MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 35/1979

Jordan Algebren

18.8. bis 25.8.1979

Die Tagung über Jordan Algebren und verwandte Gebiete stand in diesem Jahr unter der Leitung von K. McCrimmon (Charlottesville), K. Meyberg (München) und H. P. Petersson (Hagen). Als Teilnehmer konnten 43 Mathematiker aus der Bundesrepublik Deutschland, Frankreich, Israel, Kanada und den USA begrüßt werden. Das Spektrum der Vorträge umfaßte neben Themen über Strukturen allgemeiner Art (Investigations of algebras using derivations, Conservative algebras, Idempotents in the double dual algebra, Automorphisms and Jordan ideals in prime rings, The algebra of polynomial functions over an arbitrary algebra, Central localization of rings) und algebraischen Aspekten der Jordan Theorie (Zelmanov^2, Coordinatization of Jordan triple systems, Semisimple real Jordan triple systems, Exceptional simple Jordan algebras of generic matrices, Minimal exceptional Jordan algebras, Quasi-invertible dense Jordan pairs) eine Reihe von Themen der Jordan Theorie und ihrer Anwendungen im Bereich der reellen und komplexen Analysis (Jordan pairs and homogeneous Siegel domains, Theta functions for Jordan pairs, Symmetric spaces and Jordan algebras), im Bereich der Funktionalanalysis (A Gelfand-Naimark theorem for C^* triples, Derivations of Jordan C^* algebras), im Bereich der Differentialgleichungen (The Riccati differential equation in Jordan pairs, Jordan algebras and solitary waves) und im Bereich der Geometrie (Octonion planes over local rings). Zu nennen ist ferner eine Gruppe von Vorträgen über Lie Algebren bzw. Lie Tripel Systeme (Construction of Lie algebras from a class of nonassociative algebras with involution, Dynkin diagrams for Lie triple systems, Anti-Jordan pairs, Automorphisms of the Lie algebra of the polynomials in a Banach space, Some remarks about Lie algebra representations, Restricted simple Lie algebras with two-dimensional Cartan subalgebras).

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge

B.N. ALLISON: Construction of Lie algebras from a class of nonassociative algebras with involution.

Suppose $(A,{}^-)$ is an algebra with involution satisfying the operator identity $[T_z, V_{x,y}] = V_{T_z x,y} - V_{x,T_{\bar{z}} y}$ for x,y,z in A, where $V_{x,y}(w) = (x\bar{y})w + (w\bar{y})x - (w\bar{x})y$ and $T_z = V_{z,1}$. Examples of such algebras include associative algebras, Jordan algebras (with the identity map as involution), tensor products of two composition algebras, algebras constructed from Hermitian forms (just as Jordan algebras are constructed from quadratic forms), and the 56-dimensional module for E_7 with a natural binary product and involution. A Lie algebra $K(A,{}^-)$ can be constructed from $(A,{}^-)$ in a manner generalizing the Tits-Koecher construction of a Lie algebra from a Jordan algebra. This Lie algebra $K(A,{}^-)$ is central simple if and only if $(A,{}^-)$ is central simple. We discuss the structure of the central simple algebras $(A,{}^-)$ and the structure of the associated Lie algebras $K(A,{}^-)$.

G. BENKART: Investigations of algebras using derivations I

Let A be any finite dimensional algebra over a field of characteristic 0, and let L be a semisimple Lie algebra of derivations of A. Then $A = V_1 \oplus \ldots \oplus V_n$ where the V_i are irreducible L-modules. The product $V_i \times V_j \to A$ followed by the projection onto V_k induces an L-module homomorphism from $V_i \otimes V_j \to V_k$. Therefore by determining the V_i and the possible homomorphisms, we can obtain information concerning the structure of A for certain specialized types of algebras. (The investigation is joint work with J.M. Osborn, and two additional applications of this technique appear in his abstract.)

Application I - Real Division Algebras

If A is a finite dimensional division algebra over the reals, then $\dim A = 1,2,4$ or 8. We let $\operatorname{Der} A$ denote the derivation algebra of A, and prove

Theorem: (i) $\dim A = 1,2 \Rightarrow \operatorname{Der} A = 0$; (ii) $\dim A = 4 \Rightarrow \operatorname{Der} A = su(2)$ or $\dim \operatorname{Der} A = 0,1$; (iii) $\dim A = 8 \Rightarrow \operatorname{Der} A =$ compact G_2, $su(3)$, $su(2) \oplus su(2)$, $su(2) \oplus N$ where N is an abelian ideal and $\dim N = 0,1$, or N where N is abelian and $\dim N = 0,1,2$. Moreover all possibilities occur.

We then proceed using a case by case analysis and module theoretic techniques to investigate all real division algebras with a given Lie algebra as its derivation algebra for each Lie algebra listed in the theorem. The results indicate that most real division algebras having nonzero derivations are natural generalizations of the quaternions and octonions with the principal exceptions being those obtained from 3×3 skew-hermitian matrices.

J. M. OSBORN: Investigations of algebras using derivations II

We present two further applications of the technique discussed in the preceding abstract.

Application II-Algebras admitting sl(3) as derivations

We consider the class of algebras A over the real numbers which admit sl(3) as derivations and which decompose as an sl(3)-module into the direct sum of a module of dimension 1, two non-isomorphic modules of dimension 3, and an irreducible module of dimension 8. We represent this situation by

$$A = 1 + 3 + \bar{\bar{3}} + 8.$$

This class of algebras is motivated by elementary particle physics, and special cases have been investigated in detail by Domokos and Kovesi-Domokos and by Gunaydin and Gursey. In these algebras one of the 3-dimensional modules is associated with quarks and the other with anti-quarks. The 8-dimensional module is associated with gluons, which hold the quarks together. The multiplication in A can be expressed in terms of the natural multiplications between the four irreducible sl(3)-modules using a set of constants. Given an identity, such as the flexible identity, one can derive necessary and sufficient conditions on the set of constants for that identity to hold in A.

Application III – Flexible Lie-admissible algebras

Motivated by recent papers in physics, we consider finite-dimensional flexible Lie-admissible algebras over fields of characteristic 0. We can show by examples that there is no hope of developing a Wedderburn-type structure theory for those algebras, as has been done for associative algebras, Lie algebras, Jordan algebras and various classes of power-associative algebras. If A is a flexible Lie-admissible algebra and if $ad_x(y) = xy-yx$, then ad_x is a derivation of A for each $x \in A$. Thus A^- acts as an algebra of derivations on A. The decomposition of the Lie algebra A^- using the Levi decomposition may be refined further by expressing the radical of A^- as a sum of irreducible modules over the semisimple part of A^-, and we use this decomposition to investigate the structure of A. In particular we can characterize those simple flexible Lie-admissible algebras such that the radical of A^- is a direct summand of A^-. This includes all simple algebras A such that A^- is reductive. Our result shows that for any reductive Lie algebra L, there exists a simple flexible Lie-admissible algebra A such that $A^- = L$.

