MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 27|1979

Riesz Spaces and Order Bounded Operators

24. 6. bis 30. 6. 1979

Die diesjährige Tagung fand unter der Leitung von Herrn Prof. W. A. J. Luxemburg (Pasadena) und Herrn Prof. H. H. Schaefer (Tübingen) statt. Der Einladung nach Oberwolfach sind 39 Mathematiker aus 12 Ländern gefolgt. Sie nutzten die angenehme Atmosphäre des Forschungsinstitutes zu einem regen Gedankenaustausch. In 32 Vorträgen und einer Problemsitzung wurden zahlreiche Aspekte der Theorie beleuchtet. Im Mittelpunkt des Interesses standen ordnungsbeschränkte und positive Operatoren (Orthomorphismen, Fortsetzungseigenschaften, M-Matrizen, Spektraltheorie). Daneben befassten sich mehrere Vortragende mit der Struktur von Vektor - bzw. Banachverbänden (Umnormierung, Orliczräume). Erstmals wurden auch Vorträge über positive Operatoren auf C^*-Algebren und Generatoren positiver Operatorhalbgruppen gehalten.

VORTRAGSAUSZÜGE

C.D.ALIPRANTIS:

The pre-Lebesgue property in Riesz spaces

A locally solid Riesz space is said to heave the pre-Lebesgue property if every sequence $\{u_n\}$ satisfying $0 \leq u_n \uparrow \leq u$ is a Cauchy sequence. A sequence $\{u_n\}$ is called k-disjoint whenever $\inf\{|u_i| : i \in I\} = 0$ holds for eyery set I of positive integers having at least k elements. The following important theorem (due to D.H.Fremlin, P.Meyer-Nieberg and H.H.Schaefer) is considered.

THEOREM: For a locally solid Riesz space (L,τ) the following statements are equivalent:

1) (L,τ) has the pre-Lebesgue property.
2) Every order bounded disjoint sequence of L is τ-convergent to zero.
3) Every order bounded k-disjoint sequence of L is τ-convergent to zero.

The difficult part of the proof is the implication (2) $\Longrightarrow$ (3). This is proved by an inductive argument by introducing for a given $(k + 1)$-disjoint sequnce $\{u_n\}$ with $0 \leq u_n \leq u$ for each n, the k-disjoint order bounded sequence $w_1 = 0$ and $w_n = (u_n - n\sum_{i=1}^{n-1} u_i - \frac{1}{n}u)^+$ for $n \geq 2$. The details can be found in LOCALLY SOLID RIESZ SPACES (C.D.Aliprantis - O.Burkinshaw, Academic Press, 1978) pp. 64 - 70 .

T.ANDO:

Inequalities for M-matrices

A square matrix A is called a M-matrix if $A = \rho I - S$ for some $\rho > 0$ and some positive operator S such that $\rho > r(S)$, the spectral radius. With respect to the (Perron-Frobenius) order the class of M-matrices behaves quite similarly as the class of positive semi-definite matrices does with respect to the positive-definiteness order.

The aim of the talk is to analyse the basis of this similarity, and to present a different approach to various (matrix or determinant) inequalities of Ostrowski, Ptak, Fan and others. This approach can be applied to the case of operators in Banach spaces.

W. ARENDT:

Über das Spektrum von Verbandsisomorphismen

Sei T ein Verbandsisomorphismus auf einem Banachverband E. Ist $r \in \varrho(T) \cap \mathbb{R}_+$, so ist die Spektralprojektion $P = \frac{1}{2\pi i} \int_{\Gamma_r} R(z,T)dz$ $(\Gamma_r = \{z \in \mathbb{C} : |z| = r\})$ eine Bandprojektion. Als Anwendung lässt sich $\sigma(T) \cap \mathbb{R}_+$ berechnen, falls $E = C(X)$ (X kompakt). Interessant sind in diesem Zusammenhang eindeutig ergodische Homöomorphismen (das sind Homöomorphismen φ mit der Eigenschaft, dass die Cesàro-Mittel $M_n(f) = \frac{1}{n}\sum_{m=0}^{n-1} f \circ \varphi^m$ für jedes $f \in C(X)$ gleichmässig gegen eine konstante Funktion konvergieren). Sei $\text{card}X = \infty$, φ ein Homöomorphismus auf X. Jedes $h \in C(X)$ mit $h(x) > 0$ für alle $x \in X$ definiert einen Verbandsisomorphismus T_h auf $C(X)$ durch $T_h f = h \cdot f \circ \varphi$ für alle $f \in C(X)$. Es gilt:

φ ist genau dann eindeutig ergodisch, wenn für jedes $h \in C(X)$ mit $h(x) > 0$ für alle $x \in X$ $\sigma(T_h) = \{z \in \mathbb{C} : |z| = r(T_h)\}$ gilt.

S. J. BERNAU:

Orthomorphisms of Archimedean vector lattices

The lecture is concerned with showing that much of the theory of disjointness preserving linear maps of an Archimedean vector lattice into itself can be obtained independently of representation theories for Archimedean vector lattices. An example is given of a disjointness preserving map on an Archimedean vector lattice which is not an orthomorphism.

O.BURKINSHAW:

Examples of minimal topologies and L_p-spaces

The question arises naturally whether or not a space admits a minimal locally solid topology. My objective is to give examples (or show the non-existence) of minimal topologies on some classical spaces. For example, $C[0,1]$ and $L_\infty([0,1])$ do not admit a minimal locally solid topology, and the topology of convergence in measure on $L_p([0,1])$ $(0 \leq p < \infty)$ is a minimal topology. A similar result is shown for certain Orlicz spaces.

P.DODDS:

Remarks on compact kernel operators

Let (X,μ), (Y,ν) be σ-finite measure spaces. If $T: L^p(Y,\nu) \longrightarrow L^r(X,\mu)$ is a kernel operator, then T is compact if $r < \min\{p,2\}$. The condition $r < 2$ may not be omitted.

