MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 30/1978

Arbeitsgemeinschaft Algebra : Schiefkörper

16. 7. bis 22. 7. 1978

Die Tagung über "Schiefkörper" stand unter der Leitung von P.M.Cohn (London) und G.Michler (Essen) und richtete sich nicht nur an Experten, sondern sollte auch jüngeren Mathematikern eine Einführung in das Arbeitsgebiet bieten. Demgemäß nahmen Vorträge mit Überblickscharakter, in denen die hauptsächlichen Fragestellungen und Methoden erläutert wurde, einen relativ großen Raum ein. Die Vorträge von G.M.Bergman und P.M.Cohn behandelten das Problem der Einbettung eines gegebenen Ringes in einen Schiefkörper. Für gewisse Ringe, etwa die sog. Semifirs, wurde eine "universelle" Lösung dieses Problems angegeben. In den Vorträgen von S.A.Amitsur wurden Anwendungen der Theorie der Ringe mit Polynomidentitäten auf die Beschreibung von endlich dimensionalen Divisionsalgebren dargestellt. Der zahlentheoretische Aspekt der Theorie stand im Vordergrund der Vorträge von G.J.Janusz, wo für einen algebraischen Zahlenkörper K oder die Vervollständigung eines algebraischen Zahlenkörpers die Schur-Untergruppe der Brauer-Gruppe $Br(K)$ studiert wurde. Diese besteht aus den Klassen, die einen K-zentral einfachen direkten Summanden der Gruppenalgebra $K[G]$ einer endlichen Gruppe G enthalten.

Wesentlich ergänzt wurden diese Überblicksvorträge durch 15 Spezialvorträge, in denen über methodische und inhaltliche Fortschritte auf dem Gebiet der Schiefkörper und ihrer Anwendungen berichtet wurde.

Teilnehmer

Amitsur, S.A., Jerusalem
Baer, R., Zürich
Bass, H., New York
Bergman, G.M., Berkeley
Bokut, L.A., Novosibirsk
Bongartz, K., Zürich
Borho, W., Wuppertal
Brungs, H.H., Edmonton
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Dipper, R., Essen
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Vortragsauszüge

S. A. AMITSUR : Polynomial identities and finite dimensional division algebras

Polynomial identities have originated in division algebras (Dehn 1921), and the fact that matrix rings over fields satisfy identities was first given by Wagner (1936). Some of the well known identities which are satisfied by all central simple algebras (c.s.a.) of dimension n^2 are the standard polynomial

$S_{2n}[x] = \Sigma \operatorname{sg}\sigma \ x_{\sigma(1)} x_{\sigma(2)} \cdots x_{\sigma(2n)} = 0$ (Amitsur-Levitzki)

and the Capelli identity

$d_{n^2+1}[x;y] = \Sigma \operatorname{sg}\sigma \ x_{\sigma(1)} y_1 x_{\sigma(2)} y_2 \cdots y_{n^2-1} x_{\sigma(n^2)} y_{n^2} x_{\sigma(n^2+1)} = 0.$

The central polynomials of Formanek and Razmyslov have been extended, and there is a central identity which holds for all $M_n(K)$: $S[x_1, \ldots, x_{n^2}; y_1, \ldots, y_{n^2-1}]$. The identities which yield elements of the symmetric polynomial in eigenvalues of a matrix were used to prove that every c.s.a. contains a separable element, and also a generic representation of c.s.a. in the matrix ring over a maximal commutative field.

The notion of an $\mathcal{J}$-pivotal monomial yields the following characterization of division algebras of dimension n^2 : $(D: \operatorname{Cent} D)=n^2$ if n is the minimal integer such that $(1-gf^n, f)_1 = 1$ for any two polynomials $f(\lambda), g(\lambda) \in D[\lambda]$.

Another application of identities is to describe the universal solution of the problem of embedding R into an $n\times n$-matrix ring over a commutative ring. There exists such a maximal embedding and for a division ring R of finite dimension the corresponding ring is an affine domain, whose points correspond to the splitting fields of R.

