

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 29/1980

Orders and their applications

22. bis 28.6.1980

Die Tagung fand unter Leitung von Klaus Roggenkamp (Stuttgart) statt. Ein guter Teil der Vorträge stand unter dem Aspekt von Anwendungen der Darstellungstheorie von Ordnungen. Hier sind insbesondere zu nennen:

- 1.) 3 Vorträge von W.H.Gustafson über die geschichtliche Entwicklung von Ordnungen, beginnend mit der Komposition quadratischer Formen über elliptischen Kurven und komplexer Multiplikation zu Anwendungen in der Zahlentheorie,
- 2.) Vorträge über Anwendungen in der Galoismodulstruktur von A.Fröhlich, J.Queyrut, J.Ritter, L.Scott, M.Taylor, St.V.Ullom,
- 3.) Zusammenhänge mit der Topologie: Die Wall'schen Arbeiten und neuere Ergebnisse von Ch.Thomas,
- 4.) Quaternionenordnungen für minimale Modelle in der algebraischen Geometrie ternärer quadratischer Formen: J.Brzezinski,
- 5.) Grothendieckgruppen von Ordnungen, deren Studium aus der Differentialgeometrie initiiert war: H.Bass,

6.) Kristallographische Gruppen: W.Plesken.

Daneben wurden neuere Entwicklungen in der Darstellungstheorie der Ordnungen behandelt:

- 1.) Zeta-Funktionen auf Ordnungen: I.Reiner - C.Bushnell.
- 2.) Beinahe zerfallende Sequenzen, Auslander-Reiten Graphen von Blöcken und Darstellungstypen von Blöcken:
M.Auslander, Ch.Bessenrodt, M.C.R.Butler, E.Dieterichs, H.-G.Quebbemann, Th.Theohari-Apostolidi, A.Wiedemann.
- 3.) K-Theorie und Klassengruppen: B.Magurn, L.McCulloh, M.Stein, S.M.J.Wilson.
- 4.) Einheiten in Gruppenringen: G.Cliff, H.Zassenhaus; B.Sandling hat über die bisher unveröffentlichte Doktorarbeit von G.Higman berichtet, die 1940 schon sehr viele in den letzten Jahren erneut bewiesener Resultate enthält.
- 5.) Projektive Moduln und Auflösungen: J.L.Alperin, J.F.Carlson, P.J.Webb.
- 6.) Relationemoduln und Zerfall des Augmentationsideals:
W.Kimmerle, J.Williams.
- 7.) Algorithmische Bestimmung von Erzeugenden bei Hauptidealen:
O.Taussky-Todd.
- 8.) Halbgruppenarithmetik in Asano Ordnungen: E.-A.Behrens.

Zu meiner großen Freude konnte von den acht eingeladenen sowjetischen Wissenschaftlern Frau L.Nazarova kommen, die über Darstellungen von teilweise geordneten Mengen vorgetragen hat. Zu Gesprächen mit Frau Nazarova sind am Dienstag P.Gabriel und am Mittwoch-Donnerstag C.M.Ringel nach Oberwolfach gekommen.

Trotz der durch Vorträge ziemlich ausgefüllten Tage fanden mathematisch fruchtbare Gespräche statt, und auch das gesellige Leben kam nicht zu kurz.

Zusammenfassend glaube ich, sagen zu können, daß die Tagung für die Teilnehmer sowohl Initiativen für die eigene Forschung enthielt, als auch einen Überblick über die relevanten Entwicklungen und Anwendungen gab.

K.W. Roggenkamp

Teilnehmer

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Bäckström, K.-J., Göteborg	Oliver, R., Aarhus
Bass, H., New York	Plesken, W., Aachen
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Lorenz, F., Münster	Wiedemann, A., Stuttgart
Magurn, B.A., Norman	Williams, J., Garden City
McCulloh, L.R., Urbana	Wilson, S.M.J., Durham
Meyer, J., Stuttgart	Zassenhaus, H., Columbus

Vortragsauszüge

J.L.ALPERIN: Projective modules and resolutions for finite groups

A survey of recent work on complexity of modules and related topics. The latest developments involve certain affine algebraic varieties which can be attached to modules.

M.AUSLANDER: Preprojective lattices over orders over Dedekind domains

The notions of preprojective partitions and preprojective lattices were presented, together with outlines of their existence theorems. It was also shown that an order is of finite type if and only if every lattice is preprojective as well as the fact that the non-preprojective lattices have no splitting projectives and have no finite covers, at least when the order is in a simple algebra.

H.BASS: Lenstra's Calculation of $G(R[\pi])$, and Morse-Smale Diffeomorphisms

Let $f: M \rightarrow M$ be a diffeomorphism of a compact smooth manifold. Shub and Sullivan proved: f Morse-Smale \Rightarrow the eigenvalues of $f_*: \pi_1(M, 0) \rightarrow \pi_1(M, 0)$ are roots of unity. Shub and Franks found an obstruction to the converse, which lives in a group here denoted SSF ; they proposed calculating SSF . Two presentations of SSF were obtained by myself and by Dan Grayson. With neither of these however could we decide whether $SSF \neq 0$. Lenstra later solved the problem, using the first presentation, by proving that

$$\text{SSF} \simeq \bigoplus_{n \geq 1} \text{Pic}(Z[\zeta_n, 1/n])$$

This is easily deduced from the following beautiful formula, (*).

