MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 34/1980

Modulfunktionen in mehreren Variablen

27.7. bis 2.8.1980

Die Tagung fand unter der Leitung von Herrn Prof. Dr. M. Eichler (Basel) und Herrn Prof. Dr. H. Klingen (Freiburg i.Br.) statt. Schwerpunkte des Interesses waren

a) Hecke-Operatoren für Siegelsche Modulformen, ihre Wirkung auf Thetareihen und Zusammenhänge mit L - Reihen; Prof. Andrianov berichtete in einer Serie von fünf Vorträgen über seine Arbeiten zu diesen Themen.

b) tiefliegende Beziehungen zwischen automorphen Formen verschiedenen Gewichts und zu verschiedenen Gruppen (u.a. der Beweis der Saito - Kurokawa - Vermutung),

c) die Erzeugung algebraischer Zahlkörper durch Werte von Modulfunktionen in mehreren Variablen (Verallgemeinerungen der komplexen Multiplikation) und andere Zusammenhänge mit der algebraischen Zahlentheorie.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge:

A.N. ANDRIANOV:

1. Hecke Operators on Siegel modular forms

The introduction in the series of lectures on the multiplicative properties of Siegel modular forms and theta-series. The main topics were: Siegel modular forms; Hecke operators on the space of modular forms; the problems about the multiplicative structure of the Fourier coefficients of eigenfunctions of all Hecke operators and the brief formulation of the answer, which includes the Rankin zeta functions of modular forms, the problem of the behavior of the theta-series of quadratic forms under the Hecke operators, the explicit formulas and their applications to the Siegel theorem and to the problem of linear independence of theta-series.

2. Decomposition of the Hecke polynomials

The exposition of main technical tools for investigation of multiplicative properties of the Siegel modular forms which were mentioned in the previous lecture. The abstract p - Hecke ring L of the group $\Gamma = Sp_n(\mathbb{Z})$ and the similar ring L_o for the "triangle" subgroup $\Gamma_o = \{\begin{pmatrix} A & B \\ O & D \end{pmatrix} \in \Gamma\}$. The structure of the rings L and L_o . The theorems about the decomposition of polynomials over L when L is extended up to L_o . The application to the explicit decomposition of the Rankin polynomial $R(z)$.

3. Rankins ζ- functions for Siegel modular forms

The first application of the technic developed in the previous lecture: If

$$F(Z) = \sum_{\substack{{}^tN = N^{(n)} \geq 0 \\ \text{semiintegral}}} f(N)\, e^{2\pi i\, Tr(NZ)}$$ is a Siegel modular form of weight k

and order n which is an eigenfunction of all Hecke operators, then for each fixed $N > 0$ the series

$$\sum_{M \in SL_n(\mathbb{Z}) \backslash M_n^+(\mathbb{Z})} \frac{f(MN{}^tM)}{(\det M)^s} \qquad (*)$$

has an Euler product which is explicitely given in the terms of Rankin ζ - function, expressed in the terms of eigenvalues of Hecke operators associated to the eigenfunction F . The Dirichlet series (*) can be obtained as a result of an integral transformation of the form F whose kernel is expressed in the terms of theta-series of the quadratic form with the matrix N and some Eisenstein series for a congruence subgroup of the group Γ, which allow to investigate the analytical properties of the ζ - function.

4. The action of Hecke operators on theta-series

Another application of the same technic. The explicit formulae are given for the image of a theta-series

$$\Theta^n(Z,Q) = \sum_{M \in M_{m,n}(\mathbb{Z})} e^{2\pi i \, Tr({}^tMQMZ)}$$

of a quadratic form in m variables which is integral and positive, m is even and n = 1,2,... , under the Hecke operators, which expresses the image as a linaer combination of theta series of the same kind. An application of the formulae to the Siegel theorem is given.

5. Degenerations of the Hecke operators on the spaces of the theta-series

An application of the explicit formulae obtained in the previous lecture: Let Θ_m^n be the space of all Siegel modular forms of weight $\frac{m}{2}$ and degree n which are linear combinations of theta series $\Theta^n(Z,Q)$; it is proved that for each $n \geq \frac{m}{2}$ each Hecke operator from $L = L_p$ on the Θ_m^n is a polynomial in $\frac{m}{2}$ operators (a priori in n operators). This is an argument to support the conjecture which states that if $n \geq \frac{m}{2}$ the theta series $\Theta^n(Z,Q_1),\ldots,\Theta^n(Z,Q_l)$ are linearly independent if the quadratic forms Q_i are not equivalent.

