MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 23/1979

Diophantische Approximationen

27.5. bis 2.6.1979

Unter der Leitung von Herrn Professor Dr. Th. Schneider fand in der Woche vom 27.5. bis 2.6.1979 wieder die schon zur Tradition gewordene Tagung über diophantische Approximationen im Mathematischen Forschungsinstitut Oberwolfach statt. Zahlreiche Teilnehmer aus dem In- und Ausland konnten sich in insgesamt 41 Vorträgen über die neuesten Forschungsergebnisse im Gebiet der diophantischen Approximationen, der transzendenten Zahlen, der Geometrie der Zahlen und der Gleichverteilungstheorie informieren. Daneben kam ein reger Gedankenaustausch in zahlreichen persönlichen Gesprächen zustande, was nicht zuletzt auch der angenehmen Oberwolfacher Atmosphäre zu verdanken ist.

Vortragsauszüge

K. ALLADI:

On certain irrational values of the logarithm

Motivated by the ideas of R. Apéry, F. Beukers recently produced simpler proofs of the irrationality of $\zeta(2)$ and $\zeta(3)$, making use of integrals in two and three variables involving Legendre Polynomials. We investigate completely the one dimensional analogue of Beuker's method. This leads to the irrationality of the logarithm at certain arguments (involving the reciprocals of algebraic integers in an Imaginary Quadratic Field) yielding measures of irrationality for $\log(1+\frac{1}{m})$, $m \in \mathbb{Z}^+$, $\log(1+\frac{p}{q})$ for certain integers p and q, and $\pi/\sqrt{3}$. Recently Bombieri by the use of differential equations proved and measured the irrationality of $\log(1+\frac{1}{m})$ and $\sqrt[k]{m}$, $k, m \in \mathbb{Z}^+$. Our approach is related to his in some respects.

H. BAUERMEISTER:

Asymptotische Werteverteilung von Zetafunktionen

Am Beispiel der Dedekindschen Zetafunktion ζ_K wird dargelegt, wie man die Bohrsche Methode zur Berechnung des Wertevorrates der Riemannschen Zetafunktion verallgemeinern kann: Sei $K \subset \overline{\mathbb{Q}}$ mit $2 \leq n = \text{Grad } [K:\mathbb{Q}] < \infty$. Für $\sigma = \text{Rea } s > 1$ gilt: $\zeta_K(s) = \prod_{l=1}^{\infty} (1 - \mathfrak{N}(\mathfrak{p}_l)^{-s})^{-1}$ ($(\mathfrak{p}_l)_{l\in\mathbb{N}}$ Abzählung der Primideale in K). Der quadratische Mittelwert

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |\zeta_K(\sigma+it)|^2 dt$$ ist für $\sigma > 1 - \frac{1}{n}$ endlich (Chandrasekharan, Narasimhan, 1963). Hieraus läßt sich u. a. folgern, daß in $\{ s \mid 1 - \frac{1}{n} < \sigma < 1 \}$ die Summe der Logarithmen der Faktoren des Produkts von ζ_K "stochastisch" gegen einen geeignet definierten $\log \zeta_K$ konvergiert. Zusammen mit einer Wiederkehreigenschaft, die aus der (B^2)-Fastperiodizität von ζ_K folgt, läßt sich die Berechnung des Wertevorrates von $\log \zeta_K$ auf die Auswertung der

Summen $-\sum_{l=1}^{m} c_l \log(1 - p_l^{-(\sigma+it)})$, $m \in \mathbb{N}$, $(p_l)_{l\in\mathbb{N}}$ Folge der nat. Primzahlen,

c_1 gewisse nichtnegative ganze Zahlen, zurückführen. Diese gelingt aber mit der Bohrschen Anwendung des Kroneckerschen Approximationssatzes. Als Resultat bekommt man z. B. :

Satz. $1 - \frac{1}{n} < \sigma < 1 \Rightarrow \overline{\zeta_K(\sigma + iR)} = \mathbb{C}$.

Quantitative Auswertungen dieser Rechnungen ergeben eine asymptotische Verteilungsfunktion für die Werte von ζ_K.

Es läßt sich ein Beispiel angeben mit derselben asymptotischen Verteilungsfunktion wie ζ_K (auch für $K = \mathbb{Q}$), für das die Riemannsche Vermutung falsch ist.

D. BERTRAND

On some transcendence prperties related to Hilbert-Blumenthal Abelian varieties

Let A be a Hilbert-Blumenthal Abelian variety, i.e. an Abelian variety whose algebra of endomorphisms contains a totally real number field of degree dim A. Assume further that A is simple and defined over a number field, and denote by φ (resp. φ_p) a strongly normalized representation of the complex (resp. p-adic) exponential map on A.

Theorem. Each coordinate of a non zero algebraic point of φ (resp. φ_p) is transcendental.

In the complex case, the proof is an easy generalization of the method elaborated by Waldschmidt to deal with the C.-M. analogue of the theorem. "Safe multiplication formulae" in the sense of Masser are needed to treat the p-adic case.

In the complex case, the result applies to periods of φ, and yields the transcendency of the periods of cusp forms of weight two, for certain levels.

F. BEUKERS

Irrationality measures

The use of Padé-type approximations for the derivation of irrationality measures of systems of numbers $\vartheta_1, \dots, \vartheta_k$ is originally due to C. L. Siegel. Take for instance a set of analytic functions $f_1, \dots, f_k$. A Padé-type approximation is a set of polynomials $p_1, \dots, p_k$ of degree $\leq n$ such that $p_1(z)f_1(z) + \dots + p_k(z)f_k(z) = z^{kn}E(z)$ where $E(z)$ is some analytic function. By substituting small rational values a/b for z we obtain good approximations to $f_1(a/b), \dots, f_k(a/b)$ which can be used to obtain irrationality measures. It is possible to give explicit Padé-type approximations to the system

$1,\ F\left({a_1 \dots a_p \atop b_1+1 \dots b_q} \middle| z\right), \dots, F\left({a_1 \dots a_p \atop b_1 \dots b_q+1} \middle| z\right)$ of generalized hypergeometric functions. In particular one can obtain explicit Padé-type approximations to

$1,\ L_1(z), \dots, L_k(z)$, where $L_r(z) = \sum_{m=0}^{\infty} \frac{z^m}{(m+1)^2}$. An application of these approximations to the case $k = 2$ is

Theorem. Let $m \in \mathbf{Z}$, $|m| > 300$. Then

$$\left| x + yL_1\left(\frac{1}{m}\right) + zL_2\left(\frac{1}{m}\right) \right| > c\, X^{-2\frac{\log 400|m|}{\log(|m|/300)}}$$

where c is some explicit constant and $x, y, z \in \mathbf{Z}$ and $X = \max(|x|, |y|, |z|) > 0$.

A. BIJLSMA

An elliptic analogue of the Franklin-Schneider theorem

Notation: $\wp$ is a Weierstraß elliptic function with algebraic invariants. If $\wp$ possesses complex multiplication, K is the imaginary quadratic field of complex multiplication; otherwise, $K = \mathbb{Q}$.

