MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Applied Mathematical Statistics

4.11. bis 10.11.1979

Die Tagung "Applied Mathematical Statistics" fand unter der Leitung von Herrn K. Behnen (Hamburg) und Herrn G. Neuhaus (Hamburg) statt.

Der Schwerpunkt der Tagung lag auf dem Gebiet der Sequential-Analyse und Nichtparametrischen Statistik, wobei im letzteren Bereich vor allem adaptive Verfahren und Fragen der Robustheit im Vordergrund standen.

Neben dem umfangreichen Vortragsprogramm konnten viele Gespräche und Diskussionen geführt, sowie neue Kontakte geknüpft werden. Auch dieses Mal waren erfreulicherweise wieder zahlreiche ausländische Wissenschaftler anwesend, die 49 Teilnehmer stammten aus 11 verschiedenen Nationen.

Die großzügige Planung und Organisation der Tagung durch das Forschungsinstitut wurde von den Tagungsteilnehmern, insbesondere von denen, die erstmals in Oberwolfach waren, gelobt.

Vortragsauszüge

ALBERS, W.: **Asymptotic deficiencies of one-sample rank tests under restricted adaptation**

We consider adaptive rank tests for the one-sample problem. Here adaptation means that the score function J of the rank test is estimated from the sample. We restrict attention to cases with a moderate degree of adaptation, in the sense that we require that the estimated J belongs to a one-parameter family $J=(J_r|r\in I\subset R^1)$. Using asymptotic expansions, we compare the performance of such adaptive rank tests to the performance of rank tests with fixed scores. For a particular class of estimators which are related to the sample kurtosis, explicit results are obtained.

BERAN, R.J.: **Efficient robust estimates and tests in parametric models**

Let $\{P_\vartheta^n:\vartheta\in\Theta\}$, Θ an open subset of R^k, be a regular parametric model for a sample of n independent, identically distributed observations. This talk will describe estimates $\{T_n;\ n\geq 1\}$ of ϑ which are asymptotically efficient under the parametric model and are robust under small deviations from that model. In essence, the estimates are adaptively modified one-step maximum likelihood estimates, which adjust themselves according to the estimated fit of the model to the data. When the fit seems poor, T_n discounts observations that would have large influence on the value of the usual one-step MLE. The estimates $\{T_n\}$ are asymptotically minimax, in the Hâjek-LeCam sense, for a Hellinger ball contamination model and suitable loss function. The talk will also describe robust, asymptotically minimax tests for hypotheses about ϑ. One construction of such tests is based upon the estimates $\{T_n\}$.

BERK, R.H.: **Asymptotic efficiencies for sequential tests**

Various notions of asymptotic efficiency for tests are discussed in the context of sequential testing. These include the concepts due to Pitman, Bahadur, Chernoff and Hodges and Lehmann. Some of these efficiencies extend naturally to the sequential case; others present conceptual difficulties. The two best known efficiencies, those of Pitman and Bahadur, do not provide satisfactory comparisons of sequential tests.

BRAUN, H. : Stochastic approximation: A new stopping rule.

Consider a Robbins-Monro process $\{X_n\}$ determined by the recursion $X_{n+1} = X_n - a_n Y_n$, where $Y_n = Y(X_n) = M(X_n) + \epsilon_n$. We propose to take two observations on Y at each X and hence, obtain an unbiased estimate S_n^2 of $\sigma^2 = E(\epsilon_n^2)$. The stopping rule is defined by $N_k = \inf\{n : U_n(k) \le c(k)\}$ where c(k) is a constant and $U_n(k) = (\bar{Y}_{n-k+1}^2 + \cdots + \bar{Y}_n^2)/(k \cdot s_n^2)$. Intuitively, $U_n(k)$ will be close to 1/2 when $X_{n-k+1}, \ldots, X_n$ are close to 0. It is possible to show that N_k has finite moments of all orders andthat N_k/k converges in distribution to a positive random variable. Furthermore, the randomly stopped R-M process when properly normalized is asymptotically normally distributed. Small simulations show that this stopping rule works well for k=2 in a wide variety of problems.

HALL, W.J.: Nonparametric estimation of mean residual life

The mean residual life (MRL) function, for a distribution F on R^+, is defined as $e_F(x) \equiv E_F(X-x|X>x)$. It is also known as (remaining) life expectancy, and is a function of interest in actuarial studies, survivorship studies, and reliability.
Our first objective in this paper is to present a summary of the known, and several newly discovered, properties of the MRL function e_F; the latter include a characterization of the class of all MRL functions, and a study of the behavior of e 'at great age' (for large x).
The natural nonparametric estimator of e_F is the empirical version $\hat{e}_n \equiv e_{F_n}$ (F_n the empirical df). Large-sample study of it was initiated by Yang in 1978 (Ann.Statist. 6); in particular, she

considered the MRL process $n^{1/2}(\hat{e}_n - e_F)$ and showed it to be weakly convergent to a certain Gaussian process. Our second objective is to reformulate and extend this result, identifying the limiting process in terms of Brownian motion, dropping some of her assumptions, and extending her domain of convergence to R^+.

