MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 28/1979

Maßtheorie

1. 7. bis 7. 7. 1979

Die Tagung, an der 60 Wissenschaftler aus 18 Ländern teilnahmen, stand unter der Leitung von D. Kölzow (Erlangen). In ihrem Verlauf wurden insgesamt 43 Vorträge gehalten; abgeschlossen wurde sie mit einer "Problem Session".

Es ist geplant, einen Tagungsbericht zu veröffentlichen, wenn möglich wieder in den "Lecture Notes in Mathematics" des Springer Verlages.

Die Tagungsteilnehmer möchten sich an dieser Stelle beim Direktor des Mathematischen Forschungsinstituts, Herrn Professor Dr. Barner, und seinen Mitarbeitern für die große Unterstützung bedanken, die den erfolgreichen Verlauf der Tagung möglich machte.

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Vortragsauszüge

Allgemeine Maßtheorie

D. BIERLEIN

Measure extension according to functions with and without measurable neighbours

Zu gegebenem Wahrscheinlichkeitsfeld $(M, \mathfrak{A}, \mu)$ und gegebener Funktion $f: M \longrightarrow E := [o,1]$ interessiert die Menge $\mathfrak{F} = \mathfrak{F}(\mu|\mathfrak{A}, f) := \{\mu_1|\mathfrak{A}_1 : \mu_1|\mathfrak{A} = \mu|\mathfrak{A}\}$ aller (σ-additiven) Maß-Fortsetzungen von $\mu|\mathfrak{A}$ auf die von $\mathfrak{A}$ und f erzeugte σ-Algebra $\mathfrak{A}_1$. (Aussagen über $\mathfrak{F}$ in diesem Fall lassen sich ohne weiteres übertragen, wenn μ durch ein beliebiges σ-finites Maß und E durch $\mathbb{R}$ ersetzt werden).

Mit Hilfe einer measurable selection-Technik erhält man Maß-Fortsetzungen eines speziellen Typs $\mathfrak{F}_1$, bei dem jede Fortsetzung mit einem meßbaren "Nachbarn" von f korrespondiert. Im allgemeinen ist aber $\mathfrak{F} \setminus \mathfrak{F}_1$ nicht leer. Insbesondere interessiert $\mathfrak{F}$ im Fall $\mathfrak{F}_1 = \emptyset$. Für atomare und atomfreie Maße wurde eine Übersicht über das Auftreten der Fälle $\mathfrak{F}_1 \neq \emptyset$, $\mathfrak{F}_1 = \emptyset \neq \mathfrak{F}$, und $\mathfrak{F} = \emptyset$ gegeben und durch Angabe von Kriterien und Beispielen belegt.

J.P.R. CHRISTENSEN

Small ball theorems and problems

A bounded signed measure on a metric space need not be uniquely determined by its values on all balls. However, for a large class of metric spaces (including Hilbert spaces) it is true that a signed measure vanishes if it vanishes on all balls with radius less than e for some e o (small ball theorems). Some positive results were proved and many open problems discussed. While we have a satisfying "small ball theorem" for Hilbert

spaces we do not know, even for Hilbert spaces, how to find a measure if we know it on small balls (e.g. whether we can compute the measure of a set by forming infima of sums of coverings). The methods and problems are potential theoretic in spirit rather than combinatorial as it is in classical geometric measure theory. (Ref.: J.P.R. CHRISTENSEN, The small ball theorem for Hilbert spaces, Math. Ann. 237 (1978), 273-276)

G.A. EDGAR

A long James space

Properties of the James-type Banach space $J(\omega_1)$ on the first uncountable ordinal were investigated. $J(\omega_1)$ is the set of all continuous functions $f: [0,\omega_1] \to R$ with $f(0) = 0$ such that the norm $\|f\| = \sup \{ (\sum_1^n |f(\alpha_i) - f(\alpha_{i-1})|^2$ $\alpha_0 < \dots < \alpha_n$ is finite. $J(\omega_1)$ is a second conjugate space with the Radon Nikodym property (RNP) which does not embed in a weakly compactly generated space (this answers a question of P. MORRIS). The space is a dual space with the RNP, but there exists a bounded scalarly measurable function on some probability space taking values in $J(\omega_1)$ that is not weakly equivalent to a Bochner measurable function; previously known examples with these properties depend on the existence of a measurable cardinal. $J(\omega_1)$ is a dual space with RNP, but the weak and weak* Borel sets do not coincide (this answers a question of M. TALAGRAND and the author).

E. GRZEGOREK

(reporting on joint work with C. RYLL - NARDZEWSKI)

Universal null and universally measurable sets

A σ-field $\mathfrak{A}$ on a set X is called measurable if there exists a continuous probability measure on $\mathfrak{A}$.

Theorem: (i) There exists a subset X of the real line which is not Lebesgue measurable and there exists a permutation p of X such that the σ-field

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generated by $\mathcal{B}(X) \cup p(\mathcal{B}(X))$ is not measurable ($\mathcal{B}(X)$ denoting the Borel subsets of X). Remark that $\mathcal{B}(X)$ and $p(\mathcal{B}(X)))$ are measurable. (ii) If all subsets of R of cardinality less than $2^{\aleph_0}$ are Lebesgue measurable, there exists a permutation p of R such that the σ-field generated by $\mathcal{B}(R)$ and $p(\mathcal{B}(R))$ is not measurable. (iii) If X and Y are subsets of R and p is a bijection of X onto Y then the σ-field on Y generated by $\mathcal{B}(Y)$ and $p(\mathcal{B}(X))$ is not measurable iff the graph of p is a universal nullset in R*R. (iv) There exists a universal null subset of R which does not have property C. (v) There exists a universally measurable subset of R which does not belong to the σ-field generated by $\mathcal{B}(R)$ and the ideal of universal null subsets of R.

In that theorem, R can be replaced by any uncountable Borel subset of some Polish space.