If R is a noetherian ring write $G(R)$ for the Grothendieck group of finitely generated R -modules (+ exact sequences). Let π be a finite abelian group. Then there is an isomorphism

$$(*) \quad G(R[\pi]) \simeq \bigoplus_{\rho \in C(\pi)} G(R\langle \rho \rangle)$$

Here $C(\pi)$ denotes the set of cyclic quotient groups of π .

If $\rho \in C(\pi)$ has order n , and generator t , then $R\langle \rho \rangle = R(\rho)[\frac{1}{n}]$, where $R(\rho) = R[\rho]/\varphi_n(t)R[\rho]$, and φ_n is the n^{th} cyclotomic polynomial.

E.-A. BEHRENS: A non-commutative arithmetic for semigroups and its application to Asano orders

Let $\{\mathcal{O}_i, i \in I\}$ be the set of maximal orders, equivalent to the Asano order $\mathcal{O}_1 = \mathcal{O}$ in its quotient ring \mathfrak{A} . The common product $A_{hi}B_{jk}$ of two normal ideals is normal again, but if A_{hi} and B_{jk} is integral then $A_{hi}B_{jk}$ is integral iff $i = j$. Instead of Brandt's limiting the products to a partial groupoid the author extends the proper products to a new multiplication on the set \mathfrak{M} of normal ideals by defining $A_{hi} \circ B_{jk} = A_{hi} \cdot M_{ij} \cdot B_{jk}$, where M_{ij} is the distance from \mathcal{O}_i to \mathcal{O}_j . This gives a completely simple semigroup (\mathfrak{M}, \circ) , partially ordered under \subseteq , with the set \mathcal{S} of normal integral ideals being a subsemigroup. Then $(\mathcal{O}_i \circ \mathcal{O}_1) + (\mathcal{O}_k \circ \mathcal{O}_1) = \mathcal{O}_{ivk} \circ \mathcal{O}_1$ defines a V -semilattice structure on I which together with the bound function $\mathcal{O}_i \circ \mathcal{O}_k \circ \mathcal{O}_1 = (\text{bd}(i, h))\mathcal{O}_1$ from

$I \times I$ to the group \mathcal{G} of the $(0,0)$ -ideals determines the arithmetic of \mathbb{Z} completely.

The whole theory can be generalized to a convenient setting in the theory of semigroups by replacing the module theoretic arguments in the classical theory by semigroup theoretical ones.

CH.BESSENRODT: On blocks of finite lattice type

Let G be a finite group, p a prime number dividing $|G|$, R a finite unramified extension of $\hat{\mathbb{Z}}_p$.

Proposition: If D is cyclic of order p^2 then all indecomposable RD-lattices are absolutely indecomposable.

Now suppose $R/J(R)$ is a splitting field for G and its subgroups then the following main result is proved:

Theorem: Let B be a block of RG with cyclic defect group D , inertial degree t . Then

- (i) Every absolutely indecomposable RD-lattice "induces" exactly t non-isomorphic indecomposable RG-lattices in B .
- (ii) There is a vertex-preserving bijection between the set of \mathfrak{a} -orbits of absolutely indecomposable non-projective RD-lattices and the set of \mathfrak{a} -orbits of "induced" RG-lattices in B .
- (iii) All \mathfrak{a} -orbits are finite and \mathfrak{a} -orbits of induced lattices have length t or $2t$ corresponding to \mathfrak{a} -length 1 or 2 of the \mathfrak{a} -orbit of a source.

With the above proposition we have the following corollary to the theorem:

Corollary: Suppose $|D| = p^2$. Then

- (i) B has exactly $(4p+1)t$ indecomposable RG -lattices which are all generated by 2 elements.
- (ii) There is a vertex-preserving bijection between the set of Ω -orbits of indecomposable non-projective RG -lattices in B .
- (iii) All Ω -orbits of indecomposable non-projective RG -lattices in B have length $2t$. One of the orbits consists of all RG -lattices in B with vertex of order p .

J. BRZEZINSKI: Algebraic geometry of ternary quadratic forms and orders in quaternion algebras

Let $f \in A[X_0, X_1, X_2]$ be a quadratic form with coefficients in a Dedekind ring A (or, more generally, L an A -lattice on a ternary quadratic space (V, q) over the field of fractions F of A). The quadratic form f defines two objects: a projective $\text{Spec}A$ -scheme $M = \text{Proj}(A[X_0, X_1, X_2]/(f))$ and an A -order $O(f)$ in a generalized quaternion algebra Q over F - the even part $C_0(f)$ of the Clifford algebra $C(f)$ of f .

Theorem 1: The orders $O(f)$ in Q are precisely the Gorenstein orders in Q .

Theorem 2: The $\text{Spec}A$ -scheme $\text{Proj}(A[X_0, X_1, X_2]/(f))$ is regular if and only if the corresponding order $O(f)$ is hereditary.

Theorem 3: Let A/\mathfrak{p} be perfect for each prime ideal $\mathfrak{p} \in \text{Spec}A$, $\mathfrak{p} \neq (0)$, and assume that \mathfrak{p} does not divide the discriminant of the hereditary order $O(f)$. There is a one-to-one correspondence between the $k(\mathfrak{p})$ -rational points of the fiber $M_{\mathfrak{p}}$ and the integral (left) $O(f)$ -ideals with norm equal to \mathfrak{p} such that elementary transformations at two $k(\mathfrak{p})$ -rational points of $M_{\mathfrak{p}}$ give

isomorphic SpecA-schemes if and only if the right orders of the left ideals corresponding to these points are isomorphic.

We apply these results to some arithmetical questions concerning integral representations by ternary and quaternary quadratic forms.

