MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 49/1979

Nichtlineare Funktionalanalysis und partielle Differentialgleichungen

2.12. bis 9.12.1979

Die Tagung fand unter der Leitung der Herren Amann und Hess (beide Universität Zürich) statt. In insgesamt 35 Vorträgen wurde ein Ueberblick über die neuesten Entwicklungen auf diesem Teilgebiet der Mathematik gegeben, wobei sowohl konkrete Anwendungen auf nichtlineare partielle Differentialgleichungen als auch abstrakte Fragestellungen der nichtlinearen Funktionalanalysis behandelt wurden.

Unter anderem wurden Resultate aus den folgenden Spezialgebieten vorgestellt:
nichtlineare Evolutionsgleichungen, Bifurkationstheorie, Operatoren monotonen Typs, dynamische Systeme, topologische Methoden in unendlich dimensionalen Räumen etc.

Das Vortragsprogamm wurde durch anregende Diskussionen und persönliche Gespräche in harmonischer Atmosphäre ergänzt. Die vorbildliche Organisation des Instituts trug wesentlich zum Gelingen der Tagung bei.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge

C. BANDLE:

Error bounds for a class of nonlinear Dirichlet problems

Nonlinear problems of the type $\Delta u+f(u)=0$ in $D \subset \mathbb{R}^N$, $u=0$ on ∂D are considered, where f is a positive, monotone increasing convex function. Error bounds for any function smaller than the minimal solution are given in terms of the energy and the eigenvalue of the linearized problem. Symmetrization methods are then used to estimate those quantities.

N. BAZLEY:

Some Explicitly Resolvable Branching Problems

T. Küpper and the author consider a linear, self-adjoint operator B in a separable Hilbert space H, having an isolated eigenvalue λ_i with normalized eigenfunction u_i. The potential of B is given by $\psi(u)=\frac{1}{2}(Bu,u)$. Let f(x) be a real-valued, differentiable function with derivative $f'(x)=g(x)$ and defive a new potential $\psi(u)$ by $\psi(u)= f\{\frac{1}{2}(Bu,u)\}$. The solutions of grad $\psi(u)$ $= g\{\frac{1}{2}(Bu,u)\} Bu = \mu u$ are given by $u = \pm\alpha u_i$, $\alpha > o$, with norm α and eigenvalue $\mu_i=g(\frac{\alpha^2\lambda_i}{2})\lambda_i$.

If we consider the corresponding nonlinear Schrödinger equation, $-i\frac{\partial\varphi}{\partial t}= g\{(B\psi,\psi)/2\}B\psi, \psi(o)= \varphi$, the solutions are given by $\psi(t)=\sum_{j=i}^{\infty} \alpha_j e^{i\mu_j t} u_j$, where the μ_j's satisfy $\mu_j = g\{\frac{1}{2}\sum_{K=1}^{\infty} \lambda_K \alpha_K{}^2\}\lambda_j$. These considerations led G. Wake and the author to introduce a new and exactly solvable model for the thermal ignition. It is given by $-\Delta\varphi=\mu \exp(\varphi/1+\varepsilon|\varphi|)$ in G, $\varphi=o$ on ∂G, where φ denotes the maximum norm. An entirely different branching behaviour is exhibited in three regions: $\varepsilon=o$, $o<\varepsilon<\varepsilon_0$, and $\varepsilon>\varepsilon_0$.

F. BROWDER:

Strongly nonlinear parabolic problem

Let Ω be a bounded smoothly bounded domain in R^N, Q the cylinder $\Omega x[o,T]$ for T>o. We consider on Q the strongly nonlinear parabolic differential operator $\frac{\partial u}{\partial t}+A(u)+g(x,u)$ where $A(u) = \sum_{|\alpha|\leq m}(-1)^{|\alpha|}D^{\alpha}A_{\alpha}(x,t,u,\dots,D^m u)$ is a strongly elliptic operator on $W^{m,p}(\Omega)$ which satisfies the usual boundedness,ellipticity and coercivity condition in $W^{m,p}(\Omega)$. For the strongly nonlinear term g(x,u) we assume no growth condition but only the condition $g(x,o)\equiv 0$, g(x,r) non decreasing in r.

In joint work with Brézis (extending earlier joint results on strongly nonlinear parabolic equations, published in Proc.Nat.Acad.Sci. Jan 1979) we treat general variational inequalities for such operators. The basic result is the following: Let L be the operator on $L^2(G)$ which is the realization of $\frac{\partial}{\partial t}$ with domain D(L)= $\{v|v\varepsilon L^2(Q),\frac{\partial v}{\partial t}\varepsilon L^2(Q),\ v\varepsilon C(o,T,L^2(\Omega))$ with $v(o)=o\}$,

$X = L^p(o,T;W^{m,p}(\Omega)), p\geq 2$. Under the appropriate regularity and ellipticity condition on $A(u)$ and for C any closed convex subset of $W^{m,p}(\Omega)$, f an arbitrary element of X^* such that $o\in C$, there exists u in X with $u(t)\in C$ a.e. with $g(u)$ in $L^1(Q)$, $ug(u)$ in $L^1(Q)$ such that for every v in $D(L)\cap X\cap L^\infty(Q)$ and $v(t)\in C$ a.e. we have

$$\langle \frac{\partial v}{\partial t}, v-u\rangle + \langle A(u), v-u\rangle + \int_Q g(u)(v-u) \geq \langle f, v-u\rangle .$$

We may impose further conditions upon this solution in terms of the functional

$$\varphi(u) = \begin{cases} \int_Q G(u) & \text{if } u(t)\in C \text{ a.e.} \\ +\infty & \text{otherwise} \end{cases}$$

where $G(x,r) = \int_0^r g(x,s)ds$. Then for v in $D(L)\cap X$ with $\varphi(v) < +\infty$, we have

$$\langle \frac{\partial v}{\partial t}, v-u\rangle + \langle A(u), v-u\rangle + \varphi(v) - \varphi(u) \geq \langle f, v-u\rangle .$$

If A is strictly monotone, this last inequality implies that u is uniquely determined by f. An essential part of the proof is provieded by a new compactness lemma for solutions of variational inequalities which should have a broad range of applications in other types of boundary problems.