H.G. KELLERER

Baire sets in product spaces

Given an arbitrary index set I and topological spaces X_i, $i \in I$, the following problem was treated: Under which conditions is there equality between the Baire sets $\mathcal{L}(\prod_I X_i)$ of the product space and the product $\bigotimes_I \mathcal{L}(X_i)$ of the Baire σ-algebras on the factors. Main result: $\mathcal{L}(\prod_I X_i) = \bigotimes_I \mathcal{L}(X_i)$ if all finite products $\prod_T X_i$, $T \subseteq I$ finite, are Lindelöf. The explicit condition is given by " $X_i \in \{$second countable, σ-compact, Suslin$\}$ for all $i \in I$.

J. LEMBCKE

On a theorem of Bierlein

The following result, due to ASCHERL and LEHN, generalizes a theorem of BIERLEIN.

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Let $(\Omega, \mathfrak{S}, \mu)$ be a finite measure space and $\{A_i : i \in I\}$ be a family of pairwise disjoint subsets of Ω. Then there exists a σ-additive extension of μ to the σ-algebra $\mathfrak{S}'$ generated by $\mathfrak{S}$ and $\{A_i : i \in I\}$.
It was shown that one gets such a σ-additive extension by preordering the (non-empty) set $\mathfrak{F}$ of all finitely additive extensions of μ to $\mathfrak{S}'$ by $\nu \leq \nu'$ iff $\nu(A_i) \leq \nu'(A_i)$ for all $i \in I$, and taking a maximal element of $\mathfrak{F}$. Such a maximal element always exists.

W.F. PFEFFER

(reporting on joint work with R.J. GARDENER)

Some undecidability questions concerning Radon measures

Let X be a locally compact Hausdorff space, and let μ be a Radon measure defined on the Borel subsets of X. Assuming Martin's Axiom together with the negation of the Continuum Hypothesis the following statements are true:

(i) If X is meta-Lindelöf and μ is σ-finite, then μ is regular.

(ii) If X is hereditarily separable and μ is regular and diffuse, then μ is σ-finite.

On the other hand, assuming the Continuum Hypothesis, there is an example contradicting statement (i), and assuming Jensen's axiom $\diamondsuit$, there is an example contradicting (ii).

F. TOPSØE

Thin trees and the geometrical structure of Lebesgue nullsets

The process of successively halving the unit interval I, which involves the dyadic rational intervals, may conveniently be pictured by a tree.
A closed subset of I whose complement is a union of dyadic rational intervals then corresponds to a subtree whose infinite branches represent the points of the closed set. Criteria were given which ensure that there are

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so few infinite branches, that the closed set is a Lebesgue nullset. Using this idea - which also leads to a Vitali type theorem - a necessary and sufficient condition for a subset of R^N to be a nullset was given. The talk ended with a discussion of the fact that this result is not completely satisfactory. A counterexample by M. TALAGRAND to a satisfactory condition was mentioned.

R.F. WHEELER

Extensions of σ-additive measures to the projective cover

If X is a completely regular Hausdorff space, then there is an (essentially unique) extremally disconnected space E(X), called the projective cover or absolute of X, and a perfect irreducible map π of E(X) onto X. Let μ and ν be positive linear functionals on $C^*(X)$ and $C^*(E(X))$, respectively, represented as finitely additive Baire measures. Then ν is called a functional (resp., measure) extension of μ if $\nu(f \circ \pi) = \mu(f)$ for all f in $C^*(X)$ (resp., $\nu(\pi^{-1}(B)) = \mu(B)$ for all Baire sets B). Every measure extension is a functional extension, but the converse holds for all μ if and only if X satisfies a certain weak normality condition.
It was shown that if μ is σ-additive and X is either measure compact or weak cb, then every functional extension of μ must again be σ-additive. Conditions for every measure extension to be σ-additive were also obtained. Finally, connections were drawn between these ideas and the well known problem of extending a σ-additive Baire measure to the Borel sets.

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Meßbare Selektionen

M.P. ERSHOV

Some selection theorems for abstract spaces

The concept of a τ_m -operation (where m is a cardinal number) as a map from 2^{2^X} into 2^{2^X} was introduced, examples for such operations being closure of a subfamily of 2^X under union, intersection, Suslin operation and σ-algebra operations. Using this concept abstract measurable selection theorems for partitions of a set X were proved. For example:

Theorem: Let $\mathcal{A}$ be a partition of a set X, $\mathcal{X} \subseteq 2^X$, card $\mathcal{X} \leq \aleph_1$. Assume that

(i) the family $\mathcal{A} \cup \mathcal{X}$ separates points of X

(ii) for each $A \in \mathcal{A}$ the class $\{A \cap H: H \in \mathcal{X}\}$ has the finite intersection property

Then there exists a selection for $\mathcal{A}$ (i.e. a map f: X $\to$ X such that for all $A \in \mathcal{A}$: $f(A) = \{x\} \subseteq A$) such that for all $H \in \mathcal{X}$ the set $f^{-1}(H)$ belongs to the σ-algebra generated by the sets of the form $\bigcup \{A: A \cap H_1 \cap \ldots \cap H_n \neq \emptyset\}$ where $H_1, \ldots, H_n$ belong to $\mathcal{X}$.

C. GODET - THOBIE

Some results about multimeasures and their selections

Let $(\Omega, \mathcal{E})$ be a measurable space and X be a locally convex space. An X-valued multimeasure on $\mathcal{E}$ is a map M from $\mathcal{E}$ to the nonempty subsets of X which satisfies certain conditions of σ-additivity. Different definitions for infinite sums of subsets of X give rise to different notions of multimeasures (e.g. strong, normal and weak multimeasures). For a multimeasure M the set of all selections, i.e. all X-valued measures m on $\mathcal{E}$ such that $m(A) \in M(A)$ for all A, is denoted by S_M. Various conditions on M were given which ensure equality of the sets M(A) with either of the

sets $\{m(A): m \in S_M\}$, $\overline{\{m(A): m \in S_M\}}$ and $\overline{co}\,\{m(A): m \in \ddot{S}_M\}$.
Finally a result on measurable families of selections of a multimeasure was given.