R. BIX: <u>Octonion Planes over Local Rings</u>

Let O be an octonion algebra which is a free module over a local ring. We classify the subgroups of the group of norm semisimilarities of $H(O_3,\gamma)$ normalized by the group of norm preserving transformations. We define the octonion plane determined by $H(O_3,\gamma)$ and prove that every collineation between two such planes is induced by a norm semisimilarity.

H. BRAUN: <u>The Riccati differential equation in Jordan pairs</u>

As an introduction the linearization of the matrix Riccati differential equation was derived for $m \times n$-matrices (J.J. Levin, Proc.Am.Math.Soc. <u>10</u>, 1959 (512 - 524). Then the Riccati differential equation for operators in a Banach space was mentioned (L. Tartar, J. of Fu.Analysis <u>6</u>, 1974 (1 - 47)).

Let (V^-,V^+) be a Jordan pair, where V^-,V^+ are Banach spaces (for definitions and notations see O. Loos, Jordan pairs, Springer Lecture Notes 460). The triple product is denoted by

$$\{x_\sigma,y_{-\sigma},z_\sigma\} =: D_\sigma(x_\sigma,y_{-\sigma})z_\sigma \quad , \qquad \sigma = \pm$$

but the lower index will only be used when it is unavoidable. Put $\frac{1}{2}\{xyx\} =: Q(x)y$. Let I be a $\mathbb{R}$-intervall, η an initial point in I, k a given initial value, $k \in V^+$. Let $v(\xi), w(\xi)$ be given continuous functions, $v : I \to V^-$, $w : I \to V^+$ and let D and Q be continuous. The Riccati differential equation (without linear term) will be defined by

$$\frac{\partial x}{\partial \xi} = Q(x)v + w.$$

The solution $x : I \times I \to V^+$ with initial value k at the point η will be denoted by $x(\xi,\eta)$. Use the notations

$$B(u,t) := \mathrm{Id} - D(u,t) + Q(u)Q(t), u^t := B(u,t)^{-1}(u-Q(u)t),$$

for $u \in V^+, t \in V^-$ if the inverse of $B(u,t)$ exists.

<u>Theorem</u>: Let x_o be the solution of the Riccati equation with initial value $k = 0$ at $\eta = 0$, put

$$x(\xi,\eta) := x_o + h_+(k)^{h_-(z)}$$

$h_\sigma : I \times I \to \mathrm{Aut}\, V^\sigma$, $z : I \times I \to V^-$, solve the linear system

$$\frac{\partial h_+}{\partial \xi} = D(x_o,v)h_+ \; , \; \frac{\partial h_-}{\partial \xi} = -D(v,x_o)h_-$$

then $\frac{\partial z}{\partial \xi} = h_-^{-1}(v)$ with $h_\sigma(\eta,\eta) = \mathrm{Id}$, $z(\eta,\eta) = 0$.

Then $x(\xi,\eta)$ is the solution with initial value $x(\eta,\eta) = k$ (in a neighborhood of η).

Indication of the proof: Use for the functions $u(\xi)$, $t(\xi)$ the formula

$$\frac{\partial u^t}{\partial \xi} = Q(u^t)\frac{\partial t}{\partial \xi} + B(u,t)^{-1}\frac{\partial u}{\partial \xi} .$$

This formula can also be used to solve the Riccati equation explicitly for special cases.

R. BRAUN: <u>A Gelfand - Naimark theorem for C^*Triples</u>

<u>Def.:</u> Let $(A, \|\ \|)$ a $\mathbb{K}$-Banachspace ($\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$) and

$\langle\ \rangle : A \times A \times A \to A$ an $\mathbb{R}$ - trilinear map, then A is called

N-triple over $\mathbb{K}$, if

(1) $\| \langle xyz \rangle \| \leq \|x\| \|y\| \|z\| \quad \forall x,y,z \in A$

(2) $\| \langle xxx \rangle \| = \|x\|^3 \quad \forall x \in A.$

These normconditions can be used for several kinds of algebraic structures, for example for Jordan, alternative or associative triples. A C^*triple is an associative triple (of 2^{nd}kind) which is an N-triple over $\mathbb{C}$ such that $\langle\ \rangle$ is $\mathbb{C}$-linear in the outer variables and conjugate linear in the middle variable.

<u>Theorem:</u> (Gelfand Naimark for C^*triples):

Let $(A, \langle\ \rangle, \|\ \|)$ be a C^*triple, $e \neq 0$ a tripotent(e.g.$\langle eee \rangle = e$) and $A = A_{00} \oplus A_{01} \oplus A_{10} \oplus A_{11}$ the corresponding Peircedecomposition

$$A_{jk} = \{x \in A \mid \langle eex \rangle = jx, \langle xee \rangle = kx\}.$$ Assume

1) e is maximal (in the sense that $A_{00} = \{0\}$) and

2) $e + \frac{1}{2}(\langle eww \rangle + \langle wwe \rangle)$ is invertible in the associative C^*algebra A_{11} for each $w \in A_{01} \oplus A_{10}$.

Then there exists a complex Hilbertspace and a faithful representation $w : A \to L(H)$.

J. DORFMEISTER: Jordan Pairs and Homogeneous Siegel Domains

The group $\mathrm{Aut}\, D(K,S)$ of biholomorphic maps of a homogeneous Siegel domain $D(K,S)$ in $V^{\mathbb{C}} x U$ is described.
We first note $G = G_{-1} + G_{-1/2} + G_0 + G_{1/2} + G_1$ where G_λ is parametrized by P_λ. As $G^{\mathbb{C}} = (G^{\mathbb{C}} \cap \text{linear polynomials}) + G^{\mathbb{C}} \cap \text{const.}) + (G^{\mathbb{C}} \cap \text{ quadratic}) =: G'_0 + G'_{-1} + G'_1$ we get a Jordan pair $R = (R^+, R^-), R^+ = G'_{-1}, R^- = G'_1$ by setting $\{X_\varepsilon Y_{-\varepsilon} Z_\varepsilon\} = -[[X_\varepsilon, Y_{-\varepsilon}], Z_\varepsilon]$. Now put $M = (M^+, M^-)$ $M^+ = V^{\mathbb{C}} \oplus U$, $M^- = P_1{}^{\mathbb{C}} \oplus P_{1/2}$ and provide the latter with the dual complex structure), then there exist canonical isomorphisms $j_\varepsilon : M^\varepsilon \to R^\varepsilon$ turning M into a Jordan pair. We now get $(\exp X_{1/2}[w])(z,u) = ((z,u) + f(z,u,w))^w$ (the quasi-inverse in the Jordan pair M) and $(\exp X_1[x])(z,u) = (z,u)^x$. Using a theorem of W. Kaup the description of $\mathrm{Aut}\, D(K,S)$ is completed.

