MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 31/1980

Konstruktive Verfahren in der komplexen Analysis

7.7. bis 11.7.1980

Die zweite Tagung im Mathematischen Forschungsinstitut Oberwolfach über "Konstruktive Verfahren in der komplexen Analysis" wurde geleitet von D. Gaier (Giessen), P. Henrici (Zürich) und R. Varga (Kent). Es waren 41 Teilnehmer zusammengekommen, die Hälfte hatte schon an der ersten Tagung im Sommer 1978 teilgenommen. 15 Teilnehmer waren aus Uebersee angereist.

Die meisten der 32 Vorträge (von 30 bis 45 Minuten Dauer) beschäftigten sich mit konformen und quasikonformen Abbildungen und komplexer Approximationstheorie. Die konformen Abbildungen ein- und zweifach zusammenhängender Gebiete wurden mit Integralgleichungsmethoden und Schmiegungsverfahren berechnet. Für die Approximation quasikonformer Abbildungen kamen finite Elemente zur Anwendung. Als Anwendung der konformen Abbildung wurden isoperimetrische Schranken, Kapazitäten von Vierseiten und ein Strömungsproblem behandelt. Ein Teil der Vorträge über komplexe

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Approximationstheorie befasste sich mit Tschebyscheff-Approximationen, Konvergenzsätzen über rationale Interpolation und Interpolation durch ganze Funktionen. Andere Vorträge beschäftigten sich mit der Nullstellenverteilung von Partialsummen ganzer Funktionen. Auch wurden Algorithmen diskutiert zur Berechnung von Polen meromorpher Funktionen mit dem qd-Algorithmus, zur Nullstellenbestimmung bei Polynomen, zur Berechnung von Padé- und Newton-Padé-Approximationen und zur analytischen Darstellung von Nullstellenfolgen bei Exponentialsummen.

Weitere Vorträge befassten sich mit Kettenbrüchen, Koeffizientenabschätzungen für gewisse Polynome, Stetigkeit des Birkhoff-Operators, Berechnung komplexer Integrale, konjugierter Funktionen und des Szegö-kerns, Lösung gewisser transzendenter Gleichungen, nichtlinearen Randwertproblemen und Taubersätzen.

Es war zu bemerken, dass sich die Anwendung moderner Hilfsmittel, wie Splines, finite Elemente bei den konstruktiven Verfahren der komplexen Analysis durchgesetzt hat, dass aber auch vermehrt wieder auf analytische Methoden zurückgegriffen wird.

Während der ganzen Woche wurde die Tagung von schlechtem Wetter begleitet. Bei dem auf Donnerstag nachmittag verschobenen Ausflug besuchte etwa die Hälfte der Teilnehmer in strömendem Regen das Feilichtmuseum Vogtsbauernhof und liess sich dort in die Geschichte der Lebensweise der Schwarzwaldbevölkerung einführen.

Teilnehmer

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Vortragsauszüge

H. BEGEHR:

Nichtlineare Randwertprobleme für Cauchy-Riemannsche Systeme

Die grundlegenden Randwertprobleme für analytische Funktionen - das Riemannsche und das Hilbertsche Problem - lassen sich für nichtlineare elliptische Systeme erster Ordnung, komplex in der Form $w_{\bar{z}} = H(z,w,w_z)$ gegeben, auch dann noch lösen, wenn die Randbedingungen nichtlinear, aber genügend gutartig sind. Ist H linear in der letzten Variablen, so lässt sich unter geeigneten weiteren Voraussetzungen eine mit dem Newtonschen Verfahren gekoppelte Einbettungsmethode anwenden, um die Lösungen approximativ zu berechnen. Die Methode geht auf Wacker und Wendland zurück. Die Untersuchungen wurden zusammen mit G. N. Hile und G. C. Hsiao durchgeführt.

H. BRASS:

Eine Bemerkung zur numerischen Berechnung konjugierter Funktionen

Das Problem der numerischen Berechnung konjugierter Funktionen lässt sich so präzisieren: Gegeben die Werte $f(t_i)$ für $i = -n+1, \ldots, n$; $t_i = i\pi n^{-1}$ einer 2π-periodischen Funktion; gesucht Näherungen $\tilde{K}[f](t_i)$ für die Werte der konjugierten Funktion an den gleichen Stellen. Die Definition der konjugierten Funktion lässt es plausibel erscheinen, nur Näherungsformeln der Gestalt

$$\tilde{K}[f](t_i) = \sum_{k=-n+1}^{n} a_{k-i} f(t_k) \quad \text{mit} \quad \sum_{k=-n+1}^{n} a_{k-i} = 0$$

(Indizes mod 2n)

zu betrachten.

