MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 41/1979

Funktionalanalysis: Operatorfunktionen und Spektraltheorie

3o.9. bis 6.1o.1979

In der Woche vom 3o.9. bis 6.1o.1979 fand die Tagung über Operatorfunktionen und Spektraltheorie statt. Die Leitung der Tagung lag bei den Herren Professoren I. Gohberg (Tel Aviv), B. Gramsch (Mainz) und H.H. Schaefer (Tübingen).

An der Tagung nahmen 52 Mathematiker aus dem In- und Ausland teil. Das große Interesse an dieser Tagung wird schon durch den hohen Anteil ausländischer Teilnehmer deutlich: 31 Teilnehmer kamen aus dem Ausland, davon 17 aus dem nichteuropäischen. Herr Professor A. Dynin (Stony Brook), der zu der Tagung nicht kommen konnte, sandte zwei Auszüge seiner Vorträge, die in den Tagungsbericht aufgenommen wurden.

Auf der Tagung wurden in insgesamt 38 Vorträgen u.a. folgende Gebiete der Funktionalanalysis behandelt: Inversion von Operatorfunktionen, Spektraltheorie, Funktionalkalkül, invariante Teilräume, K-Theorie und topologische Algebren, Pseudo-Differentialoperatoren, nukleare Räume und Folgenräume.

Vortragsauszüge

E. ALBRECHT AND M. NEUMANN (lecture by the second author)

Automatic continuity of non-analytic functional calculi

Thm 1. Let Ω be an open subset of $\mathbb{R}$ or $\mathbb{C}$, $k=0,1,\dots,\infty$ and X a complex Banach space. Then for every homomorphism $\Phi: C^k(\Omega) \to L(X)$ with $\Phi(1) = I$ the following assertions are equivalent:

(i) $\{0\}$ is the only divisible subspace for $T := \Phi(z)$.
(ii) T is a generalized scalar operator.
(iii) The restriction of Φ to $C^{2k+1}(\Omega)$ is continuous.

As one of the consequences of this result it can be shown that every homomorphism from $C^k(\Omega)$ into a commutative Banach algebra with nil radical is necessarily continuous on $C^{2k+1}(\Omega)$. One of the essential ingredients for the proof of theorem 1 is the following general principle which also applies to the problem of continuity for local linear operators in the classical sense between spaces of ultradifferentiable functions and distributions as well as for linear operators intertwining generalized scalar operators. A mapping $\mathcal{E}$ from the family $\mathcal{F}(\Omega)$ of all closed subsets of a regular topological space Ω into the family $\mathcal{f}(X)$ of all closed subspaces of a t.v.s. X is said to be a precapacity if it is monotone and fulfills $\mathcal{E}(\emptyset) = \{0\}$.

Thm 2. Consider an (F)-space and a precapacity $\mathcal{E}_X : \mathcal{F}(\Omega) \to \mathcal{f}(X)$ such that $X = \mathcal{E}_X(\overline{U}) + \mathcal{E}_X(\overline{V})$ holds for all open $U,V \subseteq \Omega$ with $U \cup V = \Omega$. Further, let Y denote a t.v.s. with a fundamental sequence of bounded sets and let $\mathcal{E}_Y : \mathcal{F}(\Omega) \to \mathcal{f}(Y)$ be a precapacity which preserves arbitrary intersections. Then, for every linear operator $\Theta: X \to Y$ with $\Theta\mathcal{E}_X(F) \subseteq \mathcal{E}_Y(F)$ for all $F \in \mathcal{F}(\Omega)$ there exists a finite set Λ in Ω such that:

(a) $\sigma(\Theta) := \{y \in Y : \exists (x_\alpha)_\alpha$ in X with $x_\alpha \to 0$ and $\Theta x_\alpha \to y\} \subseteq \mathcal{E}_Y(\Lambda)$.
(b) Θ is closed on $\mathcal{E}_X(F)$ for all $F \in \mathcal{F}(\Omega)$ satisfying $F \cap \Lambda = \emptyset$.

Using this result, it can also be shown that every non-analytic functional calculus in several variables on an admissible (F)-algebra A is continuous on a fairly large part of A and that it is quite often possible to construct a reasonable continuous non-analytic functional calculus from a given discontinuous one.

C. APOSTOL

Invariant subspaces for subscalar operators

A subscalar operator acting in a complex Banach space is the restriction to an invariant subspace of a scalar operator in the sense of Dunford. "Every subscalar operator has a proper invariant subspace".

J.A. BALL

Interpolation Problems and Toeplitz Operators on Finitely-Connected Domains

Let R be a domain in the complex plane bounded by n + 1 nonintersecting analytic Jordan curves, let $\mathcal{E}$ and $\mathcal{F}$ be two flat unitary vector bundles over R (in the sense of Abrahamse and Douglas (Adv. Math. 19(1976), 1o6-148), and let $\Theta : \mathcal{E} \to \mathcal{F}$ be a contractive analytic bundle map from $\mathcal{E}$ into $\mathcal{F}$. A condition is given for Θ to admit a bounded left inverse, i.e. a bounded analytic bundle map D: $\mathcal{F} \to \mathcal{E}$ such that $D\Theta = I_{\mathcal{E}}$, together with an estimate for $\|D\|_\infty$. The condition involves a uniform lower bound for a class ot Toeplitz operators over R, all of which are induced (formally) by the bundle map $\overset{N}{\underset{1}{\oplus}} \Theta$ (N = rank $\mathcal{F}$). When interpreted for a finite column of analytic scalar functions, the result gives quantitative information on the corona theorem for R. The main tool is the lifting theorem for regions R proved by the author. [J. Operator Theory, 1 (1979), 3-25]. When R is the unit disk D, the result is due to Sz.-Nagy and Foias [Ann.Acad.Sci.Fenn. 2 (1976), 553-564].

H. BART

The factorization theorem for rational matrix functions and Wiener-Hopf indices

The central result of the talk can be described as follows. Let Γ be a contour in the complex plane, and let $W(\lambda)$ be a rational n x n matrix function. Suppose that o is in the inner domain of Γ and that det $W(\lambda) \neq$ o, $\lambda \in \Gamma$. It is known that $W(\lambda)$ admits a (left) Wiener-Hopf factorization with respect to Γ:

$$W(\lambda) = W_-(\lambda) \begin{bmatrix} \lambda^{k_1} & & \\ & \ddots & \\ & & \lambda^{k_n} \end{bmatrix} W_+(\lambda).$$

The numbers $k_1,\ldots,k_n$ are called the (left) partial indices of $W(\lambda)$ with respect to Γ. Assume now that $W(o) = I_n$, when I_n denotes the $n \times n$ identity matrix. Then $W(\lambda)$ admits a realization

$$(1) \qquad W(\lambda) = I_n + C(\lambda I_\delta - A)^{-1} .$$

with $\sigma(A)\cap\Gamma = \emptyset = \sigma(A-BC)\cap\Gamma$. Put

$$M = \operatorname{Im}\left(\frac{1}{2\pi i}\int_\Gamma (\lambda-A)^{-1}d\lambda\right), \qquad M^x=\ker\left(\frac{1}{2\pi i}\int_\Gamma (\lambda-A+BC)^{-1}dt\right).$$

