MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 6|1978

Einhüllende Algebren von Lie-Algebren

5. 2. bis 11. 2. 1978

Dies war die dritte Oberwolfacher Tagung über Einhüllende Algebren. Während bei der ersten (1973) noch der Fall auflösbarer Lie-Algebren im Vordergrund gestanden hatte, hat sich nun (1978) der Schwerpunkt des Interesses, wie sich bereits bei der zweiten Tagung (1975) abgezeichnet hatte, auf den halbeinfachen Fall verlagert. Die Untersuchung bestimmter Darstellungen und die Bestimmung der primitiven Ideale der Einhüllenden Algebren halbeinfacher Lie-Algebren haben große Fortschritte gemacht und gleichzeitig interessante Untersuchungen zu Fragen der algebraischen Geometrie nach sich gezogen. Im Zusammenhang mit der Untersuchung Einhüllender Algebren wurden Methoden der nicht-kommutativen Algebra verfeinert oder neue Methoden entwickelt. Außerdem sind drei neue Bereiche aufgetaucht: Verbindungen zur Analysis, Verbindungen zur Kombinatorik und das Studium von Lie-Superalgebren.

Da sich die Vorträge auf die jeweils aktuellen Ereignisse beschränkten und da das Gebiet der Einhüllenden Algebren sich in den letzten Jahren unerwartet rasch entwickelt hat, ergibt sich eine gewisse Informationslücke, bezüglich welcher wir auf die inzwischen erschienene Literatur verweisen möchten.

Im Einzelnen ergibt sich etwa folgendes Bild: Das Problem der Klassifikation der primitiven Ideale in der Einhüllenden einer halbeinfachen Lie-Algebra hat eine lebhafte Entwicklung genommen. Während 1975 nur erste Ansätze und Beispiele dazu vorgelegt wurden (vgl. die damaligen Vorträge von Borho, Dixmier, Joseph) scheinen sich 1978 bereits die Umrisse einer umfassenden, allgemeinen Theorie abzuzeichnen (vgl. insbesondere den Vortrag von Joseph). Ein Meilenstein in dieser Theorie ist das Theorem von Duflo (1976), wonach jedes primitive Ideal der Einhüllenden einer halbeinfachen Lie-Algebra der Annulator eines einfachen Quotienten eines Verma-Moduls ist. Wußte man 1975 noch sehr wenig über die (endlich vielen) primitiven Ideale mit gegebenem zentralen Charakter, so hat man heute bereits recht genaue Vorstellungen davon, wie die vollständige Klassifikation - zumindest im Falle $\underline{sl}_n$ - aussehen sollte. Dies ist der Gegenstand einer äußerst interessanten Vermutung von Jantzen (1976), zu deren Beweis Joseph wesentliche Fortschritte präsentierte.

Der erwähnte neue Satz von Duflo zeigt, daß die Theorie der Verma-Moduln (über die Verma schon 1973 vorgetragen hatte) eine noch viel wichtigere Rolle für die Idealstruktur der Einhüllenden Algebren spielt, als man bis vor zwei Jahren wußte. Dieser Umstand erhöhte noch das allgemeine Interesse an den bedeutenden Fortschritten in der Theorie der Verma-Moduln, die auf dieser Tagung zu verzeichnen waren (vgl. den Vortrag von Jantzen). Auch in anderen Bereichen der Darstellungstheorie halbeinfacher Lie-Algebren zeichneten sich interessante neue Entwicklungen ab, so etwa eine Theorie der Whittaker-Moduln, ein besseres Verständnis der Enright-Varadarajan-Moduln, Fortschritte über Harish-Chandra-Moduln und eine komplette Klassifikation der irreduziblen $\underline{sl}_2$-Moduln (vgl. die Vorträge von Kostant, Enright, Vogan, Block).

Ein anderes zentrales Thema der Tagung waren die Berührungspunkte zwischen der Theorie der Einhüllenden Algebren und dem Gebiet der algebraischen Gruppenaktionen, insbesondere der Konjugationsklassen halbeinfacher Gruppen und ihrer Lie-Algebren. Die Wechselbeziehungen zwischen diesen beiden Gebieten hatten sich 1975 erst in Ansätzen abgezeichnet (Dixmier, Joseph, Borho-Rentschler) und haben seither eine unerwartet rege Entwicklung genommen. Sie beruhen auf zwei verschiedenen, in gewissem Sinne entgegengesetzten Techniken: Einerseits auf der Orbit-Methode, die von einem Orbit $\mathcal{O} \subset \underline{g}^*$ in der koadjungierten Darstellung der Lie-Algebra $\underline{g}$ ausgeht und ihm nach Wahl einer Polarisierung eines $f \in \mathcal{O}$ (falls es eine gibt) ein primitives Ideal in der Einhüllenden Algebra zuordnet (den Annullator der induzierten Darstellung); andererseits auf der Methode der assoziierten Varietäten, welche von einem primitiven Ideal I der Einhüllenden ausgeht und ihm eine gewisse Vereinigung von Orbiten in $\underline{g}^*$ zuordnet (das Nullstellengebilde des assoziierten graduierten Ideals $gr(I)$ in der symmetrischen Algebra $S(\underline{g})$).
Dabei bietet der halbeinfache Fall den besonderen Vorteil, daß

man (mittels der Killing-Form) von der coadjungierten zur adjungierten Darstellung übergehen kann, das heißt, Konjugationsklassen in der Lie-Algebra selbst betrachtet und dort von den Ergebnissen jahrzehntelanger Forschung profitieren kann. Die Teilnahme einiger Experten aus diesem Gebiet an der Tagung hat sich deshalb als sehr fruchtbar erwiesen; nicht nur in Vorträgen (Hesselink, Kraft) sondern auch in abendlichen Diskussionen (Carter, Luna, Springer, Spaltenstein).