S. A. AMITSUR : Generic rings for classes of division rings

The ring of generic matrices $k(X)$ is a generic ring for all division algebras D of dimension n^2 over a center containing k - in the sense that "for every $0 \neq q(X) \in k[X]$, and every D with center $K \supseteq k$ and $\dim_k D = n^2$ there exists a homomorphism $\varphi : k[X] \to D$ such that $\varphi(q) \neq 0$ and $\varphi(k[X])K = D$." This homomorphism can be extended and induces an isomorphism between the residue field $k[X]_{\mathfrak{p}} / \mathfrak{p} k[X]$ of the localization of $k[X]$ at $\mathfrak{p} = \ker \varphi$ and a subdivision algebra of D of the same dimension. It has been described how to use this to show that $k(X)$ has exponent n and that $k(X)$ is not a crossed product under certain conditions on n, e.g. if $p^3 | n$ for some prime p. Other generic rings for classes of division algebras are : Saltman's ring $Q_{m,n}$, obtained with the aid of generic matrices and the universal domain of the embedding of $k(X)^{n/m}$, is a generic ring (in the previous sense) for the class of all division algebras of dimension n^2 and exponent m. This can be used to show that $Q_{m,n}$ is not a crossed product and also that it is not decomposable under certain restrictions on m and n. The idea of a generic ring for the class of division algebras which are crossed products of a fixed group G was suggested, and the group algebra $k(F/R')$ seems to be one, where $1 \to R \to F \to G \to 1$ is a representation of G by a free group F and relations R. Finally, $k(X)$ has a normal splitting field with Galois group G iff every division algebra of the corresponding class will have a splitting field with a Galois group $H \subseteq G$. This as well can be deduced from the generic properties discussed above. A corollary is that if $k(X)$ is not a p-algebra, it is not even similar to a cyclic algebra.

H. BASS : Finite dimensional representations of infinite groups: Some problems and conjectures

Let Γ be a finitely generated group with a faithful finite dimensional (f.d.) complex representation. Let X_n denote the isomorphism classes of irreducible representations $\rho : \Gamma \to GL_n(\mathbb{C})$; it is open in the affine variety $\overline{X}_n = X_1 \cup \ldots \cup X_n$. We ask about the group theoretic significance of representation theoretic invariants. Examples: 1. Interpret the asymptotic behavior of $\dim \overline{X}_n$. There are interesting arithmetic groups Γ such that $\dim \overline{X}_n = 0$ for all n . I know no other kinds of examples.

2. Suppose that the Zariski closures $\overline{\rho(\Gamma)}$ $(\rho \in \bigcup_n X_n)$ have bounded dimension. What does this imply ? E.g. is $\dim \overline{X}_n = 0$ for all n ?

3. Consider subfields $F \subset \mathbb{C}$ over which Γ has a faithful f.d. representation, and let $\operatorname{tr deg}(\Gamma) = \min_F \operatorname{tr deg}_{\mathbb{Q}}(F)$ for such F . Interpret $\operatorname{tr deg}(\Gamma)$. Is

$$\operatorname{tr deg}(\Gamma) = \max_{\Gamma'} \operatorname{tr deg}(\Gamma')$$

where Γ' ranges over solvable subgroups of Γ ?

G. M. BERGMAN : Constructing and understanding R-sfields

P.M.Cohn has shown that if R is a ring, every epic R-sfield D is determined up to isomorphism by the class of square matrices over R which have singular images over D . He gives necessary and sufficient conditions for a class of square matrices to correspond to an R-sfield, and a construction for this sfield from that data.

We look at the same data as telling what families of elements of free modules R^n become linearly dependent in D^n, give an alternative description of conditions for such "dependence-data" to come from an R-sfield, and obtain a new and simpler construction for this R-sfield.

The construction is based on considering the class of R-modules of the form R^n / K, where K is a maximal proper submodule closed with respect to the given "dependence" structure. It is shown that every nonzero map among such modules is injective, and the class of such modules has pushouts. It may be deduced that for any such module M, the category of nonzero homomorphisms $M \to N$ (N another such module) forms a directed partially ordered set. If $\tilde{M}$ denotes the direct limit of this system, one finds that nonzero endomorphisms of $\tilde{M}$ are simply transitive on nonzero elements. One deduces that this endomorphism ring is a sfield D and $\tilde{M}$ a one-dimensional D-vector space.

