai, Budapest
K. Behnen, Bremen
P. Bickel, Berkeley
S. Bjerve, Oslo
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H. Walk, Essen
W. Wefelmeyer, Köln
H. Witting, Freiburg
W.R. van Zwet, Leiden

Vortragsauszüge

R. Ahmad: On Basic Concepts in Asymptotic Methods of Statistics

It is suggested that a more natural assumption in statistics is 'partial exchangeability' or 'exchangeability' instead of the usual 'i.i.d. r.v's' structure. There is no loss of generality in this, and moreover by this approach one covers almost all (yes almost all!) practicable problems. Contiguity, which is a concept of 'nearness' or 'asymtotic absolute continuity', can be achieved via exchangeability structure. For various expansions such as Edgeworth-type and other stochastic expansions it is pointed out that, perhaps, one should employ and exploit the families of symmetric stable, stable, various L-classes (cf. Khintchine, Lévy and Urbanik), infinite divisible distributions. Some nice approximations can be achieved for some distributions by considering infinite divisibility class - and this is done for example in LeCam (1960, Contiguity paper) and Ibragimov-Linnik (1971, book). Next, sufficiency, ancillarity and various principles are discussed. Following Birnbaum (1962 JASA), Hájek (1967, 5th Berk. Symp.) and Basu (1975, Sankhya), without assuming finiteness or discreteness of sample and parameter spaces but with due regard to measure-theoretic aspects, the following result can be established.

Theorem: Let I, S, L, C, * denote respectively the principles of invariance, sufficiency, likelihood, conditionality, and their restricted (*) versions. (i) $I \Rightarrow I^*$, $S \Rightarrow S^* \Leftrightarrow L^* \Rightarrow I$, (ii) $C \Leftrightarrow L \Rightarrow S$, $L \Rightarrow I$; (iii) I^* and $C^* \Rightarrow L$, S^* and $C^* \Rightarrow L$; (iv) the essential space structure is that of locally compact, and essentially similar results hold for asymptotic structures.

M. Akahira: Asymptotic Properties of Discretized Likelihood Estimators (DLE's)

Suppose that $X_1, X_2, \ldots, X_n$, is a sequence of i.i.d. random variables with a density $f(x,\theta)$. Let c_n be a maximum order of convergence of consistent estimator of θ. We consider a solution $\hat{\theta}_n$ of the discretized likelihood equation

$$\sum_{i=1}^{n} \log f(X_i, \hat{\theta}_n + rc_n^{-1}) - \sum_{i=1}^{n} \log f(X_i, \hat{\theta}) = a_n$$

where a_n is chosen so that $\hat{\theta}_n$ is asymptotically median unbiased (AMU). Then the solution $\hat{\theta}_n$ is called a discretized likelihood estimator (DLE). If for each r,

$$\sum_{i=1}^{n} \log f(x_i,\theta+rc_n^{-1}) - \sum_{i=1}^{n} \log f(x_i,\theta)$$

is locally monotone in θ for almost all $(x_1,\ldots,x_n)$, then the asymptotic distribution of DLE $\hat{\theta}_n$ attains the bound of the asymptotic distributions of AMU estimators of θ at r. It is seen that there is at least one estimator which attains the bound. It is also shown in comparison with DLE that a maximum likelihood estimator (MLE) is second order asymptotically efficient but not third order asymptotically efficient in the regular case. Further it is seen that the asymptotic efficiency (including higher order cases) may be systematically discussed by the discretized likelihood method.

W. Albers: Testing the mean of a normal population under dependence

In testing the mean of a normal population it is well-known that the t-test may be invalidated if the observations are dependent. A modification of the t-test will be considered which has robustness of validity under m-dependence. As concerns the price for this robustness, it is shown that in case of independence the modified test asymptotically requires mu_α^2 more observations than the ordinary t-test. Here α is the size of the test and u_α is the upper α-point of the standard normal distribution function. It will also be demonstrated that similar results hold for autoregressive processes. In particular, it will be shown that the required additional number of observations under independence again tends to mu_α^2, where in this case m is the order of the autoregressive equation.

P. Ba'rtfai: On the multivariate Chernoff theorem

A Chernoff type theorem is presented for $X_1, X_2, \ldots$ i.i.d. random vectors. Introduce the notations: $R(t) = E(e^{\langle t, X_1 \rangle})$ (if the expectation does not exist, then $R(t) = +\infty$), $\rho(x) = \inf_{t \in \mathbb{R}^k} e^{-\langle t,x \rangle} R(t)$, $x \in \mathbb{R}^k$, $G \subset \mathbb{R}^k$ open set, $\rho(G) = \sup_{x \in G} \rho(x)$. Suppose that one of the following conditions is satisfied: (i) G is convex, (ii) $R(t) < \infty$ for $|t| < \varepsilon$ (iii) G is bounded, (iv) $\rho(G) = 0$, (v) $\rho(G) = 1$. Then

$$\sqrt[n]{P\left(\frac{X_1 + X_2 + \ldots + X_N}{n} \in G\right)} \to \rho(G).$$

Some properties of the Chernoff function $\rho(x)$ are analyzed by the aid of means of the convex analysis and technical details are presented.