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M. EICHLER / D. ZAGIER:

On Jacobi forms

A Siegel modular form of $Z = \begin{pmatrix} \tau & z \\ z & \tau' \end{pmatrix}$ has been expanded by Piatetskij-Shapiro into the Fourier-Jacobi sum

$$F_k(Z) = \sum_{m=0}^{\infty} \varphi_{k,m}(\tau,z)\, e^{2\pi i \, m\tau'} \qquad (k = \text{weight}).$$

They satisfy certain functional equations and have a Fourier exransion.

Two ways to define Hecke operators on Jacobi forms were discussed. The action of these operators relates the $\varphi_{k,1}$ to the Siegel modular forms of Maass' "Spezialschar", and on the other hand the $\varphi_{k,m}$ define elliptic modular forms of weight $k - \frac{1}{2}$ for some congruence subgroup of $SL(2,\mathbb{Z})$ depending on m.

The bigraded ring of Jacobian forms is not finitely generated, but the graded ring of the functions $\frac{\varphi_{k,m}(\tau,z)}{f_k(\tau)}$, where $f_k(\tau)$ denotes a holomorphic modular form of weight k, is generated by two elements.

The nature of the zeros of jacobian forms is discussed and as an application we describe the zeros of the Weierstraß $\wp$ - function.

By combining Siegel's main theorem with the Fourier-Jacobi expansion of Siegel modular forms one obtains results of Waldspurger relating numbers of representations of quadratic forms to values of L - series.

W. KOHNEN:

Values of L - series of modular forms in the middle of the critical strip

In two recent papers Waldspurger proved the following beautiful result: Let $f = \sum_{n \geq 1} a(n) q^n$ ($q = e^{2\pi i z}$) be a cusp form of even integral weight k on some congruence subgroup of $SL_2(\mathbb{Z})$. Assume f is a normalized eigenfunction of the full Hecke algebra, and let $g = \sum_{n \geq 1} c(n) q^n$ be an Hecke eigenform of weight $\frac{k+1}{2}$ corresponding to it in the sense of Shimura. Suppose that the Fourier coefficients $c(n)$ lie in the field generated over $\mathbb{Q}$ by the $a(n)$.

Then for any squarefree integer t with $(-1)^{k/2} t > 0$ the value $c(|t|)^2/t^{(k-1)/2}$ is proportional to $L(f,t,\frac{k}{2})$ with constant proportionality independent of t, where $L(f,t,s)$ is defined by analytic continuation of the L - series $\sum_{n \geq 1} \varepsilon_t(n) a(n) n^{-s}$ and ε_t is the character of $\mathbb{Q}(\sqrt{t})/\mathbb{Q}$.

Waldspurgers proof, which is presented in the language of adeles and in terms of the representation theory of the metaplectic group, is very complicated. Also, in the result as stated above, one does not know what the constant of proportionality is. Very recently Zagier and the author found an elementary proof of

Waldspurger's result in the special case of modular forms for the full modular group in which the constant of proportionality is given explicitely: it turns out to be (up to a trivial factor) the quotient of the squares of the norms of f and g in the Petersson metric. This permits to deduce as corollaries several results about the arithmetic nature of the $c(n)$ and the distribution of the values $L(f,t,\frac{k}{2})$.

S.J. PATTERSON:

Metaplectic forms on groups of higher rank

This represents joint work with Kashdan. The problem considered was the construction of automorphic forms on $\widetilde{GL}_{r,\mathbb{A}}$, where $\widetilde{GL}_{r,\mathbb{A}}$ is the metaplectic group $0 \longrightarrow \mu_n(k) \longrightarrow \widetilde{GL}_{r,\mathbb{A}} \longrightarrow GL_{r,\mathbb{A}} \longrightarrow 0$. This splits over $GL_{r,k}$. By means of the theory of Eisenstein series one constructs such a representation (following a method first used by Kubota). This can be written as $\otimes \pi_v$, where the $\otimes$ is taken over $\mu_n(k)$ (which is supposed to have n elements; k , as usual, is an $\mathbb{A}$ - field). Each π_v is a quotient of an induced representation. Then one can, following Jaquet/Langlands and Piatetski-Shapiro, derive global statements from local ones when π_v have (at least, almost everywhere) a unique Whittaker model. This occurs precisely when $r = n-1$ or n . In this way one derives automorphic forms analogous to theta functions, but only on these large groups. If $n = 3$ then one can achieve such functions on $\widetilde{GL}_2$; these can be written as Fourier series with coefficients which are Gauß sums . The more general ones are similar and represent the analytic behaviour of generating series of Gauß sums.

I.I. PIATETSKI - SHAPIRO:

L -functions for GSp_2

The aim of the talk was to define local L and ε - factors for any irreducible admissible representation of $GSp_2(k_\gamma)$, k_γ is a local field.
If $\pi = \otimes \pi_\gamma$ is an automorphic cuspidal representation of GSp_2 , then the

global L - function satisfies a nice functional equation and is analytic except for at most two poles at $s = \frac{3}{2}$, $-\frac{1}{2}$. Also, automorphic representations are described such that L has poles.

