MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 47/1979

Geometrie der Banachräume

11.11. bis 17.11.1979

Die Tagung fand unter Leitung der Herren Charles Stegall (Linz) und Ehrhard Behrends (Berlin) statt. Die Vorträge behandelten verschiedene Problemkreise aus der isometrischen und isomorphen Banachraumtheorie und deren Anwendungen, insbesondere die lokale Theorie der Banachräume, Räume vektorwertiger Funktionen und Integraloperatoren.

Vortragsauszüge

T.B. ANDERSEN

Relative width, extension constants and and characterization of simplexes.

Let Q be a closed convex subset of a compact convex set K of bounded relative width with bound N. Then each real continuous affine function on Q can be extended to a continuous affine function on K and the extension constant is less than or equal to $2N-1$.

This is used to give a characterization of metrizable compact Choquet simplexes: A metrizable compact convex set is a simplex if and only if K has the positive bounded affine extension property.

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M. CAMBERN

Problems on isometries and isomorphisms in function spaces.

Some classical results on isometries of function spaces, e.g., the Banach-Stone theorem for $C(X)$, or Lamperti's theorem for L^p-spaces, can be reviewed as consisting of two parts:

(1) a part relating the structure (topological, or measure theoretic) of the sets upon which the functions are defined, and

(2) a descriptive part, in which the relation obtained in (1) is used to determine the isometry.

Moreover, it has in some instances been shown that conclusions of type (1) may hold for isomorphisms more general than isometries.

Here, some quite recent, and yet unpublished results of this nature for spaces of differentiable functions are discussed. Both the isometric case and the isomorphic case are considered.

P.G. CASAZZA

The complemented ideals in the disk algebra.

Let A denote the disk algebra. If K is a compact subset of the unit circle with Lebesgue measure zero, let $A_K = \{f \in A \mid f|_K \equiv o\}$. The most general closed ideals in A have the form $I_F = \{F f \mid f \in A_K\}$ where F is an inner function continuous on the complement of K in the closed disk. A sequence of points $\{z_n\}$ in the unit disk will be called a Carleson sequence if the measure which assigns, for each n, a mass of $1-|z_n|^2$ to the point z_n (taking multiplicities into account) is a Carleson measure. We have the following theorem of Casazza, Pengra, and Sundberg:

Theorem: A closed ideal I_F in the disk algebra is complemented if and only if F ist a Blaschke product whose zeroes form a Carleson sequence.

This theorem gives information about the following problems:

(1) What is the norm of the best projection onto an ideal in A ?

(2) Does H^∞ have the approximation property?

(3) What are the (complemented) ideals in H^∞ ?

(4) Is A primary?

(5) Can every inner function be uniformly approximated by interpolating Blaschke products?

I.B. CONWAY

The behavior of the spectrum under small perturbations.

For a separable Hilbert space H, consider the spectrum as a function $\sigma : B(H) \to \Sigma$, where Σ is the collection of compact subsets of $\mathbb{C}$ with the Hausdorff metric. Where is σ continuous?

For A in $B(H)$, let $P_\pm(A) = \{\lambda \in \mathbb{C} : A - \lambda$ is semi-Fredholm and $\operatorname{ind}(A-\lambda) \neq 0\}$. Let $\sigma_{\ell e}(A)$ (respectively, $\sigma_{re}(A)$) denote the left (resp., right) spectrum of A in the quotient algebra $B(H)/B_0(H)$, where $B_0(H)$ is the ideal of compact operators in $B(H)$. Let $\sigma_p^0(A)$ = the set of isolated eigenvalues of A having finite multiplicity.

Theorem: σ is continuous at A if, and only if, for each λ in $\sigma(A)\setminus P_\pm(A)^-$ and for each $\varepsilon > 0$, the ball $B(\lambda;\varepsilon)$ contains a component of $[\sigma_{\ell e}(A) \cap \sigma_{re}(A)] \cup \sigma_p^0(A)$.

Similar characterizations of the Weyl spectrum, the essential spectrum, $\sigma_{\ell e}$, σ_{re}, σ_ℓ, σ_r, $\sigma_{\ell e} \cap \sigma_{re}$, and $\sigma_e \cap \sigma_r$ can be obtained.

W.J. DAVIS

The diameter of the space of n-dimensional spaces.