P. CLEMENT:

<u>On Abstract Volterra equations with completely positive kernels</u>

We consider the nonlinear problem

$$u(t) + (b*Au)(t) \ni u_o + b*g(t) \quad t>o$$

where A is a maximal monotone operator in a real Hilbert space H, $u_o \in \overline{D(A)}$, $g \in L^1(O,T,H)$ and b is completely positive i.e. satiesfies

$$\begin{cases} s(\lambda b)(t) \geq 0 \quad \forall \lambda > o \quad \forall t > 0 \\ r(\lambda b)(t) \geq 0 \quad \forall \lambda > o \quad \forall t > 0 \end{cases}$$

where s(b) is the unique solution of $u + b * u = 1$

and r(b) is the unique solution of $u + b * u = b$

In this work joint with J.A. Nohel we consider

1) characterization of completely positive kernels
2) existence of solutions
3) dependance upon the data u_o and g
4) positivity reserving properties
5) asymptotic behaviour of solutions.

We give an application for the problem of nonlinear heat flow with memory.

M. G. CRANDALL:

The initial-value problem

$$(1) \begin{cases} \frac{\partial u}{\partial t} + \sum_{i=1}^{N} \frac{\partial}{\partial x_i} f_i(u) = 0, \quad t > 0, \ x \varepsilon \mathbb{R}^N \\ u(x,0) = u_0(x) \end{cases}$$

where $f_i : \mathbb{R} \to \mathbb{R}$, has a unique weak solution $u \in C(([0,\infty], L^1(\mathbb{R}^N)) \cap L^\infty$ for each $u_o \varepsilon L^1(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$ which satiesfies the entropy condition of Vol'pert and Kružkov. For simplicity, we now set N=1. The difference approximation

$$(2) \quad v_j^{n+1} = v_j^n - \lambda \Delta_+ g(v_{j-p}, \ldots, v_{j+q}) = G(v_{j-p}^n, \ldots, v_{j+q+1}^n)$$

where $\lambda = \Delta t / \Delta x$, $\Delta_+ v_j = v_{j+1} - v_j$ is <u>consistent</u> with $u_t + f_1(u)_{x1} = o$,

$g(r,\dots,r)=f_1(r)$, it has conservation form, and it is monotone if G is nondecreasing in each argument. We explain joint results with A. Majda proving that consistent divergence form difference approximations converge to the entropy solution of (1).

Abstract results with L. Tartar showing that for integral-preserving operators, order preservation and L^1- nonexpansiveness are used in the proof. The L^∞ version of this result is explained and examples are given.

CH. C. FENSKE:

Periodic orbits to topological semiflows

A semiflow on a space X is a continuous mapping $\Phi: X \times [0,\infty) \to X$; $x,t \to \Phi_t x$ such that $\phi_0 = id$ and $\phi_{t+s} = \phi_t \phi_s$. A set $A \subset X$ is an attractor for ϕ if for each $x \in X$ and each neighbourhood U of A there is $t \geq 0$ with $\phi_t x \in U$. An attractor A is uniform if A possesses arbitrarily small invariant neighbourhoods. Assume that X is metric, that Φ has a compact attractor and that ϕ_T is locally compact for some $T \geq 0$ [these conditions are satisfied for the semiflow generated by the retarded differential equation $y'(t)=f(x(t-1))$], then Φ has a compact uniform attractor. Since periodic orbits of Φ must be contained in the attractor we now restrict ourselves to compact spaces. We investigate the behaviour of Φ in a neighbourhood of a periodic orbit and completely describe the set of periodic points in the case when there is a positive lower bound for the sets of periods and each value above this bound appears exactly once as the period of an orbit. Details are given in [Game theory and related topics, Amsterdam: North-Holland Publ. Comp. 1979, pp.157-164]

D.DE FIGUEIREDO:

<u>Double Resonance in Semilinear Elliptic Problems</u>

Let $A:D(A)\subset L^2(\Omega)\to L^2(\Omega)$ be a linear closed densely defined operator with a closed range $R(A)$ and such that its nullspace $N(A)=N(A^*)$, where A^* is the adjoint of A. Under these hypotheses we have $L^2(\Omega)=R(A)\oplus N(A)$. We also assume that $N(A)$ is finite dimensional and the restriction of A to $R(A)$ has a compact inverse. Let α be the largest positive constant such that $-\alpha^{-1}||Au||\leq(Au,u)$, for $u\in D(A)$.

Let $g:\Omega x R\to R$ be a Carathéodory function with linear growth. Suppose that the following limits exist:

1) $k_\pm(x)=\lim_{u\to\pm\infty}\frac{g(x,u)}{u}$

and suppose that $0\leq k_\pm(x)\leq\alpha$

Under these assumptions and a further hypothesis on A(in particular), if $A=A^*$) we prove the following result.

"The equation $Au+g(x,u)=h$ for a given $h\in L^2(\Omega)$, has a solution provided

2) $\int\limits_{v>o} k_+ v^2 + \int\limits_{v<o} k_- v^2 > 0 \quad \forall\ o \neq v \in N(A)$

and

3) $\int\limits_{v>o} (\alpha - k_+) v^2 + \int\limits_{v<o} (\alpha - k_-) v^2 > o \quad \forall\ 0 \neq v \in N(A+\alpha)$"

Extensions dropping the existence of the above limit are possible. Further if A is like $\Delta+\lambda_1$ it is best to replace 2) by a Landesman-Lazer type condition.