Beide Techniken haben ihre speziellen Probleme: Die erste, weil einerseits Polarisierungen nicht zu existieren brauchen und andererseits das Ergebnis des Verfahrens von der Auswahl einer Polarisierung abhängen kann, und die zweite, weil allem Anschein nach einem primitiven Ideal I nicht eine Vereinigung von Orbiten, sondern ein ganz bestimmter Orbit entsprechen sollte: Die assoziierte Varietät sollte irreduzibel sein und deshalb einen dichten Orbit enthalten.

Das nähere Studium solch heikler Fragen führt rasch auf tiefe offene Probleme schon auf der Seite der Gruppenaktionen und der Orbitstruktur. Auf diese Weise sind von den Einhüllenden Algebren wesentliche Impulse für die neuere Forschung auf diesem Gebiet ausgegangen. Dies gilt zum Beispiel für das neue Resultat von Procesi und Kraft, wonach der Abschluß eines Orbits in $\underline{g} = \underline{sl}_n$ eine normale Varietät ist: Wie man schon seit zwei Jahren wußte, impliziert dies die oben erwähnte Vermutung über die Irreduzibilität der assoziierten Varietäten, jedenfalls für den Spezialfall eines primitiven Ideals I, das mittels der Orbit-Methode konstruiert wurde. Ein vollständiger Beweis dieser Vermutung würde gemäß Josephs Vortrag den Beweis der Jantzenschen Vermutung über die primitiven Ideale der Einhüllenden von $\underline{sl}_n$ komplett machen.

Auch die Wechselwirkungen des Gebiets der Einhüllenden Algebren mit anderen Teilen der nicht-kommutativen Algebra kamen in einigen Vorträgen zur Geltung (vgl. die Vorträge von Irving, Joseph und Stafford), so etwa die Resultate von Irving und Lorenz über primitive Ideale von Gruppenalgebren, die von Joseph und Small über Goldie-Ränge und Staffords Satz über die Erzeugung von Linksidealen in Weyl-Algebren.

Das Interesse an besonderen Fragen der Einhüllenden allgemeiner Lie-Algebren und Verbindungen zu Problemen der Analysis wurden deutlich in den beiden Vorträgen von Kashiwara und Vergne. In diesem Zusammenhang sei erwähnt, daß 1976 von Duflo mit Hilfe der Theorie primitiver Ideale Einhüllender Algebren die lokale Lösbarkeit biinvarianter Differentialoperatoren auf beliebigen Lieschen Gruppen bewiesen wurde. Überraschende Aspekte der Verbindung von (gewissen unendlich dimensionalen) Lie-Algebren mit Fragen der Kombinatorik und mit Identitäten für die Dedekindsche η-Funktion wurden in einem Vortrag von Lepowsky behandelt. Über den neuen Bereich von Lie-Superalgebren wurde von Kac und Mickelsson vorgetragen.

In dieser Übersicht konnten nicht alle Aspekte der Tagung vollständig und gleichmäßig berücksichtigt werden. Für weitergehende Informationen verweisen wir auf die nachfolgenden Vortragsauszüge. Der Tagungsbericht enthält außer den Auszügen der gehaltenen Vorträge auch die zweier ursprünglich angekündigter Vorträge (V. L. Popov, Moskau und A. G. Elashvili, Tbilisi).

Die Tagung war wiederum international zusammengesetzt. Sie wurde geleitet von W. Borho (Wuppertal), J. Dixmier (Paris) und R. Rentschler (Paris).

Gefördert durch DFG Deutsche Forschungsgemeinschaft

TEILNEHMER

Bamba, S., Paris
Barou, G., Caen
Berline, N., Rennes
Block, R. E., Riverside
Carmona, J., Marseille
Carter, R. W., Coventry
Duflo, M., Paris
Enright, Th. J., La Jolla
Guimier, F., Paris
Hesselink, W. H., Groningen
Irving, R. S., Waltham
Jantzen, J. C., Bonn
Joseph, A., Orsay
Kac, V., Cambridge
Kashiwara, M., Princeton
Kostant, B., Cambridge
Kraft, H., Bonn
Krämer, M., Bayreuth
Lepowsky, J., Cambridge
Levasseur, Th., Paris
Lorenz, M., Essen
Luna, D., St. Martin d'Hères
Malliavin, M.-P., Paris
Mickelsson, J., Jyväskylä
Moeglin, C., Paris
Moscovici, H., Bukarest
Nghiem, X. H., Orsay
Raïs, M., Poitiers
Ringel, C. M., Bonn
Rohlfs, J., Bonn
Schleich, Th., Wuppertal
Schwermer, J., Bonn
Spaltenstein, N., Bures-Sur-Yvette
Springer, T. A., Utrecht
Stafford, J. T., Waltham
Strade, H., Hamburg
Vergne, M., Cambridge
Vogan, D., Princeton

VORTRAGSAUSZÜGE

G. BAROU : Local cohomology and grade for enveloping algebras of Lie algebras

Let $\underline{g}$ a nilpotent Lie algebra of finite dimension over a field of characteristic zero, $U(\underline{g})$ its universal algebra, P a prime ideal of $U(\underline{g})$ and $A = (U(\underline{g}))_P$.