Our final objective is the construction of nonparametric (asymptotic) simultaneous confidence bands for the MRL function e_F, and these are readily obtained as a corollary to our extended version of Yang's theorem. Some illustrations, to survivorship data and to reliability data, are given.
(This work is joint with Jon A. Wellner.)

HOGG, R.V.: <u>Adaptive nonparametric methods</u>

In this expository talk, a brief explanation is given describing why certain adaptive distribution-free statistical tests are also distribution-free. While not the best distribution free test for any given underlying distributional assumptions, adaptive ones seem to be quite powerful over a wide range of distributions. More generally, the adaptive idea is used with certain nonparametric robust procedures (in particular, R- and M-estimators) to create fairly efficient statistical methods. An indication is given as to the type of gains in power that can be made by the adaptive "tuning" of these robust schemes.

HUSKOVA, M.: <u>An adaptive procedure based an ranks</u>

Consider the following linear model: $Y_i = \Delta^o d_{iN} + X_i, i=1,\dots,N$, where $X_1,\dots,X_N$ are i.i.d. random variables with density f, $d_{1N},\dots d_{NN}$ are regression constants, Δ^o - an unknown parameter. The density f belongs to $\bigcup_{j=1}^{k} F_j$, where $F_j = \{f \mid f(x) = \lambda \cdot f_j(x\lambda - \omega), -\infty < \omega < +\infty, \lambda > o\}$ $j=1,\dots,K$, f_j an density with finite Fisher information.

Define $S_{Nj}(\Delta) = \sum_{i=1}^{N} d_{iN}\, \varphi(\frac{R^{\Delta}_{iN}}{N+1}, f_j)$, where

$\varphi(\omega, f_j) = - f'_j\,(F_j^{-1}(\omega)) \,/\, f_j\,(F_j^{-1}(\omega))$ and R^{Δ}_{iN} are ranks of the sequel $X_1 - \Delta \cdot d_{1N}, \dots, X_1 - \Delta \cdot d_{NN}$. The following adaptive procedure is suggested for

$H : \Delta^o = 0$ against $A : \Delta^o > 0$: We reject H if $S_{Nj}(\Delta^o) > \omega_{1-\alpha}\sqrt{I(f_j)}$, where j has the property $\max_\nu \frac{S_{N\nu}(\Delta^o+1)-S_{N\nu}(\Delta^o)}{\sqrt{I(f_\nu)}} = \frac{S_{Nj}(\Delta^o+1)-S_{Nj}(\Delta^o)}{\sqrt{I(f_j)}}$.

IRLE, A.: Transitivity in problems of optimal stopping

If we consider two families of σ-algebras $G=(G_t)_{t\in T}$ and $F=(F_t)_{t\in T}$, we may, for any real valued stochastic process X, define the value $v(X,G)$, resp. $v(X,F)$ as the maximal reward which can be achieved by optimal stopping with respect to G, resp. F. We show that these two values are equal for all X adapted to G iff G is transitive in the sense of Bahadur (1954) for F. In the case of non-transitivity we introduce a numerical quantity to measure the possible reduction in value and investigate some properties of this quantity. The consequences of a reduction by invariance are investigated in a special case.

KELLER, H.D.: Some examples of statistical inference based on the empirical characteristic function

Let $X_1, X_2, \ldots,$ be i.i.d. random vectors with distribution function F and characteristic funktion c, that is $c(t) = \int e^{i<t,x>}dF(x)$ for all $t \in \mathbb{R}^k$. Some examples are given, which show that useful statistical procedures, concerning exploratory investigation may be based on the e.c.f. $c_n(t)$, which is defined by $c_n(t) = \int e^{i<t,x>}dF_n(x) = \frac{1}{n}\cdot\sum_{j=1}^{n} e^{i<t,X_j>}$. Especially procedures based on the statistics $\sup_{||t||\leq T}|c_n(t) - c(t)|$, where $0<T<\infty$, and $\int|c_n(t) - c(t)|^2dQ(t)$, where Q is a probability measure are recommended. Furthermore let $S \subset \mathbb{R}^k$ be a compact set and $C(S)$ the Banach space of continuous functions on S endowed with the sup-norm. It seems natural to assert that $Y_n(t) = \sqrt{n}(c_n(t) - c(t)) \xrightarrow{\mathcal{D}} Y(t)$ in $C^2 = C^2(S) = C(S) \times C(S)$, where Y is a suitably defined Gaussian process. But this assertion is not true in general, as counterexamples of S. Csörgö show. Nevertheless the following positive result is shown:

For m=2,3,...,and $\varepsilon>o$ put $g_{m,\varepsilon}(x)=(\log x)\cdot(\prod_{k=2}^{m-1}\log_k x)^2\cdot$ $\cdot(\log_m x)^{2+\varepsilon}$, where $\log_j$ denotes the j-times iterated logarithm.
Let now $m \in \mathbb{N}$, $m \geq 2$ be fixed, α_m sufficiently large and

$$\int_{\{x:\alpha_m < ||x||\}} g_{m,\gamma}(||x||)\, dP^{X_1} < \infty$$

Then $Y_n(\cdot)$ converges weakly in C^2. In the special case k=1 a deeper insight may be based on strong approximations by stochastic integrals based on a sequence of Brownian bridges, which are given by the theorem of Komlôs, Major, Tusnády (1975).

KREMER, E.: Local comparison of linear rank tests in the Bahadur sense.

A theorem, stating the local optimality of general linear two-sample rank tests according to Bahadur efficiency, is derived and a condition for the existence of bounds for local Bahadur efficiency presented. Furthermore the connection of these results to former considerations on Pitman efficiency is discussed and conditions for equality of local Bahadur efficiency and Pitman efficiency given. All proofs are based on a theorem of Kremer (Ann. Statist. 1979) about the local equivalence of exact and approximate slopes.

KRENGEL, U.: Stopping points and tactics

I want to present some results concerning optimal stopping for multiparameter processes and more generally for processes $\{X_t, t\in Q\}$ indexed by a partially orderd set Q. Q is assumed locally finite, i.e. for any $q \in Q$ there exist only finitely many $t \leq q$. The process is adapted to the increasing σ-algebras F_t. Stopping points are maps $\tau : \Omega \to Q$ with $\{\tau \leq t\} \in F_t (t \in Q)$. An important class of stopping points is generated by tactics $H = \{H_{st}, s \in Q, t\in\{s\} \cup D_s\}$, where D_s are the "direct successors" of s in Q, $H_{st} \in F_s$, and for fixed s $\{H_{st}, t \in \{s\} \cup D_s\}$ is a partition of Ω. Roughly speaking, H_{ss} is the set where you stop at s if you get there, H_{st} is the set where you proceed to t. For $Q = \mathbb{N}^2$ and for Q a tree all stopping times come from tactics under a weak qualitative conditional independence condition, but not for $Q = \mathbb{N}^3$. If the σ-algebras F_t are generated by independent identically distributed $Y_s (s \leq t)$ and X_t is a nice function of $Y_s (s \leq t)$ e.g. $X_t = Y_t$, or X_t = average of all $Y_s (s \leq t)$, stopping problems for tactics can be reduced to 1-parameter problems. This yields e.g. a Wald's equation for tactics. If $X_t \geq 0$ and the X_t are independent, any stopping point can be "improved" by one taking values in a totally ordered subset of Q. This work was done jointly with L.Sucheston.

LAI, T.L.: On sequential medical trials

Anscombe (1963, T.Amer. Statist. Assoc.) proposed the following formulation of sequential design for a comparative medical trial. There are two competing treatments A and B, and a specified set of N patients are to receive either treatment A or treatment B. The trial consists of pairwise allocation of treatments to n pairs of patients, then a conclusion is made as to which treatment appears to be superior, so that the remaining N-2n patients not on trial will be given the apparently superior treatment based on the results of the first n pairs. We define the regret as the difference in mean response between the idealized procedure of using the (unknown) better treatment for all N patients and the procedure under consideration for choosing n pairs of patients for experimentation and then using the apparently better treatment for the other N-2n patients. A class of sequential procedures is proposed and analyzed in this talk, where we consider only the case of normally distributed responses. This class of sequential procedures is shown to have approximately optimal properties based on asymptotic analysis as $N \to \infty$ and on Monte Carlo simulations. Furthermore, these sequential procedures are shown to be much better than traditional fixed sample size procedures.

LORDEN, G.: Asymptotic efficiency of multi-stage tests

For the problem of testing hypotheses $\vartheta \leq \vartheta_0$ vs $\vartheta \geq \vartheta_1$, about the real parameter ϑ of an exponential family, it is proved that there are simple 3-stage tests that minimize the expected sample sizes to within $O((\log \alpha^{-1})^{1/2} \log \log \alpha^{-1})$ as α, the upper bound probabilities of error, goes to zero. This generalizes to multi-dimensional exponential families - for example, to the problem of testing $\frac{\mu}{\sigma} \leq 0$ vs. $\frac{\mu}{\sigma} \geq \delta > 0$, where μ and σ^2 are the unknown mean and variance of a normal distribution. Two-stage tests are not in general asymptotically efficient, even for testing between two simple hypotheses.