K.DONNER:

Extensions of positive linear operators in classical Banach lattices

Given two Banach lattices E, F and a linear subspace $H \subseteq E$, consider a positive (positive continuous, positive contractive) linear operator $T: H \longrightarrow F$. The following questions are answered for classical Banach lattices E, F and also in some non-classical cases:

1) Give necessary and sufficient conditions for the existence of a positive (positive contractive) linear extension $\tilde{T}: E \longrightarrow F$ of T!
2) Describe the scope of all such extensions! In particular, when do we have uniqueness of extensions?

H.O.FLÖSSER:

On the Choquet-Deny theorem and ist applications to Korovkin approximation

Let (E,τ) be a locally convex Riesz space, $V(E,\tau)$ the set of all real valued τ-continuous linear lattice homomorphisms on E and K a subset of E. The uniquness cone $e(k)_\tau$ of K is defined to be $e(K)_\tau := \{e \in E : \forall \mu \in (E,\tau)'_+ \; \forall \delta \in V(E,\tau) \; \mu \leq_K \delta \Rightarrow \mu(e) \leq \delta(e)\}$. Whenever (E,τ) is an M-space or dual atomic, K a closed convex inf-stable cone, then we have $K = e(K)_\tau$. In the case $E = C(X)$ with the topology of compact convergence, X a Hausdorff space, this is a theorem due to Choquet and Deny. Denote by τ_s the topology on E of pointwise convergence on $V(E,\tau)$ and let L be a subspace of E. If (E,τ) is an M-space or dual atomic we have $\overline{\hat{L}}^{\tau} = e(L)_\tau$ and $\overline{\hat{L}}^{\tau_s} = e(L)_{\tau_s}$, but in general $\overline{\hat{L}}^{\tau_s} \supsetneqq \overline{\hat{L}}^{\tau}$ where $\hat{L} = \{ \inf A : \emptyset \neq A \subset L \text{ finite}\}$. If L is finite dimdimensional then $\overline{\hat{L}}^{\tau_s} = \overline{\hat{L}}^{\tau} = e(L)_{\tau_s} = e(L)_\tau$.
These theorems are applied to characterize the universal Korovkin closure of a subspace of E by means of its uniqueness closure.

G.GREINER:

Über das Spektrum positiver Halbgruppen

Es gibt eine Reihe von Aussagen über das periphere Spektrum positiver Operatoren auf Banachverbänden, als da sind die Sätze von Lotz, Niiro-Sawashima, Perron-Frobenius (vgl. H.H.Schaefer: Banach Lattices and Positive Operators, Springer 1974, pp. 327 - 331). Analoge Aussagen gelten für das Spektrum des Generators einer stark stetigen Halbgruppe von positiven Operatoren. Zum Beispiel gilt folgender Satz:

Theorem: Es sei $\{T(t)\}$ eine stark stetige Halbgruppe positiver Operatoren mit Generator A, die Spektralschranke von A sei Null und ein Pol der Resolvente von A, ferner sei das Residuum in Null von endlichem Rang. Dann ist $\sigma(A) \cap i\mathbb{R}$ eine endliche Vereinigung von (additiven) Untergruppen und besteht ausschliesslich aus Polen der Resolvente.

J.J. GROBLER:

A Lebesgue convergence theorem in Riesz spaces with applications to compact operators

A subset S of a Banach lattice E is said to be of uniformly absolutely continuous norm (u.a.c. norm) whenever, given a sequence (E_n) of projection bands with $E_n \downarrow 0$ we have that $\| P_{E_n} f \| \longrightarrow 0$ uniformly on S. We show that if E is an ideal in a Riesz space M which has the principal projection property and the Egoroff property and if (f_n) is a sequence in E which is order convergent in M to an element $f \in M$, then $f \in E$ and $\| f_n - f \| \longrightarrow 0$ if $S = \{f_n : n \in \mathbb{N}\}$ is of u.a.c. norm. This fact can be used to derive compactness criteria for order bounded operators on Banach lattices. We prove for instance that if E' and F have order continuous norms, and if $T \in (E'_{oo} \otimes F)^{dd}$ then the following conditions are equivalent:

1) T is compact.
2) $T[U]$ is of u.a.c. norm, U the unit ball in E.
3) $\| P_{F_n} T \| \longrightarrow 0$ for every sequence of bands $F_n \downarrow 0$ in F.
4) $\| T P_{E_n} \| \longrightarrow 0$ for every sequence of bands $E_n \downarrow 0$ in E.
5) $\| P_{F_n} T P_{E_n} \| \longrightarrow 0$ for every sequence of bands $E_n \downarrow 0$ in E and $F_n \downarrow 0$ in F.

Conditions 1 - 5 closely resemble the conditions given by Luxemburg and Zaanen for kernel operators in Banach function spaces.

G. GROENEWEGEN:

Bochner integrals and weakly compact operators

We study operators T from a Banach function space $L_\rho(X,\mathcal{A},\mu)$ into a Banach lattice E which can be represented in the following way: there exists a strongly measurable mapping $f: X \longrightarrow E$ such that $Tg = (\text{Bochner})\text{-}\int fg \, d\mu$ for $g \in L_\rho$. We show that T has a modulus which can be represented in the same way as T, by the mapping

$|f|: X \longrightarrow E$, $|f|(x) := |f(x)|$. This result can be applied to the study of weakly compact operators $T: L^1(\mu) \longrightarrow E$. It follows that the modulus of such an operator always exists. We will describe the range spaces E for which $|T|$ is again weakly compact. We will also describe the spaces E for which the weakly compact operators form a solid subspace of $\mathcal{L}^r(L^1(\mu), E)$, the space of all regular operators from $L^1(\mu)$ into E .

U.GROH:

Das periphere Punktspektrum positiver Operatoren auf C^*-Algebren

Sei $\mathfrak{A}$ eine C^*-Algebra und sei $T \in \mathcal{L}(\mathfrak{A})$. wir nennen T positiv, falls $T(\mathfrak{A}_+) \subseteq \mathfrak{A}_+$, irreduzibel, falls keine abgeschlossene Seite in $\mathfrak{A}_+$, verschieden von $\{0\}$ und $\mathfrak{A}_+$, invariant unter T ist, einen Schwarz-Operator, falls $T(x^*x) \geq T(x)^* T(x)$ für alle $x \in \mathfrak{A}$.

