MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 30|1979

Universelle Algebra

15. 7. bis 21. 7. 1979

Die Tagung fand statt unter der Leitung der Herren W. Felscher (Tübingen), G. Grätzer (Winnipeg) und R. Wille (Darmstadt).

Die Themen der Vorträge lassen sich schwerpunktmäßig gliedern in: Varietäten universeller Algebren (Kongruenzrelationenverbände - Mal'cev-Bedingungen - modelltheoretische Fragen); Theorie spezieller Algebren; kompositionsabgeschlossene Funktionenalgebren("clones"); Verbände und geordnete Mengen; Algebren und Automaten.
Zusätzlich fand eine "Problemsitzung" statt, auf der jeweils mit einer kurzen Einführung offene Probleme vorgestellt wurden, die z.T. im Zusammenhang zu den Vorträgen standen.

Vortragsauszüge

SCHMIDT, E.T.: Congruence lattices of lattices

The congruence lattice of an arbitrary lattice is a distributive algebraic lattice, i.e. the ideal lattice of a distributive semilattice with 0. The converse of this statement is a long-standing conjecture of lattice theory. It is proved:
THEOREM. Let L be the lattice of all ideals of a distributive lattice with 0. Then there exists a lattice K such that L is isomorphic to the congruence lattice of K.
The basic step is the following: if B_i, $i\in I$, are four-element Boolean algebras and L is a bounded distributive lattice such that we have for each $i\in I$ a $\{0,1\}$ - $\vee$-homomorphism $\varphi_i: B_i \to L$, then there exists a $\{0,1\}$ $\vee$-hom. φ of the free $\{0,1\}$-distibutive product $B = \prod^* B_i$ into L satisfying $\varphi / B_i = \varphi_i$.

DAY, A.: Varieties of congruence lattices

The form of Polin's algebras and in particular the form of congruence lattices of these algebras leads to a set of functors defined on (finite) lattices. Let A be a finite Boolean algebra and L a finite lattice; $T_A(L) = \{(a,f) \in A \times L^A$: f monotone and $f(x \vee a) = f(x)\ (x \in A)\}$. Lemma 1: T_A preserves and reflects monos, epis, subdirectly irreducibleness, bounded (McKenzie). Lemma 2: $T_{A \times B} \xrightarrow{n.t.} T_A \circ T_B$. Other properties of these functors and their applications to congruence varieties are discussed.

FRIED, E.: Connection between the congruence-lattice and polynomial properties

This is a joint work with E.W. Kiss. Relations between the following properties are described: Congruence-permutability, congruence-distributivity, filtrality, semisimplicity, having complement for each (principal) congruence, (dual) discriminator variety, having restricted uniform congruence scheme, congruence extension property. Some of the main results: RUCS implies CD, FI is equivalent to CD + PCC, a variety is a discriminator-variety iff it is CP + CD + PCC. The method, used, is the polynomial description of the above conditions. These conditions give us how can one build up a chain from c to d, using principal congruences. To get equations we define Pixley conditions which is a generalization of Malcev's condition when, also, the operationsare included. These conditions describe FI or RUCS with a similar manner as the one used by B. Jónsson to describe CD. Any of these has a three- and a four-variably version. A problem, stated that CM + CC yields PC, was solved by Ch. Herrmann a few days before the meeting, in the affirmative.

HUTCHINSON, G.: Applying Lattice Word Problem Algorithms To Related Theories

The word problem for a lattice variety generated by all lattices of submodules for modules over a fixed ring R is known to be computable in terms of certain simple properties of R. Several algebraic and category theories that are closely related to the theory of modules over a fixed ring R are discussed. Classes of

problems in these theories which can be systematically reduced to lattice word problems are identified. Computational methods are discussed.

TRACZYK, T.: On a variety arising from arithmetic and set theory

Let us consider a variety $\mathcal{V}$ of algebras of similarity type (2,0) defined by the following set of identities: (1) $(xy)z = (xz)y$, (2) $x(xy) = y(yx)$. (3) $xx = 0$, (4) $x0 = x$.
It is known (K.Iseki, S.Tanaka) that variety $\mathcal{U}$ of meet semilattices with 0 is representable in $\mathcal{V}$. The representation is given by the definition (5) $x \wedge y = x(xy)$. $\mathcal{U}$ and $\mathcal{V}$ are not equivalent. However, in the finite case we have the following THEOREM. Let L be a finite member of $\mathcal{U}$. There exists an operation, $\cdot$, on L such that (1) - (5) hold if and only if, for every maximal element x in L, the interval $[0,x]$ is a product of chains.
COROLLARIES. 1. A finite semilattice is consistent either with none or with exactly one member of $\mathcal{V}$. 2. If L in $\mathcal{V}$ is finite and has the greatest element 1 then its congruence lattice is a Boolean algebra. However, the lattice of congruences of an arbitrary member of $\mathcal{V}$ is a distributive lattice, only.