Let $H^i_{\underline{m}}(-)$ the i^{th} functor of local cohomology relative to the maximal ideal $\underline{m}$ of A and K-dim (-) the dimension of Gabriel-Rentschler.

If M is a left A-module , finitely generated and not zero, then we have

1) $H^i_{\underline{m}}(M) = 0$ for $i >$ K-dim M.

2) If $H^n_{\underline{m}}(A)$ is the injective hull of the right module $A/\underline{m}$, then K-dim A $\leq$ K-dim M + $\text{grad}_A M$ and the equality holds if and only if $H_{\underline{m}}^{\text{K-dim } M}(M) \neq 0$.

3) If $M_P \neq 0$ for every prime ideal P of A containing the annihilator of M , then:
K-dim M + $\text{grad}_A M \leq$ K-dim A.

R. E. BLOCK : The irreducible representations of the Weyl algebra A_1 and of $U(\underline{sl}_2)$

The irreducible representations of the Heisenberg algebra $\underline{g}_3$ and of $\underline{sl}_2$ are determined and classified as follows.
Let $A = A_1$ be the Weyl algebra over an algebraically closed field K of characteristic 0.
Write $A = K[q,p]$ where $pq - qp = 1$, B the localization $K(q)[p]$. Also for each γ in K let B_γ be the primitive quotient of $U(\underline{sl}_2)$ with $4fe + h^2 + 2h = \gamma$, and identify B_γ with a subalgebra of A by setting $q = e$, $h = 2(qp+\beta)$, $f = -(qp + 2\beta)p$ where $\beta = \frac{1}{2}(1 + \sqrt{1+\gamma})$. For $\alpha \in K$ and $b = \sum_{j=0}^{r} b_j p^j \in B$ ($b_j \in F(q)$) define $\Theta_{\alpha,b}(\lambda) = \sum_{j=0}^{r} \{((q-\alpha)^{-\mu_\alpha(b)} - jb_j)(\alpha)\}(-1)^j \lambda(\lambda+1)\ldots(\lambda+j-1)$, where $\mu_\alpha(b) = \min\{\mu_\alpha(b_j) - j\}$ and $\mu_\alpha(b_j)$ is the multiplicity of $q - \alpha$ in b_j. Call b <u>good</u> with respect to A (resp. B_γ) if for each $\alpha \in K$,

$\Theta_{\alpha,b}(\lambda)$ has no root in $\mathbb{N}^+$ (resp. if $\alpha \neq 0$, $\Theta_{\alpha,b}(\lambda - \alpha + \beta/\alpha)$ has no root in $\mathbb{N}^+$ and (with $\delta = -1 \pm \sqrt{1+\gamma}$) $\Theta_{0,b}(\lambda - \frac{\delta}{2} + \beta)$ has no root in $\mathbb{N}$ (or in $\{0, 1, \ldots, \delta\}$ if $\delta \in \mathbb{N}$).

Theorem. <u>Each similarity class of irreducible elements of</u> B <u>contains good elements; pick one, say</u> b, <u>for each class and form the module</u> $A/A \cap Bb$ <u>(resp.</u> $B_\gamma/B_\gamma \cap Bb$); <u>also for each</u> $\alpha \in K$ <u>form the module</u> $(F[p], q - \alpha = -d/dp)$ <u>(resp. the highest weight modules</u> $L(\delta)$, δ <u>as above, and the cyclic Whittaker</u> B_γ - <u>module with eigenvalue</u> $\alpha \neq 0$ <u>for</u> e). <u>Then each of these modules is simple and every simple module over</u> A <u>(resp.</u> B_γ) <u>is isomorphic to one and only one of the above modules.</u> Similar results hold for the 2 - dimensional nonabelian Lie algebra.

T. J. ENRIGHT : Constructing lattices of modules for a semisimple Lie algebra

Let $\underline{m}$ be a reductive subalgebra of a Lie algebra $\underline{L}$ and let W denote the Weyl group of $\underline{m}$. In this lecture we give the definition of a lattice. A lattice is a certain left exact functor from $\underline{L}$ - modules to sets of $\underline{L}$ - modules

$$A \longmapsto \{A_s\}_{s \in W}.$$

The properties of this functor are described and certain questions involving its domain are raised.
An application to the theory of Harish-Chandra modules is discussed. In particular, if $\underline{L}$ is the complexification of a real semisimple Lie algebra and $\underline{m}$ the complexification of the maximal compact subalgebra then the functor

$$A \longmapsto A_1 / \sum_{s \neq 1} A_s$$

can be used to construct all irreducible Harish-Chandra modules for connected semisimple linear groups.

W. H. HESSELINK : Polarizations in the classical groups

Let G be a reductive group over a field k. Let x be a nilpotent element of the Lie algebra $\underline{g}$ of G. A polarization of x is a parabolic subgroup P of G such that $\dim (G\,x) = 2 \dim (G/P)$ and that x is an element of the Lie algebra $\underline{u}\,(P)$ of the unipotent radical $U\,(P)$ of P. The set $\mathrm{Pol}\,(x)$ of the polarizations of x is known to be finite.
Let $\mathrm{char}\,(k) \neq 2$. For nilpotent $x \in \underline{g}$ we describe the finite set $\mathrm{Pol}\,(x)$ and the action of the centralizer $Z_G(x)$ on $\mathrm{Pol}\,(x)$. It turns out that polarizations P and Q of x have conjugate Levi factors if and only if $Z_P(x) = Z_Q(x)$. We give some tables.