S. GRAF

A parametrization of measurable sections via extremal preimage measures

Let $(X, \mathfrak{A}, \mu)$ be a finite measure space, Y a Hausdorff topological space, $\mathcal{B}(Y)$ the Borel field of Y and $p: Y \to X$ a $\mathcal{B}(Y)$-$\mathfrak{A}$-measurable map. Let M denote the set of all measures ν on $\mathcal{B}(Y)$ whose image $p(\nu)$ with respect to p is the given measure μ. Generalizing a result of EDGAR (Illinois J. Math. 20(1976), 630-646) it is shown that a Radon measure ν on Y is an extreme point of M if and only if there exists an $\mathfrak{A}_\mu$-$\mathcal{B}(Y)$-measureable weak section f for p with $\nu = f(\mu)$. Further, sufficient conditions were given which ensure that every extreme point of M is the image of μ under some measurable section for p. In the case that X and Y are Polish spaces and p is $\mathcal{B}(Y)$-$\mathcal{B}(X)$-measurable, onto, and $\mathfrak{A}$ is the field of universally measurable subsets of X the following result was deduced: If E is the set of extreme points of M equipped with the narrow topology there exists a $\mathcal{B}(E) \otimes \mathfrak{A}$ - $\mathcal{B}(Y)$-measurable map g from E×X into Y such that (i) for every ν the map $g(\nu, .)$ is an $\mathfrak{A}$-$\mathcal{B}(Y)$-measurable section of p and (ii) for any measurable section f of p there exists a measure ν in E with $g(\nu, x) = f(x)$ for μ-almost all $x \in X$.

D. MAULDIN

A coanalytic set and Borel parametrization

First, a natural example of a co-analytic non-Borel set was given in the following

Theorem: Let $M := \{f \in C([0,1]): f$ does not have a finite derivative anywhere$\}$

The set M forms a co-analytic non-Borel set of $C([o,1])$.

Second, necessary and sufficient conditions were determined in order that a Borel subset of the unit square be filled up by pairwise disjoint Borel uniformizations which are "parametrized" in a Borel fashion:

Theorem: Let $B \subseteq I^2$ be a Borel set. The following are equivalent:

(i) B contains a Borel set which has perfect vertical sections

(ii) there is a map $\mu : I \times \mathcal{B}(I) \to R$ such that $\mu(x,.)$ is a Borel probability measure for all x, $\mu(.,E)$ is Borel measurable for all $E \in \mathcal{B}(I)$ and $\mu(x,B_x) > o$ for all x

(iii) B has a Borel parametrization, i.e. there is a bijective Borel measurable map $g: I \times I \to B$ such that $g(x,I) = B_x$ for all x.

M. TALAGRAND

Selections and liftings

For X compact let Ba(X) be the Baire-σ-algebra and CO(X) be the σ-algebra defined by: $A \in CO(X)$ iff there exists a sequence of open sets such that A is a union of atoms of the σ-algebra generated by that sequence. (Hence CO(X) contains all Borel sets). Let μ be a measure on X, supported by X, ρ be a linear lifting of $L^\infty(\mu)$ and $B(\rho)$ be the σ-algebra generated by $\rho(L^\infty(\mu))$. The following results were proved:

1. If ρ is strong, then for any compact Y and continuous $p:Y \to X$ onto, there exists a $B(\rho)$-$Ba(M_+^1(Y))$-measurable map $x \mapsto \Theta_x$ from X into $M_+^1(Y)$ such that $\Theta_x(p^{-1}(x)) = 1$ for all x.

2. Let card $I \geq \aleph_2$, $Z := \{o,1\}^I$ and X be the space of closed subsets of Z equipped with the Hausdorff topology. Then there is no $CO(X)$-$Ba(M_+^1(Z))$-measurable map $F \mapsto \mu_F$ such that $\mu_F(F) = 1$ for all F.

3. Let X be as in 2. Then X supports a measure and for no measure μ supported by X there is a strong lifting of $L^\infty(\mu)$ such that $B(\rho) \subseteq CO(X)$.

4. Let I be a set of regular cardinality, $\lambda <$ card I. If there is a Baire

lifting of the canonical measure on $X := \{0,1\}^I$ (i.e. $B(\mathfrak{f}) \subseteq Ba(X)$) then each continuous map $p:Y \to X$ (p onto, Y compact) has a Ba(X)-Ba(Y)-measurable section.

D.H. WAGNER

Survey of measurable selection theorems: an update

An update was given of the author's "Survey of measurable selection theorems", SIAM J. Control and Optimization 15(1977), 859-9o3, for which a "Russian literature supplement" was given by IOFFE, Ibid. 16(1978), 728-732. Emphasis was on representation, i.e., parametrization, results of the following form: Given a measurable space $(T,\mathcal{M})$, a topological space X, and $\emptyset \neq F(t) \subseteq X$ for $t \in T$, find a nice (e.g. Polish) space Z and a nice (e.g. Carathéodory or Borel) map $f: T \times Z \to X$ such that $f(t,Z) = F(t)$, $t \in T$. These are largely by IOFFE, MAULDIN, SRIVASTAVA, and GRAF, following WESLEY and CENZER & MAULDIN. Among additional topics reviewed were results on compact-valued maps, optimal measurable selections, selections for partitions, and measurable weak sections. Over 7o titles were added to the bibliography.

Liftings

A.G. BABIKER

Almost strong liftings and τ-additivity

For completely regular space X and finite topological measure, i.e. a Baire or a Borel measure, on X was discussed the existence of strong liftings for the associated topological measure space. It was shown that when X is locally metrizable, the τ-additivity of the measure relative to the given topology on X, which is always necessary for the existence of almost strong liftings, is sufficient to ensure that all liftings are

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almost strong. There were given examples of locally metrizable, non-metrizable spaces with finite, τ-additive, non-trivial topological measures on them for which the existence of a strong lifting can not be deduced from other sufficient conditions known in the literature. A stronger τ-additivity criterion for a given lifting to be almost strong was given and this was used to show that a τ-additive Lebesgue measure space may admit liftings which are not almost strong.