Let E and F be n-dimensional Banach spaces. The Banach-Mazur distance between E and F is defined by $d(E,F) = \inf\{\|T\|\ \|T^{-1}\|\ T:E \to F\}$. If $\underline{F}_n$ denotes the space of n-dimensional spaces, its Banach-Mazur diameter is unknown. All that is known is that $\sqrt{n} \le \operatorname{diam} \underline{F}_n \le n$. There has been some progress recently on distances between spaces with assumptions on their structure. For example, symmetric spaces, spaces with 1-unconditional bases, etc. Here, we give some positive results with completely different structural assumptions. The starting point follows directly an argument due to N. Tomczak-

Jaegermann:
Suppose $\dim F = \dim E = n$ and the ball of E has $(n)^{\alpha}$ extreme points and the ball of F^* has $(n)^{\alpha}$ extreme points. Then $d(E,F) \le C\sqrt{n}\log n$, where C depends only on α. Another result, similar in spirit, is: Suppose E has cotype 2 and the ball of F^* has n^{α} extreme points. Then $d(E,F) \le C\sqrt{n}\log n^2$.

F. DELBAEN

Dunford-Pettis property of spaces of analytic functions.

If X is a Banach space then L^1_X denotes the Banach space of Bochner integrable functions with values in X. We will show that L^1_A has the Dunford-Pettis property where A in the disc algebra, i.e. A in the Banach space of those functions which are continuous on the circle $\pi = \{z \mid |z| = 1\}$ and have vanishing Fourier coefficients for $n < o$. It is well known that the dual of L^1_X is the space of bounded measurable functions defined on $[0,1]$ with values in X^* and weak* measurable. (We suppose X separable). It follows that $(L^1_A) = L^{\infty}_{A^*}(w^*)$. It is also known that every functional $\varphi: A \to C$ has a unique norm preserving extension $\mu: C(\pi) \to \mathbb{C}$ and if $\varphi_n \to o$ weakly in A^* then the corresponding μ_n form a relatively weakly compact set in the space of measures $M(\pi) = C(\pi)^*$. It can be shown that μ_n can be modified in a descriptive way such that $\nu_n = \mu_n - \lambda_n$ tends to zero weakly in $M(\pi)$ and $\mu_n|_A = \varphi_n$. If given a sequence $f_n: [0,1] \to A^*$ such that $f_n \in (L^1_A)^*$ and $f_n \to o$ weakly then one can find (using the above descriptive procedure) a sequence $\tilde{f}_n = [0,1] \to M[\pi]$ such that $\tilde{f}_n$ is w*-measurable and for almost all $t: \tilde{f}_n(t)|_A = f_n(t)$ and $\tilde{f}_n(t) \to o$ weakly. To prove that a Banach space X has the Dunford-Pettis property it is sufficient to observe that $x^*_n(x_n) \to o$ as soon as $x_n \to o$ weakly in X and $x^*_n \to o$ weakly in X^*. Jean Bourgain proved, using Riesz products in a very clever way, that if $g_n \in L^1_A$ tends to zero weakly and $\tilde{f}_n$ is a uniformely bounded sequence as above then $\int < g_n(t), \tilde{f}_n(t) > dt \to o$. In particular L^1_C has the Dunford-Pettis property. The same result also gives that L^1_A has the Dunford-Pettis property. Unfortunately one could not prove that

$(L^1_A)^*$ also has the D.P. property.

J. DIESTEL

Some open problems related to the Dunford-Pettis property.
Recall that a Banach space X has the Dunford-Pettis property whenever given weakly null sequences (x_n) and (x^*_n) in X and X^*, respectively, $\lim_n x^*_n(x_n) = o$. The problems discussed centered around heredity properties of Dunford-Pettis spaces. Two problems were discussed in same detail, namely, which spaces are hereditarily Dunford-Pettis and for which subspaces Y of a space X having the Dunford-Pettis property can one conclude that X/Y has the Dunford-Pettis property. Indicative of the results known about the first of these problems we cite the

Theorem: if X is hereditarily Dunford-Pettis then X enjoys the weak Banach-Saks property.

From this it follows (with a strong helping hand from Schreier and Bessaga-Pelczynski) that a $C(K)$ space is hereditarily Dunford-Pettis if and only if K is dispersed and the ω^{th} derived set $K^{(\omega)}$ of K is empty.

Problem: classify the separable hereditarily Dunford-Pettis spaces whose duals are separable.

The discussion regarding the second problem revolved about the following simple

Theorem: If X has the Dunford-Pettis property and $Y \subset X$ has no subspace isomorphic to ℓ_1 then X/Y has the Dunford-Pettis property.

Problem: If Y is a dual subspace of L_1 does L_1/Y have the Dunford-Pettis property?

R. EVANS

A generalized multiplier theorem.