Die folgende Aussage wird bewiesen und präzisiert: In der Klasse $\{f;\ |f^{(s)}| \leqq 1\}$ ist der Maximalfehler bei jedem derartigen Verfahren $\geqq \text{const}_s\ n^{-s} \ln n$; und für das gängigste spezielle Verfahren gilt hier Gleichheit.

S. W. ELLACOTT:

On the convergence of some approximate methods of conformal mapping

Some results of approximation theory and in particular the Generalized Bernstein Lemma are used to discuss the uniform convergence of some methods for approximate conformal mapping in the case that the boundary of the region to be mapped is analytic.

P. GEIGER:

Nullstellenbestimmung bei Polynomen als Anwendung der FFT

Problem: Gesucht sind die Nullstellen $z_1, \ldots, z_n$ des Polynoms

$$p(z) = a_0 z^n + \ldots + a_n .$$

Es wird ein Algorithmus hergeleitet, der das Problem reduziert auf die Nullstellenbestimmung von Polynomen niedrigeren Grades. Sei

$\Gamma : |z| = 1$ (resp. beliebiger Kreis)

$$\sigma_k := \frac{1}{2\pi i} \int_\Gamma z^k \frac{p'(z)}{p(z)} dz = \sum_{|z_\ell|<1} z_\ell^k$$

$$= \int_0^1 \frac{p'(e^{2\pi i t})}{p(e^{2\pi i t})} e^{2\pi i (k+1) t} dt = a_{-(k+1)} \quad \text{Fourierkoeff. von } \frac{p'}{p}$$

$\Rightarrow$ Berechnung der σ_k mittels FFT möglich. Die σ_k enthalten nur die Nullstellen mit $|z_\ell| < 1$. Die für die weitere Reduktion notwendige Funktion $\frac{q'}{q}$ wird als Kettenbruch dargestellt:

$$\frac{q'(z)}{q(z)} = \frac{\sigma_0|}{|z - q_1^{(0)}} - \frac{q_1^{(0)} e_1^{(0)}|}{|z - e_1^{(0)} - q_2^{(0)}} - \cdots - \frac{q_{m-1}^{(0)} e_{m-1}^{(0)}|}{|z - e_{m-1}^{(0)} - q_m^{(0)}} .$$

Numerische Experimente zeigen, dass das Verfahren brauchbar ist, jedoch nicht effizient.

Dieses Verfahren kann auf beliebige anal. Funktionen angewendet werden.

W. B. GRAGG:

Truncation error bounds for totally positive definite T-fractions

We describe the elementary theory of totally positive definite T-fractions. $F(z) = a_1/1 + b_1 z + a_2 z/1 + b_2 z + a_3 z/1 + b_3 z + \ldots$ $(a_n > 0,\ b_n > 0)$ and its connection with the strong Stieltjes moment problem $\gamma_n = \int_0^\infty t^n d\mu(t)$, $n = 0, \pm 1, \pm 2, \ldots$ In particular, we describe the best possible inclusion lunes for $\Phi(z) = \int_0^\infty d\mu(t)/(1+tz)$ when the moments $\gamma_{-n}, \ldots, \gamma_0, \ldots, \gamma_n$ are known, and find several bounds for their diameters. It will be seen that $\sum_z^\infty (\gamma_{-n}\gamma_n)^{1/4(n-1)} = +\infty$ is sufficient for the moment problem to be determinate. We also note that a necessary condition for the moment

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problem to be solvable, with μ having infinitely many points of increase, is that the doubly infinite Hankel matrix (γ_{i+j}) be strictly totally positive (definite), and that a sufficient condition is that $\gamma_n^{(-n)} > 0$, $\gamma_n^{(-n+1)} > 0$, $n \geqq 1$, where $\gamma_n^{(k)} \equiv \det(\gamma_{k+i+j})_{i,j=0}^{n-1}$.

M. GUTKNECHT:

Real Chebyshev approximation by complex methods

In this talk the possibility and usefulness of transplanting best or near-best complex rational Chebyshev approximations from the unit circle ∂D onto the interval $I := [-1,1]$ is pointed out. When the error curve on the unit circle is nearly circular, which is typically the case, the real part of the transplanted error curve nearly equioscillates, and therefore the deviation is close to minimal. In particular, transplanting approximations computed with the Carathéodory-Fejér method turns out to be very effective for real near-best approximation on I. Its effectiveness for smooth functions is manifest both in excellent asymptotic behaviour and in numerical examples. Moreover, this transplantation reveals the connection between some examples of explicitly known best approximations on ∂D and I.