BOSBACH, B.: Quaderalgebren

$Q = (Q,*,1)$ heiße eine Quaderalgebra, wenn Q die Gleichungen $(a * a) * b = b$, $a * (b * c) = b * (a * c)$, $(a * b) * b = (b * a) * a$, $1 * a = a * a$ erfüllt.
Hauptsatz: Alle Quaderalgebren entstehen i.w. dadurch, daß man in einer abelschen Verbandsgruppe ein Element $1 \geq 0$ wählt und $a * b := o \cup (b - a)$ erklärt. Als eine spezielle Quaderalgebra stellt sich E^I ($E = \{x \mid 0 \leq x \leq 1\}$) bezüglich $a * b = \max(0, b-a)$ dar.
In dem Vortrag wird u.a. die Frage geklärt, welche Quaderalgebren Würfelalgebren, d.h. Unteralgebren von $(E^I, *, 1)$ sind. Dabei kommen maßtheoretische und topologische Aspekte mit ins Spiel. Wesentlich scheint die Bemerkung, daß die angestellten Untersuchungen nur unwesentlich an der Kommutativität der Verbandsgruppe hängen.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

ROMANOWSKA, A.: On free .-distributive bisemilattices

A bisemilattice is an algebra with two binary operations which satisfy all axioms of lattices without absorption laws. A bisemilattice which further satisfies the identity $x(y+z) = xy + xz$ is called .-distributive (.DBSL). For some years, these algebras have been investigated in papers of R. Balbes, R. Padmanabhan, J. Plonka, A. Romanowska. The known theorem about a representation of free distributive lattices as a ring of semifilters of the set of all nonempty subsets of a set can be generalized as follows. Let X be a set of the cardinality α, and $P_f(X)$ the set of all finite nonempty subsets of X. Define $B_\alpha = \{t \in P_f(P_f(X)): v, u \in t \text{ implies } u \cup v \in t\}$, and $t.s = \{u \cup v: u \in t, v \in t\}$, $t + s = t \cup s \cup ts$. Then $(B_\alpha, +, .)$ is a free .DBSL on α generators. This theorem is a base for to give a construction of free .DBSL's as semilattices of lattices of Boolean semilattices and bounds for the number of elements of finite free .DBSL's.

BANDELT, H.-J.: Median algebras (joint work with J. Hedlikovâ)

Median algebras are ternary algebras satisfying all the identities true for the median operation $(x \wedge y) \vee (x \wedge z) \vee (y \wedge z)$ in distributive lattices. It is known from results of M. Sholander and S.P. Avann that certain meet-semilattices (so-called median semilattices) can be converted into median algebras and, conversely, that every median algebra can be viewed as a median semilattice (with 0). Our main object was to give several characterizations for pairs of median semilattices with the same underlying set which give rise to the same median algebra.

TRNKOVA, V.: Algebras and automata

Generalizing universal algebras, M. Barr introduced (1970) the functorial algebras as follows: If $\mathcal{K}$ is a category and $F: \mathcal{K} \to \mathcal{K}$ an endofunctor, then F-algebra is (Q, δ), where $\delta: FQ \to Q$ is a $\mathcal{K}$-morphism. (E.g., groupoids are obtained by the choice $\mathcal{K}$ to be the category Set of all sets and mappings, F sending any Q to Q x Q.) The properties of functorial algebras heavily depend on the functor F and even in Set, strange situations occur. This has been investigated in the Prague seminar from General Mathe-

matical Structures. Some of the obtained results will be summarized (free algebras, finite congruences).
A close connection between universal algebras and tree automata was observed and investigated by Büchi, Thatcher, Wright, Eilenberg and others; Arbib and Manes extended this to functorial algebras. Some results obtained in Prague about functiorial automata will be presented.

NELSON, E.: Universal Algebraic Aspects of Computer Science

The aim of the lecture was to elucidate some of the ideas of the ADJ group (Goguen, UCLA, and Thatcher, Wagner and Wright, IBM) concerning applications of "continuous algebraic theories" to computer science. They describe flow charts as graphs whose edges are labelled with partial functions $S \to S$ (for some set S) such that edges with the same origin are labelled with partial functions whose domains of definition are disjoint. The behavior of such a flow chart is seen to be the least fixed point of a certain type of equation in the algebraic theory whose objects are the sets n x S and whose morphisms are all partial functions between them. This motivates an interest in ordered algebraic theories, and hence in ordered algebras, with appropriate completeness, as follows: Suppose for each p.o. set P that ZP is a set of subsets of P such that for any order preserving $f: P \to Q$, $f(A) \in ZQ$ whenever $A \in ZP$. Then P is called Z-complete iff every set in ZP has a join, and Z-continuous maps are those preserving Z-joins. The category $ZAlg_{\Sigma}$ consists of all partially ordered, Z-complete Σ-algebras with least element $\perp$ and Z-continuous operations, and all Z-continuous $\perp$-preserving Σ-homomorphisms. For ZP = all countable chains in P, or all subsets of P such that every pair of elements has a common upper bound in P, there is a concrete description, via trees, of free Z-continuous Σ-algebras. For arbitrary Z the question of existence of such free algebras is open.

WERNER, H.: A duality for weakly associative lattices

Let $\underset{\sim}{P}$ be a finite algebra and R a set of algebraic relations on $\underset{\sim}{P}$ (i.e. subalgebras of finite powers of $\underset{\sim}{P}$) and $\mathfrak{R}$ the class of all closed subspaced of $\underset{\sim}{P} = (P,R)_{discr.}$ which are also closed under those $r \in R$ which happen to be partial operations an P, and let $\mathfrak{A} = \mathbb{I}\,\mathbb{S}\,\mathbb{P}\,(\underset{\sim}{P})$ be the quasiveriety generated by $\underset{\sim}{P}$. Then $\underset{\sim}{A}^{+} := \{x:\underset{\sim}{A} \to \underset{\sim}{P} \mid x \text{ homomorphism}\}$ $(\underset{\sim}{A} \in \mathfrak{A})$ and $X^{*} := \{\varphi:X \to P \mid \varphi \text{ continuous and R-preserving}\}$ $(X \in \mathfrak{R})$ defines a full duality (i.e. an antiisomorphism between the categories $\mathfrak{A}$ and $\mathfrak{R}$) provided the following hold:

(i) Each R-preserving finitary operation on P is a polynomial on $\underset{\sim}{P}$.

