MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 31|1979

Harmonische Analyse
und Darstellungstheorie topologischer Gruppen

22.7. bis 28.7.1979

Die sechste Tagung über Harmonische Analyse und Darstellungstheorie topologischer Gruppen stand wieder unter der Leitung von H. Leptin (Bielefeld) und E. Thoma (München). Bei zahlreicher Beteiligung ausländischer Gäste wurde in einem umfangreichen Tagungsprogramm über neueste Forschungen berichtet. In erster Linie wurden Ergebnisse der harmonischen Analyse, Dualitätstheorie und Darstellungstheorie von Gruppen, vor allem auch von Liegruppen behandelt.

Teilnehmer

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J. Boidol, Bielefeld
M. Bozejko, Heidelberg
J.-L. Clerc, Nancy
L. Corwin, New Brunswick
M. Cowling, Genua
J. Cuntz, Heidelberg
L. De Michele, Mailand
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Vortragsauszüge

J. Boidol: **$*$-Regularity of Exponential Lie Groups**

Let G be a locally compact group, $\text{Prim}\, C^*(G)$ the primitive ideal space of $C^*(G)$ and $\text{Prim}_* L^1(G)$ the space of kernels of topological irreducible representations in Hilbert space, both equipped with the Jacobson topology. Then we have a canonical mapping

$$\Psi : \text{Prim}\, C^*(G) \to \text{Prim}_* L^1(G)\ ;\ J \to J \cap L^1(G)$$

which is continuous and surjective. We ask for what groups G Ψ is a homeomorphism and call these groups $*$-regular groups. We give a characterization of all exponential Lie groups which are $*$-regular. Let $G = \exp \mathfrak{g}$ be exponential, put for $f \in \mathfrak{g}^*$ $\mathfrak{g}(f) = \{X \in \mathfrak{g} \mid f([X,\mathfrak{g}]) = 0\}$ and $m(f) = \mathfrak{g}(f) + [\mathfrak{g},\mathfrak{g}]$ and $m(f)^\infty = \bigcap_{k=1}^{\infty} C^k\, m(f)$ where $C^k\, m(f)$ is the k-th term in the descending central series of $m(f)$. Our characterization is the following

Theorem: $\mathfrak{g}$ is $*$-regular if and only if $f(m(f)^\infty) = 0$ for all $f \in \mathfrak{g}^*$.

M. Bozejko: **Uniform amenable groups and approximation property**

A Banach space X has the uniform bounded approximation property (u.b.a.p.) if

(*) there are a k with $1 \leq k < \infty$ and a positive sequence $q_k(m)$ such that given a finite dimensional subspace $E \subset X$ there exists an operator $u : X \to X$ satisfying the following conditions

- (i) $u(x) = x$ for $x \in E$
- (ii) $\|u\| \leq k$
- (iii) $\dim u(X) \leq q_k(\dim E)$

The study of translation invariant function spaces on compact Abelian groups leads us to consider the translation invariant analogue of the u.b.a.p. Roughly speaking we modify (*) assuming that X, E and u are translation invariant.

The first result is the following

Theorem (Bożejko, Pelczynski): Every regular invariant Banach space of functions on an Abelian group has the invariant u.b.a.p..

This theorem is the consequence of the following uniform Fölner - Leptin condition:

$$\text{(UA)} \quad \forall_{\varepsilon > 0} \ \forall_{n} \ \exists_{q_\varepsilon(n)} \ \forall_{\substack{E \subset \Gamma \\ |E|=n}} \ \exists_{|K| \leq q_\varepsilon(n)} \ . \ |EK| \leq (1+\varepsilon)|K| .$$

We call such a group Γ uniform amenable. We prove the following results:

(1) Groups with uniform polynomial growth are uniform amenable.

(2) Extensions of uniform amenable groups by uniform amenable groups are uniform amenable.

(3) A subgroup of an uniform amenable group is an uniform amenable group.

(4) There are FC groups without uniform polynomial growth.

L. Corwin: Unitary Representations of the Multiplicative Group of a Locally Compact Division Algebra

Let K be a non-Archimedean locally compact field, and let D be a division algebra over K of degree m^2. Problem: compute the unitary representations of D^x. This problem has connections with parts of Langlands' program for number theory.

Let p be the residual characteristic of K. When $p \nmid m$, the unitary representations of D^x have been computed by R. Howe and me; indeed, we

even computed the characters. Howe also computed the representations when $p|m$. The general procedure is to consider the subgroup G of D of elements of the form $1 + a$, where a is in the prime ideal P of the integers of D. The representations of G are finite-dimensional and are trivial on an open subgroup G_n; one can induce them from 1-dimensional representations on certain (explicitly describable) subgroups of G containing G_n. Recently I found all the representations in the case $p^2 = m$. The situation is more delicate because algebraic complications interfere with the analysis; it turns out, however, that an analysis like that in the previous cases applies. The key result is a lemma showing that in certain cases, units in one extension of K can be closely approximated by units in a different extension. There is some evidence that the lemma (and hence the method for finding representations) applies to general D.