P. HENRICI:

The method of Burniston and Siewert for solving certain transcendental equations

Let f be analytic in $\mathbb{C}\backslash\Gamma$, where Γ is a simple smooth arc (or a finite collection of such arcs). We assume that f has at most a pole at ∞ , that f is "pole-like" at the endpoints of Γ , and that for every interior point $t \in \Gamma$ the one-sided limits $f^+(t)$ and $f^-(t)$ exist, are $\neq 0$, and define Hölder continuous functions of t . THEOREM. (i) The equation $f(z) = 0$ has at most finitely many solutions; (ii) the polynomial whose zeros are precisely these solutions can be constructed explicitly in the sense that its coefficients are rational functions of definite integrals over Γ of elementary functions of f^+ and f^- . The PROOF uses the fact that f is a solution of a Privalov problem $f^+(t) = a(t) f^-(t)$, whose general solution has a well-known representation in terms of Cauchy integrals. As an APPLICATION we find precise (non-asymptotic) information concerning the solutions of $(z-\alpha)e^z = z-\beta$.

J. HERSCH:

Konforme Abbildung und isoperimetrische Schranken für Gebietsfunktionale

Die Anwendung der konformen Abbildung zur Abschätzung von Gebietsfunktionalen in zwei Dimensionen beruht vor allem auf der Invarianz des Dirichletschen Integrals bei konformer Verpflanzung. Wird in einem Rand- bzw. Eigenwertproblem ein Gebietsfunktional durch eine Variationsaufgabe charakterisiert, so verwendet man als Probierfunktionen die konform verpflanzten Lösungsfunktionen (bzw. Eigenfunktionen) des entsprechenden Problems in einem Normalgebiet (oft die Kreisscheibe). So werden isoperimetrische Ungleichungen von Pólya, Schiffer, Szegö und anderen Autoren bewiesen, welche die Eigenwerte einer Membran mit Hilfe des Abbildungsradius abschätzen. Aehnliches tut man auch beim Stekloff-Eigenwertproblem sowie beim Problem der Torsionssteifigkeit.

—— C. Bandle hat auch eine isoperimetrische Abschätzung des kritischen Parameters eines nichtlinearen Randwertproblems mit Hilfe des Abbildungsradius gegeben.

H.-P. HOIDN:

An osculation method for the conformal mapping of simply and doubly connected regions

An osculation method (Schmiegungsverfahren) is described. The method approximates the conformal mappings which map a simply connected region onto the unit disk and a doubly connected region onto an annulus by a composition of elementary mappings. These elementary mappings map regions bounded by two or three circular arcs onto the unit disk or are the Koebe square root transformation. They are chosen so that the point on the outer boundary with the minimal modulus is pushed outwards. Therefore the boundaries (due to inversion after each iteration in the doubly connected case) become closer to circles and the image regions of the approximation converge to the unit circle or an annulus. The convergence proofs of the methods are existence proofs for the Riemann mapping theorem and the existence theorem of the mapping of a doubly connected region onto an annulus. The methods have slow asymptotic convergence rate but for a desired accuracy of 1 % they work fast even for complicated regions with slits and corners.

W. B. JONES:

Orthogonal Systems of Laurent Polynomials and Gaussian Quadrature

Orthogonal rational functions similar in many respects to the classical orthogonal polynomials are introduced. These functions are linear sums of integral (positive, negative and zero) powers of a complex variable z ; hence they are called orthogonal Laurent polynomials. As in the case of orthogonal polynomials, the theory of orthogonal Laurent polynomials arose from investigations of a class of continued fractions (in this case positive T-fractions). These continued fractions have recently been studied in connection with two-point Padé approximants and a strong Stieltjes moment problem. Results of the latter study formed the starting point of the present work. In addition to existence and uniqueness theorems for orthogonal Laurent polynomials, the talk also included Gaussian quadrature formulas and associated convergence theorems. The new results have close analogues in the theory of orthogonal polynomials, including theorems of Favard, Gauss, Markoff and Stieltjes.

