MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 26/1978

Probability in Banach Spaces

18. bis 24.6.1978

Drei Jahre nach der ersten fand in diesem Jahr die zweite Tagung über "Probability in Banach Spaces" in Oberwolfach statt. Die Leitung der Tagung hatten Prof. A.Beck und Prof. K.Jacobs, die diesmal 44 Teilnehmer, darunter 29 aus dem Ausland, begrüßen konnten. Insgesamt wurden dabei 24 Vorträge gehalten, und zudem fand wieder eine "problem session" statt, auf der noch ungelöste Probleme dargestellt und erörtert wurden. Der Schwerpunkt bei den Vorträgen lag diesmal eindeutig bei Fragen, die den Zentralen Grenzwertsatz auf Banachräumen betreffen. Während noch vor drei Jahren ein großer Teil der Vorträge Gesetze der großen Zahlen behandelte, scheint dieses Thema jetzt abgehandelt zu sein; denn diesmal gab es zu diesem Gebiet keine Beiträge. Am Rande der Tagung fanden wieder viele fruchtbare Diskussionen statt, die mit dazu beitrugen, daß die Tagung zu einem vollen Erfolg wurde.

Die Teilnehmer danken dem Direktor des Instituts, Herrn Prof.Dr. M. Barner, und seinen Mitarbeitern für die freundliche Unterstützung bei der Durchführung der Tagung.

Teilnehmer

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H.von Weizsäcker, Marburg
A.Weron, Wroclaw

Vortragsauszüge

A.de ACOSTA: On the general converse central limit theorem and processes with independent increments in Banach spaces

Let B be a separable Banach space, $\{X_{nj}\}$ a row-wise independent infinitesimal triangular array of B-valued random vectors, $S_n = \Sigma_j X_{nj}$. Our main result may be roughly stated as follows. Assume that $\{L(S_n)\}$ converges weakly. If $\{A_k : k = o,\dots,m\}$ are appropriately chosen disjoint subsets of B, $X_{nj,k}$ is the truncation of X_{nj} at A_k and $S_{n;k} = \Sigma_j X_{nj,k}$, then $\{L(S_{n;o},\dots,S_{n;m})\}$ converges weakly to a completely specified product measure on B^{m+1}. We also show that if $\Phi_k^{(n)} = \Sigma_j I_{A_k}(X_{nj})$ (k=1,...,m) then $L(\Phi_1^{(n)}, \dots, \Phi_m^{(n)})$ converges in total variation to a product of Poisson distributions. Some of the basic results of the theory of B-valued stochastic processes with independent increments follow easily from the results in this paper.

Ch.R.BAKER: Absolute Continuity for Pre-Gaussian Measures

Let $\mathcal{B}$ be a real separable Banach space, $B[\mathcal{B}]$ the Borel σ-field, μ_1 und μ_2 two pre-Gaussian Radon measures on $B[\mathcal{B}]$. We consider conditions for absolute continuity of μ_1 with respect to μ_2. Relations are obtained between absolute continuity, weak convergence, and the relative properties of characteristic functions.

R.CARMONA: Tensor Gaussian measures on tensor products of Banach spaces

Let μ_1 and μ_2 be Gaussian measures on separable Banach spaces B_1 and

B_2 and let us denote by H_1 and H_2 the corresponding reproducing kernel Hilbert spaces. First we prove that the Hilbertian tensor product $H_1 \hat{\otimes}_2 H_2$ is always embedded in $B_1 \otimes_\alpha B_2$ via a continuous one to one map, whatever the reasonable norm α is on the algebraic tensor product $B_1 \otimes B_2$. This means that there exists a Gaussian cylindrical measure, say $\mu_1 \otimes \mu_2$ on $B_1 \hat{\otimes}_\alpha B_2$, the covariance of which is the tensor product of the covariances of μ_1 and μ_2. The natural question is then the following: is $\mu_1 \otimes \mu_2$ σ-additive? The answer is known to be positive when $\alpha = \varepsilon$ the inductive norm (Simone CHEVET). We give a counter-example when $\alpha = \Pi$ and we show that if B_2 is $L^p(X,\sigma)$ for some σ-finite measure σ, then $\mu_1 \otimes \mu_2$ is σ-additive on $L^p(X,\sigma;B_1)$. The latter proves that if $\alpha = \Pi$ and $B_2 = L'(X,\sigma)$ the answer is positive, too.
Byproducts are some new results on γ-radonifying and γ-summing operators. These results are from a joint work with Simone CHEVET.