Satz: Sei $\mathfrak{A}$ eine C^*-Algebra mit Einheit $\mathbb{1}$ und sei T ein irreduzibler Schwarz-Operator auf $\mathfrak{A}$ mit $T(\mathbb{1}) = \mathbb{1}$.
Dann gilt:
1) Der Fixraum von T ist eindimensional.
2) Das periphere Punktspektrum von T ist eine Untergruppe der Kreisgruppe.
3) Jeder periphere Eigenwert α von T ist einfach und es gilt $\alpha\sigma(T) = \sigma(T)$.
4) 1 ist der einzige Eigenwert von T mit einem positiven Eigenvektor.

Da auf einer kommutativen C^*-Algebra jeder positive Operator ein Schwarz-Operator ist, ist der obige Satz eine Verallgemeinerung der Resultate von G.Frobenius und H.H.Schaefer (Math.Z. 82, 303-313, (1963)).

C.B.HUIJSMANS:

Order- and ring ideals in C(X) and generalizations

An o-ideal I in an Archimedean Riesz space L is called a z-ideal if $f \in I$, $\overline{I_f} = \overline{I_g} \Rightarrow g \in I$ ($\overline{I_f}$ = relative uniform closure of the principal ideal generated by f). In C(X) the z-ideals are the z-ideals in the Gillman-Jerison sense. The o-ideal I is called d-ideal, if $f \in I$, $f^{dd} = g^{dd} \Rightarrow g \in I$. We have:

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band $\Rightarrow$ σ-ideal $\Rightarrow$ uniformly closed ideal $\Rightarrow$ z-ideal $\Rightarrow$ o-ideal

σ-ideal $\Rightarrow$ d-ideal $\Rightarrow$ z-ideal

minimal prime ideal $\Rightarrow$ d-ideal

None of the converse implications hold. Necessary and sufficient conditions are given for the implications to hold. If L has a near unit e ($\overline{I_e}$ = L) then maximal z-ideals exist. If L is uniformly complete, an abstract version of the Gelfand-Kolmogorov theorem can be proved.
L is called z- (d-) regular if every prime z- (d-) ideal is a minimal prime ideal. Then

1) If L is uniformly complete: L is z-regular $\Longleftrightarrow$ L has σ-order continuity property and is Dedekind-σ-complete.
2) L has the principal projection property $\Longleftrightarrow$ L is d-regular and every prime ideal contains a unique minimal prime ideal (i.e. L is normal).

All these results can be translated in terms of C(X). E.g.:

C(X) is z-regular $\Leftrightarrow$ C(X) is von Neumann-regular
$\Leftrightarrow$ every zero set is open,

C(X) has σ-order continuity property $\Leftrightarrow$ every zero set is regular closed,

C(X) is d-regular $\Leftrightarrow$ $\forall f\ \exists g$ such that $\overline{X\setminus Z(f)} = \overline{\mathrm{int}Z(g)}$,

C(X) is normal $\Longleftrightarrow$ every ring ideal is order ideal.

Several other results of this kind are derived.

E.DE JONGE:

<u>Banach lattice-valued measures and their RN-derivatives</u>

Let E be a Dedekind complete Banach lattice and (Δ,Γ,μ) be an atomless measure space such that $0 < \mu(\Delta) < \infty$. By $L^1(\mu,E)$ we denote the Banach lattice of the μ-Bochner integrable, E-valued "functions" on Δ, and by $M(\mu,E)$ we denote the collection of all E-valued measures ν on Γ satisfying:
(i) ν is additve,
(ii) ν is of bounded variation,
(iii) ν is order bounded,
(iv) ν is μ-absolutely continuous,
(v) ν^+ and ν^- satisfy $\inf\limits_{\varepsilon>0}\ \sup\limits_{\mu(A)<\varepsilon} \nu^{+(-)}(A) = 0$

Then we have:

Theorem 1: Equivalent are:

(i) The measures with a RN-derivative form an ideal in $M(\mu,E)$.

(ii) E has order continuous norm.

Theorem 2: Equivalent are:

(i) $L^1(\mu,E) \cong M(\mu,E)$ in the canonical way.

(ii) c_o is not isomorphic to a closed Riesz subspace of E.

S.KOSHI:

On some operators in vector lattices

Let E be a monotone complete modular space (a generalized Orlicz space) with an almost finite modular m, and let [N] be a projection operator generated by a band N in E. Related to Nemytsky's operators or an integral functional $I(x(.),y(.)) = \int f(x(t),y(t),t)dt$, we shall consider an operator $H: E \times E \longrightarrow E$ such that $[N]H(x,y) = H([N]x,[N]y)$ for every $x,y \in E$ and every projection operator [N]. By Shimogaki's ideas, we shall prove that there exists a positive element $c \in E$ and a positive number α with $|H(x,y)| \leqslant c + \alpha(|x| + |y|)$ for every $x,y \in E$. Applications of this fact are also explained.

P.KRANZ:

Characterization of Orlicz lattices

Let (Ω,Σ,ν) be a finite positive measure space. A function $\Psi: \mathbb{R}_+ \times \Omega \longrightarrow \mathbb{R}_+$ is called a (convex) Musielak-Orlicz function if 1) $\Psi(r,s)$ is Σ-measurable for each r and there exists a set A of ν-measure zero such that for each $s \in \Omega \setminus A$ 2) $\Psi(r,s) = 0$ iff $r = 0$,

3) $\Psi(r,s)$ is a convex function with respect to the first variable,

3') $\Psi(r,s)$ is monotone and left-continuous with respect to the first variable,

4) $\Psi(r,s) = 0$ for each $s \in A$ and each r .

