MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 51/1979
(letzter Bericht 1979)

Arbeitstagung Stochastik - Stochastic Workshop

16.12. bis 22.12.1979

Die Tagung stand unter dér Leitung von H. Rost (Heidelberg). Teilnehmer waren im allgemeinen jüngere Wissenschaftler aus Westeuropa und Ungarn. Ein Ziel der Tagung war zum gegenseitigen Kennenlernen einer jungen Stochastikergeneration beizutragen. Anders als in England besteht in der BRD auf Jahrestagungen kaum Gelegenheit dazu. Zum anderen sollte die Tagung eine "Winterschool" sein, d.h. typische Arbeitsgebiete der Stochastik, auf denen gegenwärtig wesentliche Entwicklungen ablaufen, sollten vorgestellt werden. Die Herren L. Arnold (Bremen) und H. Spohn (München) waren dazu eingeladen, jeweils in einer Vorlesungsreihe eine einführende Darstellung der Gebiete "Qualitative Theorie stochastischer Differentialgleichungs-Systeme" und "Markov'sche Grenzdynamiken in der Statistischen Mechanik" zu geben. Der Tagungsleiter hatte zudem die Teilnehmer gebeten, ihre Einzelvorträge weniger als Darbietung ihrer neuesten Resultate sondern mehr als eine erste Darstellung des Arbeitsgebiets anzusehen.

Obwohl sich alle um dieses Ziel redlich mühten, wurde in einer abschließenden Diskussionsrunde die Meinung geäußert, daß manche Vorträge zu speziell waren und oft zuviel Stoff ausgebreitet wurde - auf Kosten der Zeit für Diskussionen.

Insgesamt waren aber fast alle nicht nur über die neuen Kontakte, sondern auch über die Anregungen zu neuer Lektüre erfreut.

Die Hauptschwierigkeit der Tagung war vielleicht die Kürze der verfügbaren Zeit: die beiden Übersichtsreihen nahmen etwa 9 Stunden in Anspruch; die verbleibenden 16 Stunden waren gedacht als Zeit für die Vorstellung der einzelnen Teilnehmer und ihrer Arbeitsgebiete. Bei der üblichen Dauer eines Vortrags (im Durchschnitt etwa 40 Minuten) war dies nur möglich, weil einige Teilnehmer auf einen Vortrag verzichteten und man zusätzlich am Abend Seminare mit spezielleren Themen einfügte. Die gelockerte Form des Seminars hat sich im wesentlichen gut bewährt, da von hier aus sich die Möglichkeit zur Diskussion unter Angehörigen näher benachbarter Fachrichtungen von selbst ergab. Es gab zwei größere solcher Abendsitzungen: eine über Interaktionssysteme und eine über Martingale und stochastische Differentialgleichungen.

Teilnehmer

W. Adamski, München

L. Arnold, Bremen

A. Baddeley, Cambridge

T. Barth, Hull

O. Boxma, Utrecht

T. Brown, Bath

A. Csenki , Freiburg

W. Ehm, Frankfurt

T. Eisele, Zürich

P. Embrechts, Leiden

G. Franke, Frankfurt

A. Greven, Frankfurt

Y. Higuchi, Heidelberg

P. Kallenberg, Amsterdam

W. Kliemann, Bremen

M. Kohlmann, Bonn

W. Krüger, Kaiserslautern

H. Künsch, Zürich

A. Kußmaul, Tübingen

R. Lang, Heidelberg

R. Lerche, Heidelberg

G. Lippner, Budapest

H. Pruscha, München

R. Rebolledo, Nice

U. Rösler, Göttingen

H. Rost, Heidelberg

W. Rümelin, Bremen

M. Scheutzow, Kaiserslautern

H. Spohn, München

C. Stricker, Strasbourg

A. Vetier, Budapest

Vortragsauszüge

W. Adamski

Outer Approximation of Capacitylike Set Functions by Measures

Let X be an abstract set, and let L be a lattice of subsets of X closed under countable unions and satisfying some further conditions. If $g : L \to [0,\infty]$ is monotone, σ-smooth from below, continuous on the left and supermodular, then g is the set-wise infimum of the family of L-outer regular measures on $\sigma(L)$ that are dominating g . We apply this result to some topological situations obtaining in this way various generalizations of results of B. Anger (cf. Lecture Notes in Mathematics 226, pp. 152-170).