M.G. Cowling: Semigroups and Maximal Functions

Suppose that $(T_t : t \in \mathbb{R}^+)$ is a contraction semigroup on all the spaces $L^p(X)$ $(1 \leq p \leq \infty)$. Then, following R.R.Coifman and G. Weiss, we "lift" $(T_t : t \in \mathbb{R}^+)$ to a subsemigroup of a group space $L^p(Y)$ (using Akoglu's construction) and transfer convolution operators on $L^p(\mathbb{R})$ to obtain bounded operators on $L^p(Y)$ and hence on $L^p(X)$.

Using this technique, we obtain most of the principal results of the so-called Littlewood-Paley-Stein theory. A new method for dealing with maximal functions associated with semigroups is presented, with applications to the wave equation.

J. Cuntz: C*-algebras associated with certain topological groupoids

Let $A = (a_{ij})_{1 \leq i,j \leq n}$ be a matrix with entries in $\{0,1\}$. With A we associate the C^*-algebra O_A generated by non-zero partial isometries $S_1, \ldots, S_n$ satisfying

$$S_i^* S_i = \sum_{j=1}^{n} a_{ij} S_j S_j^* .$$

This C^*-algebra does in fact not depend on the choice of the generators $S_1, \ldots, S_n$. This uniqueness has several consequences, in particular the Ext-group of O_A can be computed to be $\mathbb{Z}^n / (1-A)\mathbb{Z}^n$. Also $K \otimes O_A$ (K = algebra of compact operators on a separable infinite-dimensional Hilbert space) is invariantly - with respect to topological conjugacy - associated with the two-sided shift $\bar{\sigma}_A$ on the compact space $\bar{X}_A = \{ x \in \{1,\ldots,n\}^{\mathbb{Z}} \mid a_{x_i x_{i+1}} = 1 \ \forall i \in \mathbb{Z} \}$ and with the suspension $F_A = \bar{X}_A \times \mathbb{R} / \bar{\sigma}_A \times T_1$ (T_1 = unit translation) with its natural orientation. The conceptual reason for this is the fact that $K \otimes O_A$ is the convolution algebra of certain topological groupoids constructed from $\bar{\sigma}_A$ and F_A, respectively.

L. de Michele: Positive definite functions on free groups

A class of positive definite functions with preassigned values on the set of generators of a discrete group G is defined. These functions resemble, in many respects. The Fourier-Stieltjes transforms of the Riesz products, and include, as a special case, those defined by U. Haagerup, as $e^{-t|x|}$ where $|\cdot|$ denotes the length of the word x. A positive definite function of this class, u, is defined as follows: given a free subset $F \subseteq G$, u is zero off of the group generated by F, and such that

i) $u(1_G) = 1$ and $|u(x)| \leq 1$ for all x;

ii) $\overline{u(x)} = u(x^{-1})$ for $x \in F$;

iii) $u(xy) = u(x)\,u(y)$ if $|xy| = |x| + |y|$.

These positive definite functions are used to answer to a number of questions concerning the Fourier-Stieltjes algebra $B(G)$ and its ideals $B_o(G) = B(G) \cap C_o(G)$, $A(G)$ the Fourier algebra and $B_\lambda(G)$ (λ is the regular representation of G). In particular we exhibit functions u in B_o such that for every positive integer n are in the orthogonal complement of B_λ. This result is obtained via a general theorem resembling the Zygmund's theorem on the singularity of Riesz products.

J. Faraut: <u>Spherical distributions</u>

For the harmonic analysis on a Riemannian symmetric space, the spherical function play the main role. On a pseudoriemannian symmetric space $X = {}^G/H$ there are no more spherical functions but it is possible to define spherical distributions. They are distributions on G, biinvariant under H, and eigendistributions of the elements of the center of the universal envelopping algebra of the Lie algebra of G.

In the case of hyperbolic spaces $G = U(p,q;\mathbb{F})$, $H = U(1,\mathbb{F}) \times U(p-1,q;\mathbb{F})$ ($\mathbb{F} = \mathbb{R}$, $\mathbb{C}$ or $\mathbb{H}$) we determine the set of spherical distributions.

H.G. Feichtinger: <u>A Banach space of Distributions with Applications to Harmonic Analysis</u>

Let G be a locally compact, abelian group, and an open subset with compact closure. Then the space $S_0(G)$ is given by

$$S_0(G) := \{ f \mid f = \sum_{n=1}^{\infty} L_{y_n} f_n,\ \mathrm{supp}\ f_n \subseteq Q,\ \Sigma \|\hat{f}_n\|_1 < \infty \}.$$

For a suitable norm this space is a translation invariant Banach space of continuous, integrable functions on G. It does not depend on the choice of Q and contains the space $S(G)$ of Schwartz-Bruhat functions. Besides several other useful properties one has: Fourier transform maps $S_0(G)$ onto $S_0(\hat{G})$. Consequently $S'_0(G)$ is a Banach space of tempered distributions which is mapped onto $S'_0(\hat{G})$ by the F.T.. As most of the important properties of the space $S'(G)$ is shared by $S'_0(G)$ (e.g. the 'kernel-theorem'), but can be proved without making use of structure theory, this space appears as useful tool for Harmonic Analysis. Examples are given (multiplier-theory, transformable measures). There are also results for non-abelian groups G.