Motivated by the function module representation of a Banach space we consider the relationship between continuous embeddings of $C(K)$ [K compact] into $B(X)$ [bounded operators on a Banach space X] and ways of representing X on K. If $f \longmapsto T_f$, $C(K) \longrightarrow B(X)$ is such an embedding then $\mathrm{supp}(x) := \bigcap \{f^{-1}(o) : T_f\, x = o\}$ defines a mapping of X into the lattice of closed subsets of K possessing certain properties of the support of continuous functions, namely:

i) $\mathrm{supp}\, x = \emptyset \Leftrightarrow x = o$, ii) $\mathrm{supp}\, \lambda x = \mathrm{supp}\, x$, $\lambda \neq o$,

iii) $\mathrm{supp}\, (\Sigma\, x_i) \subseteq [\bigcup\, \mathrm{supp}\, x_i]$ for all convergent series

iv) $\mathrm{supp}\, x \subseteq G_1 \cup G_2$ (open) $\Rightarrow x = x_1 + x_2$, $\mathrm{supp}\, x_1 \subseteq G_1$, $\mathrm{supp}\, x_2 \subseteq G_2$ and $\|x_1\|, \|x_2\| \leq M\|x\|$ for a global canstant M[= norm of $f \longmapsto T_f$].

We prove that the converse is also true, any such representation by supports satisfying i) - iv) induces a continuous embedding $C(K) \longrightarrow B(X)$ which is characterised by the identity $T_f\, x = \lambda x$ whenever $f \equiv \lambda$ on $\mathrm{supp}\, x$. In special cases this reduces to known multiplier theorems [e.g. Dauns-Hofmann, Elliott].

G. GIERZ

C(X)-modules and duality.

Let $p : E \to X$ be a bundle of Banach spaces over a compact base space X. With $\Gamma(p)$ we denote the Banach space of all global continuous sections of p and with Mod $\Gamma(p)$ we denote the topological vector space of all continuous $C(X)$-module homomorphisms from $\Gamma(p)$ into $C(X)$, equipped with the topology of pointwise convergence. Note that Mod $\Gamma(p)$ is a $C(X)$-module itself, where for $f \in C(X)$ and $T \in$ Mod $\Gamma(p)$ the product fT is defined by $(fT)(\sigma) = T(f\sigma)$ for all $\sigma \in \Gamma(p)$.

Theorem: Let $\Gamma(p)$ be separable. Then the following statements are equivalent:

1. norm: $E \to \mathbb{R}$ is continuous (i.e. for all $\sigma \in \Gamma(p)$ the mapping $x \mapsto \|\sigma(x)\| : X \to \mathbb{R}$ is continuous).
2. Mod $\Gamma(p)$ is "large" in the following sense: For all $x \in X$ and for all φ in the dual of the stalk over x there is

a $T \in \mathrm{Mod}\,\Gamma(p)$ such that $\|T\| = \|\varphi\|$ and $T(\sigma)(x) = \varphi(\sigma(x))$ for all $\sigma \in \Gamma(p)$.

These conditions imply: $\sigma \to \hat{\sigma}: \Gamma(p) \to \mathrm{Mod}\,\mathrm{Mod}\,\Gamma(p)$ is a bijection, where $\hat{\sigma}(T) = T(\sigma)$. This theorem is used to give a description of the topological dual $\Gamma(p)'$ of $\Gamma(p)$ as a certain tensor product over $C(X)$ of $\mathrm{Mod}\,\Gamma(p)$ and $M(X)$.

Y. GORDON

Random factorization of operators between Banach spaces.

Let X,Y and Z be Banach spaces, and T be a given finite - rank operator from X to Z .

We are interested in finding best estimates for the factorization constant $\inf\{\|A\|\,\|B\| ; T = BA , A \in L(X,Y) , B \in L(Y,Z)\}$.

In the particular case when $X = Z = \ell_2^n$ and $T=I$ (=identity), this problem was considered by various authors, among them Figiel-Lindenstrauss-Milman, Lewis, and Figiel-Jaegerman. Using Dvoretzky's theorem it was proved for example that every K-convex space Y contains uniformly complemented copies of ℓ_2^n . We shall present a new method, using probability, which enables us to get the known results in the cases of the factorization of $I:\ell_2^n \to Y \to \ell_2^n$. Moreover, our method does not use Dvoretzky's theorem and applies, with exact estimates in many instances, to general Banach spaces X,Y and Z .

P. GREIM

Isometries of $L^p(\mu,V)$.