L. Arnold und W. Kliemann

Qualitative Theory of Stochastic Differential Systems

In this series of 5 lectures, we investigate similar problems as in the qualitative theory of deterministic systems $\dot{x} = f(x,t)$ (like invariant sets, ω-limit sets, stability, explosion, recurrence, invariant measures) for differential equations with a random right-hand side. The following subjects are treated:

1. Stability of the linear system $\dot{x}_t = A_t x_t$, A_t random: the problem; notions of stochastic stability and their relations; Ljapunov numbers, order of growth; restriction to stationary A_t ; examples : n = 1 , Mathieu equation, damped linear oscillator; approach of Chasminskij and Infante $(\omega_t = x_t/|x_t|$, $\dot{\omega} = h(A_t,\omega))$; there are at most n different Ljapunov numbers.

2. Asymptotic behaviour of nonlinear stochastic systems: $\dot{x}_t = f(x_t, \xi_t)$; ξ_t noise: the case of white noise/ Ito equations; existence of a stationary solution for white noise and real noise; necessary and sufficient condition of Chasminskij; Markovian noise ξ_t leads to the following degenerate Ito equation which from now on is the model investigated further:

$$d \begin{pmatrix} \xi_t \\ x_t \end{pmatrix} = \begin{pmatrix} a(\xi_t) \\ f(x_t, \xi_t) \end{pmatrix} dt + \begin{pmatrix} \sigma(\xi_t) \\ 0 \end{pmatrix} dW_t \, , \qquad \begin{matrix} \xi_t \in \mathbb{R}^m \\ x_t \in \mathbb{R}^n \end{matrix}$$

classification of states of a Markov process.

3. Connection between stochastic systems and deterministic control systems: the associated deterministic control system $\dot{x} = f(x, u(t))$; some control theory; translation mechanism ("tube method"); complete classification of states via their control properties; invariant measures, law of large numbers; asymptotic behaviour of trajectories for $t \to \infty$.

4. Applications: the linear system $\dot{x}_t = A_t x_t$ with stationary A_t revisited; complete solution of the stability problem for $n = 2$ and $n = 3$; example of an unstable system being stabilized by noise; systems in a random environment (noise-induced phase transitions in chemical reaction systems; Lotka-Volterra model with noisy parameters).

A. Baddeley

Absolute Curvatures: Some Recent Developments in Stochastic Geometry

Surface integrals of curvature arise naturally in stochastic geometry, as fundamental measurements of convex sets. They can be determined by averaging over lower-dimensional sections or projections. In fact, they are characterized by their invariance and additivity over convex sets. So, in applications, one often accords the curvature integrals a canonical status, excluding other measurements.

Here, however, we construct different integrals of so-called absolute curvature, having properties analogous to those above. They can be determined from sections or projections, and are characterized by invariance and additivity over surfaces. Hence they provide an alternative to the classical scheme of geometrical probability. This suggests that wider generalizations may be possible. Perhaps some deeper facts of algebraic topology underly stochastic geometry.

T. Barth

A Variational Principle in Stochastic Control Theory

A stochastic control problem with partially observable feedback control is given by the stochastic differential equation

$$dX_t = f(t,X,u(t,X))dt + G(t,X)dW_t$$

where $(W_t)_{t\in T}$ is a m-dimensional Brownian motion on $(\Omega,\mathcal{A},P)$, $T = [0,1]$, and where the admissible controls $u \in V$ are predictable for the observable σ-fields of $(X_t)_{t\in T}$. Using the

Girsanov method the equation is solved for all $u \in V$ by the same process $(X_t)_{t \in T}$ under the transformed measures P_u on (Ω, A). A terminal cost is defined by $F(u) := E_u(g \circ X_1)$ and the problem is to find ε-optimal controls $u_\varepsilon \in V$ for every $\varepsilon > 0$.

Under the distance $d(u,v) := \lambda \otimes P(\{u \neq v\})$, identifying a.e. equal controls, V becomes a complete metric space and F is continuous on (V,d). The variational minimum principle of I. EKELAND guarantees the existence of an ε-optimal control $u_\varepsilon \in V$ for all $\varepsilon > 0$ such that $F(u_\varepsilon) \leq F(u) + \varepsilon d(u,u_\varepsilon)$ for all $u \in V$. Using martingale representation results a necessary local condition is derived for an ε-optimal control u_ε. It says, that the conditional expectation of a certain Hamiltonian has to be minimized, to within ε. These results were obtained by R.J. ELLIOTT and M. KOHLMANN in 1979.