C.J.BUSHNELL - I.REINER: Zeta-Functions of Orders

Let A be a finite-dimensional semisimple Q -algebra, and Λ an order in A . Solomon induced the zeta-function

$$\zeta_{\Lambda}(s) = \sum_{L \leq \Lambda} (\Lambda:L)^{-s} \quad s \in \mathbb{C}, \operatorname{Re}(s) \text{ large}$$

where the sum is taken over all left ideals of Λ of finite index in Λ . Special cases of this have a long history, notably the Dedekind zeta-function and the Hey zeta-function (where A is simple, Λ is maximal).

Using a subscript p to denote completion at p , we have

$$\zeta_{\Lambda}(s) = \prod_p \zeta_{\Lambda_p}(s), \text{ where } \zeta_{\Lambda_p} \text{ is the obvious local analogue}$$

of ζ_{Λ} . If Λ' is a maximal order in A containing Λ ,

$\zeta_{\Lambda'}(s)$ may be computed explicitly in terms of the Wedderburn

structure of A . Moreover, we have $\Lambda_p = \Lambda'_p$, and hence

$$\zeta_{\Lambda_p} = \zeta_{\Lambda'_p}, \text{ for almost all } p. \text{ The first major result is that}$$

the "correction factor" $\varphi_p(s) = \zeta_{\Lambda_p}(s)/\zeta_{\Lambda'_p}(s)$ is a polynomial

in p^{-s} with coefficients in \mathbb{Z} . In the special case $\Lambda = RG$,

R the ring of integers in a number field, these φ_p have a symmetry,

$$\text{or functional equation } \varphi_p(s) = (\Lambda'_p:R_pG)^{1-2s} \varphi_p(\lambda-s) \text{ consistent}$$

with the functional equation of ζ_{Λ} .

The results are proved by expressing the ζ_{Λ_p} as local integrals,

as in Tate's thesis. It is easy enough to generalize Tate's work to cover the problems at hand. Alternatively, one may appeal to the much more general work of Godement and Jaquet in this direction.

M.C.R. BUTLER: Grothendieck groups and almost split sequences

Let \mathcal{L} denote either the category of finitely generated modules over an Artin algebra, or the category of lattices over an order over a complete discrete valuation ring of rank 1 in a semisimple algebra. Then \mathcal{L} possesses almost split sequences, namely the representatives of non-zero elements in the socles of functors $\text{Ext}^1(-, A)$, A indecomposable. Suppose also that \mathcal{L} has only finitely many indecomposables, so that the $\text{Ext}^1(-, A)$ functors have finite composition length. Using induction on these lengths, I showed that the Grothendieck group of \mathcal{L} may be presented as the group generated by the objects A, B, C, \dots modulo only these relations $B = A + C$ corresponding to sequences $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$ which are either split exact or almost split exact. This means that the combinatorial structure alone of the Auslander-Reiten quiver of \mathcal{L} suffices to define uniquely the composition factor structure of the indecomposables in \mathcal{L} .

J.F. CARLSON: The varieties of a module over an elementary abelian group

Let K be an algebraically closed field of characteristic $p > 0$, and let $G = \langle x_1, \dots, x_n \rangle$ be an elementary abelian p -group. To any KG -module M , we may associate two varieties $V(M)$ and $W(M)$

in K^n . The first is the zero element together with the set of all $\alpha = (\alpha_1, \dots, \alpha_n)$ such that the unit $1 + \sum_{i=1}^n \alpha_i (x_i - 1) \in KG$ does not act freely on M . The ideal of the variety $W(M)$ is related to the annihilator in $\text{Ext}_{KG}^*(K, K)$ of the class of the identity element in $\text{Ext}_{KG}^0(M, M)$. It is shown that $V(M) \subseteq W(M)$. Both varieties have dimension equal to the complexity of M . One consequence of these results is a recent theorem of Kroll which characterizes the complexity of M in terms of the order of G and of the maximal generalized subgroup of units in KG that acts freely on M .

G. CLIFF: On units of integral group rings

Let $U_1(ZG)$ denote the units of ZG of augmentation 1, for a finite group G . We consider the following problems:

- 1) Does $1 \rightarrow G \rightarrow U_1(ZG)$ split?
- 2) If so, is the kernel torsion free?

1) was raised by K. Dennis, and 2) by D. Passman. We show that 1) and 2) can both be answered in the affirmative, if G is a metabelian group having an abelian normal subgroup A , with $(|A|, [G:A]) = 1$, and $|A|$ odd. (This is joint work with S.K. Sehgal and A. Weiss.)

E. DIETERICH: Representation types of group rings over complete discrete valuation rings

Let R be a complete discrete valuation ring with valuation v , a finite group, $\Lambda = RG$, and \mathcal{L} the category of Λ -lattices.

Call Λ to be of "wild representation type" if there exists a full subcategory of \mathcal{L} which is representation equivalent to the category of matrix pairs over some field. Call Λ to be of "tame representation type", if it is neither of finite nor of wild type.

Examples have been given, where the representation type of Λ can be determined by relating \mathcal{L} to some category of representations of a poset or a species:

- 1.) If $\Lambda = RC_p$, $p > 2$, $1 < v(p) < \infty$, then Λ is of wild representation type.
- 2.) If $\Lambda = RC_p$, $p > 2$, $v(p) = 2$, and the p -th cyclotomic polynomial decomposes into two irreducible factors, then Λ is of finite representation type, and has $2p + 3$ indecomposable lattices up to isomorphism.
- 3.) If $\Lambda = RC_3$, $v(3) = 2$, and if the third cyclotomic polynomial is irreducible, then there are 7 indecomposable Λ -lattices up to isomorphism.

