Vortragsbuch

Nr. 81

18.12.88 – 18.3.89

Ein BRUNNEN Erzeugnis

DIN A 4 liniert 144 Blatt
14133 bisher 103 M 0

DM

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Inhaltsverzeichnis zum Vortragsbuch Nr. 81

GRUNDLAGEN DER GEOMETRIE
(~~19.~~ 18. – 24. 12. 1988) Fortsetzung

The length problem for transvections

Let G be a group and S a system of generators for G such that $S^{-1} \subset S$. Then every element g in G is a product of elements s_i in S: $g = s_1 \ldots s_t$. The minimal t for which g is a product of elements in S is called the length of g. F. Bachmann formulated the length problem and stressed its importance for the characterization of geometric groups. We solve the length problem for the special linear group over the quaternions which is generated by transvections.

Dec 21, 1988 Erich W. Ellers (University of Toronto)

Rings of Stable Rank 2 are Barbilian Rings.

Let M_R a free unitary module. A non-empty subset B of M_R is by definition a Barbilian ~~ring~~ set iff
$B^* := \{F \subset B \mid F$ is a free generating system of M_R with $\# F > 1\}$
satisfies: (B_I) Each $u_1 \in B$ may be completed to $\{u_1, u_2, \ldots\} \in B^*$
(B_{II}) $\{u_1, u_2, u_3, \ldots\} \in B^*$ implies $\{u_1 + u_2\alpha, u_2, u_3, \ldots\} \in B^*$ for all $\alpha \in R$.
For each Barbilian set $B \subset M_R$ holds
$B \subset B_{max} := \{u \in M_R \mid u$ may be completed to a free generating system F of M_R with $\# F > 1\}$ and B_{max} is a Barbilian set iff $B_{max} \neq \{\}$.
A ~~ring~~ ring R is by definition a Barbilian ring iff each Barbilian set B over an arbitrarily given free unitary R-module M_R coincides with B_{max}.
We show that all rings of stable rank 2 are Barbilian rings. The class of rings of stable rank 2 covers for instance the class of all semiprimarily rings, which contains all finite rings with identity.

Dec 21, 1988 Werner Leißner (Univ. Oldenburg)

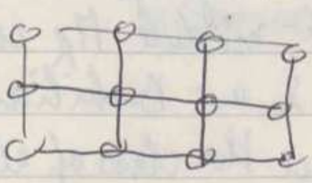
Configuration theorems and group relators

Previous work has shown that the validity of certain web configuration conditions makes free planes into planes over prime fields. If there is a known presentation of the collineation group of a free plane, then adjunction of relators (and sometimes also additional generators) equivalent to the validity of configuration conditions and of char = p yields a presentation of the collineation group of a finite plane of order p. This procedure has been shown to work for affine and projective planes. Since it is known that a minimally generated free Minkowski plane can be made into a Minkowski plane over a prime field if Benz's rectangle condition holds, and since a presentation of the collineation group of such a free plane is known, it is worthwhile to search for a relator equivalent to Benz's condition (or the stronger Miquel condition) in order to try to obtain presentations for the collineation groups of finite Minkowski planes.

Dec. 22, 1988 Rafael Artzy (U. of Haifa)

Telecommunication and incidence structures.

Question of an engineer in telecommunication:
Given a rectangular net work element with 12 points and 7 blocks. Three of these connection branches are 4-blocks and the other ones are 3-blocks.



Is it possible to connect two net work elements of this kind in such a way that any two points are in exactly one line and that all added lines are 3-lines?

Answer:

The question is positively answered for $n \in \mathbb{N}$ not work elements. To construct such system HEFFTER - difference - triples are used.

Herbert Zeitler (Bayreuth)

On the existence of strictly cyclic Steiner Quadruple Systems.

A Steiner Quadruple System SQS(v) is called cyclic if it allows a cyclic automorphismgroup C_v of order v. If the orbit of C_v have all equal length v, then the SQS(v) is called strictly cyclic - notation: sSQS(v). This term was first introduced by E. Köhler. A necessary condition for the existence of a sSQS(v) is $v \equiv 2, 10\ (24)$. In recent years progress has been made as far as the existence of these Systems are concerned (E. Köhler, Phelps, Grannell and Griggs, Colbourn and Colbourn, Cho, Piotrowski, Siemon) and it could be shown that the existence of cyclic systems could be reduced to the existence of cyclic systems with parameter $v = 2p$. The existence of sSQS(2p), $p \equiv 1, 5\ (12)$ is however not yet settled. According to Köhler sSQS(2p) is intimately connected with a graph $GS_2(2p)$ (Siemon). If $GS_2(2p)$ has a 1-factor then sSQS(2p) exists. In order to determine a 1-factor of this graph we consider an automorphism group of $GS_2(2p)$. The unit group $E(2p)$ mod 2p turns out to be an automorphism group of $GS_2(2p)$. And this group has in case $p \equiv 5\,(12)$ the number of $\frac{p-5}{6}$ orbits of equal length $\frac{p-1}{2}$. We define now an orbit graph $OGS_2(2p)$ where the orbits of $E(2p)$ are the vertices and two orbits O_1, O_2 form an edge $\{O_1, O_2\}$ iff there are $A_1 \in O_1$, $A_2 \in O_2$

so that $\{A_1, A_2\}$ form an edge ~~of these~~ in $GS_2(\mathbb{Z}_p)$. It can be shown, that $GS_2(\mathbb{Z}_p)$ has a one factor provided $OGS_2(\mathbb{Z}_p)$ has one. In the case $p \equiv 53 (120)$, $p \equiv 77 (120)$ $OGS_2(\mathbb{Z}_p)$ has a 1-factor, if $OGS_2(\mathbb{Z}_p)$ is bridgeless. A construction is given which shows that „almost all" edges lie in a circle. There are a few „exceptional" edges which cannot be handled by this construction. For these exceptional edges a number theoretic condition is established to lie in a circle.

Dec. 22, 1988 Helmut Simon

A Beckman-Quarles-Type Theorem for Coxeter's Inversive Distance

Let A and B be disjoint circles in the Möbius plane, and suppose that with respect to some point chosen at infinity, A and B have radii r_A and r_B, and have a distance d between their centres. The inversive distance δ_{AB} between them is then given by

$$\cosh \delta_{AB} = \frac{|r_A^2 + r_B^2 - d^2|}{2 r_A r_B},$$

and is independent of the choice of the point at infinity, (i.e. it is invariant under Möbius transformations). We prove the following theorem ($\mathcal{C}$ denotes the set of all Möbius circles).

<u>Theorem</u> Let ρ be a positive constant, and let $X \to X^\sigma$ denote a bijective mapping from $\mathcal{C}$ onto itself such that for all A, B in $\mathcal{C}$,

$$\delta_{AB} = \rho \quad \text{if and only if} \quad \delta_{A^\sigma B^\sigma} = \rho.$$

Then the mapping $X \to X^\sigma$ must be induced on $\mathcal{C}$ by a Möbius transformation.

Dec. 22, 1988 June Lester

Length theorems for automorphism groups of Cayley algebras

Let C be a Cayley algebra over a field F of characteristic $\neq 2$. The automorphism group of C is generated by the involutory automorphisms. Hence any automorphism of C is expressible as the product of a number of involutory automorphisms. The minimal number of involutory automorphisms needed to express the automorphism η is called the length of η. By a result of Wonenburger's any automorphism of a Cayley algebra C has at most length 3. For any automorphism η of C we have $r_\eta = \dim(\ker(\eta - id)^8) = 2, 4$, or 8. The case $r_\eta = 2$ splits into two subcases since the subalgebra B of dimension 2 fixed under η may be split or division (i.e., B is a field). Using results by Alf Neumann it is possible to determine the exact length of any automorphism of a Cayley algebra C over a special field F of characteristic $\neq 2$. Here we considered Cayley algebras over finite fields and over p-adic number fields, i.e. the Chevalley groups of type G_2 over those fields.

Dec. 21, 1988 Huberta Lausch

Characterisations of Chain Geometries ~~over~~ by their Group of projectivities.

A partial affine space is a linear space with parallelism where only whole p consisting of whole parallel classes of lines of an affine space; the point set of these two geometries beeing the same.
A weak Chain space is an incidence structure $\Sigma(P, \mathcal{C})$ consisting of points and chains, where two points are called distant if they lie on a common chain, with
(i) Through any 3 pairwise distant points there goes exactly one chain.
(ii) Some richness conditions.
A weak Chain space is a Chain space, if moreover holds:

(iii' For any $p \in P$ the residual space Σ_p is a partial affine space.

The chain geometries $\Sigma(K,R)$ – R commutative or not – are chain spaces, which all fullfill Miquels Theorem, at most by non-existence of the hypotheses of this configurational proposition. So, if one will gain something essential in a chain space, one has to formulate a strong theorem of Miquel, which at first asks for the existence of the ~~seventh~~ eigth intersection point for two of the three last chains. This implies that all chains are planar, and therefore Σ is isomorphic to the geometry of plane sections of a quadric, by a theorem of Heise.
To characterise such chain spaces which are chain geometries (namely over a quadratic algebra) one may ask for a group of automorphisms acting sharply transitively on the triples of distant points and a second condition using an idea of Hotje.

A. Herzer, Univ. Mainz.

On characterizations of quadrics

Assume, $\Pi = (P,L)$ is a projective space and F is a subset of P. We call F a 2-set, if $|l \cap F| \leq 2 \vee l \subseteq F \ \forall l \in L$ holds. A line $l \in L$ is a secant resp. a tangent of F, if $|l \cap F| = 2$ resp. $|l \cap F| = 1 \vee l \subseteq F$ holds. A point a of F is simple resp. double, if the union of all tangents passing through a is a hyperplane of Π resp. equals P. In the case $|l \cap F| \leq 2 \ \forall l \in L \ \wedge$ $|F| \geq 2$, F is called a calotte. Moreover, F is called a quadratic set, if every point of F is a simple or a double point.
As generalizations of theorems of Tits (1962) and Buekenhout (1969), and for the finite case, of Barlotti (1955) and Tallini (1956), the following theorems can be proved:
(1) Assume, Π is pappian and 4 (resp. 3) $\leq \dim \Pi \leq \infty$. Let F be a 2-set (resp. a quadratic set) of Π, which contains a simple point a. For $\nu \in \{2,3\}$,

define $L_\nu(a) := \{T \leq P \mid \dim T = \nu \wedge a \in T \wedge T \not\subseteq \tau_a\}$, where τ_a is the union of all tangents passing through a. Then, F is a quadric of Π, iff $F \cap T$ is a quadric of T $\forall T \in L_3(a)$ (resp. $\forall T \in L_2(a)$). (2) Assume, Π is pappian, $3 \leq \dim \Pi \leq \infty$, and F is a calotte, which contains a basis B of Π. Suppose, $\leq$ is a well-ordering of B, and b is simple $\forall b \in B \setminus \{b_1, b_2\}$, where $B = \{b_0, b_1, b_2, \dots\}$. For $b \in B$, $b \underset{\neq}{>} b_2$, define $L_2(b_0, b) := \{\varepsilon \leq P \mid \dim \varepsilon = 2 \wedge b_0, b \in \varepsilon \wedge \varepsilon \subseteq \overline{\{x \in B \mid x \leq b\}}\}$. Then, F is an oval quadric of Π, iff $\varepsilon \cap F$ is a quadric in ε $\forall \varepsilon \in \{\overline{b_0, b_1, b_2}\} \cup \bigcup_{b \in B, b > b_2} L_2(b_0, b)$. (This theorem is not true, if $F \cap \overline{b_0, b_1, b_2}$ is not supposed to be a quadric.)
(3) Assume $3 \leq \dim \Pi \leq \infty$ and let F be a quadratic set which contains a line l which consists of simple points only. Then, Π is pappian and F is a quadric.

E. M. Schröder, Univ. Hamburg

Generalized Affine Spaces.

Let $(\mathcal{P}, \mathcal{L}, \parallel)$ be an incidence space with parallelism, Δ be the set of all proper triangles in $\mathcal{P}$ and, for any $\underline{e} := (e_1, e_2, e_3) \in \Delta$, let $\mathcal{T}(\underline{e})$ be the set of all transversal lines of $\underline{e}$, i.e. $\forall T \in \mathcal{T}(\underline{e})$, $\forall \{i, j, k\} = \{1, 2, 3\}$: $e_i \notin T$, $\overline{e_j, e_k} \cap T \neq \emptyset$.
If $A(\underline{e}) := \{x \in \mathcal{P} \mid \{x\} = T \cap \overline{e_i, e_j}, \forall T \in \mathcal{T}(\underline{e}), \forall i, j \in \{1, 2, 3\}, i \neq j\}$, the space $(\mathcal{P}, \mathcal{L}, \parallel)$ will be called a generalized affine space if the following axioms are fulfilled.
A1. For any $\underline{e} \in \Delta$, it is:
(i) $A(\underline{e}) \neq \emptyset$;
(ii) $\forall x \in A(\underline{e}) : x \in \overline{e_i, e_j} \Rightarrow (x \parallel \overline{e_i, e_k}) \cap \overline{e_j, e_k} \in A(\underline{e})$.
A2. For any $\underline{e} := (e_1, e_2, e_3)$, $\underline{b} := (e_1, e_2, b) \in \Delta$, it is:
$$\overline{e_1, e_2} \cap A(\underline{e}) = \overline{e_1, e_2} \cap A(\underline{b}).$$
Now we can define in $\mathcal{P}$ a refined structure by setting $\forall x_1, x_2 \in \mathcal{P}$, $x_1 \neq x_2$, $\forall y \in \mathcal{P} \setminus \overline{x_1, x_2}$: $\overline{x_1, x_2}^* := (\overline{x_1, x_2} \cap A(x_1, x_2, y)) \cup \{x_1, x_2\}$, $\mathcal{L}^* := \{\overline{x_1, x_2}^* \mid x_1, x_2 \in \mathcal{P}, x_1 \neq x_2\}$ and defining a suitable parallelism relation $\parallel^*$ in $\mathcal{L}^*$. Thus we prove:

THEOREM. The triple $(\mathcal{P}, \mathcal{L}^*, \parallel^*)$ is an affine space where
(i) $\overline{x_1, x_2}^* \parallel^* \overline{y_1, y_2}^* \Rightarrow \overline{x_1, x_2} \parallel \overline{y_1, y_2}$,
(ii) the lines of $\mathcal{L} \setminus \mathcal{L}^*$ are proper affine subspaces of $(\mathcal{P}, \mathcal{L}^*, \parallel^*)$.

Mario Marchi, Udine - Italy

Die durch Epimorphismen induzierte Topologie projektiver Ebenen

Durch einen Epimorphismus φ einer projektiven Ebene $\mathcal{E}$ ist – analog wie bei bewerteten Körpern – eine Topologie T_φ auf $\mathcal{E}$ gegeben (T. Hartmann, Dissertation München 1986). Darin wurden wesentliche Eigenschaften der Bewertungs~~theorie~~topologie von von Körpern für $(\mathcal{E}, T_\varphi)$ untersucht (V-topologisch, Minimalität, Vervollständigung)

21. 12. 88

[illegible]

50 Years of WITT Designs

E. WITT (1938) characterised the MATHIEU groups as the automorphism groups of the „Steiner systems" $S(5,6;12)$ and $S(5,8;24)$, nowadays called Witt designs. They were independently discovered by R. D. CARMICHAEL 1937. Nowadays the most convenient approach to these designs is the use of the GOLAY code (1949). A short uniqueness proof of $S(5,8;24)$, using this code, i.e. linear algebra over $GF(2)$, was sketched. The Witt designs gave rise to a vast amount of splendid mathematics, in particular the famous LEECH lattice (1967) and the sporadic simple groups derived from it, finally leading to the classification of finite simple groups. 19. 12. 1988.

H. Lenz

Spaces of orderings and Witt rings of projective planes

Pfisters result on the relations between the orderings and the Witt rings of fields, axiomatic treatments of Marshall, Kula et al., and applications in semi-algebraic geometry have led to an extensive development of axiomatic and algebraic theories of reduced quadratic forms in the last 10 years. Surprisingly, it turns out that a vast amount of the related results extends to arbitrary, orderable projective planes.

We give a brief account on preorderings, orderings, Witt rings and Marshall's spaces of orderings in arbitrary planar ternary rings. In particular, any classic space of orderings can be realized over projective planes in each of the nonempty Lenz classes. Finally, we prove that there do exist projective planes of Class III that are not Moulton-planes.

Dec. 1st, 1988

Franz Kalhoff

Lie group description of kinematic spaces

After defining a product geometry on $L \times L$ for a linear space L one can show that for a group $(L, \cdot)$ on a linear space $(L, \mathfrak{G})$ with all lines greater than 2 one has a kinematic space iff $i : (x,y) \to xy^{-1}$ is collinearity-preserving. So, one has formally similar definitions for kinematic spaces and for Lie groups and the classical kinematic spaces are Lie groups. The connection between the kinematic algebras from which those kinematic spaces can be derived and the Lie algebras is investigated.

19.12.88

Herbert Hotje, Hannover

4-dimensional projective planes admitting a 7-dimensional collineation group

Theorem: Every 4-dimensional projective plane admitting a 7-dimensional collineation group is either isomorphic/dual to a translation plane or isomorphic to the shift plane by Knarr 1983, which is generated by the graph of the function $f: \mathbb{R}^2 \to \mathbb{R}^2 : (x,y) \mapsto (xy - \frac{1}{3}x^3, \frac{1}{2}y^2 - \frac{1}{12}y^4)$.

Dieter Betten (Kiel)

Automorphismen von Rechtseiträumen

Unter einem Rechtseitraum $(P, \mathcal{G}, \parallel, \equiv)$ verstehen wir einen Inzidenzraum $(P, \mathcal{G})$, auf dessen Geradenmenge $\mathcal{G}$ eine Äquivalenzrelation $\parallel$ und auf dessen Menge der Punktepaare P^2 eine Kongruenzrelation $\equiv$ gegeben ist, so daß eine Reihe von Verträglichkeitsbedingungen erfüllt sind. Jeder Rechtseitraum der Charakteristik $\neq 2$ oder der Dimension 2 läßt sich in einen euklidischen Raum einbetten. Wir zeigen, daß sich die Automorphismen eines solchen Rechtseitraumes stets eindeutig zu Automorphismen des zugehörigen euklidischen Raumes fortsetzen lassen. Weiter ist jede kongruenzerhaltende Permutation eines Rechtseitraumes der Charakteristik $\neq 2$ oder einer Rechtseitebene ein Automorphismus des Rechtseitraumes.

21.12.1988 Rotraut Stanik (Hamburg)

Algebraic and geometric aspects of of some planar configurational propositions

Let $\Pi = (P, \mathcal{L}, I)$, $I \subseteq P \times \mathcal{L}$ be a projective plane with I its symmetric incidence relation. We denote by Π^i the affine plane having i as its line at infinity. Relative to a coordinatizing quadrangle $\{X, Y, Q, E\}$, Π to be coordinatized by a set R, a ternary ring (R, F) may be obtained. Two types of addition and a multiplication may be defined on R by the identities: $a+b = F(a,1,b)$, $a * b = F(1,a,b)$ and $a \cdot b = F(a,b,0)$, $\forall a,b \in R$.

Let l and l' be two lines in π, or π^i, with two triples of collinear points $(1\,2\,3)$ and $(1'2'3')$ on them respectively. We define: $[1\,2'] \cap [1'2]$ as $3''$, $[1\,3'] \cap [1'3]$ as $2''$ and $[2\,3'] \cap [2'3]$ as $1''$. Then we consider, in addition to the two well-known forms, two minor forms of Pappus: (i) if $[1\,1']$, $[2\,2']$ and $[3\,3']$ are concurrent, then $(1''2''3'')$ is a collinear triple, and (ii) its dual. We prove:–

(1) In π^i, if every parallelogram has parallel diagonals then P_2-proposition is valid with $l \cap l'$, $2''$, $3''$ I i; and P_4 is valid with $l \cap l'$, $[1\,2'] \cap [2\,3']$, $[1'2] \cap [2'3]$ I i.

(2) The minor form of Desargues implies P_3 and P_4 propositions; thus extending a previous result of Klingenberg concerning P_1 & P_2.

(3) $(R, *)$ is an abelian group iff P_2 holds. Then using the dual plane $\tilde{\pi}$, of π, we interchange $*$ and $+$ and prove $(R,+)$ is an abelian group iff $a+b=b+a$, $\forall a, b \in R$

(4) The two operations, $+$ and $*$, are equal iff a restricted form of the minor form of Desargues holds; thus extending a result of Rashevski.

(5) If P_4 is postulated in π and $a^2=1$, $\forall a \in R$, then $(R \setminus \{0\})$ is an abelian loop.

M. W. Al-Dhahir

21.12.1988. (Univ. of Kuwait)

Projective topology of translation planes

Generalizing a result of Breuning, we show that a spread S in $P_{2n-1}(F)$, F a non-discrete locally compact skew field, defines a topological translation plane if and only if S is compact in a topology introduced by Misfeld on the projective space. To do this, we give a topological interpretation of the description of projective translation planes given by Bruck and Bose.

R. Löwen, Braunschweig

The story of the lost centre and other tales

During the work on the new edition of the volume on geometry of Tropfke's Geschichte der Elementarmathematik the elementary problem of reconstructing the lost centre of a given circle cam under consideration. The solution nowadays taught in our schools is not the one which first appeared in Euclid's elements. The history of this problem becomes interesting because one looked not only for simple constructions just by ruler and compass but also for some with more restricted tools. The Arab Abu al-Wafa in the 10th century described such a construction using a ruler and a compass with a fixed opening. According to the Dane Georg Mohr (17the century) a solution must also be possible using the compass alone; the most ingenious construction of this sort, based on the inversion is due to the French engineer Descubes (~ 1900). See: Michael Toepell: Der verlorengegangene Mittelpunkt, Didaktik der Mathematik 1989 issue 3. A similar problem is the construction of the tangents to circle from a point outside. A solution using a ruler alone has been given by Woldeck Weland 1640. Open Question: How got the „Theorem of Thales" its name (for its german meaning)?

Rudolf Fritsch, München

On orbits of collineation groups

Given a finite-dimensional desarguesian projective space $\mathfrak{P}$ we take the group Φ of all projective collineations of $\mathfrak{P}$ which fix a frame elementwise. This group is trivial if, and only if, $\mathfrak{P}$ is pappian (well-known).
We show that for every point $P \in \mathfrak{P}$ the orbit P^Φ has in a natural way the structure of a projective space. However this "orbit space" is finer in general than the trace space of $\mathfrak{P}$ which is determined by the orbit P^Φ.

20.12.1988 Hans Havlicek (Wien)

On some application of geometric results

Instead of talking about the originally chosen title "Geometric spaces, nets and indexed systems" some topics concerning applications of results from pure mathematics –

especially geometry – in applied fields are discussed.
For example: Petri Nets: theory for modeling communicating systems. The possibility of introducing geometric methods via geometric spaces is indicated (cf. non commutative geometry by J. André).
Computer Graphics: very interesting field for geometric applications. An example is given how interactive use of a graphics system can support numerics in solving polynomial equations.
Robotics: a very interdisciplinary field, a lot of pure mathematics is entering. Many hard problems arise, especially in path planning collision free shortest paths. A $2\frac{1}{2}$-D approximation of such a 3-D problem is indicated (reduction of complexity).

J. Pfalzgraf, Saarbrücken

<u>Charakteristikfreie Untersuchungen über Doppelverhältnisse in Moufangebenen</u>

Die Ergebnisse Schleiermachers über Doppelverhältnisse in echten Moufangebenen der char $\neq 2$ (1965) sollen verallgemeinert werden auf beliebige, d.h. evtl. auch desarguessche Moufangebenen beliebiger Charakteristik. Sei $\mathbb{P}$ eine Moufangebene, C der Koordinaten-Alternativkörper bzgl. (O, U, V, E) und $g := OV = C \dot{\cup} \{\infty\}$. Für je 4 paarweise verschiedene Punkte von g definieren wir das Doppelverhältnis als Konjugiertenklasse vermöge $\begin{bmatrix} a & b \\ d & c \end{bmatrix} := \langle ((a-d)^{-1}(b-d))((b-c)^{-1}(a-c)) \rangle$ (Faktoren, die ∞ enthalten, lasse man weg). Wie schon im Körperfall sind die Doppelverhältnisse zweier Quadrupel paarweise verschiedener Punkte genau dann gleich, wenn eine Projektivität existiert, die das erste auf das zweite Quadrupel abbildet. Analog hierzu operiert die Gruppe der durch Kollineationen oder Dualitäten der Ebene induzierten Permutationen von g transitiv auf genau denjenigen Quadrupeln, deren Doppelverhältnisse sich nur um einen Jordanautomorphismus von C unterscheiden. Enthält das Zentrum von C mehr als 2 Elemente, so läßt sich jede bis auf einen Jordanautomorphismus doppelverhältnistreue Permutation von g durch eine Kollineation oder eine Dualität induzieren. Andrea Blunck (Hamburg).

K-loops in the special theory of relativity

In 1965 Karzel showed that each sharply 2-transitive permutation-group G can be represented as the group of linear mappings $\alpha_{a,m}: x \longrightarrow a+m\cdot x$ of a neardomain (= Fastbereich) $(F,+,\cdot)$ which is uniquely determined (up to isomorphism) by G. The additive structure $(F,+)$ of a neardomain F is a loop which satisfies an interesting weak associative law. Such loops which were considered by William Kerby and me (in ~ 1973) we called K-loops. (The additive structure of the „pseudocorps" of Tits is not a loop).

Definition: A loop $(K,+)$, with 0 as neutral element, is called a K-loop if for each $a,b \in K$ there exists an automorphism $\delta_{a,b} \in Aut(K,+)$ such that:

$$a+(b+x) = (a+b)+\delta_{a,b}(x) \quad \forall x \in K.$$

A K-loop $(K,+)$ is called unitary if $a+b=0$ implies $\delta_{a,b} = id$.
Now A.A. Ungar recently pointed out (in Foundations of Physics Letters vol 1, 1988, p. 57-90) and in a preprint, he send to me (with the title: The relativistic noncommutative nonassociative group of velocities and the Thomas rotation) that the set $\mathbb{R}^3_c := \{x \in \mathbb{R}^3 \mid \|x\| < c\}$ of admissible velocities, where c is the speed of light, together with the relativistic velocity composition law $(*)$:

$$x * y = \frac{1}{1+\frac{x\cdot y}{c^2}}\left(x + y + \frac{1}{c^2}\cdot\frac{\gamma_x}{\gamma_x+1}\, x \times (x \times y)\right)$$

where $\gamma_x = \frac{1}{\sqrt{1-\frac{x \cdot x}{c^2}}}$ is the Lorentz-factor and $\cdot$ and $\times$

denote the usual scalar and vectorproduct in $\mathbb{R}^3$, is a unitary K-loop with the additional property: $-(a+b) = (-a)+(-b)$. Here $-a$ denotes the left inverse of a. Of course Ungar did not use the name K-loop. He is speaking of a „noncommutative nonassociat. group".

The mapping $\delta_{a,b}$ has physically a nice interpretation: it is the Thomas-rotation.

Since the calculations in $(\mathbb{R}^3_c, *)$ are complicated and lenghty, I propose to develop an algebraic theory of K-loops. First results of Kerby and me are outlined in this talk.

Heinrich Wefelscheid

Configurational Conditions and Confined Configurations in Projective Planes

For any octagon with eight distinct vertices $S_1, \dots, S_8$ and eight distinct edges $S_1S_2, \dots, S_7S_8, S_8S_1$ we can associate four main diagonal points $D_i := S_iS_{i+1} \cap S_{i+4}S_{i+5}$ as well as eight first minor diagonal lines $d_i := S_iS_{i+3}$ where indices are taken modulo 8. Using these notions we formulate the following configurational conditions:

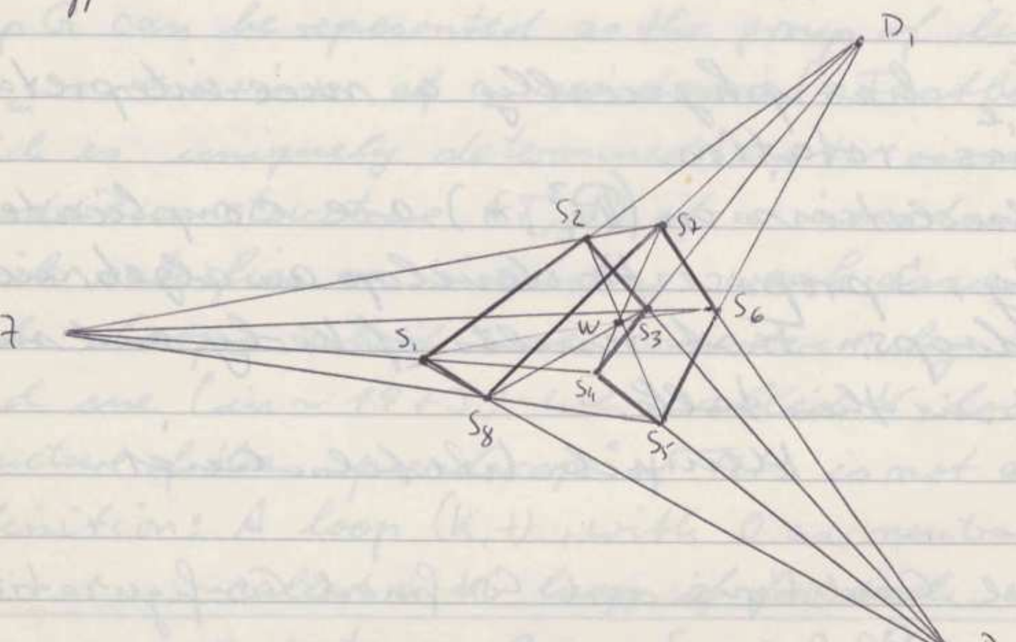
(0) If $S_1 \cdots S_8$ is a non-degenerate octagon then $D_1 = D_3$ and $d_1 \cap d_3 = d_5 \cap d_7$ imply that D_1, D_2, D_4 are collinear.

($\hat{0}$) If $S_1 \cdots S_8$ is a non-degenerate octagon then $D_1 = D_3$, $D_4 \in S_2S_3$, and $d_1 \cap d_3 = d_5 \cap d_7 =: Z$ imply $D_2 = D_4$.

Then one has:

(1) (0) and ($\hat{0}$) are equivalent to the Pappos condition.

(2) In the confined configuration induced by (6), also the remaining first minor diagonal lines d_2, d_4, d_6, d_8 are confluent in some point W. Furthermore, the twelf points $S_1, —, S_8, D_1, D_2, Z, W$, together with the eight edges and the eight first minor diagonal lines, constitute a configuration of type $(12_4, 16_3)$



Martin Funk (Potenza, Italy)

On the classification of topological planes

Let $\mathcal{P}$ be a topological projective plane with a compact point-space P of topological dimension $\dim P = 8$.

THEOREM. If $f = \dim \mathrm{Aut}\, \mathcal{P} \geq 17$, then $\mathcal{P}$ or its dual is a translation plane, or $\mathcal{P}$ is a Hughes plane (over a Tits nearfield).

All these planes are known explicitly (H. Hähl): besides the quaternion plane ($f = 35$) there are 3 one-parameter families of semifield planes with $f = 18$, a two-parameter family of proper translation planes (and their duals), and 4 one-parameter families of planes with $f = 17$.

H. Salzmann (Tübingen)

Construction of ternary rings with generalised order by formal power series

There are introduced several methods to construct generalized orderings in ternary rings, which are constructed by formal power series. These examples show that everyone of the Lenz-Barlotti-classes I 1, I 2, I 3, I 4, II 1, II 2, III 1, III 2, IVa 1, IVa 2, V, VII 2 contains ternary rings, which have at the same time proper non-trivial preorders, quarterorders, semiorders, lafforders, and orders.

Helga Tecklenburg (Hannover)

The classification of flat projective planes

The classification of all flat projective planes with 2-dimensional automorphism group, started 1976 by H. Groh, and continued by I. Schellhammer and M. Lippert, was completed by H.-J. Pohl in May 1988. His historical case diagram, showing also the planes with automorphism groups of dimensions 3, ..., 8 classified by H. Salzmann 1962-65, is being reproduced, illustrated, and interpreted. Roughly speaking, all these planes are "sums" of so-called arc planes (= shift planes in the sense of D. Betten, p. 10)

Hansjoachim Groh (Darmstadt)

Length problems in symplectic groups

Let V be a finite-dimensional K-vector space equipped with a skew-symmetric regular bilinear form (K a commutative field, char K ≠ 2). Sp(V) denotes the automorphism group. It is

generated by its involutions, provided $\dim V \geq 4$.
Result 1 (together with Nielsen). Let $\pi \in Sp(V)$. Suppose that $|K| \geq 5$, and $\dim V \equiv 0 \mod 4$ or that V contains a π-module of Huppert-type 1 (i.e. an orthogonally indecomposable regular π-module V_1 of the form $V_1 = U \oplus W$ where U and W are indecomposable π-modules having the same minimum polynomial $(x-1)^t$ or $(x+1)^t$). Then π is a product of 4 involutions of $Sp(V)$.
Result 2 (together with K. Nielsen). Let $\pi \in Sp(V)$. Suppose that $|K| \geq 5$ and $\dim V \equiv 2 \mod 4$ and $\dim V \geq 10$. Then π is a product of 6 involutions of $Sp(V)$.

There are also results in the excluded cases ($|K| = 3$; $\dim V = 6$). However, the question what is the minimal number of involutions needed in order to write a given $\pi \in Sp(V)$ as a product of involutions, is still open.

Frieder Knüppel (Kiel).

The automorphisms of normal subgroups of the collineation group of affine spaces

Let V be a (right-) vectorspace over a skew field with $2 \leq \dim V < \infty$ and $|\Sigma| \geq 3$. Denote by T the translation group of the affine geometry $AG(V)$, by $D := T \cdot (id_V \cdot \Sigma^\times)$ its dilation group and by $H := T \cdot \Gamma L(V)$ its collineation group. In a common paper with H. Siemon the following results are proved:
Theorem 1: If $\mathfrak{G} \triangleleft H$, $\mathfrak{G} \leq D$ and $T < \mathfrak{G} \cap (T \cdot id_V \cdot C^\times)$, where C is the center of Σ, then every automorphism of $\mathfrak{G}$ is induced by an inner automorphism of H, if and only if Σ is generated by $\{k \in \Sigma^\times \mid id_V \cdot k \in \mathfrak{G}\}$.
Theorem 2: If $\mathfrak{G} \triangleleft H$ with $\mathfrak{G} \not\leq D$ and with $T < \mathfrak{G} \cap D$, then

every automorphism of G is induced by an inner automorphism of H.

H. Mäurer (Darmstadt)

Multigruppen und Grundlagen der Geometrie.

Es wurden Beziehungen zwischen der Theorie der Multigruppen und den Theorien der projektiven Räume, der deskriptiven Geometrien im Sinne von Coxeter und der sphärischen Geometrien im Sinne von Kline diskutiert. Da jeder Automorphismus einer Multigruppe von einer Kollineation der assoziierten Geometrie herrührt, lassen sich viele grundsätzliche Ergebnisse der Grundlagen der Geometrie in die Theorie der Multigruppen übersetzen; insbesondere kann man mittels Bachmannscher Prinzipien metrische Multigruppen definieren.

Karl Strambach (Erlangen)

Geodesics of a product graph.

The idea of studying a structure S by counting geodesics of a suitable graph $G = G(S) = (V, E)$ related to S has been developed by S. Foldes (Discr. Math., 1977) in the case $S = (X, P(X))$ and $G(S) = (P(X)$, covering relation$)$ = the n-cube, where $n = |X|$. The same idea has been developed by the author (J. Geometry, 1984) in the case when $S = PG(n-1, q)$ (or, more generally, when S = a generalized projective space of dimension $n-1$ and order q); some improvements of the previous results have been presented by the author in a Conference in Tunxi (China, Aug. 1988) and encourage to continue to study combinatorial geometric structures "from the geodetical point of view". For example, if $S = (P, \mathcal{L})$ is a (finite, connected) partially linear space (e.g. a variety of Steiner systems, in the meaning used by prof. G. Tallini in his lecture: look to the classical case of a ruled variety, like quadrics, hermitian varieties, grassmannians, C. Segre varieties, ...), then $G(S) = (P, \sim)$ and arithmetical properties of geodesis can play a role in the characterization of some special classical case.

After giving this motivation, we present some simple results concerning the number of geodesics of a product graph; for such a graph we discuss properties like to be "function geodetic", "factorial geodetic", "distance vector geodetic", "multinomial geodetic". By the way this leads to a characterisation of Hamming graphs as graphs which are both factorial geodetic and multinomial geodetic.

Finally some similarities and differences between the classical hypercube Q_n and the q-hypercube $Q_{n,q}$ are discussed, with special regard to properties like the existence of perfect matching.

PierVittorio Ceccherini (Roma)

APPLICABLE ALGEBRA

(1.1.1989 – 7.1.1989)

Comprehensive Gröbner Bases.

The Gröbner basis method initialized by B. Buchberger is a powerful tool for the algorithmic solution of many problems concerning multivariate polynomial ideals and their zeros in algebraically closed fields. It has, however, two significant drawbacks:

1. The construction of Gröbner bases is very sensitive to variations of the coefficients of the input polynomials.
2. While lexicographic Gröbner bases admit the computation of elimination ideals, they do not provide a necessary and sufficient condition on the coefficients of a system of polynomials in order that the system has a common zero in the algebraic closure of the ground field.

Both problems can be overcome by the novel concept of a comprehensive Gröbner basis. Let K be a field, $R = K[U_1,\dots,U_m, X_1,\dots,X_n] = K[\underline{U},\underline{X}]$ a polynomial ring over K. A specialization is a ring homomorphism $\varphi: K[\underline{U}] \to K' \supseteq K$ over K. φ extends canonically to a ring homomorphism $\varphi: R \to K'[\underline{X}]$. Fix a termorder $<$ on $T(\underline{X})$. Then a finite set $G \subset R$ is a comprehensive Gröbner basis (wrt. $<$) if for all specializations $\varphi: K[\underline{U}] \to K' \supseteq K$, $\varphi[G]$ is a Gröbner basis (wrt. $<$) in $K'[\underline{X}]$.

Theorem 1. Given a finite set $F \subset R$ and a termorder $<$ on $T(\underline{X})$. Then one can construct a comprehensive Gröbner basis G wrt. $<$ such that F and G generate the same ideal in R. For a suitable notion of a reduced comprehensive Gröbner basis G' for F, G' is uniquely determined by the ideal $I(F)$ generated by F in R. Moreover, $\deg(G')$ and $|G'|$ are bounded by recursive fcts. in $\deg(F)$, $|F|$, m and n.

As a first application, we get:

Theorem 2. Let F be a finite subset of R and let G be a comprehensive Gröbner basis for $I(F)$ in R. Then G determines in an easy, explicit way boolean combinations $\delta_d(\underline{U})$ of polynomial equations ($-1 \leq d \leq n$) such that for every algebraically closed $K' \supseteq K$: $\delta_d(\varphi(\underline{U}))$ holds in K' iff $\dim V_{K'}(\varphi(F)) = d$ for every specialization $\varphi: K[\underline{U}] \to K'$. (For $d=-1$, $\dim V_{K'}(\varphi(F)) = -1$ means $V_{K'}(\varphi(F)) = \emptyset$.)

The construction of comprehensive Gröbner bases can be extended to universal, compr. Gröbner bases, i.e. to work simultaneously for all termorders, and also to one- and two-sided ideals in the non-commutative polynomial rings of solvable type studied by Kandri-Rody & Weispfenning (J. Symb. Comput.}, to apper).

Volker Weispfenning

Universität Passau.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

A computational algorithm for finding all zeros of a multivariate polynomial system

Buchberger's algorithm for the generation of a reduced Groebner Basis for n polynomials $f_i : \mathbb{C}^n \to \mathbb{C}^n$ is an elimination algorithm with a fixed elimination order which does not regard coefficient sizes. Hence it may run into considerable difficulties (cancellation of leading digits) when used in floating point arithmetic. Our aim is to compose a zero finding algorithm for multivariate polynomial systems from well-understood and stable numerical processes.

Let F be the ideal generated by the polynomials f_i. Our approach proceeds in the residue class ring $\bar{F}$ of F: We construct a basis of power products for $\bar{F}$ – under the assumption that $\bar{F}$ is finite-dimensional – and matrices $B^{(\nu)}$ which represent multiplication by a variable x_ν with respect to this basis of m PPs:

$$Z_0 := \begin{pmatrix} \vdots \\ \vdots \\ 1 \end{pmatrix} \text{ "vector of } {}^{m}\text{basis PP", in some order}, \qquad x_\nu Z_0 = B^{(\nu)} \cdot Z_0, \quad \nu = 1(1)n.$$

The root problem for F becomes an eigenproblem for the $B^{(\nu)}$: Each "zero basis PP vector" $z_0 \in \mathbb{C}^m$ generated by substituting the components of a zero of F into the basis PP vector Z_0 must be a joint eigenvector of the $B^{(\nu)}$ and each joint eigenvector of the $B^{(\nu)}$ represents a zero of F in this form. If the linear powers $x_1, \dots, x_n$ occur in Z_0, the components of the zeros of F are immediately present in the eigenvectors of $B^{(\nu)}$.

The algorithmic generation of Z_0 and the $B^{(\nu)}$ has been based on results from classical elimination theory, it consists in an elimination pass with column pivoting and exchange in a large sparse, rank-deficient matrix. The algorithm succeeds whenever the polynomial system has no zero manifolds of positive dimension (some open questions remain). Manifolds at infinity ~~more~~ may be "removed" by an appropriate modification of the algorithm in many cases.

An implementation of this algorithm generates numerical approximations for all (finite) isolated zeros of the system; these may be "improved" by a subsequent Newton step if necessary.

Hans J. Stetter, TU Wien

Constructive Arithmetic in $GF(q)[T]$

$p>2$ prime, q power of p, $\mathbb{Z}_T = GF(q)[T]$, $\mathbb{Q}_T = GF(q)(T)$, $x \in \mathbb{Q}_T$, $x = F/G$, $|x| = q^{\deg(F)-\deg(G)}$, $\mathbb{R}_T$ = completion of $\mathbb{Q}_T$ w.r.t. $|\cdot|$, $D \in \mathbb{Z}_T$, $R = \mathbb{Z}_T[\sqrt{D}]$, $K = \mathbb{Q}_T(\sqrt{D})$

The following topics were discussed:

1. Extraction of square roots in $\mathbb{Z}_T$
2. Continued fraction in $\mathbb{R}_T$
3. Continued fractions of quadratic irrationalities
4. The regulator group and how to compute in it

A. Guthmann, Univ. Kaiserslautern

The use of geometric models as a basis for the recognition of objects in two dimensional images has proven to be a practical and robust approach. Most experiments involve the use of fixed object models which are specified by numerical geometric data. Here we discuss the use of parametric object models which are represented by a system of geometric constraint equations. These constraints are expressed in terms of scalar, vector and matrix algebra. Object recognition involves symbolic manipulation of these algebraic expressions, as well as numerical optimization procedures. The numerical optimization provides a specialization of the constraint solution to the data of a specific image. The result is a new approach to object recognition.

Joseph L. Mundy
GE USA

THE TRACE OF PRIMITIVE ELEMENTS OF $GF(q^n)$
(joint work with S.A. Vanstone, to appear in J. Algebra)

The following result holds with finitely many exceptions (q,k):

Theorem Let q be a prime power and $k \geq 2$ a positive integer. Given $a \in GF(q)$ (with $a \neq 0$ for $k=2$), there exists a primitive element ω of $GF(q^k)$ with trace $T(\omega) = a$ over $GF(q)$.

In fact, there are at most 147 exceptional pairs (q,k), all with $k=2$, if one assumes $a \neq 0$. Primitive elements of $GF(q^2)$ of trace 1 can be used to construct Costas sequences of order $q-3$, as pointed out by Golomb; such sequences are useful in sonar or radar pattern construction.

D. Jungnickel, Gießen

Algebraic Methods for Automated Geometry Theorem Proving

Implemented provers following the algebraic approach to automated geometry theorem proving are discussed. The basic idea of this approach is to translate a geometry theorem into an algebraic problem and to solve the latter by computer algebra methods.

After shortly explaining the technique how to obtain an appropriate algebraic translation of a geometry theorem, the three general purpose computer algebra methods, i.e. Collins cylindrical algebraic decomposition, Buchberger's Gröbner bases method, and Ritt's characteristic sets method are investigated for their practical applicability to decide certain subclasses of geometry theorems. Explicit characterizations of what can be achieved by these methods as well as practical results on twenty representative examples are given. Then the provers of Wu, Chou, Kapur and Kutzler/Stifter, which are all based on characteristic sets or Gröbner bases, are presented and also applied to the twenty examples.

Finally, applications to constructive geometry and computer-aided design are sketched.

Bernhard Kutzler

RISC, Universität Linz, Österreich

VLSI-design and fractals.

VLSI-technology makes it possible to integrate millions of transistor functions on a chip. In order to use these new possibilities for advanced computer architecture one is led to find hardware realizations of important principles in computation theory. These principles have been developed to find very fast (parallel) algorithms. Many of them – take the divide and conquer principle as an example – lead in a very natural way to a recursive design of algorithms. In order to bring these designs on silicon a system RELACS (REcursive LAyout Computing System) has been developed in Berlin at Humboldt-Univ. and Academy of Science of GDR by L. Budach, H. Grassmann, E.G. Giessmann, B. Graw, Ch. Meinel, B. Mleczan, U. Schäfer and P. Zienicke. A RELACS-program is characterized by the fact, that not only one boolean function f but a sequence of boolean functions f_n ($n \in \mathbb{N}$) is realized by a uniform design of a sequence V_n of VLSI-layouts. It is proved that V_n converges in a certain sense to a structure V which reflects the major qualities of V_n for $n >> 0$. V can be obtained by a generalization of a methode of J.E. Hutchinson for the construction of self-similar fractals. Hutchinsons theorem is a special case of our theorem where a graph (the graph of operations) degenerates to a point.

L. Budach

Akademie der Wissenschaften der DDR

"The Unreasonable Effectiveness of Number Theory in Physics, Music and Communication"

Manfred R. Schroeder

Drittes Physikalisches Institut
University of Goettingen, D-3400 Goettingen, FRG

Number theory is often thought of as rather abstract and far removed from practical applications. Actually, however, the "higher arithmetic" provides highly useful answers to numerous real-world problems, including the design of musical scales, cryptographic systems, and special phase arrays and diffraction gratings with unusually broad scatter (with applications in radar camouflage, laser speckle removal, noise abatement, and concert hall acoustics). One of the prime domains of number theory is the construction of powerful error-correcting codes, such as those used for picture transmission from space vehicles and in compact discs (CDs). Other applications include schemes for spread-spectrum communication, "error-free" computing, fast computational algorithms, and precision measurements (of interplanetary distances, for example) at extremely low signal-to-noise ratios. In this manner the "fourth prediction" of General Relativity (the slowing of electromagnetic radiation in gravitational fields) has been fully confirmed. The quasiperiodic route to chaos of nonlinear dynamical systems, (the double-pendulum and the three-body problem, to mention two simple examples) are being analyzed in terms of continued fractions, Fibonacci numbers, the golden mean and Farey trees. Even the recently discovered new state of matter, called quasicrystals, is effectively described in terms of such number-theoretic principles. And last not least, prime numbers, whose distribution combines regularity and randomness, are a rich source of pleasing artistic designs.

The talk will highlight some of these applications drawn from the author's book *Number Theory in Science and Communication, With Applications in Cryptography, Physics, Digital Information, Computing and Self-Similarity* (2nd Enlarged Edition, Springer-Verlag, Berlin, New York 1986).

Algebraic Computations in Solid Modeling

The rôle of symbolic computation in solid modeling is discussed. While it is evident that solid modeling poses many technical problems that are quite naturally expressed algebraically, it is not clear at this time whether solving them with currently available algebraic software is competitive. This point is worked out by considering several solid modeling problems, including determining surface intersections, implicitizing parametric surfaces, and forming offset, Voronoi, and blending surfaces. It appears that current algebraic algorithms might be too general. Specializing some of them in the context of solid modeling could have decisive consequences. Moreover, fruitful approaches might be based on unconventional reformulations of problems, including computing in ideals of dimension higher than two or three.

Christoph M. Hoffmann
Purdue University, USA.

Multiplication in $GF(2^n)$

Decomposition of multiplication into shift-and-add algorithms and the translation of these algorithms into hardware architectures is discussed. For polynomial bases we have well known serial input/parallel output multipliers. Parallel input/serial output multipliers can be found when there exists an irreducible trinomial of degree n. This construction resembles the Massey-Omura multiplier for normal basis representations. Conversely we can use polynomial basis ideas to find serial input / parallel output normal basis multipliers. Their complexity is equivalent to the complexity of the Massey-Omura multiplier. Dual basis representations give rise to PISO and SIPO multipliers based on Fibonacci-type shift registers (as opposed to the Galois type registers for polynomial bases). A polynomial basis is weakly self dual when its dual basis is a permutation of the polynomial basis multiplied by a constant ($\in GF(2^n)$).
Theorem: A polynomial basis is weakly self dual iff the feedback polynomial is a

trinomial. The corresponding dual basis architectures "accept" polynomial basis data. As a matter of fact they are only a simple rearrangement of the polynomial basis multipliers.

Dieter Gollmann, Universität Karlsruhe

INVARIANT THEORY

The discriminant of a quadratic equation is zero iff the two roots coincide. Changing the variable by a fractional linear transformation will change the roots but not their being coincidental. Hence it will not change the zeroness of the discriminant. In 1830 Boole confirmed this by showing that the discriminant gets multiplied by a nonzero quantity, namely the square of the determinant of the transformation. Cayley generalized this by defining invariants of univariate polynomials of any degree, or equivalently, invariants of bivariate forms of any degree. In 1865 Gordon proved that the invariants of a bivariate form are expressible in terms of a finite number of them. In 1890 Hilbert generalized this to multivariate forms. Unlike Gordon's, Hilbert's proof was nonconstructive. Gordon's proof is based on what Young in his book on Invariant Theory (which he coauthored with Grace in 1902) has called the German Method or the Symbolic Method. The heart of this method is the FFT = the First Fundamental Theorem of Invariant Theory. The FFT says that invariants and covariants of any system of multivariate forms are expressible as meaningful symbolic expressions involving only dets and dots, i.e., determinants and dot products. The ideals of Clebsch, Gordon, Young, et. al., have ~~cates~~ culminated in the Straightening Law of Young Bitableaux which was formalized by Doubillet-Rota-Stein in 1972. Some of my own work in this direction may be found in my book entitled "Enumerative Combinatorics of Young Tableaux" published by Marcel Dekker in January 1988. Presently I am engaged in ~~redoing~~ redoing this enumerative work by bijective methods obtained by modifying the RSK correspondence, i.e., the Robinson-Schensted-Knuth correspondence as explained in the third volume of Knuth's book on the Art of Computer Programming.

Shreeram S. Abhyankar
Math Dept, Purdue Univ.
West Lafayette, IN 47907, USA

From Gröbner-Bases to the Solution of Polynomial Equations

Since the work of Trinks [1978], it has been natural to solve systems of polynomial equations via the scheme:

1) Compute a lexicographic-order Gröbner basis
2) Solve the resulting equations recursively:

for each choice of x_n satisfying the equation in x_n do
    for each choice of x_{n-1} satisfying the equation(s) in x_{n-1} and x_n do
        ....

We note that the Gianni-Kalkbrener theorem means that only one equation actually has to be solved at each stage: the lowest-order non-vanishing one. In particular, the number of zeros can be computed by purely rational arithmetic and polynomial gcds.

Where there is a great deal of structure in the system to be solved (espec. symmetry), it may be profitable to factor equations as we find them. We show examples where problems unsolved in 2 hours can now be solved in <100 seconds.

It is also possible to compute a total-degree-reverse-lexicographic order Gröbner base, & convert this to lexical order by the Faugère-Gianni-Lazard-Mora transform.

We discuss the advantages and disadvantages of the two methods, and outline directions for future research: especially the development of a fraction-free FGLM transform

James Davenport
School of Mathematical Sciences
University of Bath, BATH, U.K.

Selfdual normal bases over $GF(q)$

Starting with one normal basis $(b_0, \ldots, b_{n-1})$ of $GF(q^n) : GF(q)$ all normal basis can be constructed as $(b_0, \ldots, b_{n-1}) \cdot A$, where A runs over all invertible circulant $n \times n$-matrices over $GF(q)$.
This well known method was transfered to orthogonal circulant matrices to calculate <u>all</u> selfdual normal bases (SDNB) if one is given. (Due to a paper of Lempel, ... the problem of the existence of SDNB's is solved in full detail for all finite fields.)
By this method the number of all SDNB's can directly be calculated for any finite field.

Willi Geiselmann
Universität Karlsruhe.

<u>Computational versions of the Quillen-Suslin-Theorem</u>

We describe a constructive proof of the Quillen-Suslin theorem (Serre's conjecture) which computes an explicit free basis for a given projective $K[x_1, \ldots, x_n]$-module of finite rank. The resulting algorithm completes a unimodular polynomial matrix to a square invertible matrix. It can be implemented using Buchberger's Gröbner bases method. Applications include control theory and computational algebraic geometry.

An independent alternative algorithm has been given by J. Heintz et. al. [1988]. Using the effective Nullstellensatz, they give singly-exponential degree and complexity bounds. A combination of both methods with faster heuristics for special cases yields a practical algorithm for the Quillen-Suslin theorem.

Bernd Sturmfels, RISC-Linz

Group theoretical methods in image understanding
(画像理解における群論的方法)

The aim of image understanding is to extract, from 2D images, 3D information about the objects we are viewing — their sizes, locations, orientations, and motions in the scene. If an object model is assumed, the problem is estimation of model parameters from observations on images. If we define observable quantities of 2D images, we can derive, from the geometry of camera imaging, 3D recovery equations which relate the object model parameters with the image observables.

Since images do not have inherent coordinate systems, the choice of the observables must be essentially invariant to the rotation of the image coordinate systems (invariance to $SO(2)$). It is also shown, from the camera imaging geometry, that the 3D recovery equations must be invariant to the rotation of the camera around the center of the lens (invariance to $SO(3)$). We discuss how to exploit such invariant properties by invoking the theory of Lie groups, Lie algebras and their representations.