(ii) $\underset{\sim}{P}$ is injective in $\mathfrak{R}$.

(iii) Epimorphisms in $\mathfrak{R}$ are onto.

Using this theorem we can prove a full duality for $\underset{\sim}{W}_n$ if we choose R to be $\{\leq\}$ or $\{\leq,\delta\}$ for $n=2$ or 3, and $R = \{\leq,\delta\} \cup \mathrm{Aut}(\underset{\sim}{W}_n)$ for $n \geq 4$. $\mathfrak{R}$ can be characterized in purely topological terms.

JEZEK, J.: Medial groupoids

Groupoids satisfying the law $xy.uv=xu.yv$ are called medial. Free groupoids in the variety generated by medial cancellation groupoids and the equational theory of these groupoids are described. It is proved that for every medial groupoid G such that GG=G there exists a commutative semigroup S(+) and its two commuting automorphisms f,g such that $G \subseteq S$ and $xy=f(x)+g(y)$ for all $x,y \in G$; in the commutative case the condition GG=G can be deleted and we can demand f=g. It is proved that every medial cancellation groupoid G can be embedded into a medial quasigroup Q such that Q is generated by G as a quasigroup; this Q is determined by G uniquely up to isomorphism over G and satisfies the same identities as G. All finite simple medial groupoids are described. All minimal varieties of commutative medial groupoids are found.

GRÄTZER, G.: Finitely presented lattices

I proved with A. Huhn and H. Lakser the following structure theorem:

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

For every finitely presented lattice L there exists a congruence relation Θ such that L/Θ in finite and all congruence classes are embeddable in a free lattice.
Application: If L is finitely presented and modular, then L is finite.

POGUNTKE, W.: Finitely generated lattices

We can prove the following Theorem: There are infinitely many 3-generated lattices of width 3; only finitely many of them are infinite.
A certain infinite lattice of width 3 is called the herringbone. The above curious looking Theorem (together with an effective listing of the exceptional infinite lattices) comes out of the following recent result of B. Sands and the author: every finitely generated lattice of width 3 with an infinite descending chain contains a sublattice isomorphic to the herringbone. This result is also an important step in proving the main result of the Poguntke-Sands paper: every finitely generated subdirectly irreducible lattice of width 3 is finite.

QUACKENBUSH, R.: Tensor Products of Lattices

Let A,B be lattices with 0 and let $A \otimes B$ be the tensor product of A and B in S_0, the variety of $\vee$-semilattices with 0.
Solution to the word problem (G. Fraser): Let $a, a_1, \ldots, a_n \in A-\{0\}$; $b, b_1, \ldots, b_n \in B-\{0\}$. Then $(a,b) \leq \bigvee_{i=1}^{n} (a_i, b_i)$ iff there is a lattice polynomial p such that $a \leq p(a_1, \ldots a_n)$ and $b \leq p^d(b_1, \ldots b_n)$ where p^d is the dual of p.
Examples: 1) Let A,B be bounded distributive lattices, then $A \otimes B \cong A * B$, the free product in Bounded Dist.Lat.
2) Let B be bounded dist.lat., then $M_3 \otimes B \cong F_M({}_BM_3)$.
3) If A and B are finite, then $A \otimes B$ is a lattice.
Let F (F_A, F_B) be the free lattice (over A,B) gen. by $x_1, \ldots$; let φ_A (φ_B) be the canonical homomorphism from F to F_A (F_B). We say B is A-lowerbounded if for every $p \in F_A$, $\varphi_B(\varphi_A^{-1}(p))$ has a least element.

Theorem: Let A be locally finite and B A-lower bounded; then $A \otimes B$ is a lattice.
Cor: If A and B are locally finite or let B be distributive; then $A \otimes B$ is a lattice.
Main Thm.: Let $Con_L(C)$ be the lattice of lattice congruences of C; let A be loc. finite and B A-lower bounded; then $Con_L(A \otimes B) \cong Con_L(A) \otimes Con_L(B)$.
Cor: $A \otimes B$ is simple (s.i.) iff both A and B are simple (s.i.).