O.J. Boxma

<u>Two Queues in Series</u>

We consider a model of two single server queues in series, in which the service times of an arbitrary customer at both queues are identical. Customers arrive at the first queue according to a Poisson process.

It will be shown that a rather complete analysis is possible of this model, which is of importance in modern network design. Distributions of some variables of the model (like sojourn times in the second queue) will be derived. Finally we compare numerical results for the present model and for the model with independent, identically distributed service times at both queues.

An important role in the analysis is played by the distribution of the longest service time in a busy period of the first (M/G/1) queue. The properties of this distribution are also studied.

T. Brown

Compensators of Increasing Stochastic Processes

Consider a right continuous, increasing stochastic process, $N = \{N(t)\}_{t\geq 0}$, with $N(0) = 0$ and suppose that N is adapted to a right continuous increasing filtration $F = \{F(t)\}_{t\geq 0}$. We shall call (N,F) increasing if N is also locally integrable. In this case, there is a unique predictable increasing process, A, called the compensator of (N,F) , for which N-A is a local martingale. Let

$\mathcal{X} = \{x : x : [0,\infty) \to \mathbb{N} , x(0) = 0$, x right continuous, increasing, $x(t)-x(t-) = 0$ or

Theorem. Suppose (N_n,F_n) , $n \geq 1$, is a sequence of increasing processes with compensators A_n . If, for each $t \geq 0$,

(a) $$A_n(t) \to_p A(t)$$

where A is continuous, increasing and $F_n(0)$-measurable for all n , and

(b) $$E\{N_n(t)\, I\, [N_n(\cdot \wedge t) \notin \mathcal{X}]\} \to 0$$

then

$$N_n \to_d N$$

where $N(t) = N'(A(t))$ for a standard Poisson process, N', independent of A .

This theorem has many applications to point process convergence results. In particular, some joint results with B.W. Silverman on U-statistics with varying kernels are easy consequences. These results are useful in spatial data analysis.

A. Csenki

<u>A Theorem on the Departure of Randomly Indexed U-Statistics from Normality with an Application in Fixed-Width Sequential Interval Estimation</u>

Let $\{U_n = \binom{n}{m}^{-1} \sum_{1 \leq i_1 < \ldots < i_m \leq n} h(X_{i_1},\ldots,X_{i_m}) \mid n \geq m\}$ be a sequence of U-statistics with mean 0 based on a sequence $\{X_i \mid i \geq 1\}$ of i.i.d. random variables. Let N be an integer-valued random variable such that $P(N \geq m) = 1$ and $P(|N b^{-1}-1| \geq \varepsilon) \leq \delta$ with constants $\varepsilon \in (0,1/2]$, $b \in [2,\infty)$, $\delta > 0$ such that $b(1-\varepsilon) \geq \max\{2,m\}$. Put

$$\Delta_1 = \sup_{x \in \mathbb{R}} | P(b^{1/2} m^{-1} \sigma_1^{-1} U_N \leq x) - \Phi(x)|$$

and

$$\Delta_2 = \sup_{x \in \mathbb{R}} | P(N^{1/2} m^{-1} \sigma_1^{-1} U_N \leq x) - \Phi(x)| \quad ,$$

where $\sigma_1^2 = E(E|h(X_1,\ldots,X_m)|X_1))^2 \in (0,\infty)$

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and Φ denotes the standard normal d.f. Let r_3 be defined by $r_3 = E|h^3(X_1,\dots,X_m)| < \infty$. Then we have the following result

$$(*) \qquad \Delta_i \leq K_m(\delta + r_3\sigma_1^{-3}\, b^{-1/2} + \sigma_1^{-2/3}\, \sigma^{2/3}\, \varepsilon^{1/3}) \ , \ i = 1,2,$$

where $\sigma^2 = E\ h^2(X_1,\dots,X_m)$ and $K_m < \infty$ is an absolute constant depending only on m .

The inequality (*) is used to estimate the rate of convergence of stopping time distributions to normal distributions. This result can be applied in the theory of sequential estimation to obtain the levels of fixed width confidence intervals.