However, there are other technical (non-mathematical) issues involved. One is the question of what kind of clues should be used. This question is deeply related to psychology of human perception. We discuss typical two views of perception psychology, namely Gestalt psychology and the Gibbsonian psychology (J.J. Gibson), and their implications to the approaches of computer vision

The other issue is computational error involved in real data. Errors contained in measurement data destroy the required consistent conditions for 3D recovery. In order to cope with this difficulty, we introduce an optimization technique based on Sugihara's theory of line-drawing interpretation. Some examples of 3D reconstruction are presented, which are based on heuristics such as rectangularity of corners and parallelism of edges. We take the artificial intelligence approach of accepting reasonable hypotheses unless inconsistencies result from them

Ken-ichi Kanatani (金谷健一)
Gunma University, Japan
(群馬大学)

FFT

According to Wedderburn's Theorem the group algebra $\mathbb{C}G$ of a finite group G of order n is isomorphic ~~W G~~ to a suitable algebra of block-diagonal matrices. Every such isomorphism $W: \mathbb{C}G \to \bigoplus_{i=1}^{h} \mathbb{C}^{d_i \times d_i}$ is called a Fourier transform for $\mathbb{C}G$. Such a W links the convolution in $\mathbb{C}G$ and the multiplication of block-diagonal matrices. W.r.t. natural $\mathbb{C}$-bases, W can be viewed as an n-square matrix. The linear complexity of a matrix W is the minimal number $L_1(W)$ of $\mathbb{C}$-operations sufficient to compute $W \cdot x$ for a generic input vector x. The linear complexity of G is defined by $L_1(G) := \min\{\max(L_1(W), L_1(W^{-1})) \mid W \text{ a FT for } \mathbb{C}G\}$. The classical FFT-algorithms show that $L_1(G) = O(|G| \log|G|)$, for cyclic groups G. Trivially, $|G| < L_1(G) < 2 \cdot |G|^2$.

Theorem.

(a) $L_1(G) = O(|G|^{3/2})$.

(b) If G is metabelian ($G'' = 1$) then
$L_1(G) = O(|G| \log|G|)$.

(c) For symmetric groups: $L_1(S_n) = o(|S_n| \log^3 |S_n|)$.

The proofs of these results "nearly automatically" translate into highly regular VLSI-Designs.

Michael Clausen
Univ. Karlsruhe

The combinatorial use of finite group actions

I reported on some experiences made running the DFG-project "Molecular Structure Elucidation". The aim of this project is to solve the basic problem that gave rise to both the combinatorial theory of enumeration ("Pólya's theory") and graph theory, which is the problem of constructing all the molecular graphs that are compatible with a given chemical formula and a prescribed class of chemical substances.

Emphasis was laid on the description of a redundancy free construction of multigraphs with prescribed number of edges and multiplicities ~~as well as on~~ using double cosets in these symmetric groups as well as on the generation of graphs uniformly at random, based on an algorithm of Dixon/Wilf.

Both these methods were successfully used for cataloguing graphs with $p \leq 10$ points and for checking invariants.

A. Kerber (Bayreuth)

Linear differential equations with polynomial coefficients

A report is made on D-finite power series and their applications in simplification and combinatorics:

Let us call a power series (in finitely many variables) differentiably finite

(D-finite) if all its derivatives span an only finite-dimensional vector space over the rational functions. It is shown that D-finiteness is fulfilled for algebraic and elementary transcendental functions and preserved by addition, multiplication, Hadamard product and diagonalization (taking the diagonal power series w.r.t. two of the variables.

As an application, a canonical simplifier is presented for a huge subalgebra (containing the elementary transcendental functions) of the algebra of power series. As another application, it is shown that a wide class of sequences represented by sums that appear in combinatorics satisfy a linear recurrence relation with polynomial coefficients and therefore can be calculated fast (the first n terms of the sequence in $O(n^2(\log n)^2)$ time).

B. Haible (Karlsruhe, Germany)

On sums of characters: zero-testing and interpolation

We reported on a joint work with A. Dress, Bielefeld ([DG 89]): Many ideas and methods from the recent papers on zero-testing and interpolation of k-sparse n-variate polynomials over fields of characteristic 0 ([BT 88]) and over finite fields $GF(q)$, q a prime power, possibly allowing evaluations of elements from $GF(q^m)$ ([CDGK 88], [GKS 88]), can be unified and better understood by considering k-sums $\sum_{0 \le i < k} f_i \chi_i$ of characters $\chi_i : A \to (R, \cdot)$, where A is an abelian (semi-) group and R is an integral domain with $f_i \in R$.

The zero-test set $\{(z_0^T, \ldots, z_{n-1}^T) : T \subseteq \{0, \ldots, n-1\}, \#T \le \lfloor \log_2 n \rfloor$, $z_i^T = \begin{cases} 0, & \text{if } i \in T \\ 1, & \text{if } i \notin T \end{cases}\}$ of minimal size $\sum_{0 \le i \le \lfloor \log_2 n \rfloor} \binom{n}{i} \sim n^{\log_2 k}$ for $GF(2)$ from [CDGK 88] is constructed.

Furthermore it is shown that finding elements that distinguish the involved characters, e.g. the method of [GKS 88] using Cauchy's determinants, together with appropriate zero-test sets are the essential ingredients for efficient interpolation

algorithms.

[BT 88] Ben-Or, Tiwari. A Deterministic Algorithm for Sparse Multivariate Polynomial Interpolation, Proc. STOC. ACM, (1988).

[CDGK 88] Clausen, Dress, Grabmeier, Karpinski. On zero-testing and interpolation of k-sparse multivariate polynomials over finite fields. Techn. Rep. TR 88.06.006, Heidelberg Scientific Center, IBM Germany, (1988).

[DG 89] Dress A., Grabmeier On sums of characters, in preparation, (1989).

[GKS 88] Grigoriev, Karpinski, Singer. Fast Parallel Algorithms for Sparse Multivariate Polynomial Interpolation over Finite Fields, preprint, (1988)

J. Grabmeier, Wiss. Zentrum der IBM
Heidelberg

LM-matrix and its Combinatorial Canonical Form for Systems Analysis

A canonical form of a class of matrices is introduced, which is useful in the analysis of large-scale engineering systems such as VLSI and chemical plants.

Let $K \subseteq F$ be fields. A matrix A is called a layered mixed matrix (or an LM-matrix) with respect to K if it takes the form (possibley after a permutation of rows):

$$A = \begin{pmatrix} Q \\ T \end{pmatrix}, \qquad (1)$$

where

(i) $Q = (Q_{ij})$ is a matrix over K, and

(ii) $T = (T_{ij})$ is a matrix over F such that the set $\mathcal{T}$ of its nonzero entries is collectively algebraically independent over K.

By the admissible transformation for an LM-matrix A of (1), we means the transformation of the form:

$$P_r \begin{pmatrix} S & O \\ O & I \end{pmatrix} \begin{bmatrix} Q \\ T \end{bmatrix} P_c \qquad (2)$$

where S is a nonsingular matrix over the field K, and P_r and P_c are permutation matrices. Two LM-matrices connected by an admissible transformation are said to be LM-equivalent.

We look for a block-triangular matrix $\bar{A}$ which is LM-equivalent to a given LM-matrix A such that $\mathrm{Row}(\bar{A})$ (row-set of $\bar{A}$) and $\mathrm{Col}(\bar{A})$ are partilitioned resp. as $\{R_0; R_1, \cdots, R_r; R_\infty\}$ and $\{C_0; C_1, \cdots, C_r; C_\infty\}$ and

$$\bar{A}[R_k, C_l] = O \quad \text{if } 0 \le l < k \le \infty, \qquad (3)$$

$$\left.\begin{array}{l} |R_0| < |C_0| \text{ if } C_0 \neq 0, \\ |R_k| = |C_k| \ (>0) \text{ for } k = 1, \cdots, r \\ |R_\infty| > |C_\infty| \text{ if } C_\infty \neq 0, \end{array}\right\} \qquad (4)$$

$$\left.\begin{array}{l} \mathrm{rank}\,\bar{A}[R_0, C_0] = |R_0| \\ \mathrm{rank}\,\bar{A}[R_k, C_k] = |R_k| = |C_k| \text{ for } k = 1, \cdots, r \\ \mathrm{rank}\,\bar{A}[R_\infty, C_\infty] = |C_\infty|. \end{array}\right\} \qquad (5)$$

Theorem 1. There exists a (unique) finest block-triangular matrix $\bar{A}$ with the properties (3)-(5), which is LM-equivalent to a given matrix.

2. $\det \bar{A}[R_k, C_k]$ $(1 \le k \le r)$ is a irreducible polynomial in $K[J]$.

3. $\bar{A}$ can be computed efficiently (with $O(n^3 \log n)$ arithmetic ops in $\mathbb{K}$).

室田一雄 Kazuo Murota
Bonn 大学 Universität Bonn
東京大学 Universität Tokyo

Feedforward Functions Defined by de Bruijn Sequences
(由 de Bruijn 序列定义的前馈函数)

We show that the feedforward funtions defined by de Bruijn sequences, called as de Bruijn function, satisfy some basic cryptographic requirements. It is shown that the family of ~~those~~ de Bruijn feedforward functions could parametrized by a key space, and an appoach to parametrization is given. It is shown that de Bruijn feedforward funtions are balanced and complete. A lower bound of the degree

$$1 + \lfloor \log_2 n \rfloor \leq \deg f$$

of the de Bruijn functions is given. A certain correlational weakness of a class of de Bruijn ~~sequences~~ funtions is analyzed and an algebraic method to meliorate the weakness is also given and it will not cause any substancial drawbacks with regard to the other requirements. The lower bound given in the above is by no means ~~a~~ discouraging, yet there is hope for improving it much. So how to improve it is still a open question. (joint work with K. C. Zeng)

Zong-duo Dai 戴宗铎
Universität Karlsruhe
Institut für Algorithmen und Kognitive Systeme
(On leave from the Graduat School of USTC, Academia Sinica, Beijing, China)

Algebraic Approaches in Image Sequence Analysis

Image sequences, for example sequences of digitized video frames, allow to capture temporal variations in a scene. Algorithmic evaluation of such sequences aims at describing the 3-D (surface) structure of objects in the scene and their motion relative to the recording camera.

Given the coordinate vector $\vec{x1}_i$ at time t1 of the perspective image of a point $\vec{X}_i$ in space and the corresponding vector $\vec{x2}_i$ at time t2, these two entities are related by an equation $\vec{x2}_i^T E \vec{x1}_i = 0$ where the so-called "essential matrix" E depends only on the translation $\vec{T}$ and rotation R between camera positions and orientations at times t1 and t2. Various approaches towards the extraction of estimates for $\vec{T}$ and R from estimates of E are discussed. Recent results by Demazure, Faugeras and Maybank (INRIA 1988) describe algebraic conditions for obtaining solutions for $\vec{T}$ and R.

Attempts to study the influence of measurement noise on the estimation of translation and rotation parameters result in challenging questions for algebraic approaches.

Hans-Hellmut Nagel
Institut für Algorithmen und Kognitive Systeme
Universität Karlsruhe

The linear complexity profile of binary sequences

Stream ciphers are cryptosystems based on pseudorandom keystreams, i.e. on deterministically generated sequences of bits with acceptable properties of unpredictability and randomness. From the viewpoint of cryptology a useful measure for unpredictability and randomness is the linear complexity profile of a sequence. It measures to what extent the initial segments of the sequence can be simulated by linear feedback shift registers. We present recent results on the linear complexity profile of binary sequences relating to the following problems: (i) the construction of sequences with prescribed linear complexity profile; (ii) the behavior of the linear complexity profile for random sequences; (ii) the change in the linear complexity profile under shifts of the sequence. The relevant algebraic tools are formal power series over finite fields and their continued fraction expansions.

H. Niederreiter, Wien

FERMAT CODES

V.D. Goppa's famous method of deriving linear codes from algebraic curves can be used to construct new and interesting classes of linear codes over finite fields.

Utilizing the method of Goppa one can construct codes on the Fermat curve $x^r + y^r + z^r = 0$ where $r = p^b + 1$ and the ground field is $\mathbb{F}_{p^{2b}}$. One problem which arises in this connection is that of determining a basis for the linear space $L(A)$ of some divisor A on the curve. Letting Q be the point $(\eta, 0, 1)$ where η is a primitive $2r$-th root of unity in $\mathbb{F}_{p^{2b}}$ and fixing an integer α, $2g-2 < \alpha < n$ where g is the genus of the curve $(= \frac{1}{2}(r-1)(r-2))$ and n is the number of its $\mathbb{F}_{p^{2b}}$-rational points minus 1 $(= p^{3b})$ one has:

A basis of $L(\alpha Q)$ can be ~~[illegible]~~ parametrized by the set

$$\{(a,b) \in \mathbb{N}_0^2 \mid 0 \leq b \leq \min(a-1, r),\ 0 \leq ar - b \leq \alpha\}.$$

If C_α is the code attached to $L(\alpha Q)$ and $2g-2 < \alpha, \beta < n$ satisfy $\alpha + \beta = n + 2g - 2$, one has further

$$C_\alpha = C_\beta^\perp.$$

The computation of the exact minimal distance of these codes can be reduced to performing operations in the function field of the curve which has a very pleasant arithmetic behaviour.

M. Amin Shokrollahi

Universität Karlsruhe

Autonomous mobile robots

For several years, various autonomous mobile robots are being developed in Europe, Japan and the United States. Typical areas of application are mining, material movement, work in atomic reactors, inspection of under-water pipelines, work in outer space, leading blind people, transportation of patients in a hospital, etc. The first results of these research endeavors indicate that many basic problems still have to be solved until a real autonomous mobile vehicle can be created; e.g. the development of an integrated sensor system for the robot is a very complex effort. To recognize stationary and moving objects from a driving vehicle is several orders of magnitude more complex than the identification of workpieces by a stationary camera system. In most cases the autonomous system needs various sensors. For processing of multi-sensor signals, science has not found a solution to date. An additional problem imposes the presentation and processing of the knowledge needed for planning and following a route or trajectory which is necessary to execute an assignment. Unexpected obstacles have to be recognized, and if necessary an alternate coarse of action has to be planned.

At the University of Karlsruhe an autonomous mobile robot for the performance of assembly tasks is being developed. The assignment of the system is to retrieve parts from a storage, to bring them to a work table and to assemble them to a product. All assignments have to be done autonomously, according to a defined manufacturing plan which is given to the system.

With autonomous mobile robots it is possible to develop manufacturing plants of great flexibility. Any combination of machine tools may be selected according to a virtual manufacturing concept. E.g., an autonomous assembly system equipped with robot arms is capable of working at various assembly stations. For welding or riveting tasks, the robot can move along a large object, such as the hull of a ship and perform the desired operations. An increase in flexibility can only be obtained by the use of knowledge based planning, execution and supervision modules which are sensor supported. In addition, omnidirectional drive systems have to be conceived, capable of giving the vehicle a three-dimensional flexibility, including turning on a spot.

U. Rembold
Uni Karlsruhe

On Prolog Extensions

The term algebra used by Prolog to model domains of interest is inadequate when more exacting requirements have to be met as in modeling various phases of circuit design. Often, however, the structure of such a domain can be adequately described by a finite algebra. The characteristics of digital switching functions in n variables can be described, for instance, by a free boolean algebra in n generators. It is outlined how the expressive power of Prolog – as well as its efficiency – can be largely amplified by arbitrary finite algebras which are implemented by implementing ~~an~~ equation solvers operating on these algebras.

Wolfgang Büttner
Siemens Corporate Research, München

Well Quasi Orders and Gröbner Ideal Bases

My talk is on joint work with Andreas Dress, Bielefeld. We want to present a simple (but mainly structural and non-algorithmic) approach to the theory of Gröbner bases and some other canonical bases (e.g. by Szekeres, Rédei, G. Ayoub). We proceed by introducing suitable quasi-orders on the ground-ring K (commut. with 1), which are supposed to be simplifying for all K-ideals $\mathfrak{r}$, i.e. each residue class $\bar{u}+\mathfrak{r}$ has a (unique) least element $\underbrace{\min(\bar{u}+\mathfrak{r})}_{\lesssim}$. For a commutative monoid $(\Gamma, +, 0)$ we consider the monoid-algebra $R = K^{(\Gamma)} = \bigoplus\sum_{\gamma\in\Gamma} K\cdot X^{\gamma}$, $X^{\alpha}\cdot X^{\beta} = X^{\alpha+\beta}$.

In case $\Gamma = \mathbb{N}^{(I)}$ we have the polynomial ring $K[(X_i)_{i\in I}]$. For any partial order $\leq$ on Γ and the quasi-order $\lesssim$ on K, we introduce on $K^{(\Gamma)}$ the lexicographic quasi-order $\sqsubseteq$. If $\leq$ and $\lesssim$ are noetherian or well quasi ordered, then so is $\sqsubseteq$. If moreover on Γ the relation $\exists\gamma: \alpha+\gamma=\beta$ defines a partial order $\alpha\leq_{+}\beta$ and if $\leq$ is an addition-compatible well-ordered refinement of $\leq_{+}$, then $\sqsubseteq$ turns out to be simplifying for all R-ideals $\mathfrak{A}$. For a basis A of $\mathfrak{A}$ the appropriate reduction $\underbrace{\xrightarrow[A]{}}$ is noetherian, as $\sqsubseteq$ is. The basis A turns out to be order-adapted (i.e. Gröbner), iff for all $f, g\in R$ the relation $g = \min(f+\mathfrak{A})$ is equivalent to $\underbrace{f \xrightarrow[A]{} \cdot \longrightarrow \cdot \ldots \xrightarrow[A]{} g \text{ and no } g \xrightarrow[A]{} h}_{\sqsubseteq}$.

If $\leq_{+}$ is well partially ordered and K is a noetherian ring, then each $\mathfrak{A}$ has a finite Gröbner basis. — A preprint is available.

Gerhard Schiffels
Univ. Bielefeld

Sphere packing and signal constellations

There are many connections between the geometrical problem of packing equal spherical caps placed on the n-dimensional sphere Ω_n and the channel coding problem, i.e. the problem of design signal constellations for errorneous data transmission.

The long-standing Tammes problem of finding the densiest packing of M equal spherical caps on Ω_n is analized. This problem can be viewed as equivalent to finding an arrangement of M points on Ω_n that maximize the minimal distance between all points. This arrangement $C_B(n,M)$ is called the best spherical code. It has important application to the design of signal constellations for a band-limited channel with additive white Gaussian noise.

Using a method which consists of finding the minimum of a suitably chosen objective function of the code's distance distribution, all known conjectures for $C_B(3,M)$, $4 \leq M \leq 32$ are obtained, together with some solutions that are better than them. These solutions are expressed by means of Schlegel graphs and corresponding polytopes. Four-dimensional conjectures ($9 \leq M \leq 21$ and $M = 24, 25$) are obtained also.

Dejan Tošić
Universität Karlsruhe
Institut für Algorithmen und Kognitive Systeme
Stipendiat: Alexander von Humboldt Stiftung; von Universität Novi Sad Jugoslawien

COMPUTATIONAL ASPECTS OF COMBINATORIAL OPTIMIZATION
8. 1. – 14. 1. 1989

The Minimal Violators of Centrality

Given a system of clauses of propositional logic in conjunctive normal form, construct the clause/variable matrix with $\{0, \pm 1\}$ entries by assigning to each clause a row, to each variable a column, and by letting the ij-entry be +1 (-1) if variable j occurs (occurs negated) in clause i. Let the matrix be <u>nearly negative</u> if every row contains at most one +1. When the matrix is scalable (columns only) to become nearly negative, we call it <u>central</u>. The satisfiability problem of the system is easy if the clause/variable matrix is central. We define a <u>minor</u> of a clause/variable matrix, then characterize the minimal minors of non-central matrices. There are a total of nine minors. If the matrix is non-central, then at least one such minor can be found in polynomial time. The result is useful for solution of satisfiability problems of systems involving noncentral matrices.

Klaus Truemper
University of Texas at Dallas

Computational aspects of Combinatorial Optimization 8.1–14.1 1989

Consider a FMS which produces parts of several types. A set of problems ~~and~~ is presented and related models/algorithms are referenced such that the input of the parts in the FMS is scheduled. It also gives the sequence in the execution of the operations that does not violate certain precedence relationships, and considers constraints such that machines availability, resources availability and logical constraints. A hierarchical set of goals is considered. The combinatorial aspects of the problem are emphasized, computational experience is reported and a real life problem is described

Laureano F. Escudero
IBM T.J. Watson Research Center
Yorktown Heights, NY

Finding large independent sets in graphs with Tabu search.
(joint work with C. Friden & A. Hertz)

A metaheuristic - Tabu Search - has been designed for solving global optimization problems; it is based on ideas of F. Glover and P. Hansen.

This technique has been adapted to the search of a large independent set in a graph; we have been able to construct independent sets of $\hat{\alpha}$ nodes in random graphs having up to 1500 nodes ($\hat{\alpha}$ is a probabilistic estimation of the independence number).

An exact procedure for finding a maximum independent set was developed; it uses tabu search at some stages to reduce the number of problems generated. This technique seems to be able to handle graphs having more than 500 nodes (edge density 0.5).

D. de Werra
Ecole Polytechnique Fédérale de Lausanne
(Switzerland)

Solving an NP-hard Edge Coloring Problem by Linear Programming

We consider the problem of coloring the edges of a graph with a minimum number of colors so that any pair of edges incident to a common node have distinct colors. We formulate it as a set covering problem of minimizing the number of matchings to cover the edges. The linear programming relaxation can be solved efficiently since the pricing problem is a weighted matching problem. If the value of the LP relaxation $> \Delta$ (the largest degree) then, by Vizing's theorem, $\Delta+1$ colors are required. When the value of the LP relaxation is Δ and the solution is fractional, we augment the LP with odd circuit cuts, which say that every odd circuit must be covered by at least 3 matchings. For 3-regular graphs we show how to solve the separation problem for odd circuit constraints efficiently and we present computational results for random and difficult graphs. The time consuming part of the algorithm involves the solution of a constrained matching problem to ~~accompl~~ accomplish the pricing.

George L. Nemhauser
Georgia Institute of Technology
Atlanta, GA

Roof Duality Revisited

Together with Endre Boros we have re-examined the 1984 roof duality approach to quadratic 0-1 optimization, which provides a lower bound to the minimum, ~~of~~ as well as a polynomial algorithm to decide whether this bound is equal to the minimum. The new results reduce the determination of the ~~roof~~ duality bound to that of solving a max-flow problem in a network of $O(n)$ vertices (if ~~the~~ n is the number of variables) and establish a stronger "persistency" result (i.e. allow some of the 0-1 variables to be fixed in the optimal solution of the discrete problem, ~~following~~ to these same values as in an appropriately chosen continuous relaxation of the problem), ~~and~~ On this basis very promising computational results are reported, and it is conjectured that a constant $k (\leq 2)$ exists so that $\rho \leq f_{min} \leq k\rho$, where ρ is the roof value of the dual of f.

Peter L. Hammer
Rutgers University
New Brunswick, NJ, USA

The Boolean Quadric Polytope: Some Characteristics, Facets and Relatives.

We study unconstrained quadratic zero-one programming problems having n variables from a polyhedral point of view by considering the Boolean quadric polytope QP^n in $n(n+1)/2$ dimensions that results from the linearization of the quadratic form. We show that QP^n has a diameter of one, descriptively identify three families of facets of QP^n and show that QP^n is symmetric in the sense that all facets of QP^n can be obtained from those that contain the origin by way of a mapping. The naive linear programming relaxation QP^n_{LP} of QP^n is shown to possess the Trubin-property and its extreme points are shown to be $\{0, \frac{1}{2}, 1\}$-valued. Furthermore, $O(n^3)$ facet-defining inequalities of QP^n suffice to cut off all fractional vertices of QP^n_{LP} whereas the family described by us has at least $O(3^n)$ members. Polynomially solvable problem instances are discussed and complete polyhedral characterization is given for the case where the underlying graph is series-parallel. The relationship to vertex-packing in graphs is discussed as well.

Manfred Padberg
New York University
New York, N.Y.

<u>Speeding up parametric min-cost flow algorithms</u>

Most of the time in solving a min-cost flow problem by the primal simplex method is taken up by the last few pivot steps, when most of the edges have to be searched until a candidate for (see p. 60)

Small TSP Polytopes (joint with S. C. Boyd)

We introduce a new class of valid inequalities for the polytope of the symmetric travelling salesman problem. They generalize the clique-tree inequalities of Grötschel and Pulleyblank. We also give complete characterizations of the polytope for ~~n=~~ 6 and 7 cities. For the latter case, the new inequalities are needed. These results are related to work of R.Z. Norman in the 1950's.

W. H. Cunningham
Ottawa and Bonn.

Time-Indexed Formulations of Single-Machine Scheduling Problems (joint with J. Sousa).

We consider the formulation of the non-preemptive single machine scheduling problem using time-indexed variables. Such formulations lead to very large formulations, but give better lower bounds than other mixed integer formulations. We derive a variety of valid inequalities, and show the role of aggregation and the knapsack problem with generalised upper bound constraints as a way of generating such inequalities. Computational experience on small problems with 20/30 jobs and various objectives are presented.

L A Wolsey
CORE, Louvain-la-Neuve.

Generalized Max-Flow Min-Cut problems in the plane

Let G be a plane digraph, and $\mathcal{F}$ a collection of weighted connected subgraphs of G. We provide a polynomial algorithm that finds a min. weight subfamily of $\mathcal{F}$ whose union includes a directed circuit. We consider a dual problem, provide bounds and study complexity issues. This problem has applications to fault-tolerance.

D Bienstock, Bellcore

Node-Packing Problems with Integer Rounding Properties

We consider an integer programming formulation of the node-packing problem, $\max\{1\cdot x : Ax \le w, x \ge 0, x \text{ integral}\}$, and its linear programming relaxation, $\max\{1\cdot x : Ax \le w, x \ge 0\}$, where A is the edge-node incidence matrix of a graph G and w is a nonnegative integral vector. We give an excluded subgraph characterization quantifying the difference between the values of these two programs. One consequence of this characterization is an explicit description for the "integer rounding" case. Specifically, we characterize those graphs G with the property that for every subgraph of G and for any choice of w, the optimum objective function values of these two problems differ by less than unity.

Les Trotter, Cornell Univ.

Recent Results in Job-Shop Scheduling

It took nearly 25 years until Carlier and Pinson proved optimality of a solution of a 10x10 job-shop scheduling problem given in a book by Muth and Thompson. To get this result they developed a clever branch and bound algorithm. I is shown how this method can be combined with a block approach of Grabowski and a geometric method which reduces the problem to a shortest path problem with obstacles. Numerical results for different combinations of these three approaches are presented.

Peter Brucker
Universität Osnabrück

On order (or partial order) preserving injections
(joint with F. Margot and A. Prodon)

Let $(V_1, \underset{1}{\leq})$ and $(V_2, \underset{2}{\leq})$ be two finite posets. An injection $\varphi: V_1 \to V_2$ is partial order preserving, if $i, j \in V_1$; $i \underset{1}{\leq} j \Rightarrow \varphi(i) \underset{2}{\leq} \varphi(j)$. The question whether such an injection exists is NP-hard. If real weights are attached to the elements of $V_1 \times V_2$, the problem of finding a minimum weight partial order preserving injection generalizes several known comb. optimization problems, such as: 1- or p-machine scheduling problem under precedence constraints, capacity expansion, linear assignment problem, etc.
For the case when $(V_1, \underset{1}{\leq})$ is a chain, we give a good algorithm and a description of the convex hull of the order preserving injections.
On the other hand, if $(V_2, \underset{2}{\leq})$ is a chain, the problem remains NP-hard, even if $(V_1, \underset{1}{\leq})$ is a disjoint union of chains.

Th. M. Liebling
EPF Lausanne

Computational Results on Exact Algorithms for Large Size Knapsack Type Problems (joint with S. Martello)

Algorithms for determining the optimal solution of large size instances of single unidimensional knapsack type problems (0-1 Knapsack Problem, Unbounded Knapsack Problem, Subset-Sum Problem, Change Making Problem) are considered. The algorithms are based on the definition of an approximate "core problem", its exact solution through effective implicit enumeration methods, the comparison of the corresponding approximate solution value with tight upper bounds, and the attempt to determine the optimal value of all the variables not considered in the core problem through fast reduction procedures. Extensive computational results for different classes of large size randomly generated test problems (considering up to one million variables) are presented.

Paolo Toth
Università di Bologna (1088)

Polyhedral Study of the Capacitated Vehicle Routing Problem (joint with Farid Harche)

Given (i) a network and travel costs on each link, (ii) a fleet of identical vehicles with given capacity located at a central depot and (iii) client demands and locations, construct routes for the vehicles in order to meet the client demands at minimum travel cost while satisfying the vehicle capacity requirements. Several versions of the problem arise depending on whether the clients can be on several routes and, if so, their demands can be split among several vehicles or not. We relate polyhedra associated with these problems and characterize some of their facets.

Gerard Cornuejols
Carnegie Mellon University

Floorplanning and Global Routing Based on Circuit Partitioning

We present a method for floorplanning, i.e., placement of variable size blocks that are connected by wires. The cost measures to be minimized are area and wiring complexity. The method begins by constructing a ~~s~~ cut-tree for the circuit. Subsequent phases process the cut-tree and transform it into a minimum-area floorplan. This floorplan is optimized w.r.t. wiring complexity by incorporating hierarchical routing.

The results presented analyse and extend methods proposed by Otten and Luk et al. The work is joint with Rolf Müller and Jörg Heistermann, both at Paderborn.

Thomas Lengauer
Univ. of Paderborn

Single-machine lot-sizing and scheduling

We consider the problem of scheduling several products on a single machine so as to meet the known dynamic demand and to minimize the sum of inventory and setup cost. The planning interval is phased into many short periods, e.g. shifts or days, and setups may occur only at the beginning of a period.
We present a branch-and-bound procedure using Lagrangean relaxation for determining both lower bounds and feasible solutions. The relaxed problems are solved by dynamic programming. Computational results on a personal computer are reported for various examples with up to 50 products and 424 periods.

Bernhard Fleischmann
Universität Hamburg

Vertex Packing Algorithms Using Subgraphs with Polynomial Packing Time

We discuss a class of branch and bound algorithms for finding a maximum-weight clique in an arbitrary graph (a maximum-weight vertex packing in the complement graph), whose branching rules guarantee that every subproblem created is solvable in polynomial time. The subproblems are defined on vertex- or edge-maximal triangulated subgraphs, subgraphs with a TR-formative edge coloring, four-cycle-free subgraphs, subgraphs with a 4CF-formative edge coloring. Computational experience is discussed on weighted and unweighted graphs with up to 1,000 vertices and 150,000 edges. The work is joint with J. Xue.

A signature algorithm for the assignment problem and an application in the classification of chromosomes

An $O(n^3)$ algorithm for the linear assignment problem is presented. The iterations are guided by the valency structure of a rooted forest which represents part of the structure of the dual problem. This procedure is similar to Balinski's signature method and to a recent approach of Papamizos. Some computational experience for dense problems is described. Using a model of Tso, we applied a related algorithm to the classification of human chromosomes. Tests with sample data have shown that our results are superior to those obtained in practice by a commercial classification system.

Peter Wünschmidt
Universität Passau

Randomized Incremental Construction of Voronoi Diagrams

Abstract Voronoi Diagrams (R. Klein) are defined by a family of bisecting curves $J(p,q)$, one for each pair p, q of sites. A bisecting curve $J(p,q)$ splits the plane into two unbound domains $D(p,q)$ and $D(q,p)$. The Voronoi region $VR(p)$ of a site p is given as $VR(p) = \bigcap_{q \in S - \{p\}} D(p,q)$. Under the assumption that Voronoi regions are connected, that any two bisectors intersect only finitely often and that bisectors are computationally simple, it is shown that the Voronoi diagram for n sites can be constructed in time $O(n \log n)$ by a randomized algorithm; ~~...~~. This extends work of R. Klein. The algorithm is an instance of randomized incremental construction recently introduced by Clarkson and Shor. (joint work with St. Meiser and C. O'Dunlaing)

K. Mehlhorn, Saarbrücken

Simulated Annealing and Tabu Search Approaches to Hyperbolic Sum and Clique Partitioning

We present computational results concerning the use of these heuristics in the resolution of two combinatorial optimization problems: hyperbolic sum and clique partitioning. The motivation for studying the hyperbolic sum problem in 0-1 variables comes from its application in the formulation of a query optimization problem in information retrieval from classical databases. The clique partitioning problem has some applications, e.g. in the aggregation of binary relations. The computational results illustrate the effectiveness of both approaches, in terms of the quality of the solution they obtain. Some conclusions are also presented concerning computational times (joint work with P. Hansen, M. Poggi, J.P. Barthelémy and S. Amorim)

Celso Carneiro Ribeiro
Catholic University of Rio de Janeiro, Brazil.
January 12, 1989

Computational Complexity of Norm-Maximization

(Joint work with V. Klee) We discuss the problem of maximizing a norm for real n-space over a polytope that is presented as an intersection of m halfspaces. By work of Mangasarian and Shiau (1986) the maximization problem is known to be NP-hard for the classical p-norms. We establish NP-hardness for a considerably wider class of norms, roughly speaking, the norms for which the unit ball admits a strictly inscribed parallelotope. Further, we show that for p-norms norm-maximization is NP-hard even for parallelotopes. This, in turn, implies the NP-hardness of various other problems. We give two examples for such applications, one from pseudoboolean programming, the other from computational convexity

Peter Gritzmann (Trier)

Complementary Two-Commodity Flows: Formulations for TSPs, m-TSPs, and Vehicle Routing Problems

We describe several formulations for traveling salesman problems using different dimensions of the describing space, in particular one-, two-, and multi-commodity flow versions. Especially the two-commodity formulation provides a very useful model. Extensions to non-dense graphs are possible. Precedence constraints can easily be formulated as linear constraints. Further simple modifications succeed to model multiple traveling salesman problems and vehicle routing problems with general capacities for the vehicles

Gerd Finke
Université Joseph Fourier, Grenoble

A new class of facet-defining inequalities for asymmetric traveling salesman polytopes

We describe a class of (locally) facet-defining inequalities, which arises from digraphs whose intersection graphs are (nearly) odd K_4's. We use lifting to obtain global such inequalities. Finally, we describe a generalization (which is based on odd CAT's as introduced by Egon Balas)

Reinhardt Euler
Brest / Pittsburgh

Polynomially solvable special cases of bottleneck-TSPs

In this joint work with W. Sandholzer, Graz, we investigate bottleneck TSPs which ask for a tour for which the longest arc is as small as possible. Since this problem is NP-hard, special cases are of interest which can be handled by polynomial algorithms. We discuss two classes of such special cases. The first class concerns TSPs with symmetric cost matrix of circulant structure. Such TSPs can be solved in $O(n \log n)$ time in the bottleneck case. No efficient algorithm is known for the corresponding sum TSP.
The second class contains TSPs the cost matrix of which fulfill special algebraic properties, for example

$$\max\{c_{tu}, c_{vs}, c_{sw}\} \le \max\{c_{ts}, c_{su}, c_{vw}\} \text{ for all } 1 \le t,u,v < s < w \le n, \; t \ne u, \; t \ne v.$$

In this and similar cases there exists an optimal tour which is pyramidal and can therefore be determined in $O(n^2)$ steps

Rainer E. Burkard
TU Graz

Euclidean matching, convex hulls, rural postman: a software demonstration

We discuss and demonstrate three computer programs implementing the following algorithms:

- an $O(n \log n)$ heuristic for Euclidean perfect matching of n points in the Euclidean plane (jointly with W. R. Pulleyblank)

- an $O(n)$ expected time algorithm for determining the convex hull of n uniformly distributed points in the unit circle (jointly with G. Reinelt)

- a rural postman heuristic applied to the problem of plotting masks for real world printed circuit boards (jointly with M. Grötschel and G. Reinelt)

Michael Jünger
Universität Augsburg

A supercomputer algorithm for the 0.1 multiknapsack problem (joint work with Gérard Plateau).

We exploit the characteristics of parallel machines (vectorization, multiprocessing) in order to solve the 0.1 multiknapsack problem;
- in a first phase, a lot of tests are performed in parallel in order to reduce the size of the problem (fixation of variables, elimination of constraints)
- in a second phase, a parallel Branch and Bound algorithm allows to get an optimal solution.

Our parallel algorithm has been implemented on the asynchronous multiprocessor machine CRAY 2. Computational results on litterature's examples are reported and compared with those obtained in a sequential approach.

Catherine Roucairol
Université Paris VI
and Inria

Bicriterial Minimum-Cost Flows - Complexity and Algorithms

The problem to determine efficient (Pareto-) minimal solutions for bicriterial minimum-cost flows is considered. It is shown that the number of efficient extreme point solutions in the objective space may be exponential. Among the algorithms, ε-optimality is investigated in more detail.
A subset $S \subset X$ of feasible solutions is called ε-optimal

w.r.t. a vector valued function $f: X \to Y \subset \mathbb{R}^K$ if $\forall x \in X \; \exists z_x \in S: \; f_k(z_x) \leq (1+\varepsilon) f_k(x) \; ; k = 1, \dots, K.$
A pseudopolynomial method based on the lower approximation of the optimal points given by the sandwich algorithm is presented.
Numerical results on NETGEN-generated examples are discussed.

Günther Ruhe
Technische Hochschule Leipzig.

A cutting plane algorithm for the design of minimum cost survivable networks

We designed a cutting plane algorithm for finding a minimum cost "survivable" network in a sparse graph with connectivity constraints on the nodes (the maximal connectivity required here is 2)
We study some valid inequalities for the associated polytope. For one class of inequalities (the partition inequalities) we devised a separation heuristic based on the Gomory-Hu method.
Problem sizes can be reduced considerably by decomposition if the underlying graph is sufficiently sparse.
Computational experience on three real world LATA-networks shows that the cutting plane algorithm provides good lower bounds (within 1.5% of the optimum), and in each of the three cases the optimum was attained by adding interatively cuts from the known classes of inequalities.

Mechthild Stoer
Universität Augsburg

Speeding up parametric min-cost flow problem algorithms

Most of the time in solving a min-cost flow problem by the primal method is spent during the last few pivot step, when most of the ~~edges~~ arcs have to be scanned until a candidate for entering arc, i.e. an arc with negative reduced costs is found. For the last step, the optimality check, all arcs have to be examined.
In the parametric problem, where the costs are linear functions of one parameter and the objective function is to be determined for a range of values. In that case, essentially every pivot step is an „optimality check", and all arcs are scanned in order to find the next critical value of the parameter. We can avoid this by placing arcs whose costs are currently very high into a „bucket" where they remain until some time later and need not be examined every time.
Preliminary computational tests are very encouraging.*
Further tests must be carried out to find the best way of organizing the buckets. The idea is possibly also useful for other problems, e.g. ordinary network flow problems.

* (achieving reductions of computation time to less than 50%)

Günter Rote
Freie Universität Berlin
und Technische Universität Graz

VORONOI – TREES AND CLUSTERING PROBLEMS

A new data-structure – Voronoi trees – is introduced, which represents proximity properties in a general framework very efficiently.
Structural properties are analyzed.
Applications to the layout of flexible manufacturing systems as well as to some pattern recognition and image understanding problems are demonstrated and experimental results are reported.

Hartmut Noltemeier
Universität Würzburg

Matching Problems with side-constraints
(joint work with M. O. Ball)

We present a procedure for solving the constrained matching problem

$$\min \{ c(X) \mid X \text{ perfect matching}, \ Sx \leq t \}$$

and we discuss computational results for a set of real-world problems with a number of so-called "generalized upper bound"-constraints.
The procedure runs in three phases.
(1) Solve the Lagrange dual / LP-dual
(2) Use pairs of feasible / infeasible matchings from phase 1 to construct better feasible matchings via a so-called Ungrad-heuristic
(3) Close the gap between the lower bound from phase 1 and the upper bound by constructing the sequence of k-best matchings

for the problem

$\min\{c'x + \bar{\lambda}(Sx - t) \mid x \text{ perfect matching}\}$

with $\bar{\lambda}$ the optimal Lagrange Multiplier

Ulrich Derigs
Universität Bayreuth

Design of Minimum Cost Survivable Networks – Practical Applications

In this talk I describe certain practical problems arising in the design of fiber optic communication networks. These problems can be modeled as ~~a~~ minimum-cost network design problems with costs on links subject to certain connectivity requirements. A tool based on graphics, user interaction and fast heuristics where used to produce designs superior to ~~those~~ manual approaches currently used by network planners. Work on a polyhedral approach to these problems and computational results will also be presented at this conference.

A Polynomial Approximation Scheme for Parallel Machine Scheduling with Release + Due Dates

We show that for any $\rho > 1$, $\exists$ a algorithm that runs in polynomial time, and compute a schedule of length at most ρ times the optimum for the following problem: there are m identical machines on which n independent jobs are to be scheduled; each job j may scheduled only after its release time r_j and must be processed for p_j on one of the machines; ~~and following, pro~~ there is a delivery time q_j (that plays the role of the due date) + if a job is completed at time C_j, it is delivered at $C_j + q_j$; the objective is to minimize the maximum delivery time.

David Shmoys (MIT)

Generalized cost scaling

We describe a framework for designing minimum-cost flow algorithms based on using the maximum violation of complimentary slackness conditions as a mesure of quality of a circulation. This framework is based on better understanding of combinatorial structure of the problem, and leads to good polynomial-time bounds, as well ~~at~~ as to strongly polynomial bounds. Namely, we obtain an $O(nm \log \frac{n^2}{m} \min\{\log(nC), m \log n\})$-time bound, where n is the number of vertices, m is the number of arcs, and C is the biggest cost in the input problem.

Andrew Goldberg (Stanford)

A Production Planning Problem Solvable by Network Flow

We show how a lot-sizing problem that arises in production scheduling can be solved by a single network flow calculation in a directed graph of small size. The principal tool for recognizing and achieving the transformation is planar graph duality. It is shown that the linear programming dual of a cographic linear programming problem is graphic, and that, although most instances of the lot-sizing problem are not graphic, every instance is cographic.

Robert Bland (Cornell U.)

Fast heuristics for the symmetric traveling salesman problem

Very often in practice it is the case that approximate solutions for large scale traveling salesman problems have to be computed very fast. In such cases the usually used heuristics cannot be applied directly because they are too CPU time consuming. We present several methods exploiting underlying geometric structure for constructing sparse subgraphs and show how heuristics for computation of upper and lower bounds can be adapted to run efficiently. We present computational results and give a software demonstration.

Gerhard Reinelt
(Augsburg)

Capacity Expansion in Local Access Telecommunications Networks
(with A. Balakrishnan and R. Wong)

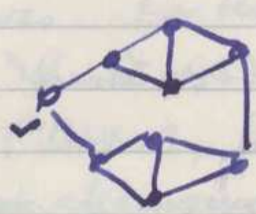
Telephone networks typically contain several components: a local area network that connects end users of the system, a switching network that connects a local geographical region, and a backbone network that carries long distance traffic. We consider a large scale mixed integer program that models capacity expansion in a local area tree network. Using a dynamic programming algorithm to solve a special version of the problem as a subproblem for a Lagrangian relaxation decomposition method, we solve several problems arising in practice: two to optimality and one to within 2.3% of optimality.

Thomas Magnanti
MIT

Adjacency Coloring: The smallest partially hard-to-color graph

An adjacency-coloring of a graph $G = (V, E)$ is a sequential coloring induced by some ordering $(v_1, \dots, v_n)$ of V for which all the subgraphs $G(\{v_1, \dots, v_i\})$ $1 \le i \le n$, are connected. A graph G is said to be hard-to-color from start-point $v \in V$ if all adjacency-colorings of G with start in v $(= v_1)$ need more than the chromatic number $ch(G)$ of colors. In this context v is called a bad start-point. G is called partially hard-to-color (a phc-graph) if there exists at least one bad start-point. G is minimal with respect to this property if there is no phc-graph having less vertices than G.

Our main result is: There is no phc-graph on less than 10 vertices except the one in the figure below. This graph has unique bad start-point. G is the unique minimal phc-graph.



Gottfried Tinhofer,
München

A Precedence Constrained Travelling Salesman Problem and Helicopter Routing

The following version of the travelling salesman problem arises when scheduling helicopters between drilling platforms, in an off-shore field. We have certain platforms which must be visited and pickups and deliveries which must be made between certain platforms. We wish to find a route which satisfies these requirements and has minimum length.

We discuss the problem, describe ~~an en~~ heuristic method which proves to be very good in practise, and present an integer programming formulation suitable for use in a cutting plane code.

This work is being done jointly with Egon Balas and Marie Timlin

W.R. Pulleyblank
Waterloo

Visualizing Combinatorial Algorithms

In this talk we present a software system for visualizing combinatorial algorithms. The idea of the system is to make the data structure as well as the underlying geometrical idea of the algorithms visible to the user and to use (interactively) heuristics as well as algorithms for a relaxation to give approximate solutions to NP-Hard problems.

The system includes features such as interactively designing graphs, applying graph operations, and has been developed so far for Shortest Path, Network Flow, Matching-, Node-Coloring-, Face-Coloring-, Vehicle-Routing-, Job-Shop-Scheduling-, Acyclic-Subgraph- and Chip-Design-problems.

Achim Bachem, Köln

On the efficiency of the Goldberg/Tarjan preflow algorithm: A brief report.

We present results of computational experiments with three fundamentally different algorithms for the maximum flow problem: the steepest edge network simplex method (Goldfarb/Grigoriadis 88), Dinic's method of blocking flows, and the recent preflow algorithm of Goldberg and Tarjan. The set of problem instances used for the experiment are those generated by RMFGEN (Grigoriadis '86) maximum flow problem generator, of sizes up to about 150,000 vertices and 850,000 arcs. For these problems, the preflow algorithm runs 4 to 15 times faster than the Dinic algorithm (DNSUB, Goldfarb/Grigoriadis, Annals of O.R. 1988). The steepest edge simplex algorithm runs faster for problems smaller than 4,000 vertices but is slower than the other two methods for larger problems.

Michael D. Grigoriadis
Rutgers University
New Brunswick, NJ, USA.

An $O(n^{1.5} \log n)$-time approximation scheme for matching points in the plane.

The problem is to compute a minimum-weight perfect matching of $2n$ points in the plane, assuming Euclidean edge weights. We propose a polynomial-time approximation procedure which consists of the following three steps: 1) Compute the Delaunay triangulation D of the given pts, 2) Compute an optimal perfect path matching of D (an odd-degree spanning forest F) and 3) Traverse F to construct the required matching. The entire procedure runs in $O(n^{1.5} \log n)$ time and produces matchings of weight a ~~at~~ most $\simeq 3$ or 5 times the optimal weight if D is computed in Step 1 w.r.t. the L_1 or L_2 norm, respectively. These bounds are not tight. Computational experiments for 120,000 instances of 8 to 1024 point sets, uniformly distributed in the plane, show the error to be almost insignificant.

Michael D. Grigoriadis
Rutgers University
New Brunswick, NJ, U.S.A.

The PLEXUS Linear Programming System

The PLEXUS LP system was described. It is a simplex based, designed to be portable, usable by a wide range of users, and particularly convenient for use as a collection of subroutines callable by integer programming applications.

Computational results ~~ohr~~ were reported showing PLEXUS to be about equally fast with MPSX /370 on a 3090/300E.
The code is written in C.

The nonnumerical parts of the code were jointly designed with Andy Boyd.

Robert E. Bixby
Rice University

On the Quality of Greedy-Algorithms for Solving the Subset-Sum-Problem from a Probabilistic View

We deal with heuristics for solving the problem
Max $\sum_{i=1}^{n} a_i x_i$ s.t $\sum_{i=1}^{n} a_i x_i \leq b$ where $x_1,..,x_n \in \{0,1\}$ and where $a_1,..,a_n, b \in \mathbb{R}^+$ are given.

When such a heuristic yields a feasible combination, then the realized objective may differ (be less) from the optimal value. This "Error" is bounded by the "Gap" which is defined as the difference between b (capacity) and realized load (sum).
Under the continuous stochastic model:
$a_1,..,a_n,b$ independent; $a_1,..,a_n$ uniformly distributed on $[0,1]$, b uniformly distributed on $[0,n]$
we calculate the conditional $(\sum a_i > b)$ gap-distribution and expected gap for seven Greedy-algorithms.
4 of them are On-line algorithms, 3 use sorting.

Karl Heinz Borgwardt
Universität Augsburg

On a class of perfect graphs and a channel routing problem *

We present a polynomial algorithm that solves a problem due to Dagan, Golumbic and Pinter [1988] about the complexity of recognizing trapezoid graphs. These graphs constitute a class of perfect graphs that naturally arise in VLSI - channel routing problems. The algorithm exploits the facts that trapezoid graphs are the incomparability graphs of partial orders of interval dimension two (i.e. the intersection of two interval orders) and that interval dimension is a comparability invariant (i.e. the same for all partial orders with the same comparability graph). It thus suffices to test if a transitive orientation of the complement of the given graph (if it exists) has interval dimension two. This is done via substitution decomposition and PQ-trees.
(* jointly with M. Habib, Montpellier)

Rolf H. Möhring, TU Berlin

A new lower bound for the quadratic assignment problem (QAP)

The QAP is formulated as a matrix optimization problem on the set of permutation matrices. We relax the domain by minimizing over the intersection of orthogonal matrices with the set of matrices having row and column sums equal one. The corresponding relaxed problem can be solved by a spectral decomposition of the input matrices (which are assumed to be symmetric). Preliminary numerical results suggest that this bound is better than existing ones.

Franz RENDL, TU Graz (jointly with H. Wolkowicz, Waterloo)

Routing in graphs

We describe a polynomial-time algorithm for the following problem arising in the design of VLSI-circuits

given: a planar graph $G=(V,E)$, faces $I_1,\dots,I_p$ of G (including the unbounded face), and curves $C_1,\dots,C_k$ in $\mathbb{R}^2\setminus(I_1\cup\dots\cup I_p)$ with end points on the boundary of $I_1\cup\dots\cup I_p$

find: pairwise vertex-disjoint simple paths $P_1,\dots,P_k$ in G where P_i is homotopic to C_i in the space $\mathbb{R}^2\setminus(I_1\cup\dots\cup I_p)$, for $i=1,\dots,k$.

The algorithm is based on a polynomial-time algorithm for finding an integer solution for a system $Ax\le b$ of linear inequalities, where A is an integer matrix satisfying

$$\sum_{j=1}^{n} |a_{ij}| \le 2 \qquad (i=1,\dots,m).$$

We also describe an extension to finding disjoint trees connecting given *sets* of points.

Mathematical Centre, Amsterdam
& Tilburg University

A. Schrijver

Design of Minimum-Cost Survivable Networks:
IP-Models and Polyhedral Investigations

In this talk we present a general integer linear programming model for the problem of designing minimum-cost survivable networks (introduced in the lecture by C. Monma). We relate this model to concepts in graph theory and polyhedral combinatorics. In particular, we consider several interesting special cases of this general model including the minimum spanning tree problem, the Steiner tree problem, and the minimum cost k-edge connected and k-node connected network design problems. We study the integer polyhedra associated with these problems and identify some classes of facets of these polyhedra. We also address the separation problems with respect to these classes of facet-defining inequalities.

Martin Grötschel, Augsburg

~~[illegible]~~

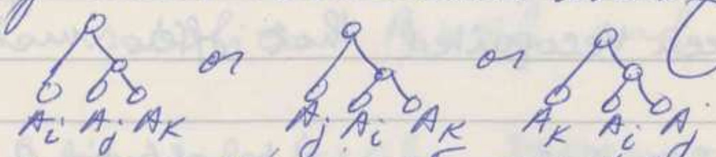
Solving large zero-one integer programming problems on distributed workstations

Commercial workstations connected by an industry-standard Ethernet network have been used to solve large-scale zero-one integer programming problems using a cutting-plane method ~~which~~ based on the polyhedral structure of the zero-one polytope. These cutting planes are embedded in a tree-search strategy that uses logical implications, heuristics, reduced-cost fixing and facial cuts to tighten the bounds at every node. The parallel implementation ~~uses~~ generates a "pool" of facial cuts which are shared by all processors. Adaptive suspension and resumption of search nodes has been implemented. The largest problems are solved with super-linear speed ups.

Karla Hoffman, Fairfax VA

Two Problems in Connexion with Rooted Trees
(In imitation of title of a paper by E. Dijkstra.)

Problem 1: A phylogenetic tree is a rooted binary tree whose leaves are labelled with "species" $A_1, A_2, \ldots, A_N$. The leaves of a subtree constitute a "phylum." Suppose, for each triple of species, A_i, A_j, A_k, one can perform an experiment to determine which of three relations holds:



How many experiments must be performed in order to determine the phylogenetic tree for N species?
Answer: $\Theta(N \log N)$.

Problem 2: The structure of a program in an Algol-like language is represented by a rooted tree in which each node represents a procedure. This tree may be augmented by two kinds of arcs:

(1) "blue" arcs extending from a node to one of its ancestors. These arcs represent references to variables.

(2) "red" arcs extending from a node to one of its ancestors, or to a child of an ancestor. These arcs represent procedure calls.

The problem is to "flatten" the tree, moving each node as close to the root as possible, while maintaining conditions (1) & (2). This can be done by an algorithm with $O(N + M\,\alpha(M,N))$ running time, where N is the number of nodes and M is the number of arcs.

Eugene L. Lawler, Berkeley

Lifting of polyhedra

Let P be a combinatorially ~~different~~ described polytope (e.g. matching, or travelling salesman), having an exponential number of facets. It has been recognized that often more complete linear descriptions can be obtained at the cost of introducing new variables. During this meeting, Lex Schrijver and I discovered that if we allow non-poly~~nomial~~hedral (but convex and computationally easy) descriptions of the lifted polyhedron, then much larger classes of polyhedra become representable this way and thereby computationally tractable. The method is to consider a system $Ax \le b$ and associate with it the set P^* of matrices Y such that

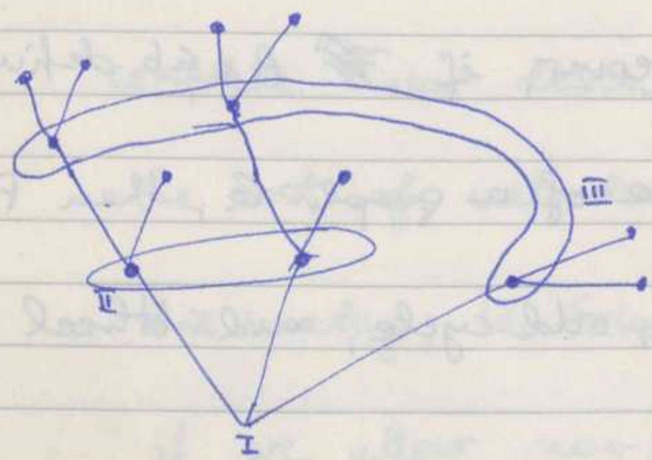
$AY \le b\bar{Y}^T$ and $Y - \bar{Y}\bar{Y}^T$ is positive semidefinite (where $\bar{Y}$ denotes the diagonal of Y). Then one can optimize any linear objective function over P' and hence also over ~~its~~ $P'' = \{\bar{Y} : Y \in P'\}$.

