MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Finite Geometries

13.4. bis 19.4.1980

Die Tagung "Finite Geometries" wurde geleitet von Herrn Lüneburg (Kaiserslautern), unter Mitwirkung der Herren Buekenhout (Brüssel) und Hughes (London). Im Vordergrund des Interesses standen Blockpläne, partielle Geometrien und Fragen aus der Kodierungstheorie. Dabei ergaben sich Verbindungen mit Graphentheorie, Kombinatorik, Zahlentheorie und Gruppentheorie. Im Rahmen einer announcement session wurde über neue Ergebnisse berichtet, vor allem über die Existenz von neuen Blockplänen und Geometrien.

VORTRAGSAUSZÜGE

TH. BETH — ON CODES IN GROUP RINGS

The problem of expressing any function $f:G \longrightarrow G$ ($(G,+)$ being a finite abelian group) with $\sum_x f(x) = 0$ as the difference of permutations allows a new proof in the prime case. This proof technique, being completely different from that given by M. Hall (Proc. AMS 1951), can be carried out using Coding Theory tools by determining the generating function G_n for the number of types of differences of permutations of $\mathbb{Z}_n (n \in \mathbb{N})$: G_n is the permanent of the circulant $n \times n$-matrix $C(x_0,\dots,x_{n-1})$ over $\mathbb{Z}[x_0,\dots,x_{n-1}]$. It can be lower estimated by its determinant, which evaluated over the cyclotomic field $\mathbb{Q}(\xi)$, (ξ a primitive n-th root of unity) allows a direct calculation if n is a prime.

A. BEUTELSPACHER — ON EXTREMAL INTERSECTION NUMBERS OF A BLOCK DESIGN

Majumdar has proved that for any two distinct blocks B,C of a 2-(v,k,λ) design the following holds:

$$\max \left\{\frac{k}{v}(2k-v), k-r+\lambda\right\} \leq [B,C] \leq \frac{2k\lambda}{r} - (k-r+\lambda).$$

We characterize all design having at most two intersection numbers, one of which is $\frac{k}{v}(2k-v)$, or $\frac{2k\lambda}{r} - (k-r+\lambda)$.

Moreover we prove the following

<u>THEOREM:</u> Let D be a smooth simple design. If D has intersection numbers $\frac{2k\lambda}{r} - (k-r+\lambda)$, or $k-r+\lambda > 0$, then D is isomorphic to the system of points and hyperplanes of a finite projective space.

In the proof the Theorem of Dembowski and Wagner is used.

As a Corollary we characterize all smooth strongly resolvable designs.

P.J. CAMERON, DUAL POLAR SPACES

A dual polar space is the geometry of points and lines associated as follows with a polar space (in Tits' sense): points are maximal subspaces, lines are next-to-maximal subspaces, and incidence is reverse inclusion. Dual polar spaces have been axiomatized; a modified version of the speaker's original axioms, due to Shult, was given. Applications include

(i) a Manning-type theorem for uniprimitive permutation groups with projective subconstituents;

(ii) characterisation of all classical dual polar spaces of rank at least 4 (and almost all those of rank 3), as distance-transitive graphs, by their intersection numbers;

(iii) characterisation of the geometry of pure spinors associated with the orthogonal geometry $O(2d+1,q)$, by properties of its embedding in projective space.

R.H.F. DENNISTON (25,9,3) DESIGNS

Seventy-eight different symmetric designs with parameters (25,9,3) have been constructed by a simple method. Work is in progress to determine whether or not this is a complete set. No automorphism group of any of these designs has a greater order than 24, nor fewer than five orbits on points. Some are self-dual but can be shown to have no polarities: others are self-dual but with trivial automorphism groups. There is one residual design which can be embedded in eight different symmetric designs.

TH. GRUNDHÖFER, GROUPS OF PROJECTIVITIES

Theorems on permutation groups are used to obtain the following results for the group Π of projectivities in some finite projective planes:

In a nondesarguesian plane of order 2^n, Π is the alternating or the symmetric group of degree 2^n+1. In a nondesarguesian Andréplane, in a nearfield plane, in some generalized André-planes and in every translation plane with quasifields of rank 3 over the kernel, Π is the alternating or the symmetric group of degree q^n+1, where q^n is the order of the plane. This is a consequence of the result that in these translation planes, the group generated by all parallel projections contains $ASL(n,q)$.

W. HAEMERS, NON-EXISTENCE OF SOME STRONGLY REGULAR GRAPHS

It is proved that there exist no strongly regular graphs with parameters (57,14,1,4) and (49,16,3,6). This makes (65,32,15,16) the smallest set of parameters for which existence of a strongly regular graph is undecided. The first non-existence proof is due to H.A. Wilbrink and A.E. Brouwer; the second one involves joint work with F.C. Bussemaker, R. Mathar and H.A. Wilbrink.

