MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 35/1980

Scattering Theory

3.8. bis 9.8.1980

Die Tagung fand unter der Leitung von Herrn P.Werner (Stuttgart) und Herrn C.H.Wilcox (Salt Lake City, USA) statt.

This was the fourth meeting on Scattering Theory to be held at Oberwolfach. The majority of the talks presented at the first three meetings (1971,1974,1977) dealt with quantum-mechanical scattering problems. The scope of this meeting was broadened to reflect recent developments in the applications of scattering theory to problems of classical and applied physics. The three years since the last meeting have again seen rapid progress in all branches of scattering theory, stimulated in part by scientific contacts which took place at these meetings.
Topics discussed this year included scattering problems of quantum and classical physics (e.g.long-range potentials, thermoelastic media, diffraction gratings, moving obstacles), and inverse problems with applications to geophysics, speech and hearing, computer-assisted tomography.

Vortragsauszüge

T.S.Angell:
Scattering control for the Robin Problem

Using a boundary integral equation formulation for the exterior Robin problem for the Helmholtz equation developed elsewhere, we consider the problem of control of scattered fields. The power in an angular sector of the far field is taken as the cost functional. The problem discussed is that of maximizing this functional over a control set of admissible impedences (Robin boundary data) consisting of a closed bounded, convex set in L_∞.

V. Barcilon:

Inverse Sturm-Liouville-like Problems

My work on Inverse Eigenvalue Problems has been motivated by the problem of inferring the internal structure of the Earth from its natural frequencies. To that effect, it was necessary to investigate inverse eigenvalue problems associated with differential systems of order 4 or higher. Since problems associated with operators of the form $L = \alpha_0(x)\frac{d}{dx}\,\alpha_1(x)\frac{d}{dx}\,\ldots\,\frac{d}{dx}\,\alpha_n(x)$, where $\alpha_i(x)>0$, are natural extensions of Sturm-Liouville operators, they provide a natural point of departure. The vibrating beam, which falls within this class of operators, is an ideal candidate for this study.
I shall present a survey of the current state of these problems and touch upon questions of existence, uniqueness and construction of solutions.

R.Colgen:

Some Remarks on Enss' Method in Quantum-Mechanical Scattering Theory

A result in abstract scattering theory (in the two space setting) is presented which provides strong asymptotic completeness of the wave operators (in the sense of the Enss' Theorem) under conditions that are quite general and easy to verify (compared with Enss' Theorem and other generalizations). The proof is based on Enss' method (and Simon's generalization) and requires little additional effort. As an application, Klein-Gordon operators are considered.

J. Cooper:

The Scattering Amplitude for Moving Obstacles

In the scattering of acoustic waves by a fixed obstacle in $\mathbb{R}^n$, n odd, the scattering amplitude is a function of the incident frequency σ and direction variables. This function has a holomorphic extension for $\operatorname{Im}\sigma < 0$ and a meromorphic extension for $\operatorname{Im}\sigma > 0$.
When the obstacle is moving, the reflection of a plane wave of frequency σ no longer has a single frequency. The scattering amplitude is now a

distribution in frequency variables σ' and σ. When $\sigma'-\sigma$ is real we find that it is still possible to construct a holomorphic extension for $\text{Im}\,\sigma < 0$.

If the motion is periodic, the scattering amplitude has a meromorphic extension for $\text{Im}\,\sigma > 0$ and may be written as a sum of partial amplitudes located at $\sigma' = \sigma + m\nu$ where m is integer and ν is the frequency of motion of the body.

J.A. DeSanto:

Coherent Scattering from Rough Surfaces

The mathematical formalism for calculating the Green's function for scattering from a rough surface is discussed with the aid of Feynam diagrams. For an arbitrary deterministic rough surface the scattering part of the Green's function satisfies a Lippmann-Schwinger-type integral equation with a non-central and complex "potential".