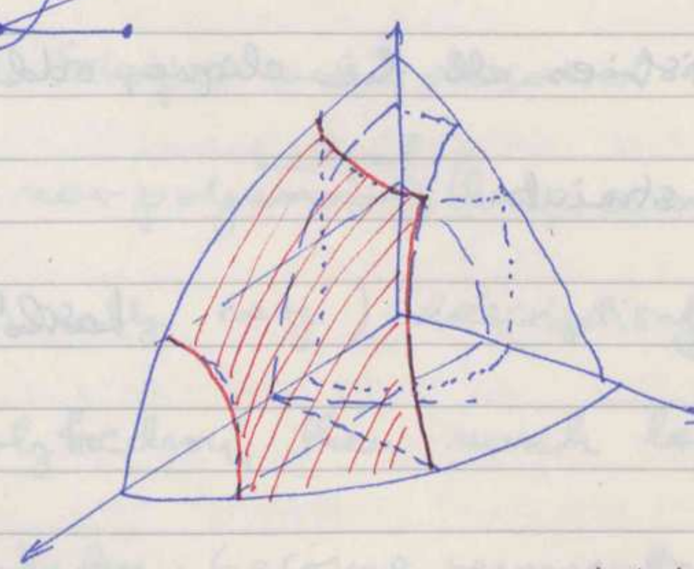
P'' satisfies $Ax \le b$ and contains all 0-1 solutions of $Ax \le b$. Moreover, if $Ax \le b$ defines the fractional vertex packing polytope of a graph G, then P'' satisfies all the clique, odd cycle, and wheel constraints.

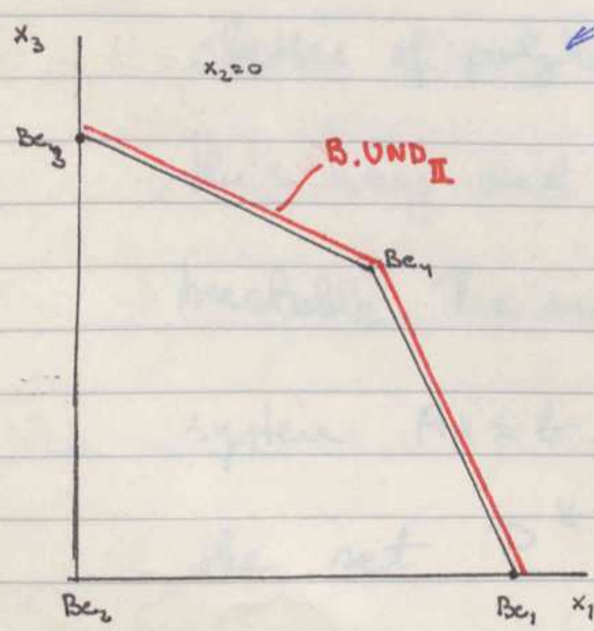
László Lovász, Budapest

Mathematical Game Theory

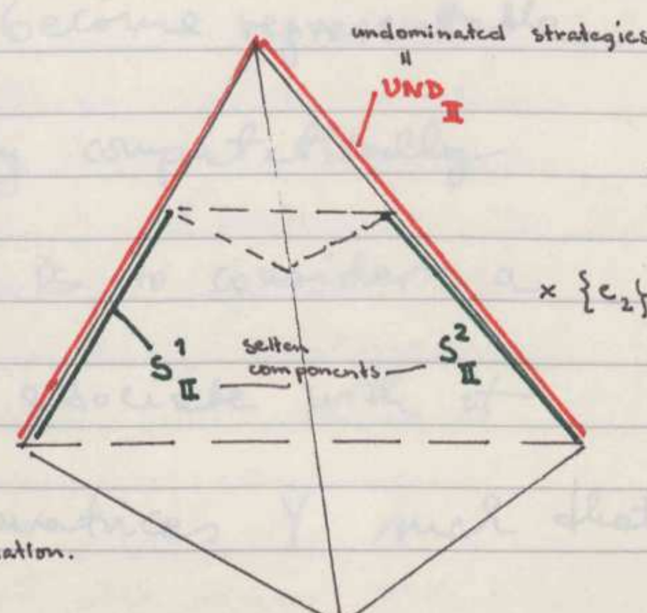
January 15. 16th – 21st, 1987







see page 81 for explanation.



Equilibrium Selection in the Spence Signaling Model

(Joint work with Werner Güth). We consider a simple version of the Spence job market signaling model, of which the data are as follows

Type	Productivity	Educ. Cost	Probability
0	0	y	$1-\lambda$
1	1	y/2	λ

The rules of the game are

- The worker (player 1) learns his type.
- The worker chooses an education level y.
- Two identical firms (the players 2 and 3) observe y, from that infer something about the worker's type, and then simultaneously offer wages $(w_2(y), w_3(y))$.
- The worker chooses a firm.
- The payoff to a worker of type t who gets the wage w after an investment y is $w - y/(t+1)$; a firm has zero profit if it does not attract the worker, the profit is $t-w$ if the firm attracts the type t worker with the wage w.

Aim of the paper is to find the Harsanyi/Selten solution of this game. It turns out that this solution is the E_1-equilibrium proposed by Charles Wilson, i.e. if $\lambda < 1/2$, the types separate (type t chooses $y = t$ and gets wage $w = t$), if $\lambda > \frac{1}{2}$ the solution is pooling at $y = 0$ (hence $w = \lambda$). The critical element of the proof is that HS gives preference to primitive equilibria (i.e. ones with 'minimal' support) of the ε-uniformly perturbed game. If $\lambda > \frac{1}{2}$ only the pooling equilibrium turns out to be primitive. For $\lambda < \frac{1}{2}$, there are many such equilibria and the solution is determined by risk dominance.

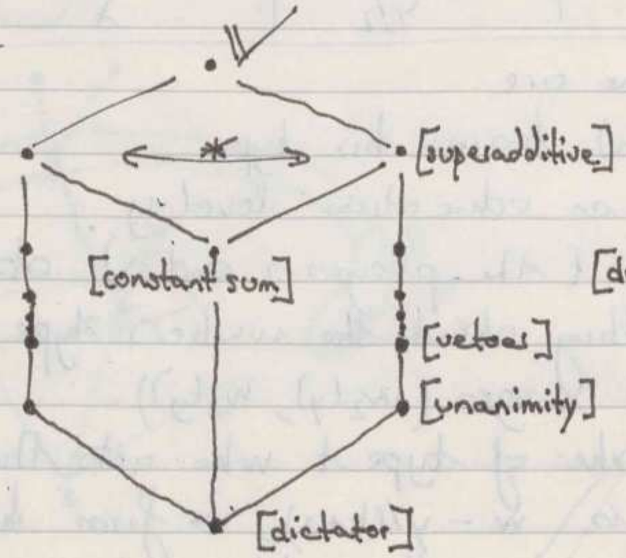
Eric van Damme, Bonn

Simple Games: On Order and Symmetry

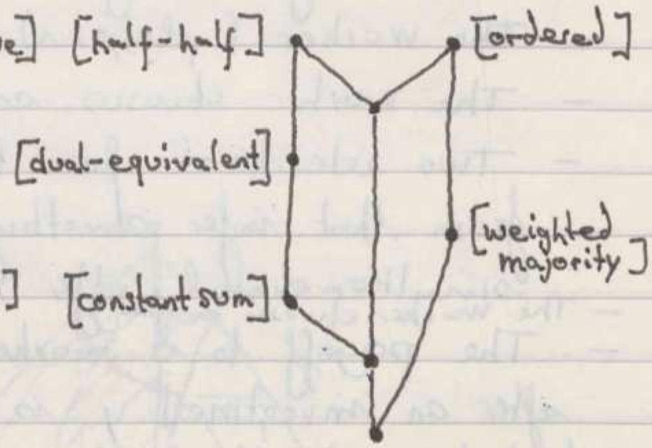
As a starting point Post's classification of boolean functions was applied to the class of (monotonic simple) games — call this class $\mathbb{V}$. Some basic facts on the Post-classes of games were reported. Let $[p] := \{v \in \mathbb{V};$ property p holds for $v\}$ and denote duality by $*$.

Proposition i $(i=1,2)$: Inclusions and intersections can be seen from the diagramm i.

diag.1



diag.2



Examples were given that separate these sets.

Next the automorphism group of a game and (sharp) τ-transitivity of a game were introduced.

Proposition 3: [transitive and ordered] $\subset$ [weighted majority] = ~~[aut v = S_n]~~
[aut $v \neq S_n$]

there are few highly transitive games not weighted majority. they can be constructed by group theoretical tools.

there exist only **13** games sharply τ-transitive not weighted majority for $\tau \geq 4$ (they are connected to Mathieu-groups, Witt-designs, cf. p. 8 this book) and they can be constructed by means of the PSL(2,11) — the number of players is 12 resp. 11.

Axel Ostmann
Saarbrücken

Majority voting in the Condorcet Paradox as a problem of equilibrium selection

Voting by majority is often viewed as undesirable since it can lead to cyclical majority decisions (Condorcet paradox). In general, there can be no transitive social ordering of alternatives based on majority decisions. Here, we do not follow the welfare theoretic attempt to derive a transitive social ordering but rather consider the situation as a game where agents select among alternatives by majority decisions. Of course, the phenomenon of cyclical majorities entails the fact that such a game has more than just one equilibrium point. But by applying the theory of equilibrium selection, one nevertheless can solve the game uniquely and thereby determine a unique public decision.

To illustrate our approach, we consider the most simple form for the so-called Condorcet paradox with 3 alternatives and 3 agents. Agents assign cardinal utilities to alternatives including the status quo which results if none of the proposals is accepted. It is an interesting fact that the set of uniformly perfect equilibrium points depends crucially on cardinal utilities although they always imply the same cyclical majorities. Furthermore, the status quo will only sustain in degenerate cases. In other words: the uniquely determined alternative is Pareto-optimal with probability 1. This indicates that the application of equilibrium selection to majority voting offers new ways to derive mechanisms of social choice. Since agents choose among alternatives and not among preference profiles, etc., such mechanisms are, in our view, much more in line with actual democratic decision processes.

Werner Güth + Reinhard Selten
Frankfurt Bonn

Reconstruction of Global Criterion

Let us consider a multicriterial decision-making problem with the set of acceptable decisions D and with ~~m~~ $n \geq 2$ criteria represented by utility functions $u_i : D \to \mathbb{R}$, $i = 1, \ldots, n$. There are two subjects influencing the choice of the decision. A decision-maker knowing the set D and all functions $u_1, \ldots, u_n$) and a manager having his own global utility function $u : D \to \mathbb{R}$ such that

$$u(d) = w_1 u_1(d) + \cdots + w_n u_n(d), \quad d \in D$$

$$w_1 + \cdots + w_n = 1, \quad w_i \geq 0, \; i = 1, \ldots, n.$$

Let us suppose that the decision-maker does not know the global utility function (i.e. the weights $w_1, \ldots, w_n$) but he aims to satisfy the manager's global demands.

For this purpose he obtains a comparative information about the global acceptability of his realised decisions. Namely, for every pair of obtained decisions $d, d' \in D$ he obtains the information whether

(*) $$u(d) \gtreqless u(d').$$

The presented contribution is devoted to the decision-maker's possibilities to estimate the weights $w_1, \ldots, w_n$ by choosing a few proper experimental decisions ~~with~~ $d^{(1)}, \ldots, d^{(m)}$. The main cases are distinguished, namely, the case when the information (*) is deterministic, and the case when it is vague, valid with some possibility $f(d, d') \in \langle 0, 1 \rangle$.

Milan Mareš
Prague

THE STRUCTURE OF THE SET OF PERFECT EQUILIBRIA
OF BI-MATRIX GAMES

The set of Nash equilibria of a bimatrix game differs considerably from the equilibrium set of matrix: it is not convex, in general, not a product set and there is no value.
This was the reason to define the Nash component of a bi-matrix game as a maximal convex subset of the equilibrium set $E(A,B)$.
The following results have been obtained for Nash components.

1. Nash components have a product structure : $N = N_I \times N_{II}$ where $N_I = \pi_I(N)$ $N_{II} = \pi_{II}(N)$.
2. Every subset $P = P_I \times P_{II} \subset E(A,B)$ is subset of at least one Nash component
3. The equilibrium set is a finite and irredundant union of the Nash components
4. The dimension relation $\# C_I(N_I) - \dim N_I = \text{rank } B|_{C_I(N_I) \times B_{II}(N_I)}$ and a similar relation for N_{II}

   $C_I(N_I) = \{ i \mid p_i > 0$ for <u>some</u> $p \in N_I \}$

   $B_I(N_{II}) = \{ i \mid e_i A q \geq e_k A q$ for <u>all</u> $q \in N_{II}$ and $k = 1, \dots m \}$
5. If $C_I \subset X_I$ (strategy set of player I) and $C_{II} \subset X_{II}$ are given, there is at most one Nash component $N_I \times N_{II}$ with $C_I(N_I) = C_I$ $C_{II}(N_{II}) = C_{II}$.

These are ~~the~~ results of Heuer, Millham, Winkels and Jansen (1974-1980)
We introduced the what we called, <u>Selten components</u> as maximal convex subsets of $PF(A,B)$, the set of perfect equilibria and proved that the properties 1, 3, 4 and 5 remain valid for Selten components
Moreover is $S = S_I \times S_{II}$ is a Selten component and N a Nash component containing S (always existing by 2) then

6. $S_I = N_I \cap \Delta_{C(S_I)}$ $S_{II} = N_{II} \cap \Delta_{C(S_{II})}$

Property 2 is no longer true for Selten components and also we cannot do the same for the set of proper equilibria. On page 76 we made a picture of some Selten components of the game

$$\begin{pmatrix} (-3,3) & (-3,0) & (-3,0) & (-3,2) \\ (3,0) & (3,0) & (3,0) & (-6,0) \\ (-3,0) & (-3,0) & (-3,3) & (-3,2) \end{pmatrix}$$

Jos Potters
Stef Tijs
Peter Borm
Mathijs Jansen
Nijmegen.

Theorems on Closed Coverings of a Simplex and Their Applications to Cooperative Game Theory

Let N, K be finite sets such that $N \subset K$, let $A := ((a_{ij}))_{i \in N, j \in K}$ be a $(\# N) \times (\# K)$ matrix such that

$$a_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \quad \text{for all } j \in N,$$

and let $\Delta^T :=$ convex hull of $\{\text{column } j \text{ of } A\}_{j \in T}$ for each $T \subset N$.

Theorem. Assume that $c \in \Delta^N$ and that the set $\{x \in \mathbb{R}^K_+ \mid Ax = c\}$ is bounded. Let $\{C^j\}_{j \in K}$ be a family of closed ~~sets~~ subsets of Δ^N such that

$$\forall\, T \in 2^N : \quad \Delta^T \subset \bigcup \{C^j \mid j \in K,\ a^j \in \text{affine hull of } \Delta^T\}.$$

Then there exists $x \in \mathbb{R}^K_+$ such that

$$Ax = c \quad \text{and} \quad \bigcap \{C^j \mid x_j > 0\} \neq \phi.$$

This theorem, its dual result, its extension are established. These theorems unify many of the theorems of the Knaster-Kuratowski-Mazurkiewicz type, including those of Scarf (1967), Fan (1968), Shapley (1973), Gale (1984) and Ichiishi (1988). Applications to cooperative game theory are also given.

Tatsuro Ichiishi, Columbus
and Adam Idzik, Warsaw.

DISCRIMINATORY VON NEUMANN-MORGENSTERN SOLUTIONS

In the context of cooperative games with side-payments, a discriminatory set is a collection of imputations representing the scenario where some players (the discriminated players) receive a fixed amount, and the group of remaining players (the bargainers) can split the rest in any way they like. These discriminatory sets appear frequently as von Neumann-Morgenstern solutions or as building blocks of vN-M solutions. The best known examples are the monotone simple games: every minimal winning coalition has a corresponding discriminatory solution that assigns 0 to each player outside the minimal winning coalition.

For arbitrary (0,1)-games this paper studies those discriminatory sets that are vN-M solutions. It turns out that the bargainers in any discriminatory vN-M solution form a minimal vital coalition (vital in the sense of Gillies) and the total amount available for the bargainers is smaller than or equal to the worth of the minimal vital coalition. Minimal vital coalitions for (0,1)-games are most easily described as minimal non-trivial coalitions with positive worth. Another result is that in case a discriminatory vN-M solution exists that assigns a positive amount to a discriminated player, then the Core of the game must be empty.

The main result of the paper is an effective characterisation to determine whether or not a proposed discriminatory set is a vN-M solution. Besides the above mentioned requirements regarding the group of bargaining players, the result also involves domination requirements for a finite set of competing discriminatory sets. These competing discriminatory sets have the same collection of discriminated players but now some of those players have lost their original allocations to the bargainers, so the competing discriminatory sets are more attractive to the bargainers than the original one. The domination requirement on a competing discriminatory set will not be fullfilled if and only if the Core of a certain attractive reduced game (for the set of bargaining players) is nonempty. The reduced game is very similar to the well-known Davis-Maschler reduced game.

Johannes (Hans) Heijmans
Univ. of Texas at Arlington.

A game-theoretical version of the maximum principle with discrete partially ordered time

1. Let we have a partial non-cooperative game
$\Gamma = \langle I, \{\mathfrak{X}_i\}_{i\in I}, \overline{\mathfrak{X}}, \{H_i\}_{i\in I}\rangle$
where $I=\{1,\dots,n\}$, $\overline{\mathfrak{X}} \subset \mathfrak{X} = \prod_{i\in I}\mathfrak{X}_i$ and $H_i : \overline{\mathfrak{X}} \to R_1$
Metrics in all $\mathfrak{X}_i$ produce metrics in all $\mathfrak{X}^i = \prod_{j\neq i}\mathfrak{X}_j$ and in $\mathfrak{X}$ as well as (Hausdorffian) metrics in all $2^{\mathfrak{X}_i}$.
For $x^i \in \mathfrak{X}^i$ we set $Z_i(x^i) = \{x_i : (x^i, x_i) \in \overline{\mathfrak{X}}\} \in 2^{\mathfrak{X}_i}$.
and label a $x^* \in \overline{\mathfrak{X}}$ as equilibrium of Γ iff
$$H_i(x^*) = \max_{x_i \in Z_i(x^{*i})} H_i(x^{*i}, x_i).$$
The set of all equilibria of Γ is denoted as $\mathscr{E}(\Gamma)$.

Theorem. Let in the partial game Γ all $\mathfrak{X}_i$ are convex compact subsets of linear topological spaces, $\overline{\mathfrak{X}}$ is also convex and compact in $\mathfrak{X}$, all correspondences Z_i are continuous, and all H_i are continuous in x and quasiconkave in x_i. Then the game Γ has equilibria ($\mathscr{E}(\Gamma) \neq \emptyset$).

2. The game Γ is said to be a production one if there ~~are~~ is a fixed structurized ressource $b=(b_1,\dots,b_n)\in R^n$ that is worked in a structurized production $x=(x_1,\dots,x_n)\in R^n$ under some fixed restriction $A(x) \leq b$; a set of such x's let be $\overline{\mathfrak{X}}$.

3. Let we have a finite oriented graph $\mathcal{G} = \langle J, G\rangle$ without loops and to every its vertex $j \in J$ is ascribed a production game Γ^j with the set of planers I. These games Γ^j are supposed to be coordinated in a natural manner: the parts x^{kj} of production x_i^k of Γ^k ($k \in G^{-1}j$) are identified with the parts of ressources b_i^{kj}; their gathering over $k \in G^{-1}j$ gives (together with the outside ressources) ~~b_i~~ the ressource b_i^j and the game Γ^j works $b^j = (b_1^j, \dots, b_n^j)$ into some

production $x^j = (x_1^j, \ldots, x_n^j) \in \bar{\mathfrak{T}}^j$ that is realized among the player of ~~the~~ games Γ^ℓ $(\ell \in G_j)$ as well as outside of the graph $\mathcal{G}$.

Uniting all games Γ^j for $j \in M \subset J$ we obtain a production game Γ^M that corresponds to the sub graph $\mathcal{G}^M = \langle M, G_M \rangle$:

$$\Gamma^M = \langle I, \{\mathfrak{T}_i^M\}_{i \in I}, \bar{\mathfrak{T}}^M, \{H_i^M\}_{i \in I} \rangle$$

If $M_1 \subset M$, the strategies x_i^M and their n-type x^M in the game Γ^M have, as their natural projections, strategies $x_i^{M_1}$ and their n-tuple x^{M_1}.

Theorem. 1° If $x^{*M} \in \mathcal{C}(\Gamma^M)$ and $M_1 \subset M$, than $x^{*M_1} \in \mathcal{C}(\Gamma^{M_1})$.

2°. If $x^{*M} \in \bar{\mathfrak{T}}^M$, $M = M_1 \cup M_2$ $(M_1 \cap M_2 = \emptyset)$ $x^{*M_1} \in \mathcal{C}(\Gamma^{M_1})$ and $x^{*M_2} \in \mathcal{C}(\Gamma^{M_2})$, then $x^{*M} \in \mathcal{C}(\Gamma^M)$.

Nicolai N. Vorob'ev
Leningrad, USSR.

Properness, balancedness and the nucleolus.

We define the nucleolus of a continuous convex map $F : X \to \mathbb{R}^m$ on a compact set X. As special cases we obtain known notions as nucleolus, prenucleolus and weighted nucleolus of a TU-game with (or without) coalition structure. Also the nucleolus of a matrix game turns out to be an interesting special case. It appears that the nucleolus of a matrix game coincides with the set of Dresher optimal strategy pairs of the game. This implies that the nucleolus consists precisely of the proper equilibria of the matrix game. To each 0,1-normalised TU-game one can construct a matrix game, which we call the excess game, such that the nucleolus of the TU-game coincides with the unique proper optimal strategy of player 2 in the excess game. Also for other nucleoli of TU-games a suitable matrix game can be constructed, where the respective nucleolus is related to the nucleolus of the matrix game. A balancedness condition is given characterizing nucleolus elements of a matrix game. It is shown that this balancedness result implies again the known balancedness characterizations of Kohlberg, Sobolev, Owen and Wallmeier.

Stef H. Tijs
Jos A.M. Potters
University of Nijmegen
The Netherlands

The compensatory bargaining set of a cooperative n-person game with side payments

The Aumann/Maschler definition of a bargaining set relies upon a stability principle imposed to the payoffs in this set: an admissible payoff belongs to a bargaining set if for every objection against this payoff, if any, there is a counter objection. Two modifications of the stability principle have been discussed in earlier papers of the author (Dragan, 1985, 1987, 1988). The present paper is considering another modification: an objection is valid only if the players who intend to move to new coalitions agree upon a prior commitment, namely that of compensating all partners who join the venture, in case of failure due to a subsequent move. The mathematical description of the model is given in the first section, where the new stability principle and the corresponding "compensatory" bargaining set M_c is defined. A feasibility theorem for the existence of a flow in a bipartite network associated to a payoff and two partial coalition structures is derived in the second section from a similar theorem by D. Gale (1957). The result is used in the third section for proving a combinatorial characterization of the non core payoffs belonging to the compensatory bargaining set. In the last section, in the set of such payoffs $M_c - C(G)$ for a 3-person game the subset of $M_c - C(G)$ consisting of individually rational payoffs is found. This subset is compared with bargaining set M of Aumann/Maschler (1964) for the same game, in order to illustrate the particularities of the new model by a comparison with a well known one.

Irinel C. Dragan
University of Texas
Dept. of Mathematics
ARLINGTON, TEXAS 76019, U.S.A.

1-17-1989

On a definition of excess in the games without sidepayments.

An axiomatic characterisation of excess relation on the set of pairs of coalitions and individually rational payoff vectors in n-person game without sidepayments is given. This relation enables to define a unique nucleolus of n-person cooperative game without sidepayments. For the sidepayment cooperative games the normalized excess functions are the utility functions representing this relation.

Elena Yanovskaya
Institute for Socio-Economic Problems Acad. Sci.
Leningrad, USSR

On Coalition Formation

(A joint work with B. Peleg)

Given a society who believes in a certain solution concept, say the nucleolus, there still is a strategic aspect while playing the game, because once a coalition forms, the game changes. So coalitions may want to rush into forming coalitions and others prefer to wait. Some players may want to leave the arena of negotiation for a while, while others would rather prefer them stay. Thus, some players may be willing to pay others to encourage them to form, or to leave or just to stay. How can one treat such situations systematically? The research presented here offers a solution to this problem. Example were given, which show that the suggested solution does indeed give intuitive prescriptions.

Michael Maschler
Department of Mathematics
The Hebrew University of Jerusalem

19-1-89

N-matrices and Univalence

(A joint work with C. Olech and G. Ravindran)

An N-matrix is a square matrix with real entries whose principal minors are negative. This concept was introduced by Inada in connection with production matrix and Stolper-Samuelson condition. Our purpose is two-fold (i) To characterize N-matrices (ii) To prove new univalence results. It is known that the inverse of an N-matrix is an almost P-matrix. We prove among other results the following: If F is a C^1 differentiable map from R^n to R^n with its Jacobian an almost P-matrix (inverse of an N-matrix) for every $x \in R^n$ then F is globally one to one in R^n. Our proof depends on the K-K-M theorem

T. Parthasarathy
Indian Statistical Institute
New Delhi - 110016,
INDIA

19-1-89

The Existence of Markov-Perfect Equilibria for Infinite-Action Games of Perfect Information

(a joint work with Martin Hellwig)

Many economic models specify decision variables (like prices or quantities) as continuous (rather than discrete) variables. Game-theoretic analysis of such models is thus required to consider infinite-action games.

The present paper investigates a broad class of perfect information games with infinite action spaces for which existence of subgame-perfect equilibrium in general history-dependent strategies has been established before. It amends those games by a Markovian state-structure and poses the general question under what additional assumptions existence of a subgame-perfect equilibrium sustained by Markov-strategies, that only condition on the present state and not on the entire past history, exists. Such equilibria are called Markov-perfect.

The answer to this question is given by set of assumptions on the dynamic structure of the game and the pay-off functions of players which is sufficient to ensure existence. They are also shown to be necessary in the sense that dropping ~~out~~ any single one them leads to the emergence of counterexamples which do not possess a Markov-perfect equilibrium.

Wolfgang Leininger
Universität Bonn
Wirtschaftstheorie III
5300 Bonn 1

Developments in Stable Set Theory

The von Neumann-Morgenstern theory of solutions (stable sets) and its extensions, as well as various bargaining sets and nucleoli, are among the most descriptive solution concepts for n-person cooperative games, especially in the case when the core is the empty set. There are also very interesting connections between the bargaining set $\mathcal{M}_1^{(i)}$ and the "symmetric-type" solutions. Some recent advances in solution theory also include new classes of finite solutions, insights into the structure of symmetric solutions (including the characterization for all symmetric solutions for all four-person games by H. Heijmans), and the characterization of discriminator solutions for symmetric games. Many interesting interpretations and insights for game experiments follow from these new theoretical results.

William F. Lucas
Claremont Graduate School
Claremont, CA 91711, U.S.A

Cooperation and Bounded Recall
(joint work with R.J. Aumann)

A two person game has common interests if there is a single payoff pair z that strongly Pareto dominates all other payoff pairs.
Assume such a game is repeated many times and that each small attaches a small but positive probability to the other playing some fixed strategy with bounded recall rather than playing to maximize his payoff.
Then the resulting supergame has an equilibrium in pure strategies, and the payoffs to all such equilibria are close to optimal (ie to z).

Sylvain Sorin
Departement de Mathématiques
Université Louis Pasteur
Strasbourg.

Equilibrium price in the one-product market.

A model of n-person market is following. Let $I=\{1,2,\dots,n\}$ be the set of players where $M=\{1,2,\dots,m\}$ is the set of producers and $N=\{m+1,\dots,n\}$ is the set of consumers of one indivisible comodity. Each $i\in M$ produces a_i units of this and each $j\in N$ demands for b_j units. The utility of unit is u_i for i-producer and w_j for j-consumer. The distribution is a vector (ξ,η): $\xi=(\xi_1,\dots,\xi_m)$, $\eta=(\eta_{m+1},\dots,\eta_n)$, where $0\le\xi_i\le a_i$, $i\in M$, $0\le\eta_j\le b_j$, $j\in N$ and $\sum_{i\in M}\xi_i=\sum_{j\in N}\eta_j$. Denote the set of all distributions $D(I)$, for $S\subset I$ put $D(S)=\{(\xi,\eta)\in D(I): \xi_i=0, i\notin S, \eta_j=0, j\notin S\}$. In distribution $(\xi,\eta)\in D(S)$ the coalition S have a profit $v(S,\xi,\eta)=\sum_{j\in S\cap N} w_j\eta_j-\sum_{i\in S\cap M} u_i\xi_i$. Put $v(S)=\max_{(\xi,\eta)\in D(S)} v(S,\xi,\eta)$. Function v describes the super-additive cooperative game $\Gamma=\langle I,v\rangle$ with non-empty core. The distribution $(\bar\xi,\bar\eta)\in D(I)$ is optimal iff $v(I,\bar\xi,\bar\eta)=v(I)$.

If p is the price of the unit of commodity, then the coalition S have the profit $x(S,p,\xi,\eta)=\sum_{i\in S\cap M}\xi_i(p-u_i)+\sum_{j\in S\cap N}\eta_j(w_j-p)$ in distribution $(\xi,\bar\eta)$. The price p is equilibrium one iff there exists such $(\xi,\eta)\in D(I)$ that $x(S,p,\xi,\eta)\ge v(S)$, $S\subsetneqq I$.

1. The optimal distribution is constructed.
2. It is proved that alredy exists such price $\bar p$, that $\bar p$ is equilibrium withe the optimal distribution $(\bar\xi,\bar\eta)$.
3. For the portial case $u_i=0, i\in M$, $w_j=1, j\in N$, $M=\{1\}$ the Shapley value and Nucleolus are calculate.

O.N. Bondareva
Leningrad State University
198904 Leningrad, USSR.

Self-Optimality and Efficiency

In a game model where each player has privat information about her type, a vector of reported types is called self-optimal if it would constitute a Nash equilibrium given that the reported types were the true types. We explore the relationship between the self-optimality concept and the incentive compatibility concept. We apply the self-optimality concept to a utility distortion game in the context of bargaining and obtain a characterization of efficient Nash equilibria.

Hans Peters
University of Limburg
Maastricht, The Netherlands

On The Existence of Equilibria in a Class of Discrete-Time Dynamic Games with Imperfect Information.

We examine the question of existence of subgame perfect equilibrium points in discrete-time dynamic games with infinite-action spaces which allow players to move simultaneously at each period. The previous literature on discrete-time dynamic games (or infinite extensive games) gave us results for games with perfect information; a situation in which simultaneous moves are ruled out. We show that when ~~one~~ restricts oneself to just continuity assumptions on the feasible action correspondences and the payoff functions we can find examples of games which do not have perfect, and therefore, sequential equilibrium points. We then show that if we allow for behavior strategies, and if one defines behavior strategies in the right way, then the behavior strategies define probability distributions over outcomes and one can then define the associated payoffs from the behavior strategies as the expected payoffs, even when the action spaces are infinite. The result is obtained by approximating the original game by finite-action games in the right way and using the equilibrium strategy combination of the finite game to define the ε-perfect equilibrium point of the original game.

If restrictions are placed on the strategy space then one can guarantee the existence of subgame perfect equilibrium points.

SUBIR K. CHAKRABARTI
DEPT. OF ECONOMICS
INDIANA UNIVERSITY
INDIANAPOLIS, IN 46202.

Resale-Proofness and Coalition-Proof Nash Equilibria

Information is freely replicatable. Thus, in trading information, a possibility of resales (of replicas) seems to be unavoidable unless resales are legally prohibited. A notion of resale-proofness was recently proposed by Nakayama, Quintas and Muto: it characterizes an information sharing pattern in which resales are never carried out even if they are freely allowed.

This study has two objectives: the first is to describe explicitly a trading manner of information and its game form; and the second is to reconsider the resale-proofness in the framework of thus described information trading game.

A main result is that resale-proof information sharing patterns are attained as equilibrium outcomes of the information trading game: perfectly coalition-proof Nash equilibria due to Bernheim, Peleg and Whinston.

Shigeo Muto
Faculty of Economics
Tohoku University
Kawauchi, Sendai 980
JAPAN

The coincidence of the prenucleolus and the ENSC-solution

Let (N,v) be a cooperative n-person game in characteristic function form. The smallest contribution of coalition $S \subset N$ with respect to the formation of $(n-1)$-person coalitions in the n-person game v is defined to be

$$m^v(S) := \min\left[\, v(N-\{j\}) - v((N-\{j\})-S) \;\middle|\; j \in N-S \,\right]$$

for all $S \subset N$, $S \neq N$,

$$m^v(N) := v(N).$$

Let the set $U(v)$ consist of efficient payoff vectors that give rise only to payoffs not greater than the relevant smallest contributions for all coalitions containing at most $n-2$ players. To be exact,

$$U(v) := \left\{ x \in \mathbb{R}^n \;\middle|\; \sum_{i \in N} x_i = v(N) \text{ and } \sum_{i \in S} x_i \leq m^v(S) \text{ for all } S \subset N \text{ with } 1 \leq |S| \leq n-2 \right\}.$$

The interrelationships between the set $U(v)$ and several solution concepts (e.g., the prekernel and the prenucleolus) are studied. The main results are as follows.

Firstly, an efficient payoff vector $x \in \mathbb{R}^n$ belongs to the set $U(v)$ if and only if the maximal excesses at x are determined by the $(n-1)$-person coalitions. Thus,

$$x \in U(v) \text{ iff } e^v(S,x) \leq e^v(N-\{i\},x)$$
$$\text{for all } i \in N \text{ and all } S \subset N-\{i\},\ S \neq \emptyset.$$

Secondly, the part of the set $U(v)$ inside the prekernel consists of at most one efficient payoff vector which equals the so-called ENSC-solution. The egalitarian nonseparable contribution (ENSC-) solution for the n-person game v is defined to be

$$ENSC_i(v) := SC_i(v) + n^{-1} NSC(v) \quad \text{for all } i \in N, \text{ where}$$
$$SC_i(v) := v(N) - v(N-\{i\}) \quad \text{and} \quad NSC(v) := v(N) - \sum_{j \in N} SC_j(v).$$

Thirdly, the prenucleolus is included in the set $U(v)$ if and only if the ENSC-solution belongs to the set $U(v)$. Furthermore, each of the two equivalent condition is sufficient for the coincidence of the prenucleolus concept and the ENSC-method.

19-01-1989

Theo DRIESSEN
Faculty of Applied Mathematics
University of Twente
P.O. Box 217
7500 AE Enschede
The Netherlands

An aspiration approach to bargaining games

this approach is based on observations in experimental bargaining chains. Considerations are restricted to 1-step games ($N = \{1,\dots,n\}$, $v: P(N) \to \mathbb{R}_+$, $v(\emptyset)=0$, $[v(S), v(T) > 0 \Rightarrow S \cap T \neq \emptyset]$, $v(\{i\}) = 0$ (all i)).

a state (x,S) is $x \in \mathbb{R}^N_+$, $S \subseteq N$, s.t. $x(S) = v(S)$ and $x_i = 0$ $(i \in N \setminus S)$. – A bargaining chain is a sequence $(x^1,S^1),\dots,(x^n,S^n)$ of states of which each dominates the preceding one

and where of a player $i \in S^t$ is at least as high as his aspiration $a_i^t := \max_{\tau < t} x_i^\tau$.
A safe bargaining chain is defined by recursion:
(1) a maximal bargaining chain (i.e. a bargaining chain which cannot be any more extended) is safe for all players in N.
(2) within the recursion we have: axiom 1: If there is a reasonable domination, then one of them will be performed. (A domination to a state (x^{T+1}, S^{T+1}) is reasonable, if the chain up to the new state is safe for all players in $S^T \cap S^{T+1}$.) axiom 2: an unreasonable domination is not performed. (A domination to a next state (x^{T+1}, S^{T+1}) is unreasonable, if there is a subsequent reasonable domination to (x^{T+2}, S^{T+2}) s.t. $S^{T+1} - S^{T+2} \neq \emptyset$, i.e. one of the dominating players is punished.) concluding the recursion: a bargaining chain is safe for player j if every domination to (x^{T+1}, S^{T+1}) with $j \notin S^{T+1}$ is unreasonable.
A stable state (x^1, S^1) is a safe bargaining chain of length 1 which is safe for all players in S^1.
examples are given, a comparison to the bargaining set approach is made. refinements of this concept are necessary if experimental results shall be explained in detail, namely:
aspirations depend on the "bloc" a player is in — players can develop "reciprocal loyalty" — dominations are only performed, when they give a minimal improvement $\Delta > 0$ — dominations with zero-improvements are possible, when the new state is "socially desirable" — no-coalition state can be entered by breaking the coalition when the expected value of the breaker is afterwards higher than ~~before~~ his value x_i^t before breaking. — the payoff structure of a selected state has to meet conditions of "prominence" — the behavior can deviate from the prediction when "ultimatum situations" arise.

Wulf Albers
Bielefeld

Easy initial states in stochastic Games.

Discrete time dynamic games are played as follows: At each period the players have to choose an action out of an available probably state dependent action set. The simultaneously chosen actions jointly determine rewards to the players and a transition distribution according to which the next state is selected. The infinite horizon model is considered under the limiting average criterion. Strategies at each period may generally depend on the history up to that period. Stationary strategies only take care of the state in which the system is arrived. It is shown that in the zero-sum case for both players there exist non-empty subsets of states which are "easy" in the sense that the players can guarantee the value by using stationary strategies. For the general sum case this result can be extended as follows: each of the players has a nonempty subset of states, which are almost easy for the players in the following sense: Starting in a state belonging to such a set, the players can play ε-equilibrium wise by using appropriate stationary strategies as long as they do not detect a deviation of one of the other players. If they do detect a debiation than they have to switch to behavioral ε-optimal punisment strategies

Koos Vrieze
Rijksuniversiteit Limburg
Maastricht, Hollad.

Weighted Reward Criteria in Markov Decision Processes and Stochastic Games

We introduce a parametrized family of Markov Decision Processes (competitive, or non-competitive) which we call "weighted Markov Decision Processes". The boundary points of this family are the now classical discounted and limiting average models. It is demonstrated that even in the noncompetitive case optimal policies may fail to exist. In this case an algorithm is given which constructs an ε-optimal "ultimately stationary" Markov policy for any $\varepsilon > 0$. In the antagonistic competitive Markov Decision Processes the weighted criterion will be either a convex combination of two discounted objectives, or of one discounted and one limiting average reward objective. In both cases we establish the existence of the game-theoretic value vector, and supply a description of ε-optimal non-stationary strategies.

Jerzy Filar
Department of Mathematics
University of Maryland,
Baltimore County, Baltimore, USA.

VOTING BY COUNT AND ACCOUNT

Let $N = \{1, \ldots, n\}$ be the set of taxpayers in a community and let w_i be the tax paid by $i \in N$. When the community has to elect an officeholder from a set of candidates several majority rules may be applicable. We may consider the symmetric simple game $(n, [\frac{n}{2}]+1) = u$ (voting by count), or the weighted majority game $v = [w^1, \ldots, w^n]$ ($S \subset N$ wins if $2 \sum_{i \in S} w^i > \sum_{i \in N} w^i$ - voting by account). A third possibility is to use the product uv - voting by count and account - which was the rule in Jewish communities in Europe during the last three hundred years or more. We prove that the Shapley value of uv Lorenz dominates that of v.

Bezalel Peleg
Department of Mathematics
The Hebrew University
Jerusalem 91904, ISRAEL

The Nucleus and the Problem of Strong Implementation

The problem of strong implementation is to determine which social choice correspondences can be obtained as the strong equilibrium correspondence of a game form. We introduce the notion of the nucleus of an effectivity function. Under certain conditions, it yields the smallest implementable social choice correspondence having that effectivity function. We contrast it with the core, which yields the largest one (as shown earlier by Moulin and Peleg), and argue that the smaller solution should be preferred when available.

Ron Holzman
Dept. of Applied Mathematics
The Weizmann Institute of Science
Rehovot, Israel

Jan. 20, 1989

Safeguards Games

Jan. 20. 1989

Safeguards problems are situations in which player I (an inspector) tries to detect "illegal" actions of player O (an Operator). He does so on the basis of observations of random variables, the distribution of which depends on player O's actions. We propose a multi-stage extensive form game to model the sequential inspection problem. Under appropriate restrictions on the payoffs and the strategy set of the operator, we prove the existence of unique Nash equilibrium. In a variant of the game in which the inspector has the possibility to commit himself publically to a certain strategy, there is again a unique Nash equilibrium which may be called the "commitment equilibrium" in which the inspector's strategy is the same as before, but the inspector's payoff is higher than in the equilibrium of the game without commitment. Therefore this may also be called a deterrence equilibrium.

For application to pollution control and nuclear material safeguards, the inspector's equilibrium strategy is shown to be exactly the statistical test commonly used in these contexts.

Shmuel Zamir
Department of Statistics
The Hebrew University
Jerusalem, ISRAEL

20. 1. 1989

Quasi-differentiable Functions in Optimization Theory.

According to V. Demyanov and A. Rubinov, a function $f: U \longrightarrow \mathbb{R}$, $U \subseteq \mathbb{R}^n$ open, $x_0 \in U$, is said to be quasi-differentiable, if its directional derivative in x_0, as a function of the direction, can be represented as a difference of two sublinear functions. We consider the directional derivative as a periodic function on the $(n-1)$-Sphere. The development into a Fourier-Series leads to an approximation by differences of sublinear functions. The norm in which this Fourier-Series converges is used to clarify the degree of differentiability. This technique is also used for higher order derivatives and overcomes the typical discontinuities which appear in non smooth analysis

D. Pallaschke
Inst. für Statistik
und Math. Wirtschaftsth.
Univ. Karlsruhe

January 20, 1989

Large Games are Market Games

We show that large finite games in coalitional form (games with "many", but a finite number, of players) are approximately market games. To model large games we use the notion of a pregame, which enables us to describe the worth of any group of players as a function of the attributes (or "types") of the members of the group. From the pregame, which is required to satisfy only mild conditions, we construct a premarket -- a space of characteristics of goods and a continuous, concave, 1-homogeneous utility function. We show that the worth of any sufficiently large coalition in any game derived from the pregame is close to the worth of the corresponding coalition (with the same player set) in the market game derived from the premarket.

We also show that games in coalitional form with a continuum of players and finite coalitions (the Kaneko - Wooders model) are equivalent to Mas-Colell differentiated-commodities market games.

Myrna Holtz Wooders
Department of Economics
University of Toronto
Canada

Fixed Preferences and Changin Tastes

The phenomenon that is colloquially referred to as "fashion" exists to some extent in the consumption of many goods and services, as well as in other aspects of human activity. To focus the attention on the main issue, we restrict our discussion to pure fashion phenomena, that is the variation over time in the share of a particular brand name or product design at the expense of other brand names or designs of the same good in the market at large or among a specific group of customers. For example, the increase in recent years in the market share of Reebok at the expense of Nike or the complete replacement of the miniskirt by the midiskirt and maxiskirt in the 1970s. The main idea is that the consumption of many commodities is, in part, a social activity. Therefore, to capture the social aspect of consumption behavior, the standard definition of a commodity, which includes its physical attributes, delivery date, location, and - in the case of contingent commodities - the state of nature, must be extended to include commodity's social attributes. We claim that the observed pattern of change in the consumption of standard commodities (e.g. Nike sneakers) is consistent with constant preferences over the space of extended-commodities.

In the present paper we implement these ideas in a dynamic game model that is reminiscent of an overlapping generations economy. Consider a game that evolves through countably many periods, $\{\cdots,-2,-1,0,1,2,\cdots\}$, without a first or a last period. In each period there ~~are~~ is a continuum of players. Every player participates in the game during finitely many consecutive periods. At the outset of each period in which he is in the game each player must select a move from a finite set of moves. The selection of moves by all ~~the~~ the players that are in the game in a given period is done simultaneously. The payoff to any given player depend on the sequence of his own moves during the periods in which he is in the game and on finitely many statistics (linear functionals) defined on the moves of all the other players during the same periods. In this model any finite set of players is negligible in so far as the relevant statistics are concerned. The sequences of moves of all players define a play of the game. A play of the game is an equilibrium play if no player can increase his utility by switching unilaterally to another sequence of moves. Existence of an equilibrium play is proved. An example with a cyclical equilibrium play is included. It demonstrates the application of the overlapping generations game to modeling changes over time in consumption due to the fashion phenomena. (This is a joint work with Edi Karni.)

David Schmeidler 21.1.1989
Tel-Aviv

Mengenlehre

22. – 28. Januar 1989

Cardinal Invariants — old and middle-aged

24. Jan. 89

"Old" cardinal invariants are those defined in van Douwen's "The Integers and Topology" (in the Handbook of Set-Theoretic Topology) and the distributivity number h. "Middle aged" refers to $g :=$ minimum number of groupwise dense sets in $[\omega]^\omega$ with no common member; see "Applications of superperfect forcing and its relatives" (to appear in STACY proceedings). The following results were presented (mostly without proof): (1) Every κ partitions $[\omega]^2 \to 2$ have a common almost (i.e. except for a finite subset) homogeneous set $\Leftrightarrow \kappa < \min\{b, s\}$. (2) For every κ partitions $\omega \to 2$ there is one $[\omega]^2 \to 2$ such that every set homogeneous for the latter is also almost homogeneous for all the former $\Leftrightarrow \kappa < \aleph_1$. [(2) is joint work with A. Taylor & P. Erdös.] (3) Let $\mathcal{C}_x$ be the Turing cone filter relative to a real x, and let $\overline{\mathcal{C}} = \bigcap_{x \in R} \mathcal{C}_x$. Then $\overline{\mathcal{C}}$ is closed under intersections of fewer than g sets. (4) For filters on ω (containing all cofinite sets) define $\mathcal{F} \leq \mathcal{F}'$ to mean that $f(\mathcal{F}) \subseteq f(\mathcal{F}')$ for some finite-one $f : \omega \to \omega$. Feeble filters are at the bottom of this order. Any filter generated by $< g$ sets is $\leq$ any non-feeble filter. In particular, if $u < g$ then all non-feeble filters are equivalent. Since (as I learned from P. Simon) there is always a non-feeble filter generated by b sets, $b < u < g$ is impossible (though $b < u$ and $u < g$ are individually consistent). (5) If $u < g$ then $d = 2^{\aleph_0}$.

Andreas Blass, University of Michigan, Ann Arbor, MI 48109.

On the foundations of mathematics.

24 Jan 1989

The talk is an attempt to explain why I do not understand certain claims made, e.g. about the "lack of certainty" in mathematics. It is based on three theses:—

1) Any piece of completed mathematics can be considered as fully formalized in some first order system, and should be so considered from the point of view of foundations.

[But there is more to mathematics, namely the justification of particular axiom systems from "heuristic pictures", "intuition", etc.]

2) "Never-never land starts early", at numbers very quickly given by the

Ackermann hierarchy [$2^{2^{2^{2^{2}}}}$ $(= 2^{2^{16}})$ was taken as an example]; in other words there can be no ontological distinction between $2^{2^{16}}$, ω, ω_1, inaccessible, measurable, huge cardinals ... (until we get to inconsistency).

[And of course I do not claim to know where inconsistency happens; I merely believe it is above measurable cardinals.]

(The point of this second thesis is that there is an area of "feasible" or "concrete" mathematics which could in fact be checked by computation; this area is bounded, and anything above that bound can only be understood as an element of some first-order axiom system.)

3) The only possible explanations for mathematical objects are
a) as concrete, feasible numbers or operations that can be directly realized in practice,
or b) as elements of [models of] some first-order axiom system,
or c) as elements of a heuristic, intuitive model or picture of some higher-order system ("real" integers, "real" real numbers, etc),

and the thesis is that the only way we can explain or understand elements of type c) is via type b): in other words by using our intuitions about the higher-order idea, to justify first-order axiom systems which approximate the higher-order intention — with the implication that the objects of type c) ~~are~~ in question are 'like' the objects of type b) thus obtained.

A point noted in the discussion is, that while there are useful distinctions that can be drawn between the methods involved in the passage from type c) to type b) objects, these affect only the strength of our belief in the consistency of the axiom systems derived, rather than the actual existence of the objects — which depends only on whether the axiom systems derived, are, in fact, consistent.

Doubts about the certainty and meaningfulness of mathematics can arise only from ignoring, first, the correlation between theory as derived in b) and practice as observed in a); and, second, the large agreement about what should be derived in b) from intuitions in c). This latter includes the soundness of 1st order logic.

Frank R. Drake, University of Leeds, Leeds LS2 9JT, U.K.

Topological Applications of Generic Huge Embeddings — 26 Jan 1989

Previously we used the Foreman-Laver collapse of a huge to $\aleph_1$ and target to $\aleph_2$ to transfer ~~results~~ paracompactness properties from $\aleph_1$ to $\aleph_2$. Now we instead collapse to $\aleph_2$ and $\aleph_3$ in order to take advantage of ~~the~~ countable closure of ~~the~~ partial order. We prove

Th 1 If X is first countable T_2 and $j(P)/P$ is countably closed, then $j''X$ is a closed subspace of $j(X)$.

Th 2 Con($\exists$ huge) $\to$ Con(first countable T_2 $\aleph_2$-paracompact spaces of size $\leq \aleph_3$ are paracompact)

Franklin D. Tall
University of Toronto
Toronto, Canada, M5S1A1

Cofinalities in Magidor's model — 23 Jan 89

$\mathrm{cof} \prod \aleph_{3n} = \aleph_{\omega+1}$, $\mathrm{cof} \prod \aleph_{3n+1} = \aleph_{\omega+2}$

T. Jech
Penn State, PA 16802

The Constructive continuum hypothesis:

Let $\Theta = \sup\{\alpha : \exists f \in L(\mathbb{R})\ f : \mathbb{R} \xrightarrow{onto} \alpha\}$

Constructive Continuum hypothesis:
$\Theta < \omega_2$

Thm (Foreman - Magidor)
If $NS(\omega_2) \restriction \mathrm{cof}(\omega_1)$ is saturated and there is a supercompact cardinal, then $\Theta < \aleph_2$.

M. Foreman
Ohio State University
Columbus, OH 43021
TS1144@ OHSTVMA.

Partitioning the Quadruples of Topological Spaces

<u>Theorem</u> Assume GCH and $\square_\kappa$ for each singular cardinal κ. For each Hausdorff space with cardinality not greater than the least weakly compact cardinal we have $X \rightarrow (Y)^4_\omega$ implies Y is discrete. In fact, for each such X there is $f : [X]^4 \rightarrow \omega$ such that each homogeneous set is discrete as a subspace of X.

William Weiss
Math. Dept. Univ. of Toronto

Homogeneity of Infinite Permutation Groups

A permutation group acting on $\kappa > \lambda$ is λ-homogeneous if for all $X, Y \in [\kappa]^\lambda$ there is a $g \in G$ with $g''X = Y$

<u>Theorem</u>: $\square_{\omega_1} \Rightarrow \exists G$ acting on ω_2 which is ω_1-homogeneous but not ω-homogeneous.

P.H. Neumann asked the problem if $\lambda > \mu$ and λ-homogeneity implies μ homogeneity and proved that the answer is yes for $\lambda \geq \omega$ and $\mu < \omega$. Independently P. Nyikos and S. Shelah with S. Thomas showed that ω_1-homogeneity does not imply ω-homogeneity under the condition that $MA + 2^{\aleph_0} > \aleph_1$ and $MA + 2^{\aleph_0} > \aleph_2$ hold respectively.

András Hajnal
Math. Institute of the Hungarian Academy of Sciences.

SOME FILTERS . Jean-Pierre Levinski - 25 January 1989

For a filter F on κ, set $B(F) = \mathcal{P}(\kappa)/F$

We look for properties $E(\kappa)$, which are very near to the measurability of κ, and are compatible with "κ is the critical cardinal" (c.c.) i.e.: $(\forall \alpha < \kappa)(2^\alpha = \alpha^+) \wedge (2^\kappa > \kappa^+)$. We had former theorems where $E(\kappa)$ is "κ bears a normal, precipitous filter". However, in these models, κ, if ineffable, is not even Π^1_3-indescribable. So, we look at the following two properties

$E_1(\kappa) \leftrightarrow \kappa$ bears a normal filter F, such that $B(F)$ is κ^+-distributive

$E_2(\kappa) \leftrightarrow \kappa$ bears an F st $B(F)$ admits a dense, κ^+-closed subset

Observe that $E_1(\kappa) \rightarrow \kappa$ is completely ineffably Ramsey.

We prove:

Theorem 1. $\mathrm{Cons}(ZFC\,M) \rightarrow \mathrm{Cons}(ZFC +$ the C.C. κ satisfies $E_1(\kappa)) \square$

Theorem 2: $\mathrm{Cons}(ZFC\,M) \rightarrow \mathrm{Cons}(ZFC + GCH + \kappa$ is measurable $+$
$\forall \alpha < \kappa$, α regular $\rightarrow 2^\alpha > \alpha^+) \square$

Theorem 2 answers an old question of Kunen and Paris

Theorem 3. $\mathrm{Cons}(ZFC + \exists \kappa, o(\kappa) = \kappa^{++}) \rightarrow$
$\mathrm{Cons}(ZFC +$ the critical cardinal κ satisfies $E_2(\kappa)) \square$

We do not know if "$o(\kappa) = \kappa^{++}$" is necessary in theorem 3

~~Theorem~~ We know that "$E_2(\kappa) \wedge 2^\kappa = \kappa^+$" $\rightarrow$ "κ is measurable".