Theorem 1. Suppose $a, b \in \mathbb{C}$ such that a and ab are not poles of $\wp$. Then there exists an effectively computable $c \in \mathbb{R}$, depending only on $\wp$, a, and b, such that only finitely many triples $(\alpha, \beta, \gamma) \in \mathbb{C}^3$ satisfy $\wp(\alpha)$, β, $\wp(\gamma)$ algebraic,

$\beta \notin K$ and

$$\max(|\wp(a)-\wp(\alpha)|, |b-\beta|, |\wp(ab)-\wp(\gamma)|) < \exp(-D^6 \log^6 H \log_2^{-5} H)$$

while $[\mathbb{Q}(\wp(\alpha), \beta, \wp(\gamma)): \mathbb{Q}] \leq D$ and $\max(e^e, H(\wp(\alpha)), H(\beta), H(\wp(\gamma))) \leq H$.

Theorem 2. For every function $g: \mathbb{N}^2 \to \mathbb{R}$ there exist $a \in \mathbb{C}$, $b \in \mathbb{C}\backslash K$, such that a and ab are not poles of $\wp$ and such that for every $c \in \mathbb{R}$ there exist infinitely many triples $(\alpha, \beta, \gamma) \in \mathbb{C}^3$ satisfying $\wp(\alpha)$, β, $\wp(\gamma)$ algebraic and $\max(|\wp(a)-\wp(\alpha)|, |b-\beta|, |\wp(ab)-\wp(\gamma)|) < \exp(-cg(D, H))$ while $[\mathbb{Q}(\wp(\alpha), \beta, \wp(\gamma)):\mathbb{Q}] \leq D$ and $\max(H(\wp(\alpha)), H(\beta), H(\wp(\gamma))) \leq H$.

W. D. BROWNAWELL

On the zeros of certain functions

This is a report on joint work with D. W. Masser on the zeors of functions of the form $P(f_1(z), \dots, f_n(z))$, where P is a polynomial of degree at most D and the f_i, $1 \leq i \leq n$ form a solution of the system of algebraic differential equations

$$f_i' = F_i(f_1, \dots, f_n), \quad 1 \leq i \leq n,$$

where the F_i are polynomials. Nesterenko has obtained the first useful general results for linear F_i.

If $\vec{f}_i$ is the solution of the above differential equation corresponding to the initial condition $\vec{\vartheta}_i = (\vartheta_{i1}, \dots, \vartheta_{in}) \in \mathbb{C}^n$ for distinct $\vec{\vartheta}_o, \dots, \vec{\vartheta}_N$ and no $P(\vec{f}_i) \equiv 0$, then

$$\sum_{j=0}^{N} \operatorname{ord}_{z=o} P(\vec{f}_j) \leq (dD)^{2^{n-1}} + N(dD)^{2^{n-2}},$$

where $\deg F_i \leq d$.

A generalization to polynomial ideals was also obtained. If $P_1, \dots, P_K$ generate a polynomial ideal of rank r, $\deg P_k \leq D$, $1 \leq k \leq K$, then in particular

$$\sum_{j=0}^{N} \min_{k=1,\dots,K} \operatorname{ord}_{z=o} P_k(\vec{f}_j) \leq (dD^r)^{2^{n-r}} + N(dD^r)^{2^{n-r-1}}.$$

The proof is from commutative algebra: Cohen-Macauley Theorem, localizations, Hilbert polynomial and Bézout's Theorem for unmixed ideals are the main ingredients.

There are various applications to transcendence theory. An application to abelian functions was obtained by Flicker and Masser, whereas we now have a quantitative form of the general Schneider-Lang theorem.

P. BUNDSCHUH

Einige Bemerkungen über Transzendenz

I. Danilov (1971), Davison (1977) sowie Adams und Davison (1977) haben reelle Zahlen gefunden, für die gleichzeitig die g-adische Entwicklung und die Entwicklung in einen regulären Kettenbruch explizit angegeben werden kann. Diesbezüglich soll hier ein umfassendes Resultat gezeigt werden.

II. Mit Hilfe der Bakerschen Theorie zeigen wir hier: Ist $A := (a_m)$ eine reinperiodische Folge algebraischer Zahlen, $f(s;A) := \sum_{m=1}^{\infty} a_m m^{-s}$ (Re s > 1) und $F(z;A) := \sum_{n=2}^{\infty} f(n;A) z^n$, so ist $F(\frac{h}{k};A)$ entweder 0 oder transzendent für jedes $(h,k) \in \mathbb{N}^2$ mit $h < k$. Einige Korollare hierzu werden angegeben, u.a. betreffend die Eulersche Konstante und die logarithmische Ableitung der Gammafunktion.

III. Bezeichnet $\gamma(n)$ für natürliche $n \geq 2$ die Anzahl der $(a,b) \in \mathbb{N}^2$ mit $a^b = n$, so wird hier $\sum_{n=2}^{\infty} \gamma(n) n^{-s}$ für natürliche $s \geq 2$ arithmetisch untersucht.

D. CANTOR

An Extension of the Definition of Transfinite Diameter

Let K be a number field and suppose $\chi = \{x_1, x_2, \ldots, x_q\}$ is a finite subset of $\tilde{K} \cup \{\infty\}$ satisfying $\sigma\chi = \chi$ for all $\sigma \in \mathrm{Aut}(\tilde{K}/K)$ $(\sigma\infty = \infty)$. Choose a set $E_v \subset \tilde{K}_v \cup \{\infty\}$ disjoint from χ satisfying $\sigma E_v = E_v$ for all $\sigma \in \mathrm{Aut}(\tilde{K}_v/K_v)$, v a completion of K. Other conditions, too lengthy to be stated here are also required. We require that $E_v = \{x \in \tilde{K}_v \cup \{\infty\};\ |x - x_i|_v \geq 1,\ x_i \in \chi\}$ (and $|x|_v < 1$ if $\infty \in \chi$) for almost all v. We define generalized Green's functions on the complement of E_v and define $\gamma_{ijv} = G_v(x_i, x_j)$ if $i \neq j$ and $\gamma_{iiv} = \lim_{y \to x_i} G_v(x_i, y) + \log(x_i - y)$ otherwise. Put $\Gamma_v = (\gamma_{ijv})$ and $\gamma_{ij} = \sum_v \gamma_{ijv}$. Almost all $\Gamma_v = 0$ and the value k of the matrix game $\Gamma = (\gamma_{ij}) = \sum_v \Gamma_v$ is the generalized Robbin's constant. The extended transfinite diameter is exp(-k).

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

In terms of this extended definition, new versions of the Polya-Carlson Theorem, Distribution of Algebraic Integers Theorems, and Polynomial Approximation Theorems hold.