Then the following equalities hold:

$$(2) \qquad -\sum_{k_i<o} k_i = \dim (M\cap M^x).$$

$$(3) \qquad \sum_{k_i>o} k_i = \operatorname{codim} (M + M^x).$$

It follows that

$$(4) \qquad -\sum_{i=1}^{n} k_i = \dim M - \operatorname{codim} M^x.$$

Taking the realization (1) to be minimal (i.e., δ is equal to the McMillan degree $\delta(W)$ of W) one can show that for $l \geq \delta(W)$

$$-\sum_{i=1}^{n} k_i=\operatorname{rank}\left[\frac{1}{2\pi i}\int_\Gamma \lambda^{l-1+i-j}\, W(\lambda)d\lambda\right]_{i,j=1}^{l} -\operatorname{rank}\left[\frac{1}{2\pi i}\int_\Gamma \lambda^{l-1+i-j}W(\lambda)^{-1}d\lambda\right]_{i,j=1}^{l}$$

An example involving the Wiener-Hopf integral equation

$$(5) \qquad \varphi(t)-\int_0^\infty k(t-s)\,\varphi(s)ds = f(t), \qquad t \geq o$$

is given to illustrate the importance of the numbers appearing in the left hand side of (2), (3) and (4). In the example these numbers are the nullity, defect and index of the Fredholm operator given by the left hand side of (5), respectively.

M. BREUER:

Ein Beweis der Kaplanskyschen Vermutung für AW^*-Algebren vom Typ II

Die Vermutung besagt, daß jede AW^*-Algebra W^* ist, falls ihr Zentrum W^* ist. Für Algebren vom Typ I bewies Kaplansky diese Vermutung in den fünfziger Jahren, für Algebren vom Typ III fanden verschiedene Mathematiker in den letzten Jahren Gegenbeispiele. 1977/78 fand ich einen neuen Existenzbeweis für die Spur einer W^*-Algebra vom Typ II, der Methoden der K-Theorie und der Homotopietheorie be-benutzt. De la Harpe (Genf) und ich bemerkten kürzlich, daß dieser Beweis auf AW^*-Algebren übertragen werden kann. Mit Hilfe bekannter Resultate von Feldmann, Kadison u.a. führt dieser Existenzberweis der Spur zum Beweis der Kaplanskyschen Vermutung für Algebren vom Typ II.

K. CLANCEY:

Spectral Multiplicity and Direct Integrals

The talk will be a report on joint work with E. Azoff concerning a canonical model for direct integrals of normal operators. The direct integral $N = \int_Z^{\oplus} N(z)d\lambda(z)$ of normal operators is written in a canonical manner as a multiplication operator on $L^2(Z \times \mathbb{C} \times \mathbb{N}: d\mu)$ where $\mathbb{C}$= complex plane, $\mathbb{N}$=natural numbers and $\mu = \int \mu_z d\lambda(z)$, with μ_z a scalar spectral measure for $N(z)$. Using this model it is possible to obtain a multiplicity function for N in terms of the multiplicity functions for $N(z)$. As an application it is shown that every self-adjoint Toeplitz operator on the half-space has uniformly infinite spectral multiplicity.

L.A. COBURN:

Wiener-Hopf operators and pseudo-differential operators in several complex variables (joint work with C. Berger and A. Koranyi)

We consider a bounded symmetric domain Ω in $\mathbb{C}^n$ with Shilov boundary $\partial\Omega$ a compact manifold with natural volume element. Let P be the orthogonal projection operator from $L^2(\partial\Omega)$ onto the subspace $H^2(\partial\Omega)$ generated by the analytic polynomials. Let M be the algebra of multiplications by smooth functions acting on

$L^2(\partial\Omega)$ and let $M[P]$ be the algebra generated by M and P. We have determined the structure of this algebra in some interesting cases, in terms of the algebras of pseudo-differential operators on certain submanifolds of $\partial\Omega$.

C. DAVIS:

Perturbation of spectrum of normal operators

A well-known result of H. Weyl says that, for hermitian matrices, $\| A - B \| \geqslant \delta(A,B)$, where $\| \ \|$ is the operator norm, and $\delta(\ ,\ )$ the naturally defined distance between the spectra (multiplicity counted). This paper treats generalizations of the theorem in several directions.

P. DEWILDE:

Exact and Approximate realizations of Second Order Stochastic Processes.

We discuss the realization-theory of second order stochastic processes using J-unitary embeddings of the spectral factor. In the process we develop an L_2 system theory. Next, we introduce Schur-recursion to obtain a recursive description of the embedding. Finally, we show how the same procedure may be applied to obtain approximate realizations. We show that those are optimal with respect to an information space described by the orthogonal complement of an invariant subspace.

R.G. DOUGLAS:

Quasitriangularity for pairs of operators

For a single operator a positive index is the obstruction to being of the form upper triangular plus compact. The obstruction for the analogous problem for almost commuting pairs of operators is much more complicated. I will discuss some recent results of my student Gail Kaplan on the case ot two essentially normal operators.

B. DROSTE:

Extension of the holomorphic functional calculus to algebras with partition of unity

Let A be a commutative Banach algebra over $\mathbb{C}$ with e. For $a=(a_1,\dots,a_n)\in A^n$ let $\sigma(a)$ denote the common spectrum of a in A, then we have $\phi_a: \mathcal{O}(\sigma(a)) \longrightarrow A$, the holomorphic functional calculus for a, ϕ_a is a continuous algebrahomomorphism with $\phi_a(1) = e$ and $\phi_a(z_j) = a_j$ for $j=1,\dots,n$. We consider the following problem : Give conditions to extend ϕ_a to an algebra bigger than $\mathcal{O}(\sigma(a))$. Some solutions of that problem are known up to now, starting with the result of Tillmann (1963) for n=1 and $\sigma(a) \subset \mathbb{R}$ and the approach of Waelbroeck (1964) for n=1 and $\bar\partial$-flat functions. Among the authors who extended these results to $n>1$ we want to mention Albrecht, Gramsch and Nguyen t.H.. The question is if it is possible to get conditions to extend ϕ_a if $\sigma(a)\subset S\subset\mathbb{C}^n$, where S is more general than $\mathbb{R}^n$, to algebras that are 'concret' and have (partly) partition of unity.