G. G. LORENTZ

Continuity of the Birkhoff Operator

The following theorem is proven. Let $P_n(f,X,E)$ be the polynomial of degree $\leq n$ which interpolates the values of the function $f \in C^{n+1}$ at points $X : x_1 < x_2 < \ldots < x_m$:

$$P_n^{(k)}(x_i) = f^{(k)}(x_i) \quad \text{for} \quad e_{ik} = 1 . \tag{1}$$

If some x_i become equal, one has to coalesce rows of the matrix E

and change accordingly equations (1). Also with this stipulation, P_n remains continuous in the uniform norm as a function of X. There are applications to quadrature formulas.

K. MENKE:

Näherung der Lösung des Dirichlet Problems durch ein Interpolationsverfahren

Mit Hilfe einer Extremaleigenschaft für Determinanten hat Curtiss ein Punktsystem $\{w_{nk}, k = 1, \ldots, 2n+1; n \in \mathbb{N}\}$ auf einer geschlossenen Jordankurve C erklärt. Für den Fall, dass C eine analytische Kurve ist, werden Aussagen über die Verteilung dieser Punkte auf C gemacht.

Weiter sei U eine auf C gegebene (mindestens) stetige Funktion, u die im Innengebiet G von C harmonische Funktion, die $U(w) = u(w)$ für $w \in C$ erfüllt. Es wird u angenähert durch harmonische Polynome H_n n-ten Grades, die $H_n(w_{nk}) = U(w_{nk})$ $(k = 1, \ldots, 2n+1)$ genügen. Dieses Interpolationsproblem ist nach einem Ergebnis von Curtiss für jedes $n \in N$ stets eindeutig lösbar. Für analytische Jordankurven werden Aussagen - in Abhängigkeit von U - über die Güte der Konvergenz von H_n gegen u auf $\overline{G}$ gemacht.

G. P. MEYER:

On the analytical representation of sequences of zeros by exponential sums

The purpose of this talk is to present a constructive method which uses a formula of Lagrange-Bürmann-Henrici type for solving transcendental equations of the form

$$w(z) := \sum_{k=1}^{m} P_k(z)\, e^{a_k z} = 0$$

($0 \neq P_k(z) \in \mathbb{C}[z]$, $a_k \in \mathbb{C}$, $a_i \neq a_j$ for $i \neq j$, $m \geqq 2$). It is well known owing to G. Pōlya and E. Schwengeler that the zeros of $w(z)$ asymptotically osculate a finite number of logarithmic lines.

We succeed in deriving convergent and asymptotic expansions for sequences of zeros of $w(z)$ "along" such a logarithmic line if on the corresponding side of the extended indicator diagram (in the sense of Pōlya-Schwengeler) there are exactly two degree-points.

G. OPFER:

Complex planar splines

There exists literature on "complex splines", which in that literature are defined on the boundary of a region in $\mathbb{C}$ and which are then extended into the interior by Cauchy's integral formula. Pars prototo we mention I. J. Schoenberg: On polynomial spline functions on the circle I + II, in G. Alexits - S. B. Stechkin (eds.): Proceedings of the conference on constructive theory of functions, Akadémiai Kiadō, Budapest, 1972, pp. 403-433.

Here we offer another approach which in spirit stems from the theory of finite elements. That means we subdivide a given region R into meshes and define a complex valued function on R piecewise on each mesh. If such a function is continuous we shall call it a "complex planar spline". Besides aspects known form the theory of finite elements there are new aspects which consist of the mapping properties of such a spline v and the mapping proberties of $f - v$ when f is a holomorphic function. For triangular grids some detailed investigations are presented. For parallelogram grids and curvilinear meshes on circular sectors a construction for planar splines is given. Besides approximating functions it should be possible to construct approximate solutions of Beltrami's equation $f_{\bar{z}} = \mu f_z + \nu \overline{f_z}$ with the means of complex planar splines.

N. PAPAMICHAEL:

The use of splines and singular functions in an integral equation method for conformal mapping

We consider the integral equation method of Symm for the conformal mapping of simply-connected domains. For the numerical solution, we examine the use of spline functions of various degrees for the approximation of the source density σ. In particular, we consider ways for overcoming the difficulties associated with corner singularities. For this we modify the spline approximation and in the neighbourhood of each corner, where a boundary singularity occurs, we approximate σ by a function which reflects the main singular behaviour of the source density. The singular functions are then blended with the splines, which approximate σ on the remainder of the boundary, so that the global approximating function has continuity of appropriate order at the transition points between the two types of approximation. We show, by means of numerical examples, that such approximations overcome the difficulties associated with corner singularities and lead to numerical results of high accuracy.