J. FAULKNER: Dynkin diagrams for Lie triple systems

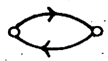
Let T be a simple Lie triple system over an algebraically closed field of characteristic zero and let H be a Cartan subalgebra of $L^+ = [T,T]$ where $L = T \oplus [T,T] = L^- \oplus L^+$ is the standard embedding of T. Let $(\ ,\ )$ be the restriction of the Killing form on L pulled over to H^*. Let $\alpha_1, \ldots, \alpha_\ell$ be a set of simple roots for L^+. If T is irreducible, let $\alpha_{\ell+1}$ be the minimal weight. Otherwise let $\alpha_{\ell+1}, \alpha_{\ell+2}$ be the minimal weights of the components of T.

Lemma 1: (1) $$A_{ij} = \frac{2(\alpha_i, \alpha_j)}{(\alpha_j, \alpha_j)} \in \mathbb{Z}$$

(2) $$A_{ij} \leq 0 \quad \text{for} \quad i \neq j$$

(3) $$\exists\, m_i \in \mathbb{Z},\ m_i \geq 0 \text{ with } m A = 0 \text{ where } A = (A_{ij}$$

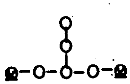
$$\text{and } m = \begin{cases} (m_1, \ldots, m_\ell, 2) & \text{if } T \text{ is irreducible} \\ (m_1, \ldots, m_\ell, 1, 1) & \text{otherwise} \end{cases}$$

Let Γ be the graph with nodes indexed by i and $|A_{ij}|$ directed edges from node i to node j. By convention o——o := 
and o⇛o :=  etc.

Lemma 2: Γ is an affine Dynkin diagram and the m_i are uniquely determined up to a scalar multiple.

Marking the vertices $\ell+1$, $\ell+2$ by ● we get a marked graph $(\Gamma, ●)$.

Theorem: $(\Gamma, ●)$ is one of 40 possibilities each corresponding to a simple Lie triple system.

Examples:  corresponds to the Jordan pair

$\mathcal{O}_{1,2} \oplus \mathcal{O}_{2,1}$ $\quad \mathcal{O}$ octonion

corresponds to the Lie triple system of trace zero $(2\ell + 1)$ square matrices.

J. C. FERRAR: Anti-Jordan pairs

A pair (P_1, P_{-1}) of vector spaces with product $M_\varepsilon x M_{-\varepsilon} x M_\varepsilon \to M_\varepsilon$ $((x_\varepsilon, y_{-\varepsilon}, z_\varepsilon) \to \{x_\varepsilon y_{-\varepsilon} z_\varepsilon\})$ satisfying i) $\{x_\varepsilon y_{-\varepsilon} z_\varepsilon\} = -\{z_\varepsilon y_{-\varepsilon} x_\varepsilon\}$ and ii) $\{\{x_\varepsilon y_{-\varepsilon} z_\varepsilon\} u_{-\varepsilon} v_\varepsilon\} = \{\{x_\varepsilon u_{-\varepsilon} v_\varepsilon\} y_{-\varepsilon} z_\varepsilon\} + \{x_\varepsilon \{y_{-\varepsilon} v_\varepsilon u_{-\varepsilon}\} z_\varepsilon\} + \{x_\varepsilon y_{-\varepsilon} \{z_\varepsilon u_{-\varepsilon} v_\varepsilon\}\}$ for all $x_\varepsilon, z_\varepsilon, v_\varepsilon \in M_\varepsilon, y_{-\varepsilon} u_{-\varepsilon} \in M_{-\varepsilon}, \varepsilon = \pm 1$, is an anti-Jordan pair.

__Theorem:__ A simple anti-Jordan pair over k algebraically closed of characteristic zero is isomorphic to one of.

a) $Sps(2n)$: $P_1 = P_{-1} = k_{1,2n}$, $\{x_\varepsilon y_{-\varepsilon} z_\varepsilon\} =$

$$(x_\varepsilon Q y_{-\varepsilon}{}^t) z_\varepsilon + (x_\varepsilon Q z_\varepsilon{}^t) y_{-\varepsilon} - (z_\varepsilon Q y_{-\varepsilon}{}^t) x_\varepsilon, \quad Q = \begin{pmatrix} O & I \\ -I & O \end{pmatrix}$$

b) $GL(n_1, n_{-1})$: $P_\varepsilon = k_{n_\varepsilon, n_{-\varepsilon}}$, $\{x_\varepsilon y_{-\varepsilon} z_\varepsilon\} = x_\varepsilon y_{-\varepsilon} z_\varepsilon - z_\varepsilon y_{-\varepsilon} x_\varepsilon$

c) $Sym(n) \subseteq GL(n,n)$: $P_\varepsilon = \{x \in k_{n,n} | x^t = \varepsilon x\}$

__Remark:__ via an embedding process analogous to the well-known Jordan pair → Lie triple system → Lie algebra we obtain an embedding sequence anti Jordan pair → anti Lie triple system (satisfying Lie triple system identities except [xyz] = [xzy] → Lie superalgebra which maps simple objects to simple objects.

P.D. GERBER: Jordan Algebras and Solitary Waves

A large number of physically relevant differential equations exhibeting solitary wave behaviors have been successfully analysed by the inverse scattering method. A key step in the development of this method was the discovery (by R. Miura, 1967) that the transformation $v = u_x + u^2$ maps the Modified Korteweg de Vries (MKdV) $u_t - 6u^2 u_x + u_{xxx} = 0$ into the Korteweg de

Vries (KdV) equation $v_t - 6vv_x = v_{xxx}$.
In order to extend the method to systems (equations in $\mathbb{R}^n$), we seek conditions on f, H and $\hat{H}$ such that $v = f(u,u_x)$ (M) maps $u_t = u_{xxx} + H(u)u_x$ (S) into $v_t = v_{xxx} + \hat{H}(v)v_x$ (T). After giving precise assumtions on (M), (S), (T), we can prove the following theorem.

<u>Theorem</u>: (M) transforms (S) into (T) if and only if (M), (S), (T) have the form

$$u_t = u_{xxx} + (R^*[-\tfrac{1}{6}u*u + Bu + c] + J)u_x$$

$$v = u_x - \tfrac{1}{6}u*u + Bu + c$$

$$v_t = v_{xxx} + (R^*[v] + J)v_x$$

where $*$ is multiplication in an algebra $Q(*)$ and

i) $Q(*)$ is a Jordan algebra
ii) $B(u^2*p) + 2u*((Bu)*p) = u^2*(Bp) + 2(Bu)*(u*p) \ \forall u,p \in Q$
iii) $3BR^*[Bu] - R^*[u](R^*[c] + J) = 3R^*[Bu]B - (R^*[c] + J)R^*[u]$
iv) $(R^*[c] + J)B = B(R^*[c] + J)$.