R. S. IRVING : Some primitive Ore domains with large centers

Let us say that a finitely generated algebra A over a field k satisfies the Nullstellensatz if, for every simple A-module M, the algebra $\mathrm{End}_A(M)$ is algebraic over k. Enveloping algebras of finite-dimensional Lie algebras, group rings of polycyclic-by-finite groups, PI-algebras, and all algebras over uncountable fields, satisfy the Nullstellensatz. But we can obtain Ore domains not satisfying it, including an enveloping algebra. Let R be a countable domain of characteristic 0, and let d be the derivation of $R[y_o, y_1, \ldots]$ defined by $d(y_n) = y_{n+1}$. Then $A = R[y_o, \ldots][x; d]$ is primitive. It is the enveloping algebra of the Lie algebra with basis $y_o, y_1, \ldots, x$ and relations $[y_i, y_j] = 0$, $[y_n, x] = y_{n+1}$. Another similar example is obtained by taking $A = R \ \ldots, y_{-1}, y_o, y_1, \ldots \ x, x^{-1}; \varphi$, where φ is an automorphism with $\varphi(y_n) = y_{n+1}$. We have $A = R[\mathbb{Z} \wr \mathbb{Z}]$. Then for R a countable domain, A is primitive.

There is also an algebra with finite Gelfand-Kirillov dimension which does not satisfy the Nullstellensatz. But no noetherian counter-example is known.

Also, other properties of primitive ideals were discussed, with a description of known results and counter-examples.

J. C. JANTZEN : Multiplicities in highest weight modules

Denote by $\underline{g}$ a semi-simple $\mathbb{C}$-Lie algebra, $\underline{h}$ a Cartan subalgebra of $\underline{g}$, $\underline{b}\supset\underline{h}$ a Borel subalgebra of $\underline{g}$, R the root system relative $\underline{h}$, R_+ the set of positive roots (with respect to $\underline{b}$), $\rho = \frac{1}{2}\sum_{\alpha\in R_+}\alpha$, W the Weyl group of $(\underline{g},\underline{h})$, $s_\alpha : \lambda \longmapsto \lambda - \langle\lambda,\alpha^\vee\rangle\alpha$ the reflection with respect to α (for $\alpha\in R$). For $\lambda\in\underline{h}^*$ and $w\in W$ set $w.\lambda = w(\lambda+\rho)-\rho$.

Use the notations (for $\lambda\in\underline{h}^*$)

$R_\lambda = \{\alpha\in R \mid \langle\lambda+\rho,\alpha^\vee\rangle \in \mathbb{Z}\}$, $R_\lambda^0 = \{\alpha\in R \mid \langle\lambda+\rho,\alpha^\vee\rangle = 0\}$,

$R_+(\lambda) = \{\alpha\in R_+\cap R_\lambda \mid \langle\lambda+\rho,\alpha^\vee\rangle > 0\}$.

$W_\lambda = \{w\in W \mid w.\lambda-\lambda\in\mathbb{Z}R\}$, B_λ = the basis of the root system R_λ such that $R_\lambda\cap R_+$ is the set of positive roots, $M(\lambda)$ the Verma module with highest weight λ, $L(\lambda)$ its unique simple quotient. For $\lambda,\mu\in\underline{h}^*$ denote by $[M(\lambda):L(\mu)]$ the multiplicity of $L(\mu)$ as simple factor in a Jordan-Hölder series of $M(\lambda)$.

Theorem 1: Assume $\lambda,\mu\in\underline{h}^*$, $R_+(\lambda) = R_+(\mu) = \emptyset$, $R_\lambda = R_\mu$, $R_\lambda^0 = \emptyset$. Then:

a) if $R_\mu^0 = \emptyset$ and $R_{\lambda-\mu} = R$ or $\langle\lambda-\mu,\alpha^\vee\rangle = 0$ for all $\alpha\in R_\lambda$ then
$[M(w.\lambda):L(w'.\lambda)] = [M(w.\mu):L(w'.\mu)]$ for all $w,w'\in W_\lambda$

b) if $R_{\lambda-\mu} = R$ and $w'\in W_\lambda$ then one knows $w_1\in W_\lambda$ such that $w_1.\mu = w'.\mu$ and
$[M(w.\lambda):L(w_1.\lambda)] = [M(w.\mu):L(w'.\mu)]$ for all $w\in W_\lambda$.

Remark: Given $\mu\in\underline{h}^*$ with $R_+(\mu) = \emptyset$, there always exists

$\lambda \in \underline{h}^*$ with $R_+(\lambda) = R^o_\lambda = \emptyset$ and $R_{\lambda-\mu} = R$.

<u>Theorem 2</u>: If $\lambda, \mu \in \underline{h}^*$, $R_+(\lambda) = R_+(\mu) = \emptyset$, $w_1 \in W$ with $B_\lambda \subset w_1 B_\mu$ and $\langle \lambda - w_1 . \mu, \alpha^\vee \rangle = 0$ for all $\alpha \in B_\lambda$ then
$[M(w.\lambda):L(w'.\lambda)] = [M(ww_1.\mu):L(w'w_1.\mu)]$ for all $w, w' \in W_\lambda$.

<u>Remarks</u>: 1) Given $\lambda \in \underline{h}^*$ with $R_+(\lambda) = \emptyset = R^o_\lambda$ there always exists $\mu \in \underline{h}^*$ with $R_+(\lambda) = R^o_\mu = \emptyset$ and $w_1 \in W$ such that $B_\lambda \subset w_1 B_\mu$ and $\langle \lambda - w_1 . \mu, \alpha^\vee \rangle = 0$ for all $\alpha \in B_\lambda$ and $\mathbb{C}R_\mu = h^*$.