KELLY, D.: On the product of lattice varieties

(Jointly with George Grätzer)
For classes $\underset{\sim}{V}$ and $\underset{\sim}{W}$ of lattices, $\underset{\sim}{V} \circ \underset{\sim}{W}$, the product of $\underset{\sim}{V}$ and $\underset{\sim}{W}$, is the class of all lattices L such that there exists a congruence Θ of L such that every class of Θ is in $\underset{\sim}{V}$ and $L/\Theta \in \underset{\sim}{W}$. If $\underset{\sim}{V}$ and $\underset{\sim}{W}$ are varieties, then $\underset{\sim}{V} \circ \underset{\sim}{W}$ is not necessarily a variety although it is closed under the formation of ideal lattices. There are continuum many varieties $\underset{\sim}{V}$ such that $\underset{\sim}{V} \circ \underset{\sim}{D}$ is a variety (including the cases $\underset{\sim}{V} = \underset{\sim}{D}$ and $\underset{\sim}{V} = \underset{\sim}{M}$). The variety $\underset{\sim}{D} \circ \underset{\sim}{D}$ contains, and is contained in, continuum many varieties of lattices. If $\underset{\sim}{V}$ and $\underset{\sim}{W}$ are nontrivial varieties of lattices, then any lattice in $\underset{\sim}{V} \circ \underset{\sim}{W}$ is contained in a subdirectly irreducible lattice in $\underset{\sim}{V} \circ \underset{\sim}{W}$. Consequently, any variety of the form $\underset{\sim}{V} \circ \underset{\sim}{W}$ is join irreducible.

RIVAL, I.: A structure theory for ordered sets

One of the most natural problems that arises in the investigation of any algebraic or relational system is that of representing the system as a whole in terms of certain distinguished subsystems, by means of canonical constructions. For relational systems (and especially ordered sets) the concept of "retract" can be combined with the "direct product" construction to fashion an effective subdirect representation theory.
Following this subdirect representation theory we are led, rather naturally, to the concept of a "variety" of ordered sets, which is a class of ordered sets closed under the formation of retracts and direct products.

Gefördert durch
Deutsche
Forschungsgemeinschaft

TUMA, J.: Combinatorial methods in lattice theory

A special type of embeddings of the partition lattice on four-element set into other partition lattices on finite sets has a combinatorial structure which can be described by the notion of polyhedron. These polyhedra have additional properties, e.g. the dual polyhedra are simplicial complexes, which are triangulations of compact topological varieties of dimension two.
Questions concerning orientability and genera of these varieties are discussed.

BRUNS, G.; Some finiteness conditions for orthomodular lattices

For an orthomodular lattice L let $\mathfrak{A}(L)$ be the set of all maximal Boolean subalgebras (blocks). We consider (with R. Greechie) the conditions: A_n : $|\mathfrak{A}(L)| \leq n$; B_n: there exists $\mathcal{B} \subseteq \mathfrak{A}(L)$ such that $|\mathcal{B}| \leq n$ and $\bigcup \mathcal{B} = L$; C_n: $|\{C(x) \mid x \in L\}| \leq n$; D_n: out of any $n+1$ elements of L at least two commute.
It is our conjecture that if X, Y stand for any two of A, B, C, D then for any n there exists m such that whenever L satisfies X_n it also satisfies Y_m. We have been able to prove:
$A_n \Rightarrow C_{2^n-1}$, $C_n \Rightarrow A_{n!}$, $B_n \Rightarrow D_n$, $D_n \Rightarrow B_{n!}$, $B_1 \Rightarrow A_1$, $B_2 \Rightarrow A_2$, $B_3 \Rightarrow A_3$, $B_4 \Rightarrow A_6$. Since $A_n \Rightarrow B_n$ holds trivially the only implication which is still missing in $B_n \Rightarrow A_m$.

WEGLORZ, B.: +-Completeness of Boolean Algebra

Known: 1) If $\mathcal{B}$ is a κ-complete B.A. and $\mathcal{J}$ is a κ-complete ideal on κ then $\mathcal{B}/\mathcal{J}$ is κ-complete.
2) If $\mathcal{J}$ is κ-complete normal ideal on κ then $\mathcal{P}(\kappa)/\mathcal{J}$ is κ^+-complete

Questions: 1) Does for each κ-complete B.A. $\mathcal{B}$ there exist a κ-complete ideal $\mathcal{J}$ such that $\mathcal{B}/\mathcal{J}$ is κ^+-complete?

Theorem: Let $\mathcal{J}$ be any κ-complete ideal on κ. Then there exists a κ-complete $\mathcal{J}' \supseteq \mathcal{J}$ such that $\mathcal{P}(\kappa)/\mathcal{J}'$ is κ^+-complete.

McNULTY, G.F.: Burnside Style Theorems for Equational Classes

In 1905 Burnside asked if every finitely generated group satisfying $x^n=1$ is finite. In 1968 Novikov and Adjan resolved this question in the negative by providing a system of finitely generated groups which are infinite and which satisfy $x^n=1$ for certain numbers n. At issue in Burnside's problem is the connection between the syntactic form of a set of equations and whether or not the set of equations possess an infinite model which is finitely generated. The present paper is concerned with this connection for arbitrary sets of equations - not just those which hold in groups. The collection of finite sets of equations which have infinite finitely generated models is not algorithmically recognizable. But some easily recognizable properties turn out to be sufficient to insure the existence of such models.

NEUMANN, B.H.: Fibonacci varieties

This is a report on some recent work carried out by Mr. Ann Chi Kim (of Busan National University, Busan, Korea) while at the Australian National University. He has studied a family of varieties of algebraic systems that are groups with an additional unary operation satisfying certain laws. The study was inspired by the theory of Fibonacci groups, and the free one-generator algebras of these varieties turn out to have, in fact, the abelianized Fibonacci groups as their underlying groups. Some natural generalisations lead to open problems.