As examples, we show how the universal sfield of a semifir, and relatively universal sfields for semihereditary rings may be obtained.

L. A. BOKUT : Jordan division rings

Theorem 1 (Zelmanov, E.I.). Any special Jordan division algebra is isomorphic to one of the following algebras: 1) An algebra $\mathcal{D}^+$, where $\mathcal{D}$ is an associative division algebra, 2) An algebra $H(\mathcal{D},*)$ of symmetric elements of a skew field $\mathcal{D}$ with involution, 3) An algebra of a symmetric bilinear form.

Theorem 2 (Zelmanov, E.I.). Any exceptional Jordan division algebra is a 27-dimensional Albert algebra over the center.

Theorem 3 (Bokut, L.A.). Let $L, L_1, \ldots, L_4$ be countable Lie algebras, such that every algebra L_i, $1 \leq i \leq 4$, is the union of an infinite increasing chain of subalgebras with the factors of dimension $\aleph_0$. Then the algebra L is embeddable into an algebraically closed Lie algebra $\mathfrak{A} = L_1 + \ldots + L_4$, which is the sum of the algebras L_i, $1 \leq i \leq 4$.

Theorem 4 (Grishnov, A.N.). Any finite-dimensional simple binary Lie algebra of characteristic zero is a Malcev algebra.

Theorem 5 (Anan'in, A.Z.). Any associative algebra with the identities $[x_1 y_1] \ldots [x_n y_n] = 0$, $[x_1, \ldots, x_n][y_1, \ldots, y_n] = 0$, $n \geq 2$, is representable (by matrices over a commutative algebra).

W. BORHO : Skew fields and enveloping algebras of Lie algebras

This was an introduction for non-specialists into some touching points between the two topics mentioned in the title. After explaining the notions of and basic facts on Weyl fields and enveloping fields, the Gel'fand-Kirillov conjecture (1966) was formulated : Every enveloping field of an algebraic Lie algebra is a Weyl field over its centre. A historical survey of the settled cases was given. To settle the classification of these skew fields up to isomorphism completely, Gel'fand and Kirillov have introduced a notion of a transcendence degree of skew fields. This is defined in a rather complicated way, but turns out to be extremely useful in various applications. For a better understanding of this notion, some ideas and results of a joint paper with H.Kraft (Math. Ann. 220, 1976) were explained. E.g. the growth invariant of a finitely generated algebra (which is an equivalence class of functions), and various facts about Gel'fand-Kirillov dimension (which is a real number or

∞, and has to be carefully distinguished from the more complicated notion of transcendence degree). As one application it was explained how knowledge on GK-dimension may sometimes imply Ore's condition for a multiplicative subset of a domain. Also, an example was given for how computations of GK-dimensions are used in the study of representations of Lie-algebras and of the skew fields occuring in this connection.
(For an introduction into this topic, cf. e.g. Sém. Bourbaki n° 489, Nov. 1976.)

H. H. BRUNGS : Right chain rings and the generalized semigroup of divisibility

Let R be a ring with unit element and without zero-divisors and let $\tilde{H}(R) = \{\tilde{x};\ 0 \neq x \text{ in } R\}$ where $\tilde{x}$ is the mapping from the set of all nonzero principal right ideals of R into itself defined by $\tilde{x}(aR) = xaR$. $\tilde{H}(R)$ is a partially ordered semigroup that can be considered as a generalization of the group of divisibility of a commutative integral domain. We study those rings for which $\tilde{H}(R)$ is totally ordered. They turn out to be localizations of right invariant right chain rings and they are right invariant if d.c.c. for prime ideals holds.

G. CAUCHON : Skew polynomial rings and applications

If A is an artinian simple ring with center K, if σ is an endomorphism of A and δ a σ-derivation, we consider the skew polynomial ring $R = A[x,\sigma,\delta]$ in which the multiplication is defined by the condition $xa = \sigma(a)x + \delta(a)$ $(\forall a \in A)$. Define $k = \{a \in K \mid \sigma(a) = a$ and $\delta(a) = 0\}$. Then, we have the following results: (I) R is P.I. $\Longleftrightarrow$ $[A : k] < +\infty$

(ii) If R is P.I., then it is a free module of finite rank n^2 over its center. Moreover, if $\sigma \in \mathrm{Inn}(A)$, R is an Azumaya algebra with constant rank n^2 over its center and, if $\sigma \notin \mathrm{Inn}(A)$, then δ may be chosen equal to 0 and, though R is not an Azumaya algebra in this case, the ring $R' = A[x,x^{-1},\sigma]$ is an Azumaya algebra with constant rank n^2 over its center.