G. V. CHUDNOVSKY

Transcendental number theory and methods of mathematical physics

We develop effective methods of construction of auxiliary functions $F(z) = P(z, f_1(z), \dots, f_n(z))$ in the Siegel-Gelfond-Schneider method for a) meromorphic functions $f_1, \dots, f_n$ satisfying algebraic differential equations and algebraic laws of addition, b) E-functions $f_1, \dots, f_n$ or c) G-functions, satisfying linear differential equations. These methods are based in the cases a) and b) on the effective (N-point) Padé approximation obtained using isospectral transformations. For classical E- or G-functions satisfying linear differential equations of the second order the construction of auxiliary functions involved classical orthogonal polynomials. In general for b), c) the construction of effective approximations is based on deformation equations for Fuchsian equations with fixed monodromy group. The new measure of irrationality and transcendency is proved for numbers $f_i(\alpha)$, $\alpha \in \bar{\mathbb{Q}}$, $\alpha \neq 0$ for $|\alpha|$ close to the origin in cases b) and c) .

P. L. CIJSOUW

Multiplicative dependence relations

Let $\alpha_1, \dots, \alpha_n$ be nonzero algebraic numbers which are multiplicatively dependent by a relation $\alpha_1^{b_1} \cdots \alpha_n^{b_n} = 1$, $b_1, \dots, b_n \in \mathbb{Z}$, $(b_1, \dots, b_n) = 1$. Then , for a suitable choice of the branches of the logarithms, $\log \alpha_1, \dots, \log \alpha_n$ are linearly dependent. Taking the conjugations $\sigma_1, \dots, \sigma_D$, $D = [\mathbb{Q}(\alpha_1, \dots, \alpha_n) : \mathbb{Q}]$, we even can see that there are values $l_{\mu\nu}(\sigma_\mu(\alpha_\nu))$ of the logarithms of $\sigma_\mu(\alpha_\nu)$ ($\mu = 1, \dots, D$; $\nu = 1, \dots, n$) such that

$$b_1 l_{\mu 1}(\sigma_\mu(\alpha_1)) + \dots + b_n l_{\mu n}(\sigma_\mu(\alpha_n)) = 0 \quad (\mu = 1, \dots, D) .$$

By using Baker's method, Bijlsma and myself could prove in a joint work that there exists a dependence relation $\alpha_1^{r_1} \cdots \alpha_n^{r_n} = 1$, $r_1, \dots, r_n \in \mathbb{Z}$, not all zero, in which $\max_\nu |r_\nu|$ is bounded by an expression in the heights of $\alpha_1, \dots, \alpha_n$ and $\max_{\mu,\nu} | l_{\mu\nu}(\sigma_\mu(\alpha_\nu)) |$. In this expression, the degree D does not occur.

T. W. CUSICK

The two and three dimensional Diophantine approximation constants

Define $c(\alpha, \beta)$ to be the infimum of those $c > 0$ such that $|x+\alpha y+\beta z| \max(y^2, z^2) < c$ has infinitely many solutions in integers x, y, z with x and y not both zero. Then the two dimensional Diophantine approximation constant c_2 is $\sup c(\alpha, \beta)$, where the supremum is taken over all real pairs α, β. The three dimensional constant c_3 is defined analogously. It was shown by Furtwängler in 1926 that $c_2 \geq 1/\sqrt{23}$ and $c_3 \geq 1/\sqrt{275}$. The lower bound for c_2 was improved to 2/7 by Cassels in 1952. We give an argument (but not a proof) for the equality $c_2 = 2/7$ and a proof via the geometry of numbers that $c_3 \geq 2/\sqrt{275}$.

K. GYÖRY

On the greatest prime factors of decomposable forms at integer points

As a common generalization of some earlier effective results we proved the following (for simplicity we state here only a special case of the result)

Theorem. Let $F \in \mathbb{Z}[x_1, \ldots, x_m]$ be a decomposable form in $m \geq 2$ variables with splitting field G over $\mathbb{Q}$ and let $[G:\mathbb{Q}] = g$. Suppose that the linear factors in the factorization of F satisfy two conditions (which will be detailed in my lecture) and let $d \geq 1$. Then there exists an effectively computable positive number $X = X(F, d)$ such that

(1) $\quad (13g+1)s\log(s+1) + (g+1)\log P > \log\log|\bar{x}|, \quad |\bar{x}| = \max_i(|x_i|),$

and

(2) $$P > (13g+1)^{-1}\log\log|\bar{x}|$$

for any $\bar{x} \in \mathbb{Z}^m$ with $|(x_1, \ldots, x_m)| \leq d$ and $|\bar{x}| \geq X$, where s and P denote the number of distinct prime factors and the greatest prime factor of $F(\bar{x})$ respectively.

The following classes of forms $F \in \mathbb{Z}[x_1, \ldots, x_m]$ satisfy all the conditions of our above theorem, and so for these forms F (1) and (2) hold:

I) F is a binary form and among the linear factors in the factorization of F at least three are distinct;

II) $F(\bar{x}) = a_0 \mathrm{Norm}_{K/\mathbb{Q}}(x_1 + \alpha_2 x_2 + \ldots + \alpha_m x_m)$, $\deg \alpha_i = n_i \geq 3$ over $\mathbb{Q}$ and

$n_2 \ldots n_m = [K:\mathbb{Q}]$;

III) $F(\bar{x}) = \mathrm{Discr}_{K/\mathbb{Q}}(\alpha_2 x_2 + \ldots + \alpha_m x_m)$, $K = \mathbb{Q}(\alpha_2, \ldots, \alpha_m)$ and $1, \alpha_2, \ldots, \alpha_m$ linearly independent over $\mathbb{Q}$;

IV) $F(\bar{x})$ is the index form of an integral basis of an order of a number field.

We established our estimates (1) and (2) for forms $F \in \mathbb{Z}_L[x_1, \ldots, x_m]$ at integer points $x \in \mathbb{Z}_L^m$, where $\mathbb{Z}_L$ denotes the ring of integers of an arbitrary but fixed algebraic number field L.

G. HELMBERG

On a theorem of Kamae on completely uniformly distributed subsequences

In the paper of Kamae "Subsequences of normal sequences" (Israel J. Math, 16 (1973), 121 - 149) the assertions

(1) Any $\alpha \in \mathrm{Nor}_p$ is a τ-collective

(2) $\mathrm{Nor}_p \circ \tau \subset \mathrm{Nor}_p$

(3) $\mathrm{Nor}_p \circ \tau = \mathrm{Nor}_p$

under the additional hypothesis

(0) $\overline{\lim\limits_{n\to\infty}}\, \tau(n)/n < \infty$

are shown to be equivalent to

(4) ϑ_τ is completely deterministic.

It is shown that (1) (and therefore also (2) and (3)) already imply (0); thus (1), (2), (3) each are equivalent to the conjunction of (4) and (0). Although unpublished as yet, however, the priority of this observation belongs to Kamae too.

