MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 29/1978

Funktionenräume und Funktionenalgebren

9.7. bis 15.7.1978

Unter der Leitung von Heinz Bauer (Erlangen) und Heinz König (Saarbrücken) fand jetzt zum ersten Mal in Oberwolfach eine Tagung "Funktionenräume und Funktionenalgebren" statt. Während auf den Funktionalanalysis-Tagungen der vergangenen Jahre ein recht breites Themenspektrum zur Sprache gekommen war, stand nun in erster Linie der Problemkreis Harmonische Funktionen, Potentialtheorie, Choquettheorie, Konvexe Analysis auf dem Programm.

In insgesamt achtundzwanzig Vorträgen wurde eine Vielzahl neuer Ergebnisse aus diesen Gebieten dargestellt. Dazwischen blieb genügend Zeit zu informativen und anregenden Gesprächen.

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Vortragsauszüge

T. W. GAMELIN: Subharmonicity for Uniform Algebras

A continuous real-valued function is subharmonic with respect to a uniform algebra A if it is a uniform limit of functions of the form $\max(-M, c_1\log|f_1|, \ldots, c_m\log|f_m|)$, where $M \in \mathbb{R}$, $c_j > o$, $f_j \in A$. The subharmonic functions form a max-stable cone. They are dual to Jensen measures.

Theorem: (N.Sibony and T.W.G): If $u \in C_{\mathbb{R}}(M_A)$ is locally subharmonic, then u is subharmonic.

First application: A theorem on the uniform approximation of plurisubharmonic functions of several complex variables, extending Bremermann's Theorem.

Second application: Jensen boundary points are local.

Third application: The cone of lacally subharmonic functions on an open subset of M_A is closed under various limit operations. Hence one can treat the analogue of Bremermann's Dirichlet problem in this context.

Fourth application: One can formulate and treat the notion of "sets of capacity zero" in the spectrum M_A of A.

M. von RENTELN: Remarks on the ideal structure of H^∞

Ideals generated by singular inner functions in the Banach algebra H^∞ of all bounded holomophic functions on the open unit disc D are considered. A singular inner function has the form

$$S_\mu(z) = \exp\left[- \int_T \frac{e^{it}+z}{e^{it}-z}\, d\mu \right],$$

where μ is a finite positive Borel measure on $T=\partial D$, which is singular with respect to Lebesgue measure on T. $\mathrm{supp}(\mu)$ denotes the (closed) support of μ.

Problem: Find conditions on the singular measures $\mu_1, \ldots, \mu_N$, s.t. $(S_1, \ldots, S_N) = H^\infty$, where $S_i = S_{\mu_i}$ $(i=1,\ldots,N)$, $N \in \mathbb{N}$.

<u>Theorem 1</u>: If $\bigcap_{i=1}^{N} \operatorname{supp}(\mu_i)=\emptyset$, then $(S_1,\dots,S_N)=H^\infty$.

<u>Theorem 2</u>: "⇐" holds in Th.1 iff N=1.

<u>Theorem 3</u>: Let N=2, $\mu_1 = \sum_{n=1}^{m} s_n\, \delta(e^{it_n})$, $\mu_2=\mu, \rho(e^{it},s) = \operatorname{dist}(e^{it}, \operatorname{supp}(\mu_1))$.

Then $\int_T \frac{d\mu}{\rho^2(e^{it},s)} < \infty$ implies $(S_1,S_2)=H^\infty$.

In the special case $\mu_1=\partial_\alpha$ condition (*) $\int_T \frac{d\mu}{|e^{it}-\alpha|^2} < \infty$ implies $(S_1,S_2)=H^\infty$.

<u>Remarks</u>:

(1) There exists a singular measure μ s.t. for every $\alpha\in T$ (*) does not hold.

(2) (*) holds iff the angular derivative $S_\mu'(\alpha)$ of $S\mu$ at the point α exists. In this case we have $|S_\mu'(\alpha)| = 2\int_T \frac{d\mu}{|e^{it}-\alpha|^2}$.

(3) (*) holds iff for every function f in the orthogonal complement of the invariant subspace $S_\mu H^2$ (where H^2 is the usual Hardy space) there exists the radial limit $\lim_{r\to 1} f(r\alpha)$ (Ahern/Clark, 1970).

(4) Condition (*) is not necessary, as a technical counterexample shows.

M. NEUMANN: <u>Automatic Continuity in Linear Systems</u>

The lecture gives a survey on some recent results which have been obtained in joint work with E. Albrecht. We are concerned with the problem of continuity for certain linear operators between spaces of functions and distributions. The following typical result is of importance in the physics of linear systems.