This is a report of a joint work with D. Hough from the Polytechnic of the South Bank, London.

Q. I. RAHMAN:

Coefficient regions for univalent trinomials

In the year 1968, Cowling and Royster (Proc. Amer. Math. Soc. 19 (1968), 767-772) proposed the problem of determining the region of variability of (a_2, a_k) for the univalent trinomial $z + a_2 z^2 + a_k z^k$, for $k > 3$. Jointly with J. Waniurski (Can. J. Math. 32 (1980), 1-20) we have shown that the trinomial

$$f(z) := z + a_p z^p + t z^q , \qquad (p < q , \quad 0 < t \leqq 1/q)$$

is univalent in $|z| < 1$ if and only if

$$a_p \in \bigcap_{0 \leqq \theta \leqq \pi/2} \frac{\sin \theta}{\sin p\theta} \overline{G}_\theta =: D(t,p,q)$$

where G_θ is the region determined by the curve

$$w(\phi) = e^{-i(p-1)\phi} + t \frac{\sin q\theta}{\sin \theta} e^{i(q-p)\phi} , \qquad 0 \leqq \phi \leqq 2\pi$$

and containing the origin.

A careful study of the region G_θ leads us to the conclusion that

$$D(t,p,q) = \frac{1}{p} \overline{G}_0$$

if $q - 1$ is not an integral multiple of $p - 1$. In other words, $f(z)$ is univalent in $|z| < 1$ if and only if it is locally univalent provided $q-1 \neq j(p-1)$, $j = 2, 3, \ldots$. If $p = 2$ then $q - 1$ is necessarily a multiple of $p - 1$. We show that in that case the above conclusion holds if $t \in (0, \frac{3}{q(q^2-4)}]$.

E. B. SAFF:

Zeros of Partial Sums of Entire Functions

It has been shown by Saff and Varga that the partial sums $s_n(z) = \sum_{k=0}^{n} z^k/k!$ of e^z are each zero-free in the parabolic region $\{z = x+iy : y^2 \leqq 4(x+1),\ x > -1\}$. Furthermore, Newman and Rivlin have shown that parabolic growth characterizes the width of the largest unbounded zero-free region for the partial sums. These results suggested the following conjecture which would generalize the classical work of F. Carlson and P. C. Rosenbloom on the zeros of partial sums of entire functions of order 0 .

Width Conjecture (Saff and Varga). Let $f(z) = \sum_{0}^{\infty} a_k z^k$. If all the partial sums $S_n(z) = \sum_{0}^{n} a_k z^k$, $n = 1, 2, \ldots$, are zero-free in the region $\{z = x+iy : |y| \leqq \beta_0 x^{1-(\lambda/2)},\ x \geqq x_0\}$ for some $x_0 > 0$, $\beta_0 > 0$, $\lambda \geqq 0$, then $f(z)$ is an entire function of order at most λ .

This conjecture is still open, and we will discuss its recent progress. Further, we describe other applications of the "parabola theorem" to Padé approximants and Generalized Bessel polynomials.

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G. SCHMEISSER:

Interpolation and approximation by entire functions of exponential type

A well-known famous theorem of Weierstrass guarantees that every continuous function on a compact interval can be approximated arbitrarily close by sequences of polynomials. Bernstein, Faber, Marcinkiewcz, Fejér, Turán and others studied under what side conditions such sequences of polynomials may already be obtained by interpolation.

If the compact interval is replaced by the whole real line, then in the approximation problem the rôle of the polynomials is taken by the entire functions of exponential type, as was shown by Bernstein in 1946. Therefore it is natural to ask whether interpolation by entire functions of exponential type leads to similar phenomena as in the polynomial case. We report on results in this direction obtained in co-operation with R. Gervais and Q. I. Rahman.

W. SEEWALD:

Der Quotienten-Differenzen-Algorithmus von Rutishauser

The Quotient-Difference Algorithm (QD Algorithm), developed by H. Rutishauser, is an algorithm for computing the poles of a meromorphic function, given by its Taylor series. Other applications concern the computation of zeros of analytic functions, especially of polynomials, and of the eigenvalues of a square matrix.

If one of the wanted poles is unique in modulus, i.e., there is no other pole of same modulus, and simple, then the corresponding column of the so-called QD scheme converges towards the reciprocal value of this pole. Otherwise, a sequence of polynomials can be constructed that converges towards the polynomial whose zeros are the reciprocal values of all those poles that have the same modulus as the wanted pole. Rutishauser did not prove this fact, though; the subject of this speech is to present a simple, direct proof.