We gave two theorems that give positive answer to this question if the differentialform $\omega=\sum_{j=1}^{n}(-1)^{j-1}u_j\cdot\bar\partial u_1\wedge\dots\wedge\widehat{\bar\partial u_j}\wedge\dots\wedge\bar\partial u_n$, where $u_1,\dots,u_n\in\mathcal{E}^{\infty}(\mathbb{C}^n\setminus\sigma(a),A)$ with $\sum_{j=1}^{n}(z_j-a_j)u_j(z) = e$, satisfies a certain exponential growth condition. One of the theorems is the following:

Theorem:

Let $\sigma(a)\subset S$, where $S\subset\mathbb{C}^n$ is compact and for some compact $K_o\subset S$ let $M:=S\setminus K_o$ be a totally-real ultradifferentiable submanifold of $\mathbb{C}^n$. Let $\{W_k\}_{k\in\mathbb{N}}$ be an open neighborhood-basis of K_o with $\overline{W_{k+1}}\subset W_k$ such that $S_k:=S\cup\overline{W_k}$ has a neighborhood-basis of domains of holomorphy for all $k\in\mathbb{N}$. Let for fixed $\alpha>o$

a) $\forall_{k\in\mathbb{N}}\ \exists_{C>o,L>o}\ \|\omega(z)\| \leq C\cdot\exp\left(\frac{L}{d(z,S_k)^{\alpha}}\right)$ for all $z\in V\setminus S_k$, V an open neighborhood of S_o and

b) $\dim_{\mathbb{R}} M = n$ or b') A semisimple $\Longrightarrow \exists!$ continuous algebrahomomorphism

$\tilde\phi_a: \mathcal{E}^{(p!^s)}\mathcal{O}(M,K_o) \longrightarrow A$ with $\tilde\phi_a|_{\mathcal{O}(S)} = \phi_a$, where $s = 1+(1/\alpha)$ and $\mathcal{E}^{(p!^s)}\mathcal{O}(M,K_o)$ is the space of functions holomorphic on (a neighborhood of) K_o and $\mathcal{E}^{(p!^s)}$ on M with the topology $\mathcal{E}^{(p!^s)}\mathcal{O}(M,K_o) = \operatorname{ind}_{k\in\mathbb{N}}\mathcal{E}^{(p!^s)}\mathcal{O}(M,W_k)$

Remark: If one has a polynomial growth condition in a) of the above theorem, $\mathcal{E}(p!^s)$ can be replaced by $\mathcal{C}^\infty$. If $a_1,\ldots,a_n$ is a commuting tuple of continuous linear operators on a Banach space, the theorem holds also for the Taylor-spectrum of $a_1,\ldots,a_n$. The results given in the lecture are part of the thesis the author is preparing at the university of Mainz.

A. DYNIN:

Complex Operator Powers and Diffusion Semigroups on Heisenberg Groups

We describe the analytic properties of complex powers and diffusion semigroups generated by some homogeneous convolution operators on Heisenberg groups. The generators are not normal in general and they have the pathological pseudo-differential type $(\frac{1}{2}, \frac{1}{2})$.

Nevertheless harmonic analysis provides a key and gives new information even in the standard example of the Kohn` Laplacian.

A. DYNIN and S. DYNIN:

Fredholm Operator families on topological vector spaces

It is well known that the standard perturbation theory fails outside of Banach frame : the operator topologies occur to be not adequate. But some trends in Numerical Analysis, PDE, K-theory et cetera give rise to operator families on rather general topological vector spaces.

An analysis of these trends leads to notion of Fredholm Operator Families on Hausdorff vector spaces. We show that in this framework all standard theory of perturbation of Fredholm operators keeps its value.

J. ERNEST:

A formulation of the spectral theorem which generalizes to non-normal operators

Let T be a normal operator on a separable Hilbert space $\mathcal{H}$. Then the spectrum $\sigma(T)$ of T admits a measure μ (unique up to equivalence of measures) and a partition $\sigma(T) = (\bigcup_{n=1}^{\infty} \sigma_n(T)) \cup \sigma_\infty(T)$ of Borel subsets of $\sigma(T)$ such that T is unitarily equivalent to a direct sum of operators

$T \simeq (\infty) T_\infty \oplus \sum_{n=1}^{\infty} (n)\, T_n$, where each T_n is defined on $L^2(\sigma_n(T), \mu)$ by $(T_n f)(x) = xf(x)$. Here we use the notation $(n) \cdot S$ (for S an operator and n a positive integer or ∞) to denote a direct sum of n "copies" of S. This statement is equivalent to the spectral theorem and the spectral multiplicity theory for normal operators. This statement persists if T is merely smooth (i.e., generates a type I or GCR or postliminal C^*-algebras) with an appropriate (operator valued) generalization of the spectrum of an operator. The formulation for arbitrary (non-normal, non-smooth) operators however requires an extension of the notation $(n)\, S$ to the case where n is any positive real number.

J.-P. FERRIER:

The infinite dimensional spectrum

We describe a spectral theory for algebras, using infinite dimenstional locally convex spaces instead of $\mathbb{C}^n$. It is connected to some problems of the following kind:

1) Let (a-s) be non invertible. Is there a character χ such that $s = \chi(a)$?

2) Let (a_n) be a bounded sequence of elements of the algebra A and let assume that the resolvent of a_n is bounded outside S_n, with bounds independent of n. Then if $(\lambda_n) \in \ell^1(\mathbb{C})$, does $\sum \lambda_n S_n$ contain the spectrum of $\sum \lambda_n a_n$, and what about bounds for the resolvent ?

The interesting case is the case of b-algebras, i.e. algebras which are inductive limits of Banach spaces, for instance algebras $\mathcal{O}(\delta)$ considered by Waelbroeck and Hörmander.

Problems (1), (2) are particular cases of a general projection problem for the spectrum. It is discussed for algebras $\mathcal{O}(\delta)$ and others.

C. FOIAS:

The Commutant Lifting Theorem and a Constructive Approach to the Corona Problem

Let $\Theta(\lambda)$ be an analytic function on $\mathbb{D} = \{|\lambda|<1\}$, the values of which are contractions: $\mathcal{E} \mapsto \mathcal{E}_*$, where $\mathcal{E}$ and $\mathcal{E}_*$ are some complex (separable) Hilbert spaces. The general Corona Problem (as it is used in theoretical electrical engineering) is the following: (a) Given $M>0$, find necessary and sufficient conditions on Θ such that there should exist an operator-valued analytic function $\Omega(\lambda)$ on $\mathbb{D}$ satisfying the following conditions:

(1) $\Omega(\lambda)\,\Theta(\lambda) \equiv I_{\mathcal{E}}$, $\sup\{\|\Omega(\lambda)\| : \lambda \in \mathbb{D}\} \leq M$.

(b) If (1) is solvable give an algorithm for constructing all the solutions of (1). A solution to Question (a) was given by B.Sz.-Nagy and C. Foias (Ann.Acad.Sci.Fenn., A.I, V.2,1976, 553-564) by connecting it to the Commutant Lifting Theorem. The aim of this talk is to prove that the next developments of this last theorem yield a complete solution to Question (b).

I. GOHBERG:

Wiener-Hopf Equations with rational symbol

A general factorization theorem is used to solve systems of Wiener-Hopf-Equations with rational symbols. The realization theorem plays a very important role. Explicit formulas are obtained.The same method is used to solve explicitely the transport equation. The main results are joint with H. Bart and M.A. Kaashoek.