P. GEORGIOU

On "idempotent" liftings

Let (T,μ) and (X,m) be two compact measure spaces with common Hyperstonean space (S,v) such that there is a continuous mapping $\omega: X \to T$ with $\mu = \omega(m)$. If (T,μ) has the strong lifting property, then there is an "idempotent" lifting of $\mathcal{L}^{\infty}_{R}(X,m)$.

Note: The "idempotent" lifting is defined as an idempotent element of the semigroup, which is defined on the set of liftings of $\mathcal{L}^{\infty}_{R}(X,m)$ (cf. Math. Annalen 208(1974), 195-202).

V. LOSERT

A Radon measure without the strong lifting property

The example, due to the author, of a Radon measure m on a compact space X which does not admit a strong lifting was presented. (For details see: V. LOSERT, A measure space without the strong lifting property, Math. Annalen 239(1979), 119-128).

In addition, it was remarked that the following can be proved in a similar way: there exist compact space X, Y, a Radon measure m supprted by X and a continuous surjective map p: $Y \to X$ which does not admit a section measurable with respect to the Baire sets on Y and the m-measurable sets on X.

Differentiation von Maßen und Integralen

M. DE GUZMAN

Some results and open questions in differentiation

Consider R^2 with Lebesgue measure. Let (θ_k) be a sequence in $[0,2\pi[$ and let B_θ be the differentiation basis of all open rectangles with one side in direction θ_k. Results: If $\theta_k = 1/k^p$, $p>0$, then B_θ does not even differentiate L^∞. If $\theta_k = 1/2^k$ then B_θ differentiates L^p for $p>1$ (STROMBERG, CORDOBA-FEFFERMAN, STEIN-WAINGER). If the θ_k determine the endpoints of intervals in the successive steps of the construction of a Cantor type set of positive measure, then B_θ is as bad as in the first case. It is unknown what happens if the θ_k arise in the construction of an ordinary Cantor set. The problem is connected with multiplier theorems for the Fourier transform.

Another problem: can one differentiate with respect to the dilatations of a fixed unbounded starshaped set of positive measure ? The answer is "yes" for L^p and $p>1$. If the fixed set satisfies a certain entropy condition, then it is also "yes" for L^1 (CALDERON, PERAL). This problem is connected with the construction of approximations of the identity by means of the dilatations of a fixed kernel in L^1.

W.A.J. LUXEMBURG

The Radon-Nikodym theorem revisited

Various forms of the Radon-Nikodym theorem were discussed with emphasis on the different ways the theorem may be proved. In particular it was shown in which case a nonstandard proof may give some additional insight. It was shown that extensions of the classical theorem to positive operators may be obtained with the help of the Maharam property.

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P. MATTILA

Differentiation of measures on uniform spaces

The question what kind of differentiation theorems for measures on uniform spaces are consequences of homogeneity properties of the basic measure was considered. A special case of the results presented is the following: Let μ be a regular Borel measure on a separable metric space X. If there are a non-decreasing function $h: (0,\infty) \to (0,\infty)$ with $\lim_{r \searrow 0} h(r) = 0$ and $c > 0$ such that $c \cdot h(r) \leq \mu(B(x,r)) \leq h(r)$ for μ-almost all $x \in X$ and for r greater than o, then for any $f \in L^p(\mu)$, $1 \leq p < \infty$,

$$(\mu(B(x,r)))^{-1} \int_{B(x,r)} f \, d\mu \to f(x) \quad \text{in } L^p(\mu) \text{ as } r \searrow 0.$$

Examples can be constructed to show that the uniform bounds for $\mu(B(x,r))$ cannot be replaced by $\liminf \mu(B(x,r))/h(r) \geq c$ or $\limsup \mu(B(x,r)) \leq 1$, and that the mean convergence cannot be replaced by pointwise converegence almost everywhere. Results of this type can also be proved for more general measures on uniform spaces.

A. VOLČIČ

On differentiation of Daniell integrals

Let I be a Daniell integral on $\mathcal{L}_1$. Define $\mathcal{L}_\infty$ by: $k \in \mathcal{L}_\infty$ iff $k \cdot f \in \mathcal{L}_1$ for all $f \in \mathcal{L}_1$. Then $\mathcal{A}_\infty := \{A: 1_A \in \mathcal{L}_\infty\}$ is a σ-algebra. Let J be the σ-ideal of sets T, such that $I(1_T \cdot f) = 0$ for all $f \in \mathcal{L}_1$. Let $\mathcal{M}_\infty$ be the $\mathcal{A}_\infty$-measurable functions and $\mathcal{M}_1$ be the Stone-measurable functions with respect to $\mathcal{L}_1$. I is called localizable if $\mathcal{A}_\infty/J$ is a complete lattice.

Theorem 1: The following are equivalent: (a) I is localizable (b) for each $I' \ll I$ there is a $g \in \mathcal{M}_\infty(I)$ such that $I'(f) = I(f \cdot g)$ for all f in the intersection of $\mathcal{L}_1(I)$ and $\mathcal{L}_1(I')$ (c) Each $K \ll I$ has a Hahn decomposition

A Radon-Nikodym derivative g of I' with respect to I is positive, iff $f \in \mathcal{M}_1(I)$ and $f \cdot g \in \mathcal{L}_1(I)$ implies $f \in \mathcal{L}_1(I')$ and $I'(F) = I(f \cdot g)$.

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Theorem 2: For every positive derivative g of I' w.r.t. I the set $\{x:g(x) = o\}$ is I' null iff there exists a strictly positive function in $\mathcal{M}_1$.