J. FREHSE:

<u>Nichtlineare Operatoren mit linearem Bildbereich</u>

Es wird gezeigt, dass Abbildungen $T:B\to B^*$ (B=reflexiver Banachraum) einen linearen Bildbereich mit endlicher Kodimension besitzen, wenn T die folgenden Bedingungen erfüllt:

(i) $T(o)=o$, (ii) T ist asymptotisch monoton,

(iii) T ist pseudo-monoton, (iv) T ist semi-koerzitiv,

(v) $(T(tw,v))$ ist differenzierbar bzgl. $t\in\mathbb{R}$ für alle w in einer dichten Teilmenge von B und alle $v\in B$, (vi) T ist stetig von der starken Topologie von B in die schwache von B*.

(vii) Ist für ein Paar $v,w\in B$ der Ausdruck $(T(w+tv),v)$ beschränkt in $t\in\mathbb{R}$, dann ist dieser Ausdruck konstant in t ("Polynombedingung").

F.E.Browder verallgemeinerte diesen Satz, was die Bedingung (iv) anbelangt, er erhielt die Linearität des Abschlusses von T(B) ohne Bedingung (iv), hierzu benötigt er die Monotonie anstelle der asymptotischen Monotonie sowie eine stärkere Differenzierbarkeitsforderung.

J. P. GOSSEZ:

Convergence of nonlinear elliptic operators with application to an implicit Signorini type problem

This talk is concerned with the following implicit Signorini type problem: find u in Ω (a bounded open set in $\mathbb{R}^N$, with boundary Γ) such that

$$(*)\quad Lu \equiv -\sum_{i=1}^{N} D^i(A_i(x,u,\nabla u)) + A_o(x,u,\nabla u) = f \text{ in } \Omega$$

$$\begin{cases} u \geq \psi(u) \text{ on } \Gamma, \\ \gamma_a u \geq 0 \text{ on } \Gamma, \\ \gamma_a u \cdot (u-\psi(u)) = 0 \text{ on } \Gamma, \end{cases}$$

where the obstacle function $\psi(u)$ is of the form

$$\psi(u) = h - \int_\Gamma \gamma_a u . \varphi$$

This problem, which has some physical origin, has been studied in the linear case by Joly, Mosco, Boccardo, Dolcetta, who have obtained the existence of a solution under an ellipticity assumption on L and either a sign or a smallness condition on φ. This is extended here to the nonlinear case under assumptions of the Leray-Lions type on L and some restrictions on φ as above. The main tool in the proof is a closure and compactness theorem for a sequence of nonlinear operators of the form(*). (joint work with M.G. Garroni).

J. HERNANDEZ:

Some existence results for reaction-diffusion systems with nonlinear boundary conditions.

We extend to the case of nonlinear boundary some known results concerning existence of solutions for parabolic systems and for the associated elliptic systems. We are able to give a simple proof of three results, by reducing the problem to a fixed point problem for a nonlinear compact operator which maps a closed bounded convex set K of a function space into itself. Then Schauder's fixed point theorem can be applied. The subset K is given by the existence of sub- and supersolutions of the corresponding problem. The same kind of methods can be used to prove that, under convenient assumptions, the elliptic problem has a unique solution and that this solution is stable in the Liapunov sense. Finally we give some examples of applications of our results to systems arising in combustion theory.

G. HETZER:

Time periodic, generalized solutions for a class of quasilinear, pseudoparabolic boundary value problems

Let $n \in \mathbb{N}, b>o, J=[o,b], \lambda \in \mathbb{R}, G$ be a nonempty bounded region in $\mathbb{R}^n$, and $g_j: G\times J\times\mathbb{R}\times\mathbb{R}^n \to \mathbb{R}$ be a function for o $\leq j \leq n$, which satiesfies Caratheodory's conditions and has sublinear growth. We consider the boundary value problem

$$\frac{\partial}{\partial t}u(x,t)-\Delta[u(x,t)+\frac{\partial}{\partial t}u(x,t)]-\lambda u(x,t)=g_o(x,t,u(x,t),\nabla u(x,t))$$

$$-\sum_{j=1}^{n}\frac{\partial}{\partial x_j}g_j(x,t,u(x,t),\nabla u(x,t)) \qquad x\in G, t\in J$$

$$u(x,o)=u(x,b) \qquad x\in G, u(x,t)=0 \qquad x\in\partial G, t\in J,$$

which is a typical example for a class of quasilinear, pseudoparabolic equations, for which the existence of generalized solutions for the time periodic Dirichlet problem can be established under some further hypotheses, if $\lambda\leq\lambda_1$, where λ_1 means the lowest eigenvalue of $-\Delta$ concerning the weak Dirichlet problem with $u|\partial G=0$.

H. HOFER:

A Multiplicity Result for a Class of Nonlinear Problems, with Applications to a Nonlinear Wave Equation

The author studies the existence of multiple solutions of the nonlinear equation

(*) $Ax=cx+Fx+h$

in a real separable Hilbert space, where A denotes a linear operator with $\dim(N(A))=\infty$ and $c>0$ an eigenvalue of A. Let $L=A-cI$. If the generalized inverse of A satisfies some compactness assumption and F has some properties similar to abstract Landesman & Lazer conditions, there exists a bounded closed set $\Omega\subset N(L)$ which is only depending on the asymptotic behavior of F, and for all $h_2\subset N(L)^{\perp}$ a bounded open set $\Omega_{h_2}\subset N(L)$ containing Ω, such that (*) is solvable for all $h=h_1+h_2$, $h_1\in cl(\Omega_{h_2})$

while for all $h_1 \varepsilon \Omega_{h_2} \setminus \Omega$ there exists at least two solutions. Moreover the range of $A-cI-F:D(A) \to H$ (H= Hilbert space) is closed. In a second part the author gives an application to a nonlinear wave equation.