E. HLAWKA

Über unendliche Reihen, die von Irrationalzahlen abhängen

Es werden Reihen von der Gestalt $\sum\limits_{h,k} \frac{\varepsilon(h)}{|h|^r}(w+2\pi i(h\alpha+k))^{-j}$ untersucht, deren Konvergenz vom Typus der Irrationalzahl α abhängt. Damit werden die von Hecke et alii eingeführten Dirichletreihen

$$\sum_l \frac{P_r(l\alpha)}{l^s}$$

(P_r: r-te Bernoullische Funktion) behandelt. Verallgemeinerungen werden besprochen.

L.-Ch. KAPPE

Commutators and some diophantine equations

Let $K(G) = \{[x, y] = x^{-1}y^{-1}xy;\ x, y \in G\}$ be the set of commutators and G' the commutator subgroup of a group G. It is well known that $G' = K(G)$ is not necessarily true. In the case G' abelian, G' and G/G' finitely generated this question can be translated into a question about the solvability of diophantine equations or congruences respectively. The method has been used successfully in the case of cyclic G' and elementary abelian G', e.g. Rodney (1974) used the method to exhibit sufficient conditions such that $G' = \langle [a, b] \rangle$ implies $G' = K(G)$ and he gave an example to show that the condition is not true in general. The author is currently investigating the case of cyclic G' with an elementary abelian automorphism group acting on G'. Macdonald (1961) and Liebeck(1974) have used the method in the case of p-groups of class 2 and G' elementary abelian of small rank. The author in addition in 1978 has investigated a generalized question about higher commutators and used variations of the method.

S. V. KOTOV

Effektive Analysis einer Klasse von diophantischen Gleichungen des Typs Normform

Seien $\alpha_1, \ldots, \alpha_m$ ganze algebraische Zahlen, sei $K_i = \mathbb{Q}(\alpha_1, \ldots, \alpha_i)$, $K_o = \mathbb{Q}$, und gelte $[K_i : K_{i-1}] \geq 3$ ($1 \leq i \leq m$). Sei $A \neq 0$ eine ganze rationale Zahl. Wir betrachten diophantische Gleichungen der Form

(1) $$\mathrm{Norm}(x_o + \alpha_1 x_1 + \ldots + \alpha_m x_m) = A$$

in den Unbekannten $x_o, \ldots, x_m \in \mathbb{Z}$.

Satz. Für die Gesamtheit aller ganzen Lösungen $x_o, \ldots, x_m$ der Gleichung (1) existiert eine effektiv berechenbare Größe σ, die nur von $\alpha_1, \ldots, \alpha_m$, m und A abhängt, so daß

$$\max(|x_o|, \ldots, |x_m|) \leq \sigma$$

gilt.

L. KUIPERS

Ring of integral quaternions - uniform distribution

In the ring J_o of integral quaternions $\{h_o + h_1 i + h_2 j + h_3 k \mid h_o, h_1, h_2, h_3 \in \mathbb{Z}\}$ one can define the notion of uniform distribution modulo a nontrivial ideal (μ) generated by some $\mu \in J_o$ with $N(\mu) \geq 2$.

A description of a complete residue system modulo μ can be given. The cardinal number of such a set is $\{N(\mu)\}^2$.

A set of characters of the group $J_o/(\mu)$ can be constructed.

Relations between uniform distribution in $\mathbb{Z}$, in $\mathbb{Z}[i]$ and in J_o resp. can be established. One has: a sequence of integral quaternions $(x_n + y_n i + z_n j + u_n k)$ is u.d. in J_o (modulo each (μ)) if and only if the sequence (x_n, y_n, z_n, u_n) is u.d. in $\mathbb{Z}^4$.

Example: If the real numbers $1, \vartheta_1, \ldots, \vartheta_4$ are linearly independent over the rationals, then the sequence $([n\vartheta_1] + [n\vartheta_2]i + [n\vartheta_3]j + [n\vartheta_4]k)$ is u.d. in J_o.

M. LAURENT

Transcendence of periods of elliptic integrals

The third of the 8 problems in Schneider's book on transcendental numbers asks for transcendence results on elliptic integrals of the third kind. The following answer has been recently found:

Theorem 1. Let ξ be an elliptic differential form defined over $\overline{\mathbb{Q}}$ with rational residues. Then the nonzero periods of ξ are transcendental.

When ξ has no residue, the result was known (Schneider) ; in this case ξ is of the first or second kind, and is associated with the extension of an elliptic curve by the additive group. When ξ has nonzero residues, it corresponds to an extension of an elliptic curve by the multiplicative group. The exponential map of such a group can be described (Severi, Serre) by means of a function $f_u(z) = \frac{\sigma(z-u)}{\sigma(z)\sigma(u)} e^{z\zeta(u)}$ which satisfies $f_u(z+w) = f_u(z)\exp(w\zeta(u) - \eta u)$. The proof of theorem 1 depends on the following

Theorem 2. Let $\wp$ be an elliptic function with algebraic invariants g_2, g_3; let u be a complex number such that no integral multiple of u is a pole of $\wp$

and $\wp(u)$ is algebraic. Let ω be a nonzero period of $\wp$, and η the corresponding quasi-period of ζ. Then the four numbers $1, \omega, \eta, \omega\zeta(u) - \eta u$ are $\overline{\mathbb{Q}}$-linearly independent.

D. W. MASSER

Mahler's method for algebraic independence

Mahler's method for transcendence and algebraic independence has been considerably extended in recent years by K. K. Kubota, J. Loxton, and A. J. van der Poorten. We describe a further extension of the method, illustrating our arguments by reference to Hecke's function $f(z) = \sum_{r=1}^{\infty} [r\omega] z^r$ for a real quadratic irrationality ω. Let U denote the set of complex numbers z with $0 < |z| < 1$. In 1929 Mahler showed that $f(\alpha)$ is transcendental for any algebraic α in U, and in 1977 Loxton and van der Poorten proved that $f(\alpha_1), \dots, f(\alpha_n)$ are algebraically independent for any algebraic $\alpha_1, \dots, \alpha_n$ in U with $|\alpha_1|, \dots, |\alpha_n|$ multiplicatively independent. It can now be shown that $f(\alpha_1), \dots, f(\alpha_n)$ are algebraically independent for any distinct algebraic $\alpha_1, \dots, \alpha_n$ in U.

M. MENDES FRANCE

A propos of the Morse sequence (joint work with Christol, Kamae, Rauzy and myself)

We establish the following result:

Theorem. Let Σ be a finite set and let $f = (f_n) \in \Sigma^{\mathbb{N}}$. The three following properties are equivalent:

(i) There exists a finite field of characteristic p and a map $\alpha: \Sigma \hookrightarrow K$ such that $(\alpha(f_n))$ is algebraic over $K(X)$ ($K^{\mathbb{N}}$ is identified with a subset of $K((X))$ in the canonical way).

(ii) f is the image of a fixed point of a p-substitution.

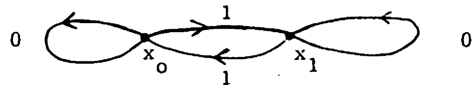
(iii) f is generated by a p-automaton.

For example, the Morse sequence is generated by any one of the following methods

(i) Let K = {0, 1}. f verifies $(1+X)^3 f^2 + (1+X)^2 f + X = 0$.