MÜLLER-FUNK, U.: On sequential signed rank statistics

Starting out from a sequence of i.i.d. r.v. $X_1, X_2, \ldots$, we consider statistics of the form $T_n = \sum_{k=1}^{n} c_k \operatorname{sgn}(X_k) g(\frac{R^+_{kk}}{k+1})$, where R^+_{kk} denotes the relative rank of $|X_k|$ among $|X_1|, \ldots, |X_k|$ and where $c_k > 0$ are constants. This class of statistics arises quite naturally in sequential analysis. The basic distribution theory necessary for that end is derived, i.e. an invariance principle and the strong law of large numbers. In order to prove the former we rely on a refinement of Hajek's projection method. It turns out that the limiting process belongs to the class of random functions $aW(t) + b \int_0^t s^{-1} W(s)ds$ where W is a standard Wiener process.

NOVIKOV, A.A.: Bounds and asymptotics for first passage time of processes with independent increments through moving boundaries

Let S_t be a process with independent stationary increments, $t \in [0,\infty)$ or $t=0,1,\ldots$, and let

$$\tau = \inf\{t: |S_t| > g(t)\}, \quad \sigma = \inf\{t: S_t < f(t)\}.$$

We consider the behavior of the probabilities

$$P\{\tau > T\}, \quad P\{\sigma > T\} \quad .$$

Using the technique due to absolute continuity of probability measures, we receive the upper and lower bounds for these probabilities. It turns out that the received bounds are close and allow to prove results about rough asymptotics of the probabilities under consideration. In particular the next result holds.

THEOREM. Let $S_n = \sum_1^n \xi_k$, $\xi_k \sim N(0,1)$ and independent.

Furthermore, let $g(n) > 0$, $\ln n = o(g^2(n))$ and

$$\sum_1^n [g(k+1) - g(k)]^2 = o(\sum_1^n g^{-2}(k)) \quad \text{as } n \to \infty$$

Then: $P\{\sigma > n\} = \exp\{-\frac{\pi^2}{8}(\sum_1^n g^{-2}(k))(1+o(1))\}, \ n \to \infty$.

When S_t is a Brownian motion some more exact results are proved. Using the result of Komlos, Major, Tusnady we can extend the above statement to the case of nongaussian random variables.
The bounds and asymptotics of $P\{\sigma > T\}$ are expressed in terms of the so called "action functional".

PFLUG, G.: On recursive estimation

Let $\{P_\vartheta\}_{\vartheta \in R}$ be a family of probability measures on a measurable space (X, A). We want to estimate the unknown parameter ϑ by observing a sequence $\{X_n\}_{n \in N}$ of independent random variables with distribution P_{ϑ_0}. A sequence of estimates $\{\hat{\vartheta}_n\}_{n \in N}$ is called *recursive*, if $\hat{\vartheta}_0$ = const, and $\hat{\vartheta}_{n+1} = \psi(n, \hat{\vartheta}_n, X_n)$. Fabian has shown in 1978, that under certain global and local (Cramér-Wald) conditions $L(n^{1/2}(\hat{\vartheta}_n - \vartheta_0)|P_{\vartheta_0}) \sim N(0, I^{-1}(\vartheta_0))$ where $I(\vartheta_0)$ denotes the Fisher-Information.
This situation (i.e. when the square roots of the densities are q.m. differentiable) is often called the "regular case". In the lecture we consider not only this case but also the so called "almost regular case" (Ibragimov/Hasminskij) and the "non regular case" (i.e. when $\lim_{\vartheta \to \vartheta_0} d_2^2(P_\vartheta, P_{\vartheta_0}) / |\vartheta - \vartheta_0|^\rho = c > 0$, $\rho < 2$; d_2 denotes the Hellinger distance).

RIEDER, H.: Robustness of rank tests

First, nonparametric hypotheses of approximate symmetry, and approximate equality of distribution functions, are formulated, to account for gross errors, and distribution freeness of rank statistics for these hypotheses is shown, which now refers to the stochastically extreme laws. Second, rank tests are investigated under local alternatives and infinitesimal neighbourhoods, which possess a subtle asymptotic fine structure; from this local viewpoint rank tests are not extra robust (e.g. unbounded scores give maximum size one and minimum power zero). Third, qualitative robustness of tests is defined, by the requirement of equicontinuous error probabilities, and necessary and sufficient criteria are derived. Rank tests prove to be qualitatively robust. Fourth, the point of final breakdown is introduced to denote the critical amount c of contamination that renders test statistics unable to distinguish any two c-contaminated probabilities, and it is determined for rank statistics (e.g. sign-test, $c = 0.5$; Wilcoxon, $c \approx 0.293$; Normal scores, $c \approx 0.239$).