NEFF, M.F.: A Variety of Near-rings

A result of J.D.P. Meldrum is that the variety generated by the distributively generated near-rings is the same as the variety $\underline{\underline{O}}$ of near-rings satisfying $0\chi = \chi 0 = 0$. A completely different proof is given, offering a construction of a "free" distributively generated near-ring and showing that it contains the free near-ring in $\underline{\underline{O}}$. The embedding presented is used to show also that the free $\underline{\underline{O}}$ near-ring is residually finite and is cancellative. A solution to the word problem in the free $\underline{\underline{O}}$ near-ring is developed.

SMITH, J.: On the uniqueness of parallelograms

Defn. A Mal'cev operation $P(x,y,z;w_i/i<\iota)$ with ι irrelevant variables $\{w_i \mid i<\iota\}$ in a Mal'cev variety $\mathcal{V}$ is an operation satisfying the identity $P(x,x,z;w_i) = z = P(z,x,x;w_i)$.

Defn. $\mathcal{V}$ satisfies the generalised uniqueness hypothesis $U(\iota)$ iff there is at most one Mal'cev operation $P(x,y,z;w_i \mid i<\iota)$ as element of $F_{\{x,y,z,w_i \mid i<\iota\}}(\mathcal{V})$.

Defn. Big Dipper identity on ternary Mal'cev op. (the parallelogram): $(y,x(x,y,z)P)P=z$.

Theorem: $\mathcal{V} = \mathfrak{Z}(\mathcal{V}) \Leftrightarrow \mathcal{V}$ satisfies $U(\omega)$ and Big Dipper.

Theorem: Let $\mathcal{V}$ have Big Dipper and type ι.

(a) If ι has one nullary, nothing bigger than ternary, let $\mathcal{V}$ satisfy $U(0)$

(b) If arities of ι are less than $4+\iota$ for $0<\iota\leq\omega$, let $\mathcal{V}$ satisfy $U(\iota)$.

Then $\mathcal{V} = \mathfrak{Z}(\mathcal{V})$, and $\mathcal{V}$'s clone is generated by parallelogram and operations at most binary.

Example. For $\mathcal{V}$ the variety of CH-quasigroups, $\mathcal{V}$ satisfies Big Dipper and $U(0)$, but $\mathfrak{Z}(\mathcal{V})$ is strictly less than $\mathcal{V}$.

Reference. On the uniqueness of Mal'cev Polynomials, Gá'bor Czédli and Jonathan Smith (TH Darmstadt Preprint # 431).

OATES-WILLIAMS, S.: Min but not Max?

In a recent paper [1] M.R. Vaughan-Lee and I gave an example of a variety generated by a finite algebra which satisfied the maximum condition on subvarieties, but not the minimum, and asked whether there was a variety generated by a finite algebra which satisfied Min but not Max, offering as a possible candidate Var(M) where M is the 3 element non-finitely based groupoid defined by Murskii [2]. We proved that it certainly did not satisfy Max, but left open the question of whether or not it satisfied Min. I have attempted to solve this problem by classifying all the subvarieties of Var(M). Let $\underline{\underline{U}}_i$ denote an element of the infinite ascending chain defined in [1], let $\underline{\underline{U}} = \bigcup_{i=1}^{\omega} \underline{\underline{U}}_i$, let $\underline{\underline{V}}$, $\underline{\underline{W}}$, $\underline{\underline{X}}$ be the subvarieties defined respectively by the laws $x^2 = x$, $xy=yx$, $(xy)z=x(yz)$. Then $\underline{\underline{U}} \cap \underline{\underline{W}} = \underline{\underline{U}} \cap \underline{\underline{X}} = \underline{\underline{U}}_1$, and I conjecture that, with

a finite number of exceptions, the subvarieties of var M are all of the form $\underline{\underline{U}}_i \vee \underline{\underline{V}}$, $\underline{\underline{U}}_i \vee \underline{\underline{W}}$, $\underline{\underline{U}}_i \vee \underline{\underline{X}}$.

[1] Sheila Oates Macdonald and M.R. Vaughan-Lee, "Varieties that make one Cross", J. Austral. Math. Soc. (Series A), 26 (1978), 368-382

[2] V.L. Murskii "The existence in three-valued logic of a closed class with finite basis not having a finite complete system of identities", Soviet.Math. Doklady 6 (1965), 1020-1024.

EVANS, T.: Some remarks on clones

We discuss some topics in the theory of clones with the main emphasis on homogeneous clones. Numerous examples are given with corresponding representation theorems for clones of functions, clones of endomorphisms on a free algebra and clones of homomorphisms of a power of an algebra. Some decision problems for clones, e.g. the word problem, are shown to be unsolvable while others are shown to be solvable. Identities in clones and varieties of clones are illustrated and connections shown between them and the hyper-identities of W. Taylor and the variety of varieties of W.D. Neumann. The use of clones in combinatorics is illustrated by the special case of the clone of all binary operations on a finite set. Combinatorical properties of the tables of binary operations are linked with algebraic properties in the clone.

CSÁKÁNY, B.: Minimal Clones of Operations on finite sets

A set of operations on a set A is a clone if it contains all the projections and it is closed under superposition. Clones on A form a lattice with respect to inclusion whose zero is the clone of all projections. The atoms of this lattice are called minimal clones an A.

The clone (of all polynomial operations) of several algebras (e.g., semilattices, rectangular bands, affine spaces over prime fields) is known to be minimal.

An operation f on A is said to be homogeneous if every permutation of A is an automorphism of the algebra $\langle A;f\rangle$.