(ii) f is invariant under the 2-substitution $0 \to 01$, $1 \to 10$.

(iii) f is generated by the 2-automaton



Using Cobham's result, we then deduce:

Corollary. Let K and K′ be two finite fields of different characteristics. Let $f = \sum_{n=0}^{\infty} f_n X^n$ and $f' = \sum_{n=0}^{\infty} f_n X^n$ where f is considered to be an element of K((X)) and f′ an element of K′((X)) (same f_n's). Then, if f is algebraic over K(X) and f′ is algebraic over K′(X), we conclude that f and f′ are both rational.

Hence the conjecture: Let $f = \sum_{n=0}^{\infty} f_n X^n$ an irrational algebraic element of K((X)). To show that $\sum_{n=0}^{\infty} \frac{f_n}{2^n}$ is transcendental (in $\mathbb{R}$). The result is actually known if f is the Morse sequence (see Mahler, Loxton, Van der Poorten, K. Kubota, Dekking, . . .).

M. MIGNOTTE

Approximation of algebraic numbers by algebraic numbers of high degree

The measure $M(\gamma)$ of an algebraic integer is defined by $M(\gamma) = \prod_{\sigma} \max(1, |\sigma(\gamma)|)$, where $\sigma(\gamma)$ runs over the conjugates of γ.

One corallary of our main result is the following:

If $M(\alpha) \leq 2$ and $\alpha \neq 1$ is an algebraic integer of degree D, then

$$|\alpha - 1| > \exp(-4\sqrt{D}\log(4D)) .$$

The proof is elementary and uses the construction of an appropriate auxiliary function.

B. NOVAK

Elementary methods on transcendental number theory

In 1975, the author proved in a very simple way that for any nonzero s, at most one of the numbers s and e^s is a Gaussian number. This result can be generalized to some fields $\mathbb{Q}(\eta)$ or to other functions (e.g. Inkeri 1976). Very simple proofs of the transcendence of the numbers e^α (α real, algebraic) and π can be obtained from a simple lemma (see Nesterenko, Comm. Math. Univ. Carolinae 1979, No. 2).

Z. PAPP

Explicit lower bounds for linear forms with algebraic coefficients

Using the Gelfond-Baker method we obtained some effective theorems for norm form equations. These theorems enable us, to establish some effective results for linear forms with algebraic coefficients. Here we mention only a Corollary of a general theorem.

Let $\alpha_1, \dots, \alpha_k$ be algebraic numbers with heights $\leq H$ and with degrees at least 3 such that the degree n of $K = \mathbb{Q}(\alpha_1, \dots, \alpha_k)$ is $\deg\alpha_1 \dots \deg\alpha_k$. Then we have for any rational integral k-tuple $(x_1, \dots, x_k)$ with $|x| = \max_i (|x_i|) > 0$

$$\| \alpha_1 x_1 + \dots + \alpha_k x_k \| > c\, |x|^{-\frac{n-\sigma-\tau}{\sigma}},$$

where $c = (2kH)^{(-2n+2\sigma-kn-1+\tau)/\sigma} \exp(-R_K - 6n)$, $\tau = (c' R_K (\log R_K^*)^2)^{-1}$, $c' = (25(r+3)n)^{15(r+3)}$, $\sigma = 1$ or $\sigma = 2$ according as K is real or not and R_K, r denote the regulator and rank of K respectively.

In particular, if k = 1, this Corollary implies the effective sharpening of Liouville's Theorem due to Feldman, with explicitly computed constants. These results were obtained in joint work with K. Győry.

A. PETHÖ

Über die nichtlineare Diophantische Approximation von quadratischen algebraischen Zahlen

Bezeichne $\| z \|$ den Abstand der reellen Zahl z von der nächsten ganzen Zahl. Unter Verwendung eines Satzes von V. G. Sprindzuk (Acta Arith. 30, (1976), 95 - 108) bewies ich:

Satz 1. Seien α eine quadratische und $\varepsilon > 0$ eine beliebige reelle Zahl. Dann existiert eine effektiv berechenbare Konstante $c_1(\alpha, \varepsilon) > 0$, so daß

$$\| q^2 \alpha \| > c_1(\alpha, \varepsilon)(\log |q|)^{t-\varepsilon} |q|^{-2}$$

gilt für jede ganze Zahl q mit $|q| > 1$, wo $t = 1/3456$.

Satz 2. Seien α, ε, t wie in Satz 1 und $\{q_n\}$ die Folge der Nenner der Näherungsbrüche von α. Sei weiter $q_n = s_n^2 Q_n$, wo Q_n eine quadratfreie ganze Zahl ist. Dann existiert eine effektiv berechenbare Konstante $c_2(\alpha, \varepsilon) > 0$, so daß

$$Q_n > n^{t/2 - \varepsilon}$$

erfüllt ist, wenn $n > c_2(\alpha, \varepsilon)$.

A. REICH

Universal functions in number theory

Let ζ_K denote the Dedekind Zetafunction of an algebraic number field K. It is shown that for suitable $\sigma_K < 1$ the function ζ_K possesses the following universal property:

Theorem. For every disk D lying in the strip $\sigma_K < \operatorname{Re} s < 1$, every function f, holomorphic in D without zeros, every $\varepsilon > 0$ there exists $m \in \mathbb{N}$ such that

$$\sup_{s \in D} | f(s) - \zeta_K(s+im) | < \varepsilon .$$

In the case of the Riemann Zetafunction one can choose $\sigma_{\mathbb{Q}} = \frac{1}{2}$.
From this universality one has immediately

Corollary 1. If $\sigma_K < \operatorname{Re} s < 1$, then
(i) $\{ \zeta_K(s+im) : m \in \mathbb{N} \}$ is dense in $\mathbb{C}$,
(ii) $\{ \zeta_K(s+im), \zeta_K'(s+im), \ldots, \zeta_K^{(n-1)}(s+im) : m \in \mathbb{N} \}$ is dense in $\mathbb{C}^n$ (for all $n \in \mathbb{N}$).

Corollary 2. $y = \zeta_K(s)$ does not satisfy any (non-trivial) differential equation of the form $\sum_{\mu=0}^{M} s^{\mu} F_{\mu}(y, y', \ldots, y^{(\mu-1)}) = 0$ with continous F_{μ}.

(Remark: According to Hilbert-Ostrowski 1900/1920 it is well known, that such a differential equation is impossible, if the F_{μ} are polynomials.)

A. SCHINZEL

Continued fractions for some transcendental numbers

Miroslav Kurošek, student of the lecturer, proved the following

Theorem. If $\delta_i = \pm 1$, ε_i are positive integers ($i = 0, 1, \ldots$), $\varepsilon_0 \geq 3$, $\sup \varepsilon_i < \infty$, $Q_0 = 1$, $Q_i = \varepsilon_i Q_{i-1}^2$ then the transcendental number

$$\sum_{i=1}^{\infty} \frac{\delta_i}{Q_i}$$

has bounded partial quotients.