But we prove

Theorem 4: $\mathrm{Cons}(ZFC\,M) \rightarrow \mathrm{Cons}(ZFC + GCH + E_3(\kappa) + \kappa$ is not measurable$) \square$

From Theorem 4, we get

Theorem 5: $\mathrm{Cons}(ZFC\,M) \rightarrow \mathrm{Cons}(ZFC + GCH + \kappa$ is not measurable $+ \kappa$ bears a normal filter F s.t. $B(F)$ is κ^+-distributive and κ^+-saturated$) \square$

Theorem 5 improves a classical result of Kunen and Paris who obtained a normal, κ^+-saturated filter on a non-measurable κ. κ was Π^1_1, but not Π^1_2-indescribable

$\longrightarrow$

The methods of proof can be used to prove theorems of the type $\mathrm{Con}(ZFC + \exists\kappa\, P(\kappa)) \to$ $\mathrm{Con}(ZFC +$ the c.c. κ satisfies $P(\kappa))$, where $P(\kappa)$ might be

(a) κ is weakly ineffable

(b) κ is ineffable

(c) κ is Π^1_n-indescribable [for any fixed $n \geq 1$ the case $n=1$ was due to Silver]

(d) κ is completely ineffable

Countable decompositions of Euclidean spaces

1. $\mathbb{R}^n$ can be colored by countably many colors so that no two monochromatic points are of rational distance from each other.
2. $\mathbb{R}^3$ can be colored by countably many colors so that the four nodes of ~~an~~ a regular tetrahedron ~~may~~ do not get the same color.

This can even be true for equilateral triangle instead of tetrahedron.

The proof consists of a mixture of set theory, geometry, and finite combinatorics.

Péter Komjáth
Dept. Comp. Sci. Eötvös U.
Budapest, Hungary

Some applications of set theory to analysis

Definition. $\mathfrak{r} = \min\{|\mathcal{R}| : \mathcal{R} \subseteq [\omega]^\omega \,\&\, (\forall A \in [\omega]^\omega)(\exists R \in \mathcal{R})(R \subseteq^* A \text{ vel } R \subseteq^* \omega - A)\}$

1. $\mathrm{cov}(\mathbb{K}) \cdot \mathrm{cov}(\mathbb{R}) \leq \mathfrak{r} \leq \min(\mathfrak{u}, \mathfrak{i})$

2. Some estimations of cardinal invariants of generalized limit were given e.g. $\min(\underline{r}, \underline{d}) \leq$ "the minimal size of a family of regular (Toeplitz) matrices such that every 0-1 sequence is summed by one of them" $\leq \underline{r}$.
3. Results comparing the strength of different convergence tests were presented too. e.g you need "only" $\underline{d}$-many divergent series to decide every one by asymptotic behaviour in contrast to $\mathrm{cof}(\mathcal{L})$ many in the best convergence test under eventual dominance.

Peter Vojtáš
Math. Inst. Slovak Acad. Sc.
Jesenná 5
041 54 Košice
Czechoslovakia

Reconstructing extenders and $\neg$GCH over measurables.

Assuming there is no inner model with strong cardinal and the $\mathcal{K}(\mathcal{F})$ has some weak covering properties we show the following:

(a) If $j: V \longrightarrow M$, ${}^{\omega_1}M \subseteq M$ and the same extender was used ω_1-times in the iterated ultrapower $j \restriction \mathcal{K}(\mathcal{F})$ then its ω_1-th image is in M.

(b) If $j: V \longrightarrow M$, ${}^{\omega}M \subseteq M$, all (κ, λ)-extenders in iteration $j \restriction \mathcal{K}(\mathcal{F})$ satisfy $\lambda < \kappa^{+\omega}$ then ~~[illegible]~~ we can replace ω_1 by ω in (a).

Moti Gitik
Dep. of Mathematics
Tel Aviv University, Tel Aviv, Israel

starting from polynomials of elementary embeddings.

Say that a set endowed by a binary operation ^ satisfying $x \wedge (y \wedge z) = (x \wedge y) \wedge (x \wedge z)$ is a clump. Then if j is an elementary embedding of some rank R_λ into itself, the set $\mathcal{P}(j)$ of all elementary embeddings constructed from j using the operation $(k, \ell) \longmapsto \bigcup_{\alpha<\lambda} k(\ell \upharpoonright R_\alpha)$ is a clump.
Conjecture. – $\mathcal{P}(j)$ is a free monogenic clump.
Clumps have so far not been much studied. An example of an infinite monogenic clump is constructed (the only one known up to now seems). A presentation of the free clumps involving a convenient extension of the braid groups $B(n)$ is given, and an algorithm related to conjugacy in free groups is given that proves left cancellation in free clumps (that should hold if the conjecture above is true).

Patrick Dehornoy
Université de Caen,
FRANCE.

THE NORMAL FILTER GENERATED BY A FAMILY OF SETS.

This talk reported on work done jointly with James Henle. For any filter F on a regular uncountable cardinal, $\Delta^2 F = \Delta^3 F$, where Δ is the diagonal intersection operator. (When F is κ-complete, $\Delta F = \Delta^2 F$). On the other hand, for filters on $P_\kappa \lambda = \{x \subseteq \lambda \mid \text{card}\, x < \kappa\}$, if $\lambda > \kappa$ and there is λ', $\kappa \leq \lambda' \leq \lambda$, such that λ' satisfies the free set existence property $(\lambda', n, \omega) \to n+1$, then there is a filter F_n on $P_\kappa(\lambda)$ such that (for $n \geq 1$) $\Delta^{n-1} F \subsetneq \Delta^n F = \text{CLUB}_{\kappa,\lambda}$. So if $\lambda \geq \aleph_\omega$ there is F on $P_\kappa \lambda$ with $F \subsetneq \Delta F \subsetneq \cdots \subsetneq \Delta^n F \subsetneq \Delta^{n+1} F \subsetneq \ldots$

CARLOS A. DI PRISCO
Instituto Venezolano de Investigaciones Científicas CARACAS.

YET ANOTHER VARIANT OF DIAMOND

the principle $\diamondsuit^{\rho}_{\kappa,\lambda}$ is defined just as the usual $\diamondsuit_{\kappa,\lambda}$, except that one is handling less than ρ many subsets of λ at a time, instead of just one. It follows from our results that, assuming the Generalized Continuum Hypothesis, $\diamondsuit^{\rho}_{\kappa,\lambda}$ holds whenever $\lambda > \rho \geq \kappa$, and $\diamondsuit^{\rho}_{\kappa,\lambda}(S)$ holds whenever λ has cofinality less than κ and S is a stationary subset of $[\lambda]^{<\kappa}$

Pierre MATET
Université de CAEN, France

JUST WHAT CAN LARGE CARDINAL HYPOTHESES PROVE ANYWAY?

Theorem Assume there are ω^2 many Woodin cardinals (below λ). Assume CBH, UBH (for countable iteration trees on V below λ). Then there are partial orders $\mathbb{P}$, $\mathbb{Q} \in V_\lambda$ such that

$$V^{\mathbb{P}} \models \text{`}\underset{\sim}{\delta}^1_2 = \omega_2\text{'}$$

$$V^{\mathbb{Q}} \models \text{`There is no precipitous ideal on } \omega_1\text{'}$$

In fact $\mathbb{Q}$ can be chosen so that

$V^{\mathbb{Q}} \models$ 'If $j : V \to M \subset V[g]$ is a generic embedding with critical point ω_1 then $|\mathbb{B}| \geq$ least Woodin cardinal where $g \subset \mathbb{B}$ is V-generic'

W Hugh Woodin
UC Berkeley / Caltech

THE IGNORANCE OF BOURBAKI

The position of the Bourbaki group with regard to logic and set theory, as set out in such papers as H. CARTAN Sur le Fondement logique des Mathématiques, Revue Scientifique 81 (1943), 3–11, J. DIEUDONNÉ, Les Méthodes Axiomatiques Modernes et les Fondements des Mathématiques, Revue Scientifique 1939, N. Bourbaki, L'architecture des Mathématiques, in Les Grands Courants de la Pensée Mathématique, Cahiers du Sud 1948, and N. Bourbaki, Foundation of Mathematics for the Working Mathematician, Journal of Symbolic Logic 14 (1948) pp 1–14, was examined and the following two questions posed:

WHY DID BOURBAKI MAKE NO MENTION OF GÖDEL?

WHY DID BOURBAKI NOT NOTICE THE INADEQUACY OF HIS CHOSEN SET THEORY (a version of Zermelo + AC) AS A FOUNDATION FOR MATHEMATICS?

It was argued in answer to the first that at whatever level of their psyche the Bourbachistes were disabled, they were not ready to face the possibility, strongly suggested by Gödel's work, that there are no foundation of mathematics in the sense proposed by Hilbert and embraced by Bourbaki; that there are no ways of grounding mathematics in logic or classes or whatever so that once a basis has thus been given in some primitive ideas, no further thought need be given to them; that though there are indeed foundational issues, they cannot be confined to Chapter One of the Great Book, for they permeate mathematics.

In regard to the second question, the stance of the Bourbachistes was compared to that of Saunders Mac Lane, and it was suggested that, like Mac Lane, they were solely interested in areas of mathematics for which Zermelo + AC is adequate, and that this area may broadly be described as geometry as opposed to arithmetic.

A.R.D. Mathias
Peterhouse, Cambridge

Determinacy and the Ramsey property

A set is ξ-$\underline{\Pi}^1_1$ for $\xi < \omega_1$ if there are $\langle A_\eta \mid \eta < \xi \rangle$ each $A_\eta \in \underline{\Pi}^1_1$ so that, setting $A_\xi = \emptyset$, $x \in A \Leftrightarrow$ least $\xi \leq \eta$ with $x \notin A_\xi$ is odd. Martin & Harrington have shown the equivalences of Determinacy for the pointclasses ξ-$\underline{\Pi}^1_1$ for each ξ not a multiple of ω^2. Martin has shown that ω^2-$\underline{\Pi}^1_1$-Det. holds if there's a measurable and asked if a Ramsey suffices. We show it doesn't by proving ω^2-$\underline{\Pi}^1_1$-Det $\Rightarrow \exists$ a mouse $M \models KP + \Sigma_1$-Separation. Such a mouse provides many inner models with c.u.b. classes of Ramseys. We conjecture this is an equivalence

Philip Welch, Dept. of Maths
Bristol GB

Canonical Partition Relations

Canonical partitions were developed by Hajnal and Galvin for proving partition relations $\alpha \to (\beta, m)^2$ for ordinals α, β and $m < \omega$, in the case where α, β are finite powers of ω. The concept generalizes to partitions of the ωth power of a cardinal κ, and if κ is Ramsey, then any partition of κ^ω can be reduced to a canonical one. This reduction is the first step in the proofs that

(1) If κ is Ramsey, then for all $m < \omega$, $\kappa^\omega \to (\kappa^\omega, m)^2$

(2) If $\lambda < \kappa$ are both Ramsey, then for all $m < \omega$, $\kappa^\omega \cdot \lambda^\omega \to (\kappa^\omega \cdot \lambda^\omega, m)^2$.

Jean Larson
University of Florida
Gainesville

Coding Over Core Models

In this talk I described what progress has been made in extending Jensen's Coding Thm. into the context of large cardinals. I began with an outline of the proof of the following result: If $\langle V, A\rangle \models ZFC + \mu$ is a measure then there is a $\langle V, A\rangle$-definable forcing for producing a real R s.t. $V[R] = L[\mu^*, R] \models \mu^*$ is a measure extending μ. Then I briefly described how the proof can be carried out as well for extender sequences $\underline{E}$ provided $o(\kappa) < \kappa^{++}$ for all κ, subject to some fine-structural facts. The latter can be established with current techniques for the case $o(\kappa) \leq 1$, all κ.

Sy D. Friedman
MIT
Cambridge, MA USA

On Core Models below a Strong Cardinal

In this lecture, I gave a definition of strong cardinals and extenders. Strong cardinals are a natural generalisation of measurability in terms of elementary embedding; extenders allow to code such "strong" embeddings. Using the notion of coherency ~~we can quickly produce~~ one can get inner models for strong cardinals. To obtain a more "L-like" hierarchisation of such models, one has to admit partial extenders in the constructing predicate: F_γ is required to be an extender on $J_\gamma[F]$ only. This is a way of obtaining condensation for the $J_\gamma[F]$-hierarchy. All the $J_\gamma[F]$ considered and $L[F]$ satisfy the axioms for mice. Finally some properties of mice are given.

Peter Koepke,
Freiburg, W-Germany.

Representable Boolean algebras

Let F be maximal almost disjoint family in ω.

A set $a \subseteq \omega$ is a partitioner of F iff $\forall_{b \in F}$ $b \setminus a$ or $a \cap b$ is finite

Let I be a natural ideal generated by F, $B_F - B_\omega$ is partition algebra of F ( ) iff B_F {a / a is partitioner for F}

B^* is representable iff there is F such that $B^* = B/I$

THM. Con (ZFC) → Con (ZFC + MA + $\exists_B$ $|B| = c$ (B is not representable)

THM. Con (ZFC) → Con (ZFC + free algebra on ω_1 generator is not representable.)

??

Some facts in matching theory

We reported some basic facts in matching theory of infinite graphs including Aharoni's Duality Theorem, an extension of Dilworth's Theorem for finite p.o. sets and an extension of Menger's Theorem. After discussing a generalization of Tutte's theorem we presented a necessary and sufficient criterion for the existence a perfect f-matching of a countable graph.

K. Steffens

Hannover

Consistency problems in the partition calculus

Assume GCH. Call $g : [\omega_2]^2 \to \omega_1$ an Erdős-Rado function if $\forall f : [\omega_2]^2 \to \omega$ $\exists X \subseteq \omega_2$ $|X| = 3$ and X is homogeneous for f and X is antihomogeneous for g, i.e., if $X = \{\alpha, \beta, \gamma\}$ then $g\{\alpha,\beta\} \neq g\{\alpha,\gamma\}$. It is consistent that CH holds, $2^{\omega_1} \geq \omega_3$ and there are no ER functions, and a recent result of Shelah seems to show it consistent that GCH holds and there are ER functions. Is it consistent that GCH holds and there are no ER functions? A related problem: Let $K = \{\{(\alpha,\beta),(\gamma,\delta)\} \in [\omega_2 \times \omega_2]^2 : \alpha < \gamma < \delta < \beta\}$. Does $K \to (3)^2_\omega$? What if ω_2 is replaced by ω_3? (Assume GCH). It is consistent that CH, $2^{\omega_1} \geq \omega_3$ and $\omega_3 \to (\alpha)^2_\omega$ for all $\alpha < \omega_2$. Does $c^{++} \to (\alpha)^2_\omega$, all $\alpha < \omega_2$? (Note that $\omega_2 \not\to (\omega_1{}^\omega)^2_\omega$ by the Milner-Rado "paradox".) Assume MA + ¬CH. It is known that $\omega_1 \cdot \alpha \to (\omega_1 \cdot \alpha, n)^2$, all $n < \omega$, if $\alpha = \omega$ or ω^2 and Larson has shown this for $\alpha = \omega^\omega$ and $n = 3$. What about other countable α? What about $\omega_1^2 \to (\omega_1^2, 3)^2$? In some ways this is similar to the famous question $\omega_1 \to (\omega_1; \omega_1)^2_2$, i.e., $\forall f : [\omega_1]^2 \to 2$ $\exists A, B \subseteq \omega_1$, $|A| = |B| = \omega_1$ and f is constant on $\{\{\alpha,\beta\} : \alpha \in A, \beta \in B, \alpha < \beta\}$.

J E Baumgartner
Dartmouth College

Borel orders and BQO theory

I reported the main known results about Borel orders, quasi-ordered under the embeddability relation. That
(i) (Harrington-Shelah) the orders (2^ξ, lexicographic ordering) for $\xi < \omega_1$ are cofinal
(ii) (Marker; Louveau) For any Borel order X and any $\xi < \omega_1$, either X is embeddable in $2^{<\omega\cdot\xi}$, or equiembeddable with $2^{\omega\cdot\xi}$, or embeds $2^{\omega\xi+1}$, and
(iii) (Louveau-Saint Raymond) if BOR_ξ denotes the set of

Borel orders embeddable in $2^{\omega.\xi}$, with the embeddability relation, then (a) ZFC $\vdash$ BOR_2 is well quasi ordered

(b) Projective Determinacy $\vdash \forall n$ BOR_n is well quasi ordered

(c) Hyper Projective Determinacy $\vdash$ BOR_ω is well quasi ordered.

We conjecture that "BOR is well quasi ordered under embeddability" is a theorem of ZFC – and in fact of 2^{nd}-order-arithmetics.

A. Louveau
Université Paris 6
et CNRS, France.

ZFJ

ZFJ is the system obtained by adding to ZF (where, as usual, AC is not included) the axioms expressing that J (a new unary function symbol) is an automorphism of the universe. The scheme of replacement is not extended to the new language. So, (M, J) is a model of ZFJ exactly when M is a model of ZF and J is an automorphism of M. This system is interesting in connection with the (still open) consistency problem for NF (Quine's system). For example, using Specker's "typical ambiguity", it is easy to show in ZFJ that:

(i) if α is an ordinal such that $\alpha < J\alpha$, then (V_α, ε) where $x \varepsilon y$ iff $x \in Jy \in V_{\alpha+1}$, is a "model" (not a real model, since the graph of ε cannot be defined as a set) of NFU (and if, moreover, J leaves fixed each element of some V_β, then this "model" is an end extension of V_β);

(ii) if ς is a cardinal such that $J\varsigma = 2^\varsigma$, then there is a "model" of NF of power ς (and if, moreover, J leaves fixed each $n \in \omega$, then this "model" also satisfies Rosser's counting axiom).

From (i), we get a simplified proof of Jensen's consistency result for NFU, and (ii) gives some interest to the conjecture that $Jc = 2^c$ is consistent with ZFJ.

Reference: my paper "ZFJ and the consistency problem for NF" in the Jahrbuch 1988 der Kurt-Gödel-Gesellschaft, p. 102-106.

M. Boffa
Mons University (Belgium).

Questions on Singular Cardinals

Q1: what is the consistency ~~of the~~ strength of the failure of the SCH?

We have $\mathrm{con}(\forall\alpha<\kappa\,\exists\nu<\kappa\; o(\nu)\geq\alpha) \leq \mathrm{con}(\mathrm{cf}(\kappa)=\omega<\kappa + 2^\kappa>\kappa^+) \leq \mathrm{con}(o(\kappa)=\kappa^{++})$.

For cofinality λ greater than ω, the ~~relation~~ condition analogous to the r.h.s. is exact: $\mathrm{con}(\mathrm{cf}(\kappa)=\lambda<\kappa + 2^\kappa>\kappa^+) \equiv \mathrm{con}(o(\kappa)=\kappa^{++}+\lambda)$.

Why is a cardinal κ singular, & can one define a witness to the singularity?

There are 5 possible reasons (assume $o(\kappa)<\kappa^{++}$ for all κ)

Reason 1 κ is singular in $K(\mathcal{F})$.

For the rest we define: $\beta\leq o(\kappa)$ is the least ordinal such that $\mathcal{F}(\kappa,\beta)$ is not generated by any set of indiscernibles.

Reason 2 ($\beta=\beta'+1$) there is a maximal Prikry sequence

Reason 3 ($\mathrm{cf}^{K(\mathcal{F})}(\beta)<\kappa$) there is a maximal sequence analogous to Magidor forcing

Q2: Is there a model over which this sequence is exactly Magidor ~~general~~ generic?

Reason 4 ($\mathrm{cf}^{K(\mathcal{F})}(\kappa)=\kappa$) There is a sequence analogous to ~~the~~ sequence $\langle C_n \mid n\in\omega\rangle$ with C_{n+1} an indiscernible for $\mathcal{F}(\kappa, C_n)$.

(for $o(\kappa)>\kappa$)

Reason 5 ($cf(\beta)>\kappa$) there is a maximal sequence $A=(a_n : n\in\omega)$ such that for any $\lambda<\beta$, & any seq $(\nu_n : n\in\omega)$ with $\nu_n < a_n$ there is a ~~set~~ $C=(c_n : n\in\omega)$ generating $\mathcal{I}(K,\lambda)$ such that $\nu_n < c_n < a_n$.

Q[3] : Is this case consistent?

Decomposing Baire functions

For any σ-class $\mathcal{A}\subseteq P(\mathcal{X})$ let $\underline{M}\mathcal{A} = \{f\in {}^{\mathcal{X}}[0,1] : (\forall a)\ f^{-1}((a,1])\in\mathcal{A}\}$, $\overline{M}\mathcal{A} = 1-\underline{M}\mathcal{A}$ and $M\mathcal{A} = \underline{M}\mathcal{A}\cap\overline{M}\mathcal{A}$.

For any classes $\mathcal{F},\mathcal{G}\subseteq {}^{\mathcal{X}}[0,1]$ let $dec(\mathcal{F},\mathcal{G}) = \min\{\kappa : (\forall f\in\mathcal{F})(\exists\{g_\alpha\}_{\alpha<\kappa})(\exists\{X_\alpha\}_{\alpha<\kappa}$ - a part. of $\mathcal{X})\ f=\bigcup_\alpha (g_\alpha\restriction X_\alpha)\}$.

THM 1. If $\mathcal{A}$ is a σ-class with universal ~~functions~~ sets, then $dec(\overline{M}\mathcal{A},\underline{M}\mathcal{A})>\omega$.

THM 2. If $\{\mathcal{A}_n\}_{n\in\omega}$ ~~given~~ σ-clases with universal sets, $\mathcal{A}\supseteq\bigcup_n(\mathcal{A}_n\cup\check{\mathcal{A}}_n)$ has reduction property, then $dec(M\mathcal{A},\bigcup_n(\underline{M}\mathcal{A}_n\cup\overline{M}\mathcal{A}_n))>\omega$.

From Thm 1 and Thm 2 it is possible to deduce all classical results of NOVIKOV, KELDYSH, LACZKOVIČ about $dec(M\Sigma^0_\alpha, M\Sigma^0_\beta)$.

Let $dec = dec(M\Sigma^0_2, M\Sigma^0_1)$. Then $cov(\mathcal{K})\leq dec$, $dec\leq d$ and $dec\leq u$ (where $cov(\mathcal{K}) = \min\{|\mathcal{X}| : \mathcal{X}\subseteq P(\mathbb{R})$ are first category sets and $\bigcup\mathcal{X}=\mathbb{R}\}$, d = minimal cardinality of dominated family in ${}^\omega\omega\}$, u = min. card. of a base of ultrafilter on ω.

It is ~~even~~ unknown if $(\forall\alpha<\beta<\omega_1)\ dec(M\Sigma^0_\beta, M\Sigma^0_\alpha) = dec$.

J. Cichoń
UNIVERSITY OF WROCŁAW

Extension of measures invariant under ctble extensions

The talk is based on join paper by Piotr Zakrzewski and A.K.

We prove among others, that there is no maximal ~~Q~~ invariant extension of the Lebesgue measure on $\mathbb{R}$. On the other hand if 2^ω is real-valued measurable, then there is a maximal σ-finite. Q-invariant measure ~~on~~ ~~R~~ proper σ-field of subsets of $\mathbb{R}$.

Adam Krawczyk (Warsaw)

* About Frege's comprehension principle.

R. Hinnion (U.L.B – U.I.A, Belgium)

The classical answer to the paradox of Russell is the system ZF, which proposes to "construct" the sets.
Another answer is NF, in which one has still "wild" comprehension, but restricted to stratified formulas.
A third way is possible: namely comprehension for (more or less) <u>positive</u> formulas. The system GPF for example has been proved to be consistent relatively to ZF (independently by E. Weydert and M. Forti (see [1])). GPF is:

- comprehension for "generalized positive" formulas (obtained in $\mathcal{L}_{ZF}$ from atomic formulas, $\wedge, \vee, \exists, \forall$, $\forall x \in y$, $\forall x\, (\Theta(x) \to \ldots)$ with $\Theta(x)$ arbitrary with 1 free variable)
- extensionality
- empty set axiom.

One can find at least 2 similarities between GPF and NF:

- they have common anti-foundation properties: for example, any finite binary structure can be embedded in any model of NF (or GPF) (see [2])
- under natural assumptions they have models in which there is a transitive set which is a model of ZF: so they can be seen as <u>superclass</u> theories over the <u>set</u>-theory ZF.

Another possibility is the "3-valued Frege", which can best be described by a model:

such a model M gives a value $v_M(\varphi(\vec{a}))$, for any φ in $\mathcal{L}_{ZF}$ and $\vec{a} \in M$; this value is $0, \frac{1}{2}$ or 1; the conditions are:

- full comprehension: $\forall \varphi$ in $\mathcal{L}_{ZF}$ $\forall \vec{a} \in M$
  $\exists b \in M \; \forall t \in M \quad v_M(t \in b) = v_M(\varphi(t, \vec{a}))$
- extensionality: $\forall a, b \in M [\forall t \in M \; (v_M(t \in a) = v_M(t \in b)) \rightarrow a = b]$
- natural ("logical") conditions:
  $v_M(\neg \varphi) = 1 - v_M(\varphi)$,
  $v_M(\varphi \wedge \psi) = \min\{v_M(\varphi), v_M(\psi)\}$,
  etc... (see [3]).

One can prove in Z_{Δ_0} the consistency of this system without extensionality (the idea of this proof appeared first in Gilmore).

Conjecture: the topological methods used by E. Weydert and M. Forti can be used to show:
$$ZFC \vdash Con\,(3\text{-valued-Frege (with extensionality)})$$

References:

[1] M. Forti & R. Hinnion
"The consistency problem for positive comprehension principles" (To appear in JSL (1989))

[2] R. Hinnion:
"Embedding properties and anti-foundation in set theory" (To appear in ZML (1989))

[3] R. Hinnion:
"Le paradoxe de Russell dans des versions positives de la théorie naïve des ensembles", C.R.Ac.Sc. Paris, 304, S.I, n°12, 1987, p. 307-310

R. Hinnion U.L.B (C.P. 135) 50, av. F. Roosevelt 1050 Bruxelles Belgium

A COMBINATORIAL APPROACH TO CODING BY A REAL

(Presented by Title)

Lee J. Stanley, Lehigh University
Dept. of Math, Bethlehem, PA, 18015

A combinatorial approach to coding by a real is sketched which uses some combinatorial consequences of Fine Structure (Squares and Scales, with a small degree of condensation coherence) and avoids direct appeal to Fine Structure or to definability notions in models associated with the combinatorics. This can be carried out over models where the cardinals coincide with the L-cardinals. Some details remain to be worked out to extend the model to all models where $0^{\#}$ does not exist. Aside from the great simplification as compared to Jensen's original method and later refinements, the forcing has some properness properties, which allow it to be iterated. This will be developed and exploited in work of Shelah and Ihoda. This is joint work with Shelah.

The analytical and topological theory of semigroups

January 29, 1989 – February 4, 1989

Semigroups on n-cells, uniquedivisibility, and matrices.

Let S be a semigroup on an n-cell, uniquely divisible (every element has a unique n^{th} root for every $n \in \omega$), with $E_S = \{1\} \cup K$ and trivial groups. Let $xS \subseteq Sx$, all $x \in S$, and assume cancellation on $S \setminus K$. Let $C(e)$, the core of e = $\{x \in S \mid xe = e = xe\}$, all $e \in K$. The possibility for dimension includes only $\dim C(e) = k$, $\dim K = n-k$, $k = 0, \dots, n-1$. Various theorems in this direction include:

1) If $k=1$, then $S \cong$ convex hull of $\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$ and a set of matrices of the form $\begin{pmatrix} & & x_{1n} \\ & 0 & \vdots \\ & & x_{n-1\,n} \\ & & 1 \end{pmatrix}$, where $(x_{1n}, \dots, x_{n-1\,n})$ represent the $\mathbb{R}^{n-1}$ coordinates of points from a compact, convex set.

2) If $k = n-1$ (so that K = arc), then S is isomorphic to $\left\{ \begin{pmatrix} x & a(1-x) & \\ 0 & 1 & 0 \\ & & y_1 & \\ 0 & & & \ddots & \\ & & & & y_{n-2} \end{pmatrix} : \begin{array}{l} x, a \in [0,1], \\ (x, y_1, \dots, y_{n-2}) \text{ are coordinates of a (multiplicative) } n-1 \text{ cone} \end{array} \right\}$

3) If $n = 4$, $k = 2$, there are 2 distinct types of examples

(a) The commutative extension of a type 1 is $\left\{ \begin{pmatrix} x & 0 & a(1-x) & 0 \\ 0 & x & b(1-x) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & y \end{pmatrix} : \begin{array}{l} y \in [x^2, x] \\ (a,b) \in \text{ convex compact set} \end{array} \right\}$

(b) a subdirect product: $\left\{ \begin{pmatrix} x & 0 & a(1-x) \\ 0 & y & b(1-x) \\ 0 & 0 & 1 \end{pmatrix} : \begin{array}{l} x, a, b \in [0,1] \\ y \in [x^2, x] \end{array} \right\}$

It is conjectured that these are the only examples –

D R Brown
Dept Math,
Univ. of Houston
77004 USA

INTERNET ADDRESS:
IN%"dbrown@uhrcc.crcc.uh.edu"

Let $I = [0,1]$ be the compact top. semigroup with multiplication max. and usual topology, $L^p(I)$, $1 \leq p \leq \infty$, denote Lebesgue space and $C(I)$ the Banach alg. of continuous functions on I with sup norm. The object $\mathrm{Hom}_{C(I)}(L^r, L^p)$, $r \leq p$, is [redacted] characterised.

H.L. Vasudeva
H.L. Vasudeva
Dept of Maths
P.U. Chandigarh
INDIA.

Lie semigroup theory.

For a Lie theory of semigroups one perceives three tasks: The infinitesimal theory, the local theory, and the global theory. This lecture mainly concentrates on the infinitesimal theory. The infinitesimal theory associated with a subsemigroup S of a Lie group yields a convex cone L(S) in the Lie algebra of of the ambient group, i.e. a closed convex cone W satisfying $e^{\mathrm{ad}\,x} W \subseteq W$ for all x in the edge. This yields the concept of a Lie wedge, generalizing the concept of a subalgebra. Lie semialgebras and invariant cones are also discussed. The details are about to appear in "Lie groups, convex cones, and semigroups, J. Hilgert, K.H. Hofmann, J.D. Lawson, Oxford University Press, 664 + 38 pp July 1989

K.H. Hofmann, TH Darmstad FRG

The embedding of infinitely divisible probability measures into continuous convolution semigroups

The speaker reports on recent developments in the theory of a problem which appears (in the classical probabilistic set up) already in the work of Paul Lévy, has been studied in detail (within the framework of locally compact groups) by K.R. Parthasarathy, Ranga Rao and Varadhan and many others as can be read in the speaker's monograph of 1977, and has recently been extended beyond the group case (within the set up of hypergroups). Hypergroups are structures X for which $\varepsilon_x * \varepsilon_y$ $(x,y \in X)$ is a probability measure with compact support on X such that (roughly speaking) the mapping $(x,y) \to \operatorname{supp}(\varepsilon_x * \varepsilon_y)$ is continuous. Hypergroups X have the property that $(M^b(X)$, convolution, involution$)$ are involutive Banach algebras, so that in the commutative case Gelfand's theory becomes applicable. The key object of the analysis is the generalized translation operator

$$T^x f(y) = \int_X f(z)\, \varepsilon_x * \varepsilon_y (dz) \quad \forall f, y \text{ and } x \in X.$$

There is a Haar measure on a commutative hypergroup, and there is the notion of a dual $X^\wedge$ of X. The following result (mainly due to M. Voit, 1988) has been discussed: (i) If $X^\wedge$ is arcwise connected, then any infinitely divisible probability measure μ on X can be embedded in the sense that there exists a continuous convolution semigroup (μ_t) in $M^1(X)$ (i.e. $\mu_t * \mu_s = \mu_{t+s}$ $\forall t,s>0$ and $\tau_v\text{-}\lim_{t\to 0} \mu_t = \varepsilon_e$) such that $\mu_1 = \mu$. (ii) In the case that X is hermitian this statement remains true if in addition $X^\wedge$ has no proper compact subhypergroup.

It was also reported that for connected Lie groups the original theory enjoyed some interesting extensions.

H. Heyer, Tübingen

The profinite group topology and the p-adic topology for the free monoid

The finite group (or profinite) topology was first introduced for the free group by M. Hall Jr and extended by Reutenauer to the case of free monoids. This is the initial topology defined by all the monoid morphisms from the free monoid into a discrete finite group. The p-adic topology is defined in the same way by replacing "group" by "p-group" in the definition. We are interested in a "descriptive" theory of this topology. That is, one restricts one's attention to "simple" subsets of the free monoid and one try to decide whether these sets are open, closed, etc., if one can compute their closure, etc. The "simple" sets we have in mind are the recognizable (or regular) sets of automata theory; these sets are completely described by a finite monoid, called the syntactic monoid of the set. We show that certain topological properties of a recognizable set are reflected by some simple algebraic properties of its syntactic monoid. We conjecture that the converse is true and we discuss these conjectures and their applications.

Reference: Topologies for the free monoid – To appear in J. of Algebra

J. E. PIN CNRS, LITP, université PARIS VI, FRANCE

Embedding local semigroups into global ones

Within the Lie-Theory of semigroups it is well known that for each Lie wedge W in a finite dimensional Lie algebra $L = L(G)$ there exist a local semigroup S which has the given wedge as its set of subtangent vectors, i.e. $L(S) = W$. (cf. the book of Hilgert, Hofmann and Lawson cited by K.H. Hofmann above). The following problem then arises immediately:

(*) Does there exist an open embedding of S into a global topological semigroup?

(Simple 3-dimensional examples show that one cannot expect an embedding into a subsemigroup T of G with $L(T) = W$).

The above mentioned problem (*) has been solved in the case of a pointed cone W, i.e. $W \cap -W = \{0\}$. There are three significant embeddings.

(1) $S \hookrightarrow S^{\infty}$: One-point-compactification

(2) $S \xrightarrow{\eta_S} R(S)$: Relatively free topological semigroup over the local semigroup S, i.e. for each partial morphism $f : S \to T$ top sgrp there exist a unique $\bar{f} : R(S) \to T$ with $f = \bar{f} \circ \eta_S$.

$$\begin{tikzcd} S \arrow[r, "\eta_S"] \arrow[dr, "f"'] & R(S) \arrow[d, dashed, "\bar{f}"] \\ & T \end{tikzcd}$$

(3) $S \xrightarrow[\varphi]{} P$, P arises as the quotient of the semigroup of all conal paths (i.e. $\dot{\alpha}(t) \in d\lambda_{\alpha(t)}(1)(W)$) modulo an appropriate homotopy relation.

Wolfgang Weiss, TH Darmstadt

Embedding compact t-semigroups into compact uniquely divisible semigroups - D.R. Brown and J.A. Hildebrant

Many of the results of this paper deal with compact holoids which are called t-semigroups. The class of compact commutative semigroups which are known to be embeddable into compact uniquely divisible semigroups is extended to include:

(i) semigroups which have a totally disconnected semilattice continuous homomorphic image whose point inverses are power-ideal semigroups;

(ii) holoids such that the down set of each idempotent is a chain and each idempotent has a neighborhood in its core which is contained in the image of the map $x \mapsto x^2$;

(iii) t-semigroups such that each idempotent has a finite down set and a t-top free neighborhood in its core; and

(iv) semigroups containing a cancellative element in their divisor.

The latter result is obtained using a construction suggested by A.D. Wallace in his 1953 unpublished notes on topological semigroups.

Conditions under which the various hypothesis for the embedding theorems can be met are discussed along with examples which demonstrate these results and their limitations. As a consequence of the final embedding theorem, it is proved that if S is a compact power-cancellative commutative semigroup such that each element of $S \setminus \{0\}$ is cancellative and $\mathbb{D}(S) \neq \{0\}$, then $S \setminus \{0\}$ is embeddable into a cone of a real topological vector space. It is also proved that if this condition on $\mathbb{D}(S)$ is replaced by finite dimensionality, then $S \setminus \{0\}$ can be embedded into a cone in $\mathbb{R}^n$.

John A Hildebrant
Department of Mathematics
Louisiana State University
Baton Rouge, LA 70803 USA
Bitnet: MMHJLD@LSUVAX

Embedding semigroups into Lie groups

Let S be a cancellative topological semigroup on a connected Euclidean manifold. One can make sense locally of left quotients $s^{-1}t$ and obtain a local group which is locally Euclidean and which admits in a natural way a local right action on S. By a result of Jacoby, the locally group is locally isomorphic to a simply connected Lie group $\tilde{G}(S)$. Form the product $S \times \tilde{G}(S)$. There exists a topology finer than the product topology on $S \times \tilde{G}(S)$ such that

$$S \leftarrow S \times \tilde{G}(S) \rightarrow \tilde{G}(S)$$

the left-hand map is a covering projection and the right-hand is a local homeomorphism. The analytic structure on $\tilde{G}(S)$ pulls back to S to make it an analytic semigroup. One component of $S \times \tilde{G}(S)$, call it $\hat{S}$, is a subsemigroup and the restriction $\hat{S} \to S$ remains a covering projection. The group $\tilde{G}(S)$ acts as deck transformations $g(s,h) = (s, gh)$, and the subgroup leaving $\hat{S}$ invariant is a countable central subgroup, call it G_S. Then $S \to \tilde{G}(S)/G_S \overset{\mathrm{df}}{=} G(S)$ is the free group on the semigroup S. It is a homeomorphism onto an open subset of $G(S)$ and $G(S)$ is a Lie group iff S is algebraically embeddable in a group. In many other cases (e.g. S simply connected) it is known to be a local homeomorphism.

Jimmie Lawson, Louisiana State University
Baton Rouge, LA 70803 USA

The Semigroup βN and its applications to Number theory.

The operations $+$ and $\cdot$ on N extend to its Stone-Čech compactification βN making βN a compact left topological semigroup. We discuss the history of the application of these operations to results in Ramsey Theory (combinatorial number Theory) including some very recent proofs of van der Waerden's Theorem.

Neil Hindman
Howard University
Washington, DC 20059 USA
Bitnet: ALAMA26 @ HUMAIN

Compact semigroups with one-sided continuity

This talk surveyed developments in the theory which took place largely in the last four years. Semigroups of the kind considered appear naturally in the theory of transformation semigroups and as solutions to many universal mapping problems. In the latter class occurs the key example of the Stone-Čech compactification βS of a discrete semigroup S, or more especially $\beta\mathbb{N}$.

We describe just one topic from the survey. A technique was described for obtaining in a simple way strong algebraic results about $\beta\mathbb{N}$ and other semigroups not superficially similar, for example that these contain copies of the free group on 2^{c} generators. The algebraic structure underlying this method has been observed (Papazyan) to correspond to a set of distinct finite sums

$$FS\langle x_n\rangle = \{x_{i_1} + x_{i_2} + \dots + x_{i_m} : i_1 < i_2 < \dots < i_m\}$$

where $\langle x_n\rangle$ is a sequence in the semigroup. If S is cancellative, then any neighbourhood V in βS of any idempotent e in $\beta S\setminus S$ contains a set of distinct finite sums in S (van Douwen, Hindman) so that V actually contains a free group on 2^{c} generators.

John Pym

University of Sheffield, England, S3 7RH.

JANET: PM1JSP @ UK.AC.SHEFFIELD.PRIMEA

Positive definite and related functions on semigroups

This talk gave a survey of positive definite and related functions on abelian semigroups with involution S, with a special emphasis on development since the appearance of the book by Berg-Christensen and Ressel: Harmonic analysis on semigroups. Springer 1984. In the integral representation of pos. def. functions on S we had earlier focused on Radon measures μ on S^* defined on the Borel σ-algebra $\mathbb{B}(S^*)$. It turns out to be fruitful to consider measures μ on the smallest σ-algebra $\mathcal{A}(S^*)$ rendering the evaluations $\rho \mapsto \rho(s)$ measurable, $\rho \in S^*$. The notions of Bisgaard and Ressel of semi-perfect and perfect semigroups were discussed, the first meaning that every $\varphi \in \mathcal{P}(S)$ is a moment function

$$\varphi(s) = \int \rho(s)\, d\mu(\rho)$$

for some μ on $\mathcal{A}(S^*)$, S^* the latter meaning that μ is in addition uniquely determined.

Christian Berg, University of Copenhagen

Dept. of Math. Universitetsparken 5

2100 Ø, Copenhagen, Denmark.

E-mail: Mathcb @ DKUCCC11. Bitnet

Weakly almost periodic functions on groups

Let G be an infinite discrete group.

(1) For $E \subset G$, let E^- be the closure of E in G^w and $\hat{E} = E^- \setminus G$. A subset E of G is called a T-set if $x\hat{E} \cap y\hat{E} = \phi$ if $x \neq y$, $x, y \in G$; E is an R_W-set if $\chi_E \in WAP(G)$ and $E^- \approx \beta E$. R_W-sets have been studied by W. Rudin and W. Ruppert. It is known that T-sets are R_W-sets. We are able to show that every G contains an R_W-set D which is not a finite union of T-sets and hence there exists $\omega \in D^- (\approx \beta D)$ such that ω is not strongly G-discrete.

Question. If $\omega \in \hat{E}$, E a T-set, is G^w homeomorphic to $G^w \cdot \omega$ under the mapping $\alpha \to \alpha\omega$, $\alpha \in G^w$?

(2) Let $DWAP(G) = \{ f \in \ell^\infty(G) : O(f)$ is relatively weakly compact$\}$ where $O(f) = \{ {}_x f_y : x, y \in G \}$, the double orbit of f. In general, $DWAP(G)$ is properly contained in $WAP(G)$. Conjecture. $DWAP(G) = WAP(G)$ iff G is an abelian-by-finite group.

Ching Chou
SUNY at Buffalo
Buffalo, NY 14214, USA

Differentiable Semigroups

A differentiable semigroup is a topological semigroup $(S, *)$ in which S is a C^1 manifold modeled on a Banach space and $*$ is C^1. We have the following results concerning the set $E(S)$ of idempotent elements of S.

Theorem. If C is a component of $E(S)$ there is an open set U containing C so that there is a C^1 retraction $\Gamma : U \xrightarrow{\text{onto}} C$ so that $x\Gamma(x) = \Gamma(x)x$ is in $H(\Gamma(x))$ (the maximal subgroup containing $\Gamma(x)$) for each x in U. It follows that C is a C^1 submanifold of S. It is also shown that each idempotent e is a member of a C^1 subsemigroup of S which is a paragroup R. Moreover, there is a neighborhood V of e in $E(S)$ which is contained in R. Examples include Banach algebras.

J.P. Holmes
Auburn University, ALABAMA USA
36849

Algebraic Varieties and Semigroups

The basic theory of linear algebraic semigroups has been developped by M. S. Putcha and myself, over the past eight years. The most interesting class of objects here is the class of irreducible monoids. The major results in the theory include
(1) a characterization of regular monoids
(2) a numerical classification of normal monoids with reductive unit group.
(3) a classification of normal (completely) regular monoids with solvable unit group.
(4) a generalization of the (group theoretic) Bruhat decomposition to reductive monoids.
(5) a determination of the conjugacy classes in reductive monoids, generalizing the classical Jordan canonical form

Some related developments are also mentioned. For example, there is an important relationship with the equivariant embedding problem for spherical homogeneous spaces (as pointed out in De Concini's (1986) ICM report).

Lex E. Renner
Univ. of Western Ontario
Canada N6A 5B7

Semigroup Compactifications

A semigroup compactification of a semitopological semigroup S is a pair (X, ψ) with X a compact right topological semigroup and $\psi: S \to X$ a continuous homomorphism. A P-compactification is a compactification with a property P. A necessary and sufficient condition (modulo some technical details) for a universal P-compactification to exist is that the property is preserved under subdirect products.

Semigroup Compactifications (continued)

With this theorem it is easy to see that a universal connected compactification, for instance, does not necessarily exist. On the other hand, many do including those defined by identities and implications.

John F. Berglund
Virginia Commonwealth University
Richmond, Va 23284-2014 USA

Non-topological groups

For G a compact, Hausdorff, right topological group ($s \to st$ continuous for all t), let $\Lambda(G) = \{t \in G \mid s \to ts$ is continuous$\}$. G is topological ($(s,t) \to st^{-1}$ is continuous) if $\Lambda(G) = G$, by Ellis (1957). We discuss examples of non-topological G, mentioning the structure theorem of Namioka (1972) for the case when $\Lambda(G)$ is dense, and the structure theorem of Ruppert (1975) for the case when the right translations $\{s \to st \mid t \in G\}$ are equicontinuous. (In Ruppert's case, $\Lambda(G)$ turns out to be closed, hence Ruppert $\cap$ Namioka = topological.) A compact, Hausdorff, right topological group falling into neither category is $G = \{\pm 1\} \times \mathbb{T}$, $(\varepsilon, v)(\delta, w) = (\varepsilon\delta, v^{\delta} w)$, where for $b > a$ $\{(1, e^{i\theta}) \mid a \le \theta < b\} \cup \{(-1, e^{i\theta}) \mid a < \theta \le b\}$ is a basis for neighbourhoods of $(1, e^{ia})$ [b varying] and of $(-1, e^{ib})$ [a varying].

Paul Milnes
Univ. of Western Ontario
London, Canada N6A 5B7.

Extension of topological semigroups by semigroups of right quotients

Let $(S, \cdot, \mathcal{T})$ be a topological semigroup, where $\mathcal{T}$ denotes the set of open sets, and $(T, \cdot) = Q_r(S, \Sigma) = \{ad^{-1} \mid a \in S, d \in \Sigma\}$ a semigroup of right quotients of S with respect to a subsemigroups Σ of S. We consider the problem to establish T with a topology $\mathcal{T}$ such that $(T, \cdot, \mathcal{T})$ is a topological semigroup and $\mathcal{T}|S \subseteq \mathcal{V}$ holds for the trace topology $\mathcal{T}|S$ of $\mathcal{T}$ on S. For the special case that one asks for the stronger conditions $\mathcal{T}|S$ and $S \in \mathcal{T}$, such a top $\mathcal{T}$ exists iff all left and right translations $a \to \alpha a$ and $a \to a\alpha$ of S defined by elements $\alpha \in \Sigma$ are open mappings. In this case $\mathcal{T}$ is uniquely determinated and can be described by the base $\mathcal{L}(\mathcal{V}, \Sigma) = \{U\alpha^{-1} / U \in \mathcal{V}, \alpha \in \Sigma\}$. In general, bases of the form $\mathcal{L}(\mathcal{V}, \mathcal{G}) = \{U\Omega^{-1} / U \in \mathcal{V}, \Omega \in \mathcal{G}\}$ where $\mathcal{G}$ is a topology on Σ are important to describe suitable topologies $\mathcal{T}$ on T. Finally, similarly questions can be answered for right quotient extensions T of S of a more general kind, for instance those due to McMorris.

Hans Weinert
TU Clausthal
D3392 Clausthal-Zellerfeld.

Strong Differentiability in semigroups

In 1938 Garrett Birkhoff showed that a local semigroup with identity, with a neighborhood of 1 homeomorphic to a Banach space and with multiplication strongly differentiable at 1 is a Lie Group. This is not true for topological semigroups. This leads us to believe that strong differentiability at the identity is a strong condition. Indeed, strong differentiability at the identity implies the existence of 1-parameter subsemigroups of local semigroups, S, based on an admissible set provided local compactness of S (in the finite dimensional case) or provided S is closed and there exists an $f: [0,1] \to S$ such that $f(0) = 0$

and f is strongly differentiable at 0 (in the B-space case). It also implies that for a local semigroup S with identity with smooth boundary, ($1 \in \partial S$) based on an admissible subset of a Banach space (in fact a half space) that the boundary of S is a subsemigroup and the maximal subgroup containing 1 is a local group in the boundary of S.

Mitch Anderson
University of Hawaii at Hilo 96720

Convolution semigroups of probability measures: Stability and self-decomposability. W. HAZOD, Dortmund

Stable, semistable and self decomposable probabilities on $\mathbb{R}$ or on $\mathbb{R}^d$ can be characterized as the possible limit distributions of suitably normalized sums of independent random variables, on the other hand by certain functional equations for the Fourier transforms. The latter can be understood as relations of the corresponding convolution semigroups.

It is possible to generalize the second concept to probabilities on locally compact groups and it turns out - at least for stable measures in full generality, that it is sufficient to consider simply connected Lie groups.

For this class of groups we obtain a description ~~of~~ of the collection of all possible stable, semistable resp. selfdecomposable measures.

Moreover, again in analogy to the vector space case, on nilpotent Lie groups stable and semistable measures can be characterized as possible limit distributions of normalized products of group-valued variables (s. S. Nobel, ph. D. Thesis, Dortmund.)

The proofs are based on the special structure of the semigroup $M^1(G)$, probability is not involved!

6 ↳

Applications of Lie semigroups in analysis / J. Hilgert, Erlangen

Let G be a Lie group and $\pi: G \to \mathcal{U}(\mathcal{H})$ a unitary representation of G. Consider analytic extensions $\hat{\pi}: \Gamma \to \mathcal{C}(\mathcal{H})$ of π where Γ is a complex manifold with Shilov boundary G and at the same time a semigroup, $\hat{\pi}$ is an extension of π (holomorphic) and $\mathcal{C}(\mathcal{H})$ is the semigroup of contraction operators on $\mathcal{H}$. Examples of this type of analytic extension have been considered by

- Kramer, Moshinsky, Seligman, Brunet ('73-'85), $G = Sp(n,\mathbb{C}) \cap U(n,n)$, π the (projective) representation associated to the CCR, as a computational device in nuclear physics.
- Howe ('87), $G = Mp(n,\mathbb{R})$ (the metaplectic group), π the metaplectic representation, as a computational device to prove some estimates for symbols of pseudodifferential operators.
- Gelfand-Gindikin, Ol'shanskii, Stanton ('77-'85), G hermitean symmetric, π in the holomorphic discrete series, in order to construct Hardy spaces on which the holomorphic discrete series can be realized in a uniform fashion

It turns out that the first two examples are essentially the same. G.I. Ol'shanskii gave a general construction of the analytic extensions for simple groups. The methods used are flexible and can be used for different groups as well.

J. Hilgert

Sublattices of $\mathbb{R}^n$ / Gierz & Stralka, Riverside, USA

Let $n \in \mathbb{N}$ be a positive integer. A family $\alpha_{ij}: [0,1] \to [0,1]$, $1 \le i,j \le n$ of ~~lower~~ upper semicontinuous monotone functions satisfying (1) $\alpha_{ij}(1) = 1$ $\forall i,j$ (2). $\alpha_{ii}(x) = x$ $\forall i, \forall x$ (3) $\alpha_{ij} \circ \alpha_{jk} \ge \alpha_{ik}$ $\forall i,j,k$ is called a <u>$\wedge$-seam</u>.

<u>Theorem</u> If $(d_{ij})_{1\le i,j\le n}$ is an n-seam, then $L = \{(x_1,\dots,x_n) \in [0,1]^n : x_i \le d_{ij}(x_j)\ \forall i,j\}$ is a closed connected sublattice containing $\vec{0}$ and $\vec{1}$. Conversely, every closed connected sublattice of $[0,1]^n$ that contains $\vec{0}$ and $\vec{1}$ is of that form.

<u>Applications</u>. A sublattice $L \subseteq \mathbb{R}^n$ is <u>full</u>, if it is compact and if the interior of L is dense in L and connected.
A point $x \in \partial L$ is a <u>$\mathcal{C}_1$-point</u>, if there is a nbhd $\mathcal{U}$ of x and a continuous $\varphi: \mathcal{U} \to \mathbb{R}$ such that (1) $\partial L \cap \mathcal{U} = \{p \in \mathbb{R}^n : \varphi(p) = 0\}$ and (2) grad $\varphi(x) \neq 0$.

<u>Theorem</u> If L is full, then $\{x \in \partial L : x \text{ is } \mathcal{C}_1\}$ is dense in ∂L. — A full sublattice $L \subseteq \mathbb{R}^n$ is a <u>lattice sphere</u> if Prime (L) = Coprime (L).

<u>Theorem</u> Up to isomorphy, there is exactly one lattice sphere in dimension 1, 2, and 3. If the dimension is at least 4, then there are uncountably many pairwise non-isomorphic lattice spheres in each dimension.

Jorland Gierz

Analysis on ordered symmetric spaces

Let S be a closed semi-group in a locally compact group G. Then $H = S \cap S^{-1}$ is a closed sub-group, and S defines an invariant ordering on the homogeneous space $X = G/H$. A causal kernel is a function $K(x,y)$ on $X \times X$ vanishing outside of $\{x \geq y\}$. The Volterra algebra $V(X)^{\natural}$ is the space of invariant causal kernels, equipped with the composition product of kernels. If there exists an involution $x \mapsto x^{\#}$ of S such that (i) $(xy)^{\#} = y^{\#} x^{\#}$, (ii) $\forall x \in H$, $x^{\#} = x^{-1}$, (iii) $\forall x \in S$ $x^{\#} \in HxH$, then $V(X)^{\natural}$ is commutative.

Gefördert durch Deutsche Forschungsgemeinschaft

Example: $G = SL(2,\mathbb{C})$, $H = SL(2,\mathbb{R})$, $S = \exp(iC)H$, where C is an invariant cone in the Lie algebra $sl(2,\mathbb{R})$.

Jacques Faraut, Strasbourg.

Topological theories for inverse semigroups and their representations

One may consider analysis of and on inverse semigroups from the viewpoint of S as a star subsemigroup of partial isometries on a Hilbert space (for example this gives easily the list of monogenic ones). The natural topology is the weak operator topology, with separately continuous multiplication and continuous involution. For S with topology it is not clear how to define $C^*(S)$ to generalize the usual $C^*(G)$ for G locally compact group. The key problem is to do so for S a semilattice. A definition is given for some classes, but the 'naturality' of the definition remains a question.

John Duncan, University of Arkansas.

The continuous extended bicyclic semigroup.

We discuss closures of certain semigroups in locally compact topological inverse semigroups. Here we assume multiplication is jointly continuous and inversion is continuous. We first make some background remarks on the free inverse semigroup on one generator and on the closures of the discrete and continuous bicyclic semigroups, as well as on the closures of the discrete and continuous extended bicyclic semigroups. We then list some examples of topological inverse semigroups on the plane to provide a setting for the following application of the previously mentioned results: If S is a topological inverse semigroup on the plane containing no nontrivial groups and whose idempotents form a line, then S ~~contains~~ is the continuous extended bicyclic semigroup.

Annie Selden / John Selden, Tennessee Technological University, Cookeville, U.S.A.