Examples: 1. There is a non-positive derivative (negative solution to a problem posed by KÖLZOW) 2. There is a Daniell integral with no strictly positive functions in $\mathcal{M}_1$.

Vektorwertige und Gruppenwertige Maße

P. MASANI

Stationary measures in Banach and Hilbert spaces

Let X be a Banach space over the real or complex numbers, G be a locally compact (additive) group, $\mathcal{P}$ be a prering of pre-compact Borel subsets of G, ξ be a finitely additive measure on $\mathcal{P}$ with values in X, and $S_\xi = \sigma(\xi(D): D \in \mathcal{P})$. We say that ξ is stationary, iff there exists a strongly continuous group of isometries U(.) on S_ξ onto S_ξ, parametrized be G, such that for all $D \in \mathcal{P}$ and all $t \in G$, $\xi(D+t) = U(t)\cdot\{\xi(D)\}$.

It was shown that every stationary measure ξ over R^q has the following canonical form, where $\mathcal{P}$ is the prering of subintervals of R^q,

$$\xi(A) = T(A)\cdot(\alpha), \quad A \in \mathcal{P}$$

Where $\alpha \in X$ and T(.) is a finitely additive S_ξ - to S_ξ operator valued measure on $\mathcal{P}$, explicitly definable in terms of U(.).

With $G = R^q$ and X = H, a Hilbert space, an explicit spectral representation for $\xi(D)$ and for the covariances $(\xi(A_1), \xi(A_2))_H$ was obtained.

P. McGILL

Elementary integrals

One of the advantages of using an elementary integration procedure is that the range space can be extremely general - usually a topological group

or a uniform semi-group. However such integrals are difficult to work with since they sometimes lack the usual properties. This talk discussed one approach which yields an integral which is quite general but nevertheless seems to possess enough structure to be useful. The definition is a generalisation of the Ito-belated integral of McSHANE using a control measure with values in a uniform semi-group. By using the work of DREWNOWSKI and SION it is possible to clarify the difficulties encountered. Relationships with other integrals were explored and a Dominated Convergence Theorem was proved.

P. MORALES

Regularity and extension of semigroup-valued Baire measures

For a non-empty class $\mathcal{A}$ of subsets of a given set, the symbols $\sigma(\mathcal{A})$, $\delta(\mathcal{A})$ will denote, respectively, the σ-ring, δ-ring generated by $\mathcal{A}$. Let X be a Hausdorff locally compact space, and let $\mathcal{K}$, $\mathcal{K}_o$ denote, respectively, the class of compact, compact G_δ subsets of X. Thus $\sigma(\mathcal{K})$, $\sigma(\mathcal{K}_o)$ are the class of Borel, Baire sets of X. Let G be a Hausdorff uniform semigroup. We say that a G-valued set function μ defined on $\sigma(\mathcal{K})$ ($\sigma(\mathcal{K}_o)$) is a Borel (Baire) measure if: (i) μ is σ-additive; (ii) the restriction $\mu|\delta(\mathcal{K})$ ($\mu|\delta(\mathcal{K}_o)$) is locally s-bounded. The main results were the following:

Theorem 1: Every Baire measure is regular.

Theorem 2: If G is complete, then every Baire measure extends uniquely to a regular Borel measure.

These results improve the well-known classical theorems, and generalise the recent group-valued results of SUNDARESAN and DAY (Proc. Amer. Math. Soc. 36(1972), 609-612) and KHURANA (Bull. Acad. Polon. 22(1974), 891-895).

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K. MUSIAŁ

A martingale characterization of the weak Radon-Nikodym property in Banach spaces

Let (S,Σ, P) be a complete probability space, X be a Banach space and $\mathcal{P}(S,\Sigma,P;X)$ be the space of all X-valued Pettis integrable functions on (S,Σ,P) endowed with the Pettis norm.

Theorem: For a Banach space X the following conditions are equivalent when holding for all complete probability spaces (S,Σ,P):

(i) X has the weak Radon-Nikodym property.

(ii) Given any directed set Π and a terminally uniformly integrable martingale $(f_\pi, \Sigma_\pi)_{\pi\in\Pi}$ of X-valued Pettis integrable functions on (S,Σ,P) then $(f_\pi,\Sigma_\pi)_{\pi\in\Pi}$ is convergent in $\mathcal{P}(S,\Sigma,P;X)$.

(iii) Given any directed set Π and a uniformly bounded martingale (f_π,Σ_π) of X-valued Pettis integrable functions on (S,Σ,P), then $(f_\pi,\Sigma_\pi)_\pi$ is convergent in $\mathcal{P}(S,\Sigma,P;X)$.

In the above conditions Π can be taken to be the set of natural numbers and the functions f_π may take only a finite number of values. Also (S,Σ,P) can be chosen to be the unit interval with Lebesgue measure.

E. PAP

Integration of functions with values in complete semi-vector spaces

Using some ideas of MIKUSINSKI's approach to the Bochner integral, an axiomatic treatment of integrals of semigroup valued functions was presented.

Let X be a commutative semigroup with neutral element 0 and with a complete metric d satisfying $d(x+y, 0)\leq d(x,0) + d(y,0)$ for x,y in X. Let K be an arbitrary set and U be a family of functions from K to X. Assume that a function $\int$ (called integral) is defined on U such that the following axioms are satisfied: (N) $0\in U$ and $\int 0 = 0$, (D) If f and g are in U, then

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$d(f,g)$ is in U and $d(\int f, \int g) \leq \int d(f,g)$, (E) If (f_n) is a sequence in U such that $\sum \int d(f_n,0) < \infty$ and the equality $f(x) = \sum f_n(x)$ holds at every point x in X where $\sum d(f_n(x),0) < \infty$, then $f \in U$ and $\int f = \sum \int f_n$.

If the metric d satisfies also $|d(x,0) - d(y,0)| \leq d(x+y,0)$ and is translation invariant, then the theory of such an integral can be enriched with: completeness of the space $\tilde{U}$ (the set of all classes), dominated convergence theorem, Riesz theorem, Fubini theorem and others.