A proof is outlined for the special case of this theorem, when $\delta_i = 1$, $Q_i = m^{2^{i-1}}$ ($i = 1, 2, \ldots$), $m \geq 3$.

H. P. SCHLICKEWEI

Small values of diagonal forms in many variables

Applying a recent result on zeros of diagonal forms with integral coefficients by W. M. Schmidt we obtain the following

Theorem. Let $k > 2$ be an integer and $\epsilon > 0$. Then there exists a constant $c_1(k, \epsilon)$, such that if $s > c_1$ and $\lambda_1, \dots, \lambda_s, \mu_1, \dots, \mu_s$ are any positive real numbers, then for any $N > 1$ the inequality

$$(*) \qquad |\lambda_1 x_1^k + \dots + \lambda_s x_s^k - \mu_1 y_1^k - \dots - \mu_s y_s^k| < M N^{-k+\epsilon}$$

has a solution in nonnegative integers $x_1, \dots, x_s, y_1, \dots, y_s$ satisfying $0 < \max(x_1, \dots, x_s, y_1, \dots, y_s) < N$, where $M = \max(\lambda_1, \dots, \lambda_s, \mu_1, \dots, \mu_s)$.

This theorem implies the forementioned result by W. M. Schmidt. The exponent $-k+\epsilon$ in (*) is essentially best possible, in the sense that it may not be replaced by any constant $c_2 < -k$.

A. L. SCHMIDT

Diophantine approximation in the Eisensteinian field

Let $K = \mathbb{Q}(\sqrt{-3})$, $\mathcal{O} = \mathbb{Z} + \mathbb{Z}\omega$, $\omega = \frac{1+\sqrt{-3}}{2}$. The approximation constant $C(\xi)$ for $\xi \in \mathbb{C}\setminus K$ is defined as $C(\xi) = \limsup_{p,q\in\mathcal{O},q\neq 0} (|q|\,|q\xi-p|)^{-1}$, and the set of all approximation constants is called the approximation spectrum. A complete description of all approximation constants less than the smallest limit point of the approximation spectrum was given, thus completing earlier work by Perron (1931) and Poitou (1953). For similar results in the fields $\mathbb{Q}(i)$ and $\mathbb{Q}(i\sqrt{11})$ see Acta Math. (1975), J. Number Th. (1978).

W. M. SCHMIDT

Diophantine approximation of forms of odd degree

It is shown that if $F(x_1, \dots, x_s)$ is a form of odd degree k with real coefficients and with $s \geq c_1(k)$, then there are integers $(x_1, \dots, x_s) \neq (0, \dots, 0)$ with $|F(x_1, \dots, x_s)| < 1$. In fact, if $s \geq c_2(k, E)$ and if $N \geq 1$, then there is a nonzero integer point $\underline{x} = (x_1, \dots, x_s)$ with $|x_i| \leq N$ and $|F(\underline{x})| \leq |F|\,N^{-E}$, where $|F|$ is the maximum modulus of the coefficients of F. This result

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is derived from a result on diagonal diophantine equations: If $s \geq c_3(k, \varepsilon)$ where k is odd and $\varepsilon > 0$, then a diagonal equation $a_1 x_1^k + \ldots + a_s x_s^k = 0$ has a nontrivial solution $(x_1, \ldots, x_s)$ with $|x_i| \leq A^{\varepsilon}$ where $A = \max(1, |a_1|, \ldots, |a_s|)$.

J. SCHOISSENGEIER

Approximation reeller Zahlen durch Quadratwurzeln

Es ist bekannt, daß es zu jedem $\beta > 0$ Konstanten $0 < c_1 < c_2$ gibt, so daß $c_1 < \sqrt{N} D_N^*(\{\beta\sqrt{n}\}) < c_2$, wo $D_N^*(\{\beta\sqrt{n}\})$ die Diskrepanz von $(\{\beta\sqrt{n}\})_{n \in N}$ bezeichnet. Für eine größere Klasse von β werden $\varliminf_{N\to\infty} \sqrt{N} D_N^*(\{\beta\sqrt{n}\})$ und $\varlimsup_{N\to\infty} \sqrt{N} D_N^*(\{\beta\sqrt{n}\})$ berechnet.

F. SCHWEIGER

Integral formulas for invariant measures

Let (B, T) be a fibered system. A fibered system (B*, T*) will be called dual to (B, T) if the following condition holds:
A block of digits $(k_1, \ldots, k_N)$ is T*-admissible iff $(k_N, \ldots, k_1)$ is T-admissible. Denote by V(k) resp. V*(k) the local inverses of T resp. T* and by $\omega(k, \cdot)$ resp. $\omega^*(k, \cdot)$ their Jacobians. Suppose (for sake of simplicity) that $T^*B^*(k)$ is a union of cylinders of rank one and put

$$D(x) = \bigcup_{T^{-1}(x) \cap B(k) \neq \emptyset} B^*(k) \quad , \text{ then}$$

$$\rho(x) = \int_{D(x)} K(x, y)\,dy$$

is a solution of Kuzmin's equation if K satisfies
$K(V(k)x, y)\omega(k, x) = K(x, V^*(k)y)\omega^*(k, y)$.

V. T. SOS

On strong irregularities of the distribution of $\{n\alpha\}$ sequences

Let (u_n) be a sequence in $[0, 1]^k$ and $\Delta_N = \sup_I |\Delta_N(I)| = \sup_I \left| \sum_{\substack{u_n \in I \\ 1 \leq n \leq N}} 1 - N|I| \right|$ (I a subinterval of $[0, 1]^k$).

By Schmidt's theorem $\Delta_N > c_1 \log N$ for all N if k = 2 while for k = 1 only $\overline{\lim}_N \Delta_N/\log N > c_2 > 0$ holds and we have sequences (e. g. the $\{n\alpha\}$ sequences) for which $\Delta_N < 1$ for infinitely many N. Inspite to this fact we have

Theorem 1. Let $u_n = \{n\alpha\}$. For every $\epsilon > 0$ and $N > N_o(\epsilon)$ $\Delta_n > c_3 \log N$ holds for all but at most ϵN values of n, $1 \le n \le N$ ($c_3 > 0$ abs. constant).

For $\{n\alpha\}$ sequences we also have

Theorem 2. $N^{-1} \sum_1^N \Delta_n > c_4 \max_{1 \le n \le N} \Delta_n$.

(which does not follow from theorem 1 for α's with "large" partial quotients.)

The "dual" theorem:

for a. a. $\beta \in [0, 1]$ $\overline{\lim}_N \Delta_N[0, \beta)/\log N > c_4$ ($c_4 > 0$ abs. constant) holds also (for $\{n\alpha\}$ sequences).