Let $\mathcal{F} = \{ x : x$ is real Σ-measurable function $\}$, and let Ψ be a (convex) Musielak-Orlicz function. If Ψ has the property: there exist a set B of ν-measure zero, a number $k > 0$ and a ν-integrable function γ such that

(+) $\Psi(2r,s) \leqslant k\Psi(r,s) + \gamma(s)$ for all r and $s \in \Omega \setminus B$,

then the function $M: L^{\Psi} \longrightarrow [0,\infty)$ given by $M(x) = \int_{\Omega} \Psi(|x(s)|,s)\, d\nu$, is a (convex) modular, where $L^{\Psi} = \{x \in \mathcal{F} : \exists\, \alpha > 0 \int_{\Omega} \Psi(\alpha|x(s)|,s)\, d\nu < \infty\}$. The space $(L^{\Psi}, \|\cdot\|_M)$ where $\|\cdot\|_M$ is an Orlicz norm is an Orlicz lattice for the ordering $x \leqslant y$ if $x(s) \leqslant y(s)$ ν-a.e..

<u>Theorem:</u> Let $(L, \|\cdot\|_{\varrho})$ be a (convex) Orlicz lattice possessing a weak unit e, and let ϱ satisfy the (Δ_2)-condition. Then there exist a positive measure space (S,Λ,μ) and a (convex) Musielak-Orlicz function $\Psi: \Omega \times \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ with the property (+) such that $(L, \|\cdot\|_{\varrho})$ is isometrically Riesz isomorphic to the space $(L^{\Psi}, \|\cdot\|_M)$.

This theorem generalizes a previous characterization of Orlicz lattices obtained by W.J.Claas and A.C.Zaanen in the case when L is assumed to be in addtion component invariant. In the theorem above neither the (Δ_2)-condition nor the (+)-condition are necessary.

J.LINDENSTRAUSS:

<u>Renorming Banach lattices</u>

The following theorem is proved:

Let E be a Banach lattice then the following are equivalent

(i) E has an order continuous norm.

(ii) E has an equivalent lattice Kadec-Klee norm.

(iii) E has an equivalent lattice locally uniformly convex norm.

As a consequence we get that if E is a σ-complete Banach lattice then E' has the Radon-Nikodym property iff E has an equivalent Fréchet differentiable lattice norm. The renorming theorem is used in giving very simple proofs to known results concerning convergence of submartingales and subadditive processes in Banach lattices.

H.P.LOTZ:

Positive Operatoren auf L^1 und messbare Transformationen

Sei μ das Lebesguemass auf $I = [0,1]$, Σ die σ-Algebra der μ-messbaren Mengen und $L^1 = L^1(I,\Sigma,\mu)$. Der folgende Satz wurde bewiesen:

Sei $T: L^1 \longrightarrow L^1$ ein positiver Operator mit $\|T\mathbb{1}\| = 1$. Dann existieren zwei messbare nicht singuläre Transformationen φ und ψ von I in I derart, dass $T = T_\psi^* \circ T_\varphi$ ist. Dabei ist $T_\varphi f = f \circ \varphi$ für alle $f \in L^1$ und T_ψ^* ist die Präadjungierte des Operators $T_\psi: L^\infty \longrightarrow L^\infty$ mit $T_\psi g = g \circ \psi$ für alle $g \in L^\infty$.

Das folgende Korollar löst ein Problem von Shiflett (Pac. J.Math. 40, 1972).

Korollar: Sei $T: L^1 \longrightarrow L^1$ ein doppelt stochastischer Operator (d.h. $0 \leqslant T$, $T\mathbb{1} = \mathbb{1}$ und $T'\mathbb{1} = \mathbb{1}$). Dann existieren zwei masstreue Transformationen φ und ψ von I in I mit $T = T_\psi^* \circ T_\varphi$.

W.A.J.LUXEMBURG:

The theory of Riesz homomorphisms

Riesz homomorphisms and ideal theory in the theory of Riesz spaces are intimately related. The purpose of the talk will be to present an overview of some of the basic results.
A great deal of attention will be paid to the so-called orthomorphisms.
Retracts of Riesz spaces will be defined and applied to obtain an extension of the famous theorem of Sikorski concerning extensions of Boolean homomorphisms.

I.MAREK:

Asymptotic behaviour of positive semi-groups

Let A be the generator of a C_o-semi-group of positive operators in a Banach lattice E. Let the semi-group operator $T(\hat{t},A)$ for a fixed value $\hat{t} > 0$ be irreducible and the spectral radius $r(T(\hat{t},A))$ be a pole of the resolvent $R(\lambda,T(\hat{t},A))$. It is shown that the asymptotic behaviour of the solutions to the Cauchy problem $\frac{d}{dt}u = Au$, $u(0) = u_o$ is nonoscillatory. This result applied to the Boltzmann linearized operator A solves an old problem of reactor physics theory.

M. MEYER:

About almost Dedekind σ-complete Riesz spaces

On sait qu'un espace vectoriel réticulé E (enabrégé e.v.r.a.) est presque σ-complet si pour toute suite croissante (x_n) , bornée pour l'ordre dans E , il existe une suite décroissante (y_n) telle que $\inf\{y_m - x_n : n,m \in \mathbb{N}\} = 0$.

Theoreme: Soit E un e.v.r.a. ; E est presque σ-complet si et seulement si, pour toute suite disjointe (x_n) de E, pour tout $u \in E$, il existe une suite disjointe (y_n) , telle que $|x_n| \wedge |y_m| = 0$ pour tout $n,m \in \mathbb{N}$ et $u \in (\{x_n\} \cup \{y_n\})^{dd}$.

Corollaire 1: Soit E un e.v.r.a. uniformément complet. Les assertions suivantes sont equivalentes:

(a) E est presque σ-complet

(b) $\forall\, x,u \in E$, il existe $g \in E$ telque $|x| \wedge |y| = 0$ et $u \in \{|x| + |y|\}^{dd}$.