CH. HERING, A COMBINATORIAL LEMMA AND SHANNON'S THEOREM

Let H and $\mathfrak{D}$ be finite sets, $d: H \times H \longrightarrow \mathfrak{D}$ a map and $\leq$ a partial ordering on $\mathfrak{D}$. For $c \in H$ and $r \in \mathfrak{D}$ we define $k_r(c) = \{x \in H \mid d(c,x) = r\}$ and $K_r(c) = \{x \in H \mid d(c,x) \leq r\}$. We assume that $|k_r(c)|$ is independent of c and denote $k_r = |k_r(c)|$ and $K_r = |K_r(c)|$. Let μ be a map of $\mathfrak{D}$ into the set of non-negative real numbers, $\mathfrak{B}$ a set of finite cardinality $M \geq 2$ and $\mathfrak{L}$ the set of all functions of $\mathfrak{B}$ into H. For $C \in \mathfrak{L}$ and $i \in \mathfrak{B}$ we define $\mathfrak{F}(C,i) = \{x \in H \mid$ there exists $j \in \mathfrak{B} \setminus \{i\}$ such that $d(x,C(j)) \leq d(x,C(i))\}$ and $P(C) = \frac{1}{M} \sum_{i \in \mathfrak{B}} \sum_{x \in \mathfrak{F}(C,i)} \mu(d(x,C(i)))$. We have

Lemma 1. $$\sum_{C \in \mathfrak{L}} P(C) = \sum_{r \in \mathfrak{D}} k_r \mu(r)\left(|H|^M - |H|(|H|-K_r)^{M-1}\right),$$

and

Corollary 1. There exists $C \in \mathfrak{L}$ such that

$$P(C) \leq \sum_{r \in \mathfrak{D}} k_r \mu(r)\left(1 - (1 - K_r/|H|)^{M-1}\right).$$

Specialising to the case that (H,d) is a binary Hamming space, we obtain Shannon's Theorem for binary symmetric channels. The same method allows to determine the exact average error probability. It turns out to be

$$\sum_{r \in \mathfrak{D}} k_r \mu(r)\left(1 - \sum_{s=1}^{M} \frac{1}{s}\binom{M-1}{s-1} k_r^{s-1}\left(|H|-K_r\right)^{M-s} \Big/ |H|^{M-1}\right)$$

for all functions $\mathfrak{B} \longrightarrow H$, and

$$\sum_{r \in \mathfrak{D}} k_r \mu(r)\left(1 - \sum_{s=1}^{M} \frac{1}{s}\binom{k_r-1}{s-1} \cdot \binom{|H|-K_r}{M-s} \Big/ \binom{|H|-1}{M-1}\right)$$

if we consider only the injective functions.

D. JUNGNICKEL, SQUARE DIVISIBLE DESIGNS

A square divisible design D with parameters n,m,k,λ consists of m classes of n points each such that points are in the same class iff they are not joined whereas $[p,q] = \lambda$ otherwise. Also, each block has k points and $b = v$. Either $m = k = n\lambda$ ("transversal design") or $k < m$ ("regular"). The intersection number 0 induces an equivalence relation and $[B,C]$ only depends on the classes of B,C. For any block B, there are $\leq n-1$ blocks disjoint from it with equality iff $[B,C] \in \{0,\lambda\}$ for all $C \neq B$. Thus D is symmetric iff each block has $n-1$ disjoint blocks. D is called "class regular" if it admits a group G acting regularly on each point class. Any class regular square divisible design is symmetric. Such D (not necessarily square) may be described by a "partial difference matrix" over G (a "generalized balanced weighing matrix"in the square case. Hadamard, Conference and balanced weighing matrixes are special cases). We discuss the known examples of GBW-matrices.

E. LANDER, SELF-DUAL CODES AND SYMMETRIC DESIGNS

It has been known for several years now that a symmetric design can give rise to a self-dual code under very particular circumstances. I will discuss a construction which greatly generalizes the existing technique: If D is a symmetric design with parameters (v,k,λ), an extended incidence matrix for D gives rise to a self-dual code over $GF(p)$ of length $v+1$, where p is any divisor of the square-free part of $k-\lambda$.

These codes can be used to obtain quick new proofs of old results as well as to prove new ones. A substantial strengthening of Hughes' Theorem on automorphisms of symmetric designs is proven. Specifically, suppose a symmetric (v,k,λ) design possesses an automorphism of odd prime order q. If some prime dividing the square-free part of $k-\lambda$ has even multiplicative order mod q, then the automorphism must fix an even number of points.