P. PONOMAREV:

Liftings of theta series

The effect of the Doi-Naganuma and Shimura liftings is determined for certain theta series associated to quaternery and ternary quadratic forms over $\mathbb{Z}$. The liftings are themselves linear combinations of theta series with coefficients which are certain representation numbers of the original quadratic form. The main tools of the proof are connections between "Anzahlmatices" for certain lattices and "Brandt Matrices" (Eichler), the "Eichler commutation relations" determining the effect of Hecke operators to theta series and Zagier's formula for the Fourier coefficients of the Doi-Naganuma lifting. Similar results are indicated for theta series with spherical harmonics.

S. RALLIS:

Problems in automorphic forms and the Weil representation

The space of cusp forms $L^2_{cusp}(O(Q)_{\mathbb{A}})$ (Q nondegenerate quadratic form over K , $\dim_K Q = m$ even) can be decomposed into an orthogonal direct sum $R_1 \oplus R_2 \oplus \dots \oplus R_m$ where each automorphic representation π in R_i can be "lifted" (via the oscillator representation of the dual pair $(Sp_i, O(Q))$) to a certain nonzero subspace in $L^2_{cusp}(Sp_{i,\mathbb{A}})$.

We make the general conjecture that the image of π in $L^2_{cusp}(Sp_{i,\mathbb{A}})$ is a nonzero multiple $\varsigma_i(\pi)$ of an irreducible automorphic representation of Sp_i. Moreover we conjecture that the map $\pi \longrightarrow \varsigma_i(\pi)$ is an injective mapping. The importance of the conjecture is that 1. the spaces R_i $(1 \leq i < \frac{m}{2} - 1)$ are the analogues of the Maaß space where the eigenvalues of certain generators of the Hecke algebra of O(Q) are prescribed nonunitary (i.e. do not satisfy the generalized Ramanujan conjecture), 2. the trace of a Hecke operator f on $L^2_{cusp}(O(Q)_{\mathbb{A}})$ will have to be compared to the sum of n Hecke operators f_i (defined in $Sp_{i,\mathbb{A}}$) acting on image $\mathcal{S}_i$ in $L^2_{cusp}(Sp_{i,\mathbb{A}})$.

We show that the conjecture above reduces to a local statement. Namely we must show a local duality conjecture for all primes and for the dual pair $(Sp_i(k_v), O(Q_v))$. We indicate several cases when this conjecture can be proved.

D. ZAGIER:

On the Saito - Kurokawa Conjecture (work of Maaß and Andrianov)

in 1978 Saito and Kurokawa conjectured that associated to each Hecke eigenform $f \in S_{2k-2}(SL_2\mathbb{Z})$ (k even) there is a Siegel modular form $F \in S_k^*(Sp_4\mathbb{Z})$ (= Maaß subspace of $S_k(Sp_4\mathbb{Z})$) whose zeta-function $Z_F(s)$ (in the sense of Andrianov) is given by $Z_F(s) = \zeta(s-k+1)\,\zeta(s-k+2)\,L_f(s)$. In 1979 Maaß and Andrianov proved a large part of this conjecture. Inthe talk it was shown how the Saito - Kurokawa conjecture can be proved completely by combining Maaß' results with results of Kohnen on the space $S^+_{k-1/2}$: It follows from the chain of isomorphisms

$$S_k^*(Sp_4Z) \simeq J_{k,1} \simeq S^+_{k-1/2} \simeq S_{2k-2}(SL_2Z)$$

where $J_{k,1}$ is the space of Jacobiforms discussed in my lecture with Eichler. The first two isomorphisms are elementary, the third, which uses a trace formula, was proved by Kohnen.

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T. ARAKAWA:

Generalized eta functions and real quadratic fields

We study the following function for any irrational real algebraic number α :

$$H(\alpha, s) = \sum_{n=1} n^{s-1} \frac{e^{2\pi i n\alpha}}{1 - e^{2\pi i n\alpha}} .$$

The infinite series is absolutely convergent if $Re(s) < 0$ by the Thue-Siegel - Roth theorem in the diophantine approximation theory. Our results are that $H(\alpha,s)$ satisfies certain transformation formulae under the action of $SL(2,\mathbb{Z})$ on α and that for irrational real quadratic numbers α , $H(\alpha,s)$ is analytically continued to a meromorphic function in the whole complex plane which has a simple pole at $s = 0$. The constant term of the Laurent expansion of $H(\alpha,s)$ at $s = 0$ is closely related to the double gamma function. Further, we are able to derive some relations of the special values of the double gamma function by using transformation formulae and the analytic continuation for $H(\alpha,s)$.