A. FRÖHLICH: Hermitian class groups

The motivation comes from the trace form on number fields or local fields. The object of study are Hermitian modules (X, h) , given by a locally free \mathfrak{A} -module X and a Hermitian form on $X \otimes_{\mathfrak{A}} A$ over A , where A is a semisimple algebra with involution, \mathfrak{A} an order on A admitting the involution. The problem solved is the generalization of discriminants of lattices to this situation.

W.H.GUSTAFSON: Orders in geometry, topology and number theory

We discuss the history of the theory of orders, with emphasis on the applications that caused its development. We hope to outline connections of the theory with the classification of elliptic curves and constructive classfield theory, with surgery of manifolds, with holonomy of flat affine surfaces, with crystallography and with the arithmetic of algebraic number fields.

W.KIMMERLE: Relative relation modules as generators

Let G be a group, $H < G$ and F a free group admitting an epimorphism λ from $F * H$ onto G such that λ restricted to H is the identity. Denote by K the kernel of λ , then $K/[K, K]$ viewed via conjugation as ZG -lattice is called a relation module of G relative to H .

Using this relativation one obtains the following integral-representation theoretical characterization of finite groups. Equivalent are

- (i) $[G, G]$ is not nilpotent.
- (ii) All relation modules of G relative to at least one maximal subgroup are generators.
- (iii) All relation modules of G relative to at least one subgroup are generators.

It should be noted that a characterization of finite groups by considering only ordinary relation modules ($n=1$) with respect to the generator property seems to be far from solution.

B.A.MAGURN: Uses of units in the Whitehead group

It is conjectured for finite groups G , K_1ZG is the direct product of SK_1ZG and stabilized units. This is proved for dihedral groups, and a simple form of representative units is used to solve realizability problems by Whitehead torsions of self-equivalences of connected finite CW-complexes, and of finite CW-pairs.

L.R.McCULLOH: Stickelberger relations in class groups of orders in abelian group rings

Let G be a product of cyclic groups of order l^n , and let H be its largest elementary abelian subgroup. The group algebra QG decomposes as $Q(G/H) \times A$. Let Λ be the image of ZG under projection $QG \rightarrow A$. (When G is cyclic, $\Lambda \cong Z[\mu_{l^n}]$.) The Stickelberger relations and associated class number formulae for cyclotomic fields are generalized to the order Λ and its class group $Cl(\Lambda)$. When G is elementary abelian, these results apply directly to $Cl(ZG)$. The Stickelberger ideal needed is a relative of one used by Kubert and Lang to describe cuspidal divisor class groups of modular function fields.

L.A.NAZAROVA: Poset representations

Let $\mathfrak{P} = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ be a finite partially ordered set (poset). A representation $S = (V, V_1, \dots, V_n)$ of the poset \mathfrak{P} over the field k is given by subspaces V_i of the finite dimensional vector space V for each $\alpha_i \in \mathfrak{P}$, such that if $\alpha_i \leq \alpha_j$, then

$$V_i \subseteq V_j .$$

It is easy to define representations of posets over rings, considering finitely generated $k[x]$ -modules instead of vectorspaces. Posets of infinite type (having infinitely many indecomposable representations) are divided into two classes: tame and wild.

A poset \mathfrak{R} has tame type (over k), if it has infinite type and for every dimension d there exists a finite set $M_d(\mathfrak{R})$ of representations of \mathfrak{R} over $k[x]$ ($k[x]$ = the ring of polynomials in one variable) such that almost all indecomposable representations (V, V_1, \dots, V_n) of \mathfrak{R} of dimension d over k are obtained by a representation $(\bar{V}, \bar{V}_1, \dots, \bar{V}_n) \in M_d(\mathfrak{R})$ in the following way: $V = \bar{V} \otimes B$, $V_i = \bar{V}_i \otimes B$, for some finitely generated $k[x]$ -module B .

Posets of tame type are naturally divided into two classes:

- 1.) finite growth, for which the number of representations in $M_d(\mathfrak{R})$ is bounded by a fixed number N for all dimensions d ;
- 2.) infinite growth or Gelfand's posets for which the number of representations in $M_d(\mathfrak{R})$ increases infinitely with increasing dimension d .

Theorem (Zavadski, Bondarenko, Nazarova): A poset \mathfrak{R} is of finite growth, if and only if it does not contain the subset



Moreover, $N = 1$ for all the posets of finite growth. A domestic criterion was given, and the complete list of faithful posets of tame type.

W.PLESKEN: Algebraic Aspects of Crystallography

Some basis problems in the theory of crystallographic groups can be solved by methods of the theory of orders. The following problems are relevant:

- (i) Find all sublattices of a ZG-lattice L , which have finite index in L and decide isomorphism (G a finite group).
- (ii) Decide isomorphism of two given ZG-lattices and construct an isomorphism, if it exists.
- (iii) Find a finite set of generators of the unit group of a given Z-order (usually the centralizer-ring of a finite subgroup of $GL(n, Z)$ in $Z^{n \times n}$).

These problems and their background in the theory of space groups were discussed.