For a random surface with homogeneous statistics the ensemble average (coherent part) of the scattered Green's function satisfies a one-dimensional singular integral equation(Dyson equation). With a Gaussian distributed surface and plane wave incidence an approximation of this equation has been solved numerically and the result compared with experimental data and single scattering theories. Results support the conclusion that contributions from multiple scattering are necessary to account for the experimental data at large roughness.

V. Enss:

Finite total cross Sections in Quantum Scattering

Using time dependent geometric methods we obtain simple explicit upper bounds for total cross sections σ_{tot} in potential- and multi-particle scattering. σ_{tot} is finite if the potential decays a bit faster than r^{-2} (in 3 dim.) or if weaker direction dependent decay requirements hold. For potentials with support in a ball of radius R bounds are given which depend on R only.

We obtain upper bounds on σ_{tot} for large coupling constant λ, the power of λ depending on the fall off of the potential. For spherically symmetric potentials the variable phase method gives a lower bound growing with the same power of λ.

In the multiparticle case for charged particles interacting with Coulomb forces the effective potential between two neutral clusters decays sufficiently fast to imply finite total cross sections for atom-atom scattering (joint work with B.Simon).

J. Hejtmanek:

The Problem of Reconstructing Density Functions from Projections as an Inverse Problem in the Scattering Theory of the Linear Transport Operator

Scattering theory of the linear transport operator was initiated by J.Hejtmanek (1975) and further developed by B.Simon (1975) and J.Voigt (1976). A survey of this theory can be found in the book: Reed, Simon, Modern Methods in Mathematical Physics, vol.3, XI.12. The linear transport equation, which describes the time behavior of the photon density function for the CT model, is a simple version of the neutron transport equation, which was the focus of much mathematical work during the last 30 years for reactor engineers and neutron physicists. It is proved that the Heisenberg operator is a multiplication operator, and that it is a one-to-one mapping from the positive cone $L^1_+(\mathbb{R}^2 \times S^1)$ onto itself. The inverse problem can be solved by the inverse Radon tranformation formula.
The aim of this lecture is to demonstrate equivalence between the following two problems: Reconstruction of density functions from projections and the inverse problem in the scattering theory of the linear transport (Boltzmann) operator.

Teruo Ikebe:

A Stationary Approach to the Completeness of the Long-Range Modified Wave Operators

A completely stationary method is proposed for the proof of the completeness of the wave operators for the Schroedinger operators with long-range potentials. No use will be made of the existence result for the wave operators already known in a time-dependent setting, but which will be shown in our approach. (This is a joint work of H.Isozaki and myself.)

A. Jensen:

Time-delay in Scattering Theory

Consider $H_0=-\Delta$, $H=-\Delta+V$ in $L^2(R^n)$ with $V(x)$ real-valued, $V\in L^2_{loc}(R^n)$, $V(x)=O(|x|^{-\beta})$ as $x\to\infty$ with at least $\beta>2$. Existence and completeness of $W\pm=\text{s-}\lim_{t\to\pm\infty} e^{itH}e^{-itH_0}$ is well known. Let $S=\{S(\lambda)\}$ be the decomposition of the scattering operator in scattering matrices in the spectral representation. The Eisenbud-Wigner time-delay operator is in the spectral rep. given by $T=\{-iS(\lambda)^* \frac{d}{d\lambda}S(\lambda)\}$. T exists for $\beta>2$ and is essentially selfadjoint, commuting with H_0.
Let P_r be the spectral projection for $[-r,r]$ for the generator of the dilation group $D=\frac{1}{2i}(x\cdot\nabla+\nabla\cdot x)$. Requiring $\beta>4$ we get for a dense set of f

$$\lim_{r\to\infty}\int_{-\infty}^{\infty}(\|P_r e^{-itH}W_-f\|^2 - \|P_r e^{-itH_0}f\|^2)dt = \langle f,Tf\rangle .$$

The integrals exist for each r and represent differences in time spent in $P_rL^2(R^n)$ for a freely and a fully evolving state. $P_rL^2(R^n)$ corresponds classically to requiring $|x\cdot p|\leq r$ in phase space.