K. Behnen: On Signed-Rank Tests with Zeros and other Ties

A unified asymptotic presentation for different methods of handling zeros and other ties for rank tests of symmetry is given. The different methods of handling ties and zeros (randomization, averaged scores, midranks, discarding zeros etc.) may be found in Conover 1973 (JASA). The results (especially on asymptotic relative efficiency) are similar to Behnen 1976 (Ann. Statist.). For example, there is a global preference of "averaging scores" to "randomization", but no global preference of "averaging scores" to "midranks" besides a practical one.

P. Bickel: Robust design against autocorrelation in time

We can observe $Y_i(t_i) = \Sigma_{j=1}^{p} \beta_j f_j(t_i) + \varepsilon_i(t_i)$, $1 \leq i \leq N$, where the ε_i are jointly normal, $-T \leq t_1 \leq \ldots \leq t_N \leq T$ and $E\varepsilon_i = 0$, $\mathrm{Var}\ \varepsilon_i = \sigma^2$, $\mathrm{Cov}\ (\varepsilon_i(t_i), \varepsilon_j(t_j)) = \gamma\sigma^2 \rho(t_i - t_j)$ where ρ is the autocorrelation function of a stationary process. If $t_i = a(\frac{i-1}{N-1})$, $i = 1, \ldots, N$, $\rho(t) = \rho_1(Nt)$, where a and ρ_1 are fixed, we derive the limiting behaviour of $\frac{N}{\sigma^2}$, Var $\hat{\beta}$, where $\hat{\beta}$ is the vector of least squares estimates. From this we derive the asymptotically optimal design a_T (which depends on T, ρ_1, γ) for linear regression. If we let $T \to 0$ the equispaced design is found to be optimal. Numerical results indicate the asymptotics are justified.

S. Bjerve: Curvature and deficiency - an example

The α-trimmed mean is efficient with respect to the M-estimator with influence curve proportional to $\Psi(x) = \max(\min(K,x),-K)$. In the location model with shape $f_o(x) = c\, e^{-x^2/2}$ when $|x| \leq K$ and $c\, e^{-K|x|+K^2/2}$ otherwise, this M-estimator is the maximum likelihood estimator of the location parameter. Valid asymptotic expansions are available both for the trimmed mean and for the M-estimator. These are employed to compute deficiencies between the estimators in a particular case. The curvature of the location model in the sense of Efron (1975) is also computed in this case.

E. Bolthausen: Applications of weak convergence of empirical processes to convergence in distribution of minimum-distance estimators

Let F_θ be a family of continuous 1-dim. distribution functions where θ is a k-dimensional real parameter, and let X_i be i.r.v. that are distributed according to F_{θ_o}, $\theta_o \in \mathbb{R}^k$. If F_n is the empirical distribution function, estimators $\hat{\theta}_n$ which minimize $\| F_n F^{-1}_{\hat{\theta}_n} - id \|$ are investigated, where $\| \quad \|$ is some norm on $D[0,1]$. Under suitable conditions $\sqrt{n}(\hat{\theta}_n - \theta_o)$ converges in distribution to $\hat{\theta}$ where $\hat{\theta}$ minimizes $\| U - \langle \hat{\theta}, g \rangle \|$ (U Brownian bridge), where $g(t) = (g_1(t),\ldots,g_k(t))$ is the vector of functions arising from partial derivatives of the family F_θ with respect to the components of θ.

A. Borovkov: Asymptotically optimal tests for close infinite-dimensional alternatives

Let $(x_1,\ldots,x_n)$, $x_i \in \mathfrak{X}$ be a sample from a distribution P_{γ_n} on $(\mathfrak{X},F)$, $\frac{dP_{\gamma_n}}{dP} = 1 + \frac{\gamma_n(x)}{b(n)}$, where P is a given distribution, $b(n) \to \infty$, $b(n) \leq \sqrt{n}$. The hypothesis $H_o:\{\gamma_n = 0\}$ is tested against

$\{\gamma_n \neq 0\}$. Let Q_n be a distribution in the set of alternatives. Then we can consider γ_n as a stochastic process. Our purpose is to find universal optimal tests, asymptotically independent of Q_n. If the set of alternatives is k-dimensional it is possible to obtain such tests for $b(n) = o(\sqrt{n})$. (See Borovkov 1975, Teoria Verojata.). In a general case this approach is not applicable. The other way is to consider minimax tests for Gaussian limit distributions of Q_n in the case $b(n) = \sqrt{n}$. The derivation of optimal tests in a general case is based on an "invariance principle", which belongs to the sphere of ideas of LeCam and in case $\Omega = \mathbb{R}$ was realized by Chibisov (Sankhya, 1969). We give some generalization of this principle. With the help of the "invariance principle" it is possible to find the explicit form of asympt. optimal test for Gaussian alternatives (γ_n converges to a Gaussian process). It shows that it is natural to consider the tests of the form

$$\Sigma_{j=1}^{L} a_j \left(\frac{1}{\sqrt{n}} \Sigma_{i=1}^{n} f_j(x_i)\right)^2 > c$$

where $\{f_j\}$ are orthonormal. In the class of such tests different asymptotically optimal tests are constructed.