We characterize the linear isometries of $L^p(\mu,V)$ onto itself, where μ is an arbitrary positive measure and V a (real or complex) Banach space.

The characterization is given for all separable V with trivial L^p-structure, thus generalizing results of Cambern and Sourour.

In the general separable case the influence of the L^p-structure of V on the isometries is shown quantitatively.

In the Hilbert space case we can drop separability.

The appropriate notion seems to be L^p-structure. In fact, the above results are immediate consequences of the determination of the L^p-structure of $L^p(\mu,V)$.

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As a very convenient tool we use Evans' integral module representation of arbitrary Banach spaces.
Some Banach-Stone theorems are given.

K. KEIMEL

Bundles of Banach spaces and function modules.

A brief discussion of the following notions and their interrelation was presented: upper semicontinuous function modules in the sense of Cunningham, vector fibrations in the sense of Nachbin, bundles of Banach spaces, sheaves with values in the category of Banach spaces, Banach space objects in topoi. The relevance of these notions for functional representations of C^*-algebras and Banach spaces was indicated. Details can be found in the papers "Sheaf theoretic concepts in analysis: bundles and sheaves of Banach spaces" by K.H. Hofmann and K. Keimel and "Banach spaces in categories of sheaves" by C.W. Burden und C.J. Mulvey which appear in a Proceedings volume "Applications of sheaves", Springer Lecture Notes Math. 735 (1969).

P. KENDEROV

Dense norm continuity of weakly continuous mappings.

Let X be a compact or complete metric space and E be a normed space. Suppose $T:X \longrightarrow (E, \text{weak})$ ist an upper semi-continuous multivalued mapping with weakly compact and convex images. Then there exists a dense G_σ - subset A of X at every point x_o of which the following continuity property is fulfilled:

(cp) | for every $t > 0$ there exists on open $U \subset X$, $U \ni x_o$, s.t. whatever $x_1, x_2 \in U$, $\inf\{\| e_1 - e_2 \| : e_i \in Tx_i, i=1,2\} \leq t$.

The corresponding result for mappings $T:X \longrightarrow (E^*, \text{weak}^*)$ is also true if every separable subspace of E has a separable dual.

H. KÖNIG

A limit case of the Hausdorff-Young inequality.

A special case of eigenvalue estimates for $(p,2)$-summing operators is the following modified form of the Hausdorff-Young inequality:

Let $p > 2$, $f \in L_{p',\infty}[0,\pi]$. Then $\hat{f} \in l_{p,\infty}$. (Here $L_{p',\infty}$ and $l_{p,\infty}$ are the Lorentz spaces and p' ist the conjugate index to p). For $p=2$, the eigenvalue estimates do not work and yield only the weaker form

$$\| (\hat{f}(j))_{|j|\leq n} \|_{l_{2,\infty}} \leq c\,(\ln(n+1))^{1/2}\, \|f\|_{L_{2,\infty}} \qquad (n\in\mathbb{N}) \qquad (1)$$

Therefore the question arises whether $f \in L_{2,\infty}$ implies $\hat{f} \in l_{2,\infty}$. It is shown that the answer to this is negative and that the $(\ln(n+1))^{1/2}$-order of growth in (1) is optimal in general.

H.E. LACEY

Local structure in Banach space which are closed under ultrapowers.

If B is a class of finite dimensional Banach spaces, I a non-empty index set, and U a free ultrafilter on I, then it is well-known that whenever the ultrapower X^I/U has local structure w.r.t. B, then so does X. It is shown that the converse is always true when U is countably complete and necessary and sufficient conditions for the converse to be true are given when U is not countably complete. In particular, it is true when X has local unconditional structure.

A. LIMA

Banach spaces with the 3.2. intersection property.

Suppose X is a real Banach space with $\dim X < \infty$. X has the 4.2. intersection property iff X is an L_∞-sum of one-dim spaces and X has the 4.3. intersection property iff X is an L_∞-sum of one- and two-dim spaces. Moreover a recent result by Hansen and Lima shows that X has the 3.2. intersection property iff X can by obtained by forming L_1- and L_∞-sums of the real line. This last structure theorem is used to show that if $B(0,1)$ has finitely many extreme points, then (1) and (2) below are equivalent:

(1) $T \in \operatorname{ext} B_{L(X,X)}(0,1)$, $x \in \operatorname{ext} B(0,1) \Rightarrow Tx \in \operatorname{ext} B(0,1)$

(2) $X = (l_1^m \oplus l_\infty^3 \oplus \ldots \oplus l_\infty^3)_{L_1}$ or

$X = (l_\infty^m \oplus l_1^3 \oplus \ldots \oplus l_1^3)_{L_\infty}$.