P. Embrechts

Mercerian Theorems in Probability Theory. An Application to the Tailbehaviour of Infinitely Divisible Distribution Functions

We work with proper distribution functions F, G etc. on $[0,\infty[$ and use the notation $F^{(n)}$ for the n^{th}-convolution of F with itself, $\bar{F} = 1-F$ for the tail of F, $\overline{F^{(n)}} = 1-F^{(n)}$, etc. We name $\mathcal{S}$ the class of subexponential F, i.e. those for which $\overline{F^{(2)}}(x)/\bar{F}(x) \to 2$ as $x \to \infty$. Let $\mathcal{R}$ denote the class of regularily varying functions, i.e. $F(x) \sim x^{-a}L(x)$ where $a \geq 0$ and L is slowly varying, hence for all $t > 0$: $L(tx)/L(x) \to 1$ as $x \to \infty$. Suppose in some probability model there is an "input" F and an "output" G . Very often one has theorems available like: $\bar{F} \in \mathcal{R} \leftrightarrow \bar{G} \in \mathcal{R} \leftrightarrow \bar{F}(x)/\bar{G}(x) \to c$ as $x \to \infty$, where c is a constant. The main problem we consider is: "How far can one enlarge the class $\mathcal{R}$, keeping the implications in force?". Moreover, can one find a class $\mathcal{K}$ such that: $\bar{F} \in \mathcal{K} \Leftrightarrow \bar{G} \in \mathcal{K} \Leftrightarrow \bar{F}(x)/\bar{G}(x) \to c$ as $x \to \infty$. The implication $\bar{F}(x)/\bar{G}(x) \to c \Rightarrow \bar{F}, \bar{G} \in \mathcal{K}$ being, by definition, a Mercerian one . We call the resulting class $\mathcal{K}$ the Mercerian class. As an example, we compare the tailbehaviour of an infinitely divisible law with that of certain functionals of its associated Lévy-measure. It turns out that, depending on the actual functional, both $\mathcal{R}$ and $\mathcal{S}$ arise as Mercerian classes.

References

[1] Embrechts,P., Goldie,C.M., Veraverbeke,N.: Subexponentiality and Infinite Divisibility. Z. Wahrscheinlichkeitsth. Verw.Geb. 49.335-347 (1979).

[2] Embrechts,P., Goldie,C.M.: Comparing the Tail of an Infinitely Divisible Distribution with Integrals of its Lévy-Measure. (To appear Ann.Prob.)

J. Franke

A Problem from Stochastic Navigation

Starting with the problem of minimizing the expected length of the path travelled by a pedestrian crossing a street and forced to prescribed evasive action by the random arrival of a car, we examine the following general optimization problem: Determine a path $p \in C_n[0,\infty)$, a control $r \in L_n^\infty[0,\infty)$ and a terminal time ζ such that for arbitrary loss function $S(x,t)$ and random time T : $ES(p(T \wedge \zeta), T \wedge \zeta) = \min!$ under the constraints

(i) $p(t) = a + \int_0^t r(s)ds$ (ii) $|r(t)| = 1$ a.s.

(iii) $Z(p(\zeta)) \leq 0$ (iv) $G_j(p(t)) \leq 0 \quad 0 \leq t \leq \zeta, \; j=1,\dots,J$,

where G_j and Z are appropriate functions (constraints). Under suitable conditions (essentially, continuous differentiability)

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on the regularity of the functions S, Z, G_j and the distribution function of T we get necessary conditions on locally optimal paths. The proof is based on the method of Dubovitskij and Milyutin for analyzing extremum problems in topological vector spaces. The method provides necessary conditions on optimal paths under quite general assumptions. A dynamic programming approach supplies an interpretation of the optimal control at time t as the direction of fastest decrease of the optimal conditional expected loss given $T > t$.

A. Greven

An Infinite System with Interaction

The following model is considered:

$$\dot{X}_k(t) = \sum_0^\infty \lambda_i (X_{k+i+1}(t) - X_{k+i}(t)) \qquad X_k(0) = X_k \text{ ;}$$

$$\lambda_{i+1} \geq \lambda_i \geq 0 \qquad \sum_0^\infty \lambda_i = 1 \text{ ; } k \in \mathbb{Z}$$

where $(X_k)_k$ are given by a stochastic process such that $X_{k+1} \geq X_k$. With an appropriate notion of solution, in the framework of processes (X_k) with stationary increments, the existence and uniqueness of these solutions is stated. Also some invariance properties are formulated.