R. Felix: Distributions invariant under a unipotent action.

Operiert eine zusammenhängende lokalkompakte Gruppe G mittels unipotenter Transformationen auf einem endlichdimensionalen reellen Vektorraum V, dann gibt es auf jeder Bahn unter dieser Operation ein G-invariantes temperiertes Radonmaß. Es wird die Frage behandelt, wann sich alle G-invarianten temperierten Distributionen auf V durch diese Maße, die man als die elementarsten G-invarianten temperierten Distributionen auf V verstehen kann, ausdrücken lassen. Im Spezialfall der koadjungierten Operation einer nilpotenten einfach zusammenhängenden Liegruppe auf dem Dual ihrer Liealgebra gestattet die Kirillov-Theorie eine darstellungstheoretische Formulierung dieser Frage. Für einige spezielle Gruppen wurden bereits teils positive, teils negative Antworten gegeben (Rothschild-Wolf, Dixmier). Wir geben eine Klasse von unipotenten Gruppenoperationen an, für die die Antwort auf die obige Frage positiv ist. Ferner zeigen wir für eine beliebige unipotente Gruppenoperation, daß sich jede invariante temperierte Distribution, die durch eine genügend hohe Potenz eines Strukturpolynoms der unipotenten Gruppenoperation teilbar ist, durch die invarianten Maße auf den Bahnen ausdrücken läßt.

M. Flensted-Jensen: Discrete series for semisimple symmetric spaces.

Let G be a semisimple Lie group (with finite center), τ an involution on G and let H be the component of the identity of the fixpoint-group of τ. Let K be a maximal compact subgroup of G and let the corresponding Cartan involution be σ. K can be chosen such that $\tau\sigma = \sigma\tau$. The following theorem is shown:

Theorem Assume that rank (G/H) = rank $(K/K\cap H) \neq 0$ then there exist non-trivial, irreducible G-invariant subspaces of $L^2(G/H)$.

As a semisimple Lie group G_1 (with finite center) may always be considered as a symmetric space G/H (where $G = G_1 \times G_1$ and $H = \{(x,x) \in G_1 \times G_1 \mid x \in G_1\}$), this provides, in particular, a new and elementary proof of Harish-Chandra's result, that G_1 has a discrete series if rank (G_1) = rank (K_1), where K_1 is a maximal compact subgroup of G_1.

R. Goodman: Conical Vectors, Whittaker Vectors, and Differential Operators of Infinite Order.

Let G be a connected, semisimple linear Lie group, with Iwasawa decomposition $G = KAN$. Set $M = \mathrm{Cent}_K(A)$, $B = MAN$. Assume that G is quasi-split, i.e. $H = MA$ is a Cartan subgroup of G (examples: $SL(n,\mathbb{R})$, $SL(n,\mathbb{C})$, $SU(2,1)$). Let h and n be the complexified Lie algebras of H and N, and let V be a G and $U(g)$ module. A conical vector u in V is an eigenvector for the solvable group B. Thus

(*) $H \cdot u = \lambda(H)u, \quad X \cdot u = 0, \quad$ for $H \in h, \quad X \in n,$ where $\lambda \in h^*$.

A Whittaker vector w in V is an eigenvector for the nilpotent group N, with non-zero eigenvalues on the simple root spaces. Thus

(**) $X \cdot w = \psi(x)w, \quad Z \cdot w = 0, \quad$ for $X \in n_1, Z \in [n,n],$ where n_1 is the sum of the positive, simple root spaces, and ψ is a "generic" linear functional on n_1.

We describe the construction of conical and Whittaker vectors for the principal series representations of G, and the existence of an infinite-order differential operator $T(\lambda)$ mapping solutions of $(*)$ to solutions of $(**)$, in the category of Gevrey differential representations.

(Joint work with Nolan Wallach, Rutgers Univ.)

F.P. Greenleaf: Fourier transform of smooth functions on certain nilpotent Lie groups.

Let N be simply connected nilpotent Lie group. Joint work with L. Corwin (Rutgers) has resulted in a version of Fourier transform allowing a characterization of Schwartz functions $S(N)$ in terms of Fourier transform data. Using the Kirillov picture of the space of irreducible representations $\hat{N}$, we study groups in which there is an ideal $\mathfrak{m} \triangleleft \mathfrak{n}$ (= Lie algebra of N) which is a polarization for generic orbits. Thus requirement is met in a number of important cases: Heisenberg groups, full upper triangular groups, and others. In the dual $\mathfrak{n}^*$ we may find a subspace $W \cong \mathbb{R}^{n-2k}$ ($2k$ = dimension of generic orbits) and a Zariski open set $\omega \subseteq W$ meeting each generic orbit once. Using $l \in \omega$ as parameter for such orbits, we let π^l be the corresponding representations in $\hat{N}$, which are all modeled simultaneously in $L^2(\mathbb{R}^k)$. For $\varphi \in S(N)$ we get various explicit formulas for the kernels $K_\varphi(u,t,l)$ of the trace class operators $\pi_\varphi = \int \varphi(n)\,\pi(n)\,dn$. The map $\varphi \to K_\varphi$ serves as a Fourier transform, but the K_φ are badly behaved. We show there is a birational change of variable $A:\mathbb{R}^{2k} \times W \to \mathbb{R}^n$ such that the "rationalized" kernels $\tilde{K}_\varphi = K_\varphi \circ A^{-1}$ have nice properties: Φ on $\mathbb{R}^n$ is $\tilde{K}_\varphi$ for some $\varphi \in S(N) \Leftrightarrow \Phi \in S(\mathbb{R}^n)$. The confusing behavior of the K_φ is entirely due to the change of variable A.