H.-G.QUEBBEMANN: Integral orthogonal and symplectic representations of the cyclic group of prime order

Let π be the cyclic group of prime order p . The classification of representations $\pi \rightarrow Sp_{2n}(Z)$ is reduced to the classification of certain skew-hermitian forms over $Z[\zeta]$, $\zeta \neq 1$, a p^{th} root of unity. The corresponding result for orthogonal representations is weaker, but allows e.g. to classify selfdual lattices $L \subset R^{p+1}$ with the property that p divides $|\text{Aut}(L)|$. If C^- = kernel of the norm map $C_{\mathbb{Q}(\zeta)} \rightarrow C_{\mathbb{Q}(\zeta+\zeta^{-1})}$ (C_K = ideal class group of K) has odd order, then there is a bijection between classes of such lattices and pairs (β, γ) , where β is

a reduced positive definite form $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$ such that $ac - b^2 = p$, $\left(\frac{a}{p}\right) = 1$, and γ is an orbit of $\text{Aut}(\mathbb{Q}(\zeta)/\mathbb{Q})$ in C^- .

J. QUEYRUT: S-Grothendieck groups and Galois module structure of rings of integers

Let N be a number field and Γ a group of automorphisms of N . The ring of integers of N , Z_N has a $Z[\Gamma]$ -module structure. The description of the structure of Z_N is achieved with the help of new Grothendieck groups. Let S be a set of primes of Z_N ; we denote by $K_0^S(Z[\Gamma])$ the Grothendieck group of the category $\mathcal{C}_{1p}^S(Z[\Gamma])$ of Z -torsion free finitely generated modules M which are locally projective outside of S and we denote by $[M]$ the class of such a module M . If S contains the primes which are wildly ramified in N , Z_N is in $\mathcal{C}_{1p}^S(Z[\Gamma])$. One can now consider the element $U_{N/K} = [Z_N] - r[Z[\Gamma]]$ of the group $K_0^S(Z[\Gamma])$, where r is the rank of Z_N . Firstly, $U_{N/K}$ is in the torsion subgroup $\tilde{K}_0^S(Z[\Gamma])$ of $K_0^S(Z[\Gamma])$. Let $\tilde{\mathcal{C}}_{\oplus}^S(Z[\Gamma])$ be the torsion subgroup of the Grothendieck group which is associated to the category of Z -torsion free, finitely generated $Z[\Gamma]$ -modules and which is defined modulo the exact sequences which split outside of S . There is a surjective homomorphism from $\tilde{K}_0^S(Z[\Gamma])$ in $\tilde{\mathcal{C}}_{\oplus}^S(Z[\Gamma])$. The image of $U_{N/K}$ is trivial. To show this result, I give a complete description of these Grothendieck groups and more generally I define and describe S -Grothendieck groups of an order in a separable algebra.

I. REINER (s. BUSHNELL)

J. RITTER: Fröhliche Klassengruppen

The aim of my lecture was to give a report on Fröhlich's description of the class group $Cl(\mathfrak{A})$ of locally free \mathfrak{A} -lattices for an arithmetic order \mathfrak{A} sitting in a semisimple algebra A .

Denote by Z the centre of A , by $J(Z)$ the idèlegroup of Z , and by $U(\mathfrak{A})$ the group of unit idèles of \mathfrak{A} , that is $U(\mathfrak{A}) = \prod_p \mathfrak{A}_p^*$; finally denote by nr the reduced norm from A to Z . Then $Cl(\mathfrak{A}) = J(Z)/Z^* nrU(\mathfrak{A})$, and, moreover, one has a cancellation property which means that two lattices of rank at least two are isomorphic if and only if they induce the same element in $Cl(\mathfrak{A})$, this fact being also true in case of rank 1 if A satisfies the Eichler condition. When $\mathfrak{A} = \mathcal{O}_K G$ which is the integral group ring of a finite group G over the integers of a number field K , one can nicely separate the components of Z by using the characters of G to get Fröhlich's fundamental isomorphism $Cl(\mathcal{O}_K G) \simeq \text{Hom}_{\mathbb{Q}}(RG, J(E))/\text{Hom}_{\mathbb{Q}}(RG, E^*) \text{Det } U(\mathfrak{A})$ and also to get the corresponding isomorphism with respect to the kernel subgroup which shows up when $\mathcal{O}_K G$ is embedded in some maximal order: $D(\mathcal{O}_K G) = \text{Hom}_{\mathbb{Q}}^+(RG, U(\mathcal{O}_E))/\text{Hom}_{\mathbb{Q}}^+(RG, \mathcal{O}_E^*) \text{Det } U(\mathfrak{A})$. There RG denotes the ring of virtual complex characters of G , E is some splitting field for G and a finite Galois extension

of K with group \mathcal{G} , and $\text{Det}(\alpha)$, for some unit idèle α , is the homomorphism which sends an irreducible character χ to $\det(T_X(\alpha))$, T_X being the corresponding representation. The + sign in the description for $D(\mathcal{O}_K G)$ indicates that only those homomorphisms are to be considered whose values at the irreducible characters of the Schur index 2 over the reals are positive at each infinite place of E . - These homomorphism groups reflect very nicely the functorial behaviour of the class groups with respect to field embeddings and various group homomorphisms. An essential example in the theory is the ring of integers \mathcal{O}_L of a Galois extension $L|K$ with group G , viewed as a $\mathcal{O}_K G$ -module. $\hat{\mathfrak{A}}$ is locally free if and only if the extension is tamely ramified, and in this case in $\text{Cl}(\mathcal{O}_K G)$ it is represented by the homomorphism $\chi \mapsto (b|\chi)/(a|\chi)$, where a generates a normal basis of $L|K$ and $(a|\chi)$ is Fröhlich's generalized resolvent, namely $(a|\chi) = \det(\sum_{\gamma \in G} a^\gamma T_X(\gamma^{-1}))$, and where b is the idèle given by $\mathcal{O}_{L,p} = b_p \mathcal{O}_{K,p} G$ and where again $(b|\chi)$ is its resolvent.