B. GRAMSCH:

Homotopy properties of regular operator functions

Let B be a complex Banach algebra with unit and B^{-1} the group of invertible elements of B and A a commutative Banach algebra of functions on $X = \mathfrak{M}(A)$. For the set of homotopy classes a theorem of Arens-Royden and Davie (1971) gives the isomorphism

$$[X,B^{-1}]_{A \otimes B} \cong [X,B^{-1}].$$

Theorem 1. For the set ϕ of Fredholm operators on a Banach space

$$[X,\phi]_{A \otimes B} \cong [X,\phi] \qquad \text{holds.}$$

For Banach analytic homogeneous spaces (Bourbaki) the Oka-principle has been proved by S. Hayes (1975/76).

Let $\mathcal{R} \subset \mathcal{L}(E)$, E Banach space be the set of operators with complemented kernels and images equipped with the topology ρ of Douady (1966).

Theorem 2. The connected components of $(\mathcal{R},\rho)$ are Banach analytic homogeneous spaces.

Let $q:B \longrightarrow B/\mathcal{K}$ be the canonical quotient homomorphism and $\mathcal{R} = \{b \in B : \exists \text{ a with bab} = b\}$.

Theorem 3. $(\mathcal{R},q,q(\mathcal{R}))$ is a fibration.
This depends on appropriate local liftings. This has applications to the theory of singular integral operators.

Theorem 4. Let X be a holomorphy region in $\mathbb{C}^n$. Then

1) $[X,(\mathcal{R},\rho)]_H \cong [X,(\mathcal{R},\rho)]$
2) $[X,\phi_k]_H \cong [X,\phi_k]$,

where H means holomorphic and ϕ_k the set of Fredholm operators T with $\dim \mathcal{N}(T) = k$.

B. HELTON:

Dilations with Jordan Operators

Nagy's theorem that every contraction has a unitary dilation and the commutant lifting theorem for two contractions are classical. This talk (on work done jointly with Joe Ball) addresses the question: Which operators T have an extension J of the form $J = S + N$ where S is self-adjoint, $N^2 = 0$, and $[S,N]=0$? This is the first step in determining which T have a Jordan dilation J. It proves to be a difficult problem which has very classical roots; namely, it is closely related to classical disconjugacy theory for Sturm-Liouiville operators. Joe Ball and I were able to make considerable headway on the problem of Jordan extensions by using Wiener-Hopf factorizations.

A. HERTLE:

A charactarization of the Radon and Fourier transforms

We study the behaviour of operators on function spaces on $\mathbb{R}^n$ under rotations U, dilation $c > o$, and translations a. For a function f we write $f_U(x) = f(Ux)$, $f_c(x) = f(cx)$ and $f_a(x) = f(x+a)$. The question is, whether an operator is determined by its behaviour under these operations.

In case of the Fourier transform F and the Radon transform R: $\mathcal{S}(\mathbb{R}^n) \to \mathcal{S}(S^{n-1} \times \mathbb{R}$ the latter being a Fourier Integral Operator (FIO), defined by

$$(Rf)(x,p) = \int_{x\cdot y=p} f(y)\,dy,$$

we have the following behaviour:

(a) $F(f_U) = (Ff)_U$ (α) $(Rf_U)(x,p) = (Rf)(Ux,p)$

(b) $F(f_c) = c^{-n}(Ff)_{1/c}$ (β) $(Rf_c)(x,p) = c^{1-n}(Rf)(x,cp)$

(c) $F(f_a) = e^{ia(\cdot)}(Ff)$ (γ) $(Rf_a)(x,p) = (Rf)(x,p+x\cdot a)$

THEOREM 1. Let $\tilde{F}$ (resp. $\tilde{R}$) be a continuous operator on $\mathcal{S}(\mathbb{R}^n)$ (resp. $\mathcal{S}(\mathbb{R}^n) \to \mathcal{S}(S^{n-1}\times\mathbb{R})$) with the properties (a) - (c) (resp. (α)-(γ)). Then up to a constant multiple, $\tilde{F}$ and $\tilde{R}$ are the Fourier and Radon transforms.

Theorem 1 holds also for $\tilde{F}: A \to B$, where $\mathcal{S} \subset A, B \subset \mathcal{S}'$ and $\tilde{R}: L^1 \to L^1$. The assumption of continuity of $\tilde{F}, \tilde{R}$ in Theorem 1 can be omitted for some spaces A and B by the following Theorem:

THEOREM 2. A dilation invariant functional T on $C^\infty(\mathbb{R})$, i.e. $T(\varphi_c) = T(\varphi)$ for $c > o$, is the δ-functional and therefore automatically continuous.

We remark that many FIO have a similar behaviour as F and R (Hilbert and Riesz transforms). So Theorem 2 gives a method to verify an inversion formula for a FIO A as follows: we only have to show that the functional $\varphi \mapsto (A^{-1}A\varphi)(o)$ is dilation invariant, which implies that $(A^{-1}A\varphi)(o) = \varphi(o)$.

M.A. KAASHOEK:

Spectral triples and Wiener-Hopf equivalence

For matrix polynomials Gohberg and Lerer have made an explicit connection between Wiener-Hopf equivalence and the problem of feedback in mathematical systems theory. In the present paper this connection is established for holomorphic operator functions in the infinite dimensional case. By applying this result to operator functions of the form $\sum_{i=1}^{n} \lambda^{k_i} P_i$, it is shown that the necessary and sufficient condition for an operator polynomial to have a Wiener-Hopf factorization, due to Rowley, is also valid for holomorphic operator functions.

Th. KAILATH:

Displacement Ranks of Operators

We introduce certain displacement operators to obtain an index of how far matrices and linear integral operators are from being Toeplitz. This concept will be used to develop new computationally efficient solution methods for linear matrix and integral equations. Some connections to the work of Livsic and Nagy-Foias have also been found.

W. KABALLO:

Operator function equations

The lecture was based on a joint paper with G.Ph.A. Thijsse (Dortmund). An example of the results obtained is the following:

THEOREM. Let G be a region in $\mathbb{C}$, A a holomorphic (or finite-meromorphic) Φ^{-}-function on G, and Ψ be a meromorphic operator function on G. Suppose that the equation $A(\lambda)\phi(\lambda) = \Psi(\lambda)$ can be solved for $\lambda \in \mathfrak{U}$, $\emptyset \neq \mathfrak{U} \subseteq G$ open. Then there exists a global meromorphic solution ϕ of $A\phi = \Psi$ on G.

Similar results are true for ϕ^+-functions, weaker versions hold in the case of several complex variables. The theorem is in general false for the equation $\phi A = \Psi$, however it is true for certain choices of the underlying spaces. For $\phi^{1,r}$-functions B. Gramsch showed in 1973, that assertions like the theorem are easy consequences of the inversion theorems for $\phi^{1,r}$-functions; hence our results may be viewed as generalizations of these inversion theorems to the case of $\phi^{\pm}$-functions.