D. Gurarie: Burnside type theorems for Banach representations

We discuss the following conjecture (Burnside type theorem): each topologically irreducible Banach representation of a LC group G (algebra L) is completely irreducible.

The following cases are considered

1) G is a LC-motion group, correspondingly L is a crossed product of a commutative B-algebra A by a compact group U. A sufficient condition on a pair (A, U) is introduced which implies the BT-theorem for L. Using it we prove it for all LC-motion groups and their representations of "finite" growth.

2) We prove BT-theorem for nilpotent and solvable Lie groups.

We also discuss the relationship of BT-theorem to some problems of harmonic analysis.

K.C. Hannabuss: Projective representations of solvable locally compact groups.

If G is a separable locally compact extension of a closed nilpotent normal subgroup N by an abelian group, and σ is a borel multiplier which is type I for both G and N, then the irreducible $*$-representations of the twisted L^1-algebra of G defined by σ can all be found by inducing characters of certain L^1-algebras by Rieffel's method, though the algebras from which one induces need not be the L^1-algebras of subgroups. This result provides a generalisation of part of the work of Auslander and Kostant for Lie groups. The proof proceeds by reduction to the case of the oscillator group and certain analogues, where the construction of the representations by Rieffel's method can be done explicitly.

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S. Helgason: Tangent space analysis for a symmetric space.

Let $X = G/K$ be a symmetric space of the noncompact type, G being a connected semisimple Lie group with finite center and K a maximal compact subgroup. Let X_0 be the tangent space to X at the fixed point o of K and let G_0 denote the group of affine transformations of X_0 generated by the translations and the natural action of K on X_0. Our purpose is to develop G_0-invariant analysis on the space X_0. We derive an integral formula for the joint eigenfunctions of the algebra $D(G_0/K)$ of G_0-invariant differential operators on $X_0 = G_0/K$ and give an explicit irreducibility criterion for the associated eigenspace representations of G_0. As tools we prove Paley-Wiener type theorems for the spherical transform and for the generalized Bessel transform on X_0.

R.W. Henrichs: The Tomita decomposition of group representations and nonseparable C^*-algebras.

Sei w ein Zustand einer C^*-Algebra A. Wenn A nicht separabel ist, ist es i.a. unmöglich ω mittels regulärer Borelmaße auf dem (top.) Raum aller faktoriellen Zustände von A zu zerlegen. Ferner kann man die Gelfand-Segal Darstellung π_ω von A i.a. nicht als topologisches direktes Integral (im Sinne von Godement) von Faktordarstellungen schreiben.
Mit Hilfe eines Resultates von M. Tomita kann man jedoch stets jedem Zustand φ im Träger des zentralen Zerlegungsmaßes μ_ω zu ω eine Faktordarstellung π^φ von A so zuordnen, daß π^φ schwach enthalten in π_φ und $\pi_\omega = \int^\oplus \pi^\varphi \, d\mu(\varphi)$ ein direktes Integral im Sinne von W. Wils ist ($\pi^\varphi = \pi_\varphi$ für μ fast alle φ, wenn A separabel ist).

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E. Hewitt: <u>Continuous singular measures with small Fourier-Stieltjes transforms.</u>

We report on joint research with Professor Gavin Brown. We consider the problem of how small pointwise the Fourier-Stieltjes transform of a continuous singular measure on the circle can be. The Riesz-Fischer theorem implies that the transform cannot be in ℓ_2. Subject to certain regularity conditions, it is shown that every even positive decreasing function on the integers majorizes a transform of a continuous singular measure. With a little more effort, one can ensure that the partial sums of the Fourier-Stieltjes series are everywhere positive, and that the measures exhibit a certain ergodic property. All measures constructed have support equal to the entire circle. Earlier constructions of this genre (Menchoff, Littlewood, Wiener & Wintner, Salem, Ivashev-Musatoff, Koerner) yield for the most part measures supported by a perfect set of measure zero.