El Mabrouk:

Scattering Theory for Linear Thermo-elasticity

We develop a scattering theory for a class of dissipative systems, which generalizes the well-known energy-conservative theory of Lax-Phillips. This is then used to investigate the problem of asymptotic behavior of the thermo-elastic waves scattered by a bounded obstacle with homogeneous Dirichlet type conditions.

B. Najman:

Scattering Theory for Matrix Operators and the Indefinite Inner Product

We investigate the operators associated to the abstract second order differential equation $\frac{d^2u(t)}{dt^2} - iK\frac{du(t)}{dt} + Hu(t) = 0$ in a Hilbert space.

This equation is a perturbation of the free equation $\frac{d^2u(t)}{dt^2} + H_0u(t)=0$; the Klein-Gordon equation is of this form. There are different operators and a whole scale of spaces associated with these equations. It is shown that the usual theorems from the scattering theory hold for these operators.
It should be noted that the operators corresponding to the perturbed equation are not selfadjoint if H is not positive definite; however they are selfadjoint in an indefinite inner product. Using the spectral theory for selfadjoint operators in Krein spaces it is possible to construct a local scattering theory.

D.B.Pearson:

Localisation of States in Position and Energy

Let H be a self-adjoint extension of $-\Delta+v(\underline{r})$ acting on $C_0^\infty(\mathbb{R}^3\setminus\{0\})$, where $v(\underline{r})\to 0$ as $|\underline{r}|\to\infty$ but v may be singular at $\underline{r}=0$. To estimate the degree to which states may be localised in the region $|\underline{r}|<R$ and in the interval Σ of the spectrum of H, define the limits

$$\mu(\Sigma) = \lim_{R\to 0} \| E_{|\underline{r}|<R}\, E_{H\in\Sigma} \| \quad \text{and} \quad \mu(\lambda) = \lim_{\substack{\varepsilon\to 0 \\ R\to 0}} \| E_{|\underline{r}|<R}\, E_{|H-\lambda|<\varepsilon} \| .$$

It may be shown that, for given λ, $\mu(\lambda) = 0$ or 1, ($\mu(\lambda)=1 \equiv$ "singular point", $\mu(\lambda)=0 \equiv$ regular point) and that $\mu(\Sigma)=0$ or 1 unless the end points of Σ are singular points.
A complete enumeration can be given of phenomena giving rise to singular points. These include breakdown of asymptotic completeness, presence of singular continuous spectrum, and a discontinuity of the scattering amplitude at a given energy.

R.S.Phillips:

Scattering Theory for the Wave Equation with a Medium Range Perturbation

One of the disadvantages of the Lax-Phillips approach to scattering has been its inability to handle perturbations which extend over unbounded domains. This paper develops just such an extension of this theory for spaces of odd dimensions. To be precise the wave equation: $u_{tt} = \partial_i(a_{ij}\ \partial_j u) - qu$ is treated when $q = O(r^{-\alpha})\ \alpha > 2$, and $a_{ij} - \delta_{ij} = O(r^{-\beta}) = \partial_i\ a_{ij}, \beta > 1$. Although the result is a well known consequence of Schrödinger scattering theory, it turns out that an adaptation of the Enss argument to hyperbolic systems is both simpler and more straightforward than the original Schrödinger argument.

A.G.Ramm:

Nonselfadjoint Operators in Diffraction and Scattering

The following questions will be discussed:

1) When have the integral operators arising in diffraction and scattering theory root system which forms a basis of a Hilbert space?
2) When have these operators no root vectors?
3) What can be said about location and properties of the complex poles of Green functions? How to calculate these poles? Stability of the poles under perturbations of the boundary of an obstacle.
4) Variational principles for spectrum of compact nonselfadjoint operators.
5) Miscellanea.
6) Unsolved problems.