S.D.CHATTERJI: The Radon-Nikodym property

We give a general survey of the Radon-Nikodym Property (RNP). Its historical origin is outlined and its relationship to martingale convergence theorems is discussed. Types of Banach spaces which have RNP are indicated. Dentability is defined and its connexion with RNP and martingale convergence analysed. It is noted that with a very general definition of RNP, every locally convex space has RNP. In the detailed report, we shall analyse the connexions of RNP with Krein-Milman and Choquet theorems from a martingale point of view. We shall also localise the discussion to subsets of a given space.

M.DENKER: On B-convex Orlicz sequence spaces

Let M be an Orlicz function and l_M be the associated Orlicz sequence space. It was observed by R.Kombrink that the following are equivalent:
(a) M and its complementary Orlicz function N satisfy the δ_2-condition
(b) l_M is B-convex
(c) l_M is reflexive
(d) l_M is uniformly convexifiable
The proof of it follows from known facts; the equivalence of (b) seems to be unknown.

R.M.DUDLEY: Lower layers in $\mathbb{R}^2$ and convex sets in $\mathbb{R}^3$ are not GB classes

In $\mathbb{R}^2$ let $\langle u,v\rangle \leq \langle x,y\rangle$ iff $u \leq x$ and $v \leq y$. A lower layer is a set

$A \subset \mathbb{R}^2$ such that if $\langle u,v\rangle \leq \langle x,y\rangle \in A$ then $\langle u,v\rangle \in A$. Let λ be Lebesgue measure on a bounded, open, non-empty set in $\mathbb{R}^2$. Let G be the Gaussian process indexed by Borel sets with $EG(A) = 0$ and $EG(A)G(B) = \lambda(A\cap B)$ (white noise).
It is proved that W has almost all sample functions unbounded on the collection $\mathcal{LL}$ of all lower layers, i.e. $\mathcal{LL}$ is not "GB". Likewise, in $\mathbb{R}^3$ the collection of all convex subsets of the unit ball is not GB.

E.EBERLEIN: <u>Dependent Banach space valued random vectors and lattices of random variables</u>

Lattices of real-valued random variables are investigated in various contexts where the variables typically are dependent. For example, in statistical mechanics one considers interacting particle systems. We introduce a certain mixing dependence structure and prove an invariance principle, i.e. the process obtained by interpolating partial sums is shown to converge weakly to a Brownian sheet. The proof is done by converting the problem into one for Banach space valued random vectors. Extending a result of Kuelbs we first show that for mixing triangular arrays of rowwise identically distributed B-valued random vectors the central limit theorem implies an invariance principle. Then we prove that the central limit theorem holds for dependent $C[0,1]$-valued random vectors arising from the lattice.

E.FLYTZANIS: <u>Linear measure preserving operators in Banach spaces</u>

We consider continuous linear operators T in a complex separable Banach space B that are also measure preserving transformations for a Borel probability measure m in B. We assume that the support of m spans B and the norm function is integrable. We prove the following:
"If T has the property that the dual space B* is spanned in its B-topology by a set of functionals $\{x^*\}$ whose orbits under T* are norm-bounded, then:
(i) T preserves a measure m as above iff B is spanned by eigenvectors of T having eigenvalues of norm 1.
(ii) If T preserves a measure m as above then:
a) All the eigenvalues of T* have norm 1 and the eigenvectors span B* it its B-topology. b) Considered as a measure preserving transformation T has complete point spectrum generated by the eigenvalues of T* ".

D.H.FREMLIN: Measures in l^∞

In collaboration with M.Talagrand, I have proved the following results. (i) There is a measure on PN, extending Cantor measure, for which every bounded additive functional on PN is measurable. (ii) Consequently, there is a scalarly measurable function from a probability space into l^∞ which has no Pettis integral. (iii) Using the same probability space, it is possible to construct a Pettis integrable function into l^∞ such that the range of the indefinite Pettis integral is not totally bounded.