If $K = R^q$ and X is a complete semi-vector space (in the generalized sense), then, if axiom (N) is replaced by axiom (H)"If $f \in U$ and $\alpha \geq 0$, then $\alpha f \in U$ and $\int \alpha f = \alpha \int f$ " and if the metric satisfies some natural conditions, a model for the HED-integral can be constructed. In special cases MIKUSINSKI's HEM-integral, the Bochner integral and the Lebesgue integral are obtained.

Stochastische Integration und Wahrscheinlichkeit

K. BICHTELER

The stochastic integral as a vector measure

Given a right-continuous process Z, consider the elementary integral it defines as a linear map $dZ: \mathcal{E} \to L^p(P)$. The collection $\mathcal{E}$ of elementary integrands is given the sup-norm topology. For dZ to have an extension $\int . \, dZ$ satisfying the Dominated Convergence Theorem, it suffices that $dZ: \mathcal{E} \to L^p(P)$ is continuous, $0 \leq p < \infty$. Daniell's method then produces the ectension. If $0 \leq p < q \leq 2$ there is a probability $P' \sim P$ so that $dZ: \mathcal{E} \to L^q(P')$ is continuous and its modulus of continuity is controlled by that of $dZ: \mathcal{E} \to L^p(P)$. This permits the <u>pathwise</u> computation of stochastic integrals $\int X \, dZ$ for left continuous integrands with right limits and of the solution of a stochastic differential eqation controlled by an arbitrary semimartingale.

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C. DELLACHERIE

A survey of stochastic integration

Let $(\Omega, \mathcal{F}, P)$ be a probability space with a filtration $(\mathcal{F}_t)_{t \in R^+}$ satisfying the usual conditions (i.e. $(\mathcal{F}_t)$ is right continuous, $\mathcal{F}$ is complete and $\mathcal{F}_o$ contains all null sets).

A survey of that part of stochastic integration which depends only on the class of null sets of P was given. The important notion of a semimartingal was introduced and it was shown, that in some strong sense, these are the only reasonable processes with respect to which it is possible to integrate. The stochastic integral with respect to a semimartingale was defined and the "calculus" for such an integral developped: ITO's formula for change of variables; existence and uniqueness of solutions of stochastic differential equations satisfying a global Lipschitz condition.

V. GOODMAN

The law of the iterated logarithm in Hilbert spaces

Let (X_i) be a sequence of identically distributed random variables with values in a separable Banach space B. Assume that for $\mu = \mathcal{L}(X_1)$ the dual of B is contained in $L^2(\mu)$ and that $\int_B \langle y,x \rangle \mu(dx) = o$ for all $y \in B^*$.

Consider the cluster set of $T := \{(\sum_1^n X_i)(2n \ln\ln n)^{-1/2}\}$ for almost all ω. By a theorem of KUELBS one has: There exists a non-random cluster set K; $S = \int_B \langle \cdot, x \rangle x \, \mu(dx)$ exists as a Pettis integral in the space of bounded operators from B^* to B and can be continuously extended to an operator defined on the closure of B^* in $L^2(\mu)$; $K' := \{\tilde{S}y : \langle \tilde{S}y, y \rangle \leq 1\}$ is bounded and contains K. For B = H a separable Hilbert space one has:

Theorem 1 (GOODMAN, KUELBS, ZINN): If S is compact and (a) $x \mapsto \|x\|^2 \ln\ln\|x\|$ is in $L^1(\mu)$ and $\sup\{t^2 \cdot \mu(\|x\| \geq t) : t > 0\} < \infty$ or (b) $x \mapsto \|x\|^2 \ln\ln\ln\|x\| / \ln\ln\|x\|$ is in $L^1(\mu)$, then T is compact with probability 1 and K = K' is compact.

Theorem 2: If S is bounded and (a) or (b) as above hold, then T is bounded.

D.A. KAPPOS

A kind of random integral

Let $(\mathfrak{B},p)$ be a probability σ-algebra and $\mathfrak{X}$ be the stochastic space of real valued random variables over $(\mathfrak{B},p)$ (see D.A. KAPPOS, Probability algebras and stochastic spaces, Acad. Press 1969). Let S be a nonempty set and $\mathfrak{F} := \mathfrak{F}(S,\mathfrak{X})$ be the space of random functions $f: S \rightarrow \mathfrak{X}$. Then $\mathfrak{F}$ is a conditionally complete lattice algebra. On $\mathfrak{F}$ the notions of o-convergence and uniform convergence are defined. Let $\mathfrak{A}$ be a Boolean σ-algebra of subsets of S. A function $\mu: \mathfrak{A} \rightarrow \mathfrak{X}$ is called a random measure iff it is positive and σ-additive w.r.t. the o-convergence. Modifying o-convergence and uniform convergence modulo μ one gets μ-convergence and almost uniform convergence w.r.t. μ. Properties of and relations between these notions of convergence were investigated.
Then the spaces of simple measurable, measurable and μ-integrable random functions were introduced. The μ-integral was extended from the space of simple functions to the space of integrable functions and - among other results - a dominated convergence theorem proved.

L. SUCHESTON

(reporting on joint work with A. MILLET)

Martingales, stopping times, Vitali conditions

Let $(\mathfrak{F}_t)$ be an increasing family of σ-algebras indexed by a directed set J. It was shown that every L_1-bounded real valued martingale converges essentially if and only if a weak type of maximal inequality holds for all martingales: $\lambda \cdot P(\text{e-lim sup } |X_t| \geq \lambda) \leq \lim E(|X_t|)$. A new covering condition C stated in terms of multivalued stopping times was introduced and characterized in terms of maximal inequalities. C was shown to be strictly weaker than the Vitali condition V, than SV (see.C.R. Acad. Sci. Paris, 288(1979), 595-598), and also sigma-SV. Under C, L_1-bounded

martingales taking values in a Banach space with the Radon-Nikodym property converge essentially. Also a point derivation version of condition C was given, sufficient to obtain Lebesgue's theorem.