B. KAWOHL:

Elliptic and parabolic equations with nonlinear mixed boudary conditions

The stationary and nonstationary solutions of the initial boundary value problem

$$u_t - Lu = f(t) \qquad \text{in } D \times [O,T]$$

$$u(o) = u_o \qquad \text{in } D$$

$$-\frac{\partial u}{\partial \nu} = \beta_l(u) \qquad \text{on } \Gamma_l \times [O,T],\ l=1,2$$

are investigated. Here L is a second order linear operator, $D \subset \mathbb{R}^2$ is bounded and may have corners, and β_l are maximal monotone mappings. In particular we give results on existence, uniqueness and regularity of the solutions. Ingredients of proof are the theory of monotone operators, convex functional-analysis, weighted Sobolev spaces and nonlinear evolution equations as well as ideas of H. Brezis.

K. KIRCHGAESSNER:

Necessary Conditions for Picard Iteration

Consider a real or complex B-space and $F \in C^1(X)$

satisfying $F(o)=o$, and denote by F^n the n-fold composition of F. Set

$$N=\{x\in X \;/\; F^n(x) \text{ converges to } o\}$$

It is well known that, if the spectral radius of $F'(o)$ (spr $F'(o)$) satisfies spr $F'(o)<1$ then N is a neighborhood of O. However, even if F is C^∞, N beeing a neighborhood of O does not imply that spr $F'(o)\leq 1$. Example is given. The following theorem is proved:
Let be F as above, moreover suppose $F'(o)$ is 1-separable (i.e. the spectrum is decomposed into two parts σ_1, σ_2, where $\sigma_2 \neq \emptyset$ and $\inf_{\sigma_2} |\lambda|>1$). Assume that one of the following conditions is satisfied

a) $\overset{\circ}{\overline{F^n(B_\rho)}} \neq o$ for all $n\in\mathbb{N}$ and all open balls B_ρ or,

b) F is analytic in X, $F'(x)$ is Fredholm for every $x\in X$ having index o and codim $F^{n}{}'(o)\leq n_o<\infty$.

Then N is of first Baire category.
In particular if X is finite dimensional, F analytic in X or, if X is infinte dimensional and $F=id+\phi$ where ϕ is compact and analytic in X then the conclusions of the theorem hold. Hence in these cases $N\subset U(\mathring{o})$ implies spr $F'(o) \leq 1$. Joint work with J. Scheurle.

A. KUFNER:

Ueber eine Klasse nichtlinearer Differentialgleichungen

Es wird gezeigt, wie man mit Hilfe der Leray-Lionschen

Theorie monotoner Operatoren Aussagen über die Existenz schwacher Lösungen von Randwertproblemen für Differentialgleichungen der Form

$$\sum_{\alpha\in E} (-1)^{|\alpha|} D^{\alpha} a_{\alpha}(x;\delta_E u(x)) = f(x) \quad ,x\in\Omega \subset \mathbb{R}^N$$

herleiten kann. Hier ist E eine Menge N-dimensionaler Multiindizes, $\delta_E u = \{D^{\beta}u \; ; \; \beta\varepsilon\ E\}$; die schwache Lösung sucht man in einem geeigneten nichtisotropen Sobolevschen Raum $W^{E,p}(\Omega)$.

V. LAKSHMIKANTHAM:

Comparison Theorems and Boundary Value Problems

Various comparison theorems relative to boundary value problems will be discussed and applied to prove existence and uniqueness results.

I. MASSABO :

On the solvability of nonlinear coupled equations on Banach spaces

Let X,Y be Banach spaces and $U \subset X \times Y$ be open and locally bounded over X. Let $\bar{f} : \bar{U} \to Y$ and $\bar{g} : \bar{U} \to X$ be compact maps. Let $f(x,y)=y-\bar{f}(x,y)$ and $g(x,y)=x-\bar{g}(x,y)$. Set $S=\{(x,y)\varepsilon X x Y: f(x,y)=0\}$, $S(x)=\{y\varepsilon Y: f(x,y)=0\}$, $S_x=\{x\} x S(x)$ and $D=\{x\varepsilon X: S_x \cap \partial U = \emptyset\}$.

Theorem A. On the above assumptions assume that

(i) there exists $r > o$ such that $B=B(o,r) \subset D$

(ii) $\deg_{LS}(f(o,-),U(o),o) \neq o$ where $U(x)=\{y \in Y:(x,y) \in U\}$.

Let $T(x) = g(x,S(x))$.

(iii) if, for $x \in \partial B$ $o \notin T(x)$ then there exists a continuous functional $x^* \in X^*$ such that $x^*(T(x)) \subset \mathbb{R}_+$ and $x^*(T(-x)) \subset \mathbb{R}-$.

Then there exists $x \in B$ such that $o \in T(x)$, or equivalently the system $\begin{cases} f(x,y)=o \\ g(x,y)=o \end{cases}$ has a solution.

From Theorem A it follows that the map $\bar{T}(x)=\bar{g}(x,S(x))$ has a fixed-point property with respect to the convex compact subsets of X. Then Theorem A can be viewed from one hand as an extension of the Liapunov-Schmidt method to the case of nonuniqueness of solutions of the equation $f(x,y)=0$ and on the other hand as an improvement to higher dimensions of the well-known connectness properties of the solution set S for one-parameter families of compact fields $\bar{f}(x.y)$.