Corollaire 2: Soit E un espace de Banach reticulé. La norme de E est continue pour l'ordre si et seulement si, pour tout $x,u \in E_+$, il existe $y \in E_+$ telque $x \wedge y = 0$ et $u = \lim_n (n(x + y) \wedge u)$.

Corollaire 3: Si T est un espace topologique normal. $C(T)$ (ou $C^b(T)$) est presque σ-complet si et seulement si pour tout F_σ ouvert O_1 de T, il existe un F_σ ouvert O_2 telque $O_1 \cap O_2 = \emptyset$ et $\overline{O_1 \cup O_2} = T$.

Des exemples d'e.v.r.a. uniformément complets non presque σ-complets sont donnés et l'on montre que les implications du corollaire 1 sont fausses si E n'est pas supposé uniformément complets.

R.NAGEL:

Strongly continuous semigroups of positive operators

We discussed the following problems and recent results on strongly continuous semigroups $\{T(t) : t \geq 0\}$ on Banach lattices E with generator $(A,D(A))$:

1) Is it true that $\{T(t)\}$ is a positive semigroup iff A satisfies the Kato-inequality $A|x| \geq \operatorname{sgn} x \cdot Ax$ for $x \in D(A)$?
   Semigroups of lattice homomorphisms are characterized by the Kato-equality (Nagel - Uhlig - Wolff).
2) Is it true that the peripheral spectrum of A is additively cyclic? The answer is "yes", if $\{\lambda R(\lambda,A) : \lambda > 0\}$ is bounded (Derndinger - Greiner).

B.DE PAGTER:

Compact Riesz homomorphisms

Given the ideals of measurable functions L and M on the measure space (X,Λ,μ) and (Y,Σ,ν) resp., where X does not possess any atoms, the Riesz homomorphisms of L into M are disjoint to the band of all order bounded kernel operators. Using the fact that on the space $L^2[0,1]$ no non-zero compact Riesz homomorphisms exist, one can find a non-zero, order bounded operator which is disjoint to all kernel operators as well as to all Riesz homomorphisms on $L^2[0,1]$.
In general, there exist non-zero compact Riesz homomorphisms from the Banach lattice L_ρ into the Banach lattice M_λ iff L_ρ^* possesses atoms. Furthermore these operators are all of the form $T = \sum \varphi_n \otimes g_n$, where (φ_n) are disjoint atoms in L_ρ^*, and (g_n) are disjoint in M_λ.
Also a description is given of all order continuous compact Riesz homomorphisms from L_ρ into M_λ, and of all compact orthomorphisms on L_ρ.

S. PAPADOPOULOU:

A geometric description of the Korovkin closure in $C(X)$

Let $\mathcal{H}$ be a closed function space on a compact space X, which contains the constants and separates the points of X. X is identified with its canonical image in the state space K of $\mathcal{H}$ and $\mathcal{H}$ with the space $\mathcal{A}(K)|X$.

Theorem: Let one of the following conditions be satisfied:

a) For each $x \in X$ the face of K generated by x is closed (e.g. $\mathcal{H}$ is finite-dimensional).

b) K is stable (that is the barycenter map from $M_+^1(K)$ onto K is open).

Then the Korovkin closure of $\mathcal{H}$ is the space of all functions $f \in C(X)$ with the property: for each $x \in X$ the function $f|X \cap \overline{F_x}$ can be extended to an affine continuous function on $\overline{F_x}$.

As an application we derive an exact description of the Korovkin closure of the space $\mathcal{H} = \mathcal{A}(K) + \mathbb{R}u$, where K is a compact convex set and u a continuous function on K.

H. H. SCHAEFER:

Positive contractions in L^p-spaces

Let T denote a positive contraction on a space $L^p(\mu)$ $(1 < p < +\infty)$. A primitive n-th root of unity ε is in the point spectrum $P\sigma(T)$ iff it is in $P\sigma(T')$, if so, the unimodular group generated by ε is in both $P\sigma(T)$ and $P\sigma(T')$. In turn, this is equivalent to the existence of n-dimensional Riesz subspaces of L^p and L^q $(p^{-1} + q^{-1} = 1)$ which are in canonical duality and on which T (resp. T') act as isometries. If, in addition, T is quasi-compact then the spectral projection associated with the unimodular spectrum of T (resp. T') is a positive contraction onto a Riesz subspace of L^p (resp. L^q) on which T (resp. T') acts as an isometry.

E.SCHEFFOLD:

Über komplexe Banachverbandsalgebren

Es werden komplexe Banachverbandsalgebren betrachtet, bei denen die Menge der komplexen Homomorphismen betragsinvariant ist. Es wird gezeigt, dass bei diesen Algebren die Menge der komplexen Homomorphismen sogar zyklisch ist. Mit diesem Ergebnis können dann im Rahmen der Gelfand-Theorie einige Zyklizitätseigenschaften des peripheren Spektrums positiver Elemente bewiesen werden.
Es wird ferner gezeigt, dass für zentrale Homomorphismen von AL-Algebren, einer speziellen Klasse von komplexen Banachverbandsalgebren, ähnliche Ergebnisse gelten, wie sie J.L.Taylor für Konvolutionsmassalgebren bewiesen hat.

A.R.SCHEP:

Positive diagonal and triangular operators

A diagonal operator on an Archimedean Riesz space is an order bounded orthomorphism. We shall present a new formula for the band projection of $\mathcal{L}_b^+(L)$ onto $\mathrm{Orth}^+(L)$, where L is a Dedekind complete Riesz space. This can then be applied to characterize order bounded generators of positive semi-groups. Then we shall present a continuity theorem for the spectral radius of a positive order continuous compact operator. This result is then used to prove the quasi-nilpotency of a class of operators, which are, among other things, triangular. Triangularity means here that the operator has a maximal chain of invariant bands. Then we determine the spectrum of the sum of such an operator and a diagonal operator.