W.L. BAILY:

Galois actions on Eisenstein series and certain abelian extensions of number fields

One may extend the results and methods of Hecke's thesis, at the same time correcting certain errors therein, to show that extensions of CM fields generated by the special values of arithmetic Hilbert modular functions are abelian extensions. One of the main tools used, not found in Hecke's thesis, is a formula giving the action of a Galois group on Eisenstein series via its action on the Fourier coefficients. This uses formulae of Kloosterman in a special case which has been significantly generalized by Karel. We have also been able to recover the q - expansion principle of Hecke (whose proof was defective in an important point) and hope to use it to prove without the use of moduli of abelian varieties, a reciprocity law like Shimura's.

H. COHN:

Explicit illustration of a modular equation in two variables

For the symmetric Hilbert modular functions over $Q(\sqrt{2})$ the modular equation with multipliers $2+\sqrt{2}$, $2-\sqrt{2}$ is now found explicitly. Use (specially chosen) generating functions (x,y) of τ and τ' ($\mathrm{Im}\,\tau > 0$, $\mathrm{Im}\,\tau' > 0$), then let (x_t, y_t) be the corresponding functions of $\tau(2+\sqrt{2})^t$, $\tau'(2-\sqrt{2})^t$. A recursive relation exists for computing (x_t, y_t) by quadratics over (x_{t-1}, y_{t-1}). This leads to a tower of relative quadratic fields for x, y algebraic. Computers are used to derive the modular equation, (Eisenstein series are all that is needed, as shown by Gundlach), and computers also assist in the discovery of singular moduli which are rational integers, e.g. for $\tau = \sqrt{(-2-\sqrt{2})}$, $\tau' = \sqrt{(-2+\sqrt{2})}$.

F.J. GRUNEWALD:

Elliptic curves over $Q(i)$ and modular forms

The aim of this lecture was to discuss the possibility of generalizing the classical Eichler - Shimura - Weil theory to imaginary quadratic number fields. Put $\Gamma = PSL_2(\mathbb{Z}[i])$ and $\Gamma_0(\mathfrak{n}) = \{(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}) \in \Gamma \mid c \in \mathfrak{n}\}$, $Q_{\mathfrak{n}}$ = normal closure of $\mathfrak{n}$ - unipotents in Γ. Furthermore put $V(\mathfrak{n}) = (\Gamma_0(\mathfrak{n})/Q_{\mathfrak{n}})^{ab} \otimes Q$. This gives a finite dimensional vector space for every ideal $\mathfrak{n}$ of $\mathbb{Z}[i]$. $V(\mathfrak{n})$ is the analog of the Eichler cohomology groups for subgroups of $SL_2\mathbb{Z}$. On $V(\mathfrak{n})$ there is a natural action of a Hecke algebra $H(\mathfrak{n})$. If $V \subseteq V(\mathfrak{n})$ is a one - dimensional eigenspace for $H(\mathfrak{n})$ one can form a Dirichlet series $L(V,s)$. This Dirichlet series converges in some halfplane and has a holomorphic continuation and a functional equation. Numerical evidence seems to suggest that for an elliptic curve defined over $Q(i)$ of conductor $\mathfrak{n}$, there is a one dimensional eigenspace $V(E) \subseteq V(\mathfrak{n})$ such that $L(V,s) = L(E,s)$, the Hasse - Weil zeta function of E. If A is an abelian variety of dimension 2 over $Q(i)$ with sufficiently many complex multiplications then $L(A,s) = (L_0(A,s))^2$ with a certain Dirichlet series $L_0(A,s)$. There are some cases where such a $L_0(A,s)$ equals a $L(V,s)$. Some possibilities were discussed how to obtain from a one dimensional eigenspace V an abelian variety of dimension 2. This program was carried out together with J. Mennicke.

K.-B. GUNDLACH:

Remarks on Hecke's thesis and "Habilitationsschrift"

