MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Analytische Zahlentheorie

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Vom 2. bis 8. November fand im Mathematischen Forschungsinstitut Oberwolfach unter Leitung der Herren Prof. Dr. H.E. Richert (Ulm), Prof. Dr. W. Schwarz (Frankfurt) und Prof. Dr. E. Wirsing (Ulm) eine Tagung über elementare und analytische Zahlentheorie statt. Die neue Rekordbeteiligung von 70 Teilnehmern aus 15 Ländern beweist das große Interesse an dieser Tagung. Neben den 53 Vorträgen trugen zahlreiche Diskussionen sowie die schon zur Tradition gewordene "Problemsitzung" zum Gelingen der Tagung bei.

Vortragsauszüge

G.J. BABU: The Riemann hypothesis and strong recurrence.
The speaker reports on some results recently obtained by B. BAGCHI that relate the zero-free regions of the Riemann zeta function to its general asymptotic behaviour in the critical strip. The terminology used in stating the results, as also the tools used in the proofs, are borrowed from the theories of topological dynamics and probability. He requires a series of definitions in order to arrive at the crucial notions of spectrum and strong recurrence.

P.T. BATEMAN: The arithmetic mean of the divisors of an integer. The following theorems were obtained in a recent paper of Bateman, Erdös, Pomerance, and Straus. In what follows $d(n) = \sum_{d|n} 1$ and $\sigma(n) = \sum_{d|n} d$.

I. The set of positive integers n for which $\sigma(n)/d(n)$ is not an integer has asymptotic density zero; in fact, if $N(x)$ denotes the number of such integers not exceeding x, we have $N(x) = x \exp(-(1 + o(1))2\sqrt{\log(2)}\sqrt{\log\log(x)})$.

II. If $d(n) = \prod_{p|d(n)} p^{\alpha_p}$ and β is a positive real number, let $\langle d(n)^{\beta} \rangle = \prod_{p|d(n)} p^{[\beta\alpha_p]}$. Then the set of n for which $\langle d(n)^{\beta} \rangle$ devides $\sigma(n)$ has asymptotic density 1 if $\beta < 2$, 1/2 if $\beta = 2$, and 0 if $\beta > 2$.

III. $\sum_{n \leq x} \sigma(n)/d(n) = (1 + o(1)) \dfrac{\lambda x^2}{\sqrt{\log(x)}}$ for a certain positive λ.

IV. $\#\{n \mid \sigma(n)/d(n) \leq x\} = (\mu + o(1))\, x \log x$ for a certain positive μ.

V. The number of distinct rational numbers of the form $\sigma(n)/d(n)$ not exceeding x is $O(x\, (\log x)^{-v})$ for a certain positive v. A more complicated argument than that given in the paper shows that v can be taken as any number less than 1 but not as 1 itself.

H.J. BENTZ: On a conjecture of Shanks.

"Conjecture: Let l_1 be a quadratic residue mod q, l_2 a non-residue mod q. Then there are "more" primes $\equiv l_2$ mod q than $\equiv l_1$ mod q." One notices that this formulation is not very precise. To understand why, it is best to look at the history of this conjecture.

a) One sense of the "more" was given by Chebyshev. He asserted (case q = 4) that (1) $\lim_{x\to\infty} \sum_{p>2} (-1)^{(p-1)/2} e^{-p/x} = -\infty$. From this one could deduce a preponderance of the

$p \equiv 3 \bmod 4$ over those $\equiv 1 \bmod 4$. But (1) is very deep, for it is fully equivalent to an analogue of the Riemann-hypothesis, namely (2) $L(s,\chi_1) \neq 0$ in $\mathrm{Re}(s) > 1/2$, χ_1 the nonprincipal character mod 4, as was shown by Hardy, Littlewood, and Landau.

b) A second case, which was investigated, is the following: $\Delta(x) := \pi_1(x) - \pi_3(x) \overset{?}{\leq} 0$, at least for $x \geq x_0$. Here $\pi_j(x)$ is the number of primes not exceeding x, which are $\equiv j \bmod 4$. Although numerical calculations show that Δ is predominantly negative, the assumption above is wrong. Hardy and Littlewood have proved that Δ is not bounded in either direction.

c) Knapowski and Turân have investigated this phenomenon with a different weight-function: (3) $\lim\limits_{x\to\infty} \sum\limits_{p>2} (-1)^{(p-1)/2} \log(p)\, e^{-\log^2(p/x)} \overset{?}{=} -\infty$. This is also equivalent to (2) - as they have shown - and therefore the answer is not known at present.