C.T.TUCKER:

Applications of the theory of Riesz spaces to real variables

Several results in the theory of Baire functions inspired by the theory of Riesz spaces are discussed. In particular, if Ω is a linear lattice of real valued functions, $B_1(\Omega)$ is the first Baire class of Ω, and φ is a positive linear functional defined on $B_1(\Omega)$, then φ is sequentially continuous for both monotone pointwise and order convergence. Two negative results about order continuity for nets are stated. In contrast, if X is a real compact topological space, then every positive linear functional on $B_1(C(X))$ is continuous with respect to pointwise convergence.

A.W.WICKSTEAD:

Extremal structure of cones of operators

Consider the space of bounded linear operators between two Banach lattices E and F. Give this the normal operator order. We consider two kinds of problems. Suppose $T:E \longrightarrow F$ is either compact or weakly compact, what conditions on E and / or F guarantee that S has the same property whenever $0 \leqslant S \leqslant T$? Consider the cones of all positive linear operators from E into F equipped with the strong operator topology and that of compact positive linear operators from E into F equipped with the operator norm topology. Under what circumstances are these cones the closed convex hulls of either their extremal elements or of the lattice homomorphisms in them ? We also introduce the notion of almost $Z(F)$-extremality and partially generalize the classical result that a positive linear functional is extremal if and only if it is a lattice homomorphism.

M.WOLFF:

Compact groups of lattice isomorphisms and an ergodic theorem

Let G denote a compact group. A Banach lattice E is called a G-lattice if there is a continuous homomorphism U^E from G into $\mathcal{L}_s(E)$ such that every U^E_g is a lattice isomorphism. A G-morphism V between G-lattices E and F is a lattice homomorphism from E into F satisfying $VU^E_g = U^F_g V$ (for all $g \in G$).

<u>Theorem 1:</u> Let E be a G-lattice possessing a topological order unit. Then there exists a compact space K_o such that $C(K_o)$ is a G-space which can be densely embedded into E by a G-morphism, in addition if C(L) is another G-lattice which is G-embeddable into E then this embedding can be factored through $C(K_o)$.

Let T be a positive operator on the G-lattice E satisfying $U_g^E T = TU_g^E$ $(g \in G)$. Then $T(F) \subset F := \{x : U_g^E x = x \ \forall\, g \in G\}$.

<u>Theorem 2:</u> If E is separable, if $\{T^n : n \in \mathbb{N}\}$ is uniformly bounded and if $T_{|F}$ is irreducible and uniformly ergodic then T is mean ergodic.

From this theorem we derive applications to C_o-semigroups of positive operators as well as to products of dependent random variables.

J.D.M. WRIGHT:

<u>A Riesz space problem arising in the theory of C^*-algebras</u>

Let (A,e) be an order-unit normed Banach space with order unit e and state space K . Then A is isometrically order isomorphic to an order-unit subspace of ℓ^∞ if, and only if, K is separabel, that is, K has a countable dense subset.

An L^∞-space, $L^\infty(\Omega,\mu)$ is said to be <u>standard</u> if there exists a Borel measure β on the real line such that $L^\infty(\Omega,\mu) \cong L^\infty(\beta)$. H.Rosenthal has proved that if the state space of $L^\infty(\Omega,\mu)$ is separable then $L^\infty(\Omega,\mu)$ is standard. Extending this result, C.A.Akemann proved that a von Neumann algebra has a faithfull representation on a separable Hilbert space if, and only if, its state space is separable. Akemann's Theorem does not extend to general C^*-algebras because I can construct a unital C^*-algebra A , with state space K , such that both K and the extreme boundary of K are separable, but A has no faithful representation on a separable Hilbert space. This counter-example is highly non-commutative because the algebra constructed has no non-trivial closed ideals. Do there exist commutative counter-examples? i.e.: Can we find a compact Hausdorff X such that $C(X)$ is embeddable as an order-unit subspace of ℓ^∞ but $C(X)$ is not embeddable as a Riesz subspace of any standard L^∞-space ?

A.C.ZAANEN:

The carrier of a positive linear functional

For φ in the order dual $L^\sim$ of the (Archimedean) space L, the carrier $C(\varphi)$ of φ is by definition the disjoint complement of the null ideal $N(\varphi) = \{f \in L : |\varphi|(|f|) = 0\}$ of φ. For φ and ψ positive it follows easily from $N(\varphi \vee \psi) = N(\varphi) \cap N(\psi)$ that $C(\varphi \vee \psi) = C(\varphi + \psi) = \{C(\varphi) + C(\psi)\}^{dd}$. The equality (1) $C(\varphi \wedge \psi) = C(\varphi) \cap C(\psi)$ does not hold generally, but it holds if one at least of φ and ψ is σ-order continuous. Furthermore, the following holds: If L has the Egoroff property, then $C(\varphi) = \{0\}$ for every singular $\varphi \in L^\sim$, and if $C(\varphi) = \{0\}$ for every singular $\varphi \in L^\sim$, then (1) holds.

Open problem: Does there exist an (Archimedean) Riesz space L such that (1) holds generally and such that there exists at least one singular $\varphi \in L^\sim$ with $C(\varphi) \neq \{0\}$?

Probleme

1. (Ando) Let $\mathcal{B}$ be a Boolean algebra of subsets of a non-empty set X and let H denote a Hilbert space. A mapping E of $\mathcal{B}$ into the subset of the orthogonal projections on H is called an orthoprojection whenever $\Delta_1, \Delta_2 \in \mathcal{B}$ and $\Delta_1 \cap \Delta_2 = \emptyset$, then $E(\Delta_1 \cup \Delta_2) = E(\Delta_1) + E(\Delta_2)$. Assume that E_1, E_2 are two orthoprojection maps.