2) Theorem 1 & 2 reduce the problem of determining all $[M(\lambda):L(\mu)]$ to that of determining them for a finite number of λ with $R^o_\lambda = \emptyset$ and $\mathbb{C}R_\lambda = \underline{h}^*$.

<u>Theorem 3</u>: Let $\lambda \in \underline{h}^*$, $R_+(\lambda) = R^o_\lambda = \emptyset$, $\alpha \in B_\lambda$, $w, w' \in W_\lambda$. Then:

a) $[M(w.\lambda):L(w'.\lambda)] = [M(ws_\alpha.\lambda):L(w'.\lambda)]$ if $w'\alpha \in R_+$

b) $[M(w.\lambda):L(w'.\lambda)] = [M(s_\alpha w.\lambda):L(w'.\lambda)]$ if $w'^{-1}\alpha \in R_+$.

Given $\lambda \in \underline{h}^*$ you can define the Bruhat ordering on W_λ as the order relation generated by

$w \leq s_\alpha w$ (for $\alpha \in R_\lambda \cap R_+$) $\Longleftrightarrow$ $w^{-1}\alpha \in R_+$. For $w, w' \in W_\lambda$ set

$r_\lambda(w,w') = \# \{\alpha \in R_\lambda \cap R_+ \mid w' \leq s_\alpha w \leq w\}$.

<u>Theorem 4</u>: Let $\lambda \in \underline{h}^*$, $R_+(\lambda) = R^o_\lambda = \emptyset$, $w, w' \in W_\lambda$ with $w' \leq w$.
Then $[M(w.\lambda):L(w'.\lambda)] = 1$ $\Longleftrightarrow$ for all $w_1 \in W_\lambda$ with $w' \leq w_1 \leq w$:
$r_\lambda(w_1, w') = \# R_+(w_1.\lambda) - \# R_+(w'.\lambda)$.

A. JOSEPH : <u>An additivity principle for Goldie rank and the Jantzen conjecture</u>

Let $\underline{g}$ be a complex semisimple Lie algebra, give $U(\underline{g})$, Prim $U(\underline{g})$, $\underline{h}^*$, R, R^+, B, W, $\hat{W}$, $P(R)$, $L(\lambda)$: $\lambda \in \underline{h}^*$ their usual meaning (Dixmier algébres enveloppantes), set $I(\lambda) = \text{Ann } L(\lambda)$ and $P(R)^{++} = \{\lambda \in P(R) : \lambda \text{ dominant and regular}\}$. For each $\hat{\lambda} \in P(R)/W$, set $\underline{X}_{\hat{\lambda}} = \{I(\mu) : \mu \in \hat{\lambda}\}$ which (Duflo) is the fibre of Prim $U(\underline{g})$ over the central character $\hat{\lambda}$. For each $B' \subset B$ set $p_{B'} = \prod \{\alpha : \alpha \in \mathbb{N}B' \cap R^+\}$, $P_{B'}$ the simple W-module generated by $p_{B'}$ and $\Omega_R \subset \hat{W}$ the set of all irreducible representations so obtained. Through a study of Goldie rank, it is shown for each regular $\hat{\lambda} \in P(R)/W$ that card $\underline{X}_{\hat{\lambda}} \geqslant \sum\{\dim \sigma : \sigma \in \Omega_R\}$. Suppose $\underline{g}$ simple of type A_{n-1}. Then $\Omega_R = \hat{W}$ and equality holds. More precisely let $St(\sigma)$ be the set of standard tableaux for $\sigma \in \hat{W}$ and $\Phi : w \longmapsto (A(w), B(w))$ the Robinson bijection of W onto $\bigcup\{ST(\sigma) \times ST(\sigma) : \sigma \in \hat{W}\}$. Then for each $-\lambda \in P(R)^{++}$ one has $I(w\lambda) = I(w'\lambda)$ iff $A(w) = A(w')$. This establishes the Jantzen conjecture up to the determination of the zero variety of gr $I(w\lambda)$. (The latter has the expected dimension). Again set $(\underline{X}_{\hat{\lambda}})_\sigma = \{I(w\lambda) : A(w) \in St(\sigma)\}$. Then there is a dense subset $\underline{D}$ of $P(R)^{++}$, a basis $\{p_i\}_{i \in St(\sigma)}$ for P_σ ($=P_{B'}$ for suitable $B' \subset B$) and $w_i \in W$ such that $rk(U(\underline{g})/I(w_i\lambda)) = p_i(\lambda) : I(w_i\lambda) \in (\underline{X}_{\hat{\lambda}})_\sigma$, for all $-\lambda \in \underline{D}$, where rk denotes Goldie rank. A further corollary is that the annihilators of simple Harish-Chandra modules for $SL(n,\mathbb{C})$ are pairwise disjoint (regular central characters only).