SOME TRENDS AND DIRECTIONS IN THE INVESTIGATION OF CONGRUENCES ON $S(\mathfrak{X})$. The latter symbol denotes the semigroup of all continuous selfmaps of the topological space $\mathfrak{X}$. Those congruences ρ for which $S(\mathfrak{X})/\rho$ is isomorphic to $S(Y)$ for some generated space Y are first determined and it is shown that for "most" spaces there are at most three such congruences. Next, the existence of a largest proper congruence and a smallest proper congruence is investigated. The semigroups of a number of spaces, including all Euclidean N-cells, have a largest proper congruence while the semigroups of many local dendrites with finite branch number do not. On the other hand, it is rare for a semigroup of continuous selfmaps to fail to have a smallest proper congruence although there are examples. Congruences called continuum congruences are then considered and $Con_c(S(\mathfrak{X}))$ the partially ordered family of all continuum congruences on $S(\mathfrak{X})$ is studied. If $\mathfrak{X}$ happens to be a local dendrite with finite branch number, then $Con_c(S(\mathfrak{X}))$ is order isomorphic to a certain family of collections of subcontinua of $\mathfrak{X}$ where $\mathcal{A} \leq \mathcal{B}$ for two such collections means that each $B \in \mathcal{B}$ is the union of copies of subcontinua from $\mathcal{A}$. This fact is then used to obtain such results as a characterization of those local dendrites with finite branch number for which $Con_c(S(\mathfrak{X}))$ is a lattice. Further properties of $Con_c(S(\mathfrak{X}))$ and natural subfamilies are also studied. For example, those Peano continua are characterized for which $Con_c(S(\mathfrak{X}))$ is a finite chain and those Peano continua for which a certain subfamily of $Con_c(S(\mathfrak{X}))$ and the partially ordered family of regular $\mathcal{J}$-classes (i.e., contains at least one regular element) are isomorphic finite lattices, are characterized. Finally, those congruences σ on $S(\mathfrak{X})$ for which $V \circ \sigma = \sigma \circ V$ (where $(f,g) \in V$ means that f and g are mutually inverse to one another) are completely determined for a great many spaces $\mathfrak{X}$. It turns out that there are two such congruences if $\mathfrak{X}$ is connected and six if $\mathfrak{X}$ is not.

Ken D. Magill, SUNY at Buffalo, Buffalo, New York, U.S.A.

Probability theory in semigroups: The aim of this talk ~~is~~ ~~to~~ show that certain problems in probability can be dealt with effectively using semigroup theory. It ~~was~~ is shown that using semigroup methods, it is possible to describe the weak convergence of the sequence of convolution iterates (μ^n) of a probability measure on finite-dimensional (and in some cases, even infinite-dimensional) matrices. Discrete time versions of the voter model (as discussed by Liggett in his book on "Interacting Particle Systems") and the contact process (also discussed in the same book) can be treated using semigroup methods. Certain results and examples in these contexts are presented.

ARUNAVA MUKHERJEA, Dept. of Mathematics, UNIVERSITY OF SO. FLORIDA, TAMPA, FLORIDA 33620-5700, U.S.A.

Amenability of semigroups: This talk summarizes some of the recent developments and open problems on amenability of discrete and semitopological semigroups.

Anthony To-Ming Lau (劉道明)
Department of Mathematics
University of Alberta
Edmonton, Alberta, Canada T6G 2G1

Invariant orders on Lie groups:

Given a Lie Group G and a semigrp. S of G satisfying $gSg^{-1} \subseteq S$ which defines an ordering $x \geq y :\iff y \in Sx$. Are the intervals $D_{ab} = \{x \in G \mid a \leq x \leq b\} = aS \cap Sb^{-1}$ compact?

1st example: Consider the semisimple Lie algebra $sl(2,\mathbb{R})$ and its simply connected Lie group $\widetilde{SL(2,\mathbb{R})}$.

The zero set of the Cartan-Killing form, which is lorentzian, determines an invariant cone in $sl(2,\mathbb{R})$ which generates a subsemigroup S of $\widetilde{SL(2,\mathbb{R})}$ which contains a whole halfspace. Of course, there are noncompact intervals. 2nd example: Let $(V, \langle\cdot|\cdot\rangle)$ be a 2n-dim. Hilbert space with the skewsymm. VS-automorph. d given by the matrix $\begin{pmatrix} 0 & -E_n \\ E_n & 0 \end{pmatrix}$. Consider the solvable Lie algebra $\mathfrak{g} = \mathbb{R} \times V \times \mathbb{R}$ with Lie bracket $[(r,v,z),(r',v',z')] = (0, r d v' - r' d v, \langle dv|v'\rangle)$ and the invariant lorentzian form $q((r,v,z),(r',v',z')) = rz' + r'z + \langle v|v'\rangle$. The zero set of q and the condition $r \geq 0$ determine an invariant cone which generates an invariant subsemigroup $S = \overline{\langle \exp W\rangle}$ in the corresponding Lie group $G = \mathbb{R} \times V \times \mathbb{R}$ with multiplication $(r,v,z)(r',v',z') = (r+r', (v + e^{rd} v'), z + z' + \frac{1}{2}\langle dv, e^{rd} v'\rangle)$ which contains the whole halfspace $[2\pi, \infty[\times V \times \mathbb{R}$. Again, there are noncomp. intervals.

Norbert Dörr, TH Darmstadt,
FR Germany

Measure Algebras on Semigroups This talk was a survey of some of the developments in this field since ca. 1980. The most significant developments (in the opinion of the lecturer) in this field have been in multipliers, Arens regularity and biduals and in weighted measure algebras. The subject of multipliers was not covered in the talk, since Vasudeva had covered the most interesting development in that field. The talk concentrated on results on Arens regularity and biduals of $L^1(G)$ and $M(S,\omega)$ and some work on representations of foundation semigroups and their measure algebras.

John. W. Baker, Dept. of Pure Mathematics, University of Sheffield, S37RH England.

SEMIGROUPS IN CONTROL THEORY:

//The object of the talk was to present some applications or possible applications to the theory of systems. First we discussed the theory of accessibility where these applications have been the most prominent. Then we talked about realisation theory where we feel the semigroup theory could help. Other domains mentioned but not discussed because of lack of time were local controllability and optimal control." I. KUPKA Department of Mathematics University of Toronto Toronto Ont M5S 1A1 CANADA

Semigroupes affines semitopologiques compacts

Ce travail a pour but de présenter quelques aspects de la théorie des semigroupes affines semitopologiques compacts. Dans les sections 4 et 5, deux types de problèmes sont abordés qui ont trait, les uns à l'existence de points de continuité à gauche pour des actions ~~de~~ séparément affines et séparément continues de semigroupes affines semitopologiques compacts (section 4), les autres à l'extrémalité des points de continuité à gauche obtenus (section 5). Ces deux sections ont ceci en commun qu'elles s'appuient l'une et l'autre sur une variante d'un résultat de Namioka concernant la dentabilité. Cette variante est l'outil principal de ce travail; elle joue un rôle comparable à celui que joue le théorème de point fixe de Ryll-Nardzewski dans l'approche originale de la presque-périodicité faible. (Le théorème de point fixe de Ryll-Nardzewski n'est pas utilisé ici; il convient cependant de rappeler que le résultat de Namioka concernant la dentabilité permet d'obtenir le théorème de Ryll-Nardzewski). Les sections 6, 7 et 8 sont consacrées à des applications des résultats établis dans les sections 4 et 5.

Troallic Jean-Pierre, {Faculté des Sciences Université Le Havre, 76600 Le Havre, (25, rue Philippe Lebon). France.

Compact Semitopological Semigroups.

Let S be a ~~compact~~ semigroup, endowed with a topology such that the translations $x \mapsto sx$ and $x \mapsto xs$ are continuous. Then S is called a semitopological semigroup. In the present talk we give an overview over the main topics of research in this area.

WAF Ruppert
Univ f. Bodenkultur Wien
Gregor Mendelstr. 33
A- 1180 WIEN
AUSTRIA / EUROPE

Globality in the Lie Theory of Semigroups

If S is a subsemigroup of Lie group G we get a tangent wedge $L(S) = \{x \in L(G) / \exp(\mathbb{R}^+ x) \subseteq \overline{S}\}$ which is a Lie wedge in $L(G)$. But ~~there~~ are Lie wedges $W \subseteq L(G)$ such that there's no subsemigroup S of G with $L(S) = W$. In the talk we give a characterisation of the Lie wedges which are global, i.e. to which there are subsemigroups of G. We describe the situation for covering morphisms of connected Lie groups and classify the global Lie wedges in groups with compact Lie algebras.

Karl-Hermann Neeb
TH - Darmstadt

Semialgebras in ~~Re~~ reductive Lie algebras

Let L be a finite dimensional real Lie algebra. A wedge W (i.e. $W + W \subseteq W$, $\mathbb{R}^+ W \subseteq W$, $\overline{W} = W$) is called a semialgebra, if there is a CH-Neighborhood B s.t. $(W \cap B) * (W \cap B) \subseteq W$, where $*$ denotes the CH-Multiplication.

Using Lawson's Theorem on tangent hyperplane subalgebras we could show:

Theorem: Let W be a generating semialgebra in a reductive Lie algebra ~~with~~ L. Then there ideals $S_1, \ldots, S_k$ ($k = 0$ may happen), all S_j isomorphic to $sl(2)$ and one ideal L^* in L such that

(i) $L = S_1 \oplus \cdots \oplus S_k \oplus L^*$

(ii) $W = S_1 \cap W \oplus \cdots \oplus S_k \cap W \oplus L^* \cap W$

(iii) All $S_j \cap W$ are (gen.) semialgebras ~~(in sl(2))~~ (in $S_j \cong sl(2)$) and $L^* \cap W$ is an invariant wedge in L^*.

This ~~stc~~ result classifies the semialgebras in reductive Lie algebra, since the semialgebras in sl(2) are well known and for a classification of the invariant cones, see the book, K.H. Hofmann mentions on page 128.

Anselm EGGERT
TH Darmstadt
W-Germany.
No Bihut (sorry!)

Representation Theory for inverse semigroups

The motivation for the study of such a theory arises from operator algebras (such as the Cuntz algebras $\mathcal{O}_n$) which are generated by inverse semigroup representations. The regular representation of such a semigroup S is faithful, so that the representations of S separate points.

The theory depends on developing a "twisted" disintegration theory based on $C^*(E)$, where E is the idempotent semilattice of S. This leads to a quasi-invariant measure on the filter completion X of E with respect to a natural action of S on X in terms of partial 1-1 maps. Associated with this set up is a natural groupoid G, whose elements consist of suitable pairs (s,x) $(s \in S, x \in X)$ and the representation theories of S and G essentially coincide. This allows the well-developed representation theory of groupoids to be applied to give information about inverse semigroup representations.

Alan L.T. Paterson,
University of Mississippi,
MS 38677, USA.

History & Applications of Compact Semilattices

We trace the development of the structure theory of compact semilattices and their applications. The development of the notion of a Lawson semilattice is described, and the structure theory of these semilattices elucidated. Applications of these objects – also known as continuous lattices – to general topology, where they arise as the open set lattice of locally compact sober spaces, and to harmonic analysis are described. In the latter, it is shown that the following holds:

Theorem For a locally compact semilattice S, TAE: 1) The algebra $M(S)$ of all finite regular Borel measures is symmetric.
2) S has compactly finite breadth; i.e. $(\forall K \subseteq S \text{ compact})\ (\exists F \subseteq K \text{ finite})\ \wedge F = \wedge K$.
3) S contains no copy of $2^{\mathbb{N}}$.

If these conditions hold, then $\Delta M(S)$ is the filter semilattice of the discrete semilattice S_d, and so the idempotent measures are direct sums of point masses, and the invertible measures are exactly the exponential measures. ◻

Michael W. Mislove
Tulane University
N.O., LA 70118 USA

Classification of invariant cones

The existence of an invariant cone W in a real Lie algebra L imposes restrictions on the structure of L. In L there is a compactly embedded Cartan algebra H. Viewing L as an H-module, one gets a root decomposition $L = H \oplus \bigoplus_{\omega} L^{\omega}$. The fine-structure of the root system was discussed. H's

Gefördert durch DFG Deutsche Forschungsgemeinschaft

remarkable that an invariant cone W is uniquely determined by its intersection C: = W∩H with the low-dimensional Cartan-algebra H. A cone C is the trace of an invariant cone in L if and only if it is invariant under the Weyl group and a certain set of rank-1-operators. The existence of such C can be read off from a modified root diagram. One looks for a classification theorem for invariant cones in the spirit of the classification of simple complex Lie algebras.

Karlheinz Spindler
TH Darmstadt
Western Germany

Восхищён приёмом, атмосферой, уровнем конференции. Плохое знание языка не позволило мне рассказать обо всём, что сделал. Надеюсь, что эти трудности преодолею в ближайшем будущем. Очень обрадован тем, как интенсивно полугруппы проникают в области, исторически завоёванные теорией групп. Очень надеюсь на дальнейший прогресс и постараюсь внести свою посильную лепту. Дома я буду ещё более настойчиво пропагандировать полугруппы.

Александр Скряго
Краснодар
СССР

MATHEMATICAL MODELS FOR INFECTIOUS DISEASES
FEBRUARY 5, 1989 – FEBRUARY 11, 1989

Analysis of infection rates

Epidemic data typically consist of the times at which individuals show symptoms. On such data one can perform a regression analysis based on a generalized linear model when one assumes that the latent and infectious periods are of constant durations. This provides an effective way of determining whether variables such as age, sex, number of infectives present, etc, affect the risk of an infection taking place. It is important to determine whether the infection rate varies with calendar time as this points to the presence of heterogeneity among susceptibles.

Variation in the infection rate ~~over~~ time can also be explored by nonparametric estimation using the martingale methods of Aalen. This can provide additional insight.

These ideas are illustrated with reference to data from an epidemic of smallpox.

Niels Becker
La Trobe University
Australia.

Multiple attractors in response to a vaccination program in a seasonal SEIR model

Though it is well known that multiple attractors may co-exist in the SEIR (susceptible/exposed/infective/recovered) epidemic model with vital dynamics and seasonally forced oscillations in transmission, the epidemiological significance of multiple attractors has been subject to debate. I show that the co-existence of attractors is relevant in using the model to study a program of vaccinating a fraction of all newborn susceptibles. When vaccination is introduced, the system may be attracted to different periodic orbits. The exact timing of the introduction and the basic reproductive rate determine which orbit is the attractor.

Joan L. Aron
Johns Hopkins University
Baltimore, Maryland, USA

Predator-Prey Interactions Influenced by Parasitic Behaviour.

The behaviour of prey populations is modified by a parasite so as to make is more susceptible to predation. The prey is divided into susceptibles and infectives, each with a different predator functional response. Criteria are obtained for the predator population to survive in the presence of parasites when otherwise extinction would occur. A technique for investigating global stability is also described. Finally, a model where the predator population also consists of infectives and noninfectives is consided. This model can exhibit both Hopf bifurcations and pitchfork bifurcations.

H. I. Freedman, U. of Alberta

On the definition and the computation of R_0

The basic reproductive number R_0 is by definition the expected number of secondary cases produced by a typical infected individual during its entire period of infectiousness, when introduced in a population which is in a steady demographic state with all individuals susceptible. Mathematically it is the dominant eigenvalue of the (linearized) next generation operator

$$(K(S)\phi)(\xi) = S(\xi) \int_{\Omega} \int_0^{\infty} A(\tau, \xi, \eta)\, d\tau\, \phi(\eta)\, d\eta$$

on $L_1(\Omega)$. Here the variable $\xi \in \Omega$ accounts for heterogeneity, S gives the steady distribution of susceptibles and A

describes the infectivity towards susceptibles in state ξ of infectives which were infected τ units of time ago while having state η.

Under certain conditions (proportionate mixing and variants thereof) one can compute or estimate the dominant eigenvalue. Some examples involving discrete groups, age, or propensity to make sexual contacts will be presented.

Odo Diekmann (CWI, Amsterdam & ITB, Leiden)

Some remarks about the modeling of AIDS.

Two stochastic epidemic models are considered to describe and predict the incidence of AIDS among a large group of homosexuals. In both models, the infectious process is assumed to be Poisson with rate α and the length of the incubation period has a random length L. Comparisons of the behavior of the epidemic under different assumptions for the distribution of the incubation period are made using the concept of partial ordering between random variables.

Malice M.P., University of KY,
Lexington, KY USA

Modeling HIV Transmission and AIDS in San Francisco

In the simulation model describing the spread of HIV in the homosexual/bisexual population, infected individuals progress through stages to AIDS and death. Parameter values are obtained so that HIV prevalences and AIDS incidences correspond to the observed values from 1978 to 1987. The model also incorporates changes in sexual behavior which are consistent with changes found in surveys. The patterns of projection into the future are similar for all parameter sets which lead to a fit of the data.

Herbert W. Hethcote
University of Iowa
Iowa City, Iowa 52242 USA

Some variants of Nåsell's model for helminthic diseases with concomitant immunity

The Ross model for malaria and the Nåsell model for helminthic diseases with concomitant immunity (in case of hermaphroditic parasites) may be analyzed as complex models build on elementary models of the Infection - Recovery type. These models are hybrid in two ways, since the infection process in the population of definitive hosts depends on the mean number of infected individuals in the population of intermediate hosts, and not of their exact number, and the infection process in the intermediate population depends on the mean number of parasites in the population of definitive hosts, and not on the exact number of these parasites. This modelisation leads to two families of independant I-R. processes. The purpose of this paper is to build models in which one of the families of I-R. processes is subordinated to the other, f.i. in supposing that the infection process in the population of definitive hosts really depend on the number of infected intermediate hosts and not any more of their expectation. Such a model is hybrid in only one way instead of two as in the usual models. The endemicity condition obtained in the new model is more difficult to met than in the classical models.

PH. PICARD
Université LYON 1
43, Bd du 11 Novembre 1918
69622 - VILLEURBANNE Cedex

Some Epidemiological Models with Periodic Solutions

Several classes of models have been developed for explaining the periodicity which is observed in data from many diseases. Periodic solutions have been found in various models with constant nonperiodic coefficients. One such class incorporates a time delay in the removed compartment, and another class assumes a nonlinear incidence rate generalizing the bilinear mass action. These two formulations are considered and models analyzed to determine their equilibria and stability. Some parameter values yield multiple equilibria with the possibility of periodic solutions arising by Hopf bifurcation. (Joint work with H.W. Hethcote)

P. van den Driessche
University of Victoria, Victoria, B.C.
Canada.

Global stability results for epidemic systems

In Beretta-Capasso (1986), the authors introduced a unified treatment of a wide class of epidemic systems by means of a general ODE system which actually includes many of the models proposed up to now by different authors and analyzed via different "ad hoc" mathematical techniques. Based on the particular structure of the ODE system, sufficient conditions for the global asymptotic stability (hence uniqueness) of the nontrivial equilibrium solution of the system were given. Here we extend the treatment to include more general structures. The new system describes a larger class of epidemic models among which continuous time delays and multigroup models are included.

Vincenzo Capasso
Università di Bari
70125 BARI, Italy

The problem of interindividual dependence in non-linear death processes

Le modèle de décès linéaire classique suppose qu'une population de N individus se comportent de manière indépendante et ont une durée de vie exponentielle de paramètre μ. Dans cette conférence, nous envisageons deux modèles de décès avec dépendance entre individus. Dans le premier, ~~cette~~ dépendance est due à l'influence du nombre d'individus présents sur la durée de vie de chacun d'entre eux. Dans le second, nous supposons que les N individus vivent dans un environnement aléatoire et sont soumis à un processus aléatoire extérieur. Nous montrons dans quelle mesure les sorts des individus sont inter-dépendants en utilisant les notions d'association et d'ortant dépendance entre variables aléatoires.

Claude Lefèvre
Université Libre de Bruxelles
Bruxelles, Belgique

Perturbation and saddlepoint approximation for simple epidemics

The saddlepoint approximation for the probabilities of the number of infectives in a simple epidemic is highly accurate for quite small populations. This enables the accuracy of perturbation approximations to the mean and variance to be evaluated for various kinds of simple epidemic, and gives some guidance on what to expect of perturbation approximations for more general epidemics where saddlepoint approximations are not available

Henry Daniels
University of Cambridge, U.K.

Global behaviour of S.I.S. epidemics in an age-structured population

A S.I.S. model, which incorporate age-structure, is presented and results on the asymptotic behaviour of the solution are reported. The problem is studied in both the limiting cases of pure INTRACOHORT transmission:

$$\text{force of infection} = K(a)\, i(a,t)$$

and pure INTERCOHORT transmission

$$\text{force of infection} = K(a) \int_0^\infty i(a,t)\,da$$

In both cases it is proved the existence of a threshold parameter which discriminates existence of a non-trivial endemic equilibrium. When this exists then it attracts any non-trivial solution, otherwise the epidemics goes to extinction.
While in the intracohort case the method of analysis rests upon reduction to an integral Volterra equation, the intercohort case requires the use of monotonicity techniques within the framework of semi-linear abstract evolution equations.
All the results have been stated in joint papers with S. Busenberg, K. Cooke, H. Thieme.

Mimmo Iannelli
Università di Trento
ITALY

A remark on global stability.

The purpose of the talk was the exposition of elementary methods for the discussion of the asymptotic behavior of some differential systems in the plane. It can be applied easily to a class of models containing the one of Ross for malaria and provides us with a simple proof of the main result of monotone system theory in the two-dimensional case.

JP Gabriel
Université de Fribourg
Switzerland

The Incubation Period for the HIV virus for blood transfusion cases.

The incubation period is defined as the time between acquisition of the HIV virus & being diagnosed as having AIDS. Using U.S. blood transfusion data, we obtain the distribution of the incubation period taking into account the probability of diagnosis & the incidence function of infected individuals. Consideration is made of various age groups & of sex.

Lynne Billard
University of Georgia
Athens, Georgia USA.

THRESHOLD CONDITIONS FOR HIV TO CHANGE THE GROWTH OF A POPULATION

A simple model is proposed for studying the interaction of the dynamics of a host population and the spread of a 'directed contact' disease like AIDS:

$$N_0' = \beta_1 N_1 + \beta_2 N_2 - [\mu_0 + \nu_1 + \nu_2] N_0$$
$$N_1' = \nu_1 N_0 - \mu_1 N_1$$
$$N_2' = \nu_2 N_0 - \mu_2 N_2 - \gamma I$$
$$I' = k(N_2 - I) I / N_2 - (\mu_2 + \gamma) I.$$

The model includes a rough age structure via a juvenile class N_0 and a rough heterogeneity by splitting the adult population into a non-core N_1 and a core N_2. The model differs from most other epidemic models by allowing the population to grow exponentially in the absence of the disease with an exponential rate λ_0. One of the main results is the following: Let $R_1 = k/(\lambda_0 + \mu_2 + \gamma)$ be the basic net reproductive number of the disease. If $R_1 < 1$, then $I(t)/N_2(t) \to 0$ for $t \to \infty$. If $R_1 > 1$, then $I(t)/N_2(t)$ is bounded away from zero for all times.

Horst R. Thieme (Arizona State University, Tempe, AZ 85287, USA)

Historical aspects of the theory of epidemics

The first chain binomial model was constructed and fitted to data of measles epidemics by P. D. En'ko, a physician at the Academy for the Daughters of the Middle-Class of the Smolnyi in St. Petersburg and published in 1889 in the weekly medical journal Vrach — more than sixty years before the Reed-Frost chain binomial model was applied to real data (See K. Dietz, Austral. J. Stat., 30A, 1988, 56-65). The socalled catalytic model of Muench (1959) is to be found in Bernoulli (1760) and Ross (1916). The socalled Kermack-McKendrick (1927) model for the SIR epidemic was analysed by Ross and Hudson (1916) and the SIR endemic was first formulated by Martini (1921) and studied by Lotka (1923).

K. Dietz
Universität Tübingen

Modeling AIDS on random graphs Ph. Blanchard Universität Bielefeld

The individuals of a given society C are considered as vertices of a graph and the edges are supposed to represent realized contact structures. A pair $(\mathcal{G}_C, \Phi)$ consisting of a space of random graphs $\mathcal{G}_C$ and a time ordering of the edges is introduced to modelize the sexual contact graph of the real life. If we consider only complete graphs or complete n-partite graphs, sexual contacts are now realized uniformly. In other words the standard modeling of epidemics using systems of ordinary differential equations appears as a special limiting case. The spread of the epidemics is described using a discrete time stochastic Markov process. We introduce first the general epidemic dynamical system $(\mathcal{G}, \Phi, X, \delta)$ on a space of random graphs and we discuss after that some special models where the underlying random graphs are generated by independent matchings (pairings of sexual partners). Moreover we study the role and meaning of the reproductive number R as a critical parameter in Random Graph Epidemics. We conclude by discussing some results obtained by computer simulation and compare for the same values of all parameters the speed of the epidemic predicted by the random graph model and by the classical models. Computer simulations show that on the graph the epidemic expands much slower

THE EFFECT OF PROSTITUTION ON SEXUALLY TRANSMITTED DISEASES

One-sex models in epidemiology implicitly assume that the numbers of infected males and females are equal and the duration of a partnership is zero. Dietz/Hadeler (1988) developed a two-sex model which takes into account, pairs where each individual is infected or susceptible to infection do not spread the disease, as long as the individuals remain together. But the model does not include liasons or prostitutes, which may be an important factor. To test the liason effect in the Dietz/Hadeler model I considered an additional class of prostitutes who interact only with the male population. The threshold condition for the stability of the noninfected state was derived.

Roland Waldstätter
Cornell University,
Ithaca, New York, U.S.A.

Effects of Social Mixing in the Spread of HIV/AIDS

Two topics are presented. First we report on the formulation and mathematical analysis of single and multiple group models for the sexual spread of the human immunodeficiency virus (HIV) which is the etiological agent for the acquired immunodeficiency syndrome (AIDS). Single group models are shown to be very robust even in the presence of variable infectivity. Multiple group models with variable population size and proportionate mixing are shown to have multiple equilibria.

Secondly, we present two new general methods for incorporating like-with-like preference into one-sex mixing models in epidemiology. The first is a generalization of the preferred mixing equation of Nold and Hethcote & Yorke, while the second comprises a transformation of a general preference function for partners of similar sexual activity levels. Both methods satisfy the constraints implicit in a mixing model. We then illustrate how the transformation preference method behaves and compare it with the standard proportionate mixing

Carlos Castillo-Chavez
Cornell University
Ithaca, New York, U.S.

Threshold conditions for the persistence of an infectious disease in a heterogeneous population

We derive necessary and sufficient conditions for disease persistence in a subdivided population where intergroup transmission is described by proportionate mixing while intragroup transmission may correspond to prefered mixing, proportionate mixing among subgroups or mixing between social and non-social subgroups. The disease persists if and only if one of the following conditions is satisfied i) The disease can persist within at least one group through intragroup contacts, ii) ~~[illegible]~~ The intergroup transmission is sufficiently high. Here the contribution from each group is weighted according to its activity level squared and to the total number of cases caused by intragroup transmission.

Viggo Andreasen
Roskilde University
Roskilde, Denmark

Freddy B. Christiansen
Århus University
Århus, Denmark.

Obtaining Incubation Information from Reported AIDS Incidence.

A joint density of the times of occurrences of the infections, $T_1, \dots, T_n$ and their respective times of diagnosis $T_1+\xi_1, \dots, T_n+\xi_n$ is constructed. A contagion function for the disease and a probability distribution for the incubation period ξ_i are used in the construction of the model. This model is used to study the effect of incubation distribution on the AIDS incidence in a finite community. Simulation results are used to compare with the monthly incidence of AIDS in San Francisco. A special case of this model is the Markovian SIR model. We use this model to examine the differences between ~~the~~ stochastic and the deterministic models for the number of infectives and susceptibles.

Grace Yang
University of Maryland,
College Park, MD USA

THE STATISTICAL ANALYSIS OF INFECTIOUS DISEASE DATA USING EPIDEMIC MODELS

Infectious disease data present several characteristics which necessitate the use of special statistical methods in their analysis. Such characteristics include dichotomous response, cluster sampling and correlated response within clusters. A probability model of infection transmission is used to analyze such data. The model is centered on households and partitions sources of transmission into those within the household and those from the community at large. Both the infectiousness of infected individuals and the susceptibility of exposed individuals is taken into account. Under certain circumstances, the model can be expressed in log-linear form. Examples from influenza epidemics are presented.

Ira M. Longini, Jr. Division of Biostatistics
Emory University, Atlanta, GA U.S.A.

Homogeneous evolution equations for sexually transmitted diseases

A system of eight differential equations for noninfected and infected singles of either sex and for the four types of pairs formed of such individuals describes the major demographic factors such as birth, death, pair formation and separation as well as the transmission of a sexually transmitted disease. The vector field is homogeneous of degree one. Hence there is a related system on the unit sphere and stationary solutions of the latter correspond to exponential solutions of the original problem. This correspondence leads to a concept of stability for exponential solutions (of homogeneous systems). In the present case there is a noninfected exponential solution with an exponent ("the demographic eigenvalue $\hat{\lambda}$") The Jacobian of the vector field at this solution determines a threshold ("the epidemic eigenvalue λ_0") The noninfected solution is stable iff $\hat{\lambda} > \lambda_0$.

K. P. Hadeler, Tübingen.

COUPLING, EPIDEMICS AND CONFIDENCE INTERVALS

We present four applications of stochastic coupling to the general stochastic epidemic. The first application enables us to derive a triangular system of linear equations for the total size distribution. In the second application we construct a sequence of epidemics, indexed by initial susceptible population size, from a birth-and-death process. This enables us to show almost sure convergence of the total size of the epidemic processes to that of a birth-and-death process, and consequently provide a new proof of the stochastic epidemic threshold theorem. In the third application we show that varying the susceptibilities of individuals to the disease slows down the spread of the epidemic. In

The final application we use a coupled family of Barnard Monte Carlo hypothesis tests to provide a Monte Carlo confidence interval for the relative removal rate, based upon the observed total size of an epidemic. The resulting confidence interval is rather wide and an alternative (shorter) interval is presented.

Frank Ball
University of Nottingham
UK

Persistent and stationary solutions in some models for parasitic infections

Three closely related models for macroparasitic diseases are discussed, which describe the dynamics of host and parasite populations. In the first model, which is due to Hadeler and Dietz, the host population is structured by age and parasite load. The parasites influence the host's mortality and fertility. The infection rate depends on the size of the host and parasite populations. The second model is a simplification of the first. It is without age structure and there is no influence of the parasite on the host's fertility. From this model a model due to Anderson & May is derived. The existence of exponential and stationary solutions is discussed for the different models depending on the exact form of the infection rate as a function of host and parasite population sizes. Anderson and May's assumption of a negative binomial distribution of the parasites on the host population is compared with results derived for the first model, for which the distribution of parasites can be calculated.

Mirjam Kretzschmar
Centrum voor Wiskunde en Informatica
Kruislaan 413
Amsterdam, NL

Modeling and Analysing HIV Transmission: The Effect of Contact Patterns

A compartmental model is developed for the spread of HIV in a homosexual population divided into subgroups by degree of sexual activity. The model includes constant recruitment rates for the susceptibles in the subgroups. It incorporates stages for the infectious period and so allows one to vary the infectiousness over the infectious period. A new pattern of mixing, preferred mixing, is defined in which a fraction of the group's contacts can be reserved for within-group contacts, the other being subject to proportional mixing. The main result is that small amounts of mixing between high and low activity groups markedly increases the spread and steady state levels in low activity groups but has only small effects on the rate of spread in high activity groups. This result was demonstrated analytically and through simulations.

Recently we have developed far more general ways of specifying many different types of non-random mixing that we call structured mixing and selective mixing. The former works at the macro level dividing the population into "structural groups" by characteristic and also by "mixing group" by place or type of mixing. The latter works at the micro level, modeling three stages of sexual partner formation: the initial contact, the attractiveness for a sexual relation, and the decision to have a sexual relation with the partner.

John Jacquez
Dept. of Physiology
University of Michigan
Ann Arbor, MI 48109

Carl Simon
Dept. of Mathematics
University of Michigan
Ann Arbor, MI 48109

Applying the Diekmann Thieme model for the spatial spread of infectious diseases

The D-T model is a spatial extension of the Kermack-McKendrick functional differential equation model for the development of a epidemic. For this model D & T independently proved the existence of an asymptotic speed of radial expansion. To apply the D-T model in practice one has to devise well fitting parameter scarce submodels for the integral kernel, and corresponding parameter estimation procedures. My talk describes the results of cooperative efforts to this end by Frank v.d. Bosch, Jan Carel Zadoks from the agricultural university in Wageningen and myself.

Estimation of malaria infection and recovery rates.

A Markov chain model for malaria infection of human hosts is established. The model allows for superinfection, relapses and false negatives. Limited superinfection is assumed, as discussed by Nåsell [1986]. For fixed number of superinfections the host is in one of three states, i.e. newly infected, relapsed or latent. Two additional states are introduced to allow for the possibility that a newly infected or relapsed individual is falsely diagnosed as not patent. The parameters in the model are estimated using the maximum-likelihood method. The estimation is based on longitudinal parasitological data from the Garki project and estimates of misclassification probabilities by Nedelman [1988]. The results are used to test the hypothesis that there is limited superinfection in malaria. The preliminary results support this hypothesis for all host ages.

Ingemar Nåsell
Dept. of Mathematics
Royal Institute of Technology
S-100 44 Stockholm, Sweden.

Mathematicals models in immunology.

The defence of an human organism against viral and bacterial infections and the response of the immune system to contamination are the basic problems of clinical medicine. We present three mathematical models of antiviral and antibacterial immune response of the following form.

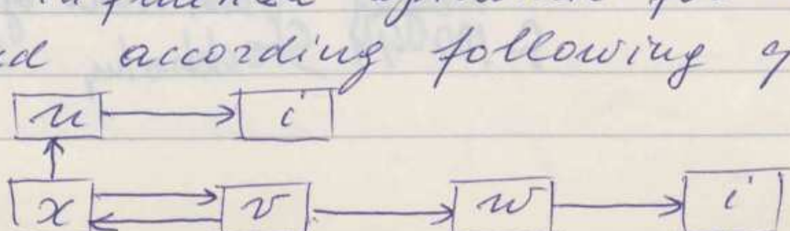
$$\begin{cases} \frac{dy(t)}{dt} = f(y(t), y^{[1]}(t-\tau_1), \dots, y^{[m]}(t-\tau_m)), \; y \in R^N, \; y^{[i]} \in R^{N_i}, \\ \qquad N_i \le N, \; i = 1,2,\dots,m; \quad t_0 \le t \le t_0 + T, \\ y(t_0) = \varphi^0 \\ y^{[i]}(t) = \varphi^{[i]}(t), \quad t \in [t_0 - \tau_i, t_0). \end{cases}$$

The simplest mathematical model of an infectious disease was used to investigate the general laws of immune system reaction to an antigens. The mathematical model of an antiviral immune response was used for modeling the acute form of viral hepatitis B. The matematical models of antiviral and antibacterial immune responses were used for investigation of biinfections in lungs.

G. Marchuk, A. Romanyukha, G. Bocharov
Department of Numerical Mathematics
USSR Academy of Sciences, Moscow,
~~Leny~~ Leninsky prospect 14, building 1.

The mathematical modelling of spreading of influenza viruses which are resistant to a chemical drug

The Rvachev's generalized mathematical model of a local influenza epidemic for a single city was used according following graph:



where x - susceptible; v and u - infected with drug-sensitive strain whom a drug is (or not) administered; w - infected with drug-resistant strain; i - immune.

Using of the desision of system equations in case of disrete time 160 experimental epidemics have been simulated with parameters of the influenza epidemic in Moscow, 1969, and have been studied for varied parameters of quota of population using a drug, of effectiveness of a drug and of frequency of virus mutation

Results. For the small effectiveness of the anty-influenza chemical drugs their unlimited usage do not influence appreciable evolution of resistance of influenza viruses. During application of a drug having the high effectiveness the selection of the resistant influenza viruses is highly probable and only reasonable limitation of drug administration do not result in the substitution of the drug-susceptible influenza virus variant on the drug resistant one.

G. G. Ivannikov

Laboratory of General Epidemiology and Cybernetics, All-Union Research Institute of Influenza, Leningrad, USSR)

RANDOM GRAPHS & HETEROGENEOUS EPIDEMICS

The themes of the talk were sensitivity (structural as well as parameter variation), the importance of expressing models in terms of basic ecological parameters; and ways of making use of the detailed structure of stochastic models. These were illustrated by examples ranging from back-of-envelope 'pre-models' to spatial and network models.

Denis Mollison
AMS
Heriot-Watt Univ.
Edinburgh EH14 4AS
[email: amsdm@uk.ac.hw.vaxa]

References: Barbour & Mollison (to appear in proceedings of Marseille meeting of Oct'88)
Cox & Durrett (1988) Stoch Procs Appl. 30, 171-191
Mollison (1986) Phil Trans. Roy. Soc. B 314, 675-693.

Mehrdimensionale Konstruktive Funktionentheorie
12 – 18 February 1989

Some negative results in multivariate approximation

The aim of this paper, which represents joint work with Norbert Kirchhoff, is to apply a general nonlinear uniform boundedness principle with rates, obtained previously, in connection with some negative results, concerned with the approximation on the N-dimensional Euclidean space $\mathbb{R}^N$ by (nontrivial) convolution processes $\{T_n\}$ of Fejér's type. More precisely, we are interested in the pointwise sharpness of well-known uniform estimates $\|T_n f\|_C = \mathcal{O}_f(n^{-\alpha})$ for elements f, belonging to some Lipschitz class in, e.g., the space $C_{2\pi}$ of functions f, defined and continuous on $\mathbb{R}^N$, 2π-periodic in each variable. The main result states that there exist counterexamples f_α in that Lipschitz class for which $\limsup_{n\to\infty} |T_n f_\alpha(x)| n^{\alpha} \geq 1$ holds true simultaneously for each $x \in \mathbb{R}^N$. Whereas the existence of complex-valued functions f_α is rather easily established, the real-valued situation is somewhat more involved. The problem is also discussed in the space $C_0(\mathbb{R}^N)$ of functions, continuous on $\mathbb{R}^N$ and vanishing at infinity. Explicit applications are mentioned in connection with the Bochner-Riesz means in $C_{2\pi}$ and the Cauchy-Poisson integral in C_0.

R. J. Nessel (RWTH Aachen)

Interpolation and Parallel Two-Dimensional Data Compression

Starting with the Paley-Wiener theorem which forms the classical result for information-preserving sequential bandwidth compression, and its Stone-von Neumann-Segal analogue for the real Heisenberg nilpotent Lie group which is at the basis of holographic reciprocity, the lecture points out a unified approach to parallel two-dimensional data compression by holographic image processing. Brief descriptions of hardware implementations are also included.

Walter Schempp (Siegen)

Asymptotic Expansions of Hankel Transforms...
and what they have to do with multivariable approximation:

Using the theory of asymptotic expansions of Hankel integrals, we establish sufficient conditions on a radial basis function $\phi : \mathbb{R}_{\geq 0} \to \mathbb{R}$ to admit cardinal interpolation on the grid $\mathbb{Z}^n$. More specifically, we aim to find a fundamental function $C : \mathbb{R}^n \to \mathbb{R}$ which is a linear combination of integer translates $\{\phi(\|x-j\|_2) \mid x \in \mathbb{R}^n\}$, $j \in \mathbb{Z}^n$. Our sufficient conditions depend on a derivative of $\phi(\sqrt{\cdot}) : \mathbb{R}_{\geq 0} \to \mathbb{R}$ being "multiply monotonic" and having prescribed asymptotic behaviour near 0 and for large argument.

Martin Buhmann, University of Cambridge.

Multidimensional Band-limited functions: Generalization of the Sampling Theorem, $L_q([-T,T]^n)$ - approximation by finite sampling sums and ε-entropy and ε-dimension

The well-known Whittaker-Kotelnikov-Shannon sampling theorem states that every signal function f which is band-limited to $[-\sigma,\sigma]$ can be completely reconstructed from its sample values $f(\pi k/\sigma)$, $k \in \mathbb{Z}$. Taking the information sence, discovered by Kotelnikov and Shannon, of this theorem as the basic idea Kolmogorov and Tikhomirov introduced the ε-entropy per "length unit" and the mean ε-dimention. These quantities were studied by Tikhomirov, Dinh-Dung and others. The ε-entropy per "length unit" and the ~~ε-entr.~~ mean ε-dimension are suitable for expression of the associated methods of approximation only in the case, when the ε-entropy or the ε-dimension of the trace on the cube $[-T,T]^n$ of a ~~set~~ function set, which we want to approximate, approximately proportional to the volume of this cube.

We suggest a new approach to the study of the ε-entropy and the ε-dimension in the space $L_q([-T,T]^n)$. for several cases when the approximate proportionality does not hold. The main result states that these quatities are approximatety proportional to some power $s>0$ of the volume of the cube $[-T,T]^n$ for some sets of band-limited functions and sets of smooth functions. In the proofs the $L_q([-T,T]^n)$ approximation by finite sampling sums plays a central rôle. We are concerned with some generalizations of the sampling theorem and analogues of Marcinkiewicz theorem for band-limited functions

Dinh-Dung
Institute of Computer Sc. and Cybernetics
LIEU GIAI BA DINH, HANOI, VIETNAM.

Index Transforms for Multidimensional DFT's

Index transforms of m-dimensional arrays into n-dimensional arrays play a significand role in many fast algorithms of multivariate discrete Fourier transforms (DFT's) (e.g. prime factor algorithm, Winograd algorithm). By an index transform of the input data, the m-dimensional DFT can be transferred to an n-dimensional DFT of „short lengths" ($n > m$). Then by efficient algorithms for the one-dimensional DFT's of short lengths, the n-dimensional DFT is computed.

C.S. Burrus (1977), H.J. Nussbaumer (1981) and J. Heleroba (1987) dealt with properties of index transforms. In a joint work with Gabriele Steidl, the nature of index transforms is explored using group-theoretical ideas. We solve the open problems posed recently by J. Hekrdla (Numer. Math. 51 (1987)).

M. Tasche (Wilhelm-Pieck-Universität Rostock, DDR)

An FFT Scheme for Boolean Sums of Trigonometric Operators

It is known that bivariate functions from a Korobov space can be well approximated by Fourier partial sums of hyperbolic type. For practical computations it is desirable to use discrete rather than transfinite Fourier coefficients. - We construct a pseudohyperbolic trigonometric Boolean sum operator which is interpolatory, yields asymptotically the same error bounds as the hyperbolic Fourier partial sum operator and whose coefficients can be efficiently computed by a bivariate FFT scheme.

Günter Baszenski (Ruhr-Universität Bochum)

Some Applications of multivariate rational approximation to Differential-Equations.

Rational Approximation can sometimes be better as polynomial approximation in the neighbourhood of certain types of singularities. We refer in this paper about experiments and calculations (mostly during the last year) for problems with unbounded domains and corners, edges a.o, especially for the Laplace equation in two and in three dimensions. It is often possible in these cases to give easy calculable lower and upper bounds for the wanted solutions. Numerical results are given. Lothar Collatz, Hamburg, University.

Shape-Preserving Quasi-Interpolation and Interpolation by Box Spline Surfaces

In joint work with Charles Chui and Harvey Diamond,

(Continued after Prof. Freeden's abstract)
p 174

Spherical (Vector) Splines

Spherical splines are defined by use of well-known properties for spherical harmonics and the concept of Green's surface functions with respect to iterated Beltrami derivatives and (invariant) pseudodifferential operators. Natural spherical splines are used to interpolate data discretely given on the sphere. An a-priori estimate in spherical spline interpolation is given dependent on the spacing of the data. As example spherical spline interpolation is discussed for the problem of determining the external gravitational potential and the geoid of the earth. Finally vector spherical splines and Green's tensors on the sphere are introduced to generalize the scalar theory to the vectorial case.

Willi Freeden, RWTH Aachen

(Continued from previous page)

we study shape-preserving approximation and interpolation of functions by box spline surfaces on three and four directional meshes. The properties of positivity, monotonicity, and convexity are considered. A characterization of the grid spacing is given which guarantees the preservation of these properties for functions in certain Lipschitz classes.

Louise A Raphael
Howard University
Washington, DC

On multidimensional Lebesgue-Stieltjes convolution operators

Let $(D_\rho)_{\rho \geq \rho_0}$ be a family of onedimensional, continuous, nonnegative and even kernel functions with integral value 1 over the whole real line which satisfy for each $\varepsilon > 0$ the approximate identity condition

$$\lim_{\rho \to \infty} \int_{|t| > \varepsilon} D_\rho(t)\, dt = 0.$$

Moreover, we consider the family of integral kernel functions $(J_\rho)_{\rho \geq \rho_0}$ generated by $(D_\rho)_{\rho \geq \rho_0}$ via

$$J_\rho(x) := \int_0^x D_\rho(t)\, dt \;, \; x \in \mathbb{R}, \; \rho \geq \rho_0.$$

In some formal analogy of the n-dimensional radial convolution operators $(\Delta_\rho)_{\rho \geq \rho_0}$,

$$\Delta_\rho(f)(x) := \left\{ \int_{\mathbb{R}^n} D_\rho(\|t\|_2)\, dt \right\}^{-1} \int_{\mathbb{R}^2} D_\rho(\|t - x\|_2)\, f(t)\, dt \;, \; x \in \mathbb{R}^n,$$

which we assume to be well-defined even on a properly chosen subset of $L_1(\mathbb{R}^n)$, we introduce the n-dimensional hyperbolic Lebesgue-Stieltjes convolution operators $(\Omega_\rho)_{\rho \geq \rho_0}$,

$$\Omega_\rho(f)(x) := (-1)^n\, 2^{1-n} \int_{\mathbb{R}^n} J_\rho\left(\prod_{k=1}^{n} (t_k - x_k) \right) df(t) \;, \; x \in \mathbb{R}^n,$$

which we show to be well-defined and useful on a properly chosen subset of $BV(\mathbb{R}^n)$.

Burkhard Lenze, FernUniversität Hagen

Box-Spline Tilings

Let $f : \mathbb{R}^d \to \mathbb{R}$ be a real analytic function with $|f(x+j)| \to \infty$ as $\mathbb{Z}^d \ni j \to \infty$ for almost all $x \in \mathbb{R}^d$. Then, the translates of the set

$$\Omega(f) := \{ x : |f(x)| < |f(x+j)| ,\ j \in \mathbb{Z}^d \setminus 0 \}$$

form a tiling of $\mathbb{R}^d$. More precisely,

(i) $\quad \overline{\Omega} \cap j + \Omega \ , \quad j \neq 0$

(ii) $\quad \text{measure} \left(\mathbb{R}^d \setminus \bigcup_j j + \Omega \right) = 0 .$

We discuss in detail the particular case

$$f(x) := (\xi \cdot x) \cdot (\eta \cdot x)$$

with $\xi, \eta \in \mathbb{Z}^2$ which arises in the characterization of functions of exponential type as limits of box-spline series.

Klaus Höllig, Univ. Stuttgart

Radial Basis Functions with the L_1-norm

We consider the subspace H of $C(\mathbb{R}^s)$ generated by the n functions $h_1(x) = \|x - x_1\|_1, \ldots, h_n(x) = \|x - x_n\|_1$ where $x_1, \ldots, x_n$ are distinct points in $\mathbb{R}^s$. We give the following necessary and sufficient condition for these functions to be suitable for interpolation (i.e. the matrix $(\|x_i - x_j\|_1)$ is invertible): the functions $h_1, \ldots, h_n$ should simply be linearly independent. If $s = 2$ there is a simple geometrical condition which determines whether the h_i are linearly independent. We also discuss the nature of H as a subspace of $C(\mathbb{R}^s)$, showing first that it is a subspace of sums of continuous univariate functions (which is elementary), and then further that it ~~is~~ consists of piecewise linear functions. Some numerical consequences of these properties are also discussed.

W. A. Light
University of Lancaster,
England.

Some results on quadratic splines
of three (and more) variables

The talk was devoted to the following question:
Given prescribed function values and gradients
at the vertices of a simplicial complex K,
is it possible to construct a refinement K' of K
such that there is a unique quadratic
C^1- spline function with respect to K',
interpolating the given data?
It was shown how to solve that problem
in the 3-dimensional case, when K
satisfies a certain geometric condition.
The underlying concept is independent
of the number of variables.

G. Worsey Univ. Wuppertal

Polyharmonic Cardinal Splines

Polyharmonic cardinal splines are distributions
which are annihilated by iterates of the Laplacian in
the complement of a lattice in Euclidean n-space and
satisfy certain continuity conditions. Here, we review
some of their properties which are remarkably similar
to the well known properties of the univariate cardinal
splines of odd degree as considered by I. J. Schoenberg,
Cardinal Spline Interpolation, CBMS Vol 12, SIAM Phila., 1973.

W. R. Madych
Univ. of Connecticut
USA

Existence of a local & stable basis for certain bivariate pp spaces.

It is shown that the <u>existence</u> of a local and stable basis for $\pi^{\rho}_{k,\Delta}$ (:= piecewise polynomials of degree $\le k$ on some triangulation Δ and in $C^{\rho}(\mathbb{R}^2)$) can already be inferred from [de B. & HÖLLIG, MATH. Z. 197 (1988), 343-363] (in the case $k \ge 3\rho+2$ in which local bases have recently been constructed by Dong, Chui & Lai, and Ibrahim & Schumaker). The argument reduces the construction task to finding a basis for $\pi^{\rho}_{2\rho,\Delta_0}$, with Δ_0 the partition ~~obtained~~ formed by all the triangles from Δ having a particular vertex in common.

CARL de BOOR,
MADISON WI U.S.A.

Homological Methods for Multivariate Splines

For a triangulated d-manifold $\Delta \subset \mathbb{R}^d$, let $\mathbb{P}^r_m$ be the sheaf of real vector spaces over Δ determined by $\mathbb{P}^r_m(\sigma) = P_m / I_\sigma^{r+1}$ for $\sigma \in \Delta$, where P_m is the space of all polynomials in d variables of degree at most m, and $I_\sigma = \{f \mid f|_\sigma = 0\}$. If $H_*(\mathbb{P}^r_m)$ denotes homology with coefficients in $\mathbb{P}^r_m$, then $H_d(\mathbb{P}^r_m) = S^r_m(\Delta)$, the space of all C^r piecewise polynomials of Δ of degree at most m. For $d=2$, we use this homology theory to derive lower bounds for $\dim S^r_m(\Delta)$ as well as the generic dimension of $S^1_m(\Delta)$.

Louis J. Billera
New Brunswick, NJ
USA

Ideals & Box Splines Theory

We introduce a map $H \to H_\downarrow$ that assigns to every finite-dimensional space of entire functions (in s variables) a corresponding space of polynomials of the same ~~deg~~ dimension. This map is dual to another map $I \to I_\uparrow$ which assigns to every ideal I which has a finite codimension in the space π of all polynomials on $\mathbb{C}^s$ a homogeneous ideal I of the same codimension. The duality is expressed by the fact that

$$I\perp_\downarrow = I_\uparrow \perp,$$

with $I\perp$ the kernel of I i.e., $I\perp = \{f \in \mathcal{D}'(\mathbb{R}^s) \mid p(D)f = 0, \forall p \in I\}$. The duality allows us to compute in certain cases the kernels of homogeneous ideals in terms of the kernels of ~~homogeneous ideals~~ non-homogeneous counterparts.

These observation are important in the analysis of the space H of all exponentials in a box spline space. In particular, we provide an algorithm for the construction of a basis for H, and give an elementary derivation of its dimension

Amos Ron
Madison, Wisconsin, U.S.A.

Problems in Multivariate Spline Interpolation

The problem of cardinal Hermite interpolation is discussed. The general problem begins with q compactly supported functions φ_ν, $\nu = 1, \dots, q$, on $\mathbb{R}^s$, and q differential operators T_ν, $\nu = 1, \dots, q$. The question is whether there is a function of the type

$$S = \sum_{\nu=1}^{q} \sum_{j \in \mathbb{Z}^s} a_\nu(j)\, \varphi_\nu(\cdot - j)$$

that satisfies the interpolation conditions

$$T_\nu S(k) = f_\nu(k), \quad \forall k \in \mathbb{Z}^s \quad \text{and} \quad \nu = 1, \dots, q,$$

for specified data $f_1, \dots, f_q$. A general solution to the problem is given, but this solution admits fundamental solutions of power growth. In the case when $\varphi_\nu = T\varphi$, φ is a box spline with certain restrictions, then a bounded $L^2(\mathbb{R}^s)$ fundamental solution is found if T_ν are successive directional derivatives. The open problem is whether there is a nice class of box splines for which Hermite cardinal interpolation of this type is correct; i.e., there is a unique solution (bounded) for bounded data.

S D Riemenschneider
Univ. of Alberta, Edmonton
CANADA

A Dual Basis for the Integer Translates of an Exponential Box Spline

Let $X = (x^1, \cdots, x^n) \subset \mathbb{R}^s \setminus \{0\}$ and $\mu \in \mathbb{C}^n$. The exponential box spline $C_\mu(\cdot|X)$ is the linear functional on $C(\mathbb{R}^s)$ given by

$$\phi \mapsto \int_{[0,1]^n} e^{-\mu \cdot u} \phi\left(\sum_{j=1}^{n} x^j u_j\right) du, \quad \phi \in C(\mathbb{R}^s).$$

When X spans $\mathbb{R}^s$, $C_\mu(\cdot|X)$ is a piecewise exponential polynomial function. In this talk we construct a dual basis $(\lambda_\alpha)_{\alpha\in\mathbb{Z}^s}$ for the integer translates $C_\mu(\cdot-\beta|X)$, $\beta \in \mathbb{Z}^s$,

$$\lambda_\alpha C_\mu(\cdot-\beta|X) = \delta_{\alpha\beta},$$

when those translates are linearly independent. The dual basis is shown to be unique in a certain sense. Our construction is based on a systematic study of the space $G_\mu(X)$ which consists of all the polynomials p such that $p(D)C_\mu(\cdot|X)$ is a bounded function.

Rong-Qing Jia, Zhejiang University, PRC
and University of Alberta, CANADA.

Constrained Interpolation

An algorithm for bivariate interpolation by quadratic splines is discussed. This is related to the algorithm of B. and Z. Ziegler which appeared in SIAM J. Numerical Analysis in 1985. An $O(\delta^2)$ estimate of the uniform error holds in general, but this can be improved in special cases. In particular at a point where $\frac{\partial f}{\partial x} > 0$ and $\frac{\partial f}{\partial y} > 0$ the error is $O(\delta^3)$.