W.A. WOYCZYNSKI

On Marcinkiewicz-Zygmund laws of large numbers in Banach space and related rates of convergence

It was shown, in particular, that for independent strongly measurable random variables (X_i) taking values in a real separable Banach space B and having uniformly bounded tail probabilities the implication " if $E(\|X_i\|^p) < \infty$, $E(X_i) = 0$ then $S_n / n^{1/p} \to 0$ almost surely " depends in an essential way on l^p not being finitely representable in B.

L^p - Räume und verwandte Gebiete

A. KATAVOLOS

Non-commutative L^p-spaces

Given a von Neumann algebra M equipped with a semifinite faithful normal trace t, one constructs the the non-commutative L^p-spaces $L^p(M,t)$ which are Banach spaces for $p \geq 1$. Given a linear mapping T between two such spaces $L^p(M_1,t_1)$ and $L^p(M_2,t_2)$, for $p \geq 2$, which maps normal elements to normal elements, and is isometric on normal elements, it was shown that, if the traces are finite and T preserves the identity, then T restricted to M_1 must be isometric, ultraweakly continuous, and the direct sum of a $*$-homomorphism and a $*$-antihomomorphism. Further it was shown that the existence of an isometric linear bijection between a non-commutative and an ordinary L^p-space implies, for $p \neq 2$, that the underlying von Neumann algebras are isomorphic as von Neumann algebras, and hence both must be Abelian. Thus the non-commutative L^p-spaces form a new class of

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Banach spaces, distinct from classical ones.

W. SCHACHERMAYER

Integral operators on L^2-spaces

Let (X, μ) and (Y, ν) be finite measure spaces. The following characterisation of integral operators was given:

Proposition: $T: L^2(\nu) \rightarrow L^2(\mu)$ is an integral operator (i.e. representable by a kernel function) iff T transforms order-bounded sets into equi-measurable sets.

The method of proof depends on the following principle: Consider a kernel $k: X \times Y \rightarrow \mathbb{C}$ as a function from X into a vector space of functions on Y.

Using the same method one can also prove:

Proposition: If $T: L^2(\nu) \rightarrow L^2(\mu)$ is integral then for each $1 \leq p \leq 2$ the composition of T with the canonical injection of $L^2(\mu)$ into $L^p(\mu)$ is compact.

D. SENTILLES

Stone space representation of vector functions and operators on L^1

An operator T on $L^1(\Omega, \Sigma, \mu)$ into a Bnach space X easily admits a weak integral representation $\langle T\nu, x'\rangle = \int_S \langle x', DT\rangle d\nu$ for $\nu \in L^1$ where S is the Stone space of $\Sigma / \mu^{-1}(o)$ and $DT: S \rightarrow X''$. T is Bochner representable on Ω as well iff $DT^{-1}(X'' \setminus X)$ is nowhere dense and T is weakly compact iff $DT(S) \subseteq X$. In either case DT is then norm continuous on an open dense σ-compact set in S and the Bochner representative of T on Ω is related to DT on S in the following way: There exist closed nowhere dense sets $C_\omega \subseteq S$, with dense union, such that T is strongly differentiable at iff $DT|C_\omega$ is constant and norm continuous (and then equal on C_ω to the strong derivative). A method of lifting $L^\infty(\Omega, \Sigma, \mu, X)$ results.

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S. TOMÁŠEK

Über einen Isomorphiesatz

Es sei $\mathcal{C}$ eine Klasse von topologischen Vektorräumen, die bezüglich der üblichen algebraischen und topologischen Operationen (Bildung der vollständigen Hülle, des assozierten separierten Faktorraums, von Unterräumen, von kartesischen Produkten) stabil ist. Es seien E und F zwei separierte Vektorräume in $\mathcal{C}$. In $E \otimes F$ wird eine Tensortopologie definiert, und zwar die projektive Tensortopologie, die durch alle $u \in \mathfrak{B}(E,F;G)$, $G \in \mathcal{C}$, erzeugt wird. Wir schreiben dann $E \otimes_{\mathcal{C}} F$.

Satz: Ist $\hat{E} \otimes_{\mathcal{C}} \hat{F}$ separiert und zu $\mathcal{C}$ gehörig, so sind die Tensorprodukte $(\hat{E}) \hat{\otimes}_{\mathcal{C}} (\hat{F})$ und $E \hat{\otimes}_{\mathcal{C}} F$ (topologisch) isomorph.

Elementare Folgerung: $L^1(\mu) \hat{\otimes} F \cong L^1_F(\mu)$, $\mu \geq 0$, F metrisierbar, lokalkonvex

$$L^1(\mu) \hat{\otimes} L^1(\nu) \cong L^1(\mu \otimes \nu), \quad \mu, \nu \geq 0$$

Integraldarstellungen

M.M. RAO

Local functionals

If F is a (linear) function space on some set, then a mapping M from F into the scalar field is called a local functional (in the sense of GEL'FAND) if M(f+g) = M(f) + M(g) for all f,g in F with f·g = o. These arise in the theory of generalized random processes taking independent values at each point, and elsewhere. It is of interest to get integral representations of such functionals under suitable conditions. The spaces F of interest for probability are the Schwartz spaces of infinitely often differentiable functions with compact support, or $F = C_{oo}(G)$, the continuous scalar functions on a locally compact space G having compact support. Elsewhere F is a Sobolev or a Lebesgue space. On each of these spaces the methods of representation are not the same, even though one may describe

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them as certain Lebesgue-Nikodym type results. An account of some of this work was presented for general locally compact G.