Various results concerning the convergence of $\dot{X}_k(t)$, $X_{k+1}(t) - X_k(t)$, $(X_k(t) - X_k(0))/t$ in $\|\cdot\|_2$-norm and with probability one, are stated. The assumptions concerning the (X_k) deal with conditions of homogeneity, moments and the dependence structure.

P.J.M. Kallenberg

Branching Processes with Continuous State Space

Talking about a branching process $\{Z_n; n = 0,1,2,...\}$ it is usual to think of Z_n as the number of individuals in the n^{th} generation of some population. Such processes are called Galton-Watson processes. It is however also possible that we want to measure the size of the population by other means, for instance in terms of its weight or volume. This gives rise to the study of branching processes with the non-negative real numbers as their state space. It is shown that such processes behave like Galton-Watson processes both on $\{Z_n \to 0\}$ if $P(Z_1=0) > 0$ and on $\{Z_n \to \infty\}$. In the remaining case, $\{Z_n \to 0\}$ and $P(Z_1=0)=0$, which can not occur in Galton-Watson processes it can be shown that Z_{n+1}/Z_n converges to the essential infimum of the offspring distribution, defined by $a = \inf\{x; P(Z_1 \leq x|Z_0 = 1) > 0\}$. We have to distinguish between the cases $a > 0$ and $a = 0$. In the former case there exists a sequence $\{c_n; n = 0,1,2,...\}$ of constants such that $c_n Z_n$ converges to a random variable Y , with $P(0 < Y < \infty | Z_n \to 0)=1$. In the latter case we can construct a function L such that $e^{-n}L(Z_n)$ converges to a random variable U , satisfying $P(0 < U < \infty | Z_n \to 0) = 1$.

Reference

P.J.M. Kallenberg [1979]:

Branching Processes with Continuous State Space, Mathematical Centre Tracts, Mathematical Centre, Amsterdam.

W. Kliemann

Boundary Value Problems for Degenerate Partial Differential Equations

The rapid development of probabilistic methods for Markov processes since the late 1950's lead to the representation of solutions of partial differential equations as expectations of functionals of diffusion processes. In this way results even for degenerate PDO's were obtained by Freidlin, Friedman, Pinsky, Stroock and Varadhan and others. In this talk we consider totally degenerate PDO's with coefficient matrix

$$a(x) = \begin{pmatrix} 0 & 0 \\ 0 & \sigma(x)\cdot\sigma^*(x) \end{pmatrix} .$$

Associating to the PDO a diffusion process x_t with diffusion matrix $a^{1/2}(x)$ we can discuss the first boundary value problem in a bounded region D for those PDO's. We give sufficient conditions for the uniformly fast exit from D and so concretize a theorem of Stroock and Varadhan. Then we investigate the way boundary values can be taken for $x_t \to \partial D$ as $t \to \infty$ and partially generalize a result of Friedman and Pinsky. The method we use consists in exploring the controllability properties of a deterministic control system associated with x_t .

M. Kohlmann

The Optimal Control of a Semimartingale

Basing on recent results from abstract martingale theory we introduce a model for stochastic optimal control which covers the continuous and the purely discontinuous problem.

(i) the quality of this model is described

(ii) optimality criteria for the partially observable control problem are derived, and

(iii) (main topic) existence results for this problem are given

(iv) the results are applied to the control of a double martingale.

H. Künsch

Almost Sure Thermodynamics for Continuous Spin Systems

Real valued random fields on a d-dimensional lattice are considered. First, a.s.-and L_1-convergence of an energy which satisfies the regularity condition is proved without assuming a pair potential. In a second step, it is shown that the density of a Gibbs state can be approximated by the conditional density if the corresponding energy is superstable and satisfies a condition slightly stronger than regularity. As corollaries we get four results:

- Convergence of pressure (new proof of a result by Lebowitz-Presutti)
- Variational principle (new proof of a result by Pirlot)
- a.s.- and L_1-convergence of entropy
- a formula connecting entropy, energy and information gain.