H. Braun: <u>Components of Goodness of Fit</u>

The problem of GF when the null hypothesis is composite is considered. A proceduce is proposed which avoids the usual difficulties associated with estimating parameters. In the case of data occuring in groups, it is suggested to test each group separately (at level α/m) using parameter estimates obtained from the entire data sets where α is the desired overall level and m is the number of groups. As $m \to \infty$, it is shown that one can use the usual significance levels appropriate to testing a simple hypothesis.
Some discussion of the problem of decomposing the GF statistic into components reflecting possible heterogenity among groups was given. In this case the natural approach is by means of the minimum

discrimination information statistic of Kullback. This of course reduces to a likelihood ratio statistic in a suitably exponded problem. Robust efficient estimates can be substituted for maximum likelihood estimates to provide robustness of validity.

D. Chibisov: <u>Asymptotic expansions for the distribution of a statistic admitting a stochastic expansion</u>

Let $(Y_{oi}, Y_{1i}, \ldots, Y_{pi})$, $i=1,\ldots,n$, be i.i.d. random vectors and $(S_n, \underline{T}_n) = (S_n, T_{n1}, \ldots, T_{np}) = n^{-1/2} \sum_{i=1}^{n} (Y_{oi}, Y_{1i}, \ldots, Y_{pi})$. Let $h_1, h_2, \ldots$ be polynomials of p+1 variables and $Z_n = S_n + \sum_{j \geq 1} n^{-j/2} h_j(S_n, \underline{T}_n)$ (finite number of terms).

The problem is to obtain an asymptotic expansion of Edgeworth type for the d.f. of Z_n, i.e. to obtain an assertion that

$$\sup_x |P\{ Z_n < x \} - \Phi_{n,k}(x)| = o(n^{-k/2}) \qquad (1)$$

with $\Phi_{n,k}$ of form $\Phi_{n,k}(x) = \Phi(x) + \sum_{j=1}^{k} n^{-j/2} Q_j(x)\varphi(x)$, Φ and φ being the standard normal d.f. and density.

<u>Theorem:</u> Suppose that (I) $E|Y_{o1}|^{k+2} < \infty$, $E\,Y_{o1}=0$, $E\,Y_{o1}^2 = 1$; (II) $\exists\ r_1, \ldots, r_p$ such that $E|Y_{11}|^{r_1} < \infty$, $1=1,\ldots p$ and for any term of $h_j(S_n, \underline{T}_n)$, say, $cS_n^{m_o} \underline{T}_n^{\underline{m}}$ the following inequality is satisfied $\sum_{1=1}^{p} (\frac{k+2}{r_1} - 1) m_1 \leq j$ $(\underline{m} = (m_1, \ldots, m_p))$, for all $j = 1,2,\ldots$ and $EY_{11} = 0$ if $r_1 \geq 1$. (III) The distribution of $(Y_{o1}, Y_{11}, \ldots, Y_{p1})$ satisfies Cramér's condition(C). Then (1) holds.

J. Durbin: <u>Saddle point approximations to distributions of serial correlation coefficients and partial serial correlation coefficients calculated from residuals from Fourier series</u>

Approximations are found to the joint distributions of non circular and circular serial correlation coefficients calculated from residuals from regression on Fourier series. For a first definition of the non-circular coefficients the series are cosine series and for a second definition the series are sine series while for circular series they are sine and cosine series. The results obtained give useful approximations to the general regression situation when the regressors are slowly changing. It is shown that when the observations are independent the partial coefficient are approximately independently distributed in forms that are the same for all odd-order coefficients and the same for even-order coefficients. This is the pattern found by Daniels (Biometrika 1956) for the circular coefficients in the non-regression situation. The results were obtained by Daniels saddle-point approximation technique. They are accurate to order n^{-1}.

F. Götze: Asymptotic expansions in the CLT in Hilbertspace

Let Q be a probability measure on the Borel sets of a separable Hilbertspace H with expectation zero such that the absolute moment of the order s exists. Then there is a gaussian probability measure Φ with expectation zero and the same covariance as Q. Suppose that the integral functions

$$h_i(a) = \int f(xn^{-1/2}+a)\ Q^{*i} * \Phi^{*(n-1)}(dx)$$

$i=0,\dots,n$ have derivatives up to the order $3(s-2)$ which are uniformly bounded on H and in $n \in \mathbb{N}$. Then there exists an Edgeworth type expansion

$$\int f(xn^{-1/2})Q^{*n}(dx) = \sum_{r=0}^{s-3} n^{-r/2} P_r(\kappa) \int f(x)\Phi(dx) + O(n^{-(s-2)/2}).$$

Applications are made for the case where f is sufficiently smooth and to the expansion of the expectation of smooth functions of the Kolmogoroff-Smirnov statistic. Finally the result is used to give a new proof of the classical results on the existence of Edgeworth expansions since the proof runs by induction on s and does not use characteristic functions.