R. SANDLING: Graham Higman's thesis

The contributions of Graham Higman's 1940 thesis, *Units in Group Rings*, are not widely known; most of the results have been rediscovered over the years. In it many yet unsolved problems are raised for the first time such as the isomorphism problem and the existence of zero divisors and non-trivial units for the case

of torsion-free groups. The classification of group rings having only trivial units, done for ZG in his earlier paper of the same title, is completed for RG , R an arbitrary ring of algebraic integers. It is based on a comparison of $U(RG)$ with $U(M)$, M a maximal order containing RG . The interpretation of RG as an order is also used for other purposes. Most of the significant results on units of finite order which are known at present appear. They are established by methods (representation theory, augmentation ideals) now taken for granted. The finite subgroups of $U(RG)$ for certain finite metabelian groups are determined: for the nilpotent case, in which the main ideas of the proof of the isomorphism problem for the general finite metabelian group are present; for the case of the affine group of a field of odd prime order, in which an explicit integral representation is constructed. For infinite groups, the two unique products condition is introduced and exploited.

L.SCOTT: Hecke actions on Picard groups

This work is joint with Klaus Roggenkamp, and was inspired by Robert Perlis' paper on arithmetically equivalent number fields. We formulate Perlis' method in terms of functors, and observe this gives improvements in class group theorems of Nehr Korn and Walter. In particular we show the class group of an abelian extension K/Q can be computed in terms of the class groups of

K^H for G/H ranging over the elementary (in Brauer's sense) quotient groups of the Galois group of K . We show, moreover, that Perlis' method is quite general and holds for Picard groups of commutative rings and usually even for schemes. In particular we prove

Theorem: Let G be a group acting on a commutative ring A . Then there is a contravariant additive functor from the category of permutation modules for G to abelian groups, which for any subgroup H of G , assigns ZG/H to Pica^H .

An important ingredient in the proof is a relative Picard group similar to the locally free Picard group introduced by Fröhlich for orders: Let (S, \mathcal{O}_S) be a ringed space and A a commutative \mathcal{O}_S -algebra. Define $S\text{-Pic}A$ to be the set of isomorphism classes of A -modules for which there is an open cover U of S with $\pi/U \simeq A/U$ for each U in U . A standard argument shows $S\text{-Pic}A \simeq H^1(S, A^*)$. Now the idea is to consider the composition of functors $ZG/H \mapsto A^{*H} \mapsto H^1(S, A^{*H})$ where G acts on A , and then to use commutative algebra to return to absolute Picard groups.

M.R. STEIN: K_2 of integral group rings

Algebraic K -theorists have recently developed several refinements of Mayer-Vietoris sequence techniques which allow the computation of the K_2 's of certain integral group rings of interest to topo-

logists and number theorists. The principal new example discussed in detail was

$K_2(\mathbb{Z}[C_2 \times C_2])$ is an elementary abelian 2-group of rank 6, where C_2 is a cyclic group of order 2.

O.TAUSSKI-TODD: Some facts concerning integral representations of ideals in an algebraic number field

An integral representation by $n \times n$ matrices given for ideals in an order of an algebraic number field of degree n via a regular representation of the field referred to a basis of integers. It is known that the ring $\mathbb{Z}^{n \times n}$ of integral matrices is a principal ideal ring and that any set of elements in this ring has a greatest common right divisor (gcd) which can be obtained by a routine method. This method goes back to Du Pasquier. The gcd is determined up to a unimodular factor on one side. Let G_0 be a representative of this set for an ideal \mathfrak{a} . It was shown by MacDuffee that G_0 is a so-called ideal matrix (a concept going back to Poincaré), i.e. a matrix which transforms a basis w_1, \dots, w_n for the order into a basis $\alpha_1, \dots, \alpha_n$ for the ideal

$$G_0 \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix}$$

The determination of G_0 is of importance for finding the generator of a principal ideal. Such a generator is an element of the map of the ideal. However, a unimodular factor of G_0 may prevent this happening here; hence further work is necessary to determine a factor U , which when multiplied by a particular G_0

commutes with all the elements of the ideal. This leads to a set of linear equations, which e.g. for $n = 2$, has a solution depending on 2 parameters. To ensure that U is also unimodular leads then to constructing a binary quadratic form in these two parameters which has to assume the value $+1$ or -1 . If the ideal is not principal then for all unimodular U 's the product of U and G_a will not commute with all the elements of the map of the ideal. Hence G_a generates as one-sided principal ideal in $Z^{n \times n}$. In the case $n = 2$, G_a for a non-principal prime ideal has eigenvalues in some quadratic field in which the corresponding rational prime number splits into principal ideal factors.

ST.V.TAYLOR: Fröhlich's conjecture and logarithmic methods

The talk started with a statement of the theorem that Fröhlich's conjecture holds for any ring of integers of E of a Galois extension of M with the property that the prime divisors of $(E:M)$ are non-ramified in E/Q . The remainder of the lecture was devoted to a description of the algebraic technique used in the proof of this result. In particular the use of a new "integral" logarithm on the group of determinants of local group rings was described.

TH.THEOHARI-APOSTOLIDI: On integral representations of twisted group rings

Let R be a complete discrete valuation ring with quotient field

K , L/K a Galois extension of finite degree with Galois group G , S the valuation ring of L and P_0 (resp. P) the prime ideal of R (resp. S).