A. SHARMA:

Interpolation in the roots of unity: An extension of a theorem of Walsh

Recently in a joint paper with Cavaretta and Varga, we extended a well-known and beautiful result of J. L. Walsh which goes back to 1931. This theorem of Walsh states that if $f \in A_\rho$ (analytic in $|z| < \rho$, $\rho > 1$) and if $p_{n-1}(z;f)$ denotes the Lagrange interpolant to f on the n^{th} roots of unity, while $P_{n-1}(z;f)$ denotes the Taylor polynomial of f of degree $n - 1$, then $p_{n-1}(z;f) - P_{n-1}(a;f)$ tends to zero in $|z| < \rho^2$. The convergence is uniform and geometric in $|z| \leqq Z < \rho^2$ and the result is best possible. Here we extend this result and the extensions thereof to functions which are meromorphic in $|z| < \rho$, with a fixed number ν of poles in $1 < \rho' < |z| < \rho$. Let P_n/Q_ν denote the rational function which interpolates f in the $(n + \nu + 1)^{th}$ roots of unity and let $R_n/S_\nu - f = O(|z|^{n+\nu+1})$ as $|z| \to 0$. If D^* denotes the region $|z| < \rho\rho'$ with the ν poles of f deleted, then $P_n/Q_\nu - R_n/S_\nu$ tends to zero in D^*. Other extensions to Hermite interpolation and to lacunary interpolation in roots of unity have also been given.

G. T. SYMM:

Two methods for computing capacitance of quadrilaterals

Given four points ABCD on a closed contour L bounding a simply-connected plane domain G, the capacitance K between AB and CD is defined by $K = \frac{1}{4\pi}\int_{AB} \phi' ds$ where ϕ is that function, harmonic in G, which takes the boundary values $\phi = 1$ on CD and $\phi = 0$ otherwise on L and the prime denotes differentiation along the normal to L directed into the domain G. Alternatively, this capacitance may be obtained from the conformal module M of the quadrilateral ABCD, where $M = -\int_{CD} \psi' ds$ where ψ is that function, harmonic in G, which takes the boundary values $\psi = 1$ on CD and $\psi = 0$ on AB with the inward normal derivative $\psi' = 0$ otherwise on L. These two methods are implemented numerically via Green's third identity.

W. J. THRON:

Analytic continuation of functions defined by means of continued fractions

This is a report on joint work with H. Waadeland. Limit periodic continued fractions $K(a_n(z)/b_n(z))$ are studied. Let $\lim a_n(z) = a(z)$, $\lim b_n(z) = b(z)$ and let $x_j(z)$, $j = 1, 2,$ be the fixed points of the mapping $w = a(z)/(b(z)+w)$. It is known that the nth approximant of the continued fraction can be obtained from a certain linear fractional transformation in w $S_n(z,w)$ by setting $w = 0$. If w is replaced by $x_j(z)$ then the sequence $\{S_n(z,x_j(z))\}$ can be used both to accelerate convergence of the sequence $\{S_n(z,0)\}$ as well as (provided $\{a_n(z)\}$

and $\{b_n(z)\}$ converge sufficiently fast) to obtain an analytic continuation of $\lim S_n(z,0)$. Regular C-fractions $K(\alpha_n z/1)$ and general T-fractions $K(F_n z/(1+G_n z))$ are studied in detail.

J. TODD:

The first problem of Zolotarev

This problem is to determine

$$\min_{(a)} \max_{-1 \leqq x \leqq 1} |x^n - n\sigma x^{n-1} + (a_2 x^{n-2} + \ldots + a_{n-1} x + a_n)|$$

where σ is a parameter which may be assumed positive.

The solution (cf. Achieser, Approximationstheorie) is a distorted Chebyshev polynomial when $0 \leqq \sigma \leqq \tan^2 \frac{\pi}{2n}$ and involves elliptic and theta functions when $\sigma \geqq \tan^2 \frac{\pi}{2n}$.

Using the "differential equation" method it is shown that the two cases are distinguished by the location of the "last" zero of the derivative of the extremal polynomial: if it is inside $[-1,1]$ we are in the trigonometrical case.

The solution in the trigonometrical case is obtained by elementary means, that in the elliptic case is more complicated and the methods to be used are illustrated in the simpler context of the approximation of $(1+x)^{-1}$ in $[0,1]$ [cf. Hornecker, Talbot, Rivlin, Achieser, Bernstein].