Ch. Hipp: Asymptotic expansions in the CLT

Let $X_1, X_2, \dots$ be a sequence of k-dimensional i.i.d. r.v's, with mean 0 and covariance matrix I (identity). Let $s \geq 3$, s_o the integral part of s, and assume $E\|X_1\|^s < \infty$. Let Q_n be the distribution of $S_n = n^{-1/2} \sum_{j=1}^{n} X_j$ and ψ be the formal Edgeworth expansion for S_n. Then

$$(*) \qquad \int f_N dQ_n = \int f_n d\Psi + o(n^{-(s_o-2)/2})$$

for $f_n = f$ satisfying

(i) $|f(x)| \leq M(1 + ||x||^s)$

(ii) f has continuous derivatives $D^\alpha f$ up to order $s_o - 2$

(iii) $|D^\alpha f(x)| \leq M(1 + ||x||^p)$ (p may be larger than s).

Furthermore, (*) holds for $f_n(x) = ||x||^s I_{\{||x|| > ((s-2)\log n)^{1/2}\}}(x)$.

J. Jureckova: Asymptotic properties of processes related to M-estimates of regression parameter

For $X_{N1}, \ldots X_{NN}$ being i.i.d. random variables, the residuals $\sum_{i=1}^{n} a_{Ni} [\varphi (X_{Ni} + \Delta d_{Ni}) - \varphi (X_{Ni})]$ which arise in connection with M-estimates of regression parameter Δ are considered as random processes with Δ as the time parameter. After a proper standardization, such processes are shown to converge weakly to the linear Gaussian process in the case of absolutely continuous φ-function and to Wiener process in the case of step-function φ; the rate of convergence is more moderate in the second case. As an application, consistent estimators of the functional $\int \varphi'(x) dF(x)$ and of the underlying density value f(s) at a specified point s are suggested and their asymptotic distributions are derived.

E. Kremer: Approximate and local Bahadur efficiency of linear rank tests

For linear rank tests of the two-sample problem, the symmetry problem, and the problem of independence the concept of approximate Bahadur efficiency (BE) is developed. For a long time the utility of the approximate approach was disregarded. But now as the main result of the lecture the local equivalence of the approximate and exact slopes is derived for general nonparametric alternatives, which shows the importance of the approximate concept for treating the exact BE of linear rank tests near the null hypothesis. This result can be used for establishing explicit formulas for the exact local BE in some subclasses of alternatives and for proving the optimality of special tests with regard to the local BE. The methods are also applicable to linear rank tests in the k-sample problem andto bivariate layer rank tests of independence.

T.L. Lai: Some asymptotic methods in sequential analysis

It is shown that by representing the sequence of test statistics in terms of a random walk plus remainder, and invoking r-quick strongs laws for the random walk (Lai, Ann. Probability 4 (1976)) and showing that the normalized remainder converges to 0 (r-quickly), on can easily obtain the asymptotic behavior of the moments of the stopping time for most sequential tests in the literature, be they based on likelihood ratios of maximal invariants, on U-statistics, or rank statistics, etc. As an example, the sequential t-test is analyzed in detail. A nonlinear renewal theory is also discussed and is shown to yield full asymptotic expansions to the expected sample sizes of these sequential tests. Moreover, this nonlinear renewal theory gives better numerical approximation to the error propabilities of likelihood-ratio-type sequential tests (such as the SPRT, power-one test, the sequential t-test).

P. Lerche: The law of the iterated logarithm for posteriori distributions and a sequential Bernstein-v.Mises theorem

A law of the iterated logarithm for posteriori distributions holds true in the case, where LeCam's conditions (Théorie asymptotique de la décision statistique) are fulfilled: It exists a constant c_o, such that $F_{x,n}(B_\theta(c_o\sqrt{(\log\log n)/n})^c) \to o$, P_{θ_o}-a.s.
This law can be used to construct tests with power 1 in very general situations. Limit theorems are available for the approximation of the error probabilities of such tests. One of these limit theorems says: Consider the posteriori-distributions as sequential observation sequences. Scale them in the right way, then under the above mentioned conditions these processes converge to the process of posteriori-distributions of Brownian motion.

R. Michel: Higher order asymptotic sufficiency

Let $P_\theta|\mathfrak{A}, \theta \in \Theta$, where $\Theta \subset \mathbb{R}^k$ is open, a family of p-measures.
Theorem: Assume that certain mild regularity conditions are fulfilled and let T_n, $n \in \mathbb{N}$, be a sequence of asymptotic maximum likelihood estimators. Then $(T_n(\underline{x}), l_n^{(2)}(\underline{x}, T_n(\underline{x})), \ldots, l_n^{(s+2)}(\underline{x}, T_n(\underline{x})))$

is asymptotically sufficient for Θ in the following sense: For each $\theta \in \Theta$ there exists a sequence of probability densities $q_n^{(s)}(\cdot,\theta)$, $n \in \mathbb{N}$, over $(X^n, \mathfrak{A}^n, v^n)$ satisfying the Neyman factorization criterion, i.e.