A.G.Ramm:

Wave Scattering by Small Bodies of Arbitrary Shape

The following questions will be discussed:

1) How does the scattering amplitude depend on boundary conditions? How to find an approximate analytical formula for scattering matrix in electromagnetic and scalar scattering problems?
2) How to treat many body problem?
3) Scattering in the medium consisting of many small particles.

P.Sabatier:

Exotic Topics in Inversion Theory

Inversion theory is the common knowledge on exact inversion methods, single ways of appraising ill-posedness of inverse problems, construction methods for solutions or quasisolutions. Exotic applications use this knowledge for purposes that have nothing or little to do with inverse problems. Four examples are given:

(1) We study a linear and irrotationnal model of water waves generation near a sloping beach by ground motions offshore. The direct problem is ill-posed and is relevant of inversion theory. One constructs a couple "source-response" in which the source is "δ-like" and the response is locally the constant depth one. It is possible to improve easily the δ-ness if the ratio maximum depth/minimum depth is bounded. In any case, provided the slope is everywhere smaller than 45^0, a good approximation can be derived.

(2) The second example uses exact inversion methods, e.g. the Gelfand Levitan method, out of the range in which they are consistent. This can yield singular potentials (e.g. Chadan's potentials, and a new exactly solvable potential at fixed energy, any 1 , a symptotic to $r^{-1}\sin 2r$ as r goes to ∞). It can also yield transformation operators that do not derive from a symmetric Kernel.

(3) The third example is a method reducing an inverse problem that is defined by a finite number of equalities for implicit functionals to a problem that is defined by a finite or infinite number of inequalities for explicit functionals of a given parameter. This can be used to get inequalities for explicit functionals of the Earth density and Lamé parameters, where the right hand side is given by experimental results.

(4) The fourth (and last) example is that which succeeded the greatest achievements: using the inverse spectral problem like an inverse transform for solving non linear partial differential equations (e.g. Korteweg-de Vries).

A.W.Sáenz:

Asymptotic completeness for Scattering by Periodic Surfaces with the Homogeneous Dirichlet Boundary Condition

Write $x\in\mathbb{R}^\nu (\nu\geq 2)$ as $x = (\tilde{x},x_\nu)$, with $\tilde{x}\in\mathbb{R}^{\nu-1}$ and $x_\nu\in\mathbb{R}$. Let Ω be a domain of $\mathbb{R}^\nu$ such that (i) $\{(\tilde{x},x_\nu)\in\mathbb{R}^\nu : x_r>1\}\subset\Omega\subset\{(\tilde{x},x_\nu)\in\mathbb{R}^\nu : x_\nu>0\}$; (ii) the periodicity property $(\tilde{x},x_\nu)\in\Omega \Rightarrow (\tilde{x}+\tau,x_\nu)\in\Omega$ holds for all $\tau\in Z^{\nu-1}$. Define the wave operators $W_\pm = W_\pm(H,H_0) = \text{s-}\lim_{t\to\pm\infty} \exp(itH)P\exp(-itH_0)$, where H_0 is the negative Laplacian in $\mathcal{H}_0 = L^2(\mathbb{R}^\nu)$, H the negative Dirichlet Laplacian in $\mathcal{H} = L^2(\Omega)$, $(D(H) = \{f \in L^2(\Omega): \Delta f \in L^2(\Omega)$ and $f \in H_0^1(\Omega)\})$, and $(Pf)(x) = f(x)$, $f \in \mathcal{H}_0, x \in \Omega$. Then: (a) $W_\pm$ are partially isometric with initial sets $\{f \in \mathcal{H}_0 : |\hat{f}(\tilde{k}, k_\nu) = 0,\ k_\nu \gtrless 0\}$ and final set $\mathcal{H}_{scatt} = \{f \in \mathcal{H}: \lim_{t \to \pm\infty} \| \exp(-itH)f \|_{L^2(\Omega_\alpha)} = 0, \alpha > 0\}$ (scattering states), where $\Omega_\alpha = |\{(\tilde{x}, x_\nu) \in \Omega : x_\nu < \alpha\}$; (b) asymptotic completeness holds: $\mathcal{H} = \mathcal{H}_{scatt} \oplus \mathcal{H}_{surf}$, where $\mathcal{H}_{surf} = \{f \in \mathcal{H}: \lim_{a \to \infty} \sup_{t \in R} \| \exp(-itH)f \|_{L^2(\Omega \setminus \Omega_\alpha)} = 0\}$ (surface states). We proved (a) and (b) by reducing the problem to a family of scattering problems in a distorted cylinder, using direct-integral methods analogous to Lyford's procedures. Our methods also work in the case of the homogeneous Neumann boundary condition.