D.J.H.GARLING: Unconditional bases and martingales in $L^p(F)$

This is a report on work of D.J.Aldous (Cambridge).
$L^p(0,1)$ $(1 < p < \infty)$ has an unconditional basis which is a martingale difference sequence (m.d.s) (the Haar system).
If F is a Banach space, when does $L^p(F)$ have an unconditional basis?
Proposition 1: If $L^p(F)$ has an m.d.s. basis, dim F = 1.
A Banach space G is of class I_p if the m.d.s. in $L^p(G)$ have uniformly bounded unconditional constants.
Proposition 2: (Maurey) If G is of class I_p, G is super-reflexive.
The converse is false (Pisier).
Proposition 3: If $L^p(G)$ has an unconditional basis, G is of class I_p.
It is also noted that L^p and c_o have the weak Banach-Saks property, but $L^p(c_o)$ does not.
The proofs are all probabilistic.

E.GINÉ: Domains of normal attraction in Banach spaces

This is a report on recent work about domains of normal attraction to stable measures by Araujo and Giné and by Marcus and Woyczinski (and by Mandrekar and Zinn in the general case).
Theorem: Let P be stable p.m. of order α in a separable Banach space B and let μ be its associated Lévy measure. Let X be a B-valued r.v. Then the condition:

$$(*) \qquad nL(X/n^{1/\alpha}) \mid B_\delta^c \to_w \mu \mid B_\delta^c$$

is:
(i) necessary in every B
(ii) sufficient if B is of type p for some p > α (Rademacher type)
for X to be in the domain of normal attraction of P. Moreover, if B is

such that (*) is sufficient for X to be in the DNA of P, for every P stable of order α, and every X, then B is of type α-stable (type $\alpha+\varepsilon$ Rademacher for some $\varepsilon > 0$).

(*) admits somewhat nicer expressions, particularly if symmetry is assumed.

S.GRAF: Losert's example of a measure space without a strong lifting

This talk gives an exposition of V.Losert's paper "A measure space without the strong lifting property". Let the following setting be assumed: S Cantor set, ν Haar measure on S, J index set with card $J = \aleph_2$, $M_1 \subset S$ closed nowhere dense with $\nu(M_1) > 0$, $M_2 \subset S$ clopen with $\emptyset \neq M_2 \neq S_p$.

$M = \{A \in \sigma(M_1, M_2) : \nu(A) > 0,\ A$ clopen or nowhere dense$\}$,

$I = \{A_{j_1} \times \ldots \times A_{j_k} \mid j_1, \ldots, j_k \in J,\ A_{j_i} \in M$ and at least one of them nowhere dense$\}$,

$T = S^J$, and μ the product of the νs on T,

$X = T \times T^I$, and for $A, B_{C_1}, \ldots, B_{C_n} \subset T$ clopen $(C_1, \ldots, C_n \in I)$:

$$\lambda(A \times B_{C_1} \times \ldots \times B_{C_n}) = \sum_{\Gamma \subset \{1,\ldots,n\}} \mu\Big(A \cap \bigcap_{k \in \Gamma} (B_{C_k} \cap C_k) \cap \bigcap_{k \notin \Gamma} (C_k)\Big) \cdot \prod_{k \notin \Gamma} \mu(B_{C_k}).$$

Then λ uniquely extends to Radon measure on X with support X and one has the following

Theorem: (X, λ) admits no strong lifting.

M.G.HAHN: Generalized Domains of Attraction of Gaussian Laws on Banach Spaces

Let B be a Banach space. A B-valued random variable X is said to be in the generalized domain of attraction (GDOA) of a Gaussian law ν on B if the laws arising from a sequential application of affine modifications to the partial sums of independent copies of X converge weakly on B to ν. The problems of characterizing the GDOA of a Gaussian and of determining appropriate affine modifications are considered for various Banach spaces.

B.HEINKEL: Central-limit theorem and law of the iterated logarithm for Banach space valued random variables

Let X be a r.v. with values in a separable Banach space $(B, \|\cdot\|)$, centered in expectation. The following result is proved:

Theorem:

If:

1) X satisfies the central limit theorem.

2) $\exists\ \alpha \in\ ]0,1[$ such that: $E\left(\frac{X^2}{(L_2 X)^\alpha}\right) < +\infty$

Then X also satisfies the law of the iterated logarithm.