We determined all minimal clones on finite A, consisting of homogeneous functions. For $2 \leq |A| \leq 4$, there is three such clones; for $5 \leq |A|$, there is two. In every case, the dual discriminator d ($d(x,y,z) = x$ if $x = y$, and $d(x,y,z) = z$ otherwise) generates such a minimal clone.

KAISER, H.: <u>Some results on interpolation in universal algebra</u>

Let A be a universal algebra and $f: A^k \rightarrow A$ ($k \in \mathbb{N}$). f is said to have the interpolation property if for every finite subset $N \subseteq A^k$ there is a polynomial function which represents f on N. If every function over A (of arbitrary arity) has the interpolation property, then we say that A has the interpolation property.
The following theorem and the relevant results one needs for its proof are presented:
Theorem: Let A be a non-trivial universal algebra which contains at least four elements and has a triply transitive automorphism group. Then A either has the interpolation property or is equivalent to an affine space over GF (2).
This result was originally abtained by L. Szabó and Á. Szendrei for finite algebras and generalized to the infinite case by H. Kaiser and L. Márki.

SCHWEIGERT, D.: <u>On local clones</u> (joint work with I. Rosenberg)

The local clone L (D) of a clone D is the clone of all functions $f: A^n \rightarrow A$, $n \in \mathbb{N}$ such that for every finite set B, $B \subseteq A$ there is a $\varphi \in D$ with $f/_{B^n} = \varphi/_{B^n}$. A local clone L (D) is complete if L(D) = F(A) where F(A) is the clone of all function of A. L(D) is precomplete if for every function $g \notin L(D)$ we have $L(\overline{D \cup \{g\}}) = F(A)$. The precomplete local clones can be described by relations of a finite arity (by a theorem of Romov). We found the following relations for precomplete clones: Type O; ϱ is a partial order such that every finite subset has a lower and an upper bound. (Concerning local order-polynom. algebra with a majority term one finds the same characterization by tolerances as in the finite case), Type C, Type Z, Type P and Type L, where also (A;+) torsion free abelian is admitted. This list is not complete.

BAKER, K.A.: Definability and Congruences

Several kinds of first-order-definability conditions are of current interest for varieties of algebras: A) Existence of finite equational bases. Recent results: C. Shallon (thesis, '79) has found a 4-element algebra, based on the graph o—o—o, that has no finite equational basis. Park and Verman have shown that the triangle $\{a,b,c\}$, with $a \vee b = b \vee a = b$ and other pairs cyclically, is finitely based as an algebra with single operation $\vee$. B) Definability of principal congruences (DPC): McKenzie showed that no non-distributive variety of lattices has DPC. For varieties generated by a finite group G, the speaker has improved a result of Burris and Lawrence to show that Var (G) has DPC iff G obeys $[[x,y],x] = 1$. C) Definability of subdirectly irreducibles (DSI), and D) Definability of finitely subdirectly irreducibles : For congruence-distributive varieties, DSI implies DFSI; the relationship of DSI and DFSI in more general settings remains to be determined. Another remaining area is to find which conjunctions of DPC, DSF, DFSI, and finite generation yield finite bases.

BURRIS, S.: Modified Boolean Powers

We introduce a construction which is useful for creating new indecomposable algebras and for proving undecidability results.
Theorem: If $V(A)$ is congruence distributive and A is subdirectly irreducible but not simple and Θ is the unique atom of Con A then $A[B,\Theta]^*$ is directly indecomposable.
Theorem: If $V(A)$ is a CD variety and A is finite, then Th $[V(A)]$ is undecidable if $V(A)$ is not semi-simple arithmetical.

GUMM, H.-P.: Factor Permutable Varieties

Factor permutable (FP) varieties are introduced as such varieties in which congruences on direct products permute with the canonical factor congruences.
It is shown that modular varieties are FP-varieties as well as those varieties studied by Fraser & Horn and by Hu. Another simple example is the variety of type (3,0) defined by $p(xxy) = y$, $p(x,o,o) = x$.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

We show that FP-varieties are well suited for a geometrical treatment where the geometry is Wille's "Kongruenzklassengeometrie". Among the results presented:

THM: If $\mathfrak{A}$ generates an FP-variety t.f.a.e.:

(i) $\mathfrak{A}$ is affine.

(ii) There is a common complement Θ of the factor-congruences on $\mathfrak{A} \times \mathfrak{A}$.

(iii) $\Delta = \{(x,x) \mid x \in \mathfrak{A}\}$ is a congruence class on $\mathfrak{A} \times \mathfrak{A}$.

Cor.: The Kronecker product of two FP-varieties is a variety of modules.

HERRMANN, C.: A cancellation theorem for congruence modular algebras and its lattice theoretic background

A cancellation theorem (up to "central" isotopy) can be proved for algebras in congruence modular varieties assuming a.c.c. for congruences and d.c.c. for the center. It relies on a special refinement theorem generalising a result of R. Baer for loops. The proof uses the analysis of the modular lattice freely generated by two complemented pairs. This analysis can be carried on to determine the "2-distributive" part of the free modular lattice with four-generators.