ROTHE, G.: Tests based of the method of n rankings

In the two-way classification model without interactions and one observation in each of the $p \cdot N$ cells a class of nonparametric tests is considered. This class corresponds to the class of projections $A \in \mathbb{R}^{p! \times p!}$ where $A \cdot 1_{p!} = 0$. In many cases, there are restrictions on the model in such a manner that it can be parametrized by a subset of $\mathbb{R}^{p!}$ where the hypotheses "there is no difference between the p treatments" corresponds to the parameter $\vartheta = 0$. The following result is presented: The "Pitman-optimal" test in the class corresponds to the projection on the tangent of this parameter set at point 0. Some special cases are presented either, e.g. if both effects can be assumed to be additive and no further information is known about the underlying distribution, Anderson's test (cf. Biometrika '59) turns out to be "optimal" in the sense of Pitman efficiency.

RÜSCHENDORF, L.: Weak convergence of empirical processes in sup-norm metrics

A certain class of sup-norm metrics is introduced on the generalized Skorohod-space D_k. It is shown that the empirical process of random variables which are uniformly distributed on $[0,1]^k$ weakly converges to a Brownian Bridge w.r.t. these sup-norm metrics. In this way it is possible to reduce the asymptotic distribution of functionals of the empirical process, which are not continuous w.r.t. the uniform metric on D_k to the distribution of corresponding functionals of the Brownian Bridge.

The proof of this result is mainly based on a Poisson-type representation of empirical processes and on a generalization of the Birnbaum-Marshall inequality.

RUYMGAART, F.H.: <u>An application of linearization in nonparametric multivariate analysis</u>

Consider independent random vectors $X_1=(\xi_1,\eta_1),\ldots,X_N = (\xi_N,\eta_N)$ with bivariate d.f.'s $F_1,\ldots,F_N$. For each $t \in [0,\pi)$ let us introduce the N independent (univariate) r.v.'s $X_{n,t} = \langle X_n, e_t\rangle$, where e_t is the unit vector in $\mathbb{R}^2$ making an angle t with the positive ξ-axis. One may investigate the bivariate random structure by means of suitably chosen univariate statistics $S_N(t)$, say, based on the linearized sample elements just introduced. These statistics form a stochastic process $S_N = \{S_N(t),\ t \in[0,\pi)\}$, certain (random) functionals of which can be used to reach decisions concerning the original bivariate sample. This situation occurs e.g.in the ordinary principle component analyis, where $S_N(t)$ is the sample variance of the $X_{n,t}$. The special problem that we have in mind here is that of testing the hypothesis $F_1 = .. = F_N=F$ (continuous); see e.g. also Puri & Sen (1970). We choose $S_N(t) = \Sigma_{n=1}^N c_n\ J(R_{n,t}/(N+1))$, where $R_{n,t}$ is the rank of $X_{n,t}$. We shall consider S_N as a random element in the Hilbert space $L_2([0,\pi),\mu)$, where μ is a convex combination of Lebesgue measure on $[0,\pi)$ and the counting measure on $\{t_1,t_2,\ldots,t_m\} \subset [0,\pi)$, i.e. the case where the process reduces to a random vector in $\mathbb{R}^m$ is covered as a special situation. It turns out that $S_N \to_w G$ on $L_2(\mu)$, where G is a Gaussian process. Unfortunately this Gaussian process has a covariance function depending on the underlying d.f. F. This covariance function can, however, be estimated so that the norm of the difference converges a.s. to zero. From this estimator we may derive a random transformation $T_N(S_N)$ of S_N such that $||T_N(S_N)||_\mu^2 \to_w \chi_r^2$, for some r under certain conditions. This means that tests based on $||T_N(S_N)||_\mu^2$ are asymptotically distribution free. The power of these tests can for well chosen μ be occasionally better than that of tests of similar type where μ is the counting measure on the set $\{0,\frac{\pi}{2}\}$.

SCHOLZ, F.W.: <u>Towards a unified definition of maximum likelihood</u>

Based on a pairwise comparison of probability measures near the observed data point, a unified definition of maximum likelihood is given. This definition applies to the dominated as well as to the undominated case, thus providing a base for nonparametric maximum likelihood problems, which so far have been solved in a more or less ad hoc fashion. The definition is based soley on the possible probability measures governing the distribution of a random element X and does not involve "appropriate" choices of the Radon Nikodym derivatives as in the Kiefer Wolfowitz definition (Ann. Math. Stat. 1956 27, 887-906). Roughly the definition looks at $\underline{\lim}\, P(N_x)/Q(N_x)$ as the neighbourhoods N_x of x shrink to the observed x, in order to decide whether P or Q is to be considered "more likely" for the observed x. It is shown that this definition coincides with the classical (dominated) definition when smooth density versions are possible. Parametric and nonparametric examples are considered and the question of consistency will be addressed.