W. HEIN: <u>Conservative algebras</u>

For an unitary commutative ring k and a k-module X let Alg X := Hom(X, End X). Obviously the elements of Alg X can be identified with the left multiplication operators of all algebras on X. For $a,b \in X$ and $A \in$ Alg X we write $aAb := A_a(b) := A(a)(b)$ to denote the multiplication in the algebra A. Alg X becomes an $\operatorname{End} X$ - Lie module by setting $a(T.A)b := T(aAb) - (Ta)Ab - aA(Tb)$ for $T \in \operatorname{End} X$, $A \in \operatorname{Alg} X$, $a,b \in X$. For $A,B \in \operatorname{Alg} X$ and $a \in X$ we define a new algebra by $AaB := -A_a . B$. The algebra $A^a := AaA$ is called the a-homotope of A (The foregoing notations are due to M. KOECHER.)

In 1972 I. L. KANTOR introduced the notion of "conservative algebra", which in the above setting means the following: An algebra A on X is called conservative, if there exists an algebra B on X such that $AaA^b = A^{aBb}$ for all $a,b \in X$. In a slightly more general sense this means, that the set of all homotopes of A is invariant under the action of the subalgebra of $(\operatorname{End} X)^-$ generated by all left multiplication operators A_a.
Let $(a,b,c) \longmapsto abc$ be a ternary product on X such that there exists an $e \in X$ with $a \longmapsto eae$ $(a \in X)$ bijective. Then the algebra $(a,b) \longmapsto aeb$ $(a,b \in X)$ is conservative, if $ab(xyz) - xy(abz) = (abx)yz - x(bay)z$ for all $a,b,x,y,z \in X$.

For any conservative algebra A we construct an anti-commutative algebra L(A) which, in case A has a left unit e, is a Lie algebra if and only if A satisfies $[a,[b,c]] + [ab,c] + [b,ac] = 0$ for all a,b,c. Conservative algebras with these conditions satisfy the "fundamental formula" $B^{\bar a} b B^{\bar c} = B^{aB^{\bar b}c}$ for all homotopes B of A and a suitable linear transformation $a \longmapsto \bar a$ $(a \in X)$. Moreover, $T \longmapsto T^* := T - A_{Te+(Te)\tau}$ for a certain linear transformation τ of X defines a homomorphism of $(\operatorname{End} X)^-$ of degree 2, and we have $T.A^a = A^{(t \circ T^* \circ t^{-1})(a)}$, $t(a) := \bar a$, $(A_b^{\bar a})^* = -A_a^{\bar b}$, $a,b \in X$.

J. HEINZE: Automorphisms of the Lie algebra of the polynomials in a Banach space

Let X be a Banach space over $\mathbb{K}$ ($\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$) and $L[X]$ be the vector space of the continuous polynomials from X to X. The (Frechet-)derivative at a point $x \in X$ for a polynomial $p \in L[X]$ is denoted by $Dp(x)$. With the multiplication

$$L[X] \times L[X] \to L[X], \quad (p,q) \longmapsto p \cdot q, \quad (p \cdot q)(x) := Dp(x)(q(x)),$$

$L[X]$ becomes a right-symmetric algebra, that means, the associator $(p,q,h) := (p \cdot q) \cdot h - p \cdot (q \cdot h)$ is symmetric in the last two arguments. Therefore $L[X]$ endowed with the commutator-product

$$L[X] \times L[X] \to L[X], \quad (p,q) \longmapsto [p,q] := p \cdot q - q \cdot p,$$

is a Lie algebra, denoted by $\mathrm{Lie}[X]$. In the following the group $\mathrm{Aut}\,\mathrm{Lie}[X]$ of automorphisms of $\mathrm{Lie}[X]$ is discussed. There exist at least two types of automorphisms:

i) $\Psi(f) : L[X] \to L[X]$, $\Psi(f)p := (f \cdot p) \circ f^{-1}$, where f is an element of the group $G[X] := \{f \in L[X];\ f(0) = 0,\ Df(0)$ invertible, $f^{-1} \in L[X]\}$. (Here is „$\circ$" the usual composition of mappings and f^{-1} the inverse of f relative to „$\circ$").

ii) $S_a : L[X] \to L[X]$, $S_a(p)(x) := p(x+a)$, where $a \in X$, the so-called group of substitution - automorphisms, denoted by $\mathrm{Subst}\, L[X]$. Obviously the group $\mathrm{Subst}\, L[X]$ is isomorphic to the additive group $(X,+)$.

- 13 -

Denote by e the <u>identity</u> mapping of X, $e(x) := x$ for all $x \in X$. We get the following results:

<u>Theorem 1:</u> The set $\tilde{G} := \{W \in \mathrm{Aut\,Lie}[X]; (We)(0) = 0\}$ is a subgroup of Aut Lie[X], and the mapping $\Psi : G[X] \to \tilde{G}$, $f \longmapsto \Psi(f)$, is a group isomorphism.

A greater subgroup of Aut Lie X is described by

<u>Theorem 2:</u> The set $G := \{W \in \mathrm{Aut\,Lie}[X];\ X \to X,\ a \longmapsto (Wa)(0),$ is surjective$\}$ is a subgroup of Aut Lie[X], and each automorphism $W \in G$ has a unique representation as $W = \Psi(f) \circ S_a$ with $f \in G[X]$ and $a \in X$.

J. LOUSTAU: <u>Idempotents in the Double Dual Algebra</u>

Let A be a non-associative algebra over a field F with identity e. There is a multiplication on $A^{**} := \mathrm{Hom}_F(\mathrm{Hom}_F(A,F),F)$, denoted by $\circ$, so that $(A^{**},\circ)$ contains A as a subalgebra. Further, the double dual of any subalgebra of A is naturally a subalgebra of A^{**}. Moreover, for A associative, $\mathrm{Hom}_F(A,F)$ is a right A-module via the dual to left multiplication and A^{**} is isomorphic to the complete ring of A-endomorphisms of this module. Hence, in the case of an associative algebra the idempotents in A^{**} correspond to complimented submodules of $\mathrm{Hom}_F(A,F)$. If A is power-associative, $x \in A$ is transcendental, then $F[x]^*$ is the direct sum of indecomposable $F[x]$-modules and associated with one of these modules is an idempotent $\varphi \in A^{**}$ such that $x \circ \varphi$ is locally nilpotent on $F[x]^*$ and $x \circ (e - \varphi)$ is invertible.

J. MAYNE: Automorphisms and Jordan ideals in prime rings

Let R be an associative prime ring and Z be its center. Let T be a function $T : R \to R$ such that $[x^T, x]$ is in Z for all $x \in R$ and T is also an endomorphism of the group R under addition. Such a mapping T is called centralizing. It is known that if R has a nontrivial centralizing automorphism or derivation, then R must be commutative. Also if U is a Jordan ideal, char $R \neq 2$, and T is a nontrivial centralizing derivation on U, then $U \subseteq Z$. Here we investigate the automorphism case.