V. KAC : Representations of Lie superalgebras

The theory of finite-dimensional representations of Lie superalgebras is discussed. The most part of the results is published in [1] Advances in Math. 26 (1977), 8-96, and [2] Comm. in Algebra 5(8)(1977), 889-897. The new results are 1) an explicit construction $\overline{V}(\Lambda)$, which gives typical [2] representations in all the cases; 2) multiplicities of the restriction of irreducible representations to the even part; 3) the following criterias for a representation $V(\Lambda)$ to be typical (see [2] for notations):

a) $L \cdot \mathrm{ch}\, V(\Lambda) = \sum \varepsilon(w) e^{w(\Lambda+\rho)}$,

b) $V(\Lambda) = \overline{V}(\Lambda)$,

c) $(\Lambda+\rho, \alpha) \neq 0$, $\alpha \in \overline{\Delta}_1^{+}$,

d) $V(\Lambda)$ splits in any finite-dimensional representation,

e) $V(\Lambda)$ is uniquely defined by χ_Λ among the finite-dimensional representations.

M. KASHIWARA : Micro-local calculus and representations

The proof of the following theorem is given.

Theorem A: The variety of the zeros of the graded ideal of a left ideal of a universal enveloping algebra is involutive.

Theorem B: The variety of the zeros of the graded ideal of a left ideal of the ring of differential operators is involutive (in the cotangent bundle).

Theorem A is obtained from theorem B by looking at the ring of differential operators on the Lie group associated with the Lie algebra. Theorem B is obtained by micro-localizing, by using the quantized contact transformation, and by reducing the case where the characteristic variety is of the form $\{(x,\xi):x_1 = \dots = x_l = \xi_1 = \dots = \xi_m = 0\}$.

B. KOSTANT : Enveloping algebras and Whittaker theory

Let $\underline{g}$ be a complex semi-simple Lie algebra and let $\underline{n}$ be the nilradical of a Borel subalgebra. Let $\eta : \underline{n} \longrightarrow \mathbb{C}$ be a non-singular homomorphism of Lie algebras. If U is the universal enveloping algebra of $\underline{g}$ and V is a U-module then a vector $v \in V$ is called a Whittaker vector if $xv = \eta(x)v$ for all $x \in \underline{n}$. A module V is called a Whittaker module if it is cyclically generated by a Whittaker vector. Let U_V be the annihilator of V and let $Z_V = U_V \cap Z$ where Z is the center of U. Theorem: The correspondence $V \longmapsto Z_V$ is a bijection of the set of all Whittaker modules (up to equivalence) and all ideals in Z_V. V is irreducible if and only if Z_V is a maximal ideal, i. e. if and only if V admits an infinitesimal character. Moreover in such a case $U_V = Z_V U$ is a minimal primitive ideal. If V is a Harish-Chandra module for U for a quasi-split real form then the ideal V contains a Whittaker vector if and only if U_V is a minimal primitive ideal.

H. KRAFT : Closures of nilpotent conjugacy classes are normal

Theorem (Procesi - Kraft) Let $x \in sl_n$ be a nilpotent element, C_x its conjugacy class. Then the closure $\overline{C_x}$ is a normal variety. Proof (outline): Define $V_i := x^i(V)$ $(V = \mathbb{C}^n)$, $L_{i,i+1} := \mathrm{Hom}(V_i, V_{i+1}) \times \mathrm{Hom}(V_{i+1}, V_i)$, $L := L_{o,1} \times L_{1,2} \times \ldots \times L_{s-1,s}$, $L_i := \mathrm{End}_{V_i}$. We have the following diagram of maps

$$\begin{array}{ccc} L & \xrightarrow{\ \varphi\ } & L_1 \times L_2 \times L_3 \times \ldots \times L_s \\ {\scriptstyle \pi}\big\downarrow & & \\ L_o & & \end{array}$$

where φ is defined by

$(\ldots(\alpha_i, \beta_i), \ldots) \longmapsto (\ldots, \alpha_i \beta_i - \beta_{i+1} \alpha_{i+1}, \ldots)$ and

by $(\ldots(\alpha_i, \beta_i), \ldots) \longmapsto \beta_1 \alpha_1$. Then one shows

a) $\varphi^{-1}(0)$ is a complete normal intersection,

b) $\pi(\varphi^{-1}(0)) = \overline{C_x}$,

c) the induced map $\pi' : \varphi^{-1}(0) \longrightarrow L_o$

is a generalized quotient (for any G-stable closed subset $S \subset \varphi^{-1}(0)$, $\pi'(S)$ is closed and $\underline{R}(\pi'(S)) = \underline{R}(S)^G$, $G := GL(V_1) \times Gl(V_2) \times \ldots$ $\underline{R} \sim$ regular functions).

b) follows from the construction; c) is proved by induction using the "first main theorem" of classical invariant theory. The proof of a) is more difficult and uses some further analysis of the construction.

J. LEPOWSKY : Lie algebras and combinatorics

Kac's classification of automorphisms of finite order of complex semisimple Lie algebras can be combined with Macdonald's unspecialized identities to produce new specializations of these identities, including a formula for $\eta(q)^{\text{rank}\,\underline{g}}$, $\underline{g}$ a complex simple Lie algebra. Another interesting specialization, called "principal specialization" because it comes from Kostant's "principal automorphism", leads to other formulas for arbitrary powers of Dedekind's η - function. These formulas all involve the denominator in the Weyl-Kac character formula for affine Lie algebras (among the generalized Cartan matrix Lie algebras introduced by Kac and Moody). Principal specialization of the characters of suitable standard modules for the affine Lie algebra $\widehat{\underline{sl}(2,\mathbb{C})}$ gives the product sides of the famous Rogers-Ramanujan identities, up to a fudge factor F (joint work with S. Milne). The classical partition function is found, surprisingly, to describe the weight multiplicities in another standard module for $\widehat{\underline{sl}(2,\mathbb{C})}$ (joint work with A. Feingold). $\widehat{\underline{sl}(2,\mathbb{C})}$ is constructed as physically interesting differential operators on a space of polynomials in infinitely many variables, using the above ideas, and F is "explained" (joint work with R. L. Wilson).