A. M. MICHELETTI:

<u>About the differentiable operators with singularities between Banach spaces</u>

Let $\Phi:\Omega \to Y$ be a C^k map $(k \geq 2)$ between Banach spaces. We consider the singular set W of Φ (the set where the derivative of Φ is not invertible) and its image (which is called critical set). There exists a situation in which these sets W and $\Phi(W)$ are locally manifolds of codimension 1. Locally Φ behaves making a fold. This situation is described by Ambrosetti-Prodi. (Am. Mat. Pura e. Appl. (93) 1973) and A.P. gave the notion of "ordinary singular points", we prove that the situation is generic. We introduce a particular notion of negligible set: "the supermeager" set. (Finallly we obtain for smooth proper Fredholm maps of index o , that the critical values are ordinary critical values)(only images of ordinary singular points), exept a supermeager set.

V. MUSTONEN:

Pseudo-monotone mappings on Sobolev-Orlicz spaces

Let Ω be a domain (not necessarily bounded) in $\mathbb{R}^N$ with segment property and consider partial differential operator

$$Au(x)=\sum_{|\alpha|\leq m}(-1)^{|\alpha|}D^{\alpha}A_{\alpha}(x,u,Du,\dots,D^{m}u),x\in\Omega$$

The functions $(x,\eta,\zeta)\to A_{\alpha}(x,\eta,\zeta)$ are assumed to satisfy the Carathéodory condition, the growth conditions of Orlicz space type and the conditions

(B_2) $\sum_{|\alpha|=m}\{A_{\alpha}(x,\eta,\zeta)-A_{\alpha}(x,\eta,\zeta^*)\}(\zeta_{\alpha}-\zeta_{\alpha}^*)\geq o$

$\forall\ (x,\eta)\in\Omega x\mathbb{R}^{N1}$ and $\zeta\neq\zeta^*$ in $\mathbb{R}^{N2}$.

(B_3) $\sum_{|\alpha|\leq m}A_{\alpha}(x\xi)\xi_{\alpha}\geq-\sum_{|\alpha|\leq m}G_{\alpha}(x)\xi_{\alpha}-G(x),\forall(x,\xi)\in\Omega x\ \mathbb{R}^{N1}x\mathbb{R}^{N2}$

where $G_{\alpha}\in E_{\bar{M}}(\Omega)$ $\forall|\alpha|\leq m$ and $G\in L^1(\Omega)$.

We show that the mapping $T:D(T)\to W^{-m}L_{\bar{M}}(\Omega)$ is sequentially pseudomonotone, where

$$(T(u),v)=\int_\Omega \sum_{|\alpha|\leq m} A_\alpha(x,\xi(u))D^\alpha v \quad \forall\ u\in D(T), v\in W_0^m E_M(\Omega),$$

$W_0{}^m E_M(\Omega) \subset D\ (T) \subset W_0{}^m L_M\ (\Omega)$. This result generalizes a result by F.E. Browder to Sobolev-Orlicz spaces and extends a result by P. Gossez for unbounded domains. Finally it gives access to existence theorems for pseudo-monotone mappings. This result is based on joint work with R. Landes.

J.W. NEUBERGER:

Projection Methods for Systems of Nonlinear Partial Differential Equations

This paper concerns a family of iterative methods for solving boundary value problems for systems of quasi-linear partial differential equations. These methods involve subtracting an average of residuals from one approximation in order to obtain a subsequent approximation. Potential theoretic considerations are involved in the choice of mode of averaging. One such process is the following: $u \to u-PL(u)u$; where P is an orthogonal projection on a Hilbert space H and L is a strongly continuous function on H so that L(u) is an orthogonal projection on H for each u in H. Convergence results are given for this and other related processes. The theory is independent of type of partial differential equation and may be used in conjunction with finite differences or finite elements. Results are given from computer calculations on problems of transonic flow, gas dynamics and minimal surfaces. Application to the study of functional partial differential equations is indicated.

J. A. NOHEL:

Abstract nonlinear Volterra equations with applications

We study qualitative properties of solutions (global existence, uniqueness, continuous dependence, and asymptotic behavior) for the nonlinear equation

(V) $u(t) + \int_0^t b(t-s)Au(s)ds \ni f(t) \quad (t \geq 0)$, under

assumptions which permit applications to problems of nonlinear heat flow in materials with "memory". In (V) $b:[o,\infty) \to \mathbb{R}$, $f:[o,\infty) \to X$ (a real Banach space) are given sufficiently smooth functions, A is a given, possibly multivalued nonlinear operator on X which is assumed to be at least m-accretive, and u is the unknown function. For heat flow in materials with memory X=H (a real Hilbert space), $Au=\partial\varphi(u)$ where $\varphi : H \to (-\infty,\infty]$ is a proper, convex. l.s.c. function satisfying physically reasonable coercivity conditions. The analysis combines techniques of nonlinear evolution equations with those for Volterra equations.

A. PAZY:

Some remarks on nonlinear Ergodic theory

Nonlinear ergodic theory consists of results concerning the convergence of averages of iterates of certain non-linear operators. It follows the main lines of the linear theory which started with the results of J. von Neumann and G. Birkhoff in 1931.

The first nonlinear ergodic theorem is due to B. Baillon (1975). He proved that if T is a nonexpansive mapping, in a Hilbert space H, having a fixed point then for every $x \epsilon H$ the averages $\sigma_n(x)=n^{-1}\sum_{j=o}^{n-1} T^j x$ converge weakly to a fixed point of T. Soon after the appearence of this result it was generalized in many directions.

In this lecture we will review and analyse the results of nonlinear ergodic theory. We will show that all these results follow from two basic facts. The first one is a geometrical oberservation consisting of the characterization of the asymptotic center of a bounded sequences or nets in a Hilbert space. The second fact follows from the contraction property and states that averages of iterates of nonexpansive mappings can only converge weakly (or strongly) to fixed points of the original mapping.