Ein berühmter Fehler in der Geschichte der Theorie der Modulfunktionen ist Heckes Behauptung in seiner Habilitationsschrift, eine spezielle nicht konstante Modulfunktion Φ zur symmetrischen Mudulgruppe $\hat{\Gamma}$ eines reellquadratischen Zahlkörpers K mit der Diskriminante $d_K \equiv 0\ (4)$ auf dem Produkt $\mathfrak{H}$ einer oberen und einer unteren Halbebene habe in $\mathfrak{H}$ keine Pole. Seit 1954 weiß man, daß eine solche Funktion nicht existieren kann; es war aber bisher nicht klar, welcher Fehlschluß zu der falschen Behauptung geführt hat. Nun zeigt sich, daß der entscheidende Fehler bereits in der Dissertation steht: Dort wird die Nullstellenmenge des Produkts J_{10} der Quadrate der 10 Thetareihen halbzahliger gerader Charakteristik durch modulare Einbettung von $\mathfrak{H}$ als analytische Menge $\tilde{\mathfrak{H}}$ in den Siegelschen Halbraum zweiten Grades $\mathfrak{H}_2$ bestimmt. J_{10} ist auf $\mathfrak{H}_2$ definiert, die Nullstellenmenge N in $\mathfrak{H}_2$ besteht aus den Punkten der Diagonale δ von $\mathfrak{H}_2$ und ihren Bildern unter der Siegelschen Modulgruppe Γ_2. Es ist jedoch nicht richtig (wie Hecke behauptet), daß ein Bild $L(N_o \cap \tilde{\mathfrak{H}})$ für einen Zweig N_o von N und ein $L \in \Gamma_2$ nur dann in $\delta \cap \tilde{\mathfrak{H}}$ liegen kann, wenn L zu einer Transformation der Hilbertschen Modulgruppe führt; diese $N_o \cap \tilde{\mathfrak{H}}$ treten letztlich als Pole von Φ in Erscheinung. Die durch diese Pole in den Beweisen der Dissertation auftretenden Lücken lassen sich schließen, wenn man berücksichtigt, daß die 10 Thetafunktionen keine gemeinsame Nullstelle haben (Igusa 1964).

M.L. KAREL:

Transformation theory of Eisenstein series

On a rational tube domain certain Eisenstein series have cyclotomic Fourier coefficients, and the Galois group of the maximal abelian extension of the rational number field acts on such automorphic forms via its action on their Fourier coefficients. Considering the Eisenstein series in an adelic setting, one finds a simple explicit description of the effect on this Galois action, and in particular one can describe how this action is related to the natural action of the rational points in the group of holomorphic automorphisms of the domain in question. In many cases, this gives an explicit description of the Galois group of modular function field extensions, and in some cases these results can be used to derive information about the transformation and multiplier equations of complex multiplication.

J. MENNICKE:

Lattice point problems in hyperbolic 3 - space

This reports on some joint work with F.J. Grunewald and J. Elstrodt. There is an analogue of theta functions, where the summation is not over a lattice in Euclidean space, but over a "lattice" in some quadric. The function has many properties in common with ordinary theta functions. The coefficients are tied up with class numbers of definite binary quadratic forms. The behaviour of the function at its singularity gives rise to information about certain sums over class numbers.

M. OHTA:

l - adic representations attached to automorphic forms

P. Deligne has constructed l - adic representations of Gal $(\bar{\mathbb{Q}}/\mathbb{Q})$ attached to elliptic cusp forms. We consider a similar problem for a division quaternion algebra B over a totally real field F such that $B \otimes_{\mathbb{Q}} R = M_2(\mathbb{R}) \times H^{r-1}$.
Starting from a representation ρ of $B^\times$ into $GL_m(\mathbb{Q})$ satisfying certain conditions, we can define: 1. a certain subspace of automorphic forms "$\mathfrak{S}(S,\rho)$" for each subgroup S of $B_A^\times$ = adelization of $B^\times$ such that $S = B_{\infty+}^\times \times S_o$ with S_o open and compact $\subset B_f^\times$ = finite adeles of $B^\times$, and 2. an l-adic representation φ_l of Gal $(\bar{F}/F)$ into $GL_{2\dim\mathfrak{S}(S,\rho)}(\mathbb{Q}_l)$ for each rational prime l . It can be shown that φ_l is unramified outside the primes dividing l , the discriminant of B , and the "level" of S . For such a prime $\mathfrak{p}$, we can prove that the characteristic polynomial of a Frobenius at $\mathfrak{p}$ with respect to φ_l coincides with the "Hecke polynomial at $\mathfrak{p}$" for $\mathfrak{S}(S,\rho)$.

S. YOSHIDA:

Abelian varieties with complex multiplication and representations of the Weil group

Let A be an abelian variety of dimension n defined over a finite algebraic number field k . We assume that the endomorphism algebra A has sufficiently

many complex multiplications, i.e. we assume that End (A) $\otimes \mathbb{Q}$ contains an isomorphic image of a commutative semi-simple algebra Λ of degree 2n over $\mathbb{Q}$. Let $\zeta(s,A/k)$ denote the (one dimensional part of) the zeta function of A over k. The previously known results due to (Weil - Deuring -) Taniyama - Shimura, which express $\zeta(s,A/k)$ as a product of L - functions with Größencharacters, were obtained under certain conditions on the field of definition of the elements of $\Theta(\Lambda) \cap$ End (A). Here θ denotes the isomorphism of Λ into End (A) $\otimes \mathbb{Q}$. The following theorem is proven without any condition on the field of rationality of $\theta(\Lambda) \cap$ End (A) :
Let W_k denote the Weil group of k. There exists a representation ϱ of W_k into $GL(2n,\mathbb{C})$ such that $\zeta(s,A/k) = L(s,\varrho,W_k)$ holds, where $L(s,\varrho,W_k)$ denotes the L - function attached to the representation ϱ of W_k.