<u>Theorem 1</u>: Every causal and translation-invariant linear operator $\theta:\mathcal{D}(\mathbb{R})\to\mathcal{D}_b'(\mathbb{R})$ is continuous and hence a convolution operator.

This theorem is just one achievement of a far-reaching new theory on automatic continuity. A central result of this theory is the following theorem which is based on a refined gliding hump technique and applies not anly to the theory in question, but also to the problem of continuity for homomorphisms, derivations, intertwining operators, local operators, etc.

Theorem 2: Consider a sequence $(X_n)_{n=o,1...}$ of F-spaces X_n and a sequence $(T_n)_{n=1,2...}$ of continuous linear operators $T_n:X_n \to X_{n-1}$. Let $(Y_n)_{n=o,1...}$ be a sequence of topological vector spaces and let $(\pi_n)_{n=1,2...}$ be a sequence of continuous linear operators $\pi_n:Y_o \to Y_n$. Assume that Y_o is the union of some sequence of bounded sets. If $\theta:X_o \to Y_o$ is linear such that $\pi_n \theta T_1 \ldots T_n:X_n \to Y_n$ is continuous for all n, then there exists some n such that $\pi_k \theta T_1 \ldots T_n:X_n \to Y_k$ is continuous for all k.

In the theory of time-invariant linear systems, one has to replace the causality condition by some dissipativity assumption to conclude:

Theorem 3: Let $\Omega \subset \mathbb{R}^n$ be open and let $\theta:\mathcal{D}(\Omega) \to \mathcal{D}'_b(\Omega)$ be linear such that $\mathrm{Re}\langle\theta f,f\rangle \geq o$ for all $f \in \mathcal{D}(\Omega)$. Then θ is continuous iff $\theta(\mathcal{D}(K))$ is locally uniformly of finite order for all compact subsets $K \subset \Omega$.

This result can be proved by making use of the following abstract theorem which, on the other hand, has applications on the continuity of involutions on certain locally convex topological algebras.

Theorem 4: Let X,Y,Z be locally convex real vector spaces and let $q:X\times Y \to Z$ be continuous and bilinear. Assume that X is barreled and that Z is ordered by a normal cone. If $T:X \to Y$ is linear such that $q(x,Tx) \geq o$ for all $x \in X$, then $(u,v) \to q(u,Tv)$ is jointly continuous on $X\times X$.

L. WAELBROECK: Quotient Banach algebras and the operational calculus

I have spoken a few times of this subject a couple of years ago. It is a question by Y. Domar (Uppsala) that prompted me to put the title back into operation. Let A be a Banach algebra, e.g. A the disc algebra, and α an ideal with disconnected hull, e.g. $\mathrm{Hull}(\alpha)=\{z_o,z_1\}$, in general $\mathrm{Hull}(\alpha)=I_o \cup I_1$. Is $\alpha=\alpha_o \cap \alpha_1$ with $\mathrm{Hull}(\alpha_o)=I_o$, $\mathrm{Hull}(\alpha_1)=I_1$?

The answer is well known to be "yes" when α is closed, applying the Shilov idempotent theorem. We prove a Shilov idempotent theorem even when α is not closed: an $e \in A$ exists such that $e^2 - e \in \alpha$, $\hat{e}=o$ on I_o, $\hat{e}=1$ on I_1, where $\hat{e}$ is the Gelfand transform of e, and this is true when A is a pseudo-Banach algebra in the terminology of Allen, Dales, and Mc Clure.

The proof of this idempotent theorem uses a holomorphic functional calculus for the quotient of a commutative Banach algebra with unit by a Banach ideal. It uses the fact that pseudo-Banach algebras and ideals are direct limits of Banach. And also the fact that the operational calculus goes well to the direct limit of the algebras and ideals (to the inverse limit of the spectra).

M. DE WILDE: Baire property in product spaces and function spaces

Let (X,t_1) and (X,t_2) be topological spaces. Assume that (X,t_2) is a complete metrizable space, that $t_1 \geq t_2$ and that X is (t_1,t_2)-regular, i.e. for every open subset w of (X,t_1) there exists another open subset w' of (X,t_1) such that $\overline{w'}^{t_2} \subset w$. Then X has a sifter (i.e. a "tamis" (CHOQUET, CRAS 1958)). In particular, it is a Baire space.