Y. Saito:

Inverse Scattering Problem for Short-Range Potentials

Let $F(k,\omega,\omega')$, $k>0, \omega, \omega' \in S^2$, be the scattering amplitude for the Schroedinger operator $-\Delta + V(y)$ in $\mathbb{R}^3$ with $V(y) = O(|y|^{-2-\varepsilon})$. Starting with a formula in which $F(k,\omega,\omega')$ is directly represented by the potential $V(y)$, we shall investigate the properties of the scattering amplitude $F(k,\omega,\omega')$ in a unified way. Further, an answer for the inverse scattering problem will be given.

C.G.Simader:

Essential Selfadjointers of Schroedinger Operators with magnetic Vector Potentials

Report on a recent joint work with H.Leinfelder (Bayreuth) is given. Consider a formal Schroedinger operator $\mathcal{H} := -\sum_{j=1}^{m} (D_j - ia_j)^2 + q$ with coefficients a_j, q defined on $\mathbb{R}^m$. Under the conditions

(C.1) $\quad a = (a_1, \ldots, a_m) \in L^2_{loc}(\mathbb{R}^m)^m,\ 0 \leq q \in L^1_{loc}(\mathbb{R}^m)$

it is proved that the maximal and the minimal form associated with $\mathcal{H}$ coincide. Further, a maximum principle holds true. If in addition

(C.2) $\begin{cases} a = (a_1,\ldots,a_m) \in L^4_{loc}(\mathbb{R}^m)^m, \text{ div } a \in L^2_{loc}(\mathbb{R}^m) \\ \text{(in the sense of distributions)}, 0 \leq q \in L^2_{loc}(\mathbb{R}^m) \end{cases}$

holds true, $\mathcal{H}$ is essentially selfadjoint on $C_0^\infty(\mathbb{R}^m)$. Various generalizations - concerning q - are given. Tha assumptions (C.1) and (C.2) as far as they concern a, seem to be possible. This work improves recent results of T.Kato (1978) and B.Simon (1980).

H. Sohr:

Remarks on Potential Scattering with a Time-Dependent Hamiltonian

We consider a time-dependent Hamiltonian of the form $H(t) = H_0+V(t)$, where $H_0 = -\Delta$ and $V(t) = V(t,\cdot)$ is a potential. Using a perturbation lemma we get the following conditions for the existence of evolution operators $U(t,s)$ $(t,s\in\mathbb{R})$ and wave operators

$W_\pm = s - \lim_{t\to\pm\infty} U(0,t)\exp(-itH_0)$: $V(t)\in C^1(\mathbb{R}^n)$, $V(t) \geq 0$, $\mathcal{D}_{V(t)} = \mathcal{D}_{V(0)}$ (for all $t\in\mathbb{R}$), $t\to V(t)(V(0)+1)^{-1}$ strongly continuously differentiable, $|\nabla_x V(t)|^2 \leq 4\alpha(V(t)+c)^3$ with constants $0 \leq \alpha < 1$, $c \geq 0$, $\| V(t,\cdot)(1+|\cdot|^{-\frac{n}{2}+1+\varepsilon+r})\|_{L_2} \leq K(1+|t|)^r$ with constants $\varepsilon>0$, $r\in\mathbb{R}$, $K>0$. This theorem applies to the case $V(t,x) = c(t)|x|^q$ with $q \geq 0$, $x\in\mathbb{R}^n$.