Let us now look at some numerical data. As already mentioned the difference Δ seems to be predominantly negative. Sign changes are found to be very rare. For example: case $q = 4$: first sign change of $\pi_1 - \pi_3$ occurs at 26861 (calculated by Leech, Wrench, and Shanks); case $q = 3$: no sign change was found up to 35000000 i.e. $\pi_1(x) \quad \pi_2(x)$ for $x \quad 35000000$ (calculated by myself); case $q = 8$: $\pi_1 - \pi_5$ has its first sign change at 588067889 (calculated by Hudson and Bays).

By these(and other) data one is forced to look for a meaning of these discrepancies. Now, I myself tried to work with another weight-function and got the following results.

Theorem 1. (case $q = 3,4$) $\lim\limits_{x\to\infty} \sum\limits_{p} \chi_1(p) \log(p)\, p^{-\alpha} e^{-(\log(p))^2/x}$ $= -\infty$ for all α in $0 \leq \alpha \leq 1/2$, χ_1 nonprincipal mod q.

Theorem 2.(case $q = 8$). Let $\varepsilon(p,q,l_1,l_2) = 1$ if $p \equiv l_1 \bmod q$, -1 if $p \equiv l_2 \bmod q$, 0 else; l_1 quadr. residue, l_2 quadr. nonresidue. Then $\lim\limits_{x\to\infty} \sum\limits_{p} \varepsilon(p,8,l_1,l_2) \log(p)\, p^{-\alpha} e^{-(\log(p))^2/x}$ $= = -\infty$ for $0 \leq \alpha \leq 1/2$.

Similar results may be obtained for other moduli as well (sometimes,however, only in a weaker form, e.g. "there is at least one non-quadr. res. class, in which there are more primes than in each of the quadr. res. classes"). The method for these theorems uses explicit formulas for L-series. The results depend on the location of the "first" zero of $L(s,\chi)$ in the crirical strip. More precise, the condition is "there is no zero in $|\mathrm{Im}\ s| \leq |\mathrm{Re}(s-1/2)|$, $0 < \mathrm{Re}\ s < 1$". This is known to be true for $q < 25$ by calculations of Davis, Hazelgrove, and Spira. A different method was suggested by J. Pintz.

Theorem 3(Bentz,Pintz). If the above condition is fulfilled for all $L(s,\chi)$ mod q, q fix, then

$$\lim_{x\to\infty} \sum_p \varepsilon(p,q,l_1,l_2) \log(p)\, p^{-\alpha}\, e^{-(\log(p))^2/x} = -\infty \text{ for}$$

$0 \leq \alpha \leq 1/2$. References: D.Shanks, Quadr. Residues and the Distribution of Primes, Math. Tables and Aids to Comp. 13 (1959) 272-284. H.J.Bentz, Discrepancies in the Distribution of Prime Numbers, to appear, H.J.Bentz,J.Pintz, Quadr. Res. and the Distribution of Prime Numbers, Monatsh. für Math. (1980).

B.C.BERNDT: <u>Chapter 5 of Ramanujan's Second Notebook</u>.

Chapter 5 of Ramanujan's second notebook contains more number theory than any other of the remaining 20 chapters. The chapter contains 94 formulas or statements of theorems Most of the results are concerned with Bernoulli numbers, Euler numbers, Eulerian numbers, and the Riemann zeta-function. As one would expect, the majority of Ramanujan's findings in these areas are not new. Ramanujan's published papers on Bernoulli numbers and irregular numbers have their genesis in this chapter. Chapter 5 also contains some interesting theorems on difference equations and an intriguing, but incorrect, power series identity involving primes.

H. DELANGE: On some subsets of $\mathbb{N}$ whose characteristic function is almost periodic B^1.

An arithmetic function f is said to be almost periodic B^1 (resp. limit-periodic B^1) if, for every positive ε, there exists a trigonometric polynomial P (resp. a periodic arithmetic function P) such that

$$\limsup_{x\to\infty} \frac{1}{x} \sum_{n \le x} |P(n) - f(n)| \le \varepsilon .$$