Proposition: Let R be a prime ring of char $\neq 2$ and T be a mapping from R to R which preserves addition and squaring. If U is a subset of R closed under addition and for all u in U, $u^2 \in U$ and $[u, u^T]$ is in Z, then $[u^T, u] = 0$.

Theorem: Suppose R is a prime ring of char $\neq 2$ and U is a Jordan ideal of R. If R has an automorphism which is nontrivial on U and such that $[u, u^T]$ is in Z for all u in U, then $U \subseteq Z$.

K.McCRIMMON: Zelmanov ^{2}

Both the Jacobson theory of Jordan algebras with d.c.c., and the Goldie theory of Jordan algebras with a.c.c., on inner ideals require the result that the nil radical of an algebra with chain condition is nilpotent. In 1977 Ephim Zelmanov

- 15 -

succeeded where others had failed, and established

<u>Zelmanov's Nilpotence Theorem:</u> If a Jordan algebra has a.c.c. or d.c.c. on inner ideals, then its nil radical is nilpotent.

Although stated for linear Jordan algebras, the result can be extended to quadratic Jordan algebras.

To make use of the chain condition we need a rich supply of annihilation inner ideals $A(X)$, such that $X \subset Y \Rightarrow A(X) \supset A(Y)$ and $X \subset A(Y) \Longleftrightarrow Y \subset A(X)$. Then $A(X) = A(A(A(X)))$, and A is an order anti-isomorphism of period 2 on annihilator inner ideals, so either chain condition implies the other. By basing proofs on such a symmetric annihilator, we can treat the a.c.c and d.c.c simultaneously.

Zelmanov introduced the annihilator $A(X) = \{z \mid z \circ X = [z,J,X] = 0\}$ or better $\{z \mid V_{z,X} = 0\} = \{z \mid V_{X,z} = 0\}$. In the quadratic case one must define $A(X) = \{z \mid V_{z,X} = V_{X,z} = U_z X = U_X z = U_z U_X = U_X U_z = 0\}$.

From the properties of this concept of annihilator one establishes the main theorem by a series of steps: in the presence of the a.c.c. or d.c.c. nil $\Rightarrow$ lower radical $\Rightarrow$ locally nilpotent $\Rightarrow$ solvable $\Rightarrow$ Penico solvable $\Rightarrow$ nilpotent.

These steps each have completely different proofs, so a single direct proof is still lacking.

- 16 -

K. MEYBERG: Coordinatization of Jordan Triple Systems

The classical coordinatization theorem of N. Jacobson for Jordan algebras asserts that a Jordan algebra with a supplementary family of nxn Jordan matrix units (for $n \geq 3$) is isomorphic to a Jordan matrix algebra $H_n(D,D_0)$ of hermitian nxn matrices with coordinates in an alternative algebra D, with nuclear involution, which is associative if $n \geq 4$. This theorem not only is fundamental in classifying the simple Jordan algebras, but also immediately describes their unital bimodules. In a joint work with K. McCrimmon we develop a similar coordinatization for Jordan triple systems. Here there are 3 distinct cases: the rectangular pxq-matrices $M_{p,q}(D)$, the symplectic nxn matrices $S_n(D)$ and the nxn hermitian matrices $H_n(D;D_0,j)$ over D. We show that a Jordan triple system with pxq rectangular grid ($p+q \geq 3$), nxn symplectic grid ($n \geq 4$) or nxn hermitian grid ($n \geq 3$) is a rectangular, symplectic, or hermitian matrix system whose coordinate algebra is associative if $p+q \geq 4$ in the rectangular case, or $n \geq 4$ in the hermitian case, and commutative associative in the symplectic case if $n \geq 4$. The key here is the fact that any two collinear tripotents are coordinatized by an alternative algebra with involution.

E. NEHER: Semisimple real Jordan triple systems

All Jordan triples considered are semisimple, real and finite-dimensional. Let V be such a triple. An involutive automorphism α of V is called a Cartan-involution if the quadratic form $v \to \text{trace}\,(u \to \{v,\alpha v,u\})$ is positive-definite.

<u>Theorem:</u> Let V' be a semisimple subsystem of V. Then every Cartan-involution of V' can be extended to a Cartan-involution of V.

Assume W is a compact Jordan triple system (i.e. id_W is a Cartan-involution) and β is an involutive automorphism of W. Then

$$W_\beta := \{x \in W; \beta x = x\} \oplus i\{x \in W; \beta x = -x\} \subset W^{\mathbb{C}}$$

is a real semisimple Jordan triple system. A Corollary of the theorem is that every real semisimple Jordan triple system arises in this way. Since W simple $\Leftrightarrow$ W_β central-simple one gets a classification of the central-simple real Jordan triple systems by a classification of the simple compact Jordan triple systems, which is known, and a classification of the involutive automorphisms of each simple compact Jordan triple system.

This second classification includes a classification of the involutive isometries of the irreducible bounded symmetric domains.

J.M. OSBORN: <u>Investigations of algebras using derivations II</u>

Text im Anschluß an G.Benkart.

H.P. PETERSSON: <u>Exceptional Simple Jordan Algebras of Generic Matrices</u>

The technique of generic matrices, due to Amitsur, has given rise to striking advances in the theory of associative algebras.

It is the purpose of this lecture to indicate what happens when this technique is applied to the setting of exceptional simple Jordan algebras. Let F be an infinite field of characteristic not 2, and write $J_s(F)$ for the split exceptional simple Jordan algebra over F.

Choose an infinite sequence $\mathcal{Y} = (Y_p)_{p \geq 0}$ of "generic matrices", i.e., of generically independent elements, over $J_s(F)$. So $\mathcal{Y}$ lives in $J_s(\Omega)$, the split exceptional simple Jordan algebra over some purely transcendental extension field Ω of F. The basic idea of Amitsur adapted to the Jordan setting now consists in looking at J' defined to be the smallest F-subalgebra of $J_s(\Omega)$ which contains 1 as well as all the Y_p's and stays invariant under the adjoint mapping. It is shown that the center, R, of J' may be viewed as a subring of Ω, which allows us to look at its quotient field $L = \text{Quot}\,R$ in Ω and to consider the "central localization" $J = L \otimes_R J'$. We prove that J is a central exceptional Jordan division algebra over L which can only be obtained from the Second Tits Construction of exceptional simple Jordan algebras. This is accomplished by using the technique of "specialization" adapted to the Jordan setting.

M. RACINE: <u>Minimal exceptional Jordan algebras</u>

A (unital) exceptional algebra is minimal if all (unital) proper subalgebras are special, exceptional division algebras being an example. Let $J = H(\mathcal{O}_3)$, $\mathcal{O}$ the split octonions, Petersson has shown that all maximal nilpotent subalgebras of J are conjugate (and of dim.12). One can show that these algebras are special. Therefore in searching for minimal exceptional algebras in J we must consider subalgebras having non-zero semi-simple part. If $A = L \oplus R$, L semi-simple, R radical, let F be the penultimate term in the Penico sequence, then F is a point space and $T(F) = 0$; $J(F) = \{x \in J \mid x \times F \subseteq F\}$ is the largest subalgebra of J containing F as an ideal. Using the classification of traceless point spaces F and of their idealizers $J(F)$ a 16 dimensional minimal exceptional algebra is obtained.