M.M.Sondhi:

Inverse Scattering Theory Applied to Problems in Speech and Hearing

The theory of one-dimensional inverse scattering can be used to infer properties of the vocal tract as well as those of the inner ear. After briefly sketching the relevant theory, I will discuss results of our recent experiments on the vocal tract. I will also discuss computations of inner-ear parameters based on measurements made at other laboratories.

F.Stenger: Siehe S. 13

H.Tamura:

The Principle of Limiting Absorption for Propagative Systems in Crystal Optics with Perturbation of Long-range Class

The principle of limiting absorption is proved for the steady-state wave problems of crystal optics in an inhomogeneous medium with perturbations of long-range class. As an application, it is shown that a solution with finite energy of the perturbed Maxwell equation has a local energy decaying property for initial data orthogonal to the eigenspace. The Maxwell equation in crystal optics is an important example of non-uniformly propagative systems. In this talk, only a special problem is dealt with, but the method here is also applicable to other wave propagation phenomena of classical physics such as elastic wave in crystals, which is also an example of non-uniformly propagative systems.

H.Ueberall:

Resonance Theory of Nuclear, Acoustic, Elastic and Electromagnetic Scattering

We demonstrate that the methods of nuclear resonance theory may be applied to classical scattering problems, and we interpret in this way the scattering of acoustic waves by elastic bodies, of elastic waves by cavities and inclusions, and of electromagnetic waves by dielectric or conducting targets. It is shown that the resonances provide information on the properties of the target, i.e. help solve the inverse scattering problem. In the nuclear domain, resonances in electron and heavy-ion scattering are considered. In all cases, the origin of the resonances is traced back to the phase matching of surface waves generated in the scattering process. (Supported by the National Science Foundation, Theoretical Physics Section, the Office of Naval Research, Code 420, and the Naval Air Systems Command, AIR-310 B).

H.Ueberall:

Theory of Mode Coupling in Sound Propagation under the Ocean

Studies on the propagation of scalar fields in laterally non-uniform ducts have been performed, with application to acoustic propagation in under-ocean sound channels. The method of calculation used was

the adiabatic mode theory of Pierce and Milder. This theory, however, in addition furnishes mode coupling terms.
We consider the following examples: (a) adiabatically: a homogeneous wedgeshaped duct; a parabolic wave number profile which opens up with range; a numerically given general profile with general range dependence: (b) including mode coupling: the parabolic profile opening up with range. A program for the generally z- and r-dependent case has also been written.
The same approach has also been used for the problem of coupled mode propagation in the ionospheric day-night transition region.
(Supported by the Office of Naval Research, Code 486, and by the Naval Research Laboratory, Washington, DC, Code 8120).

W.Wickel:

On Initial Boundary Value Problems of Linear Thermoelasticity

The initial boundary value problem for the linear thermoelastic equations in anisotropic inhomogeneous bounded and unbounded media is treated by means of semigroup theory. It is shown that for $t\to\infty$ the temperature deviation u and the expression $\frac{\partial}{\partial t} g_{ij}(x)\partial_j U(x,t)$ tend to zero (U displacement field, (g_{ij}) stress-temperature tensor). The final part contains some remarks on the "hyperbolized" case describing physical phenomena in which temperature behaves like a damped wave.