These results may be used to construct central simple algebras and give a new simple proof of a theorem of Dickson.

P. M. COHN : The universal field of fractions of a semifir

A semifir is a ring $\neq 0$ in which every finitely generated right (or equivalently left) ideal is free, of unique rank. These rings reduce in the commutative case to Bezout domains, but in general include much more, e.g. free algebras and coproducts of fields. They have the pleasant property of always possessing a field of fractions which is universal with respect to specialization. This talk discusses the form which the elements of this universal field of fractions take, in particular, it compares the different forms for a given element, and gives some applications on the structure of centralizers of elements.

W. DICKS : Sylvester's law of nullity and subrings of skew fields

The inner rank $\rho_R(A)$ of an $m \times n$ matrix A over a ring R is the least integer r such that $A = BC$ where B is an $m \times r$ and C an $r \times n$ matrix over R. P.M. Cohn has shown that a semifir R has the following properties:

(1) R satisfies Sylvester's law of nullity :

$\rho_R(AB) \geq \rho_R(A) + \rho_R(B) - n$ where A is $m \times n$ and B is $n \times$

(1') R is a subring of a skew field F in such a way that ρ_R and ρ_F agree on matrices over R.

We show that (1) and (1') are equivalent, and that they imply

(2) R has weak global dimension at most 2 and all flat R-modules are directed unions of free submodules of unique rank.

Such rings with weak global dimension at most 1 are precisely the semifirs we started with.

For two-sided Ore domains (2) implies (1), and beyond that, little is known.

S. ELLIGER : Über die galoissche Hülle einer Erweiterung einfacher Ringe

Seien $A' \supseteq B$ einfache artinsche Ringe und der (B,B)-Modul ${}_B A'_B$ halbeinfach. Notwendig und hinreichend für die Existenz einer äußer galoisschen Erweiterung $A|B$ einfacher Ringe mit $A' \subseteq A$ und $Z(A) \subseteq Z(B) = C$ ist :

i) das Tensorproduct $T_{B\,B} = A' \otimes_B \ldots \otimes_B A'$ mit $n = (A':B)$ Faktoren ist halbeinfach, etwa $\simeq S_1^{k_1} \oplus \ldots \oplus S_t^{k_t}$, S_i einfach.

ii) $\quad {}_B A_B = S_1^{n_1} \oplus \dots \oplus S_1^{n_1}$, $n_i = \dfrac{(S_i : C)}{(\text{End } S_i : C)} \leq k_i$, hat als Endomorphismenring den Gruppenring GC , G endliche Gruppe.

iii) $\quad {}_B A_B$ besitzt in der Darstellung $A'g_1 \dots A'g_n$ (nach i) und ii) als homomorphes Bild von T) die Struktur eines G-Moduls, nämlich $\left(\prod_{i=1}^{n} a_i g_i \right) g = \prod_i a_i g_i g$ wobei $G = \cup Hg_i$ die Nebenklassenzerlegung von G nach $H = \{h \in G \text{ , } a'h = a'$ für alle $a' \in A'\}$ bedeutet.

G. J. JANUSZ : <u>The Schur subgroup of the Brauer group</u>

The Schur group of a field K is the subgroup of the Brauer group, $B(K)$, consisting of those classes which contain a K-central simple algebra which is isomorphic to a direct summand of a group algebra, $K(G)$, for some finite group G. We are mainly interested in the case in which K is an algebraic number field or the completion of an algebraic number field. In these cases the full Brauer group is described by the Hasse invariants. One is interested in giving a description of the elements of the Schur group by Hasse invariants. We shall first describe the work of Yamada which gives the Hasse invariants of elements in the Schur group of a (complete) p-adic field. This is then used to describe the elements in the Schur group of an algebraic number field which is an abelian extension of the rational field.