K.H. Hofmann: <u>Some remarks on the spectral theory of C^*-algebras.</u>

How come that everybody knows that *-morphisms $f: A \to B$ between commutative C^*-algebras are nearly and functorially transported to the spectra as a perfect map $\hat{f} : \hat{B} \to \hat{A}$ between locally compact Hausdorff spaces, but that, on the other hand, no one says anything about how *-morphisms can be transported functorially in the general, not necessarily commutative situation? Perhaps one has not looked sufficiently at the lattice $L = \overline{\mathrm{Id}}\, A$ of closed 2-sided ideals in its own right. Indeed L is a <u>distribution</u>, <u>continuous</u> lattice (DCL) in the sense of Scott. Of course, $f: A \to B$ gives a function $\overline{\mathrm{Id}}\, f: \overline{\mathrm{Id}}\, B \to \overline{\mathrm{Id}}\, A, (\mathrm{Id}\, f)(J) = f^{-1}(J)$; in fact this function preserves arbitrary infs and directed sups. So let DCL denote the category of DCL-lattices and maps preserving all infs and directed sups. Then THM.I. $\overline{\mathrm{Id}}: C^* \to$ DCL <u>is a contravariant functor transforming injective limits into projective limits.</u>

(Relevant for AFC^*-algebra theory). Now each DCL-lattice L has a locally quasicompact sober prime spectrum Spec L, through which it is uniquely determined (Hofmann-Lawson) and which is a Baire space (Hofmann). Thus the prime spectrum PRIME A = Spec ($\overline{\text{Id}}$ A) of a C^*-algebra is a space of this sort. (Known if A separable since then primitive = prime). Now $f : L \to M$ in DCL induces a multivalued map F = Spec f: Spec L → Spec M which is perfect in the sense that (i) F(point) is closed, (ii) $F(\bar{S}) = \overline{F(S)}$ for all sets S in domain F, (iii) $F^{-1}(Q)$ is quasicompact for all Q quasicompact in the range of F. Let LQC_{perf} be the category of locally quasicompact sober spaces and perfect multivalued maps. Then THM II : Spec: DCL → LQC_{perf} is a functor giving an equivalence of categories. (There is an explicit inverse functor.) As a corollary: THM III. The prime spectrum of a C^*-algebra gives a contravariant functor PRIME: $C^* \to LQC_{perf}$ which transforms injective limits into projective limits. The combinatorial formalism used in the ideal theory of AFC^*-algebras is thereby generalized and systematized.

A. Hulanicki : C^k - functional calculus and holomorphy of semi-groups of measures

Theorem 1. Suppose G is locally compact compactly generated nilpotent group of class 2. Let $f = f^* \in L^1(G) \cap L^2(G)$ and let $\int |f|(1+\tau)^\alpha < +\infty$, where $\alpha > 0$, $\tau(x) = \text{mim}\{n:\ x \in A^n\}$ for a compact set of generators of G. Then there exists a number k such that C^k-functions F operate on f into $L^1(G)$.

Theorem 2. Suppose G is a locally compact compactly generated group of polynomial growth. Let $f = f^* \in L^1(G) \cap L^2(G)$ and let $\int |f|\,(1+\tau)^\alpha < \infty$ for an α large enough and τ as above. Then there exists a number k such that C^k functions operate on f into $L^1(G)$.

Corollary. If G is the Heisenberg group and $\{\mu_t\}$ is a semi-group of probability measures generated by the sum of fractional powers of Laplacians, then μ_t is a holomorphic semi-group i.e. the mapping $\mathbb{R}^+ \ni t \to \mu_t \in M(G)$ extends holomorphically to $\{z : |A \circ gz| < \frac{\pi}{2}\} \ni z \to \mu_z \in M(G)$.

J.W. Jenkins: Primary distributions on nilmanifolds.

Let N be a 1-connected nilpotent Lie group with Lie algebra n and dual n^*. Denote the Kirillov bijection between $n^* / Ad^* N$ and $\hat{N}$ by $Ad^* N \cdot \xi \leftrightarrow \pi_\xi$. Let Γ be a discrete cocompact subgroup of N. The quasi-regular representation U of N on $L^2(\Gamma \setminus N)$ is a direct sum of primary representations. Let H_ξ be the primary subspace of $L^2(\Gamma \setminus N)$ corresponding to π_ξ and let P_ξ be the projection of $L^2(\Gamma \setminus N)$ onto H_ξ. There is a $D_\xi \in S^*(\Gamma \setminus N)$ such that for all $F \in S(\Gamma \setminus N)$, $P_\xi F = D_\xi * F$. Define $\tau : S(N) \to S(\Gamma \setminus N)$ by $\tau f(\Gamma n) = \sum_\Gamma f(\gamma n)$.