C.H.Wilcox:

Scattering Theory for Diffraction Gratings

A domain $G\subset R^2$ is said to be a grating domain if $\mathbb{R}^2_h \subset G \subset \mathbb{R}^2_0$ and $G+(2\pi,0) = G$ where $\mathbb{R}^2_c = \{(x,y) : y>c\}$. Scattering theory is developed for the pair of selfadjoint operators $A = A(G)$ in $L_2(G)$ and $A_0 = A(\mathbb{R}^2_0)$ in $L_2(\mathbb{R}^2_0)$ defined by $-\Delta$ and the Dirichlet or Neumann boundary condition. The wave operators $W_\pm = W_\pm(A_0^{1/2}, A^{1/2}, J_G)$ are shown to exist and satisfy $W_\pm = \Phi_0^* \Phi_\mp$ where $\Phi_\pm^- : L_2(\bar{G}) \to L_2(\mathbb{R}^2_0)$ and $\Phi_0 : L_2(\mathbb{R}^2_0) \to L_2(\mathbb{R}^2_0)$ are unitary operators defined by complete sets of generalized eigenfunctions for A and A_0 respectively. The completeness of $W_\pm$ is shown to hold in the case where A admits no surface waves.

W.W.Zachary:

Discrete Spectrum of Schroedinger Operators for a Sum of Potentials. Connection with the Inverse Scattering Formulation of Nonlinear Evolution Equations

Solutions of the Korteweg de Vries equation have been found which are either rapidly decreasing at large distances or periodic in the spatial coordinate. However, no solutions have yet been found which embody both of these features, such as, e.g., a linear combination of the two types.

If one tries an inverse scattering solution of the latter problem similar to that known in the rapidly decreasing case, then one encounters various difficulties due to the fact that the spectral theory of Schroedinger operators is not as well known as the rapidly decreasing case.

We discuss results concerning the following aspects of the spectral theory of Schroedinger operators corresponding to potentials of the form $U = V + Q$: (1) Estimates of eigenvalues when V is periodic and $Q = 0$; (2) Asymptotic distribution of eigenvalues for V periodic and Q rapidly decreasing; (3) Upper bounds on the number of negative energy eigenvalues for V bounded below and $Q \in L^{n/2}(\mathbb{R}^n)$, $n \geq 3$; (4) Exponential decay of negative energy eingenfunctions for V bounded below, $Q \in$ suitable L^p classes.

F. Stenger:

An Algorithm for Ultrasonic Tomography Based on Inversion of the Helmholtz Equation $\nabla^2 u + [\omega^2/c^2(r)]u = 0$.

A numerical procedure is described for reconstructing the function $f(\overline{r}) = \omega^2[c(\overline{r})^{-2} - c_0^{-2}]$, where $c(\overline{r})$ denotes the speed of sound in a bounded body and c_0 denotes the speed on sound in the medium surrounding the body, for both the case of plane wave excitation, $e^{i(\overline{k}\cdot\overline{r} - \omega t)}$, and spherical wave excitation, $[4\pi|\overline{r} - \overline{r}_s|]^{-1} \exp[ik|\overline{r} - \overline{r}_s| - i\omega t]$.

Berichterstatter: P. Werner (Stuttgart)

Liste der Teilnehmer:

Dr. Hans Dieter Alber
Inst. f. Angew. Mathematik
Universität Bonn
Wegelerstr. 1o
D-5300 Bonn

Prof. T.S. Angell
Dept. of Mathematics Sci.
University of Delaware
Newark, DE 19711, USA.

Prof. V. Barcilon
Geophysical Sciences Dept.
University of Chicago
Chicago, IL 6o637, USA.

Dr. R. Colgen
Mathematisches Institut
Universität Frankfurt
Robert-Mayer-Str. 1o
6ooo Frankfurt/M. 1

Prof. Jeffery Cooper
Dept. of Mathematics
University of Maryland
College Park, MD 2o742 USA.

Dr. J.A. DeSanto
Naval Research Laboratory, Code 834o D
Washington, DC 2o375, USA.