An example of a C[0,1]-valued r.v. X is given, for which $E\|X\|_\infty^2 = +\infty$ and the hypothesis of the preceding theorem are fulfilled; in addition to the proof of this fact we construct a Gaussian process with continuous paths satisfying the majorizing measure condition and for which the entropy integral condition fails.

J.KUELBS: The rate of escape of Brownian motion in a Banach space

The rate of escape for a Brownian motion in a Banach space is examined, and some examples demonstrating the substantial difference between the finite and infinite dimensional situation are given. An example of particular interest is the rate of escape of the empirical process. That is, if $\{\eta_n\}$ is a sequence of independent uniformly distributed random variables, $F_N(s)$ the empirical distribution of $\{\eta_n\}$ at stage N, and

$$R(s,t) = t(F_{[t]}(s)-s)$$

for $0 \leq s \leq 1,\ t \geq 0$, then we obtain

$$\varliminf_{t\to\infty} \sqrt{\log\log t}\ \sup_{0\leq s\leq 1} \frac{|R(s,t)|}{\sqrt{t}} = \pi/\sqrt{8}$$

with probability one.

V.MANDREKAR: A decomposition of convergent and symmetric triangular arrays and its applications

Given a convergent and symmetric triangular array $\{X_{nj}\}$ taking values in a Banach space E we show that there exists a sequence $\{\delta_n\}$ $(\delta_n \downarrow 0)$ such that $\alpha(\sum_{j=1}^{k_n} X_{nj} 1(\|X_{nj}\| \leq \delta_n)) \Rightarrow$ Gaussian part of the limit law. As an application we complete some recent work of A.Araujo and E.Giné by proving that the statement

$$(*) \quad e\left(\sum_{j=1}^{k_n} \alpha(X_{nj})\right) \text{ tight iff } \alpha\left(\sum_{j=1}^{k_n} X_{nj}\right) \text{ tight}$$

is valid precisely in Banach spaces E in which c_o is not finitely representable.

M.B.MARCUS: Necessary and sufficient conditions for the uniform convergence of random trigonometric series

Let $\{a_k\}$ be a sequence of real numbers, $\sum_{k=o}^{\infty} a_k^2 = 1$, $\{\lambda_k\}$ a sequence of real numbers, $\{\xi_k\}$ a sequence of complex valued random variables satisfying $\sup_k E|\xi_k|^2 < \infty$ and let $\{\varepsilon_k\}$ be a Rademacher sequence. Consider the random series

$$X(t) = \sum_{k=1}^{\infty} a_k \varepsilon_k \xi_k \, e^{i\lambda_k t}, \quad t \in [0,1].$$

We give necessary and sufficient conditions for this series to converge uniformly a.s. This result was obtained jointly with G.Pisier.

W.PHILIPP: Weak invariance principles for sums of B-valued random variables

In this paper we prove theorems of the following type. Suppose that the properly normalized partial sums of a sequence of independent identically distributed random variables with values in a separable Banach space converge in distribution to a stable law. Then these partial sums converge in an appropriate sense even in probability to the corresponding stable process. A similar result holds for stationary Φ-mixing sequences of random variables.

G.PISIER: Lacunary sets and Gaussian processes

Let G be a compact Abelian group, let Γ be the dual group and let m be the normalized Haar measure. We discuss some recent applications of the Dudley-Fernique theorem (Necessary and sufficient conditions for the a.s. continuity of a stationary Gaussian process) to Harmonic Analysis and in particular to Sidon sets. It is a classical result of Rudin (1960) that if $\Lambda \subset \Gamma$ is a Sidon set, then $\forall f \in L^2(G)$ with spectrum in Λ $\int \exp|f|^2 dm < \infty$. Rudin raised the question whether this property characterizes Sidon sets. We can show that indeed this is true by combining results from harmonic analysis (Drury-Rider) with probabilistic ones (Dudlex-Fernique).

H.SATO: Hilbertian support and S-topology of probability measures on a Banach space

Let E be a real separable Banach space. Then the following are equivalent:

(1) Every Borel probability measure on E has a Hilbertian support.

(2) There exists an S-topology on E' (the topological dual of E) which is induced by non-negative symmetric bilinear forms.

In the case of $E = l_p$ $(1 \leq p < 2)$, we can explicitly construct the probability measures which do not have Hilbertian support.