Thm: Let $\xi \in n^*$ such that π_ξ occurs in U und ξ is in general position. Let $k(\xi)$ be the ideal in n generated by rad(ξ). There is a $\varphi \in L^2(\exp k(\xi))$ and $\partial \in U(n)$ with deg $\partial \leq 1 + [\frac{1}{2}(\dim Ad^* N \cdot \xi - \dim \exp k(\xi))]$ so that for all $f \in S(N)$

$$< D_\xi, \tau f > = \sum_\Gamma \int_{\exp k(\xi)} \partial f(\gamma s)\, \varphi(s)\, ds.$$

W. Kugler: The spectrum of Sobolovian algebras.

Es sei Ω ein Gebiet in $\mathbb{R}^n$. Wenn $mp > n$ ist, ist $W = W^{m,p}(\Omega)$ eine Banach-Algebra. Eine Umgebung U (bzgl. $\mathbb{R}^n$) eines Punktes x aus $\bar{\Omega}$ heißt günstig zerfallend, wenn $U \cap \Omega$ in endlich viele Zusammenhangskomponenten mit der gleichmäßigen Kegeleigenschaft zerfällt. Jeder Charakter von W induziert auf den Einschränkungen der Testfunktionen eines Obergebietes von

Ω ein Dirac-Maß. Anhand der Zusammenhangskomponenten von $U \cap \Omega$ kann man abzählen, wieviele verschiedene Charaktere dasselbe Punktmaß induzieren, vorausgesetzt U zerfällt günstig und "minimal".

G. Margulis : Amenable factors of discrete subgroups and induced representations.

We say (following Kazhdan) that locally compact group H has property (T) if the trivial one-dimensional representation T_0 of H is isolated in the space of all irreducible unitary representations of H. As H is amenable iff the closure of the regular representation of H contains T_0 (according to Hulanicki's theorem) then if H is amenable and has property (T) then it is compact. In particular if H is discret, amenable and has property (T) then it is finite. We say that a locally compact group H has property (Q) if there exists a compact subgroup $K \subset H$ such that $h^{-1} \in K h K$ for any $h \in H$.

Theorem. Let H_1 and H_2 be compactly generated separable locally compact groups having property (Q), $H = H_1 \times H_2$, Λ be a lattice in H (i.e. a discrete subgroup of H such that the volume of the factorspace $\Lambda \setminus H$ is finite) and N be a normal subgroup of H. Assume that 1) there exists a left borelian fundamental domain $X \subset G$ for Λ (i.e. $\Lambda X = H$ and $(\lambda_1 H) \cap (\lambda_2 H) = \emptyset$ for any $\lambda_1, \lambda_2 \in \Lambda$, $\lambda_1 \neq \lambda_2$) such that the set $\varphi(\gamma_X(X \cdot M))$ is finite for any compact set $M \subset H$, where $\varphi : \Lambda \to \Lambda / N$ is the natural projection and $\gamma_X(h) \in \Lambda$ is defined for any $h \in H$ from the inclusion $h \in \gamma_X(h) X$, 2) NH_i is dense in H for any $i = 1,2$. Then Λ / N has property (T) and, consequently, Λ / N is either nonamenable or finite.

The proof is based on the using of the infinite dimensional unitary representation theory.

O.C. M^c Gehee : Continuous Manifolds in $\mathbb{R}^n$ that are Helson Sets.

This talk represents joint work with Gordon S. Woodward. We are concerned with the question of when a continuous k-dimensional manifold $E \subseteq \mathbb{R}^n$ is a Helson set. Therefore we are concerned also with how the transform $\hat{\mu}$ decays at infinity when μ is a bounded Borel measure with support contained in a manifold E. The object is to understand the extent to which an E of a given dimension, and perhaps a given smoothness, can "participate" in the arithmetic structure and the harmonic analysis of Euclidean space.

J.-P. Kahane and N. Th. Varopoulos have used Baire category arguments to produce examples of interest; our productions are more nearly constructive but use similar ideas. Kahane has shown the existence of Helson curves in $\mathbb{R}^n$ for $n \geq 2$; we construct such a curve, and also a Helson surface in $\mathbb{R}^6$. Perhaps there is a Helson k-manifold in $\mathbb{R}^n$ whenever $k < n$; we do not know.

P. Malliavin: Two step factorization on $\mathcal{D}(G)$.

$\mathcal{D}(G)$ denote the smooth functions with compact support on the Lie group G. We say that factorization at step k holds then on $\mathcal{D}(G)$ iff $\forall \varphi \in \mathcal{D}(G)$, $\exists \psi_i, \chi_i$ $1 \leq i \leq k$ such that $\varphi = \sum_i \psi_i * \chi_i$. With J. Dixmier (Bull. Sciences Mathématique 1978) it was proved that factorization at step $2^{\dim(G)}$ is always true. In fact factorization at step 2 is always true. The proof is obtained by the construction of an infinite order P.D.E $h_\Lambda(-\Delta)$ where $\Delta = \sum_{k=1}^{\dim G} A_k^2$ (A_s a basis of $\mathfrak{g}$), $h_\Lambda(\zeta) = \prod_{\lambda \in \Lambda} (1+\zeta/\lambda)$ Λ subsequence of $\{2^n\}$. Then a fundamental solution J of $h_\Lambda(-\Delta)$ is obtained using Laplace transform + symbolic calculus, on the semi-group $e^{t\Delta}$ + elliptic estimates + Molchanov-Varadhan estimates: J is C^∞ on G, and belongs to Gevrey 2 outside the neutral element.