Problem. If for all $\Delta \in \mathcal{B}$ there exists a constant $o \leq \gamma < 1$ such that $\| E_1(\Delta) - E_2(\Delta) \| \leq \gamma < 1$, then does there exists a unitary operator W which interwines E_1 and E_2, i.e. , for all $\Delta \in \mathcal{B}$ we have :

$$W \circ E_1(\Delta) = E_2(\Delta) \circ W \quad .$$

If $\mathcal{B}$ is finite, the positive answer was given by Sz.Nagy. In 1976 Matsumoto showed, that the answer is always yes if $o < \gamma < \frac{1}{2}$. I showed that if $\gamma < \frac{1}{\sqrt{2}}$ and $\mathcal{B}$ is σ-complete and the orthoprojections are countable σ-additive and each E_i (i=1,2) has a cyclic vector, then the answer is yes.

2. (Bellow) Let $(\Omega, \mathcal{F}, \mu)$ be a Lebesque space of $[0,1]$ and $T: \Omega \to \Omega$ an invertible ergodic measure-preserving transformation. For $f \in \mathcal{L}^1$ define :

$$T_n f := \frac{1}{n} (f + f \circ T + \ldots + f \circ T^{n-1}) \ .$$

It is known that for each $f \in \mathcal{L}^1$, $T_n f(\omega) \to \int f d\mu$ μ-a.s. (the Individual Ergodic Theorem). Call a function $f \in \mathcal{L}^\infty$ uniform if

$$\| T_n f - \int f d\mu \|_\infty \to o \text{ as } n \to \infty \quad .$$

Let $\mathcal{U}$ be the set of all uniform functions. It is clear, that $\mathcal{U}$ is a vector space and that $\mathcal{U}$ is closed in the $\| \cdot \|_\infty$-norm. It is also clear that $\mathcal{U}$ contains a vector space $\mathcal{W}$ which is dense in L^1 for the $\| - \|_1$-norm (take for instance $\mathcal{W} = \{ c + (I - T)g \mid c \in \mathbb{R}, g \in \mathcal{L}^\infty \}$)

However $\mathfrak{M}$ is not a algebra, in fact one can show that is not a lattice (by using the Kakutani-Rokhlin skyscraper construction). There are nevertheless nontrivial algebras $\mathfrak{A} \subseteq \mathfrak{M}$. Find a functional-analytic proof for the existence of such algebras. Motivation : The construction of such algebras is essential in the Jewett-Krieger theorem.

3. (Donner) Characterize those pairs of Banach lattices (E,F), F Dedekind complete, for which the following con - ditions are equivalent for each linear subspace $H \subset E$, each positive continuous linear operator $T : H \to F$ and each $M \geq 0$, $\| T \| \leq M$:

i) For every $e \in E$ there exists a linear operator $T_e : H + \mathbb{R}e \to F$ such that $T_e|_H = T$ and $l(T_e(k)) \leq M \| k \| \cdot \| k^+ \|$ $(l \in F'_+,\ k \in H + \mathbb{R}e)$.

ii) There exists a positive extension $T_o : E \to F$ of T such that $\| T_o \| \leq M$.

If E is a L^p-space, F a L^q-space with $q \leq p$, then (E,F) is such a pair of Banach lattices. The same is true for E an arbitrary Banach lattice and F a Dedekind complete AM-space with order unit.

4. (Huijsmans, de Pagter)

1.) L Archimedean Riesz space equipped with the relatively uniform topology, $S \subset L$, then $S' =$ pseudo closure of S , $\overline{S}$ = closure of S. It is known that in general $S' \subsetneq \overline{S}$ (necessary and sufficient conditions to have equality for all $S \subset L$ were proved by T.Dodds-Chow). Give an example of a uniformly complete Archimedean Riesz space L with an element $e > 0$ such that $I_e{}' \subsetneq \overline{I_e} = L$; e.g. : in an f-algebra L for the ring unit e always $\overline{I_e} = L$.

2. Prove or disprove : For every ideal I in C(X) we have $I' = \overline{I}$ (X : completely regular Hausdorff space).

<u>Theorem</u>: $I' = \overline{I}$ for all order ideals in $C(X)$ $\Longleftrightarrow$ if $f \geq 0$, $g \geq 0$ and $Z(f) = Z(g)$ ($Z(f)$ is the zero set of f) then $f \wedge ng \uparrow f$ (r.u.).

For $X = P$ space and $X = \mathbb{R}$ we have $I' = \overline{I}$ for all ideals I in $C(X)$.

5. (de Jonge)

1.) Let $\mathcal{B}$ be a countable σ-generated Boolean σ-algebra. Consider :

i. $\mathcal{B}$ is weakly σ-distributive.
ii. The σ-measures on $\mathcal{B}$ separates $\mathcal{B}$.

Then (ii) $\Longrightarrow$ (i).

a.) Is it true that (i) $\Longrightarrow$ (ii) ?
b.) Or is it true that (i) $\Longrightarrow$ (ii) is equivalent to Souslins hypothesis ?

2.) Let $E = C(X)$, X compact Hausdorff, E Dedekind σ-complete. Let (Δ, Γ, μ) be a atomless measure space such that $0 < \mu(\Delta) < \infty$. Let $S(\Gamma, E) = \{ t: \Delta \to E \mid t = \sum_{i=1}^{n} \chi_{A_i} e_i$ $A_i \in \Gamma$, $e_i \in E \}$ an, for $t \in S(\Gamma, E)$:

$$I_\mu(t) = \sum_{i=1}^{n} \mu(A_i) e_i \ .$$

Finally, let E_c^* be the collection of sequentially order continuous linear functionals on E . Consider :

(i) E is weakly σ-distributive.
(ii) E_c^* separates E .
(iii) If $t_n \in S(\Gamma, E)$ are such that $t_n(\xi) \downarrow 0$ $\forall \xi \in \Delta$, then $I_\mu(t_n) \downarrow 0$ (in ordering).

Then (ii) $\Longrightarrow$ (i), (ii) $\Longrightarrow$ (iii). So if i.a. is true then countably σ-generated E : (i) $\Longleftrightarrow$ (ii) $\Longrightarrow$ (iii).

a.) Do we have either (i) $\Longrightarrow$ (iii) or (iii) $\Longrightarrow$ (i) ,
b.) Or doe we have for countably σ-generated E that (iii) $\Longrightarrow$ (ii) ?