R.H. LOHMAN

Generalized James spaces.

Let (x_i) be a normalized symmetric basis of a Banach space X, equipped with its symmetric norm. $J(x_i)$ is the vector space of all scalar sequences $\alpha = (\alpha_k)$, converging to zero, for which

$M(\alpha) = \sup \| \sum_{i=1}^{n-1} (\alpha_{p_{i+1}} - \alpha_{p_i}) x_i \| < \infty$, where the supremum is taken

over all increasing sequences $p_1 < \ldots < p_n$ of positive integers. Then $(J(x_i), M)$ is a Banach space whose properties will be discussed. When $(X, (x_i))$ is the dual of Tsirelson's space or Altshuler's space, both of which are reflexive and neither of which contain a copy of c_0 or ℓ_p, $J(x_i)$ is quasi-reflexive of order one and contains no copy of c_0 or ℓ_p. This also gives an example of a Banach space with a nearly perfectly homogeneous basis that does not contain c_0 or ℓ_p.

E.R. LORCH

On subalgebras of ℓ^∞.

Any subalgebra of ℓ^∞ that contains all finite sequences is isometrically isomorphic to an algebra of continuous functions $C(\hat{N})$ where $\hat{N}$ is a compactification of a discrete denumerable set N. In such an algebra, c_0 is the ideal of all functions which vanish on $\check{N} = \hat{N} - N$. It is shown that for such an algebra A one has $A = c_0 \oplus B$ where B is isometrically isomorphic to $C(\check{N})$ iff $\check{N}$ is a retract of $\hat{N}$. Since in particular, any compact metric space may be denumerably extended to the form $\hat{N} = \check{N} \cup N$ (where $\check{N}$ is the original metric space) and since $\check{N}$ is a retract of $\hat{N}$, the result applies to metric algebras $A = C(\hat{N})$.
If $\hat{G}$ represents the group of homeomorphisms of $\hat{N}$ and if $\hat{\Phi} \in \hat{G}$, then $\hat{\Phi}(N) = N$ and $\hat{\Phi}(\check{N}) = \check{N}$. If $\Phi = \hat{\Phi}/N$, then $\hat{\Phi}$ is completely determined by Φ since N is dense in $\hat{N}$. Thus $\hat{G}$ is isomorphic to a group of permutations of $\mathbb{N} = \{1,2,\ldots\}$. It can be shown that this group is complete (all automorphisms are inner). If A is metric, then any homeomorphism of $\check{N}$ can be extended to $\hat{N}$. Thus the group of homeomorphisms of $\check{N}$ (and hence of every compact metric space) is isomorphic to the factor group of G by the invariant subgroup of all homeomorphisms which leave $\check{N}$ pointwise fixed.

A fundamental problem of summability theory is: Given a positive linear functional F with associated mass μ which is concentrated in $\check{N}$ (i.e. $\mu(N) = 0$) , $\mu(\check{N}) = 1$. Can F be weak* approximated by a sequence (F_i) where F_i is positive and has associated mass μ_i concentrated on N , $\mu_i(N) = 1$. For metric algebras this is always possible.

W. LUSKY

On primary simples spaces.

We construct an example of a separable simplex space Z with non separable dual which is primary but which is not isomorphic to $C(\Delta)$, Δ the Cantor set, or to $A(S_p)$, the Paulsen simplex space. We give a condition for a class of simplex spaces $A(S)$, containing $C(\Delta)$ and $A(S_p)$, such that for any unconditional Schauder decomposition $A(S) = \sum_{n=1}^{\infty} \oplus X_n$ $A(S)$ is isomorphic to one of the factors X_n . Now, Z is an unconditional Schauder decomposition of Banach spaces C_n which are isomorphic to $C(\Delta)$. On the other hand the C_n are chosen in such a way that Z contains as a complemented subspace an example of a simplex space, due to Benyamini and Lindenstrauss, which is not isomorphic to any complemented subspace of any $C(K)$.

P. MANKIEWICZ

On the problem of isomeries.

The problem whether every surjective isometry between two linear metric spaces is affine has not been solved yet. The following results have be presented:

(i) if X is a l.c.t.v.s. with the strong Krein-Milman property then every equicontinuous group of homeomorphisms of X which contains all translations and "minus identity" consists of affine mappings only,

(ii) if X is a metrizable l.c.t.v.s. with the strong Krein-Milman property then for every translation invariant metric d on X inducing the topology of X , every isometry from X,d onto another linear metric space is affine.