$$q_n^{(s)}(\underline{x},\theta)=g_n(T_n(x),l_n^{(2)}(\underline{x},T_n(\underline{x})),\dots,l_n^{(s+2)}(\underline{x},T_n(\underline{x})),\theta)h_n(\underline{x})$$

such that for every compact subset K of Θ

$$\sup_{\theta\in K}\ \sup_{A\in\mathfrak{A}^n} |P_\theta^n(A) - \int_A q_n^{(s)}(\underline{x},\theta)dv^n(x)| = o(n^{-s/2}).$$

D.W. Müller: Asymptotically multinomial experisments and the extension of a theorem of Wald

Let $\mathcal{E}_n$ be a sequence of experiments $(\mathfrak{X},\mathfrak{A},P_{\theta.n}:\theta\in\Theta)$; let $x_1,\dots,x_n$ be i.i.d. observations from some $P_{\theta.n}$. Given a measurable partition $(A_{1.n},\dots,A_{k_n.n})$ of $\mathfrak{X}$ replace the original sample by the vector of frequencies $(f_{1.n},\dots,f_{k_n.n})$. We assume that for $n\to\infty$ the measures $P_{\theta.n}$ do not separate completely. For finite Θ a necessary and sufficient condition for the existence of approximately sufficient partitions fulfillung $\inf\{P_{\theta.n}(A_{j.n})|\theta,j,n\} > o$ is given: it says: for small $\delta > o$ and $A\subset\mathfrak{X}$ s.t. $P_{\theta.n}(A) > 1-\delta$ the conditional experiments given $x_i\in A$ $(i=1,\dots,n)$ become close to $\mathcal{E}_n^n$ (in the sense of LeCam's notion of deficiency) as $n\to\infty$. For arbitrary Θ this condition has to be replaced by a "uniform" version. Then a Wald type approximation theorem holds for $P_{\theta.n}$ being distributions on the real line having monotone likelihood ratio w.r. to $P_{\tau.n}=P_\tau$: $\mathcal{E}_n^n$ admits a sequence of approximately sufficient nondegenerating partitions. As a consequence, if in some locally nonparametric situation "improbable events do not contain much information", it admits a finite dimensional Gaussian approximation.

U. Müller-Funk: A Wiener-process approximation related to S.P.R.T.-type tests.

A Wiener-approximation to a sequence of processes on the function space $C[o,\infty)$ is considered. These processes are based on a sequence of real-valued statistics that admit a decomposition into a sum of i.i.d. random variables plus a remainder term which is supposed to tend to zero a.s. at a smooth rate over some parametrized class of distributions. This result is applied to linear signed rank statistics as well as to U-statistics in order to derive asymptotic expressions for the OC- and the ASN curve under alternatives near the hypothesis (of tests which are formal analogues to the Wald S.P.R.T.).

J. Oosterhoff: On the large deviation theorems of Sanov and Chernoff

Since the introduction of the concept of Bahadur efficiency in hypothesis testing the interest in large deviations has increased. In the present paper the connections between the classical theorems of Sanov and Chernoff are discussed and some new results are obtained. Special attenion is paid to the multinomial approximations and the limitations of this approach. A natural application is for test statistics based on Kullback-Leibler information numbers of empirical distributions with respect to the null hypothesis set, which after suitable modification have optimal Bahadur slope for all alternative distributions.

Th. Pfaff: On moments of maximum likelihood estimators

Let $(X,\mathfrak{A})$ be a measurable space, $\nu|\mathfrak{A}$ a σ-finite measure and $\{P_\theta|\mathfrak{A} : \theta \in \Theta\}$ a family of p.-measures dominated by $\nu|\mathfrak{A}$ where Θ is an open subset of $\mathbb{R}^k$. Under suitable regularity conditions it is possible to prove that for a sequence of maximum likelihood estimators for the parameter θ, $\{\theta_n : n \in \mathbb{N}\}$, for all sufficiently large numbers $\alpha > 0$ and for every $m > 0$ and every compact $K \subseteq \Theta$ there exist numbers $\kappa(m,K,\alpha) > 0$ and $N(m,\alpha) \in \mathbb{N}$ such that for all $n \geq N(m,\alpha)$ we have

$$(A_{m,\alpha}) \qquad \sup_{\theta \in K} P_\theta^n [n^{m/\alpha} \| \theta_n - \theta \|^m] \leq \kappa(m,K,\alpha).$$

The conditions admit the application to families with several location parameters and (or) scale parameters fulfilling weak assumptions. The estimation $(A_{m,2})$ is the essential tool in proving asymptotic expansions of moments, such as

$$P_\theta^n \, [n^{m/2} \prod_{j=1}^{m} (\theta_n^{(i_j)} - \theta^{(i_j)})] = \sum_{\nu=0}^{r} n^{-\nu/2} h_\nu + o(n^{-r/2}),$$

if there exists a stochastic expansion of the random vector $\sqrt{n}(\theta_n - \theta)$.