E. GOTTSCHLING:

<u>Finite presentations of discontinuous groups, in particular of $Sp(4,\mathbb{Z})$</u>

Gilt für drei Nachbarn e_1, e_2, e_3 eines Fundamentalbereichs einer diskontinuierlichen Gruppe die Gleichung $e_1e_2 = e_3$, so heißt diese eine "lokale Relation" der Gruppe. Auf H. Behr geht der Satz zurück, daß diese lokalen Relationen unter sehr allgemeinen Voraussetzungen bereits ein vollständiges Relationensystem der Gruppe bilden. Anwendungen des Behrschen Satzes scheitern jedoch im allgemeinen an dem zu bewältigenden Rechenaufwand, da die Zahl der Nachbarn z.B. mit der Dimension des Raumes, auf dem die Gruppe diskontinuierlich operiert, sehr stark anwächst und man bei n Nachbarn bereits n^2 Gleichungen zu kontrollieren hat. Hier wird für Fundamentalbereiche, die mit einer "Höhenfunktion" konstruiert werden, eine allgemeine Methode dargelegt, die den Behrschen Satz "anwendbar" macht in dem Sinn, daß der Rechenaufwand auf ein Minimum reduziert wird, das darüberhinaus kein formales Nachrechnen ist, sondern sehr stark die Struktur der Gruppe berücksichtigt.
Unter Benutzung des Minkowskischen Fundamentalbereichs kann mit dieser Methode z.B. das klassische Ergebnis über die Erzeugung der $SL(n,\mathbb{Z})$ durch Elementarmatrizen bewiesen werden, und unter Benutzung des Siegelschen Fundamentalbereichs

kann, bisher zumindest im Fall $n = 2$, ein einleuchtendes Resultat über die Präsentation der $Sp(2n,\mathbb{Z})$ durch die in ihr gelegenen Produkte von Elementarmatrizen, gewisse Kommutatorrelationen und sog. "Symbole" bewiesen werden.

K. IMAI:

On Eisenstein series for $SL(3,\mathbb{R})$

Eisenstein series appearing in the spectral decomposition of the space of square integrable functions on $SO(3)\backslash SL(3,\mathbb{R})/SL(3,\mathbb{Z})$ are:

$$E'(Y,s,v_j) = \sum_{\gamma\in\Gamma/\Gamma\cap P_1} y_1(Y[\gamma])^{-s}\, v_j(Y[\gamma]^o)$$

$$E^o(Y,s,u) = \sum_{\gamma\in\Gamma/\Gamma\cap P_1} y_1(Y[\gamma])^{-s}\, E_u(Y[\gamma]^o) ,$$

where v_j, E_u are a cusp form and Eisenstein series on $SO(2)\backslash SL(2,\mathbb{R})/GL(2,\mathbb{Z})$ respectively, s, $u \in \mathbb{C}$, $\Gamma = SL(3,\mathbb{Z})$, $Y \in \mathfrak{P}_3$ = the set of real 3×3 symmetric positive-definite matrices with determinant 1 $\cong SO(3)\backslash SL(3,\mathbb{R})$,

$$y_1(Y) = \det(Y_1) , \quad Y^o = |Y_1|^{-1/2} Y_1 \quad \text{if} \quad Y = \left(\begin{array}{c|c} Y_1 & * \\ \hline * & * \end{array}\right) \in \mathfrak{P}_3 \quad \text{and}$$

$$P_1 = \left\{ \left(\begin{array}{c|c} * & * \\ \hline 0\,0 & * \end{array}\right) \in SL(3,\mathbb{R}) \right\} .$$

Then for $f \in L^2(\mathfrak{P}_3/SL(3,\mathbb{Z}))$,

$$f(Y) = \sum_{j=0}^{\infty} \langle f,w_j\rangle\, w_j(Y) + \frac{1}{2\pi i}\sum_{j=0}^{\infty} \int_{\mathrm{Re}\, s = \frac{3}{4}} \langle f,E'(*,s,v_j)\rangle\, E'(Y,s,v_j)\, ds$$

$$+ \frac{1}{(2\pi i)^2}\frac{1}{3!} \int_{\mathrm{Re}\, s = \frac{3}{4}} \int_{\mathrm{Re}\, u = \frac{1}{2}} \langle f,E^o(*,s,u)\rangle\, E^o(Y,s,u)\, du\, ds ,$$

where $\{w_j\}_{j=1}^{\infty}$ spans the space of cusp forms and w_0 = constant.