H.L. RESNIKOFF: Theta Functions for Jordan Pairs

Following some historical observations to mark the 150th anniversary of the publication of C.G.J. Jacobi's Fundamenta nova theoriae functionum ellipticarum, theta functions associated with complex finite dimensional Jordan Pairs admitting a positive Hermitian involution were defined.

Let (V^+, V) be such a Jordan Pair with triple product $V^+ x V x V^+ \to V^+$ denoted by $(u^*, v, w^*) \longmapsto \{u\,v\,w\}$ where $u, v, w \in V$ and $* : V \to V^+$ is the involution, and we write u, w in the product in place of u^*, w^*. Let $e_1, e_2 \in V$ be tripotents such that $e_1 + e_2$ is maximal, and $V = \oplus V_{ij}$ be the corresponding Peirce decomposition. $V_{11} \oplus V_{12} \oplus V_{22} = A \otimes \mathbb{C}$ where A is a formally real Jordan algebra; write $V_{ij} = A_{ij} \otimes \mathbb{C}$ for $1 \le i \le j \le 2$. Choose lattices $L_{12} \subset A_{12}$ and $L_{10} \subset V_{10}$, an element H such that $0 < H \in A_{11}$, and let $Z = z_{22} \oplus z_{20} \in V_{22} \oplus V_{20}$

belong to the Siegel domain of type 2

$$D = \{z_{22} \oplus z_{20} \in V_{22} \oplus V_{20} : \frac{z_{22}-z_{22}^*}{2i} - \frac{1}{2}\{z_{20}, z_{20}, e_2\}^* > 0\}.$$

Let $U \in V_{12} \oplus V_{10}$. Set $\sigma(u,v) = c \operatorname{trace} D(u,v)$ where $D(u,v)w = \{u\,v\,w\}$ and the constant c is determined so that $\sigma|A\times A$ coincides with the reduced trace. Finally, put $Q(x)y = \frac{1}{2}\{xyx\}$. Then the theta function of order H associated with the lattice $L = L_{12} \oplus L_{10}$ and the Siegel domain D is

$$\Theta_L(Z,U:H) = \sum_{\substack{\lambda\in L \\ \lambda=\lambda_{12}\oplus\lambda_{10}}} \exp i\pi\sigma(Q(\lambda)Z + i\{e_1\lambda_{10}\lambda_{10}\} + \{e_1U\lambda\}, H)$$

This series satisfies analogues of all the usual functional and differential equations, and reduces to the theta function associated with a Jordan algebra when $V_{10} = V_{20} = 0$.

H. RÖHRL: <u>The algebra of polynominal functions over an arbitrary algebra</u>.

Let R be an associative, commutative, unital ring which contains an infinite field. By a R-algebra A ist meant a R-multilinear multiplication $\otimes_R^m A \to A$; the arity m is fixed throughout and is ≥ For any set X one has the algebra of formal polynomials over A with indeterminates in X, the indeterminates are not commuting, not associating, etc., and the elements of A do not commute, do not associate, etc. with the elements of X. Call this algebra $A[X]_{fl}$. Each $P \in A[X]_{fl}$ is a function $\bar{P} : A^n \to A$, for suitable $n = n(P)$. The nullfunctions form an ideal In, and we set $A[X] = A[X]_{fl}/In$

and call it the algebra of polynomial functions over A with indeterminates in X. A homomorphism of algebras, $f : A \to B$, is called nomial if it induces a homomorphism $A[X] \to B[X]$, for any X. All m-ary algebras together with all nomial homomorphisms form a category $N_R Alg_m$. It is clear that all surjective homomorphisms are in $N_R Alg_m$.

Proposition: $f : A \to B$ is nomial iff it factors surjectively through $A \to A[X]$, for some X.

Proposition: If $f_i : A_i \to B_i$ is nomial, then so is $\otimes_R^n f_i : \otimes_R^n A_i \to \otimes_R^n B_i$. If $f : A \to B$ is nomial and P is a prime ideal of B with $O = f^{-1}(P)$, then O is a prime ideal of A; it is conjectured that for any prime P, $f^{-1}(P)$ ist a prime.

Put $\gamma(A) = \{a \in A : \exists q \in \mathbb{N}$, s.t. any product in A that containes at least q factors equal to a, vanishes$\}$. For an ideal I in A, define $\sqrt{I}$ by $\sqrt{I}/I = \gamma(A/I)$.

Proposition: $\gamma(-)$ is a functor on $N_R Alg_m$.

If $R = F$ is a field and $\dim_F A = d < \infty$, there is an injective algebra homomorphism $A[X] \overset{\chi}{\rightarrowtail} F[X_1,\dots,X_d] \otimes_F A$, where $X_1,\dots,X_d$ are mutually disjoint copies of X. A is called strictly simple if this is an isomorphism. The strictly simple F-algebras form a non-empty Zariski-open set in the space of all d-dimensional algebras.

Proposition: A strictly simple $\Rightarrow$ A is radical free

$\dim_F A[X] < \infty \Leftrightarrow A$ is radical.

Theorem: A strictly simple $\Rightarrow$ $F[X_1,\dots,X_d] \supseteq I \longmapsto \chi^{-1}(I \otimes_F A)$ is a lattice isomorphism of ideals which preserves products, prime ideals, $\sqrt{}$.

This theorem implies: 1) A strictly simple, $A \hookrightarrow B$ nomial, then there is an associative, commutative unital F-algebra $\hat{B}$

with $B \cong \hat{B} \otimes_F A$ over A, and $\gamma(B)$ is the prime radical of B.

2) if, in addition, B is finitely generated over A as an algebra, then B is noetherian, $\gamma(B)$ is the Jacobson radical of B, B is a Jacobson algebra. If one permits the zeros of $P \in A[X_1,\ldots,X_m]$ to lie in $A^{F'}$, F' any algebraic extension of F, then one has

<u>Hilbert's Nullstellensatz</u>: A strictly simple, $P \in A[X_1,\ldots,X_n]$, I ideal $\subset A[X_1,\ldots,X_n] \Rightarrow$ (zeros (P) $\supseteq$ zeros (I) $\Leftrightarrow P \in \sqrt{I}$).

This implies that for F algebraically closed, and A as above, $\max\operatorname{spec} A[X_1,\ldots,X_n] \cong A^n$; and $P(a_1,\ldots,a_n) = 0 \Rightarrow P \in (X_1-a_1,\ldots,X_n-a_n)$.

<u>Theorem</u>: A strictly simple, B and C unital algebras, $f: B \otimes_F A \to C \otimes_F A$ an algebra homomorphism with $f(1 \otimes a) = 1 \otimes a$, $\forall a \in A$. Then $\exists_1 \varphi : B \to C$ s.t. $f = \varphi \otimes \mathrm{id}_A$.