H. KÖNIG (Bonn):

On the spectrum of nuclear operators

A. Grothendieck showed that the p-nuclear operators ($0<p\leq 1$) have absolutely q-th power summable eigenvalues, where $1/q=1/p-1/2$. This result is optimal for $p=1$. However, for $p<1$ it can be improved slightly to state that the eigenvalues belong to the (smaller) Lorentz sequence space $l_{q,p}$. This theorem can be improved only in special spaces like L_r-spaces, when the optimal summability exponent q of the eigenvalues of p-nuclear operators is given by $1/q = 1/p-|1/r-1/2|$. Using complex interpolation theory, Pisier proved a similar fact for r-convex and r'-concave Banach lattices ($r\leqslant 2$). Also in spaces of type r and cotype r' some improvements can be given. The optimal order is related to the order of projection constants of finite dimensional subspaces of the spaces considered. Moreover, I want to discuss the problem which l_2- sequences are sequences of eigenvalues of a nuclear operator.

J.-Ph. LABROUSSE:

The domain of regularity of an operator

E will denote a Banach space. A will be a closed,densely defined operator on E with domain D(A), nullspace N(A) and range R(A).

Def 1. A will be called quasi-Fredholm of order d ($A\in q\phi(d)$) iff the following conditions are satisfied.

(1) $\exists\, d\in\mathbb{N}$ such that:
$$\forall\, m\in\mathbb{N},\quad m\geqslant d \Longrightarrow R(A^m)\cap N(A) = R(A^d)\cap N(A)$$
$$\forall\, m\in\mathbb{N},\quad m< d \Longrightarrow R(A^m)\cap N(A) \neq R(A^d)\cap N(A)$$

(2) $R(A^d) \cap N(A)$ is complemented in $D(A)$.

(3) $R(A) + N(A^d)$ is complemented in E.

Thm 1. $A \in q\Phi(d) \Longleftrightarrow \exists M,N$ closed subspaces of E, invariant under A, such that $M \oplus N = E$ and $A|_N$ is nilpotent of order d; $A|_M \in q\Phi(o)$.

Def.2 $\lambda_o \in \text{reg}(A) \overset{}{\underset{Df}{\Longleftrightarrow}} \exists$ a neighborhood U of λ_o in $\mathbb{C}$, an operator valued analytic function B_λ on U such that: $\forall \lambda \in U \quad R(B_\lambda) \subseteq D(A)$

and $\forall u \in D(A) \qquad (A-\lambda I)\, B_\lambda\, (A-\lambda I)u = (A-\lambda I)u$

$\quad\;\; \forall v \in E \qquad\qquad B_\lambda\, (A-\lambda I) B_\lambda\, v = B_\lambda\, v.$

reg(A) is called the domain of regularity of A

Thm 2. $\lambda_o \in \text{reg}(A) \Longleftrightarrow A - \lambda I \in q\Phi(o)$.

Def.3 Let $\lambda \in \text{reg}(A)$. Then $C_o(A-\lambda I) =_{Df} \bigcap\limits_{n \geq o} R[(A-\lambda I)^n]$

$C_o(A-\lambda I)$ is closed.

Thm 3. Let $C \subseteq \text{reg}(A)$ be a connected component of reg(A). Then $C_o(A-\lambda I)$ is constant on C.

Def.4 Let E now be a Hilbert space and let A be bounded. Set $A_o = A^*$; $A_j = i[A, A_{j-1}]$ $\quad j = 1,2\ldots.$ Then A is paranormal $(A \in pN) \Longleftrightarrow \lim\limits_{j \to \infty} \|A_j\|^{1/j} = o$ and A is quasi-normal $(A \in qN)$ if it is similar to a paranormal operator.

Remark: All bounded spectral operators are quasi-normal.

Thm 4. Let $A \in pN \cap q\Phi(o)$ and let C be the connected component of reg(A) contains o. Then:

(1) $\forall \lambda \in \text{reg}(A) \qquad C_o(A-\lambda I)$ is invariant under A^* and
$\qquad\qquad\qquad\qquad\quad C_o(A^*-\lambda I)$ is invariant under A.

(2) $E = R \oplus S \oplus T$ with:

(*) $R = C_o(A) \cap C_o(A^*)^\perp$; $\quad S = C_o(A) \cap C_o(A^*)$; $\quad T = C_o(A)^\perp \cap C_o(A^*)$

and: $\qquad \rho(A/_S) \supseteq C$

$\qquad A|_R,\ (A|_T)^*$ are of the Cowen-Douglas type.

Remark: (2) is still true when it is only assumed that $A \in qN \cap q\Phi(o)$. In that case however R,S,T are no longer, in general given by $(\ast)$.

This type of decomposition is useful when looking for the solutions of some operator equations like $AA^* - A^*A = B$ with $BA = AB$ or even for more complicated ones.

G. LUMER:

Connecting of local operators, and evolution equations on networks

We define "C^2 networks in R^n", Ω, (essentially graph theoretical networks with C^2 structure, imbedded in R^n), carrying on each branch Ω_i^* of Ω a diffusion type operator

$$A_i(s_i,D) = a_i(s_i)D^2 + b_i(s_i)D + c_i(s_i) \quad , \quad D = \frac{d}{ds_i} \ ,$$

s_i = arc length on Ω_i^*, $a_i,b_i,c_i \in C(\Omega_i^*)$, $a_i(s_i) \geqslant \delta_i > o$ and $c_i(s_i) \leqslant o$ on Ω_i^*.

We connect these operators at the ramification nodes N, using "connecting operators" B_N of the form $B_N f = \sum_{\Omega_i^* \cap N \neq \emptyset} c_{N_i} \frac{df}{d\nu_{N_i}}$ ($\frac{d}{d\nu_{N_i}}$ is $\frac{d}{ds_{N_i}}$ at N, where s_{N_i} is arc length on Ω_i^* computed starting from N), imposing the condition $B_N f = o$ at each ramification node $N \subset V$ open, and continuity condition to define $f \in D(A,V)$, and so we obtain a single local operator A on Ω from the $A_i(s_i,D)$.

We obtain a necessary and sufficient condition on the c_{N_i} for A to be locally dissipative.

We show that when A is locally dissipative, $\exists$ an exhaustive family of A-Cauchy regular open sets in Ω, and that for a large class of C^2 networks Ω, if

$$|a_i| = o(r^2), \quad |b_i| = o(r),$$

r = distance to some fixed point in R^n, the Cauchy problem (sup-norm evolution problem with o-boundary conditions) can be solved for the whole (in general unbounded) space Ω.

R. MEISE:

Entire functions, nuclear power series spaces and λ-nuclearity

Inspired by results of Boland and Dineen we prove

Theorem. Let $\Lambda_\infty(\alpha)$ be nuclear, define $\beta = \beta(\alpha)$ as the increasing rearrangement of the family $(\sum_{j\in\mathbb{N}} \alpha_j m_j)_{m\in\mathbb{M}}$, where $\mathbb{M} = \{m\in\mathbb{N}_0^{\mathbb{N}} \mid m_j \neq 0$ only for finitely many $j\in\mathbb{N}\}$, and let $b : \mathbb{N} \longrightarrow \mathbb{M}$ denote a bijection with $\beta_n = \sum_{j\in\mathbb{N}} \alpha_j b(n)_j$, $n\in\mathbb{N}$. Then an entire function f on φ (the space of all finite sequences) is the restriction of an entire function on $\Lambda_\infty(\alpha)_b'$, iff the Taylorcoefficients $(a_m)_{m\in\mathbb{M}}$ of f satisfy $\lim_{n\to\infty} |a_{b(n)}|^{1/\beta_n}=0$. This proves especially $(H(\Lambda_\infty(\alpha)_b'),\tau_0) \cong \Lambda_\infty(\beta)$, where τ_0 denotes the compact open topology.