T. LEVASSEUR : Properties of some prime ideals in enveloping algebras of nilpotent Lie algebras

First it is shown that if R is a right noetherian ring and I an ideal generated by a centralizing set of elements then $\hat{R} = \varprojlim_n R/I^n$ is again right noetherian.
As a corollary, if R is a non commutative regular local ring, and $\underline{m}$ is its maximal ideal $\hat{R} = \varprojlim R/\underline{m}^n$ is regular.
For example, if $\underline{g}$ is a nilpotent Lie algebra over k a field of zero characteristic and P is a prime ideal such that: $U(\underline{g})_{\underline{P}} \cong A_m(k) \otimes_k Z(\underline{g})_{\underline{P}}$, $m \in \mathbb{N}$, $\underline{P} = P \cap Z(\underline{g})$ then $\widehat{U(\underline{g})_P} \cong D\,[[x_1,\ldots,x_n]]$ with $D \cong Fr(U(\underline{g})/P)$ is a field contained in $\widehat{U(\underline{g})_P}$ and $x_1,\ldots x_n$ are variables which commute with D.

J. MICKELSSON : Representations of step algebras and Lie superalgebras

Representations of Lie superalgebras are studied with the help of step algebras. In particular, the following case is studied in detail. Let $\underline{g} = sl(n,1)$ $(n \geqslant 4)$ be the simple Lie superalgebra in the Kac terminology. Then $\underline{g}_o = \underline{gl}(n,\mathbb{C})$, the Lie subalgebra. Let $\underline{k} = \underline{gl}(p,\mathbb{C}) \oplus \underline{gl}(q,\mathbb{C}) \subset \underline{g}_o\,(p+q=n)$ be defined in the natural way using the standard representation of $\underline{g}$.
Let $\underline{h} \subset \underline{k}$ be a Cartan subalgebra, Δ_k a positive system for $(\underline{h},\underline{k})$ and $\{\alpha_1,\ldots,\alpha_{n-2}\} \subset \Delta_k$ a basis for Δ_k such

that $h_{\alpha_1},\ldots,h_{\alpha_{p-1}} \in \underline{gl}(n,\mathbb{C})$ and $h_{\alpha_p},\ldots,h_{\alpha_{n-2}} \in \underline{gl}(q,\mathbb{C})$; then the set $\Lambda^+ \subset \underline{h}^*$ of dominant integral elements consists of those $\lambda \in \underline{h}^*$, for which $\lambda(h_{\alpha_j}) \geqslant 0$ integers (in an appropriate normalization for h_{α_j}'s). It is shown, that in a suitable (lexicographical) ordering) for Λ^+ there exists exactly one equivalence class of irreducible $\underline{k}$-finite $\underline{g}$-modules with minimal $\underline{k}$-type (λ) for all $\lambda \in \Lambda^+$ such that $\lambda(h_{\alpha_1}) > 0$ and $\lambda(h_{\alpha_p}) > 0$.

NGHIEM XUAN HAI : Reduction of semi-direct products

Let $\underline{g} = \underline{s} \ltimes \underline{r}$ be a Lie algebra, semi direct product of its solvable radical $\underline{r}$ by a Levi subalgebra $\underline{s}$. By using a Weyl subfield of the enveloping field of $\underline{r}$, the study of the enveloping field of $\underline{g}$ is reduced to that of a semi-simple Lie algebra.

J. T. STAFFORD : Module structure of Weyl algebras

The work of Dixmier, Eisenbud-Robson and Webber has given considerable information about the structure of the first Weyl algebra, A_1. We will extend their results to A_n and show, in particular, that:

a) Any right ideal of A_n can be generated by just two elements.

b) Any finitely generated torsion-free A_n-module M is isomorphic to a right ideal direct sum a free module.

c) If M is projective then M is isomorphic to either a right ideal or a free module.

d) Any finitely generated torsion Module T is a homomorphic image of a projective right ideal, and hence can be generated by two elements. (We conjecture, as is true for A_1, that T is in fact cyclic.)

H. STRADE : Irreducible representations of solvable Lie algebras of positive characteristic

Let E,S be linear forms on a solvable restricted Lie algebra L/K, $p := \mathrm{char}(K) > 2$. A Vergne-polarization Q of E is a subalgebra with the following property: there exists a chain of subalgebras $k_o \subset \dots \subset k_m = Q = L_m \subset \dots \subset L_o = L$, such that a) k_i is an ideal of L_i and $E(k_i^{(1)}) = 0$; b) $L_{i+1} = k_i^{\perp} \neq L_i$. Let $P_V(E,S) := \{\text{Vergne-polarizations } Q, \text{ s.t. } E^p(f) - E(f^{[p]}) = S^p(f) \text{ for } f \in Q\}$ and $U(L,S) := U/\langle j(f)^p - j(f^p) - S^p(f)1 \rangle$ (U the universal enveloping algebra). Define

$M(E,S,P) := K_{E+1/2\,tr(R_{L/P})} \otimes_{U(P,S|_P)} U(L,S).$

Theorem 1: a) $M(E,S,P)$ is irreducible if $P \in P_V(E,S)$.

b) Any irreducible L-module is of that kind.

Theorem 2: $M(E,S,P)$ does not depend on P.

If L is nilpotent these theorems yield a full description of irreducible L-modules in terms of linear forms.