Also if the behaviour in the neighbourhood of the zeros of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ is restricted in a certain way the global error is $O(\delta^{8/3})$

Rick Beatson
Christchurch, New Zealand

Bivariate Cardinal Interpolation by Shifted Box Splines On a 3-Direction Mesh

In this joint work with J. Stöckler and C. Chui, we show that bivariate cardinal interpolation by shifted box splines on a 3-direction mesh is correct if the shifts lie in the appropriate subset of the ½-square as described by Sivakumar. This result gives an appropriate generalization of the univariate result obtained independently by Micchelli and de-Boor-Schoenberg. The result is obtained by a careful analysis of the "symbol" of the interpolation operator.

Joseph Ward
College Station, Tx, USA

Adding Corners Sometimes Works

As a counterpart to Carl de Boor's paper "Cutting Corners Always Works" the problem of constructive subdivision to generate interpolating curves or surfaces is considered (see also the lecture given by Prof. Ulveras).
Two types of subdivision algorithms are considered and convergence of a class of rather elementary subdivision methods is proven. More elaborate algorithms involve first derivatives and practically yield nice-looking surfaces. However, the theoretical investigation of these algorithms still is incomplete.

Robert Schaback, Göttingen.

Singular p-norm distance matrices

The given work was done without my assistance by my student B.J.C. Baxter. Let $x_1, x_2, \ldots, x_m$ be distinct points in $\mathbb{R}^d$ where $m \geq 2$ and where d is any positive integer. Then the $m \times m$ matrix A with elements $A_{ij} = \|x_i - x_j\|_p$ is a p-norm distance matrix. It is known already that such matrices are always nonsingular for $1 < p \leq 2$, and this paper proves that singularity can occur for any $p > 2$. We let d be the even integer $2n$ and we let the points $\{x_i\}$ have the components $(\pm1 \ldots \pm1\ 0 \ldots 0)$ or $(0 \ldots 0\ \pm1 \ldots \pm1)$, the number of zero components being n which implies $m = 2^{n+1}$. We ask whether a nonzero vector of the form $(a \ldots a\ b \ldots b)$ can be in the null space of A. We find that p has to satisfy a single nonlinear equation that depends on the Bernstein polynomial approximation of degree n to the function $\{f(\theta) = \theta^{1/p} : 0 \leq \theta \leq 1\}$ at $\theta = \frac{1}{2}$. It follows from properties of these approximations that A can be singular for p arbitrarily close to 2. Further, by scaling some of the vectors $\{x_i\}$, it can be shown that singularity is possible for all larger values of p, which completes the proof.

M.J.D. Powell
University of Cambridge.

Methoden der Fourier-Transformation bei periodischer kardinaler Interpolation

Abminderungsfaktoren spielen bei der numerischen Fourier-Analyse eine entscheidende Rolle; hierbei werden die (äquidistant vorgegebenen) Daten zunächst einem Operator vom Faltungstyp unterworfen. Dieses Vorgehen, das inzwischen im univariaten Fall als Standardtechnik bezeichnet werden kann, wurde erst kürzlich von Gutknecht und von ter Morsche auf den mehrdimensionalen Fall übertragen. Dabei kommen kardinale Interpolationsoperatoren auf der Basis von Translaten sog. Box-Splines zum Einsatz.
Der Vortrag greift diese Fragestellung auf und diskutiert einen Algorithmus, der z.B. in der graphischen Bildverarbeitung eingesetzt werden kann. Der Diskretisierungsfehler kann hier leicht kontrolliert werden. Beispiele zeigen die Effizienz des Algorithmus.

Kurt Jetter, Duisburg

Bivariate Hermite interpolation and Algebraic Geometry

The (p,n) "Hermite interpolation" in $\mathbb{R}^2$ concerns polynomials P of total degree n. If some m interpolation knots $Z=(x_i, y_i)$, $i=1,\dots,m$ are given, we want to find a P with prescribed values of all its partial derivatives of order k, $0 \le k \le p$, at the knots. We assume that the number of coefficients is equal to the number of conditions. With purely geometric means (shifts of triangles) we prove: For $p=0,1,2,3$ all all n, all Hermite interpolation problems (with the exception of $p=1, n=2$ and $p=1, n=4$) are almost surely solvable. This means that they are solvable for almost all positions of $Z \in \mathbb{R}^{2m}$. There are applications to Hirschowitz' problems in Algebraic Geometry.

G. G. Lorentz, R. A. Lorentz

On the dimension of bivariate periodic spline spaces

Spaces of spline functions defined on uniform meshes often possess the property of being translation invariant. In the univariate case this means that translation over the mesh size does not change the underlying spline spaces; in more dimensions the invariance property concerns translations in several directions depending on the mesh shape. In these spaces the so-called exponential eigen splines play a fundamental role. In particular for "periodic subspaces" consisting of (multi) periodic functions. It seems that these subspaces are spanned by the corresponding periodic exponential eigen splines.
The problem of computing the dimension of periodic spline spaces is therefore equivalent to counting the total number of independent periodic exponential eigen splines. A survey of results with respect to the bivariate situation was presented in the talk.

H. G. ter Morsche, Eindhoven

Hans G. Feichtinger and Karlheinz Gröchenig:

Stable reconstruction of band-limited functions on $\mathbb{R}^m$ from irregularly distributed sampling values.

By the classical Whittaker-Shannon-... theorem any band-limited function $f \in L^2(\mathbb{R}^m)$ ($\operatorname{supp} \hat{f} \subseteq \Omega$, compact) can be written as

$$f = \sum_{k \in \mathbb{Z}^m} f(\alpha k)\, T_{\alpha k}\, g$$

(where α is small enough, $T_y g(z) = g(z-y)$, and $\hat{g}(x) \equiv 1$ on Ω, and having compact support, e.g. $g = \operatorname{sinc}(x) = \sin x / x$).

In the talk two irregular aspects of this theorem are given: Complete reconstruction of f from the sampling values $(f(x_i))_{i \in I}$ (including stability considerations) and secondly: expansion of f as a series of translates of g (with given translation operators). The approach is iterative, based on a spline-type approximation, followed by a smoothing operation. The convergence is in the natural norm (e.g. weighted L^p-norm), if $f w \in L^p(\mathbb{R}^m)$).

At the end of the talk connections to the modern theory of wavelets (Y. Meyer / I. Daubechies / A. Grossmann, ...) and Gabor-type expansions (based on the use of short time Fourier transform theory)

Hans G. Feichtinger

Strong uniform approximation by Bochner-Riesz means

Suppose f is a real-valued continuous function defined on $\mathbb{R}^n$ which is periodic in each variable with period 2π. The Bochner-Riesz means of f is defined by

$$S_R^\alpha(f;x) = \sum_{|m|<R} c_m(f)\, e^{imx} \left(1 - \frac{|m|^2}{R^2}\right)^\alpha$$

where m denotes n-dimensional integer and $mx = m_1x_1 + \cdots + m_nx_n$ for $x \in \mathbb{R}^n$, $|m| = (mm)^{\frac{1}{2}}$. The special value $\alpha_0 = \frac{n-1}{2}$ of the index α is called the critical index. Since we can regard $S_R^{\alpha_0}$ as an analogue of the Fourier sum of univariate. The strong uniform approximation means to estimate the quantity $\| \frac{1}{R}\int_0^R |S_r^{\alpha_0}(f) - f|^q dr \|$ in max-norm where $q > 0$ generally. Our result is the following

<u>Theorem</u>. If $\frac{n}{2} - 1 < \alpha \leq \frac{n-1}{2}$ then there exists a constant $c(n,\alpha)$ depending only on n and α such that for any f and any $R > 0$,

$$\| \frac{1}{R}\int_0^R |S_r^\alpha(f) - f|^2 dr \| \leq c(n,\alpha) \frac{1}{R}\int_0^R \omega_2(f; \tfrac{1}{r})^2 dr$$

where ω_2 denotes the modulus of continuity of order 2

Kun-yang Wang,
Universität Siegen, and
permanent: Beijing Normal University.

Slice Products and Bivariate Approximation

Suppose S and T are closed and bounded intervals in $\mathbb{R}$. Let $C(S)$, $C(T)$ and $C(S \times T)$ be the corresponding Banach spaces of all continuous real-valued functions. Let $\{s^{n_k};\ k = 1, 2, 3, \ldots\}$ and $\{t^{m_j};\ j = 1, 2, 3, \ldots\}$ be two sequences of monomials. Let G be the closed linear span of $\{s^{n_k}; k = 1, 2, \ldots\}$

in $C(S)$, let H be the closed linear span of $\{t^{m_j}; j=1,2,3,\dots\}$ in $C(T)$, and let W be the closed linear span of $\{s^{n_k}t^{m_j}; k,j=1,2,3,\dots\}$ in $C(S\times T)$. We give conditions under which $W = G \# H$, the slice product of G and H. Recall $G\# H$ is the space of all functions $f\in C(S\times T)$ such that for each $(s,t)\in S\times T$ the sections $f_s : T\to\mathbb{R}$ $f_t : S\to\mathbb{R}$ belong to H and G respectively, where $f_s(y) = f(s,y)$ for all $y\in T$ and $f_t(x) = f(x,t)$ for all $x\in S$. Abstract versions of this result are presented for S and T compact Hausdorff spaces, to provide an elementary proof that every closed subalgebra of $C(S)$ has approximation property as defined by Grothendieck.

João B. Prolla, Campinas

An algorithm for best approximating algebraic polynomial in L^p over a simplex

The problem of finding the best approximating polynomial in question of degree m in dimension d has been reduced to solving a sequence of convex minimalization problems in R^s with $s=\binom{d+m}{d}$. An algorithm for constructing a sequence of respective polynomials of degree m and approaching the best approximating polynomial for $f\in L^p$, $f\in C$ in case $p<\infty$, is given.

Zbigniew Ciesielski
Instytut Matematyczny PAN
ul. Abrahama 18
81-825 Sopot, Poland.

Periodic Spline Functions on Regular Partitions

Periodic box-splines are very useful for interpolation and approximation of smooth periodic functions. The translates of a periodic box-spline are linearly dependent unless a quite restrictive condition on the directions of the box-spline is satisfied. Using the four directional box-spline as an example we introduce different ways of modifying the periodic box-spline in order to overcome these restrictions. In the case of even multiplicities of all the directions we obtain a variational characterization for the modified box-spline.

Joachim Stöckler
College Station, Texas, USA.

Interpolatory Subdivision Schemes for Surface Modeling

We introduce a subdivision algorithm for surface generation. The method uses interpolation and iterative knot insertion. Starting from a grided surface in parametric form we introduce additional points using a curve generation algorithm in the 3-D space along the mesh curves. The process is repeated until the required visual effect is achieved or screen precision is attained. The method is local and is shown to be "shape" preserving along mesh curves. We present convergence results and several numerical examples.

Alain Le Méhauté
U. de Lille, France

Florencio I. Utreras
Universidad de Chile
Santiago, Chile.

Bernstein Quasi-Interpolants.

The classical Bernstein operator can be written as a linear differential operator on $\mathbb{P}_n$: $B_n f(x) = \sum_{i=0}^{n} f(i/n)\, b_i^n(x) = \sum_{i=0}^{n} \beta_i^n(x) D^i f(x)$ where $b_i^n(x) = \binom{n}{i} x^i (1-x)^{n-i}$ and $\beta_i^n(x) = B_n[(\cdot - x)^i / i!]$. Its inverse is also a linear differential operator: $A_n = B_n^{-1} = \sum_{s=0}^{n} \alpha_s^n D^s$, where $\alpha_s^n \in \mathbb{P}_s$ can be formally computed from the β_i^n's using $A_n \circ B_n = I$. Let $A_n^{(k)} = A_n^{[k]} = \sum_{s=0}^{k} \alpha_s^n D^s$, then $B_n^{(k)} = A_n^{(k)} \circ B_n$ is a left Bernstein quasi-interpolant and $B_n^{[k]} = B_n \circ A_n^{[k]}$ is a right Bernstein quasi-interpolant. Both of them reproduce $\mathbb{P}_k$ and $B_n^{(k)} f - f = O(n^{-[k/2]})$ when $f \in C^{k+1}$ or C^{k+2}. These results are extensions of the classical Voronovskaja theorem for B_n. The $B_n^{(k)}$ are intermediate between $B_n^{(0)} = B_n$ and $B_n^{(n)} = L_n$ = the Lagrange operator in $\mathbb{P}_n$ on $S_n = \{i/n,\ 0 \le i \le n\}$. These results are easily extended to Durrmeyer operators, Szasz operators, tensor product Bernstein operators and analogs on the triangle and more generally on the simplex.

Paul SABLONNIÈRE

laboratoire LANS, INSA de Rennes (France)

Recent Results on Complex Interpolatory Approximation

In this talk I would like to introduce some recent results on complex interpolatory approximation. The talk is divided into 5 sections.

1. Lagrange interpolation of analytic functions on compact of the complex plane.
2. Divergence and Convergence of Lagrange interpolatory polynomials on $A(|z| \le 1)$

3. Degree of approximation by interpolating polynomials in a Jordan domain

4. Birkhoff interpolation

5. Overconvergence.

Xie-Chang Shen (沈燮昌)

Peking University, Beijing

Some applications of a multivariate Horner's algorithm

by J.M. Carnicer and M. Gasca

A generalized Horner's algorithm, with interesting applications to multivariate polynomials, has been recently by the authors and submitted for publication. In this talk we apply it to evaluate efficiently some particular expressions of multivariate polynomials and their derivatives, with special emphasis in Lagrange representation of interpolating polynomials.

Mariano Gasca

Universidad de Zaragoza, (Spain)

Approximation in barycentric coordinates

Our purpose is to describe a general process which provides:

1) a "quasi automatic" construction of classical piece-wise functions as finite elements or polynomial splines, and also many others new type of piece-wise functions.

2) a generalization of the notion of polyhedral splines

3) the possibility of extending, for example, the notion of generalized divided differences.

Our main tools are Algebraic topology and Hilbertian kernels.

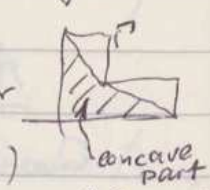
Marc Atteia

Laboratoire d'Analyse Numérique, Université Paul Sabatier, Toulouse (France)

2ter Februar 1989

Multivariate Splines: Polynomial Degree & Approximation Order.

Consistent families of polynomials (Appell sequences) have a property, $D^\beta h_\alpha = h_{\alpha-\beta}$, (1) that leads to a Taylor expansion $h_\alpha = \sum_{\gamma\in E} v_\gamma m_{\alpha-\gamma}$, (2) in terms of monomials $m_\beta = x^\beta/\beta!$. Here, equation (1) makes sense if $\{h_\alpha : \alpha\in\Gamma\}$ is given where Γ is a lower set. Otherwise we need the more general definition of consistency: $D^\beta h_\alpha = D^{\beta'}_{\alpha'}$ if $\alpha-\beta = \alpha'-\beta'$. Then we can extend $\{h_\alpha\}$ for $\alpha\in\Gamma-\mathbb{Z}^s_+$ by $h_\alpha = D^\beta h_{\alpha+\beta}$, $\alpha+\beta\in\Gamma$, $\beta\in\mathbb{Z}^s_+$. The (backward) Taylor formula (2) is now proved by induction, with $E\subset\Gamma^* = \{\alpha\in\Gamma\setminus\mathbb{Z}^s_+ : h_\alpha\neq 0\}$, where the typical induction step involves removing a maximal element from Γ^* and representing h_α as integral of its derivatives $D_j h_\alpha$. Now, if $\gamma\in E \Rightarrow \gamma\geq 0$ then $h_\alpha\in\pi_\alpha$ and in this case there is the translation representation, $h_\alpha = \sum_{j\in\Delta} a_j m_\alpha(\cdot - j)$. These families are called strongly consistent. Consistent families are LMG (local monomial generating) if $m_\alpha = \sum_{\ell\in I}\sum_{j\in\mathbb{Z}^s} h_{\ell,\alpha}\varphi_\ell(\cdot - 1)$, $\alpha\in\Gamma$, relative to a set $\Phi = \{\varphi_\ell : \ell\in I\}$ of basic functions, 'splines', with a second index ℓ added. Let $\partial\Gamma = \{\alpha\in\mathbb{Z}^s_+ : \alpha\notin\Gamma, \exists j\in\{1,2,\cdots,s\}, \alpha-e^j\in\Gamma\}$, the "outer fringe" of Γ. If $\{h_{\ell,\alpha}\}$ is strongly consistent LMG, then the operator



$u_\eta = Q_\eta u = \sum_{\ell\in I}\sum_{j\in\Delta}\sum_{k\in\mathbb{Z}^s} a_{\ell j}\, u(\eta(k-j))\,\varphi_\ell(x/\eta - k)$ achieves the optimal order $\|u-u_\eta\|_{L^p(\mathbb{R}^s)} \leq K\max\{\eta^\beta : \beta\in\partial\Gamma\}$. Note $\max\{\eta^\beta : \beta\in\partial\Gamma\}$ ignores all but the concave part of Γ.

Lemma: $\eta^\alpha/\max\{\eta^\beta : \beta\in\partial\Gamma'\}$ is unbounded iff $\alpha\in$ concave part of Γ. The converse holds: A strongly consistent LMG family $\{h_{\ell,\alpha}\}_{\ell\in I,\,\alpha\in\text{concave part of }\Gamma}$ exists if the local (de Boor - Jia) approximation order $\max\{\eta^\beta : \beta\in\partial\Gamma\}$ is given. For non-strongly consistent LMG no similar result is possible, therefore, although examples of such (with higher degree) can be given.

H. G. Burchard
Stillwater, Oklahoma

On the reconstruction of multivariate rational functions by interpolation.

A typical problem for algorithms which use exact arithmetic is the intermediate coefficient swell. Sometimes the algorithm runs out of memory even if it is known that the result has coefficients of moderate size. If the coefficients of the input depend on parameters, the algorithm can be applied to a series of specified values of parameters. If it is known, that the coefficients of the output are rational functions of the parameters, they can - if sufficiently many specified values are known - be reconstructed by rational interpolation (r.i.)

This reconstruction requires the construction of rational interpolating functions of increasing (numerator and denominator-) degree, which become after a finite number of steps identical to the function to be constructed.

We present a new bivariate r.i. method and compare it to the known methods of Siemaszko, Kuchminskaya, Cuyt, and Verdonk with respect to the reconstruction aspect.

H. Michael Möller
FernUniversität Hagen
D 5800 Hagen 1

On optimal recovery for periodic functions —
by H.L. Chen, C.K. Chui and C.A. Micchelli.

We ~~are~~ studied the following problem: Does the ratio of sampling error and n-width converge (or diverge) when n tends to infinity? to be more precise, the problem to be considered is that under which conditions does the radio

$$\frac{\inf_{A_n} D(\mathcal{F}, \Delta_n)}{i_n(\mathcal{F})} = \frac{\inf_{A_n} \sup_{f \in \mathcal{F}} \| f - A_n(f(x_1), \dots, f(x_n)) \|_2}{\inf_{A_n, I_n} \sup_{f \in \mathcal{F}} \| f - A_n(I_n f) \|_2}$$

converges when n tends to infinity?

Where I_n : any continuous linear mapping from $\mathcal{F}$ into R^n.

A_n : any mapping from R^n into $\mathcal{F}$.

$\{x_j\}_1^N \in Q_2$ $\quad Q_2$ Unit square.

$\mathcal{F}$: certain class of functions.

Δ_n : $\{x_j\}_1^n$

~~We got one~~ (theorem 1) ~~a result~~ states that for some function classes the ratio converge, on the other hand, we also got a theorem which states that the optimal sampling point set is not always optimal. We listed many examples.

(H. L. Chen)

Characterization + Calculation of Quasi-interpolants
(joint with Charles Chui)

We work in the space $\mathcal{S}(\Phi) = \{ s(x) = \sum c_j \Phi(x-j) \}$, $x \in \mathbb{R}^s, j \in \mathbb{Z}^s$ where Φ is locally supported. Assume that Φ satisfies the Strang-Fix condition of degree n so that $\Pi_n \subset \mathcal{S}(\Phi)$ (Π_n = polynomials of total degree $\leq n$).

Let $\mathcal{Q}$ denote the set of quasi-interpolation operators Q of the form $Qf := \sum \lambda f(\cdot + j) \Phi(x-j)$; let $\Lambda_{\mathcal{Q}}$ be the set of corresponding λ's. Then we prove that

$$\lambda_1, \lambda_2 \in \Lambda_\Omega \Rightarrow \lambda_1(p) = \lambda_2(p) \;\forall p \in \Pi_n.$$

Likewise, if $\lambda_1 \in \Lambda_\Omega$ and $\lambda_2(p) = \lambda_1(p)\; \forall p \in \Pi_n$ then $\lambda_2 \in \Lambda_\Omega$.

Applications include

a) Formulation of the problem of "minimally supported" $\lambda \in \Lambda_\Omega$.

b) More general quasi-interpolants of the form $\sum \lambda_j f(\circ + j)\Phi(x-j)$, $\lambda_j \in \Lambda_\Omega$ which can be used for

i) uniform approximation on compact domains $K \subset \mathbb{R}^s$ where f is not known off of K

ii) Approximation of functions from scattered data.

We also have the result that

$$\lambda \in \Lambda_\Omega \Leftrightarrow \sum_j f(j)\, \lambda\Phi(\circ + x - j) \text{ is a quasi-interpolant for the space } \mathcal{S}(\Psi),\; \Psi = \lambda\Phi(\circ + x)$$

This allows us to produce functions Ψ with desired properties of smoothness / support-size by choosing appropriate $\lambda \in \Lambda_\Omega$.

Harvey Diamond
West Virginia University

On iterates of variation-diminishing operators and Characterization of Bernstein-type approximation

The phenomenon of the good shape-preserving property and low convergence rate ~~of~~ in Bernstein-type approximation is investigated by iterates.

Theorem 1 shows the iterate process of any L.V.D. approximant on $C[0,1]$ reproducing the linears is convergent. and the limiting function is linear or piecewisely linear. As a result, we found that the first eigenvalue with modulus less than 1 of a v.D. operator is a quantity which characterize the contradiction phynomenon mentioned above.

The concept of asymptotic v.D. operator introduced in this paper is crucial in characterizing Bernstein-type v.D. operator, and it hopefully be useful in studying v.D. property and convexity for higher demention approximation.

16. Feb. 1989. Yingsheng Hu (胡莹生)
(Institute of Math. Academia Sinica
Beijing, China

Problems and results in the calculation of extremal fundamental systems for sphere and ball.
A nodal system is called extremal fundamental system if the related evaluation functional are independent and if the Lagrangians w.r. to a special function space are of minimal uniform norm. There exists a Remez-type algorithm for calculating such systems, if a reproducing kernel is well known. However, though convergence is guaranteed theoretically, there occur problems in the numerical practice by the fact that there may exist nodal systems which satisfy all the necessary conditions without yielding the global result. An explicite example for this can be given in a space of spherical harmonics. On the other hand side, for polynomials with degree μ over the ball B^3, the algorithm is successfull. The related quadrature is very precise (relative error about 10^{-8} in case of $\mu = 6$, 84 nodes). Earlier results for the sphere S^2 could be applied successfully in tomography.

Manfred Reimer

Approximation by Riemann sums in generalized Orlicz spaces.

Let $R_n(f,y) = \frac{1}{|n|}\sum_{k=0}^{n-1} f\left(\frac{y+k}{n}\right)$, $0 \le y \le 1$, $y \in \mathbb{R}^m$, $n \in \mathbb{N}^m$, $|n| = n_1 \cdots n_m$ for $n = (n_1, \ldots, n_m)$, be the equidistant Riemann sums of a function f in $[0,1]^m \subset \mathbb{R}^m$, and let $I_\varphi(f) = \int_{[0,1]^m} \varphi(x, |f(x)|)\,dx$, where $\varphi: [0,1]^m \times \mathbb{R}_+ \to \mathbb{R}_+$ be a generalized Orlicz function satisfying suitable conditions. The error $\Omega_n(f) = I_\varphi(R_n(f,\cdot) - \int_{[0,1]^m} f(x)\,dx)$ is estimated for any $f \in L^\varphi([0,1]^m)$; in case of a usual Orlicz function φ, $\Omega_n(f) \le \omega_\varphi(2^m c f, 1/|n|)$, where ω_φ is the integral φ-modulus of continuity. The result was obtained jointly with A. Kamińska.

16. Febr. 1989 Julian Musielak

A. Mickiewicz University, Poznań, Poland

Simultaneous Approximation by n-th Order Blending Operators.

The method of n-th order blending was introduced by Delvos and Posdorf and constitutes a generalization of Gordon's discrete blending approach. In our talk we shall investigate the degree of approximation of bivariate functions given on a rectangle by n-th order blending operators. Our aim is to give a fuller description than in the literature by using mixed moduli of higher orders as introduced by Marchaud. The results include certain permanence principles which explain how generalized n-th order blending operators inherit quantitative properties from their univariate building blocks. Moreover, for univariate spline interpolation we prove an extension of a theorem due to Swartz and Varga which includes our previous generalization of the Sharma-Meir theorem on the degree of approximation by cubic spline interpolators.

Feb 17, '89 H. Gonska

Universität Duisburg

Rational Approximation in Signal Processing and System Theory

Matrix-valued rational functions not only provide important tools for approximation but also represent realizable models of digital and analog filters and feedback control systems. However, since stability is an essential issue in these applications, the poles of the rational approximants must be restricted to certain regions such as the unit disk for digital systems and the right half plane for analog systems. Hence, best approximation in the Hankel norm is very appropriate, since boundedness of the finite rank Hankel approximants is equivalent to stability of the corresponding rational approximants. Recent development based on the AAK theory is surveyed, and in particular, results in the four-block problem and minimum-norm interpolation problem are discussed.

Feb. 17, 1989 Charles K. Chui
Texas A&M University, College Station

Multivariate Splines

Some problems in the analysis of MS spaces were described. In particular the dependence of the dimension on the geometry of the triangulation was illustrated and a link between bivariate splines and higher dimensional splines was sketched. We won't understand high dimensional splines of high degree before we understand bivariate splines of low degree. A basis of a super-spline space was also given.

2/17/89 Peter Alfeld
Univ. of Utah.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Generalized Bochner - Riesz Means of Fourier Integral

Suppose $f(x) \in L^2(\mathbb{R}^n)$, $\hat{f}(u)$ is its Fourier Transform. If $\hat{f}$ and $|x|^r \hat{f}(x)$ belong to $L^2(\mathbb{R}^n)$, We say $f(x) \in B^r$, Bessel Potential Space of order r.

Generalized B-R Means is defined as

$$B_R^{\delta,\lambda}(f)(x) = \int \hat{f}(u) e^{iu\cdot x} \left(1 - \frac{|u|^\lambda}{R^\lambda}\right)^\delta du.$$

We prove If $f \in B^r$, then

$$B_R^{\delta,\lambda}(f)(x) - f(x) = o(R^{-r}), \quad \text{if } \sigma < r < \lambda,$$

$$B_R^{\delta,\lambda}(f)(x) - f(x) = O(R^{-r}) \quad \text{if } \lambda = r,$$

only if $\alpha > 0$.

The estimate is sharp.

Feb. 18. 1989. Tianping Chen
Fudan University, Shanghai,
P. R. China

Boolean lattice rules

Lattices rules are important methods in multidimensional numerical quadrature. We apply Boolean methods of multivariate interpolation to construct Boolean sums of lattice rules. Using duality theory of Boolean algebra we derive remainder formulas for Boolean lattices rules and we show that these rules are good formulas in the sense of Korobov.

17. 2. 1989 Franz-Jürgen Delvos
Universität - GH - Siegen

A trivariate Boolean cubature scheme

The talk is concerned with the numerical integration of smooth periodic functions in three dimensions. Using parametric extensions of the univariate trapezoidal rule we construct a trivariate cubature scheme of interpolatory type being related to the concept of discrete blending function interpolation.
Besides an explicit representation formula we derive an error estimation for functions of the trivariate Korobov-space E_3^α, $\alpha > 1$.
We show the the cubature error is of the order $O(\ln^{2\alpha+2}(N) / N^\alpha)$ where N is the number of evaluation points.

17. 02. 89 Helmut Nienhaus
Universität - GH - Siegen

Medical Statistics:

Statistical Methods in Epidemiology

19 – 25 February, 1989

A comparison of populations self-selected and randomly selected for coronary risk factor screening

The comparison of a random sample with two self-selected samples shows, as expected, that self-selected samples are not representative for the target population. But in spite of this the risk profile is quite similar in all three samples. As most people are not aware of their cholesterol level and high cholesterol level is mostly not associated with symptoms it is quite unlikely that cholesterol is a self-selecting factor. Therefore the data of such screening programs may also be used to study the dependence of biological parameters on demographical and environmental factors.

21. 2. 89 Jürgen Berger

Universität Hamburg FB Medizin

Cumulative Damage Models in Cancer Epidemiology

Analogous to the mode of proceeding in physics, where different theoretical models have been developed for different levels of reality, a phenomenological carcinogenesis model is suggested for the macroscopic consideration of disease by epidemiology. Investigations indicate that "cumulative damage models" (CD-models) possess essential qualities desirable for such a model. This concerns theoretical qualities such as, for example, the fact that they incorporate clear concepts for exposure towards environmental factors as well as for the damage process with the respective host system and also practical qualities with regard to fitting empirical data. Important fundamental consequences for epidemiology follow from the fact that these models offer measures for the description of environment-induced damages which are not dependent upon a baseline category, linear with regard to the exposure period and additive with independent exposures. All traditional measures of epidemiology can easily be calculated with the aid of the CD model.

21.2.89 Nikolaus Becker
DKFZ, Heidelberg

Multiplicative modelling of additive excess hazard

The hazard for a group of exposed persons is often contrasted with a fixed hazard prevailing in a general population comparable with respect to, e.g., sex, age and calendar time period, it may be of interest to investigate which variables are responsible for an eventual excess in the hazard and to quantify the effects. Especially with patient materials it may not always be reasonable or possible to assume that the events concerned are rare, ~~especially~~ particularly when large patient materials are in question.

Estimation of various models for an increased hazard may be achieved by GLM. For cancer patient materials, multiplicative modelling of the excess hazard is particularly advantageous in terms of interpretation and fit of the model. According to this model the total hazard μ may be decomposed as follows:

$$\mu = \mu^* + \nu,$$

in which μ^* is the hazard in the general healthy population and ν is the excess that may depend as μ^* on sex, age and calendar period, and additionally, e.g., on stage and histology of the tumour and on treatment. It certainly also depends on follow-up time. At many sites, $\nu = 0$ after a number, e.g. 5 or 10 years of follow-up, i.e., the living patients are cured. Multiplicative modelling for ν means that

$$\nu = e^{x\beta},$$

in which $x = (x_1, x_2, \ldots, x_m)$ is a vector of independent variables and $\beta = (\beta_1, \beta_2, \ldots, \beta_m)^T$ a vector of the corresponding parameters.
The main alternative way of modelling is

$$\mu = \mu^* \theta$$

in which

$$\theta = e^{x\beta}$$

thus making the excess hazard $\nu = \mu^*(\theta - 1)$ proportional to the expected mortality μ^*. Results with materials from population-based cancer registries are used for illustration of model fitting when grouping has been performed with respect to follow-up time.

21.2.1989 Timo Hakulinen
Finnish Cancer Registry,
Helsinki (visiting
scientist at the DKFZ,
Heidelberg)

Nonparametric of changes in hazard rates for censored survival data

Instead of considering the parametric change-point model $h(x) = c_1 1_{\{0 \le x \le \tau\}} + c_2 1_{\{\tau < x \le T\}}$, it is suggested to analyze an approximative model with a smooth hazard rate by means of nonparametric hazard function estimators. As such, kernel estimators based on the Nelson (1972) estimator for curve and first derivative are proposed, and the notion of a point of most rapid change θ, where $h^{(1)}(\theta) = \sup_x h^{(1)}(x)$ is suggested. The properties of $\hat\theta$: $\hat h^{(1)}(\hat\theta) = \sup_x \hat h^{(1)}(x)$, where $\hat h^{(\nu)}$ is the ν-th derivative of the kernel estimator, are derived and an asymptotic confidence interval for θ is ~~derived~~ found. The analysis is based on an i.i.d. representation of the Kaplan-Meier estimator (Lo & Singh 1986). Further results for estimating in the discontinuous change-point model with one-sided kernels and for tests for change-points are discussed. (joint work with J.L. Wang)

21.2.89, Hans-Georg Müller, UC Davis

Assessment of adverse reactions during sporadic drug use.

Side effects of drugs which are taken occasionally and repeatedly for periods of varying length are modeled. This approach is built on the distribution of the duration of individual exposures and leads to risk measures of the individuals as well as to risk measures of the population. The occurrence of adverse reactions during or immediately after drug use is of special interest and possible confounders are also considered. The parameter estimation is conducted by the maximum likelihood method. As an example, the risks of analgesics to induce the agranulocytosis-disease is investigated.

22.7.89 Uwe Feldmann
Universität Heidelberg

On the analysis of biased case-control data

We assume that in a (matched or unmatched) case-control study one case group and two control groups are collected. For each individual a vector of covariables $X=(X_1, \dots, X_k)$ is collected. A subset of these covariables collected from the second control group is assumed to be biased. Under this assumption the classical methods of analysis (conditional or unconditional logistic regression) would lead to biased estimates of the regression coefficients. Ignoring the second control group completely is correct but information on the unbiased

factors is not used and therefore this design is not fully efficient.
Two methods of analysis are proposed:
- a polychotomous logistic regression model in which additional parameters in order to correct for the bias are to be estimated. This model is efficient however it is constraint by certain conditions on confounding
- a two-stage-design like analysis. This is an analysis proposed by Breslow & Cain. It was shown that it is applicable under the present situation.

Heiko Becher
German Cancer Research Center,
Heidelberg

Statistical problems in longitudinal studies on respiratory diseases in children

Short-term effects of air pollutants, meteorologic variables and viruses on respiratory diseases in preschool-children - especially croup syndrome - are investigated. Data from 18 500 cases in different areas of the FRG, collected over 2-3 years, are analysed on a daily basis. Stepwise linear regression is used after data transformation.
The problem of missing or insufficient specification of confounders is discussed and correction strategies (filter techniques, dummy variables, seasonal restrictions) are introduced.
Finally an upper and a lower approximation for air pollution effects is proposed which gives the limits in which the 'real' estimates are expected.

U. Emil Wichmann 22.2.85
University of Wuppertal

Latent variable models for non metric dependent variables

The classical approach to latent variables assumes that latent variables (factors) that are continuous random variables are connected to continuous observed r.v.'s by a linear model in the form $y = \Lambda\eta + \varepsilon$. The latent variables may be structured by a structural equation model of the form $\eta = B\eta + \Gamma x + \zeta$ in which x are exogenous variables. This approach is extended to include non-metric discrete r'v's as indicators for η. This is done by defining latent indicators $y_i^* = \Lambda_i\eta + \varepsilon_i$ which are connected to observed variables by ~~to~~ models such as threshold or random utility maximization models, f.i. $y = y^*$ for $y^* > \tau$ and $y = \tau$ for $y^* \leq \tau$ in a censored model where τ is a known threshold. Dichotomous, censored, ordinal and multinomial indicators can thus be used for latent variable models. Estimation strategies and applications to epidemiology are discussed.

Gerhard Arminger,
Bergische Universität Wuppertal, FB 6

Misclassification in general relative risk models

Misclassification is a common problem in the analysis of epidemiological data and there has been extensive work in evaluating of its effect in estimating the odds-ratio. In this paper the effect of misclassification for non-multiplicative models is investigated, mainly for the additive model and mixture models. Different assumptions about the structure and the magnitude of the error were considered to estimate the bias. Even more than in multiplicative models small errors can yield important biases in the estimation of the relative risk and its variance. In mixture models the mixture ~~model~~ parameter λ can be seriously biased, therefore the use of the mixture parameter to distinguish between the two models should be used with caution if misclassification errors are present.

Maria Blettner, University of Liverpool.

Variable Selection with the Bootstrap-method

In clinical and epidemiological studies we often have the problem of many correlated variables and we have to search for the 'important' ones. The Stepwise procedures are the mostly used methods for variable selection in regression models, although some problems are well known.
The bootstrapping method can be used as a flexible instrument for getting ideas about model stability and it may be used as a method for the selection of variables in a variety of models.
The method will be presented and some basic ideas for a selection strategy in the Cox-model will be illustrated with an example of a clinical trial.
Theoretical results will be difficult to obtain, but the first practical results show that it may be a useful tool to improve the variable selection techniques. More insight has to be gained in a simulation study.

Willi Sauerbrei,

Inst. f. Med. Biometrie u. Informatik, Uni. Freiburg

Nonparametric estimation of Dietz & Schenzle's transmission potential from current-status data.

In a steady-state population an immunizing infection is assumed to happen with intensity (age-dependent incidence rate) $\lambda_0(a)$. The age-dependent mortality $\mu(a)$ is assumed to be the same for susceptibles and infected and the age-specific vaccination rate is denoted by $q(a)$. Under suitable assumptions (see K. Dietz' lecture) the transmission potential is well approximated by

$$R_\varphi = \int_0^\infty e^{-\int_0^a \mu(u)du} \lambda_0^2(a)\, e^{-\int_0^a \varphi(u)du}\, da \Big/ \int_0^\infty e^{-\int_0^a \mu(u)du} \lambda_0^2(a)\, e^{-\int_0^a \lambda_0(u)du}\, da .$$

Modern survival analysis methods are used to estimate $\lambda_0^2(a)$, and hence R_φ, from current-status data. The methods are applied to Bulgarian hepatitis data and Danish measles data.

Niels Keiding (Univ. Copenhagen)

Attributable Risk Estimation from Case-Control Data via Logistic Regression

By fitting an 'unconditional' logistic regression model to unmatched case-control data an estimate of the joint population attributable risk λ for the factors included may be obtained from the intercept parameter δ via the relation

$$\delta = \log(n_1/n_0) + \log(1-\lambda),$$

where n_1 and n_0 denoting the numbers of cases and controls, respectively, sampled for the study. A variance estimate can easily be obtained from the estimated variance of the intercept parameter for 'prospective' sampling. The method is generalized to stratified data with large strata. In that case the stratum-related intercept parameters enable the calculation of stratum-specific attributable risks. The method is applied to data from a German case-control study on lung cancer

Karsten Drescher
University of Bremen

Evaluation of vaccination strategies

In order to determine the minimum coverage necessary for reducing the transmission to zero one has to estimate the basic reproduction number (or transmission potential) R of an endemic virus disease, i.e. the number of secondary cases which one case could generate during the entire infectious period if the population were completely susceptible. For the equilibrium a formula can be derived (Dietz & Schenzle, 1985) which expresses R as a function of the age-dependent infection rate. In a cross-sectional survey one only knows the age of an individual and the serological status which tells whether the individual is still susceptible or has had already the infection, i.e. one has to estimate the age-dependent infection rate based on observations which are all either left or right censored. The solution to this statistical problem is given in the contribution to this conference by Niels Keiding (see p. 207)

K. Dietz (Univ. Tübingen)

Model selection in large data sets

A multiple test procedure for inferring the dimension of a general finite parameter model is proposed which consists of individual tests of each of these parameters. If the critical limits of the individual tests are allowed to depend on the sample size in an appropriate way, the test procedure provides a weakly consistent estimate of the minimal "correct" subset of model parameters. The procedure is applied to a large sample of autiometric measurements in workers exposed to noise

P. Bauer
University of Cologne

Methods for data distortion to improve the confidentiality of data files.
Under the present law on confidentiality protection in the FRG it is almost impossible to obtain statistical data, collected by the statistical offices, in the form of microdata files. The reason given is that individual records in the file can be identified too easily. Several procedures have been proposed to modify the data records in such a way that the risk of reidentification becomes negligible. The talk is concerned with the statistical implications of such modifications.

S. Schach
Univers. Dortmund

While therapy research has resolved the problem of confounding by adoption of the randomized, double-blind clinical trial etiological research in cancer has resorted to the use of multiple regression models. It is proposed that the uncritical use of these models does not provide progress in understanding the causes of cancer, and that the problems are further compounded by measurement error in risk factors. Statisticians therefore need to become more closely involved in study design, and in particular in the choice of study populations and the incorporation of measurement error quantification. Further use of matching would also be appropriate. At the stage of analysis, more attention must be paid to covariate structure. Fitting of measurement error models and stratum specific analyses are also encouraged.

J. Kaldor
International Agency for Research on Cancer, Lyon - FRANCE

Some applications of Bayesian methods in cancer epidemiology.

Several problems which involve large numbers of parameters were discussed including variation in individual susceptibility, exposure measurement error, and testing multiple hypotheses. Many of these problems lead to complex models for which no analytic solutions are available and numerical methods using the E-M algorithm are difficult. Furthermore, empirical Bayes approaches tend to underestimate the variability in the parameters of interest owing to incorrectly treating the estimates of the hyperparameters as known with certainty. Bayes empirical Bayes approaches put a further prior on the hyperparameters and seek to estimate the joint posterior distribution of all parameters. This is easily implemented using the IP algorithm of Tanner and Wong (JASA, 1987). The method was described in the context of estimating several Normal means, with unequal variances and a prior consisting of a mixture of a Normal and a spike at zero, and applied to data on cancer and occupational exposures (Thomas et al, Am J. Epidemiol, 1985). More recent work on multiple regression with an exchangeable mixture distribution for the regression coefficients were described. It is suggested that this approach provides a means of selecting variables and quantifying the uncertainty in the selection of the "best" model.

Duncan C. Thomas
Univ. of Southern California, Los Angeles
(on sabbatical at MRC Biostatistics Unit
Cambridge England)

Robust regression: methods and implementation

In classical regression, the error model distribution is Gaussian, and the parameters are estimated by maximum likelihood (i.e. least squares). It is well known that this procedure is sensitive to outlying data, especially "influential points" or outliers in the covariate space. During the past two decades, estimation and testing procedures that are resistant to outliers and stable with respect to deviations from the given model have been developed. These procedures are called "robust". Among these, procedures based on M-estimates (of Huber, Mallows and Schweppe-Type) and "high-breakdown" estimates are receiving great attention. Their theory has been partly extended to generalized linear models. The numerical algorithms for their computation have been studied and implemented in a pakage called ROBETH-ROBSYS. The talk give an overview of the methods and algorithms included in this package.

A. Marazzi

Inst. Méd. Soc. Prév., Université, Lausanne

Tests for Spatial Clustering in Inhomogeneous Populations

A new method is proposed for detecting spatial clustering in populations with non-uniform density. The method is based on selecting controls from the population at risk and computing interpoint distances for the combined sample. Non-parametric tests are proposed which are based on the number of cases among the k-nearest neighbours of each case.

The performance of these tests are evaluated and the method is applied to a dataset on the locations of cases of childhood leukaemia and lymphoma in a defined geographical area. In particular the impact of the choice of k and the ratio of cases to controls on power is examined.

Finally a score statistic, which is a linear combination of the test statistics for values of $k = 1, 2, 4, 8$ is proposed, as

a single statistic to test for spatial clustering.
(This work has been in conjunction with Jack Cuzick of the ICRF)

Rob Edwards
Imperial Cancer Research Fund
London

es).

Nonparametric analysis of dose-response relationships in epidemiology

The proof of a dose-response relationship is an important criterion on the way to establish causality. As an alternative to the common approaches a nonparametric method, the isotonic regression, is proposed. Up to two variables can be considered. In the case of two variables it can be investigated either the influence of a second variable given the influence of the first one or the form of the interaction of both variables. An example is the form of interaction of smoking and age on developing lung cancer.

Kurt Ulm
Technische Universität München

Analysis of data from a cohort of HIV$^+$ men.

Various methods which may be appropriate to such cohort data are proposed. A mixture model to estimate the probability of developing AIDS after HIV infection is investigated. The possibility of left truncation and an uncertain time of infection are incorporated into the analysis. The efficiency losses associated with uncertain infection times are investigated. In addition, the use of relative risk regression models is illustrated with particular attention focussed on the role of lymphadenopathy.

Vern Farewell
University of Waterloo

How to apply an index of dental health to prehistoric populations

The scientific field of paleopathology investigates diseases and their spread in prehistoric populations. As is the case for the epidemiology of today some major aspect is the impact of socioeconomic factors on health. But in contrast to modern epidemiologists who can properly design their investigations conclusions of paleopathologists are restricted to their findings from excavated scelets with all the practical difficulties such as missing entities.
In our paper we confine to dental health and discuss the feature of caries affection and intravital losses. In this context archeologists often compute very strange quantities which may reach values of 100% and cannot be interpreted in any meaningfull way.
With the use of a simple stochastic model and Bayes' formula we derive a meaningfull index. It is a counterpart to the well-known DMFT-index being widely used in our days e.g. to evaluate the effect of dental prophylaxis. Thus prehistoric populations may even be compared to populations of today.
The index may be presented either as the spectrum for the various types of teeth or in a condensed form for the global affection. In any case it may be considered as estimator for the respective

parameter of some distribution but must properly be adjusted for age (at time of death) which is a confounding factor of dental health.

Reinhard Hilgers
Abt. Mediz. Statistik
Universität Göttingen

Overdispersion in quasilikelihood models

Motivated by problems in overdispersed Poisson regression analysis, we study quasilikelihood models where the mean is specified by $\mu(x;\beta)$ and the variance by $V(\mu;\theta)$. Two versions of the standard errors and score test statistics for mean value parameters are investigated, one calculated from the usual model based covariance matrix whose validity depends on correct specification of the variance function, and another using an empirical covariance matrix that has a more general asymptotic justification. Monte Carlo simulations demonstrate that these procedures yield approximately unbiased estimates of regression coefficients and their standard errors, and that model based Wald, score and deviance tests approximate the nominal size at the 5% level for moderate sample sizes. The empirical standard errors and score test perform adequately for larger samples. Variance parameters are not particularly well estimated by the moment equations used for them and their estimated standard errors do not adequately convey the true degree of uncertainty about them. Applications are given to the analysis of viral pock counts and the fitting of age-period-cohort models in Epidemiology.

Norman Breslow
University of Washington, Seattle

The German Cardiovascular Prevention Study (GCP): Design and some related methodological problems

The GCP is a multicenter community-based intervention study for the primary prevention of ischemic heart disease and cerebrovascular diseases. It comprises a total of 1.23 persons in five study regions spread all over the FRG. Risk factor reduction is being assessed in three health examination surveys at the start, at mid-term and at the end of the study. Age-specific mortality in men and women aged 25-69 years in the intervention regions shall be reduced by at least 8% as compared to the rest of the Federal Republic. Only official mortality can be used for the final analysis of this endpoint. Two methodological problems that have occurred during the course of the mortality evaluation approach are considered

- the problem of 5 years age groups for determining age-specific rates. It is shown how the use of five years groups may give misleading results depending on the force of mortality within the age group
- for the analysis of temporal and spatial trends of CV mortality a Random Regression Coefficient (RCR)-model is proposed that allows to answer the question whether

higher starting levels imply stronger decrease of mortality. This is demonstrated for the FRG and US data as well.

Karl-Heinz Timm
Bremer Institut für Präventionsforschung und Sozialmedizin (BIPS)

Two-Sample Comparisons with Multiple Endpoints Controlling the Experimentwise Error Rate

Medical trials are often concerned with the comparison of two treatment groups with multiple endpoints. As alternatives to the commonly used methods, the T^2 test and the Bonferroni method, O'Brien (1984) proposes tests based on statistics, which are simple resp. weighted sum of the endpoints. This approach turns out to be powerful if all treatment differences are uniformly in the same anticipated direction (compare Pocock, 1987). The disadvantage of these multivariate methods is that they are only suitable for demonstrating a global difference, whereas the investigator is further interested which specific endpoints or sets of endpoints actually causes this difference. It is shown here that all tests are suitable for the construction of a closed multiple testing procedure which controls the experimentwise error rate. This procedure is just as powerful as the ~~set~~ related multivariate test and furthermore it enables to detect significant differences between single endpoints or sets of endpoints.

Walter Lehmacher
GSF - Medis, Neuherberg bei München

Statistical procedures for the construction of a cut-off point for a quantitative diagnostic test

For the evaluation of a quantitative diagnostic test, Greenhouse and Mantel (Biometrics 1950) and Linnet (Stat. in Med. 1987) proposed parametric and nonparametric statistical tests for a null-hypothesis of the form H_0: "specifity ≤ SP or sensitivity ≤ SE". More precisely, the alternative hypothesis is equivalent to the existence of a cutoff point having spec. > SP and sens. > SE (SP and SE are prespecified values). These statistical tests thus do not yield confidence statements on a concrete cutoff estimated from the data, as would be desirable for practical applications.
We consider the following procedure to achieve a concrete construction: First, perform the above existence test. In case of a significant result, proceed to construct the cutoff point as a (suitably) weighted mean of the SP-percentile estimated from the sample of non-diseased and of the (1-SE)-percentile estimated from the sample of diseased individuals. The (adequately defined) type I error risk of this procedure is markedly increased over the choosen nominal α level of the existence test. We show that it may be bounded to the desired level α by adjusting the nominal α_{nom} of the existence test. (Explicit asymptotic solution in the Gaussian case, Monte Carlo for small sample sizes). The resulting "level-α-construction-procedure" requires a 25% increase of sample sizes over the mere existence test, which we consider worthwhile for the gain of a concrete cutoff point.
A tentative (since not efficient) nonparametric construction procedure is also outlined.

Helmut Schäfer
Inst. f. Med. Biometrie Univ. Heidelberg
24.2.1989

Multivariate Median Tests.

N-dimensional k-sample median tests are proposed on the basis of the componentwise ("arithmetic") median. For $k=2$, the observed frequencies of the combined sample in all 2^N rectangular sub-spaces – where each subspace is spanned by N half axes originating from the common median – allow the construction of N orthogonal contrasts related to the N components of the median. Under the null hypothesis of the equality of the medians of the two probability distributions, these contrasts have expectations zero and can exactly be tested by calculating N corresponding binomial probabilities. Since these probabilities originate from independent comparisons, they may be combined into one single p-value under the null hypothesis. The test is shown to be possible and exact for all $k=2m$, $m=1,2,\ldots$ For $k=2m+1$, $m=1,2,\ldots$, a χ^2-approximation is possible.
The construction of the one-sample median test ($k=1$) follows in a simple way. The general procedure ($k=1,2,\ldots$) has to be modified if some of the 2^N rectangular subspaces are empty.

Klaus Abt, Frankfurt
Klinikum Universität

Wellenerscheinungen mit geophysikalischen Anwendungen vom 20.-24.02.1989

1. Dr. A. Nesis: Physical Conditions of the Solar Overshoot layers

Zusammenfassung:

Die Untersuchung der Sonnenatmosphäre erfolgt experimentell durch das Studium der Photosphäre. Konvektive Instabilitäten unterhalb der Photosphäre beeinflussen wesentlich die Dynamik der Photosphäre. Die Wechselwirkung von Konvektionsbewegungen unterhalb der Photosphäre mit der Photosphäre wird als "Overshoot" bezeichnet. Vorgänge in diesen "overshoot layers" haben turbulenten Charakter und führen zu Wellenerscheinungen, die für das Aufheizen der Chromosphäre verantwortlich sind.
Mit Hilfe von Absorptionslinien-Untersuchungen lassen sich die physikalischen Zustände der "overshoot layers" beschreiben, wobei Linienverbreiterung in Zusammenhang gebracht werden kann mit hydrodynamischen und thermodynamischen Prozessen.
Die durchgeführten Experimente liefern Spektren der rms-Geschwindigkeiten V_z und V_x (Vertikal- und Horizontalkomponente). Es lassen sich die Gradienten $\frac{dV_x}{dz}$ und $\frac{dV_x}{dz}$ in der Photosphäre bestimmen und in Zusammenhang bringen mit dem Energie- und Impuls-Transfer in den "overshoot layers". Damit ist ein wichtiger Beitrag geleistet zum Verständnis der physikalischen Vorgänge unterhalb der Photosphäre (turbulente Konvektionszone) und innerhalb der "overshoot layers".

2. K. G. Roesner: Experimentelle und theoretische Ergebnisse zum Wirbelaufplatzen

Zusammenfassung:

Die Wirbelbewegung und der Zerfall von Wirbeln kann als ein elementarer Prozeß angesehen werden, der zum Verständnis turbulenter Strömungen wesentlich beiträgt. Der Übergang von geordneten laminaren Strömungen zu turbulenten Bewegungen ist gekennzeichnet durch geordnete Zwischenzustände, bei denen die Ausbildung von Wirbeln eine wichtige Rolle spielt. Bekannt sind die Phänomene: Taylor-Görtler-Wirbel an konkaven Wänden, Wirbelaufplatzen bei Tragflügeln von Δ-Form, Wirbel in geschlossenen Kanälen (Strömung mit Drall).

Zu der experimentellen Untersuchung von Wirbeln in geschlossenen Hohlräumen, wie in Zylindern oder Kugeln, zählt die Erscheinung der äquatornahen Toruswirbel bei engen Spalten zwischen einer rotierenden Innenkugel und der ruhenden äußeren Kugelschale. Darüber hinaus wurden aber auch "polnahe Ringwirbel" beobachtet, die auch dann auftreten können, wenn der Spalt zwischen den Kugelflächen groß ist (Außenradius : Innenradius = 2 : 1). Diese Wirbelbewegungen sind numerisch durch Rechnungen von P. Bar-Yoseph, A. Solan und K. G. Roesner simuliert worden und konnten experimentell für exzentrisch gelagerte Innenkugeln nachgewiesen werden.

Bei einer Drallströmung in einem geschlossenen Kreiszylinder, dessen Deckel und Boden relativ zueinander gedreht werden können, treten dieselben wirbelartigen Rückströmungsgebiete auf wie im Kugelspalt.

Die Abhängigkeit des Auftretens solcher Rückströmungsgebiete vom Bewegungszustand der Deckel- und Bodenfläche wurde für verschiedene Höhen-Radienverhältnisse des Zylinders experimentell untersucht:

Ein in derselben Richtung wie der Deckel mitrotierender Boden "begünstigt" das Auftreten von Rückströmungsgebieten.

Gegenrotation unterbricht die Wirbelbildung, wirkt also destabilisierend auf die Ausbildung solcher Sekundärbewegungen.

Analytische Ansätze zur Modellbildung des Wirbelverhaltens in Drallströmungen sind bislang nur für Rohrströmungen diskutiert worden. Dabei spielt die Annahme von 2 Strömungsbereichen [parabolisch-zäh (in Wandnähe) und elliptisch-reibungsfrei (in Achsennähe)] eine wesentliche Rolle.

Eine Übertragung auf Drallströmungen in geschlossenen Hohlräumen ist nicht unmittelbar möglich.