For many sequences α it is possible to calculate β (up to equivalence). This leads to a number of examples, e.q. $(H(s'),\tau_0)\cong s$, $(H(\mathcal{D}_b'),\tau_{0,bor})\cong \mathcal{D}$, $(H(H(\mathbb{C}^N)_b'),\tau_0) \cong \Lambda_\infty((\ln n)^{\frac{N+1}{N}})$, $(H(\Lambda_\infty(\alpha)_b'),\tau_0) \cong \Lambda_\infty(\gamma_{[\ln n]}(\ln n)^{p+1})$ if $\alpha_n = \gamma_n n^p$ for any stable increasing sequence γ and any $p > 1$.

The sequence β can also be used to prove the following generalizations of the nuclearity result of Boland and Waelbroeck, which are optimal in a sense.

Theorem. Let E be a quasi-complete l.c.space with (E',τ_0) $\Lambda(\alpha)$-nuclear, where Λ stands for Λ_1, $\Lambda_{\mathbb{N}}$ or Λ_∞. We assume that $\Lambda_\infty(\alpha)$ is nuclear if $\Lambda=\Lambda_{\mathbb{N}}, \Lambda_\infty$ and that $\Lambda_1(\alpha)$ is nuclear if $\Lambda = \Lambda_1$. Then $(H(\Omega),\tau_0)$ is $\Lambda(\beta)$-nuclear for any open set Ω in E, provided that for $\Lambda = \Lambda_{\mathbb{N}}$ there exists $D > 0$ such that for any $p\in\mathbb{M}$ $\lim\sup_{n\in\mathbb{N}} \frac{\beta_{pn}}{\beta_n} \leq D$. (This condition is satisfied by any stable sequence α and also by many others.)

E. MEISTER:

Wiener-Hopf operators on three-dimensional wedgeshaped regions

If $P = P_{G(\gamma)}$ denotes the space projector on $L^2(\mathbb{R}^3)$ corresponding to multiplication by the caracteristic function $\chi_{G(\gamma)}$ of a wedge $G(\gamma) \subset \mathbb{R}^3$ with angle $0<\gamma<\pi$

the Wiener-Hopf-Eq.

$$(1)\quad T_p(A)u \equiv u(x) - \int_{G(\gamma)} k(x-y)u(y)dy = v(x) \in L^2(R(P)) \cong L^2(G(\gamma))$$

with $k \in L^1(\mathbb{R}^3)$ is investigated.

Theorem. $T_p(A)$ is invertible on $R(P)$ iff $T_p(R)$ is Fredholm or, equivalently, iff all W HOs $T_{P_\Gamma}(A_{x_3})$ where $A_{x_3} := F^{-1}_{1,2}\,\sigma_A(\xi_1, \xi_2, x_3)$. $F_{1,2}$ is a family of two-dimensional operators are pointwise, with respect to x_3, invertible. $\sigma_A := 1-\hat{k} = 1-Fk$. Due to a result by DEVINATZ & SHINBROT (1969) a new sufficient condition for invertibility is derived, e.g. for the right-angled wedge: Let the symbol $\sigma_A(\xi_1, \xi_2, x_3)$ be factorizable into $\sigma_- \cdot \psi_A \cdot \sigma_+$ where $\sigma_+(\cdot,\cdot,x_3) \in \mathfrak{M}^1_+ \otimes \mathfrak{M}^1_+$; $\sigma_-(\cdot,\cdot,x_3) \in \mathfrak{M}^2_+$; $\mathfrak{M}^2_+$ the m-dim.Fourier images of the plus Wiener algebras on $\mathbb{R}^m$, $\psi(\cdot,\cdot,x_3)$ being pointwise strongly elliptic and all factors continuous in x_3. - Generalization to "N-part composible W HOs:

$$W_N := \sum_{j=1}^{N} B_j P_j \ ;\quad P_j P_k = \delta_{jk} P_k \ ;\quad \sum_{j=1}^{N} P_j = I \text{ on } \mathbb{R}^3;\ P_j = \chi_{G(\gamma_j - 1, \gamma_j)}$$

$\gamma_0 = 0 < \gamma_1 < \gamma_2 < \dots < \gamma_N = 2\pi$ with certain conditions on the $B_j = B_0 C_j H_j$ and σ_{c_j}.

R. MENNICKEN:

Analytic perturbation of Φ^+-operators in F-spaces

Let X and Y be F-spaces, Γ_X and Γ_Y be systems of continuous seminorms generating the topology on X or Y respectively. Let T be a closed linear operator from X to Y with $\overline{D(T)} = X$. Apart from the "global" dual spaces X',Y' and the global adjoint T', the "semiglobal" dual spaces $X'^p = (X,p)'$, $Y'^q = (Y,q)'$ and the "semi-global" adjoints $T'^{(p,q)}$ are considered.

Theorem 1. T is open iff for each $p \in \Gamma_X$ there exists a $q \in \Gamma_Y$ such that 1) $T'^{(p,q)}$ is (q,p)-open and 2) $R(T') \cap X'^p \subset T'(Y'^q)$.

This theorem generalizes one of the basic theorems in Treves' book "Locally convex spaces and linear partial differential equations" published in 1966.

A rather simple proof of theorem 1 is given using some "polar formulas" which are the essential implements.

Theorem 2. Let Γ_Y consist of a denumerable number of seminorms $q_1 \leqq q_2 \leqq \cdots$. Assume that Y'^{q_n} is a q_{n+1}-closed subspace of $Y'^{q_{n+1}}$ for $n \in \mathbb{N}$. Let G be a region in $\mathbb{C}^n$. Suppose that $T(z)$ is a holomorphic function from G to $L(X,Y)$ provided with the topology of pointwise convergence and has only values in $\Phi^+(X,Y)$. Finally assume that for all $z_0 \in G$ there exist $r > o$ and $n \in \mathbb{N}$ such that $N(T(z)')$ is contained in Y'^{q_n} for $|z-z_0| < r$. Then

$$\Sigma^+ := \left\{ z \in G: \text{def } T(z) > \min_{\xi \in G} \text{def } T(\xi) \right\}$$

is an analytic set in G.

The proof of this theorem is based on theorem 1 and on the fact that X'^p, Y'^q are B-spaces.

Perturbations of Φ^+-operators by uniformly locally bounded and uniformly compact operator bundles are also discussed and the theorems 1 and 2 are applied to linear PDO's in local Sobolev-spaces.