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O.A. Nielsen: The Topological Frobenius Property for Nilpotent Lie Groups

The topological Frobenius reciprocity property (usually called (FP)) was first studied by J.M.G.Fell, who showed that locally compact abelian groups and compact groups satisfy this property. In the last few years various authors have considered the problem of which groups satisfy (FP), and one now knows which discrete groups and which Lie groups do so. In particular, any connected and simply-connected non-abelian nilpotent Lie group fails to satisfy (FP), and this talk will be concerned with the extent of this failure. If G is such a group and if H is a closed connected subgroup of G then it will be shown that the pair (G, H) satisfies (FP) if and only if H is normal in G.

J. Nourrigat: Hypoellipticity and representations of Nilpotent Lie Groups.

We consider an homogeneous left invariant differential operator on a connected nilpotent Lie group G with a stratified Lie algebra $\mathfrak{g}$, and a closed subgroup H preserved by dilations. If π is the projection of G on $H \setminus G$, we conjecture that the differential operator $\pi(P)$ is hypoelliptic (with some inequalities) on $H \setminus G$ if and only if, for every non trivial unitary irreducible representation $\tilde{\pi}$ of G, which is in the spectrum (or support) of π, the operator $\tilde{\pi}(P)$ is injective in the space of C^∞ vectors of $\tilde{\pi}$. We have proved the necessity of this condition, and the equivalence when the length of the graduation of $\mathfrak{g}$ is ≤ 3, or when H is the trivial subgroup. This problem will perhaps be useful for studying hypoellipticity of differential operators in a manifold, which can be expressed as polynomial of vector fields satisfying Hörmander's condition.

D. Poguntke: Symmetry and nonsymmetry of a class of exponential Lie groups.

An involutive Banach algebra is called symmetric if every element of the form a^*a has a nonnegative spectrum. Several authors have investigated the question for which locally compact groups G the convolution algebra $L^1(G)$ is symmetric. Even for simply connected Lie groups a characterization of those which have a symmetric group algebra is not known. Let G be such a group, and let $G = S \ltimes R$ be its Levi decomposition. Then the compactness of S is a necessary condition for the symmetry. If S is compact then the symmetry of $L^1(R)$ is a sufficient condition for the symmetry of $L^1(G)$. So, the problem is essentially reduced to the case of a simply connected solvable Lie group G. While the case of groups with polynomially growing Haar measure is completely settled (they always have symmetric group algebras) the case of exponential groups seems to be much harder - and this is the topic of the talk.

For a subclass of the class of exponential groups the characterization problem can be completely solved. An exponential group G belongs (by definition) to [EA] if its Lie algebra $\mathfrak{g}$ allows a decomposition $\mathfrak{g} = \gamma \ltimes \mathfrak{n}$ with a nilpotent ideal $\mathfrak{n}$ and a commutative subalgebra γ where γ acts by semisimple derivations on $\mathfrak{n}$. For these groups we have the following

Theorem. Let $G \in$ [EA]. $L^1(G)$ is symmetric iff for every real functional f on $\mathfrak{g}$ the ideal $m(f)^\infty$ is contained in the kernel of f where $m(f)^\infty$ denotes the smallest ideal $\mathfrak{a}$ in $\mathfrak{g}(f) + [\mathfrak{g},\mathfrak{g}]$ such $\mathfrak{g}(f) + [\mathfrak{g},\mathfrak{g}]/\mathfrak{a}$ is nilpotent.

T. Pytlik: Gevrê classes operate on weighted algebras.

On a locally compact group it is considered a class of weights ω (so called polynomial weights) which satisfy

$$\omega(xy) \leq C\,(\omega(x) + \omega(y))$$

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for a constant C. To each weight ω there corresponds a convolution Banach *-algebra $L^1(G,\omega)$ of all measurable functions f on G such that

$$\|f\|_\omega = \int_G |f(t)|\ \omega(t)\,dt$$

is finite.

The main result is: Let G be a locally compact group and ω a polynomial weight on G. Let u denote the analytic function $u(z) = e^{iz} - 1$. If $1/\omega \in L^p(G)$ for a $1 \leq p < \infty$, then for every hermitian $f \in L^1(G,\omega) \cap L^2(G)$ we have

$$\|u(nf)\|_\omega = 0\ (e^{n^\gamma})$$

for every $\gamma > \frac{p+1}{p+2}$.

This by classical results implies that the Gevrê classes $G(1/_\gamma)$, $1 > \gamma \geq \gamma_0 > 0$ operate on hermitian elements of $L^1(G,\omega) \cap L^2(G)$.

H. Rindler: Uniform distribution and Harmonic Analysis

Two concepts of uniform distribution in locally compact groups are considered (the first one is based on H.Weyl's classical concept, the second one on the concept of uniform distribution in probability theory). There is a strong connection with invariant means. Some recent results are presented.

"Uniform distribution and the Mean ergodic Theorem" by Losert, Rindler, Inventiones Math. 50, 65-74 (1978)

"Asymptotisch gleichverteilte Netze von Wahrscheinlichkeitsmaßen auf lokalkompakten Gruppen" by Maxones, Rindler, Coll. Math. XL, 131-145 (1978).