A. Kußmaul

A Vector Valued Martingale Proof of Belayev's Alternative

Let X(t) be a stationary process with spectral representation

(*) : $X(t) = \int_0^\infty e^{itn}\, dM_n$, where (M_n) is a square integrable martingale with independent increments. Then for each bounded interval $T \subseteq \mathbb{R}$ $X_s(\cdot) := \int_0^s e^{i\cdot n} \, dM_n$ can be considered as an $L^2(T)$-valued martingale which, according to the 0-1 law for processes with independent increments, satisfies the following assertion: The probability of X_∞ taking its values in $L^\infty(T)$ is either 0 or 1 . In the latter case the martingale convergence theorem, applied to the $L^\infty(T)$-valued process (X_s), shows that X_∞ in fact takes its values in the space $C(T)$ of continuous functions on T . This proves Belayev's alternative for stationary processes of the form (*) (including stationary Gaussian processes): Either with probability 1 all paths of $X(t) = X_\infty(t)$ are continuous or unbounded on each nondegenerate interval.

R. Lang

Phase Transition and Symmetry Breaking

The talk is an introduction into the problems of phase transition and symmetry breaking, no new results are given. The role of the dimension and of the symmetry group acting on the spins is explained by a comparison of the Ising-Ferromagnet (discrete symmetry group) with the classical Heisenberg model (continuous symmetry group).

For two-dimensional lattice models with spins in a rather general space the following theorem of Dobrushin and Shlosman (Comm. Math.Phys. 42, 31-40, 1975) is presented: if the potential U is invariant under a connected compact Lie group G , acting on the configuration space, then every Gibbs measure with potential U is also G-invariant. The proof is based on an ingenious reduction of the problem to a local central limit theorem.

Open problems in the case of point processes are mentioned.

R. Lerche

On the Sample Size of Sequential Tests of Power One

An introduction to the theory of tests with power 1 is given. Especially we point out that in the one-sided testproblem $H_o : \theta \leq 0$ against $H_1 : \theta > 0$ no uniformly best test procedure exists (in fact any test is a stopping time and stopping means rejection of the hypothesis): For every level α-test T there exists another level α-test T' such that the inequality $E_\theta T \leq E_\theta T'$, $\forall \theta > 0$, is violated. To overcome this lack there are several approaches to optimality in the literature. We discuss that one of Farrell (AMS, 1964) and give in the case of Brownian motion with drift the asymptotic distribution and their moments for the stopping time.

Theorem: Let g be an increasing concave function, which is continuously differentiable and is $\sqrt{2t \log_2 t} \lessapprox g(t) = O(t)$ if $t \to \infty$. Let a_θ be the point fulfilling $g(a_\theta) = \theta a_\theta$ and assume $g'(a_\theta)/\theta \to \beta$ if $\theta \to 0$ with $0 < \beta < 1$.

Then for any sequence $t_\theta \to \infty$ with $t_\theta = o(a_\theta)$, if $\theta \to 0$, the following statements hold:

(a) $\lim_{\theta \to 0} \mathcal{L}(T/a_\theta | T > t_\theta) \xrightarrow{D} \delta_1$

(b) $E_\theta(T^\alpha | T > t_\theta) \sim a_\theta^\alpha$

(c) $E_\theta T^\alpha \sim P_0(T=\infty)\, a_\theta^\alpha$.

This result describes a new kind of renewal theory and is the base for a deeper study of first exit distributions of tests with power 1 .

H. Pruscha

Functions of Markov Chains: Cluster Analysis and Parameter Estimation

Two situations are very common in practice when one considers functions $g(X_n)$ of a Markov chain X_n :

(1) the observer expects to get more insight into the structure of the process by grouping of states;

(2) the observer has only incomplete information about the present state of the process.

In both situations the process $Y_n = g(X_n)$ can be described by a model which was first introduced by Blackwell (1957) and which describes several applied stochastic models, e.g. linear learning models. In situation (1) the entropy is chosen as cluster criterion. The entropy is calculated by using a Shannon-McMillan theorem. The procedure is illustrated by examples of behaviour research. In situation (2) we study the computability and consistency of the M LE for the transition matrix of the Markov chain X_n obtained from the process Y_n .

R. Rebolledo

On Weak Asymptotic Methods in Stochastic Calculus

A partial survey on weak asymptotic methods in stochastic calculus is given in two lectures.

Two main problems are treated: general tightness conditions for sequences of processes with trajectories in the Skorokhod space $D = D(\mathbb{R}_+, \mathbb{R})$ and the central limit theorem for local martingales.

Some general ideas about the construction of solutions of (semi)martingale problems are also developed.

The tightness conditions we treat are based on the asymptotic behaviour of some particular stopping times. Necessary and sufficient conditions are derived for D-valued sequences. Some particular criteria (sufficient conditions) are obtained from this general theorem. Moreover necessary and sufficient conditions for C-tightness are also developed.