F. KIRCHHEIMER:

Explicit presentation for certain Hilbert modular groups

I reported on two papers, the first jointly with J. Wolfart (Crelle 315), the other to appear in Crelle.- We developed a method which is a variant of the classical Poincaré idea to obtain a presentation from the "neighbour structure" of the fundamental domain F of a group Γ acting discontinuously on a simply connected topological space, provided F has "neat" geometric properties. This situation applies to a wide class of groups, especially to all Hilbert modular groups of real number fields with class number one. By specializing to this case and considering the stabilizers of certain $2n-1$ - dimensional boundary components of F in the "floor", one obtains the following type of presentation for Γ:

1. Generators and relations of the upper triangle group B,
2. generators and relations describing the stabilizers of the components of the floor and their relationship to B,
3. relations obtained from the action of Γ on the $2n-2$ - dimensional boundary components of the floor (mod B).

In case F has a "simple floor" (H. Cohn), the relations consist only of Steinberg relations and Steinberg symbols (in K - theoretic language). This applies to the Hilbert modular groups of $\mathbb{Q}(\sqrt{5})$ and the real cyclic cubic field of conductor 7. The Hilbert modular groups for $\mathbb{Q}(\sqrt{8})$ and $\mathbb{Q}(\sqrt{13})$ have similar presentations. In the case $\mathbb{Q}(\sqrt{12})$ one more relation is needed.

F.W. KNÖLLER:

Multiplizitäten von Spitzen und Klassifikation von Modulflächen

Die Multiplizität eines lokalen (analytischen) Ringes $(R,\underline{m})$ der Dimension N ist der höchste Koeffizient $\mu(R)$ des Hilbert-Samuel-Polynoms

$$\dim R/\underline{m}^k = \frac{1}{N!}\mu(R)\, k^N + O(k^{N-1}) \quad .$$

Sie ist ein grobes Maß für die Komplexität einer Singularität. Für Hilbertsche Spitzen (M,V) wird diese Invariante durch Vergleich der $\underline{m}$ - adischen Filtrierung mit einer zweiten Filtrierung (I_k) auf elementare Weise bestimmt (Mh. Math. 88). Diese lokale Invariante läßt eine differentielle Interpretation zu, die schließlich zu einer unteren Abschätzung der Plurigeschlechter der Hilbertschen

Modulräume $\overline{H^N/SL(2,\mathcal{O})}$ führt. Im Falle reellquadratischer Zahlkörper $\mathbb{Q}(\sqrt{p})$, $p \equiv 1$ (4), erhält man so - auf andere Weise - den wohlbekannten Satz, daß die zugehörigen Flächen von allgemeinem Typ sind, sofern nur p hinreichend groß ist.

M. OZEKI:

Explicit formulas for the Fourier coefficients of Eisenstein series of degree 3

Let $a_k(T_n)$ be the Fourier coefficients of Eisenstein series of degree n and of weight k for the full modular group of degree n, where T_n are the positive semi-definite semi-integral n - ary matices. Explicit values of $a_k(T_1)$ are the classical result. Maass treated the case $a_k(T_2)$ in two papers. Here the explicit computation of $a_k(T_3)$ is described for primitive types of T_3. At first it is necessary to classify the integral quadratic forms locally, then the $a_k(T_3)$ are computed by Siegel's infinite product formula for $a_k(T_3)$ according to local types.

H.L. RESNIKOFF:

Optics and modular forms

Using the Fraunhofer diffraction formula, it was shown how to realize the Hardy - Landau formula $\sum_0^{t}{}' r_2(k) = \pi t - \sum_1^{\infty} r_2(n)\left(\frac{t}{n}\right)^{1/2} J_1(2\pi\sqrt{nt})$ by means of an optical system and how to obtain the functional equation of the theta function. It was further shown that the Fresnel diffraction formula realizes the Segal - Shale - Weil representation of the metaplectic group $Mp(1)$, and a generalization to $Mp(n)$ was scetched. Finally, a general construction of intertwining kernels which realize certain group representations was applied to produce a generalization of the classical Fourier - Bessel transform, viz.:

$$f \longmapsto \int_0^{\infty} e^{\frac{i}{2}(x+y)\cot\vartheta} J_\alpha\left(\frac{\sqrt{xy}}{\sin\vartheta}\right) f(x)\,dx = (T_{e^{-2i\vartheta}} f)(y) ,$$

$\omega \longmapsto T_\omega$ is a representation of $U(1)$ on $L^2(\mathbb{R}^+)$.