M. VERGNE : Campbell-Hausdorff formula and invariant distributions (with M. Kashiwara)

Let L be the free Lie algebra generated by two indeterminates x and y and $\hat{L}$ its completion, $e^x = 1+x+\frac{x^2}{2} + \ldots + \frac{x^n}{n!} + \ldots \in \hat{L}$.

We conjecture the following.

Conjecture (C): For any Lie algebra $\underline{g}$, there exists F and $G \in \hat{L}$ such that

a) $x+y-\log e^y e^x = (1-e^{-\mathrm{ad}\,x})F(x,y)+(e^{\mathrm{ad}\,y}-1).\,G(x,y)$

b) F and G give $\underline{g}$-valued convergent power series

c) $\mathrm{Tr}_{\underline{g}}(\mathrm{ad}\,x)\,\partial_x F+\mathrm{tr}_{\underline{g}}(\mathrm{ad}\,y)\,\partial_y G = \frac{1}{2}\mathrm{tr}_{\underline{g}}\left(\frac{\mathrm{ad}\,x}{e^{\mathrm{ad}\,x}-1}+\frac{\mathrm{ad}\,y}{e^{\mathrm{ad}\,y}-1}-\frac{\mathrm{ad}\,z}{e^{\mathrm{ad}\,z}-1}-1\right)$

where $z = \log e^x e^y$ and $\partial_x F$ is the End $\underline{g}$-valued real analytic function given by $\frac{d}{dt} F(x+ta,y)\Big|_{t=0}$. We prove this conjecture when $\underline{g}$ is solvable.

Let G be a Lie group and $\underline{g}$ its Lie algebra. We show that under this conjecture, the Duflo isomorphism between the ring of biinvariant differential operators on G and the ring $I(\underline{g})$ of constant coefficient operators on $\underline{g}$ invariant by the adjoint action of G extends to an isomorphism between the corresponding algebra of distributions.

D. VOGAN : Tensor products and structure of representations of semisimple Lie algebras

Let G be a connected semisimple Lie group, and H a Cartan subgroup. Write $\underline{h}$ for the complexified Lie algebra of

Fix a positive root system $\Delta^+ \subset \Delta(\underline{g},\underline{h})$, and some translate Λ of the lattice of differentials of weights of finite dimensional representations.

Theorem (joint with B. Speh): Suppose $\{\Theta(\lambda) \mid \lambda \in \Lambda\}$ is a coherent family of virtual characters of G in the sense of Schmid. Fix a nonsingular dominant weight $\lambda \in \Lambda$, and a simple positive root α such that $\frac{2\langle\alpha,\lambda\rangle}{\langle\alpha,\alpha\rangle} \in \mathbb{Z}$. Suppose $\Theta(\lambda)$ is the character of an irreducible representation. Then either a) $\Theta(s_\alpha \lambda) = -\Theta(\lambda)$, or

b) $\Theta(s_\alpha \lambda) = \Theta(\lambda) + \Theta_o$, with Θ_o the character of a representation (which may be reducible or zero).

A proof was given, using Duflo's theorem on primitive ideals to reduce to a problem about Verma modules. Applications to the structure of irreducible representations were mentioned.

* * *

A. G. ELASHVILI : Polarizations on simple Lie algebras of the exceptional type

The classification of polarizable nilpotents in simple Lie algebras of the exceptional type is presented and the types of parabolic subalgebras polarizing a nilpotent are given. The solution of problem 1 is based on two lemmas.

L e m m a 1. Let $\underline{g}$ be a semisimple Lie algebra and x be a nilpotent which is contained in the parabolic-type reductive subalgebra $\underline{a}$ of the algebra $\underline{g}$ and which is not polarizable in $\underline{a}$; then it is not polarizable in $\underline{g}$ either.

L e m m a 2. Let x be an odd nilpotent of the algebra $\underline{g}$, such that $g_{x,h}$ (h is a characteristic of x) contains a simple three-dimensional subalgebra $\underline{a}$ and $\underline{g}^1 = \{x \in \underline{g} : [h,x] = (2k+1)x\}$ (k is an integer) considered as an $\underline{a}$-modulus be the direct sum of $\underline{a}$-moduli which are isomorphic to the irreducible two-dimensional modulus. Then if $\dim(g_x)^1 \not\equiv 0 \pmod 4$, x is not polarizable in $\underline{g}$.

Problem 2. is solved by a direct construction of the Richardson nilpotent for each parabolic subalgebra of a given type.

V. L. POPOV : Quasihomogeneous algebraic varieties

The definition of quasihomogeneous algebraic varieties (q.a.v.), The theorems of embedding of q.a.v.; the connection with the closures of orbits on linear representations. Complete and affine q.a.v.. Complete case: The classification of q.a.v. with two ends (by D. N. Abiezer) and with two orbits. Affine case: The classification of two and three dimensional q.a.v.. The connection with the classification of invariant finitely generated subalgebras of the algebra of regular functions on the group. The criterion for the subgroup to be the stabilizer of dense orbit on affine q.a.v. (F. Bogomolov, Suchanow). The classification of q.a.v. of several special types: 1) The q.a.v. of algebraic tori (toroidal embeddings by D. Mumford); 2) affine q.a.v. of sl_2; 3) the classification of affine q.a.v. with the dense orbit of the type G/H where H contains the maximal unipotent subgroup of G (by Popov and Vinberg); algebro-geometrical and arithmetical properties of such q.a.v.. The connection of the theory of affine q.a.v. with 14th Hilbert problem (by F. Grosshans).

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