SEN, P.K.: <u>Asymptotically risk efficient nonparametric sequential point estimation procedures.</u>

Nonparametric sequential point estimation of estimable parameters based on U-statistics and von Mises' differentiable statistical functions is considered. The asymptotic risk efficiency of this procedure is established under very mild regularity conditions. In this context, the estimated variance of a U-statistic is expressed as a linear combination on reverse martingales on which certain moment inequalities are used to yield the desired convergence results in a very simple way. Certain auxilliary results on U-statistics having interest of their own are also considered and incorporated in the study of the main results. Applications of jackknifing in this context has also been studied. Asymptotic normality of the stopping time has been studied under weaker regularity conditions.

SENDLER, W.: <u>A functional limit theorem in connection with order statistics</u>

Let $(U_{ni}, 1 \leq i \leq n)$ be the ordered vector of n independent U(0,1) distributed r.v.'s; we consider the random function

$T_n(x) := \frac{1}{n} \cdot \sum_{i=1}^{n} c_{ni}(x)\, g_n(U_{ni})$, $0 \leq x \leq 1$. The following result is presented.

THEOREM: Under suitable regularity and smoothness conditions on the g_n as well as the functions $c_{ni}(\cdot)$ the processes $H_n(\cdot) = \sqrt{n}\,(T_n(\cdot) - \mu_n(\cdot))$ (where $\mu_n(\cdot)$ are appropriate centering functions) tends weakly to a Gaussian limiting process H_o. In most cases of interest the convergence is w.r.t. Skorohod's M_1, M_2 or the sup-norm-topology.
The method of proof is a functional version of Shorack's (1972) approach.

SIEGMUND, D.O.: Sequential χ^2 and F tests and the related confidence intervals

Sequential χ^2 and F tests are proposed for testing equality of the means of several normal populations with common variance. Approximations are given for the significance level and power, and approximate confidence intervals are obtained for the non-centrality parameter ϑ. For each fixed ϑ a confidence region is given for the angle ω which the treatment effect vector in canonical coordinates makes with a fixed direction, and hence an over-all confidence region is obtained for the pair (ϑ,ω).

SMITH, A.F.M.: Unsupervised sequential learning procedures

We shall consider the following general problem. A sequence of (possibly vector-valued) observations $x_1, x_2, \ldots, x_n$, is obtained, each belonging to one of $k(\geq 2)$ exclusive populations $\Pi_1, \Pi_2, \ldots, \Pi_k$. For each n, the n-th observation is to be classified on the basis of the observations $x_1, x_2, \ldots, x_n$, and without any feedback concerning the correctness, or otherwise, of previous classifications. This class of so-called unsupervised learning problems embraces a large number of important applications to on-line Pattern Recognition, Signal Detection, and the Tracking of manoeuvring targets or signal sources in noisy environments.
Uncertainty about the distribution of a particular observation leads to a mixture form, which precludes a closed-form updating and decision procedure and requires the implementation of some form of approximation.
A unified review of proposed methods for handling such problems will be presented and their use illustrated. Convergence properties

of various schemes will also be considered.

STADJE, W.: Some secretary problems

Two generalisations of the classical secretary problem are studied:
1. We no longer assume that the n objects are totally ordered; let there be m criteria according to which they have to be compared. The optimal stopping problem is then explicitly solved for monotone utility functions, and asymptotic results for some concrete cases are given.
2. We formulate the problem of stopping k times "optimally", where again optimality is defined as maximal expected utility. Specializing the utility function, we obtain optimal "k-stopping rules" for the problem of maximizing the probability that the ℓ absolutely best objects are among the k chosen ones and for the problem of minimizing the expected rank sum. For special cases the asymptotic behaviour of these stopping rules is computed for n and for k tending to ∞.

STEINEBACH, J.: Non-error rates in discrete sample-based classification

Let k disjoint population groups $G_1,\dots,G_k$ mix in a large population. An individual selected randomly from the mixed population may be regarded as a random vector (X,I), where X denotes the individual's characteristics and I indexes its group. Consider the case of X taking values in a finite space $\mathcal{X} = \{x_1,\dots,x_M\}$. Then the distribution of X is given by $\phi(x)=P(X=x) = \sum_i q_i f_i(x)=\sum_i g_i(x)$, where $q_1,\dots,q_k$ are the mixing proportions of the groups and $f_1,\dots,f_k$ denote the group-conditional distributions, i.e. $f_i(x)= = P(X=x \mid I = i)$. A classification rule D is an ordered partition $D = (D_1,\dots,D_k)$ of $\mathcal{X}$, which allocates an individual to group G_i iff $X \in D_i$. Let $r(D^*)$, $r(\hat{D})$, $\hat{r}(\hat{D})$ be the optimal and (sample-based) actual resp. apparent non-error rate as defined by Glick (Biometrics 1973). Then, extending Glick's results, we prove

(1) $0 \leq P(r(D^*) - r(\hat{D}) > 0) \leq m(k-1)(1-d^2)^n$

(2) $0 \leq r(D^*) - E\{r(\hat{D})\} \leq (kr(D^*) - 1)(1-d^2)^n$

where m denotes the number of points x with at least two different discriminant scores $g_i(x)$ and

$d^2 = \min_i \min_{g_i(x) < \max_j g_j(x)} |\sqrt{g_i(x)} - \sqrt{\max_j g_j(x)}|^2 > 0$. Furthermore,

(3) $0 \leq E\{\hat{r}(\hat{D})\} - r(D^*) \leq M(k-1)(1-d^2)^n$,

if all M points of X have at least two different discriminant scores.