H. LENZ — SELF-ORTHOGONAL LATIN SQUARES WITH SELF-ORTHOGONAL SUBSQUARES

A Latin square is called self-orthogonal (an SOLS) if it is orthogonal to its transpose. Let L be the set of orders of SOLS's and S(u) the set of orders v of SOLS's with a sub-SOLS of order $u < v$. It is well-known that $L = \mathbb{N} \setminus \{2,3,6\}$. A comparatively simple proof was sketched. A necessary condition for $v \in S(u)$ is $v \geq 3u+1$. It is unknown whether it is sufficient.
D. DRAKE and the author proved:
$S(u) \supset \mathbb{N}^{\infty}_{4u+3}$ for $u \geq 304$; and $v \in S(u)$ if $v \equiv u \bmod 3$, $v \geq 3{,}25u + 2{,}25$ for large u. By difference methods (quasi - difference-matrices which occurred already with BOSE-SHRIKHANDE-PARKER and WILSON) some further partial results are obtained:
For $t \in \mathbb{N}$

$18t - 1 \in S(6t - 1)$, $18t + 5 \in S(6t + 1)$ [K. HEINRICH],
$18t - 2 \in S(6t - 2)$, $18t + 4 \in S(6t)$ if $t > 1$.
$9m - 1 \in S(3m - 2)$ if $6m+1$ is a prime power, and of course
$3t + 1 \in S(t)$ for $t \in L$ (BOSE-SHR.-P., K. HEINRICH).

Many questions remain open, e.g.: Is $14 \in S(4)$? Simpler proofs for 10, 12, 14, $18 \in L$ would be desirable.

R. LIEBLER — AUTOTOPISM GROUP REPRESENTATIONS

Group representation theory is applied to the study of autotopism groups of finite projective planes (in Lenz class V). If such a group G fixes a subplane, it must act freely on the natural modules. This theorem may be viewed as a generalization of a result of Ganley which deals with the case $|G| = 2$. It follows that the representations associated with an arbitrary autotopism group of a finite plane are projective.

This tends to support the celebrated conjecture that a finite autotopism group must be solvable, since it follows, for example that A_5 could only be an autotopism group if the plane has dimension divisible by 60.

J.H. VAN LINT

PARTIAL GEOMETRIES

This is a survey of recent progress in the study of partial and semi-partial geometries.

I. Constructions

a) Wilbrink, Higman and Thas have constructed s.p.g. by taking projective geometries with a quadric, taking special points and some lines and also by considering a hexagon H(q) and taking ideal lines as lines.

b) pg(6,6,2) was constructed by Schrijver and van Lint. It is the only sporadic partial geometry.

c) pg(9,8,4) was constructed by Cohen and later by Haemers and van Lint. Subsequently De Clerck, Dye and Thas showed it was the first of a sequence $pg(2^{2n-1}+1.\ 2^{2n-1},\ 2^{2n-2})$. Thas generalized this to $q = 3$.

II. Nonexistence theorems

a) For completeness we mention that the strongly regular graph on 49 points with valency 16 does not exist.

b) s.p.g. (2,6,1,4), i.e. the strongly regular graph $n = 57$, $k = 14, \lambda = 1$ does not exist.

c) a s.p.g. with $\alpha \geq 2$, $s > \alpha$, $\mu = \alpha^2$ is known or it does not exist (Brouwer and Wilbrink)

d) a s.p.g. with $s = \alpha = 2$ is known or it does not exist (Seidel).

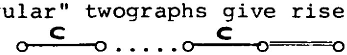
A. NEUMAIER

COMPLETELY REGULAR TWOGRAPHS

There are at most ten parameter sets for which a twograph $\mathcal{C}$ with the properties:

(1) $\mathcal{C}$ contains a regular clique;

(2) For all integers $i \geq 2$, the number of (i+1)-cliques containing a given i-clique is a constant a_i.

Five parameter sets have a unique solution, each with a doubly point-transitive automorphism group: S_6 on 10 points, $2^4 S_6$ on 16 points, Sp(6,2) on 28 and 36 points, Con.3 on 276 points; five cases with 96, 288, 1128, 640 and 3160 points are undecided. All these "completely regular" twographs give rise to a geometry with Buekenhout diagramm .

J.A. THAS

MOUFANG CONDITIONS FOR FINITE GENERALIZED QUADRANGLES

A generalized quadrangle $S=(P,B,I)$ of order (s,t) is said to satisfy the Moufang condition $(M)_p$ for some point p provided the following holds: for any two lines A,B of S incident with p, the group of collineations of S fixing A and B pointwise and p linewise is transitive on the lines ($\neq A$) incident with a given point x on A ($x\neq p$). Those S satisfying $(M)_p$ for a prescribed set of points p, and in some cases certain other hypotheses are studied. Sample results: If s is prime and S of order (s,s^2) satisfies $(M)_p$ for some point p, then S is isomorphic to the generalized quadrangle $Q(5,s)$ arising from the elliptic quadric in $PG(5,s)$. If S satisfies the dual condition $(\hat{M})_L$ for every line L through some coregular point p, and if $s>1$ and $t>1$, then $s=q^{h'}$ and $t=q^{h''}$ with q a prime power and $h'=h''$ or $h''a=h'(a+1)$ with a odd. If in particular s is prime, then S is isomorphic to $Q(5,s)$ or to the generalized quadrangle $Q(4,s)$ arising from the non-singular quadric in $PG(4,s)$; if in particular every line is regular, then $t=s^2$ or S is isomorphic to $Q(4,s)$.

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