L. N. TREFETHEN:

Chebyshev approximation on the unit disk

If an analytic function $f(z)$ is Chebyshev approximated on the unit disk by a rational function $r^*_{mn}(z)$, the error curve is often exceedingly close to a perfect circle of winding number $m + n + 1$. It turns out that if the space of approximations is extended to a certain infinite dimensional space, then a corresponding best approximation $\tilde{r}_{mn}$ of f can be constructed explicitly from a singular value decomposition of a Hankel matrix of Taylor coefficients of f, and $\tilde{r}_{mn}$ has a perfectly circular error curve. This theory is due to Carathéodory and Fejér, Adamian Arov and Krein, and others.

We use $\tilde{r}_{mn}$ to derive a nearby "Carathéodory-Fejér approximation" r^{cf}_{mn} that is rational and has a nearly circular error curve, hence is near best. From this it is shown that if f is approximated on small disks $|z| < R$, then as $R \to 0$ the relative variation from a constant of the modulus of the error curve is $O(R^{m+n+2})$.

B. A. TROESCH:

A fluid dynamics problem and the velocity potential at a corner

The solution of the Dirichlet problem has an intricate behaviour at a corner. Free fluid oscillations in a canal are also governed by the Laplace equation so that again fractional powers and logarithms may appear at the edge of the canal. Nevertheless, for any eigenvalue $\lambda = \omega^2/g$, for any acute angle at the edge, and for any width x_0 there exists a canal with a velocity potential $\phi = \mathrm{Re}\{f(z)\}$ where $f(z)$ has no singularity at the edge. However, for the isoperimetric problem (i.e., the λ is a maximum for a fixed cross section area of unknown shape) $f(z)$ must be singular at the edge if $\lambda x_0 \geqq 0.89$, and probably for all $\lambda x_0 > 0$. There exists an efficient method to compute the isoperimetric shape despite the singularity.

The conformal mapping is available for the isochronus container $y(x) = -(1/\lambda)\log\cos\lambda x$, but not for the isoperimetric problem. Conformal mapping and hodograph plane considerations seem to be helpful for free boundary problems only if the flow extends to infinity.

R. S. VARGA:

Width conjecture

It is shown (A. Edrei, E.B. Saff, R.S. Varga) that the width conjecture is valid for the Mittag-Leffler functions, i.e., the partial sums of $f_\lambda(z) := \sum_{j=0}^{\infty} z^j/\Gamma(1+j/\lambda)$ have no zeros in the region $\{x+iy : |y| \leqq Kx^{1-\lambda/2}\}$ for all n sufficiently large.

Eneström-Kakeya Theorem

The classical Eneström-Kakeya Theorem for obtaining bounds for polynomials with positive coefficients is extended to a larger class of functions, and the extensions are shown to be best possible.

J. WALDVOGEL:

Quadrature in the Complex Plane by Means of the Trapezoidal Rule

Methods for the numerical calculation of line integrals in the complex plane are discussed. It is assumed that the path of integration C is an analytic curve containing no singularities of the integrand $f(z)$.

If C is a closed curve, as in applications like the calculation of residuals, the trapezoidal rule may be applied directly. The approximation error is expressed in terms of the Fourier coefficients of a periodic analytic function, and is shown to be of the order $O(e^{-cn})$, $c > 0$, where n is the number of evaluations of the integrand. In an implementation using successive halving of the mesh previously collected evaluations can be used efficiently.

To cope with singularities near the path C , or to handle strong oscillations of the integrand a change of variables, or the deformation of C , or both is suggested.

If C extends to infinity on both sides (e.g. in some integral transforms) the trapezoidal rule is well suited, too. Then the error is given by Poisson's summation formula.

D. D. WARNER:

Some Interpolation Results for Generating Functions of Polya frequency series

A generating function for a one-sided Polya frequency series is a function

$$f(z) = e^{\gamma z} \frac{\prod_{k=1}^{\infty} (1 + \alpha_k z)}{\prod_{k=1}^{\infty} (1 - \beta_k z)}$$

where $\alpha_k \geq 0$, $\beta_k \geq 0$, $\gamma \geq 0$ and $\Sigma(\alpha_k + \beta_k) < \infty$. Let $a = \max\{-1/\alpha_k\}$ and let $b = \min\{1/\beta_k\}$. Let $X = \{x_0, x_1, \ldots\}$ be a sequence of points in the open interval (a,b) . We establish two results. The formal Newton series for $f(z)$ and X is totally positive. Second, the corresponding Newton-Padé table is paranormal. These results are natural extensions to the Newton-Padé table of earlier work by Schoenberg and Karlin. Paranormality guarantees the existence of the recursive algorithms for the Newton-Padé approximants.