HEIDEMA, J.: Axiomatising miniaturised classes of models in large infinitary languages (joint work with W.A. Labuschagne)

In any $L_{\alpha\beta}$ it is impossible to give first-order axiomatisations of, e.g., the following classes of structures: complete semilattices, complete lattices, complete Boolean algebras, topological spaces, compact spaces or complete uniform spaces.
Suppose that there exists a model of set theory in which there are two inaccessible cardinals, $\Theta_1 < \Theta_2$, and the two resulting subuniverses U and V of all sets of rank $< \Theta_1$ and rank $< \Theta_2$ respectively. A set is called U-small if it belongs to U. $L_{\Theta_2 \Theta_2}$ is an infinitary language with conjunctions of $< \Theta_2$ formulas, quantification over sets of variables of cardinality $< \Theta_2$ and predicates of arity $< \Theta_2$. It then becomes possible to specify theories in $L_{\Theta_2 \Theta_2}$ which

axiomatise the sets of all U-small complete semilattices, all U-small complete lattices, etc. This leads to model-theoretic methods of, i.a., constructing complete Boolean algebras free with respect to all U-small complete Boolean algebras, and of proving the Tychonoff product theorem for topological spaces. Since U is itself a model of set theory, there is no great loss of generality.

ROSENBERG, I.G.: Functional completeness; subalgebras of direct powers of partial algebras

We present the following results:

1. Characterization of functionally complete finite algebras in congruence distributive varieties.
2. Functional completeness of single generated or surjective algebras.
3. Large classes of functionally complete algebras.
4. Subalgebra systems of direct powers of partial algebras.

The first three are adaptions of a primality criterion while the last translates the problem to full algebras by adjoining an absorbing element to the universe.

SICHLER, J.: Quotients of rigid algebras

Theorem (M.E. Adams, J.S.) For a connected algebra $A = (X;\alpha,\beta)$ with α,β unary the following are equivalent:

(i) The class of all algebras having A as a quotient forms a binding category;

(ii) A is a quotient of a rigid algebra;

(iii) There is no homomorphism from A into the free one-generated algebra.

The above theorem holds also for all binding subvarieties of the variety of all $(X;\alpha,\beta)$ with $\alpha^2=\alpha$, $\beta^2=\beta$.

Problem: Does it remain valid in every binding variety of bi-unary algebras?

STONE, M.G.: Abstract representation results for subalgebras and endomorphisms (joint work with N. Sauer)

A monoid M and a lattice L are algebraic if for some algebra $\mathfrak{A}$ the endomorphisms End$\mathfrak{A} \cong$ M and the subalgebras Su$\mathfrak{A} \cong$ L. For the two element chain, 2, it is well known that if M and 2 are algebraic then M and L are algebraic also, for every algebraic lattice L $\geq$ 2. B. Jónsson poses the problem (Topics in Universal Algebra, Springer Notes, p. 147) as to whether or not the same statement is true with 2 replaced by 3. We use concrete representation results to prove the following:
Theorem: If L is an algebraic lattice with at least one atom a, then M and 3 algebraic implies M and L algebraic also..
Further, we give an example of a monoid M with M and 3 algebraic, and a lattice L (atomless) with L $\geq$ 3 for which M and L are not algebraic. The answer then to the question posed be Jónsson is "no" in general, but "yes" for, at least, L finite.

SAUER, N.: On the structure of the lattices of subalgebras and the monoids of endomorphisms of universal algebras; versatile lattices.

We say that an algebraic lattice L and a monoid M are algebraic if there exists a universal algebra $\mathcal{A} = \langle A ; P \rangle$ with Su$\mathcal{A} \cong$ L and End $\mathcal{A} \cong$ M. It is known that every monoid which is algebraic with the 2-chain is algebraic with every lattice containing at least two elements and that each element in such a monoid is either a right zero or right cancellative. M. Gould who did most of that work called such monoids versatile. We will call a lattice L versatile if the only monoids which are algebraic with L are the versatile monoids. Quite surprisingly there exist infinitely many such lattices and we characterise all of them in purely lattice-theoretical terms.
We will point out that the methods used to gain those results are of a more general nature and probably basic in any problem concerning the structure of a lattice L which is algebraic with a monoid M.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

ADÁMEK, J.: <u>Categorial constructions of algebras and automata</u>

In a simple categorical setting, an algebra is a set X equipped with an "operational" map $d: FX \longrightarrow X$, where F is a given set-functor (called the signature). Several closely related problems are

(i) free completion of partial algebras;

(ii) free products and colimits of algebras;

(iii) minimal realization of behaviors of algebraic automata.

We exhibit iterative categorical constructions which solve these problems (depending on the properties of the signature-functor F). Some of these result hold generally for algebras and automata in an arbitrary category K (the signature is here a functor $F: K \longrightarrow K$). Yet, even in "nice" categories, like topological spaces and posets, pathological cases arise: e.g. the existence of free algebras does not guarantee cocompleteness.