A similar descent theorem is valid for derivations.

<u>Corollary</u>: A strictly simple $\Rightarrow$ the category $(A, N_R\mathrm{Alg}_m)$ of nomial A-algebras is equivalent to the category of associative, commutative (not nec. unital) F-algebras. In particular, $\mathrm{Aut}(B,A) \cong \mathrm{Aut}(C_A(B),F)$, where B is a nomial extension of A, and $C_A(-)$ is the above equivalence of categories.

The corollary leads to a generalization of Wedderburn's Principle Theorem etc.

A general A-module M is a R-module together with $\otimes_R^{i-1} A \otimes_R M \otimes \otimes_R^{m-i} A \to M$, $i = 1,\ldots,m$. M is called nomial if $A \to A \oplus M$ (split null extension) is nomial.

<u>Theorem</u>: The category $N_A\mathrm{Mod}$ of nomial A-modules is a Grothendieck category with enough injectives, projectives, and free objects. The free objects are exactly the homogeneous degree one piece $A[X]^{(1)}$ of $A[X]$. A itself and all ideals of A are in $N_A\mathrm{Mod}$.

<u>Theorem</u>: If A is a finite dimensional F-algebra, then for some finite field extension F_0 of F, $A \cong \prod_{\text{finite}}$ (strictly simple alg's) x null algebra $\Leftrightarrow$ for all finite field extensions F' of F,

- 23 -

$\mathrm{gl\,dim}\,({}_{A}N_{F}\text{-}\mathrm{Mod}) = 0.$

Theorem: If A is strictly simple, then $\mathrm{gl\,dim}\,({}_{A}N\,\mathrm{Mod}) = nd$ where $d = \dim_F A$.

H. STRADE: Some remarks about Lie algebra representations

Sei L eine endlich dimensionale Lie-p-algebra über algebraisch abgeschlossenem Körper der Charakteristik $p > 2$, U die Universelle Einhüllende und Z das Zentrum von U. Dann gilt:

Satz: 1) Sei $n := \max\{\text{Dimension irreduzibler Darstellungen von } U\}$.

Dann gilt $n^2 = \dim_{Q(Z)} U \otimes_Z Q(Z)$

2) Sei $\mathcal{M} := \{\text{maximale Ideale } M \text{ von } U, \text{ so daß } \dim U/_M = n^2\}$.

Dann gilt: $\forall M \in \mathcal{M} \quad \exists \alpha \in Z - M$, so daß $U_{(\alpha)}$ eine zentral separable $Z_{(\alpha)}$-Algebra ist.

3) $(Z \cap M)U = M \Longleftrightarrow M \in \mathcal{M} \Longleftrightarrow (Z \cap M)U \in \mathrm{Spec}\,(U)$.

Satz: Sei M ein irreduzibler L-modul. Dann gibt es eine p-Unteralgebra Q, so daß gilt

1) Es gibt genau einen irreduziblen Q-Untermodul M_0 von M. M wird induziert von M_0. (Sei φ_0 die Darstellung)

2) Es gibt ein Ideal N von Q, so daß $\mathrm{rad}(^R/_N) = (0)$, $N^{(1)} \subset \varphi_0^{-1}(\mathrm{Kid})$

3) $N/_{\varphi_0^{-1}(\mathrm{Kid})} = (0)$ oder $N/_{\varphi_0^{-1}(\mathrm{Kid})}$ ist direkte Summe irreduzibler p-Moduln von $Q_{/N}$ mit gerader Dimension.

Korollar: Die Bestimmung der irreduziblen Darstellungen einer Lie-algebra läßt sich zurückführen auf diejenige von Lie-algebren G, deren Radikal gerade das eindimensionale Zentrum ist.

A. TILLIER: Symmetric Spaces and Jordan Algebras

Let Inv(U) be the open set of invertible elements of U, where U is a finite-dimensional Jordan algebra over $\mathbb{R}$ with unit e. Then Inv(U) is a symmetric space, pseudo-Riemannian if U is 1/2-simple. Inv(U) is not connected; we give a definition of the connected components in the formal real case, and relations between their groups of transvections and the group generated by the quadratic representation.

H. UPMEIER: Derivations of Jordan C^*-algebras

Derivations of Jordan C^*-algebras *(JB-algebras)* are important to mathematical physics (quantum mechanics) and complex analysis (bounded symmetric domains). By the structure theory of Alfsen, Shultz and Størmer, the study of JB-derivations can be reduced to the case of *JC-algebras* (Jordan algebras of Hilbert space operators). Our *extension theorem* shows that all derivations of *reversible* JC-algebras can be extended to derivations of C^*-algebras and are therefore induced by Hilbert space operators. Here reversibility can not be omitted as examples of spin factors show. We prove that JB-algebras with Banach predual have only *inner derivations* (except algebras of type I_2). For JB-algebras in general, each derivation is a limit of inner derivations in a certain topology *(approximation theorem)*. As an application we obtain fundamental algebraic properties *(semi-simplicity, simplicity)* of JB-derivation algebras.

- 25 -

K.S. WATSON: QUASI-INVERTIBLE DENSE JORDAN PAIRS

The class of quasi-invertible dense Jordan Pairs is introduced and discussed. Let $V = (V^+, V^-)$ be a Jordan Pair over a ring k. For $a \in V^\sigma$, $b \in V^{-\sigma}$ $(\sigma = \pm)$, let $V_\sigma(a,b) = \{x \in V^\sigma : (a+x,b)$ is quasi-invertible$\}$. Then the linear fractional topology of V^σ is defined to be the topology on V^σ which has the $V_\sigma(a,b)$, $a \in V^\sigma$, $b \in V^{-\sigma}$ as a sub-basis. We call V^σ quasi-invertible dense (qid) if each $V_\sigma(a,b)$ is dense in V^σ with the linear fractional topology. The Jordan Pair V is itself qid if both V^+ and V^- are qid. We show that a semi-simple Jordan Pair V over a ring k with dcc on principal inner ideals is qid provided that kx is infinite for all $x \in V^\sigma, \sigma = \pm$. The proof makes use of the socle of a non-degenerate Jordan Pair.

R.L. WILSON: Restricted simple Lie algebras with two-dimensional Cartan subalgebras

The following theorem has been proved (in joint work with R. Block): Let F be an algebraically closed field of characteristic $p > 7$. Let L be a finite-dimensional restricted simple Lie algebra over F. Assume that H contains a two-dimensional toral Cartan subalgebra H. Then L is either classical (of type A_2, C_2, or G_2) or L is isomorphic to the $2p^2$-dimensional Jacobson-Witt algebra W_2 $(= \mathrm{Der}\, F[x_1,x_2]/(x_1^p,x_2^p))$ In the course of the proof of this theorem it is necessary to determine all finite-dimensional semisimple Lie algebras L over F containing a two-dimensional toral Cartan subalgebra.