V. K. KHARCHENKO : Noncommutative Galois Theory

Let R be a domain, $R_{\mathcal{F}} = \varinjlim_{I \neq 0} \mathrm{Hom}(I,R)$ its left ring of quotients in the sense of Martindale and let $Q = \{q \in R_{\mathcal{F}} \mid \exists\, 0 \neq I \triangleleft R \,;\; qI, Iq \subseteq R\}$ and $\mathcal{C}$ = the center of Q (which is a field). If G is a group of automorphisms of R, then it can be continuated to Q and we denote by $B(G)$ the $\mathcal{C}$-subalgebra of Q generated by elements corresponding to inner automorphisms of Q. We say that G is regular iff any inner automorphism of R corresponding to an element of $B(G)$ belongs to G. The number $\dim_{\mathcal{C}} B(G) \cdot |G_i : G_{int}|$ is called reduced order of G which will be supposed finite.

A subring S of R is called anti-ideal if from the inclusions $sx \in S$, $0 \neq s \in S$, $x \in R$ it follows that $x \in S$.

Theorem 1. Let G be a reduced-finite regular group of automorphisms of the domain R. Then there exists a Galois correspondence between regular subgroups of the group G and the anti-ideals of the domain R, containing the ring of invariants R^G.

This theorem admits a generalization in the case of prime rings and semi-prime rings. This theorem together with the fact that $Q(F) = F$ for any non-commutative free algebra allows us to obtain an interesting corollary for free algebras.

Theorem 2. Let G be a finite group of linear automorphisms of the free algebra F. Then there exists a Galois correspondence between all the subgroups of G and all the free subalgebras containing F^G.

M. LORENZ : The heart of prime ideals in group algebras of polycyclic groups

Let K[G] be the group algebra of the polycyclic-by-finite group G . We associate to every prime ideal P in K[G] a commutative field $\mathcal{H}(P)$, called the heart of P , which is defined to be the center of the semisimple artinian ring of quotients $\mathcal{Q}(K[G]/P)$. We show that in case the field K is non absolute (i.e. not algebraic over a finite field), the primitive ideals of K[G] are precisely those prime ideals P in K[G] such that the field extension $\mathcal{H}(P)/K$ is algebraic. Using this characterization of primitive ideals one can prove the following ideal theoretic version of Clifford's classical restriction theorem : Given a primitive ideal P in K[G] and a normal subgroup N of G , then there exists a primitive ideal Q of K[N] such that $P \cap K[N] = \bigcap_{x \in G} Q^x$. Also, one can give a formula for the height of prime ideals P in group algebras of certain polycyclic-by-finite groups G involving the transcendence degree of $\mathcal{H}(P)$ over K and some group theoretic term pl(G;P) .

As to the structure of $\mathcal{H}(P)$, we give some sufficient conditions that imply the equality $\mathcal{H}(P) = \mathcal{Q}(\mathcal{Z}(K[G]/P))$ and an example showing that this equality does not hold in general. At least, $\mathcal{H}(P)$ is always a finitely generated field extension of K of transcendence degree at most h(G) , the Hirsch number of G . - The results presented in this talk were obtained in joint work with D.S. Passman.

K. MATHIAK : Valuations of ordered skew fields

A valuation ring B of a skew field K is called subinvariant if there exists a valuation ring which is contained in B and is invariant under the group of inner automorphisms of K . The value group of a valuation is linearly ordered iff the valuation ring is subinvariant. A valuation of a linearly ordered skew field is called compatible with the order of the field if the valuation ring is a convex subset of K . These valuations are subinvariant.

C. M. RINGEL : Problems on division algebras arising from the representation theory of artinian rings

Theorem: A hereditary artinian ring is of finite representation type if and only if a corresponding graph is the disjoint union of the Coxeter graphs A_n , B_n $(=C_n)$, D_n, D_6, E_7, E_8, F_4, G_2, H_3, H_4 and $I_2(p)$. In case of an algebra, the cases H_3, H_4, $I_2(p)$ $(p=5$ or $p \geq 7)$ cannot occur and it is an open problem whether these graphs can occur at all. A ring of type H_3, H_4 exists if and only if there exists a division ring F with a division subring G such that $\dim F_G = 2$, $\dim {}_G F = 3$ and $\dim {}_G\mathrm{Hom}({}_G F_F, {}_G G) = 1$.