C. STEGALL: RNP and Frêchet differentiability

Some inequalities of Smulian are applied to prove the following:

Theorem: Let D be a closed, bounded subset of the Banach space E. Then, the gauge functional ρ_D is Frechet differentiable on a dense subset of int(S(D)) where $S(D) = \{x^* : x^*$ attains its sup on D$\}$. This result improves somewhat results of Lindenstrauss, Trojanski, Phelps and Bourgain.

L.SUCHESTON: Martingales, amarts, Vitali conditions, and derivation

$(F_t)_{t\in J}$ is a net of increasing σ-fields indexed by a directed set J. T are simple stopping times; $\sigma,\tau \in T$. Th.1 TFAE: (1) Vitali condition V (= V_∞); (2) ∀ adapted (X_t), $(X_\tau)_{\tau\in T}$ converges stoch. ⇒ $X_t \to$ essentially; (3) Every L^1-bounded amart converges ess., (4) The same for sub pramarts, i.e., nets (X_t) such that stoch. $\overline{\lim}_{\sigma\leq\tau} (X_\sigma - E(X_\tau | F_\sigma)) \leq 0$. (5) If $f(\sigma,\tau)$, $\sigma \leq \tau$, is a family of functions meas. w.r.t. F_σ, $f(\sigma,\tau) = f(s,\tau)$ on each $\{\sigma=s\}$, and $f(\sigma,\tau) \xrightarrow{stoch} f_\infty$, then $f(\sigma,\tau) \xrightarrow{ess} f_\infty$. The equivalence (1) ⇔ (3) is due to Artbury, other results were obtained with A.Millet. There are similar results for a) stopping times with totally ordered ranges, and V'; b) Multivalued stopping times, and V^P.

M.TALAGRAND: A Banach space which is not a Radon space

There exists a Banach space E and a σ-additive measure μ on the Borel σ-algebra of B for the weak topology, which is supported by the unit ball, but has no barycenter, and hence is not Radon. (The Borel σ-algebra of E for the weak topology is strictly included in its Borel σ-algebra for the norm topology. Our example uses no special axioms of set theory).

A.WERON: Stationary processes in Banach spaces

The object of the talk is to give the basic formalism and some new results in the theory of stationary processes with values in Banach spaces.
Let B be a complex Banach space, $H = L^2(\Omega)$, CL(B,H) - the class of continuous linear operators from $B \to H$. A family $(X_t)_{t\in T} \in CL(B,H)$ is a stationary B-process over a group T if $X_s^* X_t = K(t,s)$ depends only on t-s.
By using the methods of dilation theory it is possible to find unitary equivalent image of the time domain of the stationary B-process. It is a space of Hellinger square integrable measures $m : \Sigma \to B^*$. In the case $B = \mathbb{C}$ this integral reduces to the one introduced by Hellinger (cf. Proc.Lond.Math.Soc. 2(1920), 249-256).
Here, the Aronszajn-Kolmogorov Kernel theorem plays an important role. This theorem is not true in all locally convex (l.c.) spaces, but a characterization of l.c. spaces for which it is true is obtained. The class of such spaces, called "spaces with the factorisation property" is large and contains for example barreled spaces.

Problem Session

Während der "problem session" wurden die folgenden Probleme gestellt:

1. $(E,\mathcal{B})$ sei ein meßbarer Vektorraum; d.h.: E ist Vektorraum und $\mathcal{B}$ eine σ-Algebra auf E, so daß folgende Abbildung meßbar ist:

$$E \times E \times \mathbb{R} \to E$$
$$(x,y,\lambda) \to x + \lambda y$$

wobei $E \times E \times \mathbb{R}$ mit der Produkt-σ-Algebra versehen wird. Nun sei $\mathcal{B}$ weiter punktetrennend, P ein W.-Maß auf $\mathcal{B}$ und P_x durch $P_x(A) = P(A-x)$, $x \in E$, $A \in \mathcal{B}$, definiert.

Problem: Gibt es ein W.-Maß P auf $\mathcal{B}$, so daß $P_x = P$ für ein $x \in E \setminus \{0\}$ gilt.
(Dudley)

2. $(E,\mathcal{B})$ sei wieder ein meßbarer Vektorraum. Ein W.-Maß P heißt stabil, falls es für alle unabhängigen, (P)-verteilten Zufallsvariablen X,Y und für alle $A,B \in \mathbb{R}$ eine Zahl $C \in \mathbb{R}$ und einen Vektor $x \in E$ gibt, so daß gilt: $L((AX+BY)C+x)=P$

P heißt zentriert stabil, falls oben x=0 gewählt werden kann.