The purpose of this lecture is to give some results about the representation type of the twisted group ring $\Lambda = S^\circ G$. This is well known ⁱⁿ the case of tame extensions, but here the general case is examined, including the wildly ^{ramified} extensions. The main theorem is the following:

Λ is of finite representation type if and only if the first ramification group ^{G_1} of the prime ideal P in the extension L/K is of order 1 or 2. If $G = G_1$ and $|G|=2$ the integral representations of Λ are characterized by the three non-isomorphic indecomposable Λ -lattices Λ , S , P . In this last case $\text{rad} \Lambda = \text{rad} \Gamma$, where Γ is the intersection of maximal orders in $L^\circ G$ containing Λ . This permits easily the computation of ^{the} ζ -function of Λ .

CH. THOMAS: Integral representations in the study of finite Poincaré complexes

Let Γ be a finite group and $D(Z\Gamma)$ the subgroup of the projective class group consisting of elements $[P]$ such that P becomes free over a maximal order $m_r \subseteq Q\Gamma$. If (r, Σ) denotes the rank 1 projective module generated by r (coprime with $|\Gamma|$) and Σ , the sum of the group elements, the classes $[(r, \Sigma)]$ form a subgroup $T(Z\Gamma)$ of D . Under favourable hypotheses (such as $H_*(\tilde{Y})$ nilpotent over $Z\Gamma$), the Wall finiteness obstruction $\sigma(Y)$ for a finitely dominated CW-complex lies in $D(Z\Gamma)$ - provided

we allow multiplication by the Artin exponent $A(\Gamma)$, when Γ is not nilpotent. Furthermore there are circumstances in which the module (r, Σ) can be used to modify the homotopy type of Y . In one direction this can be used to test whether (r, Σ) maps to 0 in $\tilde{K}_0(Z\Gamma)$, thus using representation theory over C for the group D_{pq}^* , the extension of a cyclic group of order pq by one of order q , we can show that $(r, \Sigma) \sim 0$ if $r \equiv S_1^q \pmod{p}$ & $r \equiv S_2^q \pmod{q^2}$. In the other direction algebra can be used to prove non-trivial topological results - thus when $h^+(p)$ is odd, there exists a free action of D_{4p}^* ($q=2$ above) on S^{4k-1} ($k \geq 2$), such that the orbit manifold is not homotopy equivalent to a space of constant positive curvature. (Besides knowledge of the subgroup $T(Z\Gamma)$ in this case, one needs to have the structure of the surgery obstruction group $L_3^S(Z\Gamma)$.)

ST.V.ULLOM: Galois module structure for intermediate extensions

We compare the Galois module structure of the ring of integers of tamely ramified extensions N and N' of F satisfying

- (1) $\text{Gal}(N/F) \simeq \text{Gal}(N'/F) =$ quaternion group H_{4n} , and
- (2) N and N' are quadratic over a field K containing F .

Let E be a quadratic extension of F such that K/E is cyclic, Δ_A^n cyclic group of order 2^n , $O = \text{int } E = \text{int } E$. The class of the ratio $(\text{int } N')(\text{int } N)^{-1}$ in the class group $\text{Cl}(O\Delta)$ is related to the ratio of Artin root numbers W_N/W_N for quaternion characters.

P.J.WEBB: Restriction to elementary subgroups for lattices

We describe a theorem which bounds the ranks of ZG-lattices, where G is a finite group, in terms of the ranks of their non-projective cores on restriction to elementary abelian p -subgroups of G .

Theorem: Let G be a finite group. There exists a constant B with the property that if M is a ZG-lattice there is a prime $p \mid |G|$ and an elementary abelian p -subgroup E of G for which $\text{rank}_Z \text{core}(M) \leq \text{rank}_Z \text{core}(M|_E)$.

This result is an integral version of a recent theorem of J. Carlson. It can be used to prove an integral version of a theorem of Alperin and Evens concerning the complexity of modules.

A.WIEDEMANN: Auslander-Reiten graphs of orders and blocks of cyclic defect two

Let R be a complete Dedekind domain and let Λ be an R -order in a separable algebra over the quotient field of R .

Criterion for finite lattice type: Assume Λ to be twosided indecomposable. If \mathcal{C} is a finite component of the Auslander-Reiten graph $\mathfrak{A}(\Lambda)$, then Λ has finite lattice type, and $\mathcal{C} = \mathfrak{A}(\Lambda)$.

The components of stable Auslander-Reiten graphs with a periodic vertex were described. Moreover, a classification of those orders with loops in their Auslander-Reiten graph was given in case $R/\text{rad } R$ is finite.

Let G be a finite group, R the p -adic integers. In the second

part, the Auslander-Reiten graph of a block \mathfrak{B} of RG with cyclic defect group of order p^2 was indicated. Using the above criterion one gets immediately the number $(4p + 1) \cdot e$ of indecomposable \mathfrak{B} -lattices ($e = \#$ simple \mathfrak{B} -modules).

J. WILLIAMS: Prime graph components of finite groups

Let G be a finite group, define the prime graph as follows: The vertices are the primes dividing the order of G and an edge joins p, q iff G contains an element of order pq . A subgroup H of G is isolated (CCT) if $H \cap H^g = \langle 1 \rangle$ or H and $C_G(h) \subseteq H \forall h \in H^\#$.

Theorem: Let G be a group all of whose composition factors are either simple Chevalley groups or a sporadic group or C_p , let π_1 be a closed connected component of the prime graph and suppose $\pi_1 \not\subseteq \pi(G)$ and $2 \notin \pi_1$. Then either G is Frobenius or 2-Frobenius or G contains an isolated Hall π_1 -subgroup which is nilpotent.