Let (Y,d) be a complete metric space. Define the uniform topology on Y^X as the topology associated to the distance $d_u(f,g) = \sup_{x \in X} d(f(x),g(x))$. If J is a collection of nonnegative functions on X, filtered for $\leq$ and such that, for each $x \in X$, $\alpha(x) > o$ for at least one $\alpha \in J$, define t_J as the topology associated to the pseudometrics $d_\alpha(f,g) = \sup_{x \in X} \alpha(x) d(f(x),g(x))$. If $o < c \leq \alpha$ for some $\alpha \in J$, then Y^X is (t_1,t_2)-regular, hence (Y^X, t_J) is a Baire space. So are also its uniformly closed subspaces, in particular C(X,Y) when X is a topological space.

If X is locally compact and σ-compact, we may replace t_u by the compact open topology of Y^X. We get then that C(X,Y) is a Baire space for any topology t_J when J consists of continuous functions.

M. SCHIRMEIER: The theorem of Banach-Stone for adapted spaces

The following theorem was presented:
Let (X,E) be an adapted space in the sense of Choquet, linearly separating the points, $\tilde{E}$ be a vector space of real-valued functions on a topological space $\tilde{X}$ and $\varphi:E\to\tilde{E}$ linear such that $\varphi(E)$ is nondegenerate. If either E is a vector lattice and φ is a homomorphism of vector lattices or E is an algebra and φ is a positive homomorphism of algebras, then there exists a unique continuous map $\psi:\tilde{X}\to X$ and a continuous strictly positive function $\tilde{f}$ on $\tilde{X}$ such that $\varphi(p)=\tilde{f}(p\circ\psi)$ holds for all $p\in E$.

If $(\tilde{X},\tilde{E})$ is a linearly separating adapted space too, then:
1) $\varphi(E)$ is cofinal in $E \Rightarrow \psi$ is a proper map.
2) φ is surjective $\Rightarrow \psi$ is a homeomorphism of $\tilde{X}$ onto the closed subset $\psi(\tilde{X})\subset X$.

As an application we get the classical Banach-Stone theorem for vector lattices and algebras as well as some Banach-Stone type theorems for spaces $C(X)$, X locally compact and σ-compact, and $K(X)$ (the spaces of continuous functions with compact support), X locally compact.

R. R. PHELPS: Generic differentiability of convex functions on Banach spaces

A real Banach space E is called an Asplund space [weak Asplund space] if for every continuous convex function φ on E, the set of points at which φ is Frêchet [Gateaux] differentiable contains a dense G_δ subset of E. The Asplund spaces can be characterized as those spaces whose duals have the Radon-Nikodým property, and they satisfy almost all of the standard stability properties. Some analogous results can be proved for weak Asplund spaces, but for the most part the obvious questions remain open. For instance, is a subspace of a weak Asplund space again such a space? Or, if E is a weak Asplund space, is the same true of $E\times\mathbb{R}$?

A prize of one bottle of Scotch whisky is offered for the answer to the following question: Let $D[o,1]$ denote the space of real functions on $[o,1]$ which have left-hand limits and are right continuous, with the supremum norm. Is this a weak Asplund space?

E. ALBRECHT: Automatic continuity for local operators

The results of this lecture have been obtained in joint work with Michael Neumann. Theorem 2 of the talk of Neumann (this meeting) is used to derive continuity properties for local operators on spaces of functions, distributions and ultradistributions. Thus, we obtain generalizations of a theorem of Peetre (1960). The results are presented in the framework of generalized local operators, which gives us the possibility to obtain generalizations of theorems on continuity for certain algebra homomorphisms (by Bade and Curtis) and of theorems for linear operators commuting with generalized scalar operators (due to Johnson, Sinclair, and Vrbovà).

I. CNOP: Growth conditions and spectra

The holomorphic functional calculus of L.Waelbroeck for elements of commutative unital algebras not admitting a compact spectrum involves, and has applications to certain algebras of holomorphic functions with growth conditions. The aim of the talk is to present an overview of this interplay.

B. FUCHSSTEINER: Production and Distribution

Methodes from convex analysis and from the theory of sublinear functionals are used to treat a supply-demand model in Mathematical Economics with infinite commodity space X. To be more precise we sketch the mathematical theorems on which the model is based. Given finite tight measures α and β on $K(X)$ (space of nonempty compact subsets of a metric space X equiped with the Hausdorff metric) such that $\int_{Z\subset X} d\beta(Z) \leq \int_{Z\cap Y\neq\emptyset} d\alpha(Z)$ for all compact $Y\subset X$, then α and β are called the supply- und the demand measure respectively. It is shown that we can then find a tight measure π (called global production plan) on X such that

$$\int_{Z\subset X} d\beta(Z) \leq \int_Y d\pi \leq \int_{\{Z\mid Z\cap Y\neq\emptyset\}} d\alpha(Z) \quad \text{for all measurable } Y\subset X.$$

By application of suitable desintegration techniques we find furthermore measures p and v on XxK(X) which correspond to the notions of "individual production plan" and "distribution plan" and which fulfill reasonable inequalities with respect to α,β,π.