WALK, H.: Stochastic iteration for a constrained optimization problem

A Lagrangian stochastic iteration method for an optimization problem with constraints, where the economic function and the constraint functions are noise-corrupted, has been investigated by Kushner and Sanvicente (1975), Hiriart-Urruty (1976) and Kushner and Clark (1978). The results of these authors on almost sure and first mean convergence are sharpened and a functional central limit theorem is given which yields fixed width asymptotic confidence domains for the optimal solution.

WETHERILL, G.B.: Normal sequential sampling

Some results on decision boundaries and risks are obtained by dynamic programming for multiple and full sequential Bayesian sampling plans; and various properties of and tables for them are presented here and compared with results for single and double sampling. Robustness of these plans with respect to errors in some of the parameters are investigated as well as methods for reducing the computation for the sequential plans. The latter include an evaluation of, and application of Chernoff's asymptotic approximations in terms of a partial differential equation.

WIJSMAN, R.A.: Sequential confidence sets

Families of invariant sequential probability ratio tests can be used to generate confidence sets for parameters in the presence of nuisance parameters. These confidence sets are capable of covering the true value of the parameter and not covering a specified false value, both with pre-assigned probabilities. Examples will be given and some properties discussed, in particular, the behaviour of the stopping time.

WITTING, H.: On sequential correlation rank tests

The remainder term in the Chernoff-Savage representation of correlation rank statistics is shown to be of order $n^{-1/2-\gamma}$, $\gamma=\gamma(\kappa)>0$, with probability $1-n^{-\kappa}$ for all $\kappa>0$ uniform over the class of all continuous bivariate distribution functions. At the crucial point of the proof a linear programming approach is employed. The theorem gives various weak convergence results and a law of the iterated logarithm. Moreover, a class of nonparametric SPRT-type tests for testing independence is given. Asymptotic OC- and ASN-curves are derived. These tests are justified by asymptotic comparison with the corresponding SPRT.

VAN ZUILEN, M.C.A.: Empirical distributions and rank statistics

Some important classes of statistics will be discussed, including simple linear rank statistics, rank statistics for testing independence and linear combination of order statistics.
It is well-known how in a Chernoff-Savage approach certain properties of the empirical distribution function can be used to obtain the asymptotic distribution of these statistics in the i.i.d. case and in particular the convergence to zero in probability of the remainder term. We shall strengthen these properties in order to obtain a stronger type of convergence to zero of the remainder term in the non-i.i.d. case. Moreover, it will be discussed how so - called Kendall-type statistics do fit in this framework.

VAN ZWET, W.R.: On efficiency of first and second order

It has been noted by a number of authors that if two tests are asymptotically efficient for the same testing problem, then typically their powers will not only agree to first but also to second order. In a paper entitled "First order efficiency implies second order efficiency" which appeared in the Hâjek memorial volume, Pfanzagl proved the first general result of this type under a large number of technical conditions. Here our concern will be to establish a theorem of this kind under rather mild assumptions and to provide an intuitive understanding of the phenomenon.
The research reported was done jointly with P.J.Bickel (Berkeley) and D.M. Chibisov (Moscow).

Liste der Tagungsteilnehmer

Antille, A.,	Departement de Mathématique, Université Fribourg, Fribourg/Pérolles, Swice - 1700
Albers, W.,	Department of Mathematics, Twente University of Technology, P.O. Box 217, Enschede, The Netherlands
Behnen, K.,	Institut für Mathematische Stochastik, Universität Hamburg, Bundesstraße 55, 2000 Hamburg 13
Beran, R.J.,	Department of Statistics, University of California, Berkeley, California 94730, USA
Berk, R.H.,	Department of Statistics, Rutgers University, New Brunswick, New Jersey 08903, USA
Braun, H.,	Department of Statistics, Princeton University, Fine Hall, P.O. Box 37, Princeton, New Jersey 08540,USA
Eicker, F.,	Abteilung Statistik, Universität Dortmund, Vogelpothsweg, 4600 Dortmund
Fieger, W.,	Institut für Mathematische Statistik, Universität Karlsruhe, Englerstraße, 7500 Karlsruhe
Hall, W.J.,	Department of Statistics, University of Rochester, Rochester, New York 14627, USA
Hogg, R.V.,	Department of Statistics, University of Iowa, Iowa City, Iowa 52242, USA
Holm, S.,	Department of Mathematics, Chalmers University of Technology, 40220 Göteborg, Schweden
Huskova, M.,	Department of Probability and Statistics, Charles University, Sokolovská 83, 18600 Prague 8, Czechoslowakia
Irle, A.,	Institut für Mathematische Statistik, Universität Münster, Roxeler Str. 64, 4400 Münster
Krafft, O.,	Institut für Statistik und Wirtschaftsmathematik, Universität Aachen, Pontstr. 51, 5100 Aachen
Keller, H.D.,	Abteilung Statistik, Universität Dortmund, Vogelpothsweg, 4600 Dortmund