V. G. SPRINDZUK

An effective version of Hilbert's irreducibility theorem

Let F(x, y) be an absolutely irreducible polynomial with integral coefficients, $\deg_y F(x, y) \ge 2$, $F(0, 0) = 0$, $\frac{\partial}{\partial y} F(0, 0) \ne 0$, a and b integers, (a, b) = 1, a_p the maximal power of a prime p in a, $\max(|a|, |b|) < C\, a_p^{n/(n-1)-\delta}$, $C > 0$, $0 < \delta < n/(n-1)$. Then the polynomial $F(\frac{a}{b}, y)$ is irreducible in $\mathbb{Q}[y]$, if a_p exceeds a computable value depending on C, δ and F(x, y). It follows that if (x_o, y_o) is a rational point on the curve F(x, y) = 0, $x_o = \frac{u}{v}$, (u, v) = 1, $|v| \le C'|u|$, p any prime divisor of u and u_p the maximal power of p in u, then $u_p \ll |u|^{1-(1/n)+\epsilon}$ for any $\epsilon > 0$, where the symbol $\ll$ implies a constant depending on C', ε, and F(x,y), but not on p.

C. L. STEWART

On the representation of an integer in two different bases

Let a, b, and n be integers larger than 1 and let α and β be integers satisfying $0 \le \alpha < a$, $0 \le \beta < b$. We denote by $L_{\alpha, a}(n)$ the number of digits in the expansion of n to base a which differ from α and we put $L_{\alpha, a, \beta, b}(n) = L_{\alpha, a}(n) + L_{\beta, b}(n)$. We have proved the following extension of a result of H. G. Senge and E. G. Strauss:

Theorem. If $\log a/\log b$ is irrational then

$$L_{\alpha, a, \beta, b}(n) > \frac{\log\log n}{\log\log\log n + C} - 1, \quad \text{for } n > e^2,$$

where C is a positive number which is effectively computable in terms of a and b only.

Results of a similar character have been obtained concerning the radix representation of numbers which are terms of certain recurrence sequences.

P. SZÜSZ

On the length of finite continued fractions

The proof of the following theorem is sketched:

Let N be a given natural number; further for any $1 \leq a < N$, $(a, N) = 1$ set

$$(*) \qquad \frac{a}{N} = [0; a_1, \dots, a_r] \quad (a_r \geq 2)$$

We call $r = r(a, N)$ the length of the continued fraction (*) and write $r = l(a)$, since N is considered to be fixed. Then we have for $N > N_0(\varepsilon)$

$$(1 - \varepsilon)\frac{\log N}{\log \gamma} < l(a) < (1 + \varepsilon)\frac{\log N}{\log \gamma},$$

with the exception of at most $N^{1-\varepsilon}$ values of a. Here $\gamma = \exp(\frac{\pi^2}{12\log 2})$.

The result improves previous ones by Heilbronn (1968) and Dixon (1970).

R. TIJDEMAN

Sets of bounded discrepancy

Let U be the unit interval $[0, 1]$ and let $\omega = \{\xi_1, \xi_2, \dots\}$ be a sequence of numbers in this interval. For $\alpha \in U$ and $n \in \mathbb{N}$ we write $Z(n, \alpha)$ for the number of integers i with $1 \leq i \leq n$ and $0 \leq \xi_i < \alpha$. We put $D(n, \alpha) = |Z(n, \alpha) - n\alpha|$. Mrs. Van Aardenne-Ehrenfest showed in 1946 that $D(n) = \sup_{\alpha \in U} D(n, \alpha)$ cannot be bounded as $n \to \infty$. W. M. Schmidt (Copositio Math. 24 (1972), 63-74) showed that the set of α such that $D(n, \alpha)$ is bounded as $n \to \infty$ is countable. The opposite is also true: for every countable set S there exists a sequence ω such that $D(n, \alpha)$ is bounded for every $\alpha \in S$. For the set S of rational numbers an effective bound for $D(n, \alpha)$ can be given which in a sense is best possible. The set S is said to be a set of bounded discrepancy when a sequence ω and a positive integer M exist such that $D(n, \alpha) \leq M$ for every $n \in \mathbb{N}$

and $\alpha \in S$. Schmidt proved that the (4M+1) th derivative of S has to be empty. This condition is not sufficient. In the talk the problem of characterizing sets of bounded discrepancy will be discussed.

K. VÄÄNÄNEN

Remarks on p-adic G-functions

Recently Flicker proved an interesting p-adic analogue to the main results of Galochkin and Nurmagomedov concerning the arithmetic properties of Siegel G-functions. Our purpose is to make this result of Flicker more explicit. Let p denote a prime number, and let $P(x_1, \ldots, x_s) \neq 0$ be a polynomial with rational integer coefficients of degree D and height H. We consider certain p-adic G-functions $g_1(z), \ldots, g_s(z)$, and for example, find a lower bound for $|P(g_1(p^n), \ldots, g_s(p^n))|_p$, where $p^n > c$ $(n \in \mathbb{N})$, and here a positive constant $c = c(g_1, \ldots, g_s, s, D)$ is explicitly given. Our results can be applied to p-adic logarithms $\ln(1+a_1 z), \ldots, \ln(1+a_s z)$, where $a_1, \ldots, a_s$ are distinct nonzero algebraic numbers. In particular, if $p^n > e^{811}$, then $h_1 + h_2 \ln(1+p^n) + h_3 \ln(1-p^n) + h_4 \ln(1+p^n)\ln(1-p^n) \neq 0$ for any rational integers h_1, h_2, h_3, h_4, not all zero.

A. VAN DER POORTEN

Linear forms in logarithms

This work appears to have reached a plateau and it is now possible to collect all the various refinements and ideas so as to produce lower bounds that include all earlier results. In the homogeneous case with rational coefficients we can now prove in a fairly elegant manner that if

$\Lambda = b_1 \log \alpha_1 + \ldots + b_n \log \alpha_n$ then $\Lambda \neq 0$ implies

$$|\Lambda| > \exp(-2^{5n+7}(n+1)^{2(n+1)}(n!)^{-1} D^{n+2} (\log E)^{-(n+1)} V_o \ldots V_n \log(\frac{B}{V_n} + \frac{|b_n|}{V_1})).$$

Here $\alpha_1, \ldots, \alpha_n \in K$ with $[K:\mathbb{Q}] = D < \infty$ and satisfy an independence condition $[K(\alpha_1^{1/2}, \ldots, \alpha_n^{1/2}) : K] = 2^n$; if α_i satisfies $a_o \alpha_i^{d_i} + \ldots + a_{d_i} = 0$ $a_j \in \mathbb{Z}$; $(a_o, \ldots, a_d) = 1$, $a_o > 0$ write $V_i = V(\alpha_i) = \max(|\log \alpha_i|, \log\|\alpha_i\|)$ where $\|\alpha_i\|^{d_i} = a_o \prod_\sigma \max(1, |\sigma\alpha|)$ (so $\log\|\alpha\|$ is the absolute logarithmic

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height); set $B = \max_{1\le i\le n-1} |b_i|$ and $E = eDn(\Sigma|\log\alpha_i|/V_i)^{-1}$; finally V_o is log of the quantity $2^{5n+7}(n+1)^{2(n+1)}(n!)^{-1}D^{n+2}(\log E)^{-(n+1)}V_1\dots V_{n-1}$. We know we can obtain a similar result in the p-adic case. The present work depends actually on inputs actively from Loxton, Waldschmidt, Mignotte and myself as well as from Stewart, Shorey, Cijsouw, ... and many earlier workers in the field.