Berichterstatter: W. Poguntke, Darmstadt

Liste der Tagungsteilnehmer

Baker, Prof. K.A., Dept. of Mathematics, University of California, Los Angeles, California 90024, USA

Bandelt, Dr. H.-J., Fachbereich IV der Universität, Ammerländer Heerstraße 67-69, D-2900 Oldenburg

Bosbach, Prof. B., OE 03 Mathematik, Gesamthochschule, Heinrich-Plett-Str. 40, D-3500 Kassel

Bruns, Prof. G., Dept. of Mathematics, McMaster University, Hamilton, Ontario L8S 4K1, Canada

Burris, Prof. St., Dept. of Mathematics, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada

Csákány, Prof. B., J. Attila Tudományegyetem, Bolyai Intézete, H 6720 Szeged, Hungary

Day, Prof. A., Dept. of Math. Sciences, Lakehead University, Thunder Bay, Ontario P7B 5E1, Canada

Evans, Prof. T., Dept. of Mathematics, Emory University, Atlanta, Georgia 30322, USA

Felscher, Prof. W., Mathematisches Institut der Universität, Auf der Morgenstelle 10, D-7400 Tübingen

Fried, Prof. E., ELTE TTK, Múzeum krt 6-8, H 1088 Budapest VIII, Hungary

Ganter, Prof. B., Fachbereich Mathematik - AG 1, Technische Hochschule, D-6100 Darmstadt

Grätzer, Prof. G., Dept. of Mathematics, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada

Gumm, Dr. H.P., Fachbereich Mathematik - AG 1, Technische Hochschule, D-6100 Darmstadt

Heidema, Prof. J., Dept. of Mathematics, Rand Afrikaans University, Auckland Park, P.O. Box 524, Johannesburg, 2000, South Africa

Herrmann, Dr. C., Fachbereich Mathematik - AG 1, Technische Hochschule, D-6100 Darmstadt

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Hutchinson, Dr. G., Dept. of Health, Education, and Welfare, Bethesda, Maryland 20014, USA

Ježek, Dr. J., MFF - UK, Sokolowská 83, CS 18600 Praha, CSSR

Kaiser, Dr. H.K., Institut für Algebra u. Math. Strukturtheorie, Technische Universität Wien, Argentinierstr. 8, A-1040 Wien

Kalmbach, Prof. G., Abteilung Mathematik III, Universität Ulm, Oberer Eselsberg, D-7900 Ulm

Kamara, Dr. M., Dept. of Mathematics, Bayero University , Kano, Nigeria

Keimel, Prof. K., Fachbereich Mathematik - AG 1, Technische Hochschule, D-6100 Darmstadt

Kelly, Prof. D., Dept. of Mathematics, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada

McNulty, Prof. G.F., Dept. of Mathematics, University of South Carolina, Columbia, SC 29208, USA

Mitschke, Dr. A., Paosostr. 105, D-8000 München

Müller, Prof. W.B., Institut für Mathematik der Universität, Universitätsstr., A-9010 Klagenfurt

Neff, Prof. M.F., Dept. of Mathematics, Emory University, Atlanta, Georgia 30322, USA

Nelson, Prof. E., Dept. of Mathematics, McMaster University, Hamilton, Ontario L8S 4K1, Canada

Neumann, Prof. B.H., Australian Nat. University, P.O. Box 4, Canberra, 2600 , Australia

Oates-Williams, Prof. S., Dept. of Mathematics, University of Queensland, St. Lucia, Brisbane, 4067, Queensland, Australia

Poguntke, Dr. W., Fachbereich Mathematik - AG 1, Technische Hochschule, D-6100 Darmstadt

Pudlak, Dr. P., CSAV Matematický ustar, Zítná 25, CS 1100 Praha 1, CSSR

Quackenbush, Prof. R.W., Dept. of Mathematics, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada

Rival, Prof. I., Dept. of Mathematics, University of Calgary, Calgary, Alberta T2N 1N4, Canada

Romanowska, Dr. A., Inst. Matematyki, Politechnika Warszawska, Pl. Jednosci Robotniczej 1, 00 644 Warszawa, Poland

Rosenberg, Prof. I.G., C.R.M., Universitê de Montreâl, Montreâl, Quebec H3C 3J7, Canada

Sands, Prof. B., Dept. of Mathematics, University of Calgary, Calgary, Alberta T2N 1N4, Canada

Sauer, Prof. N., Dept. of Mathematics, University of Calgary, Calgary, Alberta T2N 1N4, Canada

Schmidt, Prof. E.T., MTA MKI, Reâltanoda u. 13-15, H 1053 Budapest, Hungary

Schulte-Mönting, Dr. J., Mathematisches Institut der Universität, Auf der Morgenstelle 10, D-7400 Tübingen

Schweigert, Dr. D., Fachbereich Mathematik der Universität, Pfaffenbergstr. D-6750 Kaiserslautern

Sichler, Prof. J., Dept. of Mathematics, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada

Smith, Dr. J.D.H., Fachbereich Mathematik - AG 1, Technische Hochschule, D-6100 Darmstadt

Stone, Prof. M.G., Dept. of Mathematics, University of Calgary, Calgary, Alberta T2N 1N4, Canada

Traczyk, Prof. T., Inst. Matematyki, Politechnika Warszawska, Pl. Jednosci Robotniczej 1, 00661 Warszawa, Poland

Trnkovâ, Prof. V., MFF-UK, Sokolowskâ 83, CS 18600 Praha, CSSR

Tuma, Dr. J., Katedra Matematiky FJFI CVUT, Horskâ 2, CS 12800 Praha 2, CSSR

Węgłorz, Prof. B., Dept. of Mathematics, University, Wrocław, Poland

Wenzel, Prof. G.H., Fakultät für Mathematik u. Informatik der Universität, A 5/C 101, D-6800 Mannheim

Werner, Prof. H., OE03 - Mathematik, Gesamthochschule, Heinrich-Plett-Str. 40, D-3500 Kassel

Wille, Prof. R., Fachbereich Mathematik - AG 1, Technische Hochschule, D-6100 Darmstadt