There are close relations between these results and results concerning flows in networks.
(Part of this is joint work with Antonius Schröder).

K. YABUTA: Problems of prediction theory type in uniform algebras (Helson-Sarason type theorems)

A report on some extensions to uniform algebras setting of Helson-Sarason theorems on prediction theory. Namely:

Let A be a uniform algebra on a compact Hausdorff space S and ϕ a complex homomorphism of A with a unique representing measure m. Let $A_o=\{f\in A|\phi(f)=o\}$, $A_o^n=$ linear span of $\{f_1f_2\ldots f_n,\ f_j\in A_o\}$ and H the weak-*-closure of A, $H_o=\{f\in J|\int f dm=o\}$. For a finite positive regular Borel measure μ on X set

$\rho_n(\mu)=\sup\{|\int fg d\mu|,\ f\in A, g\in A_o^n$ with $\int|f|^2 d\mu\leq 1$ and $\int|g|^2 d\mu\leq 1\}$. Then

1) If H_o is not simply invariant, then $\lim\limits_{n\to\infty}\rho_n(\mu)=o$ iff $d\mu=cdm$.

2) If H_o is simply invariant, then $\lim\limits_{n\to\infty}\rho_n(\mu)=o$ iff $\mu\ll m$ and there exist a polynomial $P(z)=a_o+\ldots+a_kz^k$ and $u,v\in C(T)$ such that $d\mu|dm=|P(Z)|^2\exp(u(Z)+V(Z))$, where $Z\in H$ with $H_o=ZH$ and V is the usual conjugate function of v. Also such a characterization of measures with $\rho_n(\mu)<1$ and other related results.

G. LEHA: Relative Korovkin-Sätze und Ränder

Für zwei Funktionenräume H,L auf kompaktem Grundraum X mit $H\subset L\subset C(X)$ sei $K(H,L)=\{f\in L|(T_i)_{i\in I}$ Netz linearer positiver Operatoren $T_i:L\to L$ mit $\lim\limits_{i\in I} T_i(h)=h$ für alle $h\in H\Rightarrow\lim\limits_{i\in I} T_if=f\}$
der relative Korovkin-Abschluß von H bezüglich L.

Mit Hilfe der beiden Begriffe "relativer Choquet-Rand von H bezüglich L" und "determinierende Menge für L" werden hinreichende Bedingungen angegeben dafür, daß ein $f\in L$ in $K(H,L)$ liegt und insbesondere dafür, daß H Korovkin-Raum bezüglich L ist, das heißt $L=K(H,L)$. Ist speziell L ein abgeschlossener simplizialer Funktionenraum in $C(X)$, so wird der relative Korovkin-Abschluß von H bezüglich L mit Hilfe des Shilov-Randes von L charakterisiert. Insbesondere ergibt sich in diesem Fall, daß H genau dann Korovkin-Raum bezüglich L ist, wenn der Shilov-Rand von L im relativen Choquet-Rand von H bezüglich L enthalten ist.

H. H. SCHAEFER: Comparison of Spectra

Suppose $A=P[t]$ is the ring (over $\mathbb{C}$) of polynomials in one indeterminate t. If $z\to\|z\|_i$ are two norms making A a normed algebra, A_1,A_2 the respective completions, and if B is the completion of A with respect to $\|z\|:=\|z\|_1 \vee \|z\|_2$ then $\|w\|=\|\varphi_1(w)\|\vee\|\varphi_2(w)\|$ for $w\in B$, φ_i denoting the continuous extension of $A\to A_i$ to B.

It is shown that $\sigma_{A_1}(t)\cup\sigma_{A_2}(t)=\sigma_B(t)$ if $(\sigma_{A_1}\cup\sigma_{A_2})^c$ is connected. Moreover, if φ_i are injective und σ_{A_i} countable $(i=1,2)$ then $\sigma_{A_1}(t)=\sigma_{A_2}(t)$.

Application: Let (X,Σ,μ) be a finite measure space. If T is bounded, linear $L^1(\mu)\to L^p(\mu)$ for some p, $1<p\leq\infty$, then T induces an oparator $T_q: L^q\to L^q$ for all q, $1\leq q\leq p$, such that $\sigma(T_q)=\sigma(T_1)$ for all q, $1<q\leq p$. (Dayanithy (M.Z.179, 1978) has given an example of an operator $T_2: L^2\to L^2$ which induces an operator T_q on each L^q, $2<q\leq\infty$, such that $\gamma(T_2)=1$ while $\sigma(T_q)=\{o\}$ for all q, $2<q\leq\infty$.)