J. P. PUEL:

A model of buckling for a thin elastoplastic plate

We present here a simple model for the buckling of a thin plate made out of an elastoplastic material. We have the following hypothesis: the relation between the flexion stress tensor and the curvature tensor is linear.

The relation between the tension stress tensor and the nonlinear (uncopmlete) plane strain tensor is nonlinear and corresponds to a Hencky law (a variational inequality associated with a convex set of pasticity C.) This model has been introduced by C. Do.

Writing the equilibrium equations for the displacement ζ orthogonal to the plate we get an equation of the form:

(1) $A\zeta - B(\lambda)\zeta + N(\lambda,\zeta) = f$ where λ is the loading parameter, A is a 4^{th} order operator, $B(\lambda)$ is a 2^{nd} order operator depending only continuously on λ and $N(\lambda,\zeta)$ is a nonlinear operator which is nondifferentiable in ζ.

We show that (1) is a variational problem, and we look for critical points of a functional for which (1) is the "Euler equation".

Under geometrical hypothesis on the convex set C we show an existence theorem for the problem (1) and also a bifurcation result for the homogeneous associated problem, showing that if λ_0 is the "first eigenvalue" of $A\zeta = B(\lambda)\zeta$, then (λ_0, o) is a bifurcation point.

P. RABINOWITZ:

Subharmonic solutions of Hamiltonean systems

Consider the Hamiltonean system of ordinary differential equations : $\dot{z}=\mathfrak{J}H_z(t,z)$ where H is T periodic in t. We seek subharmonic solutions of the equation, i.e. solutions which are k T periodic in t. We show that if H is appropriately super- or subquadratic in z as $z\to\infty$, there exist infinitely many distinct such solutions. Some other qualitative properties of the set of solutions is described.

F. ROTHE:

Global existence of solution branches for a simple reaction-diffusion system

We consider the reaction-diffusion system

$$u_t=\mu u_{xx}+f(u)-v$$
$$\varepsilon v_t= v_{xx}+u-v$$

Because it is a gradient system, the global existence of infinitely many solution branches can be shown by the method of Ljusternik-Schnirelmann.

M. SchATZMAN:

Singular Hamiltonean systems

Consider the following problem

$$\frac{d^2u}{dt^2} + \partial\varphi(u) \ni f, \quad u(o) = u_o, \quad \dot{u}(o) = u_1,$$

where u lives in a finite dimensional space $\mathbb{R}^N$.

This problem can be precisely formulated as:

Find u, a lipschitz continuous function with values in $\mathbb{R}^N$ such that

$$(*) \begin{cases} \dfrac{d^2u}{dt^2}, \quad \text{is a measure with values in } \mathbb{R}^N \\ \langle f-\dfrac{d^2u}{dt^2}, v-u\rangle \leq \int_0^T (\varphi(v)-\varphi(u))dt \quad \forall v \in C^o([o,t];\mathbb{R}^N) \\ \frac{1}{2}\left|\dfrac{du}{dt}(t+o)\right|^2+\varphi(u(t)) = \frac{1}{2}\left|\dfrac{du}{dt}(t-o)\right|^2+\varphi(u(t)) = \\ \frac{1}{2}|u_1|^2+\varphi(u_o)+\int_0^t (f,\dot{u})ds \\ u(o)=u_o \\ \dfrac{du}{dt}(o+o)-u_1+N_k(u_o)\ni o \end{cases}$$

Here φ is lower semi continuous, convex & proper; $K=\overline{\text{dom } \varphi}$, $N_k(u_o)$ is the normal cone to K at u_o, $u_o \in \text{dom } \varphi, u_1 \in \mathbb{R}^N$, and $f \in L^2(o,T,\mathbb{R}^N)$.

Then one can prove that (*) possesses a smooth solution, by studying the system

$$(*)_\lambda \begin{cases} \dfrac{d^2u_\lambda}{dt^2} + \partial\varphi_\lambda(u_\lambda) = f \\ u_\lambda(o) = u_o, \dot{u}_\lambda(o) = u_1 \end{cases}$$

where φ_λ is a convenient smooth approximation of φ.

Nevertheless the solution of (*) is not unique, and several examples are given.

To get some information about the flow associated to (*), one considers the semi-group associated to $(*)_\lambda$ which is defined by:

$s_\lambda(r)f(q,p)=f(F_\lambda(r,q,p))$

where F_λ is the flow of $(*)_\lambda$.

This semi-group is in fact a unitary group as F_λ is measure-preserving, and it operates in $L^2(\Omega)$ where Ω = int $K \times \mathbb{R}^N$, if we assume $\overline{\text{dom }\varphi_\lambda}=K$.

The generator of S_λ is A_λ defined by

$$D(A_\lambda)=\{f \in L^2(\Omega) \mid p\frac{\partial f}{\partial q}-\partial\varphi_\lambda(q)\frac{\partial f}{\partial p} \in L^2(\Omega)\} \;\&\; A_\lambda f=p\frac{\partial f}{\partial q}-\partial\varphi_\lambda(q)\frac{\partial f}{\partial p}$$

It can be shown that S_λ converges strongly to a semi-group S with generator A given by

$$D(A)=\{f \in L^2(\Omega) \mid p\frac{\partial f}{\partial q} - \partial\varphi(q)\frac{\partial f}{\partial p} \in L^2(\Omega) \;\&\; f(q,p)|_{\partial\Omega}=f(q,-p)|_{\partial\Omega}\}.$$

Using the mulitiplicative property of S, there exists a measurable flow such that $S(r)f(q,p)=f(F(r,q,p))$. Therefore, the flow defined by (*) is measurable.