J. Pfanzagl: On certain 2nd order efficient nonparametric procedures

A minimum contrast functional $\kappa : \mathfrak{P} \to \mathbb{R}^m$ is defined by $P(f(.,\kappa(P))) = \min\{P(f(.,t)) \mid t \in \mathbb{R}^m\}$. It is shown that $\kappa(Q_n^X)$ -with Q_n^X being the empirical probability distribution - is second order efficient in the following sense: If $\{F_n(t_o) > o\}$ is the critical region of level $\alpha + o(n^{-1/2})$ for the hypothesis $\kappa_o(P) = t_o$, obtained from $\kappa(Q_n^X)$ by asymptotic standardization and $\varphi_n(t_o)$ any sequence of critical functions of level $\alpha + o(n^{-1/2})$ for this hypothesis, then $P_o^n(\varphi_n(\kappa_o(P_o) - \Delta n^{-1/2})) \leq P_o^n\{F_n(\kappa_o(P_o) - \Delta n^{-1/2}) > o\} + o(n^{-1/2})$ provided the family $\mathfrak{P}$ is sufficiently large.
As a corollary one obtains the second order efficiency of tests based on the maximum likelihood estimator for estimating the parameter of any parametric family $F_\theta, \theta \in \Theta \subset \mathbb{R}^k$ with Θ open, and the second order efficiency of tests based on minimum contrast estimators for minimum contrast functionals on nonparametric families of probability measures. From this, corresponding optimum properties of asymptotically median unbiased estimators are obtained which improve earlier results of Levit (Theor. Ver. 1975).

B.L.S.P. Rao: On some problems of inference for Markov processes

A survey of some recent results obtained by the author in the area of parametric as well as nonparametric estimation for discrete time stationary Markov processes is presented. Properties of density estimators for discrete time stationary Markov processes using delta-sequences are discussed. In the area of parametric estimation results on rates of convergence of Bernstein - von Mises approximation and asymptotic equivalence of Bayes and maximum likelihood estimtors are mentioned. These results generalize some recent results of Walter and Blum (1976), Hipp und Michel (1971) and Strasser (1977) for i.i.d. random variables. The following theorem is proved for density estimators for Markov processes $\{X_n, n \geq 1\}$ satisfying Doeblin's condition. We assume that the process is stationary and the initial distribution is the stationary distribution. Let $f(x)$ be the density of the stationary distribution. A sequence $\{\delta_m\}$ of $L_\infty(\mathbb{R})$ is said to be of positive type $\alpha > o$, if $\exists A, B > o$ such that (i) $|1 - \int_A^B \delta_m(x)dx| = o(m^{-\alpha})$, (ii) $\delta_m(x) \geq o$, $m \geq 1$, $x \in \mathbb{R}$, (iii) $\sup\{|\delta_m(x)| : |x| \geq m^{-\alpha}\} = o(m^{-\alpha})$ and (iv) $\|\delta_m\|_\infty \simeq m$.

Let $\hat{f}_{nm}(x) = n^{-1} \Sigma_{i=1}^{n} \delta_m(x-X_i)$.

Theorem 1: If $f \in \text{Lip}(\Lambda)$ for some $o < \Lambda \leq 1$, then $\sup_{x \in \mathbb{R}} E(\hat{f}_{nm}(x)-f(x))^2$

$$\leq c_o m n^{-1} + c_1 m^{-2\alpha\Lambda}$$ where c_o, c_1 are constants independent of n.

It is mentioned that one can show that Bayes estimators β_n and minimum contrast estimators θ_n satisfy $|\beta_n - \theta_n| \leq c_k n^{-1}$ in the parametric case under suitable regularity conditions. These results generalize those of Strasser (1977).

L. Rüschendorf: On estimators of the mode

The Chernoff estimator of the mode is generalized to the k-dim. case and in the context the problem of determining the distribution of a random variable which maximizes a Wiener process with k-dimensional unbounded time is stressed.
In the more regular case a general type of kernel estimator with kernels approaching the δ-function is considered and some asymptotic properties of these estimators and of the corresponding estimators of the mode are proven. The results hold also true in the φ-mixing case.

F.H. Ruymgaart: Some aspects of large deviation probabilities and curved exponential families

Suppose we are given a sample of size N from a curved exponential family determined by a curve $\Gamma = \{\theta(t), t \in [o,\infty)\}$ in the natural parameter space Θ, with $\theta(o) = o \in \text{int}(\Theta)$. Our interest is in testing $H_o: t = o$ (i.e. $\theta = o$) versus $H_1: t > o$ (i.e. $\theta \in \Gamma - \{o\}$). We shall restrict attention to tests of asymptotic size $\alpha \in (o, 1/2)$ that have closed convex acceptance regions $C_N \downarrow C$, C closed convex. The criterion of comparison will be the behaviour of $N^{-1} \log \Pr$(error type II) for large N under a fixed alternative. We call a sequence of the tests just described of Type (a) if $\text{int}(C) \neq \emptyset$, and of type (b) if all C_N are bounded and $C = \{o\}$. Typically, locally most powerful tests are of type (a) and so are likelihood ratio tests (over Γ) for certain curves Γ. The extended likelihood ratio tests (over Θ), however, are of type (b). The following theorem is nothing but an illustration of the "first heuristic principle" formulated in Brown (1971, Ann.Math.Statist).