B. MITYAGIN

B-splines and rational pencils

For any partition $t = \{o = t_0 \leqslant t_1 \leqslant \ldots \leqslant t_n = 1\}$ of the unit interval S_{tk} denotes the spline-subspace of all C^{k-1}-functions f whose restrictions $f|\Delta_\alpha$ to any subinterval $\Delta_\alpha = [t_\alpha, t_{\alpha+1}]$, $o \leqslant \alpha < n$, are polynomials of degree $\leqslant 2k-1$, and $p = P_{tk}: C[o,1] \rightarrow S_{tk}$ denotes the orthogonal projection (in $L^2[o,1]$). We can investigate this operator on $L^\infty_{tk} = \{ f \in L^\infty : f|\Delta_\alpha \in \mathcal{P}_{2k-1}, \ o \leqslant \alpha < n \}$ only: $\| P|C \rightarrow S_{tk} \| \leqslant \mathcal{A}(k)$. $\| P| L^\infty_{tk} \rightarrow S_{tk} \|$, where $\mathcal{A}(k)$ depends on k and does not depend on t.

By the special split-procedure we get a decomposition

$$L^\infty_{tk} = A \oplus B = (\bigoplus_{o}^{n-1} A_\alpha) \oplus (\oplus B_\alpha),$$

and an operator $T: B \rightarrow A$ such that supp A_α = supp $B_\alpha = \Delta_\alpha$, $o \leqslant \alpha < n$

$$S_{tk} = \text{Graph } T = \{(a,b) : a=Tb, \exists\, b \in B\}$$

Then $S_{tk}^{\perp} = \text{Graph }(-T^*)$ and P can be explicitly written in terms of T and T^*. This representation has special block-properties, and it gives the possibility to prove the uniform boundedness of $P:C \to S_{tk}$ for wide classes of the partitions t, In particular, there exist constants m(k) and M(k), s.t. if

$$\Delta_{\alpha+1}/\Delta_{\alpha} \geqslant m(k),\ o \leqslant \alpha < n, \quad \text{then } \|P:C \to S_{tk}\| \leqslant M(k),$$

if m is fixed, $o < m < \infty$, and $\Delta_{\alpha+1}/\Delta_{\alpha} = m$, $o \leqslant \alpha < n$, then $\|P:C \to S_{tk}\| \leqslant M(k)$, M(k) does not depend on n or m. This case is essentially based on the analysis of the rational pencil

$$W_x(z) = -\, xXSz + (1+S^*S+xX^2) - S^*X/z,\ x=\frac{1}{m},$$

where x is a parameter, $X = \begin{pmatrix} 1 & & & 0 \\ & x & & \\ & & x^2 & \\ & & & \ddots & \\ 0 & & & & x^{k-1} \end{pmatrix}$ and S is the mewn (kxk)-matrix to construct the operator $T:B \to A$ above. This pencil is "positively-defined", more precisely,

$$\mathrm{Re}\langle W_x(z)h, h\rangle \geqslant \frac{2}{3}\langle h,h\rangle,\ |z| = 1, \text{ for any } h \in \mathbb{C}^k \text{ and } x,\ o < x \leqslant 1.$$

C. PEARCY:

<u>Contractions with rich spectrum have invariant subspaces</u>

Let $\mathcal{H}$ be a separable, infinite dimensional, complex Hilbert space, and let $\mathcal{L}(\mathcal{H})$ denote the algebra of all bounded linear operators on $\mathcal{H}$. As usual, a subspace $\mathcal{M}$ of $\mathcal{H}$ is said to be a <u>nontrivial invariant subspace</u> for an operator T in $\mathcal{L}(\mathcal{H})$ if $(0) \neq \mathcal{M} \neq \mathcal{H}$ and $T\mathcal{M} \subset \mathcal{M}$. Recall that a <u>contraction</u> is an operator T in $\mathcal{L}(\mathcal{H})$ satisfying $\|T\| \leqslant 1$. The spectrum $\sigma(T)$ of a contraction is said to be a <u>dominating set</u> if the intersection of $\sigma(T)$ with the open unit disc D is sufficiently large that for every function h belonging the the algebra H^{∞} (of the unit disc), $\sup_{\lambda \in \sigma(T) \cap D} |h(\lambda)| = \|h\|_{\infty}$. In a paper of the same name that appeared in the Journal of Operator Theory, Vol.1 (1979),pp.123-136,

S. Brown, B. Chevreau, and the speaker proved the following: Theorem. Every contraction T in $\mathcal{L}(\mathcal{H})$ whose spectrum is a dominating set has a nontrivial invariant subspace. A sketch of the main ideas of the proof will be given.

J.D. PINCUS:

"Index" on the Essential Spectrum

We show that there is a natural relationship between the Jensen function of certain analytic almost periodic functions h and the principal function of an associated Toeplitz operator W_h in a type II_∞ von Neumann algebra. We show that the principal function is a generalized winding number of a part of the essential spectrum g W_h with Hausdorff dimension one, and we use facts from geometric measure theory to analyze the stability of the classical Lagrange mean motion for an exponential polynomial.

This is presented in the context g a more general extension g index onto a "thick" essential spectrum.

D.PRZEWORSKA-ROLEWICZ:

Right inverses and Volterra Operators

Suppose that X is a linear space. An operator $A \in L(X)$, dom A = X, is a Volterra operator, if the operators $I-\lambda A$ are invertible for all scalars λ. The set of all Volterra operators acting in X will be denoted by V(X).

Suppose that $D \in L(X)$ is right invertible and that F_0 is an initial operator for D corresponding to a Volterra right inverse R_0 of D, i.e. a projection onto ker D such that $F_0 R_0 = 0$. We assume here that dim ker D $\neq 0$ and dom $R_0 = X$. Denote by R(X) the set of all right invertible operators in L(X) and $\mathcal{R}_D$ will stand for the set of all right inverses of a $D \in R(X)$. The following question arises: Do all right inverses of D be also Volterra operators ? In other words: Does $\mathcal{R}_D \subset V(X)$? We shall see that the answer is, in general, negative. We shall examine also some other relations between right inverses and Volterra operators, for instance,the existence of exponential elements (i.e. eigenvectors of D), in particular, in linear rings.

J. ROVNYAK:

An operator-theoretic approach to theorems of the Pick-Nevanlinna and Loewner type

The lecture will give an account of joint work with Marvin Rosenblum. An operator method is used to obtain generalizations of Loewner's characterization of real valued functions on $(-1,1)$ which are restrictions of holomorphic functions $f(z)$ defined for $y>0$, $y<0$, and across $(-1,1)$ such that $\operatorname{Im} f(z)>0$ for $y>0$. The method is based on the lifting theorem and yields other interpolation theorems.

B. SZ.-NAGY:

A structural property of the functional model of contractions

Let $\Theta(\lambda)$ be an analytic function on the open unit disc D, whose values are contractive operators from a Hilbert space $\mathcal{E}$ into a Hilbert space $\mathcal{E}_*$. Setting $\Delta(e^{it}) = [I-\Theta(e^{it})^* \Theta(e^{it})]^{1/2}$ (where $\Theta(e^{it})$ denotes radial limit on the circle) consider the function spaces

$$K_+ = H^2(\mathcal{E}_*) \oplus \overline{\Delta L^2(\mathcal{E})}, \qquad \mathcal{H} = K_+ \ominus \{\Theta w \oplus \Delta w : w \in H^2(\mathcal{E})\}$$ and the

operator $S(\Theta)$ defined on $\mathcal{H}$ by

$$S(\Theta)(u \oplus v) = P_{\mathcal{H}}(\lambda u \oplus e^{it}v);$$

it is known that $S(\Theta)$ is the functional model of (completely non-unitary) contractions.