J. BLIEDTNER, W. HANSEN: Simplicical Cones in Potential Theory

Applications of "Choquet Theory" to potential theory are given: Let X be a compact subset of a $\mathfrak{P}$-harmonic space (X,H), and $G\subset X$ a finely open set. It is shown that the function cone

$$S(X,G) = \{s\in C(X) \mid s \text{ "finely superharmonic" on } G\}$$

is a simplicial function cone and that the corresponding minimal representing measures are obtained by balayage of the unit masses to the Choquet boundary $CH_{S(X,G)}X$. Furthermore, a potential-theoretic characterization is given of $CH_{S(X,G)}X$: it is the greatest finely closed subset of G^c which is not thin at any of its points, i.e. $CH_{S(X,G)}X=X\cap\beta(G^c)$.

The set $M_x(S(X,G))$ of representing measures of $x\in X$ with respect to $S(X,G)$ is characterized and a complete description of its extreme points is given. As a consequence several relations between different cones of type $S(X,G)$ are established. In particular, the following theorem of Runge type is obtained: Let U be an open subset of a $\mathfrak{P}$-Brelot space which is non-compact and connected. Then $\mathcal{f}(Y)_{|U}$ is dense in $\mathcal{f}(U)$ with respect to uniform convergence on compact subsets if and only if no connected component of U^c is compact.

(Inventiones math. 29, 83-110(1975), 46, 255-275 (1978)).

I. NETUKA: <u>The space of harmonic functions with continuous boundary values</u>

This is an expository lecture about the function space $H=H(U)$ which plays an important role in classical and abstract potential theory. Here U is an arbitrary relatively compact open set in a harmonic space and $H(U)$ constists of all functions continuous on the closure of U and harmonic on U. (The solutions of the Laplace equation or the heat equation serve as standard examples.) By way of introduction, a few words about the axiomatic potential theory will be said and the following topics will be discussed: When $H(U)=H(V)$, $H|\partial U=C(\partial U)$? How to learn that $F\in H|\partial U$? What is the Choquet and Shilov boundary of H? (Choquet points, exposed points, Shilov boundary, simpliciality, lattice properties.) What is the right solution of the Dirichlet problem? (Keldyš operator and Keldyš set.) About continuation and approximation of functions in H. Are $H|\partial U$ or H ever function algebras?

F. ALTOMARE: Opérateurs de Lion sur le produit d'espaces compacts, semi-groupes d'opérateurs positifs et problème de Dirichlet

A partir d'une famille d'espaces compacts sur chacun de quels est défini un opérateur de Lion, on prouve l'existence d'un opérateur de Lion défini sur le produit topologique de la famille des compacts. Par conséquence on donne une condition nécessaire et suffisante pour la résolution du problème de Dirichlet abstrait sur l'espace produit tensoriel ou affine d'une famille de simplexes de Lion. Les résultats obtenus en appliquent à la théorie des semi-groupes d'opérateurs markoviens positifs sur un espace compact et à la théory des familles résolvantes. Finalement on donne une représentation du produit tensoriel de particulières familles de sous-espaces de fonctions continues.

G. WITTSTOCK: Eine ordnungstheoretische Charakterisierung von C*-Algebren und Anwendungen in der axiomatischen Quantenmechanik

Wir benutzen den 'operational approach' zur Beschreibung quantenmechanischer Systeme. Ein System I werde durch eine Dualität $\langle A_I, V_I \rangle$ eines vollständigen ordnungseinsgeordneten Raumes A_I und eines vollständigen basisnormierten Raumes V_I beschrieben. Für ein weiteres System II einfacher bekannter Struktur wurde der übliche Hilbertraumformalismus vorausgesetzt: $A_{II}=L(H)$ und $V_{II}=T(H)$. Eine Analyse des gekoppelten Systems ergibt, daß A_I und V_I eine Matrixordnung tragen. Dies kann man nun benutzen, um in A_I eine C*-Algebrastruktur zu erzeugen. Unter den Operationen des Systems I sei P_∞ die Menge der p-Projektionen im Sinne von Alfsen und Shultz, die mit der Matrixordnung verträglich sind. Ein quasikomplementäres Paar P,P' liegt in P_∞ wenn $P\otimes Q_1$ und $P'\otimes Q_1$ p-Projektionen auf $\langle A_I\otimes M_n, V_I\otimes M_n\rangle$ $(n\in\mathbb{N})$ sind. Dabei ist Q_1 eine p-Projektion im System II, die auf einen reinen Zustand projeziert. U_∞ sei die Menge der zugehörigen projektiven Einheiten.