If f is a.p.B^1, then, for every real λ, $\lim_{x\to\infty} \frac{1}{x} \sum_{n \le x} f(n)\, e^{-2\pi i \lambda n}$ exists and is finite, $= C_f(\lambda)$ say. Of cause $C_f(\lambda)$ depends only upon the fractional part of λ. The spectrum of f, which we denote by Sp f, is the set of those $\lambda \in [0,1[$ for which $C_f(\lambda) \neq 0$. If infinite it is denumerable. The Fourier series of f, which we denote by F_f, is $\sum_{\lambda \in \mathrm{Sp}\, f} C_f(\lambda)\, e^{2\pi i \lambda n}$.

f is l.p.B^1 if and only if it is a.p.B^1 and Sp f contains only rational numbers. Then each term of $F_f(n)$ is of the form $C_f(h/q)\, e^{2\pi i (h/q) n}$, where $q \in \mathbb{N}$, $1 \le h \le q$, and $(h,q) = 1$. We say that $F_f(n)$ is a "Ramanujan series" if $C_f(h/q)$ depends only upon q, so that, by grouping together the terms corresponding to the same q, it may be written as $\sum_q a_q c_q(n)$ where $c_q(n)$ is Ramanujan's sum. It is known that, if f is real-valued and a.p.B^1, then it has a limit distribution. If σ_f is the corresponding distribution function, then for each x where σ_f is continuous, the set of those n for which $f(n) < x$ has the natural density $\sigma_f(x)$. In other words the function $I_x \circ f$ has the mean - value $\sigma_f(x)$; here $I_x(t) = 1$ if $t < x$ and 0 else.

In a recent paper in the Proceedings of the Academy of Japan, J. Mauclaire states the following result: Let f be a real-valued multiplicative function such that $f(n) \ge 1$ for every n, and suppose f is l.p.B^1. Then for each x where $\sigma_f(x)$ is continuous, $I_x \circ f$ is l.p.B^1 and its Fourier series is a Ramanujan series. As examples he quotes $f(n) = n/\varphi(n)$

and $f(n) = \sigma(n)/n$. I prove the following result.

Theorem 1. Let f be a real-valued arithmetic function. (1) If f is a.p.B^1, then for each x where σ_f is continuous $I_x \circ f$ is a.p.B^1; (2) If f is l.p.B^1, then for each f where σ_f is continuous, $I_x \circ f$ is l.p.B^1; (3) If f is l.p.B^1 and if its Fourier series is a Ramanujan series, the for each x for which σ_f is continuous, $I_x \circ f$ is l.p.B^1 and its Fourier series is a Ramanujan series.

Examples of (3): f such that $\sum |f'(n)|n^{-1} < \infty$, where $f'(n) = \sum_{d|n} \mu(d)f(n/d)$. In particular, f multiplicative such that $\sum_{p,r} |f(p^r)-f(p^{r-1})|p^{-r} < \infty$ (which includes the functions quoted by Mauclaire). - Any real-valued multiplicative function which is a.p.B^1 and has a non-zero mean value. - Any real-valued additive function which is a.p.B^1.

I also prove another theorem which gives subsets of N whose characteristic function is l.p.B^1 with a Fourier series which is a Ramanujan series.

Theorem 2. Let $f = F \circ g$, where g is a complex-valued arithmetic function such that $\sum_{g(n)\neq 0} n^{-1} < \infty$ and F is defined on g(N), and bounded. f is l.p.B^1 and its Fourier series is a Ramanujan series.

Corollary. Let g be a complex-valued arithmetic function such that $\sum_{g(n)\neq 0} n^{-1} < \infty$. If E is any subset of $\mathbb{C}$, then the characteristic function of the set of those n for which $g(n) \in E$ is l.p.B^1 and its Fourier series is a Ramanujan series. In particular, if a is any element of $g(\mathbb{N})$, the set of those n for which $g(n) = a$ has a natural density δ_a say. It is easy to see that $\sum \delta_a = 1$.

Examples: 1. f additive such that $\sum_{f(p)\neq 0} p^{-1} < \infty$ 2. f such that $f(n) = f(q_n)$, where $q_n = \prod_{p^2|n} p^{v_p(n)}$.

J.M. DESHOUILLERS: Additive number theory and integral points on curves. The aim of the lecture was to underline connection between the number of representations of an integer as a sum of two elements from a given sequence and the number of integral points on a curve.

Let P be a positive integer and denote by $B^{(P)}$ the class of $C^{(P)}$-class functions $a: \mathbb{R}^+ \to \mathbb{R}^+$, $a(0) = 0$, $a(1) = 1$, $a^{(p)} > 0$ $(p = 1,\dots,P)$. Define $r_h(t) = \{(n_1,\dots,n_h) \mid t = a(n_1) + \dots + a(n_h)\}$, $A = a^{-1}$.