Es müssen alle sich bildenden Grenzschichten miteinander in Wechselwirkung treten.

Vorerst lassen sich diese Bewegungen mit den vollen Navier-Stokes-Gleichungen nur numerisch simulieren.

Teilnehmer des Arbeitskreises:

Krozer
Nesis
Roesner
Mellner
Wasner
Würfel

3. S. Krozer: Modellbildung zur Widerstandsreduktion bei Strömungen mit hochpolymeren Zusätzen

Zusammenfassung:
Der Umschlag in der Grenzschicht vom laminaren in den turbulenten Zustand kann durch den Zusatz langkettiger Moleküle beeinflußt werden und führt so möglicherweise zu einer Verringerung des Reibungswiderstands.
Grundlegend für das Studium dieser Erscheinung ist die Modellbildung für das Verhalten von langkettigen Molekülen in Lösungen bei niedriger Konzentration der Zusätze.
Es wurden Modellgleichungen hergeleitet, die die Wechselwirkung von bewegtem Lösungsmittel und der Deformation der Makromoleküle beschreiben.
Vom Typ her sind es stochastische Differentialgleichungen, die die Bewegung eines Moleküls in einem Schergeschwindigkeitsfeld beschreiben.
Abschätzungen der Größenordnungen von Termen in den DGlen. haben gezeigt, daß die Trägheitsglieder beweglicher Teile des Moleküls nicht vernachlässigt werden dürfen gegenüber Reibungseinflüssen zwischen umgebendem Fluid und Molekülsegmenten.
Eine numerische Untersuchung der gekoppelten DGlen. für die Langevin-Verteilungsfunktion und den Ortsvektor für die Molekülsegmente ist grundsätzlich möglich, steht aber noch aus.

On Strengths and Weaknesses of the Method of Backcalculation for Projecting the Incidence of AIDS Cases and for Estimating Seroprevalence and Trends in Seroprevalence

This presentation represents joint work with Phil Rosenberg, Ron Brookmeyer, Robert Biggar and Jim Goedert.

The method of backcalculation permits one to estimate the infection curve (number of cases infected per unit time) from serial data on AIDS counts and from knowledge of the incubation distribution of times from infection to development of clinical AIDS. We represent the infection curve as a linear combination of known functions from "flexible families". We prefer those families because they allow recent AIDS counts to influence estimates of the infection curve, whereas many previously used parametric forms for the infection curve are largely determined by the early AIDS count data only.

We performed a sensitivity analysis to determine how much uncertainties in our knowledge of the incubation distribution and uncertainties in AIDS counts affect our estimates of the cumulative ~~projected~~ number of ~~AIDS~~ AIDS cases projected through January 1993 (P93), the cumulative number infected through January, 1985 (N85), the cumulative number infected through April, 1988 (N88), and the difference between the average infection rates from January 1985 through April, 1988 and from January 1981 through January, 1983 (trend). We presented data from non-intravenous drug using homosexual ~~men~~ on the West Coast of United States, for whom it is thought that the rate of infection has decreased.

We estimated P93 = 46,000, N85 = 87,000, N88 = 93000 and trend = −22,500 per year, which indicates a decreasing infection rate. Taking into account both stochastic error, which derives from random variation in AIDS counts, and systematic errors that derive from misspecification of the incubation distribution or from distortion of AIDS counts from reporting delay, changing definitions of AIDS, and underreporting, we find that P93 (± 22%) and N85 (± 30%) are relatively precisely estimated, whereas uncertainty is larger for N88 (± 44%) and trend (± 84%). An increase in the hazard rate for the first two years in the incubation distribution leads to a sharp decrease in the magnitude of the trend, but trend remains negative. An increase in the hazard of the incubation distribution for years 2-8, which induces a decrease in median incubation, decreases P93, N85 and N88 substantially. An increase in AIDS counts for the years 1977-1985, where underreporting and effects of definitional changes are likely, increases estimates of the rate of infection in 1981-1983 and exaggerates the negative trend. Other perturbations produce little change, and, in particular, the right tail of the incubation distribution has little effect on these results.

Mitchell Gail
National Cancer Institute
Bethesda, Maryland USA

Strategies for the Study of Diet and Disease

Various approaches to the study of dietary factors and disease incidence will be reviewed and illustrated. These include such aggregate data methods as international correlation, time trend and migrant studies as well as such analytic methods including cohort, and case-control studies and intervention trials. Aggregate data methods are usually regarded as having hypothesis generating potential only, while analytic methods also have severe limitations for certain aspects of diet owing to dietary homogeneity in populations available for study, and especially owing to measurement error in individual dietary assessment. Hence no replicable epidemiologic method short of an intervention trial exists for the study of dietary hypotheses of major public health potential.

The above viewpoints will be illustrated by means of various data sets relating dietary fat to selected cancers. Also options for enhancing the reliability of the study types mentioned above will be addressed. These include refined measurement error methods for analytic studies using partial likelihood methods and validation datasets; international dietary and risk factor survey data to be used in conjunction with linear relative risk models and random effects assumptions, and the judicious application of intervention trials for primary disease prevention.

Ross Prentice
Fred Hutchinson Cancer Research Center
and University of Washington
Seattle, Washington USA

KOMBINATORIK

Feb 26 - Mar 3 1989
4

Conceptual measurement and finite structures

The aim of conceptual measurement is to understand the conceptual structure of data sets by comparison with given patterns of concept systems. Our approach to conceptual measurement uses the framework of formal concept analysis (cf. B. Ganter, R. Wille, J. Stahl: Conceptual measurement and many-valued contexts. In: W. Gaul, M. Schader (eds.): Classification as a tool of research. North Holland, Amsterdam 1986, 169-176). A scale is defined as a context $\mathbb{S} := (G_{\mathbb{S}}, M_{\mathbb{S}}, I_{\mathbb{S}})$ with a clear conceptual structure which reflects some meaning. The $\mathbb{S}$-measures of an (empirical) context $\mathbb{K} := (G, M, I)$ correspond to $\bigvee$-preserving maps from the concept lattice $\underline{\mathfrak{B}}(\mathbb{K})$ into the concept lattice $\underline{\mathfrak{B}}(\mathbb{S})$ respecting objects. Most important in measurement is the problem: By which scales can a given context be measured? Answers are given by measurability theorems which describe the use of considered finite structures for analyzing data. An example of such a theorem is:

Theorem: A finite context $\mathbb{K}$ admits a full measure into a direct product of one-dimensional ordinal scales if and only if $\mathbb{K} \cong (P, P, \not\geq)$ for some finite ordered set P.

Rudolf Wille (TH Darmstadt)

The Order Dimension of Convex Polytopes and Planar Maps.

Let M be a convex polytope in $\mathbb{R}^3$. Associate with M a poset P_M consisting of the vertices, edges and faces of M partially ordered by inclusion. Our goal is to find the order dimension, denoted $\dim(P_M)$, of this poset. Recall that the order dimension, $\dim(P)$, of a poset P is the least t for which P is the intersection of t linear extensions. Alternately, $\dim(P)$ is the least t for which there exists an assignment $x \mapsto (x_1, x_2, \ldots, x_t) \in \mathbb{R}^t$ $\forall x \in P$ so that $x \leq y$ in $P \iff x_i \leq y_i$ in $\mathbb{R}$ for $i = 1, 2, \ldots, t$. One can ask this question in a general setting. Find the order dimension of the face lattice of a convex polytope in $\mathbb{R}^n$. However for $n \geq 4$, there is no bound which depends only on n. This is due to the existence of "cyclic" polytopes which can contain arbitrarily large sets of vertices with each pair on an edge. Spencer had shown that the dimension of the poset consisting of all one and two element subsets of an m-element set is at least $\log\log m$. So the problem only makes sense in $\mathbb{R}^n$ for $n \leq 3$. For $n \leq 2$, the answer is clearly $n+1$, but for $n = 3$, it is not at all easy to see that there is any upper bound. However we show that for every convex polytope M in $\mathbb{R}^3$, the dimension of P_M is exactly 4. Our techniques extend to show that if M is any planar map (loops and multiple edges allowed), then $\dim P_M \leq 4$. Our proof contains a polynomial time algorithm for producing the coordinatization, and we believe that this may happen some value as a data structure for the vertex/face incidence relation.

This is joint work with Graham Brightwell (Cambridge) and Klaus Reuter (Darmstadt)

William T. (Tom) Trotter

Upper Bounds for Block Codes from Polyhedral Theory

Let $A(n,d,q)$ denote the largest size of a block code of words of length n over an alphabet with q letters and minimum (Hamming) distance d. We transform the problem of calculating $A(n,d,q)$ into a stable set problem and use methods of polyhedral theory and linear programming to compute upper bounds for $A(n,d,q)$. This way we can give new interpretations of known bounds and we obtain – in some cases – improvements over the best upper bounds known to date.

Martin Grötschel (Augsburg), joint work with E. Zehendner

Facets for the complete cut cone

We present results on the facets of the complete cut cone, i.e. the cone C_n of dimension $n(n-1)/2$ generated by the cuts of the complete graph on n vertices. We describe some operations on facets, in particular, a lifting procedure for constructing facets of C_{n+1} from given facet of lower dimensional cone C_n. We present several new classes of valid inequalities for C_n and we prove facetness for some subclasses. The elements of the complete cut cone C_n admit the following geometric characterization: they are exactly the semi-metrics on n points which are isometrically embeddable into L^1. The results presented follow from a joint work with M. Laurent

Michel Deza (Paris, CNRS) and Univ. of Tokyo

Posets with Maximal Möbius Function

Let P be a poset of length $\ell+1$, bounded, with $n+2$ elements. Then the Möbius function of P satisfies

$$|\mu(P)| \leq \max_{k\leq \ell} \max_{p_1+\dots+p_k=n} \prod_{i=1}^{k}(p_i-1).$$

This bound is sharp: for every P there is a poset P^* with $\#P=\#P^*$, $\ell(P^*)\leq \ell(P)$ that achieves equality, and the posets achieving equality are classified. The right-hand side is evaluated, yielding $|\mu(P)|\leq 4^{n/5}$. This solves a problem of R. Stanley.

The analogous problem is solved over graded posets (of given size and length) and attacked for lattices.

Günter M. Ziegler (Augsburg)

Designs, Linear Programming and Ramsey Theory

A (D,c) coloring of the complete graph K_n is a coloring of the edges with c colors such that all monochromatic connected subgraphs have $\leq D$ vertices. Resolvable block designs with c parallel classes and with block size D are natural examples. However, (D,c)-colorings are more relaxed structures.

We investigate the largest n such that K_n has a (D,c)-coloring. The main tool is the fractional matching theory of hypergraphs.

Zoltán Füredi

Math. Inst. Hungarian Academy, Budapest

On two Conjectures of Demetrovics, Füredi, and Katona, concerning partitions

Is it possible to find n partitions of an n-element set whose pairwise intersections are just all atoms of the partition lattice? Demetrovics, Füredi, and Katona verified this for all $n \equiv 1$ or $4 \pmod{12}$ by constructing a series of special Mendelsohn Triple Systems. They conjectured that such triple systems exist for all $n \equiv 1 \pmod 3$ and that the problem on the partitions has a solution for all $n \geqq 7$. We prove that both conjectures are true, except for finitely many n.
This is joint work with B. Ganter, TH Darmstadt, FRG.

Hans-Dietrich Gronau
Ernst-Moritz-Arndt-Universität Greifswald, GDR

OZ and Unimodality

The O'hara-Zeilberger identity:

$$\begin{bmatrix} n+j \\ j \end{bmatrix} = \sum_{\lambda \vdash j} q^{\sigma(\lambda)-j} \prod_{i=1}^{j} \begin{bmatrix} (n+2)i - L_{i-1} - L_{i+1} \\ \lambda_i - \lambda_{i+1} \end{bmatrix},$$

$$\sigma(\lambda) = \lambda_1^2 + \ldots + \lambda_j^2, \quad L_i = \lambda_1 + \ldots + \lambda_i, \quad \begin{bmatrix} n+j \\ j \end{bmatrix} = \prod_{i=1}^{j} \frac{(1-q^{n+i})}{(1-q^i)}, \ n \geq 0$$
$$= 0, \ n < 0$$

implies unimodality of the Gaussian polynomials since each summand is a unimodal polynomial (by inductive hypothesis) with mode at $nj/2$.

OZ is easily proven by demonstrating that

$$\sum_{\substack{\lambda \vdash j \\ \ell(\lambda) \leq k}} q^{\sigma(\lambda)} \left(\prod_{i=1}^{k-1} \begin{bmatrix} ni - L_{i-1} - L_{i+1} \\ \lambda_i - \lambda_{i+1} \end{bmatrix} \right) \begin{bmatrix} n_k - L_{k-1} - L_k \\ \lambda_k \end{bmatrix}$$

is the generating function for partitions with j parts $< n$ such that if f_i = # of parts of size i then $f_{i-1}+f_i \le k \ \forall i$ and $f_{i-1}+f_i = k \Rightarrow f_i + 2\sum_{t>i} f_t + ik \le n_k$.

David M. Bressoud
Penn State

Algebraic combinatorics: The use of finite group actions.

The basic tools are the Cauchy-Frobenius and Burnside's lemma, both in constant and weighted form. These were presented and it was shown how they apply to enumeration of symmetry classes of mappings. Then a redundancy free construction of orbit representatives using double cosets in symmetric groups was mentioned as well as the method of Dixon/Wilf for generating orbit representatives uniformly at random was described. Specific applications are the construction of chemical isomers and the evaluation of catalogs of graphs with $p \le 10$ points. Emphasis was laid on the fact that these methods apply in many other cases, too.

A. Kerber (Bayreuth)

Enumeration of tableaux by number of columns

For $\frac{1}{2}(n-3) \le m \le n-1$, we prove that the number of involutions on $\{1,\dots,n\}$ whose longest increasing subsequence has length m is

$$\sum_{\substack{i,j \ge 0 \\ 2i+j \le n-m}} (-1)^{n+m+i+j} \binom{i+j}{i}\binom{n}{i+j} \mathrm{Inv}(j) \qquad (1)$$

where $\mathrm{Inv}(j)$ is the number of involutions on $\{1,\dots,j\}$, and that the number of permutations on $\{1,\dots,n\}$ whose longest increasing subsequence has length m is

$$\sum_{\substack{i,j,\ell \ge 0 \\ i+j+\ell \le n-m}} (-1)^{i+j} \ell! \binom{i+\ell}{i}\binom{j+\ell}{j}\binom{n}{i+\ell}\binom{n}{j+\ell}. \qquad (2)$$

The proof of (1) uses the Schensted correspondence to express this number as the sum of degrees of all irreducible representations of the symmetric group corresponding to partitions λ with largest part equal to m. This sum is thus the coefficient of $x_1 \cdots x_n$ in $\sum s_\lambda(x_1, \ldots, x_n)$, where s_λ is a Schur symmetric function, and the sum is over partitions with largest part m. The Schur function sum is evaluated using an idea of I.G. Macdonald, yielding (1) as well as more complicated formulas for smaller values of m relative to n. The proof of (2) proceeds similarly and involves the sum $\sum s_\lambda(x_1, \ldots, x_n) s_\lambda(y_1, \ldots, y_n)$, again restricted to partitions λ with largest part m.

The simple form of (1) and (2) suggests that a nice constructive proof exists, and it is hoped that such a construction would lead to new results in symmetric functions

Ian Goulden, Waterloo, Canada

Connections Between Hall-Littlewood Functions and the Rogers-Ramanujan Identities

There exist several identities from the theory of Hall-Littlewood functions that can be viewed as multi-variate generalizations of multiple basic hypergeometric series (i.e., q-series). Included in this list of q-series that can be generalized are some extensions of the Rogers-Ramanujan identities originally due to G. Andrews and D. Bressoud. The most important part of Hall-Littlewood function theory that is relevant to this development involves the adaptation of a technique of I. Macdonald first used in the proof of plane partition conjectures of MacMahon and Bender-Knuth.

John Stembridge
University of Michigan
Ann Arbor

Quasi-Symmetric Designs

A quasi-symmetric t-design is a t-design with two block intersection sizes p and q (where $p<q$). We describe algebraic invariants for quasi-symmetric designs that are similar to the invariants for symmetric designs that are given by the Bruck-Ryser-Chowla theorem. We also settle a conjecture of Sane and Shrikhande by classifying quasi-symmetric 3-designs with $p=1$: the method is to reduce the classification problem to that of finding all integer points on the elliptic curves $y^2 = x^3 - 11x^2 + 32x$ and $y^2 = x^3 - 4x + 4$.

Robert Calderbank
AT&T Bell Labs
Murray Hill NJ.

Solution of an extremal problem for sets using resultants

A short and completely new proof is given of the following, fundamental, theorem of Bollobás: let $A_1, \dots, A_h$ and $B_1, \dots, B_h$ be collections of sets with $\forall i: |A_i| = r, |B_i| = s$ and $|A_i \cap B_j| = 0$ if and only if $i = j$. Then $h \leq \binom{r+s}{s}$. The proof immediately extends to the generalizations of this theorem obtained by Frankl, Alon and others.

The proof uses resultants of polynomials: Associated to each A_i (B_j) is a polynomial $a_i(x)$ ($b_j(x)$) sa $[a_i(x) = \prod_{\alpha \in A}(x-\alpha)]$. S.t. $R(a_i, b_j) = 0 \Leftrightarrow i \neq j$. Then properties of R are used to obtain the bound

A. Blokhuis
Eindhoven
The Netherlands

Log Concave Sequences of Symmetric Functions & Analogs of the Jacobi-Trudi Determinant

Let $x = \{x_1, x_2, \dots\}$ be an infinite set of variables & consider the polynomial ring $\mathbb{R}[x]$. If $\le$ is a partial order on $\mathbb{R}[x]$ then a sequence $(f_n(x))_{n\ge0} = f_0(x), f_1(x), \dots$ is strongly log concave w.r.t. $\le$ iff $f_{k-1}(x) f_{\ell+1}(x) \le f_k(x) f_\ell(x)$ $\forall\, 0 \le k < \ell$

$\iff \begin{vmatrix} f_\ell(x) & f_{\ell+1}(x) \\ f_{k-1}(x) & f_k(x) \end{vmatrix} \ge 0$ for $0 < k \le \ell$, a "Jacobi-Trudi" determinant.

For example, define $\le_x$ by $f(x) \le_x g(x)$ iff $g(x) - f(x) \in \mathbb{R}^+[x]$ and consider $e_k(n) = e_k(x_1, \dots, x_n)$ & $h_k(n) = h_k(x_1, \dots, x_n)$ which are the k^{th} elementary and complete homogeneous symmetric functions in the variables $x_1, \dots, x_n$.

Thm (special case of Jacobi-Trudi) For fixed n the sequences $(e_k(n))_{k\ge0}$ & $(h_k(n))_{k\ge0}$ are strongly log concave w.r.t. $\le_x$.

For our analogs of this theorem, we use a standard partial order on $\mathbb{R}[x]$, i.e., one satisfying ① $f(x) \in \mathbb{R}^+[x] \Rightarrow f(x) \ge 0$
② $f(x) \le g(x) \Rightarrow f(x) + h(x) \le g(x) + h(x)$ $\quad \forall h(x) \in \mathbb{R}[x]$
③ $f(x) \le g(x) \Rightarrow f(x) \cdot h(x) \le g(x) \cdot h(x)$ $\quad \forall h(x) \ge 0$

Note that $\le_x$ is standard. Our new results are

Thm 1 Let $\le$ be standard & suppose $(x_n)_{n\ge0}$ is strongly log concave w.r.t. $\le$, then so are the sequences (i) for fixed k: $(e_k(n))_{n\ge0}$ & $(h_k(n))_{n\ge0}$
(ii) for fixed n: $(e_k(n-k))_{k\ge0}$ & $(h_k(n-k))_{k\ge0}$ □

The method of proof involves lattice path arguments à la Gessel-Viennot. As a special case, consider a single variable q & the ring $\mathbb{R}[q]$. Let $[n] = 1 + q + \dots + q^{n-1}$ & define q-binomial coefficients: $\left[{n \atop k}\right] = \frac{[n]!}{[k]!\,[n-k]!}$ where $[n]! = [n][n-1]\cdots[1] \Rightarrow \left[{n \atop k}\right] = q^{-\binom{k}{2}} e_k(1, q, \dots, q^{n-1})$
$= h_k(1, q, \dots, q^{n-k})$

signless q-Stirling #s of the 1^{st} kind: $c[n,k] = c[n-1,k-1] + [n-1]\, c[n-1,k]$ $(n>0)$, $c[0,k] = \delta_{0k} \Rightarrow c[n,k] = e_{n-k}([1], \dots, [n-1])$

q-Stirling #s of the 2^{nd} kind: $S[n,k] = S[n-1,k-1] + [k]\, S[n-1,k]$ $(n>0)$, $S[0,k] = \delta_{0k} \Rightarrow S[n,k] = h_{n-k}([1], \dots, [k])$

Cor 2 Define $f(q) \le_q g(q)$ by $g(q) - f(q) \in \mathbb{R}^+[q]$. Then the following are strong log concave w.r.t. $\le_q$
(i) $\left(\left[{n \atop k}\right]\right)_{n\ge0}$, $(c[n+k,k])_{k\ge0}$, $(S[n+k,k])_{k\ge0}$
(ii) $\left(\left[{n \atop k}\right]\right)_{k\ge0}$, $(c[n+j, k+2j])_{j\ge0}$, $(S[n,k])_{n\ge0}$ □

For $k\times k$ determinants, define $(f_n(x))_{n\ge0}$ to be PF w.r.t. $\le$ if the matrix $[f_{j-i}(x)]_{i,j\ge0}$ has all of its minors ≥ 0 $(f_n(x) = 0$ if $n<0)$

Thm 3 Let $\le$ be standard & suppose $x_{k-1} x_{\ell+1} = x_k x_\ell$ $\forall\, 0 < k \le \ell$. Then the sequences of Thm 1 (i) & (ii) are PF w.r.t. $\le$. □

Cor 4 $\left(\left[{n \atop k}\right]\right)_{n\ge0}$ and $\left(\left[{n \atop k}\right]\right)_{k\ge0}$ are PF w.r.t. $\le_q$. □

Bruce Sagan, Michigan State University, East Lansing

HALVING STEINER TRIPLE SYSTEMS

When does there exist a Steiner triple system $(V,\mathcal{B})$ of order v (STS(v)) which admits a partition of the set of its triples $\mathcal{B} = \mathcal{B}_1 \cup \mathcal{B}_2$ such that $(V,\mathcal{B}_1)$ and $(V,\mathcal{B}_2)$ are isomorphic hypergraphs? An obvious necessary condition is $b = |\mathcal{B}| \equiv 0 \pmod 2$, i.e. $v \equiv 1, 9, 13$ or $21 \pmod{24}$. This condition is not sufficient: we prove that an STS(v) with the above property exists iff $v \equiv 1$ or $9 \pmod{24}$. On the other hand, almost all STSs do not have the above property. – We also prove that when b is odd, i.e. when $v \equiv 3, 7, 15$ or $19 \pmod{24}$, there exists an STS(v) $(V,\mathcal{B})$ and a triple $t \in \mathcal{B}$ such that there exists a partition of $\mathcal{B} \setminus \{t\} = \mathcal{B}_1 \cup \mathcal{B}_2$ with $(V,\mathcal{B}_1) \simeq (V,\mathcal{B}_2)$.

Alexander Rosa
McMaster University
Hamilton, Ontario

Intersection numbers of difference sets

Let D be a (v,k,λ) difference set in G, $U \trianglelefteq G$, $[G:U] = u$. Let $H = G/U$. For $x \in H$, let $s_x = |\{d \in D : d+U = x\}|$. The numbers s_x are called the intersection numbers of D relative to U. We prove:

<u>Theorem</u> With notations as above, assume G/U is abelian of exponent u^*. Let p be a prime not dividing u^* and assume $tp^f \equiv -1 \pmod{u^*}$ for some numerical G/U-multiplier t of D & some nonnegative integer f. (We can always take $t=1$, for instance). Then (i) if $p^{2j} \| n$, then $s_x \equiv y \pmod{p^j}$ $\forall x \in H$. (ii) One has $yu \equiv k \pmod{p^j}$. If y_0 is the smallest nonnegative solution of this congruence, then $y_0 u \leq k$.

<u>Remark</u> $p^{2j} \| n$ for some j, follows from a theorem of Jungnickel & Pott.

As applications we prove the nonexistence of certain nonabelian difference sets. We also correct erroneous proofs of Lander for the nonexistence of certain $(352, 27, 2)$ abelian difference sets.

K.T. Arasu
Wright State Univ
Dayton, Ohio, U.S.A.

Degree Sequences of Graphs

Let $w: E(G) \to \{1,\dots,m\}$ be a weighting of the edges of a graph G; w is admissible if all weighted degrees $W(x) = \sum_{e \ni x} w(e)$ are distinct ($x \in V(G)$). The irregularity strength $s(G)$ = min m for which an admissible weighting is possible. A survey is given on the number $s(G)$. In particular: Thm 1 (A., Triesch): Let T be a tree on n vertices, then $s(T) \le n-2$ except when T is a star, then $s(T)=n-1$. Thm 2 (A., Triesch): Let G be a graph on n vertices, G connected. Then $s(G) \le n-1$ except for K_3; $s(K_3)=3$. The method of proof uses partitions of the additive group $\mathbb{Z}_n$ and alternatively, theorems from the geometry of numbers. "Graceful" conjecture: Let T be a tree on n vertices, then there always is an admissible weighting which uses all the numbers $1,\dots,n-1$.

M. Aigner (Berlin)

Antipodal distance regular covers of complete graphs.

A graph of diameter d is antipodal if any two distinct vertices at distance d from a third are themselves at distance three. An antipodal distance regular graph of diameter three is a covering graph of a complete graph. They are related to group divisible designs; some of the chief constructions are also geometric. We survey the basic theory of these graphs, discuss a new construction method and new existence constraints for a class of covers.

Chris Godsil, University of Waterloo

The chromatic index of a projective space

The chromatic index of a linear space is the least number of colours such that the lines can be coloured in such a way that no two intersecting lines have the same colour. This number is called the chromatic index χ'.
The conjecture of ERDŐS-FABER-LOVÁSZ says that χ' is not greater than the number of points.
We present recursive constructions and direct constructions.
As consequences we have
- the conjecture is true if q d is odd.
- The conjecture is true for some even d.

Albrecht Beutelspacher (Gießen)

Sparse Ramsey Theorems (joint work with H. J. Prömel)

My talk reports on results from the following papers: Ramsey theorems for finite graphs I/II, submitted to JCT B and A sparse Graham-Rothschild theorem, Trans. AMS (1988). The following results are discussed:
Theorem A (sparse graph Ramsey theorem) For all positive integers k, m, r and g there exists a graph G with the following properties: (1) for every r-coloring of the K_k-subgraphs in G there exists a mono-chromatic K_m-subgraph, (2) the set of K_m-subgraphs do not form ~~short~~ cycles with respect to intersection in K_k-subgraphs of length shorter than k.
Theorem B (sparse partition theorem for Boolean lattices) For all positive integers $m \neq 1$, r and g there exists an n and a set $S \subseteq \underline{2}^n$ with the following properties: (1) for every r-coloring $\Delta : S \to r$ there exists a monochromatic $\underline{2}^m$-sublattice in S, (2) the set of $\underline{2}^m$-sublattices in S do not form cycles

of length shorter than g.

Theorem C (Sparse Hales-Jewett theorem) Given a finite set A and a positive integer r and g there exists a positive integer n and there exists a set $S \subseteq A^n$ such that the set of parallel and combinatorial lines in S doesn't form cycles of length shorter than g but still $S \to (1)_r$.

As a corollary we resolve a conjecture of J. Spencer (1975):

Corollary (Sparse van der Waerden theorem) Given k, r and g there exists a set S of positive integers such that for every r-coloring $\Delta : S \to r$ there exists a monochromatic arithmetic progression of length k in S but these k-term AP in S do not form cycles of length less than g.

B. Voigt
(Bonn)

THRESHOLD FUNCTIONS FOR EXTENSION STATEMENTS

We dub the POISSON PARADIGM that if Z counts many rare fairly independent events and $E[Z] = \mu$ then $Pr[Z=0] \sim e^{-\mu}$. E.g. in the random graph $G(n,p)$ let Z be the number of K_4's so $\mu \sim \binom{n}{4} p^6$ and $Pr[G \not\supseteq K_4] \sim e^{-\mu}$. This was known in the original Erdős–Rényi paper via Brun's Sieve. We give a general Correlation Inequality that under fairly general conditions, $M \leq Pr[\wedge \bar{A}_i] \leq M e^{2\Delta}$ where $M = \prod Pr[\bar{A}_i]$ and $\Delta = \sum Pr[A_i \wedge A_j]$, summed over "dependent" pairs. This allows a quick proof of the Erdős–Rényi theorem and holds even when $\mu \to \infty$ if appropriately slowly. We also indicate a proof of Bollobás' Thm. that if $n^{d-1}p^d = \ln(\binom{n}{2}/c)$ then G has diameter d with probability e^{-c}.

Joel Spencer
(Courant Inst. NY)

Graph Theoretic Codes

Let G be a graph with p vertices, q edges and girth d. It is known that the cycle space of G gives rise to a binary (q, q-p+1)-code and that this code is majority logic decodable. Such codes are usually referred to as graph theoretic codes. In this lecture we describe a decoding procedure for graph codes based on k-regular graphs having a particular type of 1-factorization. We consider a 1-factorization $\mathcal{F}$ such that there exists an automorphism of G which acts cyclically on the 1-factors of $\mathcal{F}$. In addition we would like $\mathcal{F}$ to have the property that the union of any two of its 1-factors does not contain a 4 cycle. In particular these properties permit an efficient method to decode complete graph codes. The algorithm corrects all single and double adjacent errors and all double errors confined to a 1-factor.

Scott Vanstone (Waterloo).

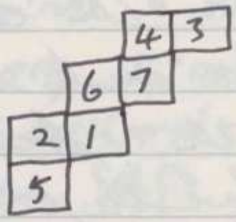
On families with prescribed intersection properties

Let $(X, \wedge, \vee)$ be a ranked finite lattice. For given intersection sets $I \subseteq \mathbb{N}$ the maximum cardinalities of families $\mathcal{F} \subseteq X$ satisfying $rk(F \wedge F^*) \in I$ for all $F, F^* \in \mathcal{F}$, $F \neq F^*$ are considered. For homogeneous lattices – for each two elements $x, y \in X$ of the same rank the cardinalities of their k-shadows are the same for all $k \in \mathbb{N}$ – which satisfy the van der Monde property it is $|\mathcal{F}| \leq \sum_{i=0}^{|I|} \binom{rk(1)}{i}_X$. Using the concept of generalized Stirling numbers due to Voigt it follows that this result is valid for several lattices, e.g. powerset-lattices (Frankl, Wilson), linear and affine lattices, partition lattices and Graham-Rothschild lattices. Moreover, if the intersection set is $I = [\bar{0}, s] \cup [t, n]$, $s < t$, the maximum cardinalities of such families $\mathcal{F} \subseteq \mathcal{L}(n,q)$ in the linear lattice are given.

Hanno Lefmann (Bielefeld)

Permutations with balanced patterns

We consider "balanced rim-staircase tableaux", for example



or (equivalently) permutations $x_1, x_2, \ldots, x_n$ of $1, 2, \ldots, n$ having no peaks or valleys in even positions. (A peak is an element x_i such that $x_i \geq x_{i-1}, x_{i+1}$, and a valley is an element x_i such that $x_i \leq x_{i-1}, x_{i+1}$.) If b_n = number of such permutations, for each n, we show that

$$B_0(x) = \sum_n b_{2n}\, x^{2n}/2n! = \frac{1}{1 - x/\sqrt{2}\,\tanh x/\sqrt{2}}$$

$$B_1(x) = \sum_n b_{2n+1}\, x^{2n+1}/2n+1! = \frac{\sqrt{2}\,\tanh x/\sqrt{2}}{1 - x/\sqrt{2}\,\tanh x/\sqrt{2}}$$

We note that Gessel has considered the related problem of enumerating permutations with no valleys in even positions. If g_n = number of such permutations, Gessel obtains

$$G_0(x) = \sum_n g_{2n}\, x^{2n}/2n! = \frac{\operatorname{sech} x}{1 - x \tanh x}$$

$$G_1(x) = \sum_n g_{2n+1}\, x^{2n+1}/2n+1! = \frac{\tanh x}{1 - x \tanh x}$$

It follows (comparing generating functions) that $g_{2n+1} = 2^n\, b_{2n+1}$, a fact for which we have no simple direct combinatorial explanation.

Curtis Greene (Haverford)

Affine difference sets

We present some recent existence tests for abelian affine difference sets which allow to prove the prime power conjecture for order up to 10.000.

Dieter Jungnickel (Giessen)

Maps on orientable surfaces of arbitrary genus

Let $M_{\mathcal{A}}(u,x,y,z)$ be the generating function for $m^{\mathcal{A}}_{g,i,j,k}$, the number of rooted maps of genus g with i vertices, j faces, k edges and no faces of degree not in $\mathcal{A} \subseteq \{1,2,3,\dots\} = \mathbb{N}$. Let $R_{\mathcal{A}}(x,y|z)$ be the generating function for $r^{\mathcal{A}}_{i,j,k}$, the number of rotation systems $\nu \in \mathfrak{S}_{2k}$ with i cycles such that $\varepsilon_k \nu$ has j cycles, none of whose lengths are not in $\mathcal{A}$. For $f \in \mathbb{Q}[[x,y,z]]$, let $\Omega_u f(x,y,z) = 2u^2 z \frac{\partial}{\partial z} f(xu^{-1}, yu^{-1}, \frac{1}{2}zu)$. Then $M_{\mathcal{A}}(u,x,y,z) = \Omega_u \log R_{\mathcal{A}}(x,y|z)$. Let $\Pi_{\mathcal{A}}$ be the set of all (integer) partitions with no part not in $\mathcal{A}$, and let $H_\theta(x) = \prod_{1 \le i \le l(\theta)} (x-i+1)^{(\theta_i)}$ (rising factorial) where θ is a partition. We use the group algebra $\mathbb{C}\mathfrak{S}_N$, the embedding theorem, and the character theory of the symmetric group to prove

Thm 1: $R_{\mathbb{N}}(x,y|z) = \sum_{n \ge 0} \frac{z^n}{n!(2n)!} \cdot \sum_{\theta \vdash 2n} f^\theta \chi^\theta_{[2^n]} H_\theta(x) H_\theta(y)$. $\square$

where χ^θ is an irreducible (ordinary) character of $\mathfrak{S}_N$ and f^θ is the degree of the rep. associated with $\theta \vdash N$.

Lemma 2: Let $\Pi_{k,n}$ denote the set of all k-tuples $(\theta^{(1)}, \dots, \theta^{(k)})$ of partitions whose weights sum to n, and let $\mathcal{B}_{k,n}$ be the set of all $\theta \vdash kn \ni \chi^\theta_{[k^n]} \ne 0$. Then $\exists\ \Delta_k : \mathcal{B}_{k,n} \xrightarrow{\sim} \Pi_{kn} \ni \chi^\theta_{[(kb)^n]} = \pm n! \prod_{j=1}^{k} \frac{1}{n_j!} \chi^{\theta^{(j)}}_{[b^{n_j}]}$ for $\theta \in \mathcal{B}_{k,n}$, the sign depending only on θ, where $\Delta_k(\theta) = (\theta^{(1)}, \dots, \theta^{(k)})$ and $n_j = |\theta^{(j)}|/b$. Moreover, $H_1^\theta(x) = H_k^{\theta^{(1)}}(x) H_k^{\theta^{(2)}}(x-1) \cdots H_k^{\theta^{(k)}}(x-k+1)$, where $H_j^{(\theta)}(x) = \prod_{1 \le i \le l(\theta)} (x-ji+j)^{(j\theta_i)}$. $\square$

With the aid of this we prove:

Thm 3: $M_{\{4\}}(u^2,x,y,z) = \frac{1}{2}\{M_{\mathbb{N}}(4u^2, x+u, x, z^2 y) + M_{\mathbb{N}}(4u^2, x-u, x, z^2 y)\}$.

When $g=0$: $M_{\{4\}}(0,x,y,z) = M_{\mathbb{N}}(0,x,x,z^2 y)$. This is the generating function form for Tutte's bijection between rooted quadrangulations on the sphere and rooted maps on the sphere. His construction does not generalise to the torus, since 4-regular maps on the torus are not 2-face colourable. Thm 3 indicates that there may be some kind of constructive proof of a generalisation of Tutte's bijection to surfaces of higher genus.

The theory can be extended in a natural way to hypermaps to obtain other bijections for surfaces of arbitrary genus. [Joint work with T. Visentin].

David Jackson (Waterloo)

A strengthening of Mann's theorem on difference sets (joined work with D. Jungnickel)

The main tools to prove the non-existence of certain (v,k,λ)-difference sets are multipliers and a theorem due to MANN (1964). There are several proofs of Mann's result. We simplify Lander's proof and generalize his results. We obtain new non-existence results even for non-abelian difference sets.

In particular, we obtain:

Thm. Let D be a (v,k,λ)-difference set in G, $H \trianglelefteq G$, G/H abelian, $\exp G/H =: u^*$. Then the following holds:

If $p^i \equiv -1 \bmod u^*$ (p prime), then $p^{2j} \| n$ (i.e. $p^{2j+1} \nmid n$)

(generalization of Mann's thm to non-abelian groups.

Corollary: (i) $p^j \leq |H|$

(ii) $|G/H| > k \Rightarrow p^{2j} \mid v\lambda$

For instance, we prove: $\nexists$ abelian $(704, 38, 2)$-d.s. if $\exp Syl_2(G) \leq 4$

$\nexists$ $(7^3, 19, 1)$-d.s. (this holds also for the non-ab. groups of order 7^3)

Alexander Pott, (Gießen)

The restricted Ramsey theorem for graphs

Apparently P. Erdős was the first to ask whether there exists a graph F such that $F \to (K_3)_2^{K_2}$ but F has small clique size $cl(F)$, where $cl(F)$ denotes the maximal size of a complete subgraph in F. Answering this question J. Folkman (1970) constructed a graph F with $F \to (K_3)_2^{K_2}$ and $cl(F) = 3$. This result was a starting point for Ramsey Theory for graphs and hypergraphs.

One of the key results in this area is the restricted Ramsey theorem for graphs and hypergraphs due to Nešetřil and (1977, 1983). A hypergraph $(X, \mathcal{E})$ is called irreducible if for any two vertices $x, y \in X$ there exists an edge $E \in \mathcal{E}$ such that $x, y \in E$. Observe that with respect to ordinary graphs cliques are the only irreducible ones. Let $\mathcal{F}$ be a family of irreducible hypergraphs. Then $\mathrm{Forb}(\mathcal{F})$ denotes the set of all hypergraphs which do not contain any member of $\mathcal{F}$ as an induced subgraph. Let $G, H \in \mathrm{Forb}(\mathcal{F})$. Then Nešetřil and Rödl proved that there exists an $F \in \mathrm{Forb}(\mathcal{F})$ such that $F \rightarrow (G)^H_2$.
The original proofs of this result are quite involved and conceptually not that easy to understand, even in the case of ordinary graphs. The aim of the talk given here is to present a short and simple proof for the restricted Ramsey theorem for hypergraphs. This proof was obtained jointly with B. Voigt and will appear in the J. Combin. Theory Series A.

Hans Jürgen Prömel
(Universität Bonn)

Exchange properties and elimination processes

The Gaussian elimination algorithm is besides the Euclid algorithm probably among the most famous and certainly among the most used algorithms in mathematics. It turns out that its combinatorial basis, i.e. the sequence of its pivot elements is nothing but a combinatorial exchange structure, namely a special greedoid. This greedoid has neither the interval nor the transposition property. Thus it seems to have less structure. However, we can give some nice algorithmic, duality and polyhedral results: Gauß greedoids can be characterized by the optimality of the greedy for linear objective functions; they are closed under an appropriate duality operator (of which matroid duality is a special case). Finally, we give some polyhedral characterizations. For special Gauß greedoids we can linearly describe the convex hull of its characteristic vectors completely, and there is some hope to extend these results to general Gauß greedoids. This lecture reports on some earlier results of my student O. Goecke and recent joint work with L. Lovász and R. Schrader

Bernhard Korte (Bonn)

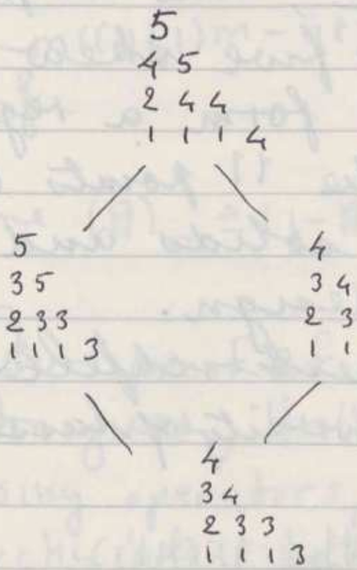
Permutations are tableaux and tableaux are permutations

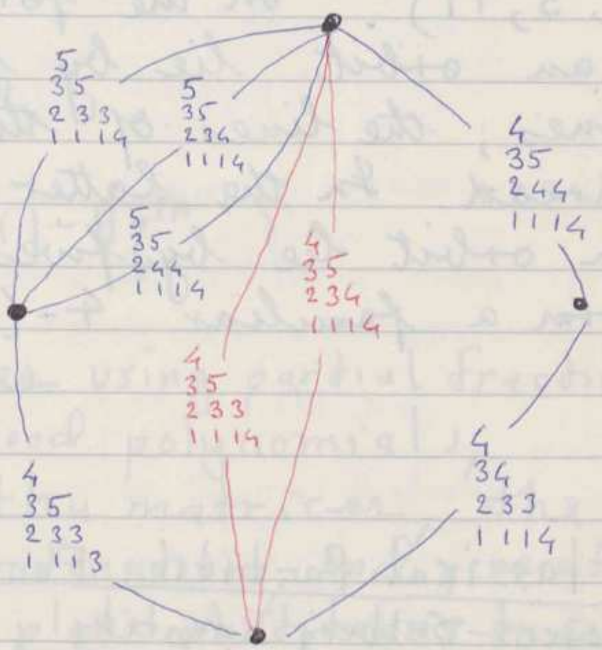
A. Lascoux & M.-P. Schützenberger

A decomposition of a permutation μ is any product $\sigma\sigma'..\sigma''$ of simple transpositions which is equal to it. Taking the subwords of $\sigma\sigma'..\sigma''$ produces the permutations smaller than μ for the Ehresmann order (also called strong or Bruhat order).

To any permutation μ, we can associate the tableau $K(\mu)$ whose columns are the successive left (reordered) factors of μ considered as the word $\mu_1\,\mu_2\ldots$ The Ehresmann order is just the componentwise order on the special tableaux $K(\mu)$ called <u>keys</u>.

Conversely, given any tableau t, pushing successively each of its columns to the right by the jeu de taquin or by Schensted algorithm, gives a key $k_+(t)$; symmetrically, we get on the left another key $k_-(t)$, and one has $k_-(t) \le t \le k_+(t)$. Thus, we can add to the graph of the Ehresmann order an edge, labelled by t, joining the vertices $k_-(t)$ and $k_+(t)$. This new is Eulerian (see Séminaire Lotharingien, sept 88) and has many properties generalizing those of the Ehresmann/Bruhat/strong order, in connection with the geometry of flag varieties (see Minneapolis meeting of Combinatorics, June 88, to appear in Springer L.N.).





Ehresmann order for the four permutations

41523

31524 41325

31425

6 edges to add at distance 1

2 edges " " 2

Lascoux @ FRCIRP 71

LITP, Université Paris 7
75251 Paris Cedex 05

Projective spaces of square size

If $|PG(n,q)| = rs$ and T is a Singer cycle, then there are many cases in which the orbits of $\langle T^r \rangle$ give interesting subsets of the space. When $r = s$ the only possibilities for (n, q, r) with $n > 1$ are $(3, 7, 20)$ and $(4, 3, 11)$. In the former case, the 20 points of an orbit lie by fours on five skew lines; the lines of the orbits form a regular spread. In the latter case, the 11 points of an orbit lie by fives in 66 solids and form a familiar $4-(11,5,1)$ design.

James Hirschfeld
(University of Sussex)

Classical Partition Functions and the $U(n+1)$ Rogers-Selberg Identity

We show that after suitable specialization the "balanced" side of the $U(n+1)$ Rogers-Selberg identity gives the generating function for all partitions whose parts differ by at least $(n+1)$. A similar specialization yields the additional condition that the parts must be $\geq n+1$. The case $n=1$ is the sum side of the pair of classical Rogers-Ramanujan-Schur identities.

This connection between classical partition functions and the $U(n+1)$ Rogers-Selberg identity depends upon the identity

$$(1)\quad \sum_{\substack{m_1+\dots+m_n=m \\ m_i\geq 0}} \Big\{ \Big(\prod_{1\leq r<s\leq n} (q^{nm_r} - q^{(s-r)+nm_s}) \Big) \cdot \Big(\prod_{i=1}^{n} (q)_{((i-1)+nm_i)} \Big)^{-1}$$

$$\cdot\, q^{(n(n+1)/2)(m_1^2+\dots+m_n^2)} \cdot (-1)^{(n-1)m}$$

$$\cdot \Big(\prod_{i=1}^{n-1} q^{(i-n)(n+1)m_i} \Big) \Big\}$$

$$= q^{m((n+1)m-n+1)/2} \cdot \frac{1}{(q)_m},$$

where $(A)_m = (1-A)(1-Aq)\cdots(1-Aq^{m-1})$

Our proof of (1) involves using partial fraction techniques, Hall-Littlewood polynomials, Raising operators, q-Kostka matrices, the Cauchy identity for Schur functions, and generating functions for column-strict plane partitions to solve a general q-difference equation. One out-come of this proof is a new class of symmetric functions, analogous to Hall-Littlewood polynomials, that interpolates between Schur functions and complete homogeneous symmetric functions.

Stephen C. Milne
(University of Kentucky)

~~II~~ Two Coloring Problems

I We show that any 3-hypergraph uniform of degree 3 on n vertices can ~~be~~ have its vertices colored by 3 dimensional 0-1 vectors such that the colors on any edge span. (with Füredi, Griggs, + Holzman)

Does this hold for $k=4,5$? If the hypergraph instead of 3 has a cyclic vertex transitive symmetry thus ~~became unate~~, does this hold for all k? This would imply a conjecture of Chung + Graham. Aart Blokhuis at this meeting gave a counterexample to the general question for $k \geq 6$.

II Any planar graph can be colored in 3 colors so that two color classes form forests and the third is an independent set.

Daniel Kleitman (MIT)

Complexity theory for fast growing functions

By establishing the complexity of the Ketonen Solovay function another example of a true theorem of Peano arithmetic which is not provable in PA is given

W. Deuber (Bielefeld)

KOMBINATORIK, Oberwolfach, Feb. 1989

BASES AND ORIENTATIONS IN MATROIDS

Michel LAS VERGNAS, C.N.R.S., Paris

The structure of oriented matroid abstracts the main combinatorial properties of signed linear dependence over ordered fields. Classical examples include : cycle spaces of directed graphs, configurations of points and (dually) arrangements of hyperplanes in Euclidean spaces , arrangements of pseudolines in the projective plane and generalizations in higher dimensions (this last example being generic by the Folkman-Lawrence Topological Representation Theorem). Oriented matroids provide several ways to encode the different combinatorial types of configurations of points or hyperplanes.

Theorem A [Las Vergnas 1975] : The number of acyclic reorientations of an oriented matroid M (or, equivalently, the number of maximal covectors, or the number of regions of the Folkman-lawrence representation) is given by the evaluation $t(M;2,0)$ of its Tutte polynomial.

Theorem A generalizes Stanley's theorem (1973) on acyclic orientations of graphs and contains Zaslavski's theorem (1975) on the number of regions of an arrangement of hyperplanes. It can be generalized to oriented matroid perspectives, oriented matroid counterpart of linear applications [Las Vergnas 1977]. A further generalization of Theorem A deals with the notion of activities.

Theorem B [Las Vergnas 1982] : denoting by o_{ij} the number of reorientations with activities i,j of an oriented matroid on an ordered set, we have $t(M;\zeta,\eta) = \sum_{ij} 2^{-i-j} o_{ij} \zeta^i \eta^j$.

Comparing Theorem B with

Theorem [Crapo 1969, generalizing works of Tutte for graphs] : denoting by b_{ij} the number of bases with internal activity i and external activity j of a matroid M on a totally ordered set, we have $t(M;\zeta,\eta) = \sum_{ij} b_{ij} \zeta^i \eta^j$

we get the equality $o_{ij} = 2^{i+j} b_{ij}$.

This equality suggests a question : Is there a natural correspondence between bases and reorientations of an oriented matroid compatible with these equalities for all i,j ? Our purpose in the present talk is to describe such a correspondence.

The Complexity of Knots and Colourings

We show that the problem of determining the Jones polynomial of an alternating knot is #P-hard. This follows from studying the complexity of the Tutte polynomial of the associated graph universe. This is found to be #P-~~co~~hard except when we are evaluating it along a special hyperbola of the form $(x-1)(y-1) = \alpha$ when it <u>may</u> be in polynomial time.

Examples of this are when $\alpha = 1$ (all graphs and matroids), $\alpha = 2$ (planar graphs).

Evaluation of the Tutte polynomial at a ~~special~~ point (a, b) turn out to be as hard as evaluation along the whole special hyperbola through the point except when (a,b) is one of the 9 special point $(0,0)$, $(-1,-1)$ $(1,1)$, $(-1,0)$, $(0,-1)$ $(i,-i)$ $(-i, i)$, (j, j^2), (j^2, j). where $j = e^{2\pi i/3}$. At the six of these special points which lie on the Jones curve the Jones polynomial has a known evaluation. In view of these results we believe that there is no other point in the plane at which the Jones polynomial can be evaluated in polynomial time.

<u>Open problems</u>

1) Is counting 4-colourings of a planar map #P-hard?

2) Is determining the Tutte polynomial of a planar map along $x = 1$ #P-hard?

<u>Note</u> The above results are joint with F. Jaeger and D.L. Vertigan.

Dominic Welsh
Mathematical Institute
Oxford.

Partielle Differentialgleichungen

5. März — 11. März 1989

Elliptic differential inequalities with an application to gradient bounds.

Let $Lu = \sum_{i,j=1}^{n} a_{ij}(x) u_{x_i x_j} + \sum_{i=1}^{n} b_i(x) u_{x_i}$ in $D \subset \mathbb{R}^n$ open and bounded and assume that $\exists\, \alpha, K > 0$ such that $\alpha \xi^2 \le \xi^T a(x) \xi \le K \xi^2$, $|b_i| \le K$, $-K \le c(x)$ (no upper bound on c). A generalization of the classical strong maximum principle is proved:

Theorem. Assume that D satisfies a uniform interior ball condition and that there exists $h \in Z = C^2(D) \cap C^1(\bar D)$ such that $Lh + ch \le 0$ in D and $h > 0$ in D. Then $u \in Z$,

$Lu + cu \le 0$ in D, $u \ge 0$ on ∂D implies (i) $u = \beta h$ ($\beta < 0$), or (ii) $u \equiv 0$, or (iii) $u > 0$ in D.

The case (i) signifies that u is an eigenfunction for the operator $L + c$ corresponding to the eigenvalue $\lambda = 0$. Two applications:

1. Let $Lu = \sum_{i,j} D_j (a_{ij}(x) D_i u)$, and let (λ_1, θ_1) be the first eigenpair for $Lu + \lambda u = 0$ in D, $u = 0$ on $\Gamma = \partial D$. Then

$Lu + cu \le 0$ in D, $u \ge 0$ on Γ, $c(x) \le \lambda_1$, $c(x) \not\equiv \lambda_1 \Rightarrow u \equiv 0$ or $u > 0$ in D.

This is a best theorem, since it becomes obviously false if $c(x) \equiv \lambda_1$.

2. Consider the nonlinear elliptic equation (1) $F(\varphi, \varphi_x, \varphi_{xx}) = 0$ ($\varphi_x = \operatorname{grad} \varphi$, φ_{xx} = Hessian) with $(\partial F / \partial r_{ij})$ pos. definite. The following theorem generalizes results by Pucci (1987) and Weinberger (1987).

Theorem. Let $\varphi \in C^1(\bar D) \cap C^2(D)$ be a solution of (1) and let $\varphi_\xi = \xi \cdot \varphi_x$ be the directional derivative in the direction ξ ($|\xi| = 1$). Then

$\varphi_\xi \ge 0$ on Γ implies $\varphi_\xi \ge 0$ in D

under each of the following conditions:

L as above with $a_{ij} = \partial F / \partial r_{ij}$, $b_i = \partial F / \partial p_i$, $c = \partial F / \partial u$, Argument $(\varphi(x), \ldots)$

(i) there exists h: $Lh + ch \le 0$ in D, $h > 0$ in D.

(ii) there exists η with $|\eta| = 1$ such that $\varphi_\eta \ge 0$ in all of D.

Wolfgang Walter
Universität Karlsruhe/Germany

Asymptotic behaviour of solutions of dissipative systems

We consider parabolic resp. damped hyperbolic systems of the following type: $u_t + A^k u = 0$ / resp. $u_{tt} + A^k u + u_t = 0$, where $A = -\Delta$ or $A = -\partial_i a_{ik}(x)\partial_k$ elliptic with $A = -\Delta$ outside a ball, in an exterior domain $\Omega \subset \mathbb{R}^n$, together with initial and suitable boundary conditions. To get the desired result, namely the decay behaviour of $L^q(\Omega)$-norms of the solution u, $2 \leq q \leq \infty$, we make an ansatz with generalized eigenfunction expansions and are led to the study of pointwise estimates of solutions of exterior boundary value problems for the operator A. As further motivation we discuss the importance of these estimates for the corresponding nonlinear systems.