Herr K.H. Werner (Saarbrücken) hat nun gezeigt, daß es eine C*-Algebra $A_\infty \subset A$ gibt, deren Projektoren gerade die Menge U_∞ sind. Ist

$A=V^*$, so ist A_∞ eine W*-Algebra. A ist ein A_∞-Modul. Ist $A_\infty=L(H)$ und projeziert jeder minimale Projektor im Verband P_∞ auf einen reinen Zustand, so ist $A=A_\infty=L(H)$.

B. KRAMM: A functionalanalytic characterization of Stein algebras

A complex-analytic space (X,O) is a Stein space iff the natural map $X\to{}_oO(X)$, $x\to\partial_x$, $\partial_x(f):=f(x)$, is a homeomorphism. Here $O(X)$ stands for the algebra of all holomorphic functions on X, and ${}_oO(X)$ for its spectrum endowed with the Gelgand-topology. A topological Algebra A is called a Stein algebra if there exists a Stein space (X,O) such that A is top-iso to $O(X)$.

Theorem: A uniform Fréchet-algebra with locally compact spectrum is isomorphic to $O(R)$, for some Riemann surface R, iff all closed maximal ideals are principal.

Theorem: 1) A strongly uniform Fréchet-Schwartz-algebra with locally compact and pure-dimensional spectrum is a pure-dimensional Stein algebra, and vice-versa.

2) A strongly uniform Fréchet-Schwartz-algebra with locally compact spectrum, such that all closed maximal ideals are (topologically) n-generated, is a regular Stein algebra, and vice-versa.

The proofs use theorems of Gleason, Carpenter, Basener/Sibony and the author. The above theorems have appeared resp. will appear in Pac. J. 76, Stud. Math. Vol.66.3 and Adv. Math.

There is an example $A=\varprojlim O(X_n)$, X_n onedimensional Stein spaces, which satisfies all hypotheses of the above theorems except the local compactness of its spectrum.

W. A. J. LUXEMBURG: On a class of entire functions of exponential type whose Borel transforms are in H^2

Let $f=f(z)$, $z\in\mathbb{C}$, be an entire function of exponential type and let $\psi(w)=\int_0^\infty f(t)e^{-tw}dt$ be its so-called Borel transform. The classical

Paley-Wiener theorem suggests to investigate the problem under what conditions on f is its Borel transform of class H^2 on the complement of its conjugate indicator diagramm? Some results and examples concerning this problem will be presented.

S. PAPADOPOULOU: <u>On the stationary Korovkin closure in C(X)</u>

Let X be a compact space, H a function space on X. The problem of determining the stationary Korovkin closure of H in C(X), that is the space $\tilde{H}=\{f\in C(X)\mid$ for any positive operator $T:C(X)\to C(X)$ with $T|H=Id$ one has $Tf=f\}$, is examined. Conditions are given, under which $\tilde{H}$ coincides with the space $\hat{H}$ of the H-affine functions. Examples show that $\tilde{H}$ can be the whole space S(X), even if the Choquet boundary of H is not dense in X. This is even possible for finite-dimensional spaces H and for spaces of the form A(S), where S is a simplex.

A. CLAUSING: <u>Completely p-convex functions</u>

A function $f\in C^\infty[0,1]$ is called completely p-convex for $p\in \mathbb{N}$, if $(-1)^n f^{(pn)} \geqq o$ for $n=o,1\ldots$

Classical cases are p=1 (the completely monotonic functions of Bernstein) and p=2 (the completely convex functions of Widder). For p>2, there is no integral representation known which holds for the cone C_p of all completely p-convex functions. Inspired by work of Berg and of Leening and Sharma, we consider certain subcones of $C_p(p>1)$ which are given as follows. Let $\Delta=\{\partial_1,\ldots,\partial_p\}$ be a subset of the set of functionals $f\to f^{(j)}(x)$ on $C^p[o,1]$, where $x\in\{o,1\}$ and $j=o,\ldots,p-1$. Suppose there is a unique set $\{f_1,\ldots,f_p\}$ of polynomials fulfilling $f_j^{(p)}=o$, $\partial_i(f_j)=\partial_{ij}$ $(i,j=1,\ldots,p)$. Then the f_j do not change sign in [0,1], and putting $\epsilon_j=\operatorname{sgn} f_j$, we define a subcone of C_p by $C_\Delta=\{f\in C_p\mid (-1)^n\epsilon_j\partial_j(f^{(pn)})\geqq o,\ j=1,\ldots,p,\ n=o,1,\ldots\}$. It is shown that C_Δ has a compact base B which is a Bauer simplex whose set of extrem points is homeomorphic to $\overline{\mathbb{N}}$. This is used to give a polynomial expansion for the functions in C_Δ. The proof is functional analytic and avoids complex methods.