E.G.F. THOMAS

Integral representation in convex cones

Let F be a real locally convex Hausdorff space which is quasi-complete and $\Gamma \subseteq F$ a closed convex proper cone. ext Γ denotes the set of extreme generators of Γ and (in case ext$\Gamma \neq \emptyset$) S a fixed subset of extΓ, not containing o, meeting each extreme ray in precisely one point and satisfying some measurability condition. The following definition does not depend on the choice of S : $a \in \Gamma$ has a (unique) integral representation by means of extreme generators iff there exists a (unique) Radon measure m on S such that for all $l \in F'$: $l \in L^1(m)$ and $l(a) = \int l(x)\, m(dx)$.

Problem: For which cones Γ does every $a \in \Gamma$ have a (unique) integral representation by means of extreme generators.

Generalizing a classical theorem of CHOQUET and theorems by EDGAR and the author, the following result was obtained:

Theorem: Let Γ be γ-conuclear, the sets in γ being bounded convex Suslin sets with the Radon-Nikodym Property, then every point in Γ has an integral representation by means of extreme generators. This representation is unique for each point if and only if Γ is a lattice in its own order.

Integraltransformationen von Maßen

A. HERTLE

The Radon transform of measures

The Radon transform (RT) defined by $(Rf)(x,p) = \int_{\langle x,y\rangle = p} f(y)dy$ can be considered as an operator from $L^1(R^n)$ to $L^1(S^{n-1} \times R)$. First it was shown that the inversion problem for the RT cannot be properly posed on L^1. Guided by that result, the RT was extended to finite measures on R^n and

on separable Hilbert spaces H as follows :

$$(Rm)(g) = - \int_S \int_R \frac{\partial}{\partial p} g(x,p) \int_{\langle x,y\rangle < p} dm(y)\, dp\, d\mu_S(x)$$

Here, μ is a Gaussian measure on H and μ_S the surface measure induced by μ on the sphere S of H (in the case $H = R^n$ μ is the normal distribution). After that, the inversion problem for the RT is well-posed. In particular, a function on H is uniquely determined by its μ-surface integrals over all hyperplanes in H. Among other things, theorems of HELGASON and JOHN can be generalized from functions on R^n to measures on H.

Verschiedenes

C. CONSTANTINESCU

Spaces of multipliable families in Hausdorff topological groups

Let G be a Hausdorff topological group, I be a linearly ordered set, $\mathcal{P}(I)$ be the power set of I endowed with the compact topology obtained by identification with $\{0,1\}^I$, and let l be the set of families $(x_i)_{i\in I}$ in G such that $(x_i)_{i\in J}$ is multipliable for every $J \subseteq I$.

Theorem 1: For every (x_i) in l the map $J \longmapsto \prod_{i\in J} x_i$ from $\mathcal{P}(I)$ into G is continuous.

Theorem 2: If $((x_{n,i})_{i\in I})_{n\in N}$ is a sequence in l such that $(\prod_{i\in J} x_{n,i})_n$ converges for every $J\subseteq I$, then the convergence is uniform in J, the family $(\lim x_{n,i})_{i\in I}$ belongs to l and $\prod_{i\in J}(\lim_n x_{n,i}) = \lim_n \prod_{i\in J} x_{n,i}$ for every $J \subseteq I$.

Some corollaries of these theorems were presented, among them generalizations to the non-commutative case of results of ANTOSIK, DREWNOWSKI, KALTON, LABUDA and THOMAS.

T.E. DUNCAN

A geometric approach to some stochastic problems

The solutions to some problems of estimation and control for linear pure delay time systems were given. Such a system can be viewed as a system over a ring of polynomials formed from the delays and the corresponding algebraic vector bundle can be formed. The estimation and control problems are formulated in the symplectic vector bundle obtained from the Lagrangian Grassmannian description of these problems in each fiber of the vector bundle that describes the system. In addition, the infinite time estimation problem was shown to be well posed given a natural observability condition in the fibers of the vector bundle. Some geometrical remarks were also made on some other related stochastic problems.

F.Y. MAEDA

A convergence property for solutions of Euler equations of certain integral functionals

Let U be an open set in R^n and $f(x,t,\tau)$ be a real function on $U \times R \times R^n$ which is convex and C^1 in (t,τ). The Euler (-Lagrange) equation for the integral functional $I(u) = \int f(x,u,\nabla u)dx$ is formally written as $Lu = -\operatorname{div} \nabla_\tau f(x,u,\nabla u) + D_t f(x,u,\nabla u) = o$. Let H be the set of all weak solutions of Lu = o on U. The problem here is to see whether H is "closed" in the following sense: If $u_n \in H$, $\{u_n\}$ is locally uniformly bounded and $u_n \to u$ almost everywhere on U, then $u \in H$.
If f satisfies certain structural conditions, then H has this property.

W. SŁOWIKOWSKI

Abstract path spaces

One considers a unitary group with reflection, which is a triplet $(H, U(.), H_o)$, where H is a Hilbert space, H_o a subspace of H and U(.) a

unitary group on H such that the set of translations of H_o by U(.) is total in H. Assume the reflection principle, i.e. $E_oU(t)E_o$, where E_o is the projection onto H_o, is selfadjoint for all t. Then the reflection $\sim$ is introduced, which is the identity on H_o and intertwines U(t) and U(-t) for all t. Denote by E_+ the projection onto the linear span of $U(t)H_o$, $t \geq o$. Require positivity of $E_+^{\sim}E_+$ which is called the Osterwalder-Schrader condition. Let F_o be the orthogonal projection of $H_+ = E_+H$ onto the orthogonal complement of the null space of $E_+^{\sim}E_+$. $F_oU(t)$, $t \geq o$, extends to a contraction semigroup on the completeion F of F_oH_+ with respect to the norm $\|(E_+^{\sim}E_+)^{1/2}\cdot\|$, where $\|\cdot\|$ is the norm in H. By use of spectral theory it was proved that every contractive semigroup of selfadjoint operators on a Hilbert space F which moves a subspace H_o over a total subset of F, up to unitary equivalence, originates in a unique way from a unitary group with reflection as described above. This result was connected with a result of KLEIN (Journ. Funct. Anal. 1978) on measure preserving groups. All groups and semigroups are assumed strongly continuous.

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