Theorem 1. Let α be in $]0,1]$ and $P > 0$; one has (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii)

(i) $\forall a \in B^{(P)}$: $r_2(t) \ll (A(t))^{\alpha}$

(ii) $\forall f \in B^{(P)}$: $\#\{n \le x,\ f(n) \in \mathbb{N}\} \ll \left(\int^x \sqrt{1 + (f'(t))^2}\ dt\right)^{\alpha}$

(iii) $\forall a \in B^{(P)}$: $(\forall p \le P : (-1)^p A^p > 0) \Rightarrow r_2(t) \ll (A(t))^{\alpha}$.

In the special case P = 2 sentences (i) and (iii) are equivalent, and it is even possible, by a direct construction to prove Theorem 2: In theorem 1, for P = 2, (i) and (ii) hold with $\alpha = 2/3$ and implied"constant"$(3/\pi^2)^{1/3}(1+o(1))$, and these are best possible.

The case P = 3, in conjunction with a result by Swinnerton - Dyer leads to Cor. 1: For $P \ge 3$, $r_2(t) \ll_\varepsilon (A(t))^{3/5 + \varepsilon}$.

Cor. 2. For $P \ge 3$, $r_h(t) \ll_\varepsilon (A(t))^{h - 7/5 + \varepsilon}$.

It is however impossible to go much further in this direction

Theorem 3. $\exists\, a \in B^{(\infty)}$: $r_2(t) = \Omega((A(t))^{1/2})$; $\forall \ell \ge 3\ \exists a \in B^{(\infty)}$: $r_h(t) = \Omega((A(t))^{h-2})$.

E. FOUVRY: On the level of distribution of some sequences.

For $a \ne 0$ the Bombieri - Vinogradov theorem implies

$$\forall \varepsilon > 0\ \forall A \quad \sum_{\substack{q \le x^{1/2-\varepsilon} \\ (q,a)=1}} \left|\pi(x;q,a) - \frac{\pi(x)}{\varphi(q)}\right| \ll x\,(\log x)^{-A}.$$

The constant 1/2 has never been improved, and the purpose of this lecture was to explain the construction of sequences approaching the sequence of primes with a better constant.

Theorem A. Let $z \le x^{1/883}$ and $1 \le |a| \le x$, then for any $A > 0$

$$\sum_{\substack{q \le x^{11/21} \\ (q,a)=1}} \left| \sum_{\substack{n \le x \\ n \equiv a \bmod q \\ p \mid n \Rightarrow p > z}} 1 - \frac{1}{\varphi(q)} \sum_{\substack{n \le x \\ (n,q)=1 \\ p \mid n \Rightarrow p > z}} 1 \right| \ll x\,(\log x)^{-A}.$$

(this theorem has been proved by H. Iwaniec and the speaker).
Let us recall R^* - hypothesis, conjectured by C. Hooley:
For $1 \leq A_2 - A_1 < r$, we have

$$\sum_{\substack{A_1 < s < A_2 \\ (s,r)=1}} e\left(\frac{k\bar{s}+ls}{r}\right) \ll (A_2 - A_1)^{1/2} (k,r)^{1/2} r^{\varepsilon} .$$

If we have to study the level of distribution of the sequence $mn - a$, where m and n belong to particular sequences, we must study

$$\text{(E)} \quad \sum_{\substack{q \leq N^{1-\varepsilon} \\ (q,a)=1}} \ \sum_{\substack{m < M \\ (m,q)=1}} \left| \sum_{\substack{n \leq N \\ n \in P \\ mn \equiv a \bmod q}} 1 - \frac{1}{\varphi(q)} \sum_{\substack{n \leq N \\ n \in P \\ (n,q)=1}} 1 \right| \ll$$

$MN(\log(MN))^{-A}$ for $M \leq N$, $1 \leq |a| \leq MN$, and $P \subset N$.

We proved the following:

Theorem B. On R^*-hypothesis: If P satisfies, for $(b,q)=1$ and for all B,

$$\sum_{\substack{n \in P \\ n \leq x \\ n \equiv b \bmod q}} 1 = \frac{1}{\varphi(q)} \sum_{\substack{n \in P \\ n \leq x}} 1 + O(x(\log x)^{-B}),$$

and if $N \leq M^{1/3 - \delta_0}$, $\delta_0 > 0$, then (E) is true.

Theorem C. If P is the sequence of primes, and if $N \leq M^{10/9 - \delta_0}$, then (E) is true.