G. WAGNER

Solution of a problem of Erdös on diophantine approximation

P. Erdös mentioned the following problem in 1974 at a conference in Marseille: Consider an arbitrary sequence $z_1, z_2, \dots$ of points on the unit circle C: $|z| = 1$, not necessarily distinct. For each N define the function $f_N(z) = \prod_{i=1}^{N} |z-z_i|$, $z \in C$, and let $d_N = \max_{z\in C} f_N(z)$. Does there exist a sequence $\{z_n\}$ for which the sequence $\{d_N\}$ is bounded?

The speaker has shown that the answer is negative by proving that there exists a constant $\delta > 0$ such that, for any sequence $\{z_n\}$, there are infinitely many $d_N > (\log N)^{\delta}$. The proof is based on a modification of a method introduced by W. M. Schmidt.

It is conjectured that there exists a $\gamma > 0$ such that one always has $d_N > N^{\gamma}$ for infinitely many N.

M. WALDSCHMIDT

Baker's theorem by Schneider's method in several variables

Let $\log\alpha_1, \dots, \log\alpha_n$ be $\mathbb{Q}$-linearly independent logarithms of algebraic numbers; then $\log\alpha_1, \dots, \log\alpha_n$ are $\overline{\mathbb{Q}}$-linearly independent. This result was proved by A. Baker in 1966 by means of a generalization of Gel'fond's solution of Hilbert's seventh problem. The auxiliary function used by Baker was a polynomial in $\alpha_1^z, \dots, \alpha_n^z$; it involves only one variable and the derivatives played an important role.

Here we use a generalization of Schneider's solution of Hilbert's seventh problem to prove that $\log\alpha_1, \dots, \log\alpha_n$ are linearly independent over $\overline{\mathbb{Q}}\cap\mathbb{R}$.

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Our auxiliary function is a polynomial in $z_1, \dots, z_{n-1}$ and $\alpha_1^{z_1} \dots \alpha_{n-1}^{z_{n-1}}$, and there is no derivative (Recently, J. C. Moreau succeeded to extend the proof to the linear independence of $1, \log\alpha_1, \dots, \log\alpha_n$ over $\overline{\mathbb{Q}} \cap \mathbb{R}$; of course he uses derivatives). The main step is a Schwarz lemma in several variables which rests on a lemma on polynomials due to Masser and Moreau, and on Schmidt's generalization of the Thue-Siegel-Roth theorem. This method enables one to study the algebraic points of the graph of an n-parameter subgroup of an algebraic group. Finally we describe a general conjecture on Schwarz lemma concerning finitely generated subgroups of $\mathbb{C}^n$.

R. WALLISSER

A generalized Liouville theorem in function fields

Let k be an arbitrary field, $k[X]$ the ring of all polynomials in X, $k(X)$ the field of rational functions in X and $k\langle X\rangle$ the field of all formal series $z = a_n X^n + a_{n-1} X^{n-1} + \dots$. In $k\langle X\rangle$ one has the nonarchimedean valuation $|z| = e^n$ if $a_n \neq 0$. Then we have the following generalization of the Liouville theorem in the field of power series:

Let $z_1, \dots, z_n$ be power series, which are algebraically dependent over $k(X)$, $f \in k[X, X_1, \dots, X_n]$, $f \neq 0$, be a polynomial in the variables $X_1, \dots, X_n$ with coefficients in $k[X]$ and $f(X, z_1, \dots, z_n) = 0$. Then there exists a constant $c(z_1, \dots, z_n, f)$, such that for all polynomials $p_1, q_1, \dots, p_n, q_n$ with $q_1 \dots q_n \neq 0$ we have either $f(X, p_1/q_1, \dots, p_n/q_n) = 0$ or

$$\max_\nu |z_\nu - p_\nu/q_\nu| > c(z_1, \dots, z_n, f)\, |q_1|^{-d_1} \dots |q_n|^{-d_n},$$

where d_i denotes the degree of f in X_i. As a corollary, we have the algebraic independence over the field of rational functions of certain power series with gaps, for example $z_\nu = \sum_{i=1}^{\infty} X^{-k_{i,\nu}}$ $1 \le \nu \le n$, where $k_{i,\nu}$ is a strictly increasing sequence of positive integers such that $\lim_{i\to\infty} (k_{i,\nu+1}/k_{i,\nu}) = \infty$ and $\lim_{i\to\infty} (k_{i+1,\nu}/k_{i,\mu}) = \infty$ ($1 \le \nu, \mu \le n$).

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G. WÜSTHOLZ

Zur algebraischen Unabhängigkeit von Werten von Funktionen, welche algebraischen Differentialgleichungen genügen

Sei $L = \mathbb{Q}(\theta, \vartheta)$, θ transzendent, ϑ algebraisch über $\mathbb{Q}(\theta)$. Wir wollen die folgende Fragestellung betrachten. Gegeben seien die meromorphen Funktionen f_1, f_2, f_3 einer Wachstumsordnung $\leq \rho$, welche den algebraischen Differentialgleichungen mit Koeffizienten in L genügen mögen:

$$p_i(f_1, f_2, f_i') = 0, \quad i = 1, 2$$
$$p_3(f_1, f_2, f_3, f_3') = 0.$$

Dann betrachten wir die Menge $S = \{ s \in \mathbb{C},\ f_i(s) \in L,\ i = 1, 2, 3 \}$.

Satz. Seien $g_1(z), g_2(z), g_3(z)$ ganze Funktionen, so daß $g_i f_i$ ganz für $i = 1, 2, 3$ ist. Es gelte

$$\text{(i) } \operatorname{typ}(f_i(s)) \ll R^{\rho}, \quad s \in S_R = S \cap B(0, R),$$
$$\text{(ii) } \log |g_i(s)| \gg R^{-\rho}, \quad s \in S_R,$$
$$\text{(iii) } R^{\mu} \ll \# S_R \ll R^{\mu}, \quad \mu \in \mathbb{R}, \ \mu > 0.$$

Dann gilt $\mu < 8\rho$.

Dieser Satz verallgemeinert in gewisser Hinsicht einen Satz von Th. Schneider, und als Anwendung wollen wir das folgende Korollar geben.

Korollar. Sei $\wp(z)$ die Weierstraß'sche $\wp$-Funktion, u ein algebraischer Punkt von $\wp(z)$. Ist $\alpha \in \overline{\mathbb{Q}}$ vom Grad 16, so sind mindestens zwei der Zahlen

$$\wp(u\alpha), \ldots, \wp(u\alpha^{15})$$

algebraisch unabhängig.

Dieses Resultat ist ein elliptisches Analogon eines Ergebnisses von Gel'fond über die algebraische Unabhängigkeit von Werten von Exponentialfunktionen.

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft

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