Furthermore we introduce the concepts of domination and contiguity for sequences of processes. These concepts enable us to get some useful criteria of tightness by comparing one sequence with another.

The central limit theorem for local martingales is a particular version of a more general result concerning semimartingales. It roughly states that under a weak condition on the asymptotic behaviour of jumps the convergence of the quadratic variations of the local martingales (in probability) towards a deterministic increasing continuous process A is equivalent to the convergence in distribution of the sequence of local martingales towards a continuous Gaussian martingale with associated increasing process A and another "property on subsequences" of the sequence of local martingales.

Semimartingale problems are considered in terms of local characteristics. The method to solve these problems is based on the study of sufficient conditions for weak convergence of semimartingale distributions to a probability measure, which satisfies the requirements of the analyzed semimartingale problem.

U. Rösler

Martin Boundary for the Two-Dimensional Ornstein-Uhlenbeck Process

Let X_t be a two dimensional diffusion with a generating operator L

$$L = 1/2 \sum_{i=1}^{2} \frac{\partial^2}{\partial x_i^2} + \sum_{i,j=1}^{2} b_{ij} x_j \frac{\partial}{\partial x_i}$$

R.S. Martin showed a one-to-one correspondence between harmonic non-negative functions h and Borel measures on some set Δ given by

$$h(x) = \int_{\Delta} K(x,y)\, \mu(dy) .$$

Our main result is the identification of Δ in the transient case with the usual boundary of the unit circle. Further we calculate K(x,y) and give various probabilistic interpretations.

Reference

R.S. Martin:
Minimal Positive Harmonic Functions TAMS 49 (1941) pp 137-172.

W. Rümelin

Numerical Treatment of Stochastic Differential Equations

We define general Runge-Kutta approximations for the solution of stochastic differential equations (sde). These approximations are proved to converge in quadratic mean to the solution of a sde with a corrected drift. The explicit form of the correction term is given.

Concerning the order of convergence we show that in general it is impossible for the quadratic mean of the one step error to be of an order greater than $O(h^3)$. This order is attained e.g. by the stochastic analogue of Heun's method. In the n-dimensional case the highest order of convergence is in general only $O(h^2)$, attained by Euler's method. The order $O(h^3)$ can only be reached if $(\nabla_x \sigma^i)\sigma^j = (\nabla_x \sigma^j)\sigma^i$ for the columns of the diffusion matrix σ .

H. Spohn

Limit Dynamics in Classical Statistical Physics

Since Boltzmann the use of kinetic and hydrodynamic equations is a common tool in nonequilibrium statistical mechanics. We discuss how their validity can be understood on the basis of a microscopic particle model: The initial state of the particles is given by a probability measure on phase space. The particles interact pairwise by a given force and their dynamics is governed by Newton's equation of motion.

The Lorentz gas is discussed in detail. There a single particle moves through spacially randomly distributed scatterers. The motion of this particle is then described by a non-Markovian stochastic process. Under certain scalings (low density, weak coupling, mean field, hydrodynamic) this process is approximated by a Markov process. The forward equation of this limiting dynamics turns out to be the usual kinetic equation.

As an example for the limiting dynamics of an interacting system we discuss a hard sphere gas in the low density limit.

C. Stricker

Behaviour of Semimartingales in $]0,+\infty[$

The space $\mathcal{L}^c_{open}$ is the space of all processes $(M_t)_{t>0}$ such that for all $n \in \mathbb{N}$, $(M_{t+1/n})_{t\geq 0}$ is a continuous local martingale with respect to the filtration $(F_{t+1/n})_{t\geq 0}$. I give a new proof of the following result due to Sharpe: (M_t) can be extended to a local martingale (at 0) iff M_{0+} exists. The one-sided-maximal inequality of Burkholder gives immediately this result. Then we study the more general problem of semimartingales on $]0,+\infty[$ and we solve this problem by proving the following result:

If (X^n) is a sequence of semimartingales (on $[0,1]$ for example) and (A_n) a sequence of predictable sets such that for all n, $X^n_t = \int_0^t 1_{A_n} \, dX^{n+1}$ for all t, then there exists a semimartingale X satisfying : $X^n_t = \int_0^1 1_{A_n} \, dX$ iff for all predictable sets A, $\int_0^1 1_A \, dX^n$ converges in probability.

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