In a paper written in collaboration with Prof. C. Foias (to appear in Acta. Sci.Math., 41) conditions are given under which every function $f \in L^1$ (on the unit circle) can be represented modulo a function in H^1_0 in the form

$$f(e^{it}) = (h(e^{it}), h'(e^{it}))_{\mathcal{E}_* \oplus \mathcal{E}} , \quad \text{where } h,h' \in \mathcal{H} .$$

This representation easily implies existence of a non-trivial invariant subspace for $S(\Theta)$. It holds e.g. if a) $\dim \mathcal{E}_* = \infty$, b) there exists a "dominant" subset M of D and a number ϑ $(0<\vartheta<1)$ such that

$$\inf_{\mathcal{A}\in\Phi} \|\Theta(\mu)^*|\mathcal{A}\| \leqslant \vartheta \text{ for every } \mu\in M.$$

Here Φ denotes the family of subspaces $\mathcal{A}$ of $\mathcal{E}_*$ with finite codimension.

F.-H. VASILESCU:

Stability of the Euler characteristic for Banach complexes

We consider a class of (cochain) complexes of Banach spaces (whose morphisms are closed linear operators) called semi-Fredholm and define a notion of index by means of the Euler characteristic. We investigate then the stability of the index under small or compact perturbations, extending the classical stability theorems valid for closed operators. A Fredholm theory for systems of closed transformations, commuting in a certain sense, can be derived from these results. Another application is the study of the small perturbation of the $\bar{\partial}$-operator, when acting in strongly pseudoconvex manifolds.

D. VOGT:

Splitting theorems for exact sequences of (F)-spaces and their applications

Let $0\to E\to F\to G\to 0$ be an exact sequence of nuclear (F)-spaces. G is said to have property (DN), iff there exists a basis of seminorms with $\| \ \|_k^2 \leqslant \| \ \|_{k-1} \cdot \| \ \|_{k+1}$, E is said to have property (Ω) iff there exists a basis of seminorms with $\| \ \|_k^{*2} \leq \| \ \|_{k-1}^* \cdot \| \ \|_{k+1}^*$, where $\| \ \|_k^*$ is the dual (extended valued) norm for $\| \ \|_k$. The following theorem is an immediate consequence of D. Vogt: Math. Z. 155(1977), 1o9-117 and D.Vogt - M.J. Wagner: Studia Math. 67, to appear.

Thm: If G has (DN) and E has (Ω), then the sequence splits. Applications are given on the existence of 1. extension operators for C^∞-functions on compact sets in $\mathbb{R}^n$ (Tidten), 2. extension operators for holomorphic functions on analytic (especially algebraic) subvarieties of $\mathbb{C}^n$. 3. the existence of right inverses for differential operators. Further developments in this theory are presented for the case of E,G being power series spaces. These lead for example to necessary conditions and sufficient conditions for the solvability of certain systems of linear equations with infinitely many unknown variables.

D. VOICULESCU:

Some results on norm-ideal perturbation of Hilbert space operators

The talk will contain results concerning diagonability after norm-ideal perturbations and concerning the invariance up to unitary equivalence of the absolutely continuous part under norm-ideal perturbations, for n-tuples of commuting hermitian operators.

L. WAELBROECK:

Analytic function, galbs, and tensor products

Let U be a finite dimensional complex manifold, and E a complete topological vector space. A mapping $f:U\to E$ is convex-analytic if it factors $U\to E_1\to E$ with $U\to E_1$ analytic and E_1 locally convex, and with $E_1\to E$ continuous and linear.

Let $\varphi:E\to F$ be a continuous linear mapping of topological vector spaces, and $(\lambda_n)\in l_{1.+}$. Then $(\lambda_n)\in G(\varphi)$ if $\forall V\in\mathcal{V}_F \quad \exists U\in\mathcal{V}_E : V\supseteq \Sigma'\lambda_n U$ where

$$\Sigma'\lambda_n U = \bigcup_k \Sigma^k\lambda_n U$$

Consider now $f: U\to E$, $g: V\to F$ convex analytic, $a: E\times F\to G$ continuous and bilinear, and $\varphi: G\to G_1$ with $\lambda\in G(\varphi)$. Then $\varphi\circ a(f,g): U\times V\to G_1$ is convex-analytic if $\lambda_k = a^{k^{1/n}}$ where $n = \min(\dim U, \dim V)$. And this is the weakest statement involving only $G(\varphi)$ which ensures the convex analyticity of $\varphi\circ a(f,g)$. If the galb generated by λ does not contain $\lambda_k = a^{k^{1/n}}$, a non convex-analytic mapping $\varphi\circ a(f,g)$ can be constructed, with $\lambda\in G(\varphi)$, a continuous and bilinear, f and g convex analytic.

The above results seem remote from operator theory, but they are not. Multilinear mappings pervade the foundations of operator theory. Think of the mappings

$\mathcal{L}(E) \times E \to E$, $\mathfrak{A} \times \mathfrak{A} \to \mathfrak{A}$, $\mathfrak{A} \times \alpha \to \alpha$, with E a TVS, $\mathfrak{A}$ a TA, α a topological ideal. The research above was motivated by results of Kaballo, who succeeded in constructing meromorphic α-valued functions satisfying relations, where α is a small ideal of $\mathcal{L}(H)$.

M. WOLFF:

On the generators of C_o-semigroups of positivity preserving operators

Sei E ein Banachverband (über $\mathbb{R}$) und A ein dicht definierter Operator mit Definitionsbereich D(A).
A heiße sublokal, wenn gilt: Ist $x \geqslant o$ aus D(A), $y \geqslant o$ aus E und $\inf(x,y) = o$, so ist $\inf((Ax)^-, y) = o$ $\quad (z^- = \sup(-z, o))$.
A heiße stark sublokal, wenn gilt: Ist $x \geqslant o$ aus D(A), so ist $(Ax)^- \in \overline{E_x}$ (das von x erzeugte abgeschlossene Ideal).

Satz 1: Sei A Generator einer C_o-Halbgruppe (T_t) und D(A) sei ein Verband. Äquivalent sind

(a) $T_t \geqslant o$ (d.h. $|T_t x| \leqslant T_t |x|$) f.a. $t > o$

(b) (i) A ist sublokal und (ii) es gibt $\lambda > o$, so daß aus $x \geqslant o$ und $A x \geqslant \lambda x$ stets $x = o$ folgt.

Satz 2: Sei E = C(X) (X kompakt) der Raum der stetigen Funktionen auf X und A Generator einer C_o-Halbgruppe (T_t). Äquivalent sind

(a) $T_t \geqslant o$ f.a. $t > o$ (b) A ist stark sublokal.

Verbindungen mit der Kato-Ungleichung, dem Maximumprinzip und mit Ergebnissen von R.S.Philipps u. K. Sato werden hergestellt.

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