Problem: P sei ein symmetrisches und stabiles W.-Maß. Ist P dann auch zentriert stabil?
(Dudley)

3. Es sei $(\Omega,\mathcal{F},P)$ ein Wahrscheinlichkeitsraum, $L_E^1 = \{f : \Omega \to E \mid \|f\| \in L^1\}$
Für reellwertige Zufallsvariablen gilt der folgende Satz von Komlós:
$\{f_n\} \subset L_{\mathbb{R}}^1$ sei eine Folge $L_{\mathbb{R}}^1$-beschränkter Zufallsvariablen. Es gibt dann eine Z.V. $\bar{f} \in L_{\mathbb{R}}^1$ und eine Teilfolge $\{f_{n_k}\}$, so daß für jede weitere Teilfolge $\{f_{\bar{n}_{k_l}}\}$ von $\{f_{n_k}\}$ gilt:

$$\lim_{m\to\infty} \frac{f_{n_{k_1}} + \qquad + f_{n_{k_m}}}{m} = \bar{f} \qquad \text{f.s.}$$

Es stellt sich nun die Frage, ob es möglich ist, den Satz von Komlós auf Banachräume zu verallgemeinern. In diesem Zusammenhang sind folgende Resultate schon bekannt:

1) Damit im Banachraum E der Satz von Komlós gilt, ist es notwendig, daß E die Banach-Saks-Bedingung erfüllt.

2) Falls E ein Hilbertraum ist, gilt der Satz von Komlós in E.

Problem: Man charakterisiere die Klasse der Banach-Räume, in denen der Satz von Komlós gilt.
(Bellow)

4. $(E,\|\cdot\|)$ sei Banachraum, μ,ν Radon W.-Maße auf $\mathcal{B}$
$B(x,r) = \{y \mid \|x-y\| \leq r\}$

Problem: Gilt die folgende Implikation,

$\mu(B(x,r)) = \nu(B(x,r)) \Rightarrow \mu = \nu$

In folgenden Spezialfällen gilt die obige Aussage,

i) E besitzt eine differenzierbare Norm

ii) $E = L^1(\xi)$, ξ nicht atomares Maß

iii) $E = C(X)$

iv) μ und ν Gaußmaße

Kein Resultat ist bisher für $E = l^1$ und μ,ν beliebige Radon-Maße bekannt. Man kann das obige Problem noch weiter verschärfen, indem man sich fragt, ob schon $\mu = \nu$ gilt, wenn μ und ν auf beliebig kleinen Kugeln übereinstimmen. Man erhält so das "small ball problem": $\exists\, r_o$, so daß $\forall\, x$, $\forall\, r \leq r_o$ gilt:

$$\mu(B(x,r)) = r(B(x,r))$$

Folgt daraus, daß $\mu = \nu$?

Falls μ und ν Gauß'sche Maße sind, ist die obige Aussage wahr, mehr noch:

$$\lim_{r \to o} \frac{\mu(B(x,r))}{\nu(B(x,r))} = 1 \quad \forall\, x \;\Rightarrow\; \mu = \nu \qquad \text{(C.Borell)}$$

(Hoffmann-Jørgensen)

5. μ sei ein W.-Maß auf dem Banachraum E, $\varepsilon > 0$

Problem:

a) packing problem

Gibt es disjunkte Kugeln $B(x_i,r_i)$ mit $r_i \leq \varepsilon$, $i=1,2,\ldots,$ so daß gilt

$$\sum_{1}^{\infty} \mu(B(x_i,r_i)) \geq 1 - \varepsilon \;?$$

b) Covering problem

Gibt es Kugeln $B(x_i,r_i)$ mit $r_i \leq \varepsilon$, $i=1,2,\ldots,$ so daß

$$\bigcup_{i=1}^{\infty} B(x_i,r_i) = E \quad \text{und} \quad \sum_{i=1}^{\infty} \mu(B_i) \leq 1 + \varepsilon$$

(Hoffmann-Jørgensen)

Herold Dehling, Göttingen