The conditional theorem B gives the exponent $4/7 - \varepsilon$ with the optimal choice of N and theorem C the exponent $10/19 - \varepsilon$.

J. FRIEDLANDER: Sifting short intervals.

Let $g(x)$ denote a real-valued function tending to ∞ with x. Heath - Brown (1978) proved that for all y in $[0,x]$ apart from a set of measure $o(x)$, the interval $(y, y+\varphi(y))$ contains a P_3, where $\varphi(y) = (\log y)^{35} g(y)$ for arbitrary g as above. Wolke (1980) proved a corresponding result for P_2 with $\varphi(y) = (\log y)^A$

for a rather large value of A. We attempt, by sacrificing the number of prime factors, to reduce the length of the intervals.

Theorem. The corresponding result holds for P_4 with $\varphi(y) = (\log y)^5 g(y)$.

Remark. We are currently unable to further reduce the length of the intervals even for P_r with large fixed r.

S.W. GRAHAM: A class of extremal functions for the Fourier transform. Let λ be a positive number, and define

$$M_\lambda(x) = \left(\frac{\sin \pi x}{\pi x}\right)^2 \left\{ \sum_{n=0}^{\infty} \frac{e^{-\lambda n}}{(x-n)^2} - \sum_{n=1}^{\infty} \frac{\lambda e^{-\lambda n}}{x-n} + \sum_{n=1}^{\infty} \frac{\lambda e^{-\lambda n}}{x} \right\}.$$

Then $M_\lambda(x)$ has the following properties:

(1) $M_\lambda(x) \geq e^{-\lambda x}$ for $x \geq 0$,

(2) $M_\lambda(x) \geq 0$ for $x < 0$,

(3) $\hat{M}_\lambda(t) = 0$ for $|t| \geq 1$,

(4) of all functions that satisfy (1), (2), and (3), $\hat{M}_\lambda(0)$ is minimal.

We give some applications of this result. One application is a generalization of the Wiener - Ikehara tauberian theorem. Other applications involve generalizations of inequalities usually proved by the large sieve. The results represent joint work with J.D. VAALER.

G. GREAVES: Weighted sieves and their applications.
Last year in Durham I reported some results on the 1-dimensional sieve problem. For simplicity suppose $\#\{a \in A_x,\ d|a\} = \frac{x}{d} + O(1)$ and write $y = x(\log x)^{-2}$. The results imply that if $a < y^g$ for all a in A_x then: if $g < R - \delta_R$, then some a has at most R prime factors. Here the (positive) numbers δ_R are defined by a theorem leading to the following numerical approximations: $\delta_2 = 0.06373\ldots$, $\delta_3 = 0.10000\ldots$, $\delta_R \to 0.12482\ldots$ as $R \to \infty$.
I give some details how these approximations were obtained.

The differential - difference problem

$$\frac{d}{ds}(sT(s)) + T(s+1) = 0 \quad (\mathrm{Re}\ s > 0),\ sT(s) \to 1 \text{ as } s \to 0,$$

has a solution analytic in the half plane Re $s > 0$:

$$T(s) = \int_0^\infty e^{-sx} \exp\left(-\int_x^\infty \frac{e^{-t}}{t}\,dt\right) dx .$$

I needed approximations to $T(1)$, $T'(1)$, $T''(1), \ldots, T^{(n)}(1), \ldots$. I worked with the difference - differential equation rather than with the integral representation of its solution. We remove successive singularities by setting $\psi_0(s) = 1/s$, $T(s) = 1/s - f_0(s)$, $s\psi_j(s) = \int_0^s \psi_{j-1}(w+1)\,dw$, $f_j(s) = \psi_j(s) - f_{j-1}(s)$; then $\frac{d}{ds}(sf_j(s)) + f_j(s+1) = \psi_j(s+1)$ (Re $s > -j-1$) so that $T = \psi_0 - \psi_1 + \psi_2 - \ldots$ in a sense we describe. The equation for f_j gives $sf_j^{(n+1)}(s) + (n+1)f_j^{(n)}(s) + f_j^{(n)}(s+1) = \psi_j^{(n)}(s+1)$. Take $s = 0$ and represent $f_j^{(n)}(0)$ by its Taylor series about 1. Replace $f_j^{(n)}$, $\psi_j^{(n)}$ for $n > N$ by 0 and solve what is left (having computed the $\psi_i(1)$). The error in the resulting estimate for $f_j^{(n)}$ does not exceed $(1/\log 2)^n \frac{N!}{j!} \left(\frac{j+1}{N+2}\right)^j \left(\frac{\log 2}{j}\right)^{N+1}$ ($\to 0$ as $j \to \infty$ if $N \geq 1$).