This yields to following application to integral representations.

Theorem: Let G be a group all of whose composition factors are simple groups of K-type, then the following are equivalent:

- (a) Z is ^{not} Heller
- (b) \mathfrak{G} , the augmentation ideal, decomposes
- (c) $\pi(G)$ has more than one component
- (d) G contains an isolated subgroup.

S.M.J. WILSON: Miyata's Theorem on the transfer map from the class group of a dihedral group to that of a cyclic group

Miyata has proved the following theorem:

"Let G be a cyclic group and $D_0 = C_0 \rtimes \Gamma$ be the corresponding dihedral group then the restriction map $Cl(ZD_0) \rightarrow Cl(ZC_0)$ is injective."

In this talk I indicated a proof of a generalization of this result where C_0 is assumed only to be finite abelian with cyclic 2-torsion (the conclusion remaining the same), Γ acting by inversion.

My proof, which is, reputedly, shorter than that of Miyata, involves some simple facts about twisted group rings; uses the idelic formula for the class groups as given in my paper "Reduced norms in the K-theory of orders" J.Alg.1977, and employs the inclusion $ZD_0 \subseteq \Lambda D \oplus ZD$ corresponding to the cartesian square

$$\begin{array}{ccc} Z D_0 & \rightarrow & Z D \\ \downarrow & & \downarrow \\ \Lambda D & \rightarrow & (Z/nZ)D \end{array}$$

where D is the Sylow 2-subgroup of D_0 , $n = |D_0/D|$ and $\Lambda = Z(D_0/D)/(\Sigma \text{ groupelts})$. (Miyata's proof also uses this diagram). Among other applications, one can use this result to deduce that every D_0 -extension L/K of algebraic number fields has a normal integral Z -basis. (This uses the result of Taylor for abelian extensions.)

H.ZASSENHAUS: On F.C.subrings of rings

A theorem of A.Williamson (1978) "The supercentre of a group G defined as $SC(G) = \{x | x \in G : |x^{U(ZG)}| < \infty\}$ is a characteristic subgroup of G containing the centre $C(G)$ of G . It con-

tains $C(G)$ properly precisely if there is an element x of order 4 and another element c of G such that $c^2 = x^2 = (cy)^2$ for all y of $C_G(x)$. In this event either $G = SC(G)$ is a hamiltonian 2-group or else $SC(G) = \langle x, C(G) \rangle$ " suggested to the speaker jointly with Sudarshan K. Sehgal the definition of the F.C. subring of an arbitrary ring Λ as $FC(\Lambda) = \{x | x \in \Lambda : |x^{U(\Lambda+Z)}| < \infty\}$ where $\Lambda + Z$ is the unital ring generated by Λ . $FC(\Lambda)$ is a characteristic subring of Λ containing the centre $C(\Lambda)$ of Λ . If Λ is finite or commutative then $FC(\Lambda) = \Lambda$. If Λ is infinite simple then $FC(\Lambda) = C(\Lambda)$. $FC(\bigoplus_{i=1}^S \Lambda_i) = \bigoplus_{i=1}^S FC(\Lambda_i)$.

Theorem 1: If Λ is a Z -order then the maximal ideal $[\Lambda : FC(\Lambda)] = [FC(\Lambda) : \Lambda]$ of Λ contained in $FC(\Lambda)$ is the intersection of the kernels of the irreducible representations Δ of Λ over C for which $Q \Delta \Lambda$ is a totally definite quaternion algebra, and $FC(\Lambda) / [\Lambda : FC(\Lambda)] = C(\Lambda) / [C(\Lambda) : FC(\Lambda)]$. $FC(\Lambda)$ is commutative $\Leftrightarrow (U(\Lambda) : U(C(\Lambda))) < \infty$, otherwise the unitgroup $U(\Lambda)$ of Λ contains a free subgroup on 2 generators. The proofs use only Dirichlet's unit theorem, in particular the remark that the number of fundamental units of an algebraic number field E and a proper subfield F coincide precisely if F is totally real with E as totally complex quadratic extension.

Theorem 2: If Λ is the group ring ZG of a finite group G then the kernels occuring in theorem 1 are in 1-1-correspondence with the factor groups of G of the form $\langle a, b \rangle$ with defining relator sets $a^2 = b^m = (ab)^n$ and representations Δ defined by

$$\Delta(a) = \begin{bmatrix} & -1 \\ 1 & \end{bmatrix}, \quad \Delta(b) = \begin{bmatrix} \cos \frac{\pi}{m}, \cos \frac{\pi}{n} + bi \\ -\cos \frac{\pi}{n} + bi, \cos \frac{\pi}{m} \end{bmatrix}$$

(b real, $b^2 + \cos^2 \frac{\pi}{m} + \cos^2 \frac{\pi}{n} = 1$) such that

I. $m \geq 2$, $n = 2$ (projective dihedral groups), II.- IV. $m = 3$, $n = 3, 4$ or 5 projective tetrahedral ($SL(2,3)$), octahedral or icosahedral ($SL(2,5)$) groups. - As a corollary an order-theoretic proof of Williamson's theorem is obtained. An alternative specially group ring theoretic argument was sketched already in S.K. Sehgal's book on group rings.

In this connection a simplified proof characterizing $SL(2,5)$ as nontrivial finite group such that every subgroup of order pq is cyclic - p and q rational primes - is given (Zassenhaus 1934).

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