The Dedekind completion of $C([0,1])$ does not satisfy (i), (ii) or (iii) .

6. (Lindenstrauss) A Banach lattice E is called λ-injective ($\lambda \geq 1$) whenever for each B-lattice F such that $E \subset F$ there exists a positive projection P of F onto E such that $\| P \| \leq \lambda$. For $\lambda = 1$, the 1-injective B-lattices were characterized by Cartwright and Haydon. For the case of finite dimensional B-lattices it can be shown that the most general injective B-lattices are of the form

$$(*) \qquad G = (\sum_{j=1}^{k} \oplus 1_1^{(n_j)})_\infty$$

For a given finite dimensional λ -injective lattice E determine $M(\lambda) := \inf d(E,G)$, where G is of the form $(*)$.

In order to show that such a constant M exists one is led to the following purely combinatorial problem :

For each $\lambda \geq 1$ does there exists a constant $M = M(\lambda)$ having the following Property : Given any collection of subsets $\{\sigma_i\}_{i=1}^{\infty}$ of the integers such that for each i, card $\sigma_i = k$ (k=1,2, ..) and such that if $\sigma_{i_1} \cap \sigma_{i_2} \cap \sigma_{i_3} \cap \dots \cap \sigma_{i_\ell} \neq \emptyset$ it follows that $\mathrm{card}(\bigcup_{j=1}^{\ell} \sigma_{i_j}) \leq \lambda \cdot k$, there exists disjoint sets of integers $\{G_s\}_{s=1}^{\infty}$ with the following properties:

(i) For each s = 1,2 .. $\mathrm{card}(\bigcup_{i \in G_s} \sigma_i) \leq M(\lambda)k$.

(ii) If $\sigma_{i_1} \cap \sigma_{i_2} \cap \dots \cap \sigma_{i_\ell} \neq \emptyset$ then $\{i_1, i_2, \dots, i_l\}$ is contained in at most $M(\lambda)$ sets G_s.

7. (Luxemburg) An Archimedean Riesz space L is said to have the automatic order boundness property (a.o.b.) whenever every linear transformation T of L into L which preserves bands is order bounded. For instance, C([0,1]) has the a.o.b. property. $\mathcal{M}$ the space of all Lebesque measurable functions on [0,1] does not have the a.o.b. property. I have recently shown, that if L is non-atomic Dedekind σ-complete and none of its principal bands are latterally complete, then L has the a.o.b. property.

Conjecture: In the above theorem Dedekind σ-completeness may be replaced by complete in the relative uniform topology. In general find necassary and sufficient conditions for an Archimedean Riesz space to have the a.o.b. property.

8. (Nagel) Construct a lattice dilation of a strongly continuous semigroup $\{T(t) \mid t \geq 0\}$ of positive contractions on $L^1(X,\mu)$, i.e. does there exists a probality space $(\hat{X},\hat{\mu})$, a lattice injection J, a conditional expactation Q and a strongly continuous group of lattice isometries $\{\hat{T}(t) \mid t \geq 0\}$ such that the following diagramm commutes for all $t \geq 0$:

$$\begin{array}{ccccc} & L^1(X,\mu) & \xrightarrow{\quad T(t) \quad} & L^1(X,\mu) & \\ J & \downarrow & & \uparrow & Q \\ & L^1(\hat{X},\hat{\mu}) & \xrightarrow[\quad \hat{T}(t) \quad]{} & L^1(\hat{X},\hat{\mu}) & \end{array}$$

9. (Schaefer) Let $1 \leq p < \infty$ and let T be a positive linear operator from $L^p(\mu) \to L^p(\mu)$ such that $\|T\| = 1$. If $\Gamma = \{ z \mid z \text{ complex}, |z| = 1 \}$ and $\sigma(T) \subseteq \Gamma$, does it foollows that T is a isometry ?

10. (Schep) Let E be a Banach lattice and let T be a positive operator from E into E with spectral radius $r(T) = 0$.

Q1 . Is T the norm limit of a sequence of positive nilpotent operators ?

Q2. Is the answer to question 1 perhaps in the affir - mative if T is compact ?

Q3. Does there exists an increasing system $T_\alpha, \alpha \in \mathcal{A}$ of positive nilpotent operators such that $\sup T_\alpha = T$?

For operators on a Hilbert space every quasi-nilpotent operator is the norm limit of a sequence of nilpotent operators (C. Apostel, D. Voiculescu, C. Pearcy).

11. (Wolff) Let H denote a finite dimensional linear sub - space of a given Banach lattice E. Set $H^\wedge = \{\inf A \mid \emptyset \neq A, A \leq H, A \text{ finite}\}$. Then $\mathcal{H}(H) = \overline{H}^\wedge \cap (-\overline{H^\wedge})$ is called the space of all H-harmonic elements. It is always contained in the Korovkin hull $\mathrm{Kor}(H)$ of H.

Question : Does $\mathcal{H}(H)$ always be equal to $\mathrm{Kor}(H)$? $\mathrm{Kor}(H)$ is defined as follows : $x \in \mathrm{Kor}(H)$ iff for all equicontinuous nets (T_α) of positive linear operators on E $\lim T_\alpha x = x$ holds whenever $\lim T_\alpha y = y$ for all $y \in H$ holds.

12. (Zaanen) The existence of the absolute value $|h|$ of $h = f + ig$ in the complexification $E + iE$ of a uniformly complete Riesz space E is usually proved by representing the principal ideal generated by $e = |f| + |g|$ as a space of continuous functions. This is not necessary; there exists a proof showing that $|h|$ is the e-uniform limit of an e-uniform Cauchy sequence in E. It would be desirable to have also a direct proof of the Riesz decomposition theorem in $E + iE$ (i.e., if $h = f + ig$ and $|h| \leq f_1 + f_2$ for $f_1, f_2 \in E^+$, then there exist h_1, h_2 in $E + iE$ such that $h = h_1 + h_2$, $|h_1| \leq f_1$, $|h_2| \leq f_2$)

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