P. MALLIAVIN: Uniform algebras and methods of optimal stochastic control

Gaveau proved the following representation for the solution of the Monge Ampère equation in a strictly pseudoconvex domain D. (J.F. Analysis Vol. 25, 1977, 391-411)

Let $f \in C(\overline{D})$, $\varphi \in C(\partial D)$. A kählerian control is a nonanticipative map $\sigma : \Omega(\mathbb{C}^n) \times \mathbb{R}^n \to M_n^+$, ($\Omega(\mathbb{C}^n)$ the probabilistic span of the Brownian on $\mathbb{C}^n$, M_n^+ the positive definite hermitian matrices with $\det(m) \geq 1$)

Define

$$u(z_o) = \inf_{\sigma} \{ E_{x_o}(\varphi(X(T_{\sigma,z_o})) - \int_o^T f(X_{\sigma,z_o}(t)\ dt \}$$

where $X_{\sigma,z_o}(t) = z_o + \int_o^t \sigma_j^i\ ds^j$.

Then u fulfilled in Bedford Taylor's sense (Invention 76):

$$\begin{cases} (\det(\frac{\partial^2 u}{\partial z_i \partial \overline{z}_j}))^{1/n} = cf \text{ on } D \text{ (c numerical constant)} \\ u = \varphi \text{ on } D. \end{cases}$$

(ii) Using the theory of Gamelin of Hartog's function (Pacific Journal of Math. Vol. 62, 401-417) it is possible to prove

Theorem: Let $A(\overline{D})$ be the uniform algebra of holomorphic functions on the ball D, continuous on $\overline{D}$. Let $x_o \in D$ and let $J'_D(x_o)$ be the set of Jensen measures on D of x_o. Consider the Poisson kernel P_a associated to the Laplacian

$$\Delta = \sum_{i,j} a^{i,j}(z) \frac{\partial^2}{\partial z^i \partial z^j} \qquad \text{(with Trace } a \geq 1).$$

Then $P_a \in J'_D(x_o)$ (trivial) and $\{P_a\}_a$ generates by weak-*-closure of convex combinations $J'_D(x_o)$.

(iii) Extremal potential. D as before (ii). Let $K \subset D$ be compact. Denote $u(z) = \inf_{\sigma} \{ E(II_D(X_{\sigma,z}(T)) \}$.

Theorem: u is P.S.H. in D and $u(z) \to 1$ $(z \to z_o)$ whenever $z_o \in D$, $u(z) \to o$ $(z \to z_o)$ whenever $z_o \in K$, regular for the newtonian potential.

Theorem: Suppose u is continuous. Then $\{z | u(z) = o\}$ = convex polynomial hull of K.

(iv) Theorem: Let K a compact of $\mathbb{C}$. Then a), b), c) are equivalent for every $h \in C(K)$:

a) h is Hartog relatively to R(K).
b) $h(x_o) \leq \int h(x) d\alpha$ for all $x_o \in K$, $\alpha \in J(x_o)$.
c) h is finely subharmonic.

(v) Rickarts subharmonic functions (which contain the Hartog class) are also discussed and an abstract theory is constructed with Arens Singer measures.

G. LUMER: Evolution equations on function spaces: New developments and their applications to P.D.E.

We use the theory of local operators (see [1]), the Trotter theorem on approximation of semigroups (with a sequence of Banach spaces X "approaching" a Banach space X), and a priori estimates in the classical context or corresponding abstract assumptions in the general context of locally compact spaces, to prove a result on approximation of solutions of evolution equations (posed in the sup-norm O-boundary-values context of [1]). In the classical context of open sets V in $\mathbb{R}^n$, the approximation result says roughly that solutions of problems with no regularity assumptions on V, and little regularity assumption on the operator A (A induced by an A(x,D) which is as in [2], section II), can be approximated uniformly by solutions computed in "very regular" (C^∞) domains $G_n \uparrow V$, with respect to approaching operators $A_n(x,D)$ with C^∞coefficients (the initial values in G_n beeing obtained by restricting to $\bar{G}_n$ the original initial value f, and this so that $A_n(x,D)$ and F are chosen and for all given A(x,D),V, independent of the particular initial value considered. One has a similar result in the general context (with abstract assumptions replacing a priori estimates).