K. MUSIAL

A characterization of the weak Radon-Nikodym property in terms of the Lebesgue measure.

It is shown that if a Banach space has the weak Radon-Nikodym property with respect to the Lebesgue measure then it has the property with respect to all finite complete measure spaces.

S. NEGREPONTIS

A non-linear extension of the Amir-Lindenstrauss method.

The results have been obtained jointly with A. Tsarpalias. The main theorem is the following non-linear version of the Amir-Lindenstrauss result.

Theorem: Let X be a Banach space, K a weakly compact subset of X, whose closed linear hull is X. Let Y be a Banach space, and a linear operator $T : X^* \to Y^*$ continuous with respect to the norm and weakly* continuous. Then there is a set Γ and a function $s : T(B_{X^*}) \to c_o(\Gamma)$ weakly*-weakly continuous, one-to-one such that

$$\| sT(x^*) \| \leq \sup_{x \in K} |x^*(x)| \quad \text{for} \quad x^* \in B_{X^*}$$

(where B_{X^*} denotes the unit ball of X^*).

A.J. PACH

Recent results on girth and flatness.

Let X be a Banach space and S_X its unit sphere. The inner metric δ on S_X is defined by taking for $\delta(x,y)$ the infimum of the lengths of all curves in S_X connecting x and y. We define $m(X) = \inf\{\delta(x,-x) : x \in S_X\}$. Then $m(X) \geq 2$ and $m(X) > 2$ iff X is superreflexive (i.e. iff X has an equivalent uniformly convex norm) (James-Schäffer 1972). If $\delta(x,-x) = 2$, then x is called a flat spot. If there is a curve in S_X connecting x and $-x$ and having length 2, then this is called a girth curve, and X is called flat. If X is the closed span of a girth curve, X is called completely flat.

Theorem 1: (Pach 1978) If $m(X) = 2$, then every $x \neq o$ is a flat spot for some equivalent norm.

Theorem 2: (van Dulst-Pach 1979). X is completely flat iff X is the completion of $L^1[0,1]$ in a norm satisfying certain specific conditions.

A. PELCZYNSKI

Factorization theorems.

Among other facts it is shown that there exists an n-dimensional space K such that every factorization of the identity operator $i : L_n^\infty \to L_n^1$ through K has norm $\geq c \sqrt{n}$ where c is a universal constant. Here L_n^1 denotes the n-dimensional vector space with the norm $\|x\|_1 = \frac{1}{n} \sum_{j=1}^{n} |x(j)|$ for $x = (x(1), x(2), \ldots, x(n))$.

This result improves a result due to C. Schütt.

N. POPA

Projizierbare Untervektorräume und Untervektorverbände der nicht-lokal-konvexen Lorentzräume.

Es wurde das Problem der Existenz eines Untervektorraumes E in $d(w,\beta)$ $(o < \beta < 1)$ behandelt, für den E projizierbar und isomorph zu ℓ_p ist.

Satz: Sei $p = \frac{1}{k}$, $1 < k \in \mathbb{N}$ und sei $w_1^k < \frac{1}{n} \left(\sum_{i=1}^{n} w_i \right)^k$ für alle $n > 1$. Wenn $E \subset d(w,\beta)$ ein Untervektorverband ist so daß eine positive und kontraktive Projektion $P : d(w,\beta) \to E$ existiert, dann ist E ein Ordnungsideal von $d(w,\beta)$, folglich ist $E \not\approx \ell_p$.

W. RUESS

Compactness and weak compactness in spaces of operators.

Starting point is the following refinement of Schauder's Theorem on compact operators:

Given Banach spaces X and Y and $u \in L(X,Y)$, the following are equivalent:

(i) $u \in K(X,Y)$ (ii) $u^* \in K(Y^*,X^*)$ (iii) $u^* \in K((Y^*,bw^*),X^*)$

(Here $K((Y^*,bw^*),X^*)$ denotes the space of compact linear operators from Y^*, endowed with the bounded weak*-topology, into X^*, so that (iii) means, that u^* not only sends B_{Y^*} into something relatively compact in X^*, but even a bw*-zero neighbourhood.)

The object of the talk is to use this result together with a corresponding new representation of $K(X,Y)$ and $K^*(X,Y)$ for deriving unified proofs and extensions of results on

a) weak compactness in $K(X,Y)$ (Brace/Friend, Kalton,...)

b) compactness in $K(X,Y)$ (Holub) and

c) the dual of $K(X,Y)$ (Feder/Saphar) and reflexivity of $L(X,Y)$ (Ruckle, Holub, Kalton,...) .