I. SATAKE:

On zeta-functions associated to self-dual cones

The purpose of my talk was to generalize the method of Shintani of evaluating the special values of zeta-functions to the case of those functions associated to self-dual homogeneous cones. Let Ω be such a cone (irreducible) in $U = \mathbb{R}^n$ and let $G = G(\Omega)^o$, $r = \mathbb{R}\text{-rk}\, G$; fix $e \in \Omega$ s.t. $K = G_e = G \cap O(n)$. Let further L be a lattice in U , $U_{\mathbb{Q}} = L \otimes_{\mathbb{Z}} \mathbb{Q}$ and assume that G and e are $\mathbb{Q}$ - rational. Γ is a torsion-free arithmetic subgroup of G s.t. $\Gamma L = L$.
We consider zeta-functions

$$\zeta_\Omega(s;\Gamma,L) = \sum_{u \in \Gamma\backslash \Omega \cap L} N(u)^{-s} \qquad (\mathrm{Re}\, s > 1) .$$

By reduction theory ζ_Ω can be expressed as a finite sum of "partial zeta functions" corresponding to a rational simplicial cone $C = \{v_1,\ldots,v_l\}_{\mathbb{R}^+}$ and $a \in C \cap L$

$$\zeta_\Omega(s;(v_j),a) = \sum_{\nu_j \in \mathbb{Z},\, \geq 0} N\Big(a + \sum_{j=1}^{l} \nu_j v_j\Big)^{-s} .$$

Then, imitating Shintani's method, one obtains

$$\Gamma_\Omega(s)\,\zeta_\Omega(s;(v_j),a) = c\, r! \int_{t_1 > t_2 > \cdots > t_r} \cdots \int \Big(\prod_{i=1}^{r} t_i\Big)^{\frac{n}{r}(s-1)} \Big(\prod_{i<j} (t_i - t_j)\Big)^d F(t_1,\ldots,t_r) \prod dt_i ,$$

where $\Gamma_\Omega(s)$ is the Γ - function of Ω (defined by Koecher), $d = \dfrac{2(n-r)}{r(r-1)}$,

and
$$F(t_1,\ldots,t_r) = \int_K \frac{e^{-\langle a, k \sum t_i e_i\rangle}}{\prod_{j=1}^{l} \left(1 - e^{-\langle v_j, k \sum t_i e_i\rangle}\right)}\, dk .$$

When $\bar{C} \subset \Omega$ and d is even, the above integral can be transformed (after a suitalble change of variable) into a contour integral. It follows that ζ_Ω is meromorphic on the whole plane $\mathbb{C}$, and the value of ζ_Ω for $s = \frac{r}{n}(1-\nu)$, $\nu \in \mathbb{Z}_+$ is a $\mathbb{Q}$ - linear combination of

$$I(\nu_{ij}) = \int_K \prod_{\substack{1 \leq i \leq r \\ 1 \leq j \leq l}} \langle v_j, k e_i\rangle^{\nu_{ij}}\, dk ,$$

where $\nu_{ij} \in \mathbb{Z}$ and $\nu_{ij} \geq 0$ unless $i = 1$. When the number of negative exponents ν_{ij} is ≤ 2 (and $r = 2$), it can be shown that $I(\nu_{ij}) \in \mathbb{Q}$. But in general the computation of $I(\ _{ij})$ seems to be rather difficult.

M. SCHMIDT:

Über Indizes bei Siegelschen Stufengruppen

Es sei $T = \operatorname{diag}[t_1,\ldots,t_n]$ eine Stufe ($t_1,\ldots,t_n \in \mathbb{N}$, $t_\nu | t_{\nu+1}$ für $\nu = 1,\ldots,n$). Bezeichnet man mit 1_n die n - reihige Einheitsmatrix und mit $M_n(\mathbb{Z})$ den Ring aller $(n\times n)$ - Matrizen mit ganzzahligen Koeffizienten, so ist

$$\Gamma(T) := \begin{pmatrix} T & 0 \\ 0 & 1_n \end{pmatrix} M_{2n}(\mathbb{Z}) \begin{pmatrix} T^{-1} & 0 \\ 0 & 1_n \end{pmatrix} \cap Sp_{2n}(\mathbb{Q})$$

die Siegelsche Stufengruppe oder paramodulare Gruppe zur Stufe T . Aus einem Satz von Allan folgt, daß es zu $\Gamma(T)$ genau eine maximale diskrete rationale Erweiterung $\Phi(T)$ gibt. $\Gamma(T)$ ist genau dann schon selber maximal, wenn $t_n t_1^{-1}$ quadratfrei ist. Für beliebige Stufen T hat G. Köhler die Gruppe $\Phi(T)$ explizit angegeben; sie ist isomorph zu einer maximalen Stufengruppe $\Gamma(F)$. Der Gruppenindex $[\Phi(T) : \Gamma(T)]$ läßt sich induktiv berechnen. Dabei werden Methoden verwendet, die von U. Christian und von H. Klingen entwickelt wurden.

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Gefördert durch DFG Deutsche Forschungsgemeinschaft

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