Dipl.Math. Godher Eberlein
Fachbereich Mathematik
Universität Darmstadt
Schlossgartenstr. 7
61oo Darmstadt

Prof. Volker Enss
Mathematisches Institut
Universität Bochum
4630 Bochum

Herr Markus Faulhaber
Math. Institut A
Universität Stuttgart
Pfaffenwaldring 57
7 Stuttgart 8o

Dr. Otto Gartmeier
Höhenkirchenerstr. 42
8o11 Hohenbrunn b.München

Prof. Dr. J. Hejtmanek
Math. Institut
Universität Wien
Strudlhofgasse 4
A-1090 Wien

Prof. T. Ikebe
Faculty of Science
Kyoto University
Kyoto 606, Japan

Prof. A. Jensen
Math. Department
University of Kentucky
Lexington, Kentucky, USA.

Prof. T. Kato
Dept. of Mathematic
University of California
Berkeley, CA 19711, USA

Prof. S.T.Kuroda
College of General Education,
Pure & Applied Science,
University of Tokyo,
381 Komaba Meguro Ku
Tokyo (153), Japan

Prof. N.Latz
Math. Institut
Techn. Universität Berlin
Strasse des 17. Juni, 135
1 Berlin 12

Prof. Rolf Leis
Inst. f. Angew. Mathematik
Universität Bonn
Wegelerstr. 10,
5300 Bonn

Dr. Mongi El Mabrouk
Dept. de Mathématique
Service de Mécanique
Univ. des Sciences et Technique de Lille
B.P.36, 59650 Villeneuve d'Ascq, France

Prof. E. Meister
Fachbereich Mathematik
Technische Hochschule
6100 Darmstadt

Dr. B. Najman
Dept. of Mathematics
University of Zagreb
POB 187, 41001 Zagreb, Jugoslawien

Prof. D.B.Pearson
Dept. of Applied Math.
University of Hull,
Hull HUG7RX, England

Prof. Ralph S. Phillips
Dept. of Mathematic
Stanford University
Stanford CA 943o5, USA

Dr. Rainer Picard
Inst. für Angew. Mathematik
Universität Bonn,
Wegeler Str. 1o
53oo Bonn

Dr. K.S.Rao
Inst. f. theoret. Kernphysik
Universität Bonn
Nussallee 45,
53oo Bonn

Prof. A.G. Ramm
Dept. of Mathematics
University of Michigan
Ann Arbor, MI 481o4, USA.

Prof. B.C.Sabatier
Laboratoire de physique mathématique
Université des Sciences de Languedoc
34o6o Montpellier Cédex, France

Dr. W.A.Sâenz
Naval Research Laboratory,
Code 66o35
Washington, DC. 2o375, USA.

Prof. J.Saito
Dept. of Mathematics
Osaka City University,
Sugimoto-cho, Sumiyoshi-ku
Osaka, Japan

Prof. Dr. C.G.Simader
Fakultät f.Mathematik und Physik
Universität Bayreuth
Postfach 3oo8,
858o Bayreuth

Prof. H.Sohr
Mathematisches Institut
Gesamthochschule Paderborn
479o Paderborn

Dr. M.M. Sondhi
Bell Telephone Laboratories
Murray Hill NJ. o7974, USA.

Prof. F.Stenger
Dept. of Mathematics
University of Utah
Salt Lake City, Utah 84112, USA.

Prof. H. Überall
Dept. of Mathematics
Catholic University of America
Washington DC 2oo64, USA.

Prof. Dr. N. Weck
Mathematisches Institut
Universität Essen
43oo Essen

Dr. Karl-Joseph Witsch
Inst. f. Angew. Mathematik
Universität Bonn
Wegelerstr. 1o
53oo Bonn

Dr. W.W.Zacharay
Naval Research Laboratory
Washington DC 2o375, USA.