W. SCHACHERMEYER

Representation of integral operators as sums of Carleman operators and orderbounded operators.

Let $(X,\underline{X},\mu)$ and $(Y,\underline{Y},\nu)$ be finite measure spaces and k a scalar valued measurable function on $X \times Y$. We say that k defines an integral operator $\mathrm{Int}(k)$ from $L^q(\nu)$ to $L^p(\mu)$, if

(1) for $g \in L^q(\nu)$ $\quad g(\cdot).K(x,\cdot) \in L^1(\nu)$

(2) for $g \in L^q(\nu)$ $\quad \mathrm{Int}(g)(x) = \int_Y g(y)\, K(x,y)d\nu(y) \in L^p(\mu)$.

$\mathrm{Int}(k)$ is called a Carleman-operator if in addition for μ-a.e. x $k(x,\cdot) \in L^{p'}(\nu)$, where $\frac{1}{p'} + \frac{1}{p} = 1$, and $\mathrm{Int}(k)$ is called orderbounded if it transforms orderbounded into orderbounded sets. Using theorems of L. Weiss and E. Nikishin we prove:

Theorem: Let $1 \leq q \leq \infty$, $q \geq p \geq 0$ and $\mathrm{Int}(k)$ an integral operator from $L^q(\nu)$ to $L^p(\mu)$. Then we may write k as $k^c + k^o$, such that $\mathrm{Int}(k^c)$ is a Carleman operator from $L^q(\nu)$ to $L^p(\mu)$ and $\mathrm{Int}(k^o)$ is orderbounded from $L^q(\nu)$ to $L^q(\mu)$ (hence in particular from $L^q(\nu)$ to $L^\infty(\mu)$).

If $q > 1$ we may choose this representation such that $\mathrm{Int}\,|k^o|$ is a compact operator from $L^q(\nu)$ to $L^q(\mu)$ of arbitrarily small norm.

By some examples we show that in the case $q < p$ the theorem need not to hold and also that for $q = 1$ the second part of the theorem need not to hold.

C. SCHÜTT

Projection constants of symmetrie Banach spaces.

The projection constant of an n-dimensional Banach space E is given by

$$\lambda(E) = \inf\{\|P\| \mid P \in L(\ell^\infty, E),\ P \text{ is projection}\}$$

and the isomorphic distance between two n-dimensional Banach spaces by

$$d(E,F) = \inf\{\|I\| \, \|I^{-1}\| \mid I \in L(E,F),\ I \text{ is isomorphism}\} .$$

An outline for the proof of the inequality

$$d(E,\ell_n^\infty) \le c(\log_2 \lambda(E))^{7/2} \lambda(E)$$

for symmetric spaces is given. It is pointed out that $\lambda(E)$ and $d(E,\ell_n^\infty)$ are up to some power of $\log_2 \lambda(E)$ proportional to

$$\min_{1\le\ell\le n} \max_{\frac{n}{\ell}\le k\le n} \sqrt{\frac{n}{k\ell}} \left\| \sum_{i=1}^{k} e_i \right\| \left\| \sum_{i=1}^{\ell} e_i^* \right\|$$

where $\{e_i\}_{i=1}^n$ denotes a symmetric basis of E and $\{e_i^*\}_{i=1}^n$ its dual basis. Relationships to the hypergeometric distribution are observed.

A.R. SOUROUR

Isometries of certain spaces of Hilbert space operators.

Let I be a symmetrie norm ideal of operators on Hilbert space other than the Hilbert Schmidt class.

Theorem: A linear map $\tau: I \to I$ is an isometry of I onto I if and only if there exist unitary operators U and V such that $\tau(X) = UXV$ or $\tau(X) = UX^tV$, where X^t is the transpose of X relative to a fixed orthonormal basis.

The proof proceeds by first showing that a linear map η on I is hermitian (i.e., the numerical range of η is real, or equivalenty $i\eta$ generates a one-parameter group of isometries) if and only if there exist self-adjoint operators A and B such that $\eta(X) = AX + XB$. Next, we use the fact that $\tau\eta\tau^{-1}$ must be hermitian whenever η is hermitian to show that τ is the product of the left multiplication by a unitary and a map α which lifts to a $*$-automorphism or a $*$-antiautomorphism of $B(H)$.

N. TOMCZAK-JAEGERMANN

Computing 2-summing norm with few vectors.