Theorem: Let $\{A_N\}$ be any sequence of type (a) and $\{B_N\}$ any sequence of type (b). Then we have

$$\liminf_{N\to\infty} \left(\frac{1}{N} \log P_\theta(\overline{X} \in A_N) - \frac{1}{N} \log P_\theta(\overline{X} \in B_N)\right) \geq o,$$

for each $\theta \in (\Gamma - \{o\}) \cap A^c$.

S. Schach: Asymptotic properties of certain estimators in a regression model with random coefficients

We consider the model $y_{ij} = b_{oi} + b_{1i} t_{ij} + \ldots + b_{pi} t_{ij}{}^p + e_{ij}$, $(i = 1,\ldots,n;\ j = 1,\ldots,m_i)$, where the $b_i := (b_{oi}, b_{1i},\ldots,b_{pi})$ are independent and identically distributed random vectors with $E\, b_i = \beta$, $\Sigma_{b_i} = \Lambda$ and the e_{ij} are i.i.d. random variables with $E\, e_{ij} = 0$, Var $e_{ij} = \sigma^2 < \infty$. Estimators for β, Λ, σ^2 are proposed, which are unbiased and consistent as $n \to \infty$ under fairly general conditions. Under somewhat more restrictive assumptions the estimator of β is also asymptotically normally distributed with a covariance which can be estimated from the observations.

W. Schäfer: Some new nonparametric tests for comparing variances

A generalization of a theorem of Behnen about local asymptotic unbiasedness of two-sample rank-order-tests is given. From this gener. it is derived that, except for location problems, linear rank tests cannot behave in a satisf. manner. Hence a new class of nonparametric tests is investigated. These tests essentially rest on reduction by invariance. Their test-statistics are functions of the ranks and of certain values of the emp. distr. fct. and therefore require certain multivariate score-gen. functions. Monotonicity conditions for such functions are established, which ensure that the corresponding tests are unbiased and consistent for their respective alternatives and asymp. optimal for suitable subclasses of contiguous alternatives. Finally, these results are applicated to some problems in comparing variances.

P.K. Sen: Asymptotic Theory of Nonparametric Life Testing Procedures

In the context of life testing procedures, censoring, truncation and progressive censoring schemes are often adapted to limit experimentation from cost and time considerations. In this context, the parametric procedures usually involve a partial sequence of linear combinations of functions of order statistics. Also rank procedures involve a partial sequence of censored rank statistics. The basic idea is to represent such a partial sequence in terms of a martingale plus remainder term (converging at a faster rate) and then use some invariance principles for such martingales to have parallel results for the basic sequence. Complications due to staggering entry and random withdrawals and also concomitant variates can also be taken care of by using weak convergence to appropriate Brownian sheet processes.

W. Sendler: Some asymptotic results in concentration measurement

Let X be a random variable with $P(X > 0) = 1$ and distribution function F. The Lorenz-curve is defined by $\mu_F^{-1} \int_0^1 1_{[0,\alpha]}(t) F^{-1}(t)dt$, the Gini-coefficient by $\kappa = \mu_F^{-1} \int_0^1 (2t-1)F^{-1}(t)dt$ ($\mu_F = EX$, $0 \leq \alpha \leq 1$). These parameters are special cases of parameters of type $\varphi = q_2/q_1$, where $q_k = \int_0^1 I_k(t)F^{-1}(t)dt$, $k = 1,2$. To estimate these class of parameters, define $Q_k^{(n)} := \frac{1}{n} \sum_{i=1}^{n} a_{in}^{(k)} X_{in}$, $k = 1,2$.

Theorem 1 Under certain smoothness conditions on the $a_{in}^{(k)}$ and F we have for

$\hat{\varphi}_n := \dfrac{Q_2^{(n)}}{Q_1^{(n)}}$ the result $\sqrt{n}(\hat{\varphi}_n - \varphi) \rightarrow N(0,\sigma^2)$, where $\sigma^2 = \dfrac{\tau^2}{q_1^2}$ with

$\tau^2 := \int_0^1 \int_0^1 (s \wedge t - st)\, I_\varphi(s) I_\varphi(t) dF^{-1}(t) dF^{-1}(s)$ and $I_\varphi := I_2 - \varphi I_1$.

Theorem 2 Under the conditions mentioned in Th. 1 we have with

$$\hat{\tau}_n^2 := \sum_{i,j=1}^{n-1} \left(\frac{i}{n} \wedge \frac{j}{n} - \frac{ij}{n^2}\right) g_n\left(\frac{i}{n}\right) g_n\left(\frac{j}{n}\right) (X_{n,i+1} - X_{n,i})(X_{n,j+1} - X_{n,j})$$

a strongly consistent estimate for τ^2, $(g_n(t) := I_2(t) - \hat{\varphi}_n I_1(t))$. From Th. 2 we derive that $\hat{\varphi}_n \frac{Q_1^{(n)}}{\hat{\tau}_n} \longrightarrow N(0,1)$, which can be used for asymptotic confidence intervals for φ.