[1] G. Lumer, "Problème de Cauchy...", Annales Inst.Fourier, 25 (1975), fasc. 3 et 4, 409-446.
--------, C.R. Acad. Sci. Paris, 284 série A (1977),1435-1437.

R. HAYDON: Dual L^1-spaces

It is a theorem of Gelfand that $L^1[0,1]$ is not isomorphic (that is to say, is not linearly homeomorphic) to any dual Banach space. A refinement of this result, due to Hagler and Stegall, tells us that if X is separable and X* is isomorphic to an L^1-space, then X* is isomorphic either to l^1 or to C[0,1]*. We consider the problem of determining the isomorphism classes of non-separable dual L^1-spaces. The obvious examples are spaces of the type C(S)*, and among these the only ones which come naturally to mind do in fact have the property that C(S)* is isomorphic to an l^1-direct sum of spaces of the type $C([0,1]^K)^*$. We can therefore pose two problems:

Let X be a Banach space and suppose X* is isomorphic to an L^1.

Problem A: Does there exist a compact space S such that X* is isomorphic to C(S)*?

Problem B: Is X* isomorphic to a space of the form $(\sum_{\alpha\in A}^{\circledast} C([0,1]^{K_\alpha})^*)_{l^1}$?

Our results all depend on the Continuum Hypothesis.

Theorem 1: There is a compact space T such that C(T)* is isometric to $(L^1([0,1]^{\aleph_1}) \circledast C[0,1]^*)_1$, hence the answer to B is "no".

Theorem 2: If the density character of X* is smaller than $\aleph_\omega$ the answer to A is "yes".

D. KÖLZOW: A survey of Reproducing Kernel Hilbert Spaces

The survey included:

1) the basic results of Moore and Aronszajn,
2) the Bergman Space and its application to conformal mappings,
3) the Fisher Space, the operational calculus of Newman and Shapiro, the Fisher decomposition, and Bergman's solution of the representation problem for the canonical commutator relation,
4) the connection with elliptic differential equations,
5) the classical Wiener space, and the Sobolev Space $W_n^{m,2}$ for 2m>n, as a generalized Reproducing Kernel Hilbert space.

U. BAUERMANN: Harmonic and analytic functions

Let X be a locally compact space with countable base, $\tilde{H}^*$ be a sheaf of hyperharmonic functions such that $(X,\tilde{H}^*)$ is a $\mathfrak{P}$-harmonic space in the sense of Constantinescu-Cornea. Let $\tilde{A}=(\tilde{A}(U))_{U\subset X \text{ open}}$ be a sheaf of algebras of continuous complex-valued functions. $(1\in\tilde{A}(U)$, $\tilde{A}(U)$ separates the points of $U)$.

Assume the following relations between $\tilde{H}$ and $\tilde{A}$:

It exists a base B of connected regular sets such that

(i) $\overline{\mathrm{Re}\ A(U)} = H(U)$
for all $U\in B$,
(ii) $\sigma(A(U)) = \overline{U}$

where $A(U) = \overline{\{f\in C(U) \mid f_{|U}\in\tilde{A}(U)\}}$, $H(U) = \{h\in C_{\mathbb{R}}(U) \mid h_{|U}\in\tilde{H}(U)\}$.

Theorem: X is a Riemann surface, $\tilde{H}$ is the sheaf of usual harmonic functions, and for any U open $\subset X$ the closure of $\tilde{A}(U)$ w.r.t. uniform convergence on compacta is the set of holomorphic functions on U.

H. SCHIRMEIER: On Keldych's Theorem

Stating Keldych's theorem means giving a uniqueness condition for the extension in the theorem of Hahn-Banach, applied to the situation

$$\begin{tikzcd} E \arrow[dr, "T"] & \\ H \arrow[u, bend left] \arrow[r, "K"'] & F \end{tikzcd}$$

where the canonical mapping K is established by different restriction from $H(U)$. If you take $H:=H(U)_{|\partial U}$, then $K(h_{|\partial U}):=h_{|U}$ for all $h\in H(U)$.

Setting $E=C(\partial U)$, $F=H_b(U)=\{$all bounded harmonic functions on $U\}$ we get the following:

1) In general, K has different positive linear extensions T, called Keldych-operators. In that case, there are even nonlinear monotone extensions, called K-operators.
2) Uniqueness of T can be described in terms of the boundary of U: K has a unique monotone extension iff U is a Keldych-set.
3) Exhausting (possibly nonlinear) K-operators approximate the Perron-Wiener-Brelot-Operator H^U.

Remarks on the "non-relatively compact" case (concerning U) and examples were given.

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