L. RISMAN : Group rings and series : The arithmetic of division algebras

We base our study of group rings on group extensions and valuation theory. Let D be a division ring and T a group. Consider a group extension $1 \to D^* \to E \to T \to 1$, where T acts by ring automorphisms of D . The group ring of E is the ring whose basis over D is $\{ \hat{t} \mid t \in T \}$ and in which elements $d\hat{t}$ multiply as in E .

A morphism of group extensions induces a homomorphism of group rings. The group ring of E is generic with respect to certain crossed product algebras, generalizing and simplifying previous results.

For T on ordered group, the group ring of E can be embedded in a division ring of series. Valuation and ramification theory characterize the structure of subfields of this division ring. This characterization is applied in proving the non-crossed product theorems.

S. ROSSET : Group extension and division algebras

Let Γ be a virtually free abelian group, i.e. Γ is an extension $1 \longrightarrow A \longrightarrow \Gamma \longrightarrow G \longrightarrow 1$ where A is free abelian and G is finite. Assume G is faithfully represented on A. Let k be a field. Then the group ring $k\Gamma$ has a total ring of fractions $k(\Gamma)$. Denote the field of fractions of kA by L $(=k(A))$. In L^*, A can be identified as the group of monomials, G acts on L and $i : A \longrightarrow L^*$ is a G module map. We prove $i_* : H^2(G,A) \to H^2(G,L^*) \subset Br(K)$ is injective. Here $K = L^G$. It is easily seen that $k(\Gamma)$ is central simple over K and its Brauer class is $i_*\alpha$ where $\alpha \in H^2(G,A)$ represents Γ. Thus the order of $[k(\Gamma)]$ in BrK equals that of α.

Theorem 1. Given a finite group G, $|G| = n$, an integer m such that $m|n$ and m,n have the same prime factors, there is a (unirational!) extension K of k and a division algebra over K of dimension n^2 of order m (in BrK).

By the above this results from

Theorem 2. Given G and m as above, there exists a G module A and $\alpha \in H^2(G,A)$ such that (a) A is free abelian (f.g.), (b) α represents a torsion free extension, (c) G acts faithfully on A, (d) order $(\alpha) = m$.

M. SCHACHER : Brauer groups of function fields

We are concerned with the following question: which abelian torsion groups can arise as the Brauer group $B(F)$ of some field F? One does not know, for example, whether the cyclic group of order 3 is the Brauer group of some field. A conjecture of Auslander-Brumer says that if $B(F)$ has non-zero elements of order p, $2 \neq p$ a prime, then $B(F)$ contains a p-divisible subgroup; this would say a finite Brauer group must be a 2-group.

In this talk we report on some joint work with B.Fein on these questions. We prove:

Theorem 1. Let G be a countable torsion group with $2G$ divisible. Then $G \cong B(F)$ for some field F algebraic over the rational field Q.

Theorem 2. If k is a global field of characteristic $q \neq p$, p a prime, and $t_1, \ldots, t_n$ are indeterminates over k, then $B(k(t_1,\ldots,t_n))$ contains reduced elements x of order p which have infinite height (the equation $p^n y_n = x$ has solutions y_n for all n). We determine the Ulm length of these groups, and partial information about the Ulm invariants.

Finally, we show any divisible torsion group whose p-rank is infinite for all primes p does arise as the Brauer group of some field.

L. W. SMALL : Polynomials over division rings

The following theorems were discussed :

Theorem 1. If D is a division ring and $D[x_1,...,x_n] = R$, polynomials in commutating variables, then simple R-modules are finite-dimensional over D .

Theorem 2. R and D as in Theorem 1. R is primitive, if and only if $M_t(D)$ contains a subfield of transcendence degree $> n$ (over the center of D).

Examples are given of division rings, $D^{(i)}$, such that $D^{(i)}[x_1,...,x_i]$ is primitive, but $D^{(i)}[x_1,...,x_u]$, $u > i$, is not primitive. - The above is joint work with S.A.Amitsur.

M. Lorenz (Essen)

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