Finally, define $Q_k^{(n)}(\alpha) := \int_0^1 I_k(\alpha,t)F_n^{-1}(t)dt$, $k = 1,2$ and consider the empirical ratio process

$$R_n := \sqrt{n}\left(\frac{Q_2^{(n)}(\cdot)}{Q_1^{(n)}(\cdot)} - \varphi(\cdot)\right),$$

where $\varphi(\alpha) := q_2(\alpha)/\ q_1(\alpha)$. Possibilities for the proof of weak convergence of R_n to a Gaussian limit process are discussed.

J. Steinebach: <u>Exponential convergence rates of large deviation probabilities in the multidimensional case</u>

Let $\{W_n\}_{n=1,2,...}$ denote a sequence of k-dimensional random vectors on a probability space $(\Omega,\mathfrak{A},P)$ and $\{k_n\}_{n=1,2,...}$ be a sequence of positive real numbers with $k_n \to \infty$ $(n \to \infty)$. Using moment-generating function techniques two theorems are proven on the existence of limits $\rho(A) = \lim_{n\to\infty} [P(W_n \in k_n A)]^{1/k_n}$ for certain subsets $A \subset \mathbb{R}^k$. The first theorem is concerned with sets A of the form $A = x + C$, where C is a cone in $\mathbb{R}^k$ and x is a fixed vector, and the second theorem deals with the existence of $\rho(A)$ for complements A of bounded subsets of $\mathbb{R}^k$. Both theorems can immediately be applied to partial sumes of independent, identically distributed random vectors with finite moment-generating functions.

W.Wefelmeyer: <u>An asymptotically complete class of tests</u>

Consider the class of sequences of asymptotically similar critical regions of the form $\{S_n > 0\}$, $n \in \mathbb{N}$, where the test-statistic S_n admits a certain stochastic expansion. It is shown that for such test-sequences first order efficiency implies second order efficiency (i.e. efficiency up to an error term $o(n^{-1/2})$. Moreover, the asymptotic power functions (under local alternatives) of first order efficient test-sequences are determined up to an error term $o(n^{-1})$, and a class of critical regions is specified which is minimal essentially complete up to $o(n^{-1})$.

W. Wefelmeyer: A third order optimum property of the maximum likelihood estimator

Let $\theta^{(n)}$ denote the maximum likelihood estimator of a vector parameter, based on an i.i.d. sample of size n. The class of estimators $\theta^{(n)}+n^{-1}q(\theta^{(n)})$, with q running through a class of sufficiently smooth functions, is essentially complete in the following sense: For every estimator $T^{(n)}$ there exists a function q such that the risk of the corrected maximum likelihood estimator $\theta^{(n)}+n^{-1}q(\theta^{(n)})$ exceeds the risk of $T^{(n)}$ by an amount of order $o(n^{-1})$ at most, simultaneously for all loss functions which are symmetric, bowl shaped and bounded. If q^* is chosen such that $\theta^{(n)}+n^{-1}q^*(\theta^{(n)})$ is unbiased up to $o(n^{-1/2})$, then this estimator minimizes the risk up to an amount of order $o(n^{-1})$ in the class of all estimators which are unbiased up to $o(n^{-1/2})$. The results are obtained under the assumption that the estimators admit stochastic expansions and that either their distributions have -roughly speaking- densities with respect to the Lebesgue measure, or the loss functions are sufficiently smooth.

W. van Zwet: A strong law for linear functions of order statistics

Let $U_1, U_2, \ldots$ be i.i.d uniform $R(0,1)$ random variables defined on a single prob. space, $U_{1:N} < U_{2:N} < \ldots\ldots < U_{N:N}$ the ordered $U_1,\ldots,U_N$. We have Borel functions $J_N: (0,1) \to \mathbb{R}$, $h: (0,1) \to \mathbb{R}$. Define $h_N(t) = h(U_{[Nt]+1:N})$ for $t \in (0,1)$ and

$$M_N := \int_0^1 J_N(t)\{h_N(t)-h(t)\}\,dt.$$

Theorem: Let $1 \leq p \leq \infty$, $\frac{1}{p}+\frac{1}{q}=1$, $J_N \in L_p$, $N = 1,2,3$ and $h \in L_q$. If either

(i) $1 < p \leq \infty$ and $\sup_N \| J_N \|_p < \infty$, or

(ii) $p = 1$ and $\{J_N,\ N=1,2,3,\ldots\}$ uniformly integrable, then $M_N \to 0$ for $N \to \infty$, with prob. 1. If moreover, $\int_0^t J_N(s)ds \to \int_0^t J(s)ds$ for $J \in L_p$ and all $t \in (0,1)$ than

$$\int_0^1 J_N(t)h_N(t)dt \to \int_0^1 J(t)h(t)dt$$

with prob. 1. This theorem generalizes parts of the results in Wellner AS 5 (1977).

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