For any 2-summing operator $u : X \longrightarrow Y$ (X,Y Banach spaces) we define the sequence of ideal norms of u , which converges to $\pi_2(u)$: for every positive integer n $\pi_2^{(n)}(u)$ is the smallest number C such that inequality

$$\left(\Sigma \| ux_j \|^2\right)^{1/2} \leq C \cdot \sup_{\| x^* \|_{X^*} \leq 1} \left(\Sigma |(x^*,y_j)|^2\right)^{1/2}$$

holds for all sequences $x_1,\dots,x_n$ in X .

Of course $\lim_{n\to\infty} \pi_2^{(n)}(u) = \pi_2(u)$. In many situations which occur in the local theory of Banach spaces rather the norms $\pi_2^{(n)}(\cdot)$ than $\pi_2(\cdot)$ are used.

Theorem: If rank $u = n$, then $\pi_2(u) \leq 2\pi_2^{(n)}(u)$.

We also introduce similar gradations for the type 2 and cotype 2 constants of a Banach space X . For every positive number n , $K^{(2,n)}(X)$ and $K_{(2)}^{(n)}(X)$ are the smallest positive numbers such that the following inequalities hold

$$\frac{1}{K_{(2)}^{(n)}(X)} \left(\Sigma \| x_j \|^2\right)^{1/2} \leq \left(\int_0^1 \| \Sigma\, r_j(t)\, x_j \|^2 dt\right)^{1/2} \leq K^{(2,n)}(X) \cdot \left(\Sigma \| x_j \|^2\right)^{1/2} .$$

for all sequences $x_1,\dots,x_n$ in X . There $r_j(\cdot)$ denotes the j-th Rademacher function on $\langle 0,1\rangle$ $(j=1,2,\dots,n)$.

Theorem: Let X be a normed space, $\dim X = n$.

Then

$$K^{(2)}(X) \leq 2\, K^{(2;n)}(X)$$

$$K_{(2)}(X) \leq 2\, K_{(2)}^{(n)}(X) ,$$

where $K^{(2;n)}(X)$, $K_{(2)}^{(n)}(X)$ denote the type 2 and cotype 2 constants of X .

S. TROYANSKI

On equivalent norms in reflexive spaces.

An example is given of a reflexive Banach space which fails to have either an equivalent norm that is uniformly convex in every direction or equivalent norm that is uniformly differentiable in every direction.

J.J. UHL, Jr.

Weak compactness in $L_\infty(\mu,X)$.

Let X and Y be B-spaces and let (Ω,Σ,μ) be a finite measure space. If $T : Y \to L_\infty(\mu,X)$ is a weakly compact operator, then there exists a function $g : \Omega \to L(X,Y)$ such that

$$Ty(\cdot) = (\cdot)(y)$$

for all y in Y. If X has the Schur property, then g can be taken to be measurable for the operator norm. As a consequence of this and factorization, it is shown that if X has the Schur property, and $W \subseteq L_\infty(\mu,X)$ is weakly compact, then for each $\varepsilon > 0$ there is a set A in Σ with $\mu(\Omega\setminus A) < \varepsilon$ such that $\chi_A \cdot W$ is a norm compact subset of $L_\infty(\mu,X)$. (These results were obtained in joint work with Professor Kevin T. Andrews.)

L. WEIS

A change of density result with applications to integral operators.

Let $T : L_p(\Omega,\Sigma,\mu) \to L_p(\Omega,\Sigma,\mu)$ be an order - bounded operator, (Ω,Σ,μ) a σ-finite measure space. Then, after an appropriate change of density, T acts as a bounded operator in all L_q-spaces. More precisely: There exits a strictly positive g, $\int g \, d\mu = 1$, such that $\varphi_p T \varphi_p^{-1} : L_q(\nu) \to L_q(\nu)$ for all $1 \leq q \leq \infty$ where $d\nu = g\, d\mu$ and $\varphi_p(f) = f \cdot g^{-1/p}$ is an isometry from $L_p(\mu)$ onto $L_p(\nu)$.
We use this, to give simple proofs to recent characterizations of integral operators due to Bukhvalov and Schachermayer and a more direct proof to a result of Maurey, König, Johnson and Retherford:
$\int(\int|k(s,t)|^{p'} dt)^{p/p'} ds < \infty$ implies that the induced operator has p-summing eigenvalues for $p \geq 2$.
Another consequence: Every integral operator $T : L_p(\mu) \to L_p(\mu)$

has nice compact restrictions: for every $\varepsilon > 0$ there exists an $E \in \Sigma$ with $\mu(\Omega - E) \leq \varepsilon$ such that $P_E T$ is order - bounded and compact. This implies for example that every such integral operator factors through ℓ_p .

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