J. WEISEL:

Numerische Ermittlung quasikonformer Abbildungen mit finiten Elementen

Die Klasse der "p-analytischen" quasikonformen Rechtecks- und Kreisringabbildungen kann durch Extremalprinzipien charakterisiert werden, welche eine Ritz-Approximation mit finiten Elementen ermöglichen. Für Gebiete mit polygonalen Randkurven werden Fehlerabschätzungen in verschiedenen Normen sowohl für stetig differenzierbare als auch für stückweise konstante Funktionen p vorgestellt. Ueber numerische Experimente wird berichtet.

W. WENDLAND / W. NIETHAMMER:

Ein Iterationsverfahren zur Berechnung des Szegö-Kerns mit der Integralgleichung von Kerzman und Stein

Die Autoren verwenden ein von W. Schempp und W. Niethammer entwickeltes Limitierungsverfahren [aeq. matematicae 5 (1970)], um die von N. Kerzman und E.M. Stein [Math. Ann. 236 (1978)] aufgestellte Integralgleichung für den Szegö-Kern auf einer einfach geschlossenen genügend oft differenzierbaren Randkurve Γ iterativ zu lösen. Man erhält die immer konvergente Iteration

$$u_m := s_0 \mathbb{A} u_{m-1} + s_2 u_{m-2} + s_0 f \,, \qquad u_{-1} = u_0 = f \,, \tag{1}$$

$m = 2, 3, \ldots$ mit $s_0 = 2/(1 + \sqrt{1 + \beta^2})$ und $s_2 = 1 - s_0$. Die Konstante β kann aus Krümmung und c-Eigenschaft der Kurve direkt berechnet werden. Nach G. Bruhn und W. Wendland [Internat. Series Num. Math. 7 (1967)] hat man die Fehlerabschätzung

$$\| u - u_m \| \leqq \max_{|\eta|=1/q} \| (I - (s_0\eta/(1-s_2\eta^2))\, \mathbb{A})^{-1} \| \; \| f \| \; q^{m+1}/(1-q) \,. \tag{2}$$

Hierbei ist $\sqrt{s_2} < q < 1$ und $\| \cdot \|$ irgendeine der Normen von $C^0(\Gamma)$, $L_2(\Gamma)$ oder Sobolev-Norm auf Γ. Diskretisiert man die Integralgleichung und damit auch (1) mit der Integralformelmethode oder mit Galerkin-Kollokation für Splines auf Γ, so bleibt (2) i.w. gültig unabhängig von der Schrittweite h. Schliesslich werden asymptotische Fehlerabschätzungen für o.g. Approximationen angegeben.

J. WILLIAMS:

Least Squares Rational Approximation in the Complex Plane

The paper is concerned with some theory and practice of rational least squares approximation in the complex plane. Questions of existence and uniqueness of best approximations are first discussed. Then, as in Cheney and Goldstein [Math. Zeit., 95 (1967)] where the real case is treated, a local best approximation is characterized. The application of the Osborne-Watson algorithm involving local linearisation is then described and numerical examples of approximation of analytic functions on the unit disc and semi-disc are given.

Finally, a brief mention is made of attempts to extend some of the above work to deal with two problems of practical interest. Firstly, the numerical inversion of Laplace Transforms and secondly, a difficult problem involving the approximation of complex data on the imaginary axis.

K. ZELLER:

Taubersätze und komplexe Analysis

Bei der Behandlung von Taubersätzen sind funktionentheoretische Hilfsmittel nützlich, vor allem Integration (Riesz, Ikehara, Gaier), der Satz von Vitali (Hardy-Littlewood) und der Satz von Montel (Bowen, Delange), auch in Verbindung mit quadratischer Transformation (Wiener u.a.). Diese Methoden werden erläutert und verglichen, einige Beweise werden vorgeführt (mit Neuerungen, z.B. Einsatz von Bessel-Funktionen, Approximationsprinzipien, BE-Formeln). Die Untersuchungen führen weiter zu der grundlegenden funktionalanalytischen Aussage, dass das Borel-Verfahren die Eigenschaft AKB besitzt (Beweis z.B. mittels Verfahren vom Valiron-Typ). Dies regt an zu allgemeinen strukturellen Betrachtungen (insbesondere über Kompaktheit, Perfektheit und Hardy/Saks-Räume).

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