J. Rosenberg: The 'Mackey Machine' and the Effros-Hahn Conjecture.

When G is a (second countable) locally compact group with a closed normal subgroup N, one is often interested in determining the irreducible unitary representations of G from knowledge of the representation theory of N

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and of the action of G on N by conjugation. When N is type I and "regularly embedded" in G, an algorithm for doing this is provided by the celebrated "Mackey machine". However, it has long been a problem to understand the more complicated cases of non-type I or non-regularly embedded N. We therefore explain how the recent proof of the "generalized Effros-Hahn conjecture" (joint work of the lecturer and E. Gootman, to appear in Inventiones Math.) can be used to classify the irreducible representations of G up to "weak equivalence", which coincides with unitary equivalence exactly when G is type I. When G is not type I, classifying the irreducible representations of G up to unitary equivalence is usually hopeless. Applications are given to theorems of Pukanszky on connected Lie groups and of Howe on nilpotent locally compact groups.

I.E. Schochetman : Generalized Group Algebras and their Bundles.

Our objective is to combine the notions of Banach *-algebraic bundle, generalized L^1-algebra and twisted group-algebra into a unified theory. (The latter two notions are known to be equivalent to each other, but skewly related to the first.) We accomplish our objective by: (1) extending the Banach *-algebraic bundle construction to the setting where the product and involution are just measurable, i.e. not necessarily continuous.
(2) introducing the *-algebra operations into such a bundle by means of operator fields and studying the smoothness of the operations in terms of the smoothness of the fields.

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S. Stratila : Representations of the unitary group $U(\infty)$.

The talk was a report on a joint work with Dan Voiculescu appearing in Lecture Notes in Mathematics No. 486 and Mathematischen Annalen 235 (1978), 87 - 110, concerning the representation theory of the non-locally compact group $U(\infty)$ = the direct limit of the classical unitary groups $U(n)$. This study was suggested by the work of A.A.Kirillov (Doklady Akad. Nauk, SSSR, 1973).
We have associated an Af C^*-algebra $A(U(\infty))$ so that the factor representations of $A(U(\infty))$ correspond in a canonical way to factor representations of $U(\infty)$ or of some $U(n)$. Using the diagonalization of AF-algebras that we developed (see also the works of W.Krieger, O.Bratteli, Jean Renault), we labelled the primitive ideals of $A(U(\infty))$ by two sequences of integers ("signatures"):

$$+\infty \geq L_1 \geq L_2 \geq \dots \geq L_n \geq \dots \geq M_n \geq \dots \geq M_2 \geq M_1 \geq -\infty$$

namely

$$L_j = \sup \{\sup \{ m_j^n ;\ n \geq j \}\} , \quad M_j = \inf \{\inf \{m_{n-j+1}^n ;\ n \geq j \}\}$$

is the signature which determines $\operatorname{Ker} \rho$ when ρ is a factor representation of $U(\infty)$ and the sup and inf are taken over all signatures $(m_1^n \geq \dots \geq m_n^n)$ appearing in the decomposition of $\rho \mid U(n)$.
On the other hand, let $\alpha = \{a_n\}$, $\beta = \{b_n\}$ be two orthonormal sequences in the separable infinite dimensional Hilbert space H and consider the natural representations ρ^α and ρ^β of $U(\infty)$ on the infinite tensor product Hilbert spaces along the sequences α and β, respectively. We proved that these are type II_∞ factor representation and that

$\rho^\alpha \simeq \rho^\beta \iff$ there exist finite sets $F_\alpha \subset \mathbb{N}$, $F_\beta \subset \mathbb{N}$, a bijection $\sigma : \mathbb{N} \setminus F_\alpha \to \mathbb{N} \setminus F_\beta$ and a sequence $\{\theta_n\}$ of unimodular complex numbers such that

$$\lim_n \| b_n - \theta_n a_{\sigma(n)} \| = 0.$$

An open problem is to find a good necessary and sufficient condition for the unitary equivalence $\rho^\alpha \simeq \rho^\beta$. In the particular case when α and β are contained in the same orthonormal basis of H, we proved that $\rho^\alpha \simeq \rho^\beta$ if and only if there exists n_0 such that $a_n = b_n$ for all $a \geq n_0$.

Y. Weit : On invariant subspaces of functions on $\mathbb{R}^N$.

Let A be a linear topological space and F a class of continuous linear transformations of A into itself. We say that the class F is of Wiener's type, if every closed subspace of A, invariant under F, contains an irreducible closed subspace of A.

We have studied this problem with A being the space $L_\infty(\mathbb{R}^n)$, equipped with the w*-topology and $F = M_0^{(r)}(\mathbb{R}^N)$, the space of all rotation-invariant Radon measures having compact supports, acting in $L_\infty(\mathbb{R}^N)$ by convolution. The fact that $M_0^{(r)}(\mathbb{R}^N)$ is not of Wiener's type implies that numerous sets of right maximal closed ideals in the Euclidean motion group fail to be sets of spectral synthesis. In particular, every right maximal closed ideal contains infinitely many right-primary ideals.

J. Boidol (Bielefeld)

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