Reinhard Racke (Bonn)

A family of torsional creep problems

Subject of the lecture is the study of solutions to the problem

$$-\Delta_p u = -\operatorname{div}(|\nabla u|^{p-2}\nabla u) = 1 \text{ in } \Omega$$
$$u = 0 \text{ on } \partial\Omega$$

as $p \to 1$ or as $p \to \infty$. For $1 < p < \infty$ the problem has a unique solution $u_p \in W_0^{1,p}(\Omega)$. The limiting case $p \to \infty$ models perfectly plastic torsion. It is shown that $\lim_{p\to\infty} u_p(x) = d(x, \partial\Omega)$, the distance function to $\partial\Omega$. The limiting equation is no longer elliptic. The case $p \to 1$ is of independent geometric interest because it leads to interesting free boundary problems. Even if Ω is a ball in $\mathbb{R}^n$ there are surprises: If the ball has radius less than n, u_p tends to zero as p goes to 1, but if the radius is greater than n, u_p blows up to ∞ everywhere in Ω. This phenomenon is linked to the isoperimetric inequality between perimeter and volume of Ω. It can be explained for general domains if one solves the geometric problem: Given Ω find $D \subset \Omega$ such that surface area of D minus volume of D becomes minimal.

Bernhard Kawohl (Heidelberg)

A $C_{\alpha,\beta}$ - Theory for Parabolic Equations

The well-known $C_{\alpha,\alpha/2}$ - Theory for parabolic equations

$$(*) \quad \frac{\partial u}{\partial t} - \sum_{i,j} a_{ij}(x,t)\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_i b_i(x,t)\frac{\partial u}{\partial x_i} + c(x,t)u = f(x,t) \text{ on } \Omega\times(0,T)$$

$$u = 0 \quad \text{on } \Sigma_T = \partial\Omega\times(0,T) \cup \bar\Omega\times\{0\}$$

does not give the complete picture of the dependence of the regularity of the solution from the data. We assume data from function spaces $C_{\alpha,\beta}$ — α-Hölder-continuous with respect to x and β to time, with $\beta = 0$ included — and give various interior and global estimates for the solution in corresponding spaces. By counterexamples it is shown, that additional information on the data along the parabolic boundary is needed to prove global Hölder-estimates. On the other hand we prove, that $(*)$ has a classical solution u in $C_{2,\alpha}(\overline{\Omega\times(0,T)})$ (including the boundary), if for some $\alpha > 0$ the data are in $C_{\alpha,0}$, $\partial\Omega \in C_{2+\alpha}$, $f(x,0) = 0$ on $\partial\Omega$ and as the only assumption concerning time-regularity: For $x \in \partial\Omega$ the form $\sum_{i,j} a_{ij}(x,t)\nu_i\nu_j$ (ν the outer normal) is δ-Hölder-continuous with respect to time for some $\delta > 0$.

Michael Wiegner (Bayreuth)

On the positive solutions of the Emden equations in cones

Let (r, θ) be the polar coordinates of a point x in $\mathbb{R}^N$ and let $\mathcal{C} = \{(r,\theta) : \theta \in \Omega \subset S^{N-1} = \{|x| = 1\}\}$ be a cone. It is known that the problem $\Delta u + r^{\sigma} u^p = 0$ in $\mathcal{C}$, $u = 0$ on $\partial\mathcal{C}$ with $p > 1$ and $\sigma \in \mathbb{R}$ has solutions of the form $u = r^{-(2+\sigma)/(p-1)} \alpha(\theta)$ for a range of $p \in (p^*, p^{**})$. It turns out that for $p \leq p^*$ no regular or singular solutions exist. The asymptotic behavior at the vertex and at infinity can be computed for certain classes of solutions by means of a potential theoretical approach. These results were developped in a common paper with M. Essén.

C. Bandle (Basel)

The Cauchy problem for the time dependent nonlinear Schrödinger equation

(joint work with T. Cazenave)

We consider the problem

$$\left.\begin{aligned} iu_t + \Delta u &= f(u) \\ u(0,x) &= \varphi(x) \end{aligned}\right\} \text{(NLS)}$$

where $u = u(t,x) \in \mathbb{C}$, $t \geq 0$, $x \in \mathbb{R}^n$, $f(u) = \lambda |u|^\alpha u$, $\lambda \in \mathbb{R}$, and $\alpha > 0$. If $\alpha = 4/(n-2)$, the critical power, this problem is well-posed in $H^1(\mathbb{R}^n)$; and if $\|\nabla\varphi\|_{L^2}$ ~~is~~ is sufficiently small, the solution is global. For $\alpha > 4/(n-2)$, we fix s, $0 \le s < n/2$, so that $\alpha = 4/(n-2s)$. Then, subject to the technical restriction $[s] < \alpha$, (NLS) is well-posed in $H^s(\mathbb{R}^n)$; and if $\|(-\Delta)^{s/2}\varphi\|_{L^2}$ is sufficiently small, the solution is global. In fact, for this result we may consider $\alpha \geq 4/n$; and in particular if $\alpha = 4/n$, (NLS) is well-posed in $H^0(\mathbb{R}^n) = L^2(\mathbb{R}^n)$. If $\|\varphi\|_{L^2}$ is small, the solution is global.

Fred B Weissler
(Paris / College Station)

Evolution equations in noncylindrical domains.
(joint work with P. CANNARSA and J.P. ZOLÉSIO)

We consider the problem:

$$(1) \quad \begin{cases} u_t = \Delta u + f(t,x) \ ; & t \in [0,T] \ , \ x \in \Omega_t \subset \mathbb{R}^N \\ u(0,x) = u_0(x) \end{cases}$$

where Ω_t depends (smoothly on t). We reduce (1) to an abstract problem, by setting:

$$H = L^2(\mathbb{R}^N)$$

$$D(A(t)) = \{ u|_{\Omega_t} \in H^2(\Omega_t) \cap H^1_0(\Omega_t) \text{ and}$$

$$u|_{\Omega_t^c} \in H^2(\Omega_t^c) \cap H^1_0(\Omega_t^c) \} \ ; \Omega_t^c = \mathbb{R}^N \setminus \Omega_t$$

$$A(t)u = z \ ; \quad \int_{\mathbb{R}^N} z\varphi \, dx = \int_{\mathbb{R}^N} u \Delta\varphi \, dx$$

for all $\varphi \in C_0^\infty(\mathbb{R}^N)$ such that $\varphi = 0$ on T_t, the boundary of Ω_t.

Problem (1) reduces to

$$(2) \qquad u'(t) = A(t)\,u(t) \quad , \quad u(0) = u_0$$

We prove that $\{A(t)\}_{t \in [0,T]}$ fulfils the Kato-Tanabe hypothesis

$$\begin{cases} \| (\lambda - A(t))^{-1} \| \leq \dfrac{K}{|\lambda|} \\ \| \dfrac{\partial}{\partial t} (\lambda - A(t))^{-1} \| \leq \dfrac{K}{\sqrt{|\lambda|}} \end{cases}$$

and solve (2).

We consider also the damped wave equation:

$$(3) \qquad \begin{cases} u_{tt} = \Delta(u + u_t) + f(t,x) & ; \; x \in \Omega_t \\ u + u_t = 0 & \text{on } T_t \\ u(0) = u_0 \; , \; u'(0) = u_1 \end{cases}$$

Giuseppe Da Prato.

(Scuola Normale Superiore di Pisa (Italy))

Eigenvalues of the Schrödinger operator $H - \lambda W$ in a spectral gap of H.

(Joint work with S. Alama and P. A. Deift)

We consider Schrödinger operators $H = -\Delta + V$, acting in the Hilbert space $L_2(\mathbb{R}^\nu)$, where the bounded, measurable function V is such that the spectrum of H has a gap (a,b). We then ask for the eigenvalue branches of the operator family $H - \lambda W$ (here W is a relatively compact perturbation of H and λ a real coupling constant). Such operators arise in the quantum theory of solids as a model for crystals with localized impurities.

In the present talk, we concentrate on the case $W \geq 0$ and describe the asymptotic distribution of eigenvalue branches of $H \pm \lambda W$, as λ goes to $+\infty$: we define (for E in the gap (a,b))

$$N_\pm(\lambda) := \#\{0 < \lambda_j < \lambda;\ E \in \sigma(H \mp \lambda W)\},$$

for $\lambda > 0$, and discuss the relationship between the asymptotic behaviour of $N_\pm(\lambda)$, as $\lambda \to \infty$, and the volumina of the related classically allowed regions in phase space. While the semi-classical approximation gives the correct answer for the asymptotics of N_+, the asymptotics of N_- may be determined in some cases by means of the integrated density of states of H and will not in general agree with the asymptotics of the associated phase space volume.

Rainer Hempel (Univ. München)

Sturmian theory for second order elliptic equations

The well-known comparison theorem by Sturm and Picone (in the version of Leighton (1962)) for ordinary, self-adjoint, second order differential equations is extended to self-adjoint elliptic differential equations. The basic domain G and the coefficients of the equation are not necessarily bounded, and no regularity hypotheses on the boundary ∂G are required. As an application of the theory the number of the nodal domains of the eigenfunctions of an elliptic differential operator (Friedrichs extension) belonging to the lower eigenvalues of the spectrum is estimated.

Erich Müller-Pfeiffer
(Erfurt, G.D.R.)

The distribution of eigenvalues of boundary value problems

Consider an eigenvalue problem for a self-adjoint elliptic differential operator of order $2m$ on a compact n-dimensional manifold with a boundary. Let $N(\lambda)$ denote the eigenvalue distribution function i.e. the number of eigenvalues λ_k smaller than a given λ. The asymptotic formula

$$N(\lambda)=a\lambda^{n/2m}+o\left(\lambda^{n/2m}\right), \quad \lambda \to +\infty, \quad (1)$$

is a well-known classical result. A refined

Two-term asymptotics

$$N(\lambda)=a\lambda^{n/2m}+b\lambda^{(n-1)/2m}+o(\lambda^{(n-1)/2m}),\ \lambda\to+\infty,\ (2)$$

is established by the author; coefficient b takes account of the boundary conditions.

Formula (2) holds when a certain geometrical condition is fulfilled. This geometrical condition is formulated in terms of a branching Hamiltonian billiard associated with the differential operator. In this respect periodic, absolutely periodic and deadend trajectories are investigated.

Dmitri G. Vasil'ev
(Moscow, USSR).

The asymptotic of the Weyl-Titchmarsh m-function on the spectrum of perturbed Hill's equation.

In the last time appeared some works in wich the asymptotic of the Weyl-Titchmarsh m-function is studied in the angle $0<\varepsilon<\arg z<\pi-\varepsilon$ ($|z|\to\infty$). A more difficult problem consist in studing of the asymptotic of the m-function on the spectrum. Of course, it is possible only in the case when the limit $m(z)$, $\operatorname{Im} z\to 0$ exist.

In the report it will be given an account of some results about the asymptotic expansion of the m-function of the m-function for $\lambda\to+\infty$ ($\operatorname{Im}\lambda=0$) in the case of perturbed Hill's equation: $-y''+[p(x)+q(x)]y=\lambda y$ $(-\infty<x<\infty)$, $p(x+1)\equiv p(x)$ a smooth periodic function, $q(x)$ – a smooth function with the condition

$$\int_{-\infty}^{\infty}(1+|x|)|q(x)|\,dx<\infty.$$

B. M. Levitan (Moscow University).

A Semigroup Approach to Parabolic Equations in Hölder Spaces

We consider a parabolic initial-boundary value problem in $[0,T]\times\overline{\Omega}$, where $\Omega\subset\mathbb{R}^n$ is a bounded open set with regular boundary $\partial\Omega$:

$$u_t(t,x) = A(t,x,D)u(t,x) + f(t,x) \quad , \quad 0\le t\le T,\ x\in\overline{\Omega}$$
$$u(0,x) = u_0(x) \quad , x\in\overline{\Omega}$$
$$B_j(t,x,D)\,u(t,x) = g_j(t,x) \ , \ 0\le t\le T,\ x\in\partial\Omega \ , \ j=1,..,m$$

Here $A(t,x,D)$ is an elliptic $2m$-order operator, and $B_j(t,x,D)$, $j=1,..,m$, are boundary differential operators, satisfying roots and complementing conditions.
Such problems have been recently studied by means of the theory of analytic semigroups in $C(\overline{\Omega})$, adapted to the case of non dense domain and nonhomogeneous boundary conditions.
The classical theory of Solonnikov concerning optimal ~~reg~~ Hölder regularity in (t,x) has been recovered by Lunardi, Sinestrari, & Von Wahl. Hölder continuity with respect to the space variable x has been studied by Lunardi, whereas Hölder continuity with respect to time has been studied (in the homogeneous case) by Sinestrari & Von Wahl, and by Acquistapace & Terreni.

alessandra lunardi
(cagliari, Italy)

Estimates of the eigenvalues of the Dirichlet Laplacian on a domain with fractal boundary.

We study the eigenvalues of $\begin{cases} -\Delta u = \lambda u & \text{in } \Omega \subset \mathbb{R}^n \\ u = 0 & \text{on } \partial\Omega \end{cases}$

we know that these eigenvalues are real, positive... and we are interested in the asymptotics of the counting function $N_0(\lambda, \Omega)$ (the number of eigenvalues less than λ) as λ tends to $+\infty$, when $\partial\Omega$ is fractal.

we know that when Ω is bounded the following formula holds

(*) $N_0(\lambda, \Omega) \sim a_n |\Omega|_n \lambda^{n/2}$ as $\lambda \to +\infty$

$|\cdot|_n$ denotes the Lebesgue measure in $\mathbb{R}^n$ and $a_n = (2\pi)^{-n} \omega_n$ where ω_n is the volum of the unit ball in $\mathbb{R}^n$.

This formula is due to Weyl (1911) when $\partial\Omega$ is smooth.

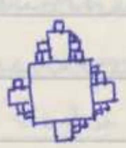
For the Neumann problem, even if Ω is bounded, (*) is not necesary valid; ~~the~~ boundary can be "too long".

The length of the boundary arises in the "remainder term":

$\varphi(\lambda, \Omega) := N_0(\lambda, \Omega) - a_n |\Omega|_n \lambda^{n/2}$

since, when $\partial\Omega$ is smooth $\varphi(\lambda, \Omega) = -b_n |\partial\Omega|_{n-1} \lambda^{(n-1)/2} + o(\lambda^{(n-1)/2})$.

In 1980 Berry's suggest to replace $n-1$ by h, the Hausdorff dimension, in the expresion of $\varphi(\lambda, \Omega)$ for domains with fractal boundary. In 1986 Brossard and Carmona ~~built~~ a counter example (a union of squares) where they ~~showed~~ conjectured that the Hausdorff dimension has to be replaced by δ, the Minkowski's one.



Here we prove that (joint work with M. L. Lapidus 1987)

$N_0(\lambda, \Omega) = a_n |\Omega|_n \lambda^{n/2} + O(\lambda^{\delta/2})$

where δ is the Minkowski dimension of the boundary.

More precisely we asume that there exists $0 < \mu < \infty$ such that

$\varepsilon^{-(n-\delta)} |\Omega_\varepsilon| \xrightarrow[\varepsilon \to 0]{} \mu$

with $\Omega_\varepsilon = \{x \in \mathbb{R}^n / \operatorname{dist}(x, \partial\Omega) < \varepsilon\}$.

The method of the proof if the "courant's method" and a previous work of G Métivier.

J. Fleckinger
univ. Toulouse I
pl A. France
31042 Toulouse cedex
- France

The Riemann-Roch Theorem on algebraic curves

(Joint work with J. Brüning and N. Peyerimhoff)

An analytic proof of the following generalization of the Riemann-Roch theorem on algebraic curves was sketched:
Let $C \subset \mathbb{C}P^N$ be an algebraic curve, $\pi : S \to C$ the Noether normalization and $\Sigma \subset C$ the singular locus. Given a vectorbundle E of rank k over C, holomorphic over $C \setminus \Sigma$, and equipped with an Hermitian metric which is constant near Σ one has the "twisted" Cauchy-Riemann operator

$$\bar{\partial}_E : C_0^\infty(E|_{C\setminus\Sigma}) \to C_0^\infty(\Lambda^{0,1}E|_{C\setminus\Sigma}).$$

$\bar{\partial}_E$ is closable with domain in $L^2(E)$ and range in $L^2(\Lambda^{0,1}E)$ where the metric on $C\setminus\Sigma$ comes from restricting the Fubini-Study metric of the ambient projective space. All closed extensions are Fredholm operators and correspond to the subspaces W of the finite dimensional space

$$W_0 = \mathcal{D}(\bar{\partial}_{E,max}) / \mathcal{D}(\bar{\partial}_{E,min}).$$

The corresponding extension $\bar{\partial}_{E,W}$ has index

$$\operatorname{ind} \bar{\partial}_{E,W} = \operatorname{ind} \bar{\partial}_{E,min} + \dim W,$$

with

$$\operatorname{ind} \bar{\partial}_{E,min} = k \cdot \chi(S) + \int_{C\setminus\Sigma} c_1(E)$$

and

$$\dim W_0 = k \sum_{q\in\pi^{-1}(\Sigma)} (n(q)-1).$$

Here $\chi(S) = 1-g$ is the arithmetic genus of the compact Riemann surface S, and $n(q)$ is the multiplicity of the branch of C, which is determined by q.

Helmut Schröder (Augsburg)

Quasilinear parabolic equations with nonlinear boundary conditions.

We consider the following problem:

$$\text{(P1)}\quad u_t - \sum_{l,m=1}^{N} A_{lm}(x,t,u(x,t),\nabla u(x,t))\cdot u_{x_l x_m} = F(x,t,u(x,t),\nabla u(x,t)) \quad \text{in } \Omega\times(0,T],$$

$$\text{(P2)}\quad \sum_{l,m=1}^{N} A_{lm}(x,t,u(x,t),\nabla u(x,t))\cdot n_m(x)\cdot u_{x_l} = g(x,t,u(x,t),(|\nabla u(x,t)|^{\gamma})_{1\le r\le N}) \quad \text{in } \partial\Omega\times[0,T],$$

$$\text{(P3)}\quad u(x,0) = \Psi(x) \text{ on } \bar{\Omega}$$

Here $\Omega\subset\mathbb{R}^N$ is a bounded domain; n is the outward unit normal to Ω; $\gamma\in(1,\infty)$. Let (A_{lm}) be strongly elliptic, and assume that A_{lm}, F, Ψ, g satisfy certain (rather low) smoothness conditions. Moreover, assume the following compatibility conditions:

$$(*)\quad \Psi_{x_l}(x) = 0 = g(x,0,\Psi(x),(|\Psi_{x_r}(x)|^{\gamma})_{1\le r\le N}) \quad \text{for } x\in\partial\Omega,\ 1\le l\le N.$$

Then we can show that a solution to (P1)–(P3) exists, locally in time. We further show by a counterexample that condition (*) may not be replaced by the "natural" compatibility conditions. Further counterexamples prove that local continuation and uniqueness are not possible in our context.

P. Dewing (Bayreuth)

Globally regular solutions to the u^5-Klein Gordon equations

The Cauchy problem for the semilinear wave equation

$$(1) \qquad u_{tt} - \Delta u + u^5 = 0 \quad \text{in } \mathbb{R}^3 \times \mathbb{R}_+$$

$$u|_{t=0} = u_0 \,, \quad u_t|_{t=0} = u_1$$

for radially symmetric initial data $u_0(x) = u_0(|x|) \in C^3(\mathbb{R}^3)$, $u_1(x) = u_1(|x|) \in C^2(\mathbb{R}^3)$ is shown to admit a unique, global, radially symmetric solution $u(x,t) = u(|x|,t) \in C^2(\mathbb{R}^3 \times \mathbb{R}_+)$.

Essential tools are the local (small time) existence and uniqueness results of Jörgens, the a priori estimates of Rauch for solutions with small initial energy, and a decay estimate for solutions near a singularity. We heavily exploit invariance of (1) under scaling $u \mapsto u_R(x,t) = R^{1/2} u(Rx, Rt)$. Moreover, the energy inequality is used extensively.

Michael Struwe

Pressure jumps for the dam problem

Stephan Luckhaus, Bonn

This talk presents joint work with G. Gilardi on the dam problem in the free boundary formulation. To save writing let all the constants be one. Denote by p the pressure, by $-e_2$ the vector of gravity, s the relative water content of an earth dam Ω. The flow equations in strong formulation are

$$\partial_t s - \nabla\cdot(\nabla p + s e_2) = 0 \text{ in } \Omega$$

$$0 \le s \le 1,\quad 0 \le p,\quad p\cdot(1-s) \equiv 0 \text{ in } \Omega$$

$$\partial_\nu p + s\nu e_2 = 0 \text{ in } \Gamma_N,\quad p = p_D \text{ in } \Gamma_D,\quad \partial_\nu p + s\nu e_2 \ge 0 \text{ in } \Gamma_D \cap \{p_D = 0\}$$

where $\partial\Omega = \Gamma_N \cup \Gamma_D$

Under suitable conditions on the data, the most important being $\nu_2 \le 0$ in Γ_N, we prove that $\partial_t p$ is a measure and $\partial_t p_-$ bounded (in the interior). $\partial_t p^+$ can indeed be a singular measure as an example shows. A key ingredient in the proof is a lemma on nonnegative subharmonic functions φ.

Lemma:
Suppose $p \geq 0$, $\Delta p \geq 0$ in B_ρ then

$$\int_{B_\rho} \Delta p > \frac{c}{\rho} \int_{B_\rho} p \; |\{p=0\} \cap B_\rho|^{1-\frac{1}{n}} . \;\square$$

This allows a rigorous estimate from below for $\partial_t p$, the time derivative of the pressure, even though one has no knowledge of the smoothness of the free boundary.

Stephan Luckhaus

Asymptotic properties of solutions of the elasticity system. V.A. Kondrat'ev (Moscow)

We consider the equation:

$$\sum_{|\alpha| \le m} A_\alpha D^\alpha u(x) = f(x) \qquad (1)$$

where $A_\alpha : X_{|\alpha|} \to X_0$, $X_m \in X_{m-1} \in \dots \in X_0$, $u(x) \in X_m$, $f(x) \in X_0$, $x = (x_1, \dots x_n)$, $D^\alpha = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1} \dots \partial x_n^{\alpha_n}}$, X_k - all Hilbert's spaces. Suppose, that exists $P(\lambda) = \sum_{|\alpha| \le N} a_\alpha \lambda^\alpha$ such that $P(\lambda)\left(\sum_{|\alpha| \le m} i\lambda^\alpha A_\alpha\right) = P(\lambda) R(\lambda)$ is analytic for $|\operatorname{Im} \lambda| \le C$ and bounded. Then $\|u(x)\|_{X_m} \le C_1 e^{-c_2|x|} \int_{\mathbb{R}^n} e^{c_2|x|} \|f(x)\| dx$

A number of questions of theory of elliptic equations is reduced to the investigation of equation (1). For example, consider the system of elasticiti $\sum_{i,j,h,k=1}^{n} \frac{\partial}{\partial x_i} a_{ij}^{hk} \frac{\partial u^k}{\partial x_j} = f_h$, $k = 1, \dots n$, $x \in \Pi$, $\Pi = \{x : 0 < x_n < 1, (x_1, \dots x_{n-1}) \in \mathbb{R}^{n-1}\}$.

Problem I. To find the weak solution of the system elasticit, such that $E(u, \Pi) = \sum_{i,j=1}^{n} \int_\Pi \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2 dx < \infty$ and $\sigma_h(u) = \sum_{i,j,h,k=1}^{n} a_{ij}^{hk} \frac{\partial u^k}{\partial x_j} \cos(n, x_i) \Big|_{\partial\Pi} = 0$. Some theorems of uniqueness of solutions are proved.

Kondrat'ev

The Cauchy problem with small data for non-linear second order hyperbolic differential equations

For non-linear perturbations of the wave equation in $\mathbb{R}^{1+n}$

$$\Box u = G(u, u', u'')$$

where $G \in C^\infty$ vanishes of second order at 0, it is now well established that if $u_j \in C_0^\infty(\mathbb{R}^n)$ there is a global solution with Cauchy data

$$u = \varepsilon u_0, \quad \partial_t u = \varepsilon u_1 \quad \text{when } t = 0,$$

for small $\varepsilon > 0$ if $n \geq 5$. When $n = 4$ this is also true if there is no u^2 term, that is, $G(u,0,0) = O(u^3)$ as $u \to 0$; for any G there is a constant c such that a solution exists for $0 \leq t \leq e^{c/\sqrt{\varepsilon}}$. (Unpublished; $\sqrt{\varepsilon}$ can probably be replaced by ε with some additional work.) Assume now that $n = 3$. When G is a function of u' and u'' only, then F. John and S. Klainerman have proved that a solution exists for $t \leq e^{c/\varepsilon}$. A lower bound for c has been given by F. John and the speaker when G is linear in second derivatives; it agrees with an upper bound given by F. John in a special case. The lecture was mainly devoted to the extension of this result to fully non-linear equations. The main point is the determination of the lifespan of the solution of an approximating Cauchy problem in $\mathbb{R}^2$,

$$\frac{\partial u}{\partial t} = a\left(\frac{\partial u}{\partial x}\right)^2 + 2bu\frac{\partial u}{\partial x} + cu^2; \quad u(0,x) = u_0(x);$$

the case $a = c = 0$ (Burgers' inviscid equation) was sufficient for the earlier result.

Lars Hörmander

Helmholtz decomposition and the weak Neumann-problem in L^q.

Report ~~to~~ given on joint work with H. Sohr (Paderborn).
Let $G \subset \mathbb{R}^N$ denote either a bounded or an exterior domain and $\mathcal{D}_0^\infty(G) := \{\phi \in C_0^\infty(G)^N \mid \operatorname{div}\phi = 0 \text{ in } G\}$. For $1 < q < \infty$ let $\mathcal{D}^q(G) := \overline{\mathcal{D}_0^\infty(G)}^{\|\cdot\|_q}$ and $G^q(G) := \{\nabla p \mid p \text{ measurable}, p \in L^q(G_R)\ \forall R > 0, \nabla p \in L^q(G)\}$ where $G_R := G \cap \{|x| < R\}$. Then the Helmholtz decomposition states that $L^q(G) = \mathcal{D}^q(G) \oplus G^q(G)$ ($q \neq 2$ direct decomposition, $q = 2$ orthogonal). Further there is a constant $k > 0$ such that $\|u + \nabla p\|_q \geq k(\|u\|_q + \|\nabla p\|_q)$, $u \in \mathcal{D}^q$, $\nabla p \in G^q$. The equivalence to the weak Neumann Problem in L^q is indicated: Let $\hat{\hat{H}}^{1,q}(G) := \{u : G \to \mathbb{R} \mid u \text{ measurable}, u \in L^q(G_R)\ \forall R > 0, \nabla u \in L^q(G)\}$ and define $\hat{H}^{1,q}(G) := \hat{\hat{H}}^{1,q}(G)/\mathbb{R}$. Equipped with norm $\|\nabla \cdot\|_q$, $\hat{H}^{1,q}(G)$ is a reflexive Banach space. Then there is a constant $C > 0$ such that for $u \in \hat{H}^{1,q}(G)$
$C\|\nabla u\|_q \leq \sup\{\langle \nabla u, \nabla\phi\rangle \mid \phi \in \hat{H}^{1,q'}(G), \|\nabla\phi\|_{q'} \leq 1\}$ where $q' := \frac{q}{q-1}$. Further for $F \in (\hat{H}^{1,q'}(G))^*$ there exists a unique $u \in \hat{H}^{1,q}(G)$ such that $F(\phi) = \langle \nabla u, \nabla \phi\rangle$ $\forall \phi \in \hat{H}^{1,q'}(G)$.

C. G. Simader (Bayreuth)

Symmetry breaking for semilinear elliptic equations

Consider the Dirichlet problem
$$\Delta u + \lambda f(u) = 0$$
in the n-dimensional unit ball B^n;
$$u = 0 \text{ on } \partial B^n.$$
Due to the theorem of Gidas, Ni, Nirenberg positive solutions are always radially symmetric. The question if symmetry

breaking bifurcation from branches of radially symmetric solutions changing sign occurs was studied by Pospiech, Smoller and Wasserman using techniques of local bifurcation with symmetry. In a joint paper with K. Schmitt a result of Pospiech obtained by global arguments could be improved to nonlinearities f which are asymptotically linear at infinity, positive in 0 and whose primitive has exactly one minimum. Consider a connected component of fully symmetric solutions in function space and a bounded domain B in the parameter-function space and assume that there is a door D to this box B through which the component enters and leaves the box. Assume that on the door there are only fully symmetric solutions and that the degree can be computed on the door and does not vanish. Then there must be symmetry bifurcation inside the box. This idea is made precise and is applied to the special boundary value problem. Bifurcation from infinity is used to study the branches of radially symmetric solutions on which symmetry breaking bifurcation takes place.

Willi Jäger (Heidelberg)

Global Existence for Quasilinear Reaction-Diffusion Systems

We report on existence theorems for classical solutions of quasilinear parabolic systems whose principal parts are in divergence form. We are in particular interested in the question of global existence. In the special case of upper-triangular systems ("chemotaxis systems") it is shown that an L_∞-bound implies global existence.

Herbert Amann (Zürich)

Nonlinear equation Schrödiger-type.

Consider equation

$$-\Delta\psi + U(x)\psi = A\psi\,; \quad \psi\big|_{\partial\Omega} = 0$$

Here

$A = \lambda$ classical Schrödinger equation

$A = \psi^2$ "ψ-cube" equation

$A = \int_\Omega K(x,y)\psi^2(y)\,dy$ Pekar-Bogolubov polaron equation (electron in crystall)

Particular case $K(x,y) = a(x)\,b(y)$. In this case $A = a(x)\int_\Omega b(y)\psi^2\,dy$; Putting $\lambda = \int_\Omega b(y)\psi^2\,dy$ we got formally linear equation $-\Delta\psi + U(x)\psi = \lambda a(x)\psi$.

Scaling ψ, $\psi = \beta\varphi$, we can reduce problem to linear one strictly

$$\left\{\begin{array}{l} -\Delta\varphi + U(x)\varphi = \lambda a(x)\varphi \\ \varphi\big|_{\partial\Omega} = 0 \\ \int_\Omega \varphi^2\,dy = 1 \end{array}\right. \qquad \left\{\begin{array}{l} B = \int_\Omega b(y)\,\varphi^2(y)\,dy \\ \beta = \sqrt{\dfrac{\lambda}{B}} \end{array}\right.$$

$$\Psi = \beta\varphi$$

Therefore spectrum of nonlinear ~~system~~ equation is arbitrary (for $\lambda B > 0$) subset of the spectrum of linear equation. Generalisations are very possible.

Molchanov A.M. (Puščino near Moscow)

09.03.89г.

The Schrödinger equation with a constant, weak magnetic field.

We describe some joint work with B. Helffer, related to earlier works by Avron-Simon, Nenciu, Bellissard, Guillot-Ralston-Trubowitz and many physicists.

Let $e_1, \dots, e_n$ be a basis in $\mathbb{R}^n$, $\Gamma = \bigoplus_1^n \mathbb{Z} e_j$, $V \in C^\infty(\mathbb{R}^n; \mathbb{R})$ with $V(x+\gamma) = V(x)$, $\forall \gamma \in \Gamma$. Let $b_{j,k} = -b_{k,j}$, $1 \le j,k \le n$ be real and constant. With $A_k(x) = \frac{1}{2}\sum b_{j,k} x_j$, $B := d(\sum A_k dx_k) = \frac{1}{2}\sum\sum b_{j,k}\, dx_j \wedge dx_k$, we put

$$P_{B,V} = \sum_1^n (D_{x_k} + A_k(x))^2 + V(x).$$

Let $E_0(\theta) \le E_1(\theta) \le \dots$ be the Floquet eigenvalues of $P_{0,V}$, for $\theta \in \mathbb{R}^{n*}$. If:

$$(*) \qquad \sup E_{k-1} < \inf E_k \le \sup E_k < \inf E_{k+1},$$

for some k, the Peierls substitution in solid state physics says that for energies close to $[\inf E_k, \sup E_k]$ and for $|B|$ small, "$P_{B,V}$ is well described by the pseudodifferential operator $E_k(D_x + A_k(x))$". Our first theorem justifies this, in the sense that it gives a corresponding reduction of the spectrum of $P_{B,V}$. We can even drop the assumption $(*)$. Then the reduced operator is a matrix of pseudodifferential operators.

The second theorem treats the case $n=3$. Under suitable assumptions, we find singular oscillations in the density of states measure. They are related to the de Haas - van Alphen effect, explained heuristically by Onsager.

Johannes Sjöstrand

Multiple solutions of the nonlinear Dirichlet problem with L^2-boundary data.

The purpose of this talk is to describe the existence of multiple solutions for the Dirichlet problems

$$Lu + b(x)u^+ - a(x)u^- = s\,\varphi_1(x) + h(x) \text{ in } Q,$$
$$u(x) = \varphi(x) \text{ on } \partial Q$$

and

$$Lu + b(x)u^+ - a(x)u^- = h(x) \text{ in } Q$$
$$u(x) = t\,\varphi(x) \text{ on } \partial Q,$$

where $\varphi \in L^2(\partial Q)$, $h \in L^2(Q)$, s and t are parameters, L is a selfadjoint operator and φ_1 is the first eigenfunction of the operator $L+b$. If a and b intersects with the spectrum of the operator L then both problems admit multiple solutions for s and t large. Since $\varphi \in L^2(\partial\Omega)$, these solutions belong to a weighted Sobolev space.

Jan Chabrowski (University of Queensland Australia)

Some aspects of spectral theory on locally symmetric spaces

We consider a locally symmetric space $X = \Gamma\backslash G/K$ of finite volume. Here G is a semi simple Lie group of non compact type, K a maximal compact subgroup and $\Gamma \subset G$ a discrete subgroup of co-finite volume. The Laplacian Δ of X is essentially selfadjoint in L^2 with domain $C_c^\infty(X)$. Let $\bar{\Delta}$ be the unique selfadjoint extension of Δ in L^2. One of the basic problems is to analyze the spectrum of $\bar{\Delta}$. This problem is closely related to various fields in mathematics such as representation theory, algebraic number theory, theory of PDE, ... The investigation of this problem has been started by Selberg and Roelcke. Very important contributions to this field have been made by Gelfand and Langlands. In particular, the so-called "Langlands program" has emerged from the investigation of Eisenstein series, which is part of the spectral theory. The easiest case is that of a Riemann surface $\Gamma\backslash H$ of constant curvature -1 and finite volume. This case has been treated by Selberg. It is known that the spectrum $\sigma(\bar{\Delta})$ consists of a point spectrum $\sigma_p(\bar{\Delta})$

and an absolut continuous spectrum $\sigma_c(\bar{\Delta})$. The absolut continuous spectrum is the interval $[1/4, \infty)$ with finite multiplicity (= # of cusps of $\Gamma \backslash H$). The point spectrum consists of eigenvalues of finite multiplicity with ∞ the only possible point of accumulation. There exist many subtle questions related to eigenvalues even in this case. The main tool to study eigenvalues is "Selbergs trace formula". A modern approach to the investigation of the continuous spectrum uses scattering theory. The case of a Riemann surface is similar to potential scattering on the real line and a very effective method is the method of Enss.

In general, the continuous spectrum has been analysed by Langlands via the theory of Eisenstein series. This may be regarded as stationary approach to scattering theory. It would be very interesting to develop other methods which can be generalized to other spaces. Also note that the problem of studying the continuous spectrum is similar to the study of the continuum in the case of the N-body Schrödinger operator. Concerning eigenvalues, we prove the following result: Let $N(\lambda) = \#\{\lambda_i \le \lambda\}$ be the counting function. Then there exist constants $C > 0$, $N \in \mathbb{N}$, such that

$$(*) \qquad N(\lambda) \le C(1+\lambda^N), \quad \lambda \ge 0.$$

This implies that for all $\rho \in \mathcal{S}(\mathbb{R})$, $\sum_{i=1}^{\infty} \rho(\lambda_i) < \infty$, and therefore, Selberg's trace formula can be developed in the general case. This has been a well-known conjecture in the theory of automorphic forms. The methods, which we used to prove $(*)$ yield also results on automorphic L-functions.

Werner Müller (Berlin).

Asymptotic behavior of eigensolutions and the spectrum of Schrödinger operators

Pointwise decay or growth properties of solutions to the equation $-\Delta v + qv = \lambda v$ are closely related to the spectrum of the corresponding Schrödinger operator. Bounds on eigenfunctions, well known for q_- (the negative part of q) bounded or in L^n, extend to $q_- = o(|x|^2)$, as do estimates for the distance of λ to the essential spectrum, depending on the rate of growth of non-L_2-solutions v. The case $q_- = O(|x|^2)$ appears as a border line with a striking example turning up, the crucial point of which is still an open problem (and remained open during the meeting).

Andreas M. Hinz (München).

Local Existence of a parabolic equation with fully nonlinear boundary condition arising in the theory of heat conduction; an L_p-approach.

The problem under consideration is:

$$
\begin{array}{lll}
u_t(x,t) - \operatorname{div}_x \underline{a}(x,t,u(x,t),\nabla_x u(x,t)) = f(x,t,u(x,t),\nabla_x u(x,t)) & \text{in } \Omega_T := \underset{\substack{\cup \\ \mathbb{R}^n\ (\text{bd., } C^2 \text{ bdr.})}}{\Omega} \times (0,T) \\
\langle \underline{a}(\xi,t,u(\xi,t),\nabla_x u(\xi,t)), \underline{\nu}(\xi)\rangle = \psi(\xi,t) & \text{on } \partial\Omega_T \\
u(x,0) = \varphi(x) & \text{in } \Omega
\end{array}
$$

($\underline{\nu}$:= outer normal to Ω)

We construct a local (in time) solution in $W_p^{2,1}(\Omega_T) := \{u \mid \partial_x^\alpha u, \partial_t u \text{ (distr. sense)} \in L_p(\Omega_T)\ \forall |\alpha| \le 2\}$, $p > n+2$. The underlying estimates for the linear problem are essentially due to SOLONNIKOV. Problems of the type above were also considered by ACQUISTAPACHE, P./TERRENI, B. (1987); these authors work in different function spaces (Hölder in t, Sobolev in x)

Peter Weidemaier (Bayreuth)

Spektrum der Maxwell Gleichungen

Untersucht wird das Verhalten der Eigenlösungen u der Maxwell Gleichungen im Außenraumgebiet mit long range Dielektrizität und Permeabilität vom Typ $p_i(|x|) + O(r^{-\delta})$ für ein $0<\delta\leq 1$ und entsprechender Bedingung an die Radialableitung. Für beliebiges δ erhalten wir exponentielles Abklingen der Eigenlösungen im L^2-Mittel und für $\delta = 1/2$ sogar die Freiheit von Punkteigenwerten auf der reellen Achse. In engem Zusammenhang mit diesem Problem steht das Prinzip der eindeutigen Fortsetzbarkeit, das wir unter der Voraussetzung, u klinge in 0 von unendlich hoher Ordnung ab, und einer Bedingung an die Radialableitung an die Permeabilität vom Typ $O(r^{\varepsilon-1})$, beweisen können.

N. Vogelsang (Clausthal-Z.)

L^∞ - bounds for parabolic systems with cross-diffusion

An invariance theorem for reaction-diffusion systems

$$u_t = A(t,x,u)Lu + f(t,x,u,u_x) \quad \text{in } G \subset \mathbb{R}^{n+1}$$
$$u = \varphi \quad \text{on } \Sigma_1 \qquad , \quad \Sigma_1 \cup \Sigma_2 = \partial_p G$$
$$B(t,x,u)\,u_\nu = g(t,x,u) \quad \text{on } \Sigma_2$$

is established.

As an application, global a priori bounds in L^∞ for solutions to the two component competing species system

$$u_t = [(c_1 + d_1 v)u]_{xx} + u(e_1 - a_1 v - b_1 u) \quad \text{in } t>0,\ 0<x<1$$
$$v_t = c_2 v_{xx} + v(e_2 - a_2 u - b_2 v)$$

with $u_x = v_x = 0$ for $x = 0,1,\ t>0$ and initial values $u_0, v_0 \geq 0$ is derived.

R. Redlinger (Karlsruhe)

MATHEMATISCHE STOCHASTIK

12.03. – 18.03.1989

Convergence of infinitely divisible laws

A purely probabilistic proof of an equivalent form of Gnedenko's classical criterion (1939) of convergence of infinitely divisible laws was sketched.

Sándor Csörgő (Szeged)

The work of A.N. Kolmogorov on strong limit theorems

The historical setting is briefly reviewed, from J. Bernoulli's Ars Conjectandi (1713), through to Hilbert's 6th problem (1900), the work of Borel, Cantelli and others. Next, the weak law is considered (Khinchin, Lévy, Kolmogorov, ...) and random series (three-series theorem, Kolmogorov inequalities, ...).

Kolmogorov's work on the strong law is considered (Comptes Rendus (1930), Grundbegriffe der Wahrscheinlichkeitsrechnung (1933)), together with its generalisations (ergodic theorem, martingale convergence theorem, L_p version, Banach spaces, ...).

Next, Kolmogorov's work on the law of the iterated logarithm (1929) is discussed, and its applications, eg to the Hartman–Wintner LIL (1941).

Finally, we discuss randomness and computational complexity. The basic references for this section are the papers of Kolmogorov & Uspensky (Tashkent 1986 / Th. Prob. Appl. 1987), and Vovk (TPA 1987). Connections with von Mises' theory of collectives are briefly mentioned.

N.H. Bingham (RHBNC, London)

Global extrapolation of local efficiency.

Given an EDF-test for goodness of fit it is shown how the second derivative of the asymptotic power function can be employed to obtain global upper bounds for the efficiency. These bounds are least in the class of tests with convex and centrally symmetric acceptance region. The proof is based on a linear optimization lemma for isotonic critical functions.

Helmut Strasser (Bayreuth)

A law of the iterated logarithm for trimmed sums

Let $X_1, X_2, \ldots$ be a sequence of independent random variables with a common (continuous) distribution function F. For $0 \le k \le n-1$ let $S_n(k)$ denote the "modulus trimmed sums" formed when the k summands largest in absolute value are excluded from the partial sums $X_1 + \ldots + X_n$. When F is stochastically compact and symmetric about 0 and k_n are positive integers with $k_n / \log\log n \to \infty$ and $k_n / n \to 0$, the law of the iterated logarithm is shown to hold for the trimmed sums $S_n(k_n)$. This result, which is joint work with D.M. Mason, answers a question posed by P.S. Griffin, Probab. Th. Rel. Fields 77, 241-270 (1988).

Erich Haeusler (Munich)

Pointwise Bahadur-Kiefer type theorems.

Let $U_1, U_2, \ldots$ be an iid. sequence of uniformly distributed r.v's on $(0,1)$, and let $F_n(x) = n^{-1} \# \{U_i \le x : 1 \le i \le n\}$, $F_n^{inv}(x) = \inf\{y : F_n(y) \ge x\}$, $F_n^{inv}(0) = 0$, $\alpha_n(x) = \sqrt{n}(F_n(x) - x)$ and $\beta_n(x) = \sqrt{n}(F_n^{inv}(x) - x)$.

Recently, David Mason and the lecturer were able to prove the Kiefer (1970) conjecture, i.e. by showing that

$$\lim_{n\to\infty} n^{1/4} (\log n)^{-1/2} \|\alpha_n + \beta_n\| / \|\alpha_n\|^{1/2} = 1 \text{ a.s.},$$

where $\|f\| := \sup_{0<x\leq 1} |f(x)|$.

Other theorems of this type have been proven for the sum of the partial sum process and its inverse. Moreover uniform limsup results are provided by Einmahl and Mason (1989) for $\|\alpha_n + \beta_n\|_{t_n}$, where $\|f\|_t = \sup_{0<x\leq t} |f(x)|$.

We present here the limiting behavior of $\|\alpha_n(t_n) + \beta_n(t_n)\|$, where $0<t_n<1$ is a nonincreasing sequence. The case where $t_n = t \in (0,1)$ is due to Kiefer (1967), who proved that

$$\limsup_{n\to\infty} \pm \|\alpha_n(t)+\beta_n(t)\| / \{2^{5/4} 3^{-3/4} (t(1-t))^{1/4} n^{-1/4} (\log\log n)^{3/4}\} = \pm 1 \text{ a.s.}$$

We show how this result extends to the case where $t_n \downarrow 0$. The cases $t_n/\log\log n \to \infty$ (resp. $c \in (0,1)$, resp. 0) are of special interest here. Also, we explain the origin of the constants which appear in the statements.

Paul Deheuvels (Univ. of Paris VI)

Applications of Nonparametric Estimation to Parametric Estimation in Regression Theory

If we have the general linear model $Y = A^t(X)\theta + \varepsilon$ where A is a functional of $\mathbb{R}^q \to \mathbb{R}^p$ and ε is the random error of mean zero, using the fact that $\alpha(x) = E[Y/X=x] = A^t(x)\theta$ is the regression function, we can define a nonparametric estimator $\hat{\alpha}_n(x)$ in function of an initial sample $\{(X_1,Y_1),\ldots,(X_n,Y_n)\}$ and construct general estimators for θ minimizing $\Psi(\theta) = \int (\hat{\alpha}_n(x) - A^t(x)\theta)^2 \, d\Omega_n(x)$ with Ω_n a general weighting function. This methodology is applied to Random Design model, Fixed Design model, Sequential model, Dependent Data model and Censored Data model. Some simulations results showing a promissing behaviour are also presented.

Wenceslao González Manteiga (Santiago de Compostela)

Multivariate Bernoulli Distribution

Let $X_1, X_2, \ldots, X_n$ be an arbitrary sequence of random variables taking only the values 0 or 1. Using the machinery of the Kronecker product from multilinear algebra we can give a simple representation of $p_{k_1,k_2,\ldots,k_n} = P\{\bigcap_{i=1}^{n}[X_i = k_i]\}$ in terms of combined simple and/or centralized moments. Extensions to binomial distributions are possible. Our representation should provide an alternative to the traditional log-linear model.

Jef L. Teugels (Leuven, Belgium)

Improving S-estimators.

It may be postulated that a good robust estimator should be (1) globally robust (high breakdown point), (2) locally robust (Fréchet differentiable), (3) efficient at the assumed model and (4) should exhibit no pathologies off the assumed model. The properties (1)-(3) may be obtained using Rousseeuw's minimum volume ellipsoid to obtain the high breakdown point and then a two-step M-estimator to improve the local properties. It is shown that the rate of convergence of the minimum volume ellipsoid is $n^{-1/3}$. Property (4) cannot be obtained using the minimum volume estimator as it may not be well-defined off the model. This can be overcome using a smooth S-estimator but such estimators are not practical as there is no real chance of calculating them for a given data set.

Laurie Davies (Essen, W. Germany).

The Darling-Erdős theorem for sums of iid random variables

Let $\{X_n\}$ be a sequence of iid r.v.'s with $EX_1 = 0$, $EX_1^2 = 1$. Darling and Erdős (1956) have shown that one has with the additional assumption of a finite third absolute moment for appropriate sequences $\{a_n\}$, $\{b_n\}$:

$$a_n \max_{1 \le k \le n} \sum_1^k X_m / \sqrt{k} - b_n \xrightarrow{\mathcal{D}} E,$$

where E is an extreme value distribution.

They raised the question whether this result can hold under the sole assumption of a finite second moment. Using a skillful truncation argument due to Feller (1946) we show that one can obtain a general Darling-Erdős type theorem when slightly changing the normalizing sequence $\{\sqrt{k}\}$. We infer that the Darling-Erdős theorem holds in its classical formulation if and only if $EX^2 1\{|X| \ge t\} = o((LLt)^{-1})$ as $t \to \infty$. As a by-product we are able to reprove fundamental results of Feller (1946) dealing with lower and upper class functions in the Hartman-Wintner LIL.

Uwe Einmahl, East Lansing, U.S.A.

On the sample path behaviour of the first passage time process of a Brownian motion with drift

Let $\{W(t); t \geq 0\}$ be a standard Wiener process, and consider the Brownian motion with positive drift $\mu > 0$ and variance $\sigma^2 > 0$ defined by $X(t) = \mu t + \sigma W(t)$, $t \geq 0$. We shall be concerned with the first passage time process $\{M(t); t \geq 0\}$ of $\{X(t); t \geq 0\}$, i.e. $M(t) = \inf\{s \geq 0 : X(s) \geq t\}$. Strong limit theorems are established on the behaviour of the sample path modulus of $\{M(t); t \geq 0\}$, characterized by the maximal and minimal increments $\Delta^{\pm}(T, K) = \pm \sup_{0 \leq t \leq T-K} \pm \{M(t+K) - M(t)\}$ for $0 \leq K \leq T$. The case where $K = K_T = O(\log T)$ as $T \to \infty$ is of particular interest here. The results are derived from their corresponding analogies for partial sums of inverse Gaussian random variables, which are developed first.

Josef Steinebach (Hannover)

Some more remarks on the Cramér-Rao lower bound.

In the talk (based on joint work with F. Pukelsheim and H. Witting) two theorems supplementing the Cramér-Rao inequality as stated in Witting (1985), for instance, are presented.

(1) A streamlined version of a result due to Wijsman (1973) is given ("global attainment of is possible iff the class underlying is an exponential family")

(2) Sufficient conditions for L_2-differentiability in the sequential case are stated. As a corollary, the sequential Cramér-Rao-inequality is proved.

Ulrich Müller-Funk (Münster)

On a new goodness of fit test to detect rotating cosmic pulsars of γ-rays.

A new goodness of fit test is proposed for the analysis of circular data in order to detect rotating cosmic pulsars of γ-rays. The test is invariant under rotations and consistent for a broad class of alternatives. The limiting distribution of the test statistic is derived under both the null-hypothesis and the above-mentioned alternatives. Also, the test has high Pitman asymptotic relative efficiency with respect to the Greenwood spacings test. Small sample studies indicate that the proposed test performs better than well-known tests in the literature, such as Watson's test, with respect to power.

(Jan W.H. Swanepoel, Potchefstroom, South Africa)

Limit theorems for adaptive regression estimates of kernel type

Let (X,Y) be a bivariate random vector. Adaptive kernel estimates of the regression function $\mathbb{E}(Y|X=t) = r(t)$ can be written in the form

$$r_n(t) = \sum_{i=1}^{n} Y_i K((t-X_i)/A_n(t)) \Big/ \sum_{i=1}^{n} K((t-X_i)/A_n(t))$$

where (X_i, Y_i), $i=1,\dots,n$ is a sample of i.i.d. r.v.'s, K is a kernel function and $A_n(t) = A_n(t; (X_1,Y_1),\dots,(X_n,Y_n))$ is a sequence of bandwidths depending on the data and t. It is shown that the estimate $r_n(t)$ is asymptotically normal (at a fixed point t) and that the distribution of a weighted integrated squared error of r_n (properly normalized) tends to the standard normal distribution. On the basis of these limit theorems optimality properties of adaptive estimates r_n are investigated and connections to the optimality of the Nadaraya-Watson-estimate are discussed.

Hannelore Liero (Berlin, GDR)

On the rate of weak convergence of the prepivoted sample quantile

It is shown that the rate of weak convergence of the prepivoted sample q-quantile to the uniform distribution on (0,1) is exactly $O(n^{-1/2})$. Consequently, this is also the level error of confidence intervals for the underlying q-quantile which are derived by bootstrapping the sample q-quantile. In view of the poor rate of convergence of the bootstrap estimate of the distribution of the sample q-quantile, this is an unexpected high accuracy.
A confidence interval of even more practical use is derived by using backward critical points. The resulting confidence interval has the same length as the one derived by ordinary bootstrap but it is distribution free and has higher coverage probability.

Michael Falk (Siegen)

Second order asymptotic distribution of M-estimators.

Let $Y_1, \dots, Y_n$ be independent random variables, $Y_i \sim F(y - \underset{\sim}{x}_i' \underset{\sim}{\beta})$, $i=1,\dots,n$, where $\underset{\sim}{\beta} \in R^p$ is the parameter of interest, $\underset{\sim}{x}_i = (x_{i1}, \dots, x_{ip})' \in R^p$, $i=1,\dots,n$, are given vectors. M-estimator $\underset{\sim}{M}_n$ of $\underset{\sim}{\beta}$ is defined as a solution of the minimization $\sum_{i=1} \rho((Y_i - \underset{\sim}{x}_i' \underset{\sim}{t})/\varphi_n) := \min$ w. r. to $\underset{\sim}{t} \in R^p$, where $\varphi_n = \varphi_n(Y_1, \dots, Y_n)$ is translation invariant and scale-equivariant, $\sqrt{n}(\varphi_n - \varphi(F)) = O_p(1)$; ρ is absolutely continuous and such that $h(t) = \int \rho\left(\frac{x-t}{\varphi}\right) dF(x)$ has a unique minimum at $t=0$.
The asymptotic study of $\underset{\sim}{M}_n$ is based on the process

(random field) $S_n(t, u) = n^{-1/2}\sum_{i=1}^{n} x_i[\psi((Y_i - x_i'\beta - n^{-1/2}x_i't)/(\sigma_0 \exp\{n^{-1/2}u\})) - \psi((Y_i - x_i'\beta)/\sigma)]$, $\psi = \rho'$, $(t, u) \in R^p \times R^1$.
We shall show some limiting properties of $S_n(t, u)$ (as $n \to \infty$) and their applications to the asymptotic theory of M-estimators. We shall also discuss some open problems.

Jana Jurečková, Prague, Czechoslovakia

Tail estimation for stochastic processes

The present work, joint with L. de Haan and M.R. Leadbetter is concerned with one of the many problems in the area of statistical methods for extremes of dependent data. This is to estimate the (small) probabilities of very large values and the distribution of the maximum over very long intervals. The solution hinges on estimating the extremal index θ, where $1/\theta$ is the limiting mean length of clusters of exceedances and a parameter β defined as the limiting mean height of and exceedance of a high level, given it is non-zero. Both θ and β are estimated by obvious "moment" estimators, i.e. by the number of clusters divided by the number of exceedances (this has been studied by T. Hsing) and by the average height of exceedances (the "Hill estimator") respectively. A central limit theorem for the estimators is obtained under suitable mixing and tail conditions, and the results are applied to water level data from den Helder, Holland and to acid rain measurements from Pennsylvania, USA.

Holger Rootzén (Lund, Sweden)

An adaptive nonparametric peak estimator

Kernel estimators for the location and size of a peak of a regression function are considered. The problem is how to estimate local bandwidths (which seem to be more appropriate than global bandwidths for this application) for these kernel estimators. The estimated coordinates of the peak are the coordinates of the peak in the kernel estimate of the regression function. A stochastic process in local bandwidths for location and size of the peak is shown to converge weakly to a Gaussian limit process. This result is applied to establish efficiency of a variety of data-driven local bandwidth selection procedures. The bandwidths for location and size of the peak have to be chosen differently. Simulation results indicate the superiority of local over global bandwidth choice for this application.

Hans-Georg Müller (UC Davis)

MATH. STOCH. - ABSTRACTS continued in the next book

(12.03.-18.03.89)