H. R. THIEME:

Asymptotic behaviour of solutions of a partial differential equation in population dynamics

We discuss the question as to wether the solutions of the following PDE (which describes the dynamics of a spatially distributed population with age structure)

$$(*)\quad \begin{aligned} &(\partial_t+\partial_a-D(a)\Delta_x+\mu(a))\, u(t,a,x)=0 &&; \quad t,a>0, x\in\Omega \\ &u(0,a,x)= u_o(a,x) &&; \quad a\geq 0, x\in\Omega \\ &u(t,0,x)= f(x,\textstyle\int_{a_1}^{a_2} u(t,a,x)da) &&; \quad t>0, x\in\Omega \\ &u(t,a,x)= 0 &&; \quad t,a>0, x\in\partial\Omega \end{aligned}$$

converge towards a solution of the PDE

$$(\partial_a - D(a)\Delta_x + \mu(a))\ v(a,x) = 0 \qquad ;\ a>0, x\in\Omega$$
$$v(0,x) = f(x, \int_{a_1}^{a_2} v(a,x)da) \qquad ;\ x\in\Omega$$
$$v(a,x) = 0 \qquad ;\ a>0, x\in\partial\Omega$$

Hereby $0<a_1<a_2<\infty$.

Among several assumptions imposed on Ω ,$D,\mu,u_o f$(e.g. Ω a bounded domain in $\mathbb{R}^N$, $N \in \mathbb{N}$, D, μ bounded and positive, u_o bounded and non-negative, and appropriate smoothness assumptions) the following assupmtion for $f:\Omega x[0,\infty)\to[0,\infty)$ is a crucial one:

For any $y\in\Omega$, $f(y,r)/r$ strictly monotone decreases and $rf(y,r)$ strictly monotone increases as $r>0$.Even $f(y,r)/r\to 0$ for $r\to\infty$, uniformly in $y\in D$.

We obtain the following alternative.

Either

(i) $v\equiv 0$ is the unique bounded non-negative solution of (**) and, for any bounded non-negative solution u of (*),

$$u(t,a,x)\to 0 \text{ for } t\to\infty,$$

pointwise in $x\in\Omega, a\geq 0$.

Or

(ii) there exists a unique bounded positive solution v of (**) and, for any non-trivial bounded non-negative solution u of (*),

$$u(t,a,x)\to v(a,x) \text{ for } t\to\infty,$$

uniformly in $x\in\Omega, a\in[0,a_o], a_o>0$.

F. TOMI:

Fredholm-Abbildungen mit negativem Index und eine Anwendung auf Minimalflächen

Der Vortrag behandelt Ergebnisse, die in einer gemeinsamen Arbeit mit A. Tromba erzielt wurden. Zunächst wird ein allgemeiner funktionalanalytischer Satz vorgestellt, welcher besagt, dass das Bild einer differenzierbaren eigentlichen Fredholmabbildung $f:M \to N$ vom Index ≤ -2, M und N Banach - Mannigfaltigkeiten, keine zusammenhängende offene Teilmenge von N trennt. Sodann werden mögliche Anwendungen auf das Plateau-Problem besprochen. Man erhält z.B., dass eine generische Jordankurve im $\mathbb{R}^N$, $n \geq 4$, stets eine ungerade Anzahl von minimalen Immersionen berandet.

R. E. L. TURNER:

Internal waves in fluids with rapidly varying density

Internal waves in a density-stratified, incompressible, inviscid fluid, which travel without changing form correspond to solutions of the Long-Yih equation. If the stream function is used as an independent coordinate the equation reduces to en eigenvalue problem for a quasilinear partial differential equation which is nearly elliptic. Using a variational principle and techniques from quasilinear elliptic theory we show the existence of periodic and solitary waves in regimes with rapidly varying density distributions.

H. BEIRAO DA VEIGA:

Local existence in time of a classical solution for the motion of a non-viscous compressible barotropic fluid in domains with boundary

We consider the equations of the motion of a non-viscous compressible barotropic fluid in a domain Ω of the euclidean space $\mathbb{R}^3$:

$$(*)\quad \begin{cases} \rho\left[\dfrac{\partial v}{\partial t} + \sum_{i=1}^{3} v_i \dfrac{\partial v}{\partial x_i} - f\right] = -\nabla p & \text{in } Q_T \equiv\,]o,T[\times\Omega \\ \dfrac{\partial \rho}{\partial t} + \operatorname{div}(\rho v) = o & \text{in } Q_T, \\ v(o) = a & \text{in } \Omega \\ \rho(o) = \rho_0 & \text{in } \Omega \\ v\cdot n = o & \text{on } \Sigma_T \equiv\,]o,T[\times\Gamma \end{cases}$$

where Γ is the boundary of Ω and n is the unit outward normal to the boundary. We assume that the initial velocity $a(x)$ and the initial density distribution $\rho_0(x)$ are given. Moreover (barotropic fluids) the pression $p=p(\rho)$ depends on ρ and $p'(\xi)>0$ if $\xi>o$. If $a(x)$, $\rho_0(x)$ and $f(o,x)$ verify the compatibility conditions(they are necessary conditions) we prove the existence of a classical solution $v(t,x)$, $\rho(t,x)$ for system (*) during a time interval $[o,T]$, $T>o$.

O. VEJVODA:

Eine Verallgemeinerung des Satzes über implizite Funktionen

Man untersucht in einem Banach-Raum die Existenz der Lösungen der Gleichung $Nu=\varepsilon Fu$, wobei man voraussetzt, dass die F- Derivierte im allgemeinen eine nichttriviale (auch unendlich dimensionale) Nullmenge hat. Einige einfache Beispiele werden angeführt.