Kremer, E., Institut für Mathematische Stochastik, Universität Hamburg, Bundesstraße 55, 2000 Hamburg 13

Krengel, U., Institut für Mathematische Statistik und Wirtschaftsmathematik, Universität Göttingen, Lotzestraße 13, 34 Göttingen

Kurotschka, V., Institut für Mathematik III, Freie Universität Berlin, Arnimallee 2-6, 1000 Berlin 33

Lai, T.L., Department of Mathematical Statistics, Columbia University, New York, New York 10027, USA

Lorden, G., Department of Mathematics, California Institute of Technology, Pasadena, California 91125, USA

Mammitzsch, V., Fachbereich Mathematik, Universität Marburg, Lahnberge, 3550 Marburg

Morgenstern, D., Lehrstuhl für Mathematische Stochastik der TU, Welfengarten 1, 3000 Hannover 1

Müller, D.W., Sonderforschungsbereich 123, Universität Heidelberg, Im Neuenheimer Feld 294, 6900 Heidelberg 1

Müller-Funk, U., Institut für Mathematische Stochastik, Universität Freiburg, Hermann-Herder-Str. 10, 7800 Freiburg

Neuhaus, G., Institut für Mathematische Stochastik, Universität Hamburg, Bundesstraße 55, 2000 Hamburg 13

Novikov, A.A., Steklov Mathematical Institute, Moscow, Vavilovstr.42, USSR 117966 GSP-1

Pflug, G., Institut für Statistik, Universität Wien, Rathausstraße 19, A 1010 Wien, Austria

Rieder, H., Institut für Mathematische Stochastik, Universität Freiburg, Hermann-Herder-Str. 10, 7800 Freiburg

Rothe, G., Abteilung Statistik, Universität Dortmund, Vogelpothsweg, 46 Dortmund

Rüschendorf, L., Institut für Statistik und Wirtschaftsmathematik, Universität Aachen, Pontstr. 51, 5100 Aachen

Ruymgaart, F.H., Department of Mathematics, Catholic University Nijmegen, Toernooiveld, Nijmegen, The Netherlands

Schach, S., Abteilung Statistik, Universität Dortmund, Vogelpothsweg, 46 Dortmund

Schmitz, N., Institut für Mathematische Statistik, Universität Münster, Roxeler Str. 64, 4400 Münster

Scholz, F.W., 637 NW 84th, Seattle 98117, USA

Scott, D., Mathematisches Institut, Universität München, Theresienstraße 39, 8000 München 2

Sen, P.K., Department of Biostatistics, University of North Carolina, Berkeley, California 94720, USA

Sendler, W., Abteilung Statistik, Universität Dortmund, Vogelpothsweg, 46 Dortmund

Siegmund, D., Department of Statistics, Stanford University, Stanford, California 94305, USA

Smith, A.F.M., Department of Mathematics, University of Nottingham, University Park, NG7 2RD, England

Stadje, W., Institut für Mathematische Statistik und Wirtschaftsmathematik, Universität Göttingen, Lotzestraße 13, 3400 Göttingen

Steinebach, J., Institut für Statistik und Dokumentation, Universität Düsseldorf, Universitätsstraße 1, 4000 Düsseldorf

Varbanova, M., Institut der Bulgarischen Akademie der Wissenschaften, 1090 Sofia, POB 373, Bulgarien

Vogel, W., Institut für Angewandte Mathematik, Universität Bonn, Wegelerstraße 6, 5300 Bonn

Walk, H., FB 6 Mathematik, Gesamthochschule Essen, Universitätsstraße 2, 4300 Essen

Wetherill, G.B., Mathematical Institute, Cornwallis Bldg., The University, Canterbury CT2 7NF, Kent, England

Wijsman, R.A., Department of Mathematics, University of Illinois Urbana, Illinois 61801, USA

Witting, H., Institut für Mathematische Stochastik, Universität Freiburg, Hermann-Herder-Str. 10, 7800 Freiburg van Zuijlen, M.C.A., Department of Mathematics, Catholic University Nijmegen, Toernooiveld, Nijmegen, The Netherlands

van Zwet, W.R., Institute of Applied Mathematics and Computer Science, University of Leiden, Wassenaarseweg 80, 2300 RA Leiden, The Netherlands

Berichterstatter: E. Kremer (Hamburg)

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