- 26 -

R. WISBAUER: Central localization of rings

The object of this talk is to show, how associative module theory can be applied to obtain certain results on (nonassociative) rings.

I. Associative module theory

Let R be an associative ring with unity, R-mod the category of unitary left R-modules. $\sigma[M]$ denotes the full subcategory of R-mod, whose objects are all modules subgenerated by the R-module M.
We are studying the torsion theory defined by the injective hull $\hat{M}$ of M in the Grothendieck category $\sigma[M]$.

A submodule $K \subset M$ is called rational in M, if M/K is a torsion module in this torsion theory, i.e. $\mathrm{Hom}_R(M/K,\hat{M}) = 0$ or – equivalently – $\mathrm{Hom}_R(V/K,M) = 0$ for all $K \subset V \subset M$.

Theorem: If every essential submodule is rational in M, then

(1) $\mathrm{End}_R(\hat{M})$ is a regular, left injective ring

(2) $\varinjlim_{K \text{ essential in } M} \mathrm{Hom}_R(K,M) = \mathrm{Hom}_R(M,\hat{M}) = \mathrm{End}_R(\hat{M})$.

II. Ring theory

Consider any ring A (nonassociative, without 1) as module over its multiplicationring $M(A)$. Applying I we have the subcategory $\sigma_{M(A)}[A]$ of $M(A)$-mod and

<u>Lemma</u> If A is a semiprime ring, then every essential ideal is rational in A.

Observing that $\mathrm{End}_{M(A)}(A)$ is the centroid of A we obtain by the theorem above properties of the <u>extended centroid</u> of the ring (= $\mathrm{End}_{M(A)}(\hat{A})$).

Berichterstatter: Kurt Thilo

Liste der Tagungsteilnehmer

B. N. Allison
Department of Mathematics, University of Alberta,
Edmonton, Alberta T 6 G 2 E 1, Canada

G. Benkart
Department of Mathematics, University of Wisconsin,
Madison, Wisconsin 53706, USA

R. A. Bix
Department of Mathematics, University of Michigan-Flint,
Flint, Michigan 48503, USA

E. Bönecke
Mathematisches Seminar der Universität Hamburg,
Bundesstr. 55, D-2000 Hamburg 13

H. Braun
Mathematisches Seminar der Universität Hamburg,
Bundesstr. 55, D-2000 Hamburg 13

R. Braun
Mathematisches Institut der Universität Tübingen,
Auf der Morgenstelle 10, D-7400 Tübingen

R. Carlsson
Mathematisches Seminar der Universität Hamburg,
Bundesstr. 55, D-2000 Hamburg 13

J. Dorfmeister
Mathematisches Institut der Universität Münster,
Roxeler Str. 64, D-4400 Münster

J. R. Faulkner
Department of Mathematics, University of Virginia,
Charlottesville, Virginia 22901, USA

J. C. Ferrar
Department of Mathematics, Ohio State University,
Columbus, Ohio 43210, USA

P. D. Gerber
IBM T. J. Watson Research Centre, PO Box 218,
Yorktown Heights, New York 10598 , USA

J. Goldman
Department of Mathematics, De Paul University,
Chicago, Illinois 60614 , USA

W. Hein
Fachbereich Mathematik der Gesamthochschule Siegen,
Hölderlinstr. 3, D-5900 Siegen 21

J. Heinze
Mathematisches Institut der Universität Münster,
Roxeler Str. 64, D-4400 Münster

K.-H. Helwig
Institut für Mathematik der Technischen Universität München,
Arcisstr. 21, D-8000 München 2

Y. Ilamed
Israel Atomic Energy Comm, Soreq Res. Estab.,
Yavne, Israel

F. D. Jacobson
Department of Mathematics, Albertus Magnus College,
New Haven, Connecticut 06511, USA

N. Jacobson
Department of Mathematics, Yale University,
New Haven, Connecticut 06520, USA

G. Janssen
Institut A für Mathematik der Technischen Universität Braunschweig
Pockelstr. 14, D-3300 Braunschweig

M. Koecher
Mathematisches Institut der Universität Münster,
Roxeler Str. 64, D-4400 Münster

O. Kühn
Mathematisches Institut der Universität Münster,
Roxeler Str. 64, D-4400 Münster

J. A. Loustau
Department of Mathematics, City University of New York
Hunter College,
New York 10021, USA

J. H. Mayne
Department of Mathematics, Loyola University,
Chicago, Illinois 60626, USA

K. McCrimmon
Department of Mathematics, University of Virginia,
Charlottesville, Virginia 22901, USA

R. Meinert
Bramfelder Str. 20, D-2000 Hamburg 60

K. Meyberg
Institut für Mathematik der Technischen Universität München,
Arcisstr. 21, D-8000 München 2

E. Neher
Mathematisches Institut der Universität Münster
Roxeler Str. 64, D-4400 Münster

J. M. Osborn
Department of Mathematics, University of Wisconsin,
Madison, Wisconsin 53706, USA

H. P. Petersson
Fachbereich Mathematik der Fernuniversität - Gesamthochschule -
Hagen
Lützowstr. 125, D-5800 Hagen

M. Racine
Department of Mathematics, University of Ottawa,
Ottawa, Ontario K1N 6 N 5, Canada

H. L. Resnikoff
Department of Mathematics, University of California,
Irvine, California 92664, USA

G. Rhinow
Fachbereich Mathematik der Gesamthochschule Siegen
Hölderlinstr. 3, D-5900 Siegen 21

H. Röhrl
Department of Mathematics, University of California,
San Diego, La Jolla, California 92037, USA

P. Schnehagen
Doescherstr. 20, D-2000 Hamburg 76

C. Sprengelmeier
Fachbereich Mathematik der Gesamthochschule Siegen,
Hölderlinstr. 3, D-5900 Siegen 21

H. Strade
Mathematisches Seminar der Universität Hamburg,
Bundesstr. 55, D-2000 Hamburg 13

A. Thedy
Abteilung Mathematik der Universität Dortmund,
Neubau Mathematik Hauptbaufläche, D-4600 Dortmund 50

K. Thilo
Fachbereich Mathematik der Fernuniversität - Gesamthochschule - Hagen
Lützowstr. 125, D-5800 Hagen

A. Tillier
Départment de Mathématiques, Université de Lyon I,
86 Rue Pasteur, F-69365 Lyon

H. Upmeier
Mathematisches Institut der Universität Tübingen,
Auf der Morgenstelle 10, D-7400 Tübingen

K.S. Watson
Fachbereich Mathematik der Fernuniversität - Gesamthochschule - Hagen
Lützowstr. 125, D-5800 Hagen

R.L. Wilson
Department of Mathematics, Rutgers University,
New Brunswick, New Jersey 08903, USA

R. Wisbauer
Mathematisches Institut der Universität Düsseldorf,
Universitätsstr. 1, D-4000 Düsseldorf