A. VIGNOLI:

A topological method for solving nonlinear equations

Let E,F, be normed spaces and $\Omega \subset E$ be open and bounded. A continuous map $f:\overline{\Omega} \to F$ with $f(x)\neq 0$ on $\partial\Omega$ is called 0-epi (zero-epi) if the equation $f(x)=h(x)$ is solvable for any compact map $h:\overline{\Omega}\to F$ such that $h(x)=0$ on $\partial\Omega$. The main properties of 0-epi maps are the following.

Existence property. Let f be 0-epi then $f(x)=0$ is solvable in Ω.

Homotopy property. Let $H:\overline{\Omega}x[0,1]\to F$ be continuous and such that a) $H(x,0)=f(x)$, b) $H(x,1)=g(x)$, c) $H(.,0)-H(.,.)$ is compact. Then f and g are either both 0-epi or both not 0-epi.

Normalization property. The inclusion $i:\overline{\Omega}\hookrightarrow E$ is 0-epi if $0 \in \Omega$.

Pertubation proberty. Let $f:\overline{\Omega} \to F$ be 0-epi and proper and let $h:\overline{\Omega}x[-1,1]\to F$ be compact and such that $h(x,0)=0$ for all $x \in \overline{\Omega}$. Then there exists $\varepsilon>0$ such that $f(.)+h(.,\lambda)$

is 0-epi for all $|\lambda|<\varepsilon$.

The class of 0-epi maps contains, among others, the following classes of maps: compact vector fields having nonzero topological Leray-Schauder degree; α-contractive vector fields with nonzero Nussbaum topological degree; couples of maps having nonzero Mawhin's coincidence degree; monotone countinuous operators with bounded null-set and proper on bounded and closed sets.

W. VON WAHL:

Semilinear Evolution Equations

First we treat semilinear evolution equations $u'(t)+A(t)u(t) + M(u(t)) = 0$, $u(0)=\varphi$, in a Banach space B. The closed operators A(t) with constant domain of definition $D(a(t)) = D(A(0))$ are supposed to generate an evolution operator $\mathfrak{U}(t,s)$. If M is a Lipschitz continuous mapping form B into itself we can construct a weak solution to the differential equation above, namely the solution of the integral equation

$$u(t) = U(t,0)\varphi + \int_0^t U(t,s)M(u(s))ds, \quad \varphi \in D(a(0)),$$

on an interval $[0,T_{max})$. It is shown now that u is in fact a solution of the differential equation on its interval of existence if B is reflexive. If B is not necessarily reflexive u is only Lipschitz continuous in t.

In the second part we introduce the concept of an analytic mapping between Banach spaces. We show that these mappings have interpolation properties which allow us to apply the notion of analyic mappings to nonlinear partial differential equations, in particular to the Navier-Stokes equations. We show that for every initial value φ in $P(L^p(\Omega))^n$, where p is number > n and where P is the projection of the divergence free part of $(L^p(\Omega))^n$, the n-dimensional Navier-Stokes equation $u' - \nu\nabla u + u \cdot \nabla u + \nabla p = f$, $\nabla \cdot u = 0, u|\partial\Omega = 0$, $u(0)=\varphi$ over $(0,T)\times\Omega$ has a solution u with $u' \in C^o((0,T_{max}), D(A^\rho))$, $u \in C^o((0,T_{max}), D(A^{1+\rho}))$, $u \in C^o([0,T_{max}), P(L^p(\Omega))^n)$. Here $A = -\nu P\Delta, \rho<\frac{1}{2p}$. This result was announced by Kato-Fujita in their paper in Archive Rat.Mech.Anal.16.

W. WALTER:

<u>Remarks on the Cauchy Problem for parabolic systems of the form</u> $u_t = A\Delta u + f(t,x,u,u_x)$.

In this talk two simple existence proofs for the Cauchy problem

$$u_t^k = a_k \Delta u^k + f_k(t,x,u,u_x) \text{ in } S=(0,T]\times R^n,\ u^k(0,x) = \varphi^k(x)$$

(k=1,....,m) were presented. In the first ("primitive") method the corresponding integral equation

$$u(t,x) = \int_{\mathbb{R}^n} \Gamma(t,x-\xi)\varphi(\xi)d\xi + \int_0^t \int_{\mathbb{R}^n} \Gamma(t-\tau,x-\xi) f(\tau,\xi,u,u_x)d\tau d\xi$$

$$\Gamma(t,x) = \text{diag}(\gamma(a_1 t,x),\ldots,\gamma(a_m t,x)),\ \gamma(t,x) = (4\pi t)^{-n/2}\exp(-x^2/4t)$$

is considered as an operator equation u=Su in the Banach space E of functions u continuous in $\bar{S}$ with continuous derivative u_x in S and finite norm.

$$||u||_\alpha = \max\{\sup_S |u(t,x)| e^{-\alpha t}, \sup_S \sqrt{t}|u(t,x)|e^{-\alpha t}\}.$$

If f(t,x,z,p) is Lipschitz continuous in z and p, then S is a contraction in E, $|| Su-Sv||_\alpha \leq q || u - v ||_\alpha$, where $o\leq q<1$.

The second method, which generalized an approach of Rauch and Smoller (Advances in Math. 1978) is based on a Banach space B of functions from R^n in R^m with the property that the familiy (τ_ξ) of translation operators, $(\tau_\xi w)(x) = w(x+\xi)$, is a strongly continuous group of operators. The corresponding integral equation

$$u(t) = K(t)\varphi+\int_0^t K(t-s)f(s,u(s))ds,$$

$$K(t) = \text{diag } (H(a_1t),\dots,H(a_mt)), \quad H(t)v= \int_{\mathbb{R}^n}\gamma(t,\xi)\tau_\xi v \, d\xi$$

is again solved by reduction to the contraction principle, using a weighted maximum norm. An advantage of this approach is that it gives at the same time existence, uniqueness, growth conditions of the solution and continuous dependence on the initial value, on f and on the a_k.

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