G. HALÁSZ: Distribution of additive and mean values of multiplicative functions. A short introduction is given to the probabilistic theory of additive functions. It is shown how the characteristic function method leads to investigating mean values of multiplicative functions and for this an analytic approach is scetched. Some applications, such as generalization of a formula of Sathe - Erdös - Selberg concerning local behaviour of the number of prime divisors and large deviation versions of the Erdös - Kac - Kubilius theorem are discussed.

E. HEPPNER: Multiplicative numbertheoretical functions in several variables. At first the results of a joint paper with W. Schwarz on "related" functions are generalized to the case of several variables. Then, under some weak assumptions, it is shown that a multiplicative function in two variables has a mean value different from zero if and only if the two functions $f_1(n) = f(n,1)$ and $f_2(n) = f(1,n)$ have non zero mean values. (There is no difficulty in extending this result to more than two variables.)

J. HINZ: Eine Anwendung der Selberg'schen Siebmethode in algebraischen Zahlkörpern. Es sei K ein algebraischer Zahlkörper über $\mathbb{Q}$ vom Grade $n = r_1 + 2r_2$ (in der üblichen Bezeichnung). Die n Konjugierten einer Zahl $\xi \in K$ werden mit $\xi^{(k)}$, $k = 1,\dots,n$, bezeichnet. Eine Zahl $\omega \in K$ heißt prim, wenn das Hauptideal (ω) ein Primideal in K ist. Für die reellen Zahlen $p_1 \geq 1,\dots,$ $p_n \geq 1$ mit $p = p_1 \cdots p_n$ gelte $p_k = p_{k+r_2}$ für $k = r_1+1,\dots,r_1+r_2$. Ferner sei $R = \{\omega \in K,\ \omega \text{ prim},\ \omega^{(k)} > 0\ ,\ k = 1,\dots,r_1,\ |\omega^{(k)}| \leq p_k\ ,\ k = 1,\dots,n\}$.

Satz. Es sei $F(x,y) \in K[x,y]$ ein von x und y abhängiges irreduzibles Polynom vom Grade $g \geq 1$ mit ganzen algebraischen Koeffizienten und ohne festen Primteiler. Für ein Primideal $\mathfrak{p}$ in K bezeichne $L(\mathfrak{p})$ die Anzahl der ganzzahligen Lösungen mod $\mathfrak{p}$ von $F(\alpha,\beta) \equiv 0 \bmod \mathfrak{p}$ mit $(\alpha,\mathfrak{p}) = (\beta,\mathfrak{p}) = 1$. Es sei $L(\mathfrak{p}) < (N\mathfrak{p} - 1)^2$ für alle Primideale $\mathfrak{p}$ mit $N\mathfrak{p} \leq g+1$. Dann gilt:

a) $$\sum_{\omega \in R} \sum_{\substack{\omega' \in R \\ F(\omega,\omega')\ \text{prim}}} 1 \leq 2\, c_0(F,K)\, p^2/(\log p)^3 \left\{1 + O\left(\frac{\log\log p}{\log p}\right)\right\},\ p \geq 3$$

b) $$\sum_{\omega \in R} \sum_{\substack{\omega' \in R \\ \Omega(F(\omega,\omega')) \leq g+1}} 1 \geq \frac{2}{3}\, c_0(F,K)\, p^2/\log^3 p\ ,\ p \geq p_0(F,K).$$

Dabei ist $c_0(F,K)$ eine von F und K abhängige Konstante. $\Omega(\alpha)$ bedeutet die Anzahl der mit ihrer Vielfachheit gezählten Primidealteiler des Hauptideals (α).

Die untere Abschätzung stellt die Verallgemeinerung eines Resultates von G. Greaves dar. Zum Beweis werden Ergebnisse von W. Schaal zum linearen Selbergschen Sieb in algebraischen Zahlkörpern für größere Anwendungsbereiche hergeleitet. Ferner benötigt man eine geeignete Verallgemeinerung des Satzes von Barban und Davenport - Halberstam auf Zahlkörper.

K.-H. INDLEKOFER: Some results on the behaviour of additive and multiplicative functions. In this talk the spaces $L_\alpha := \{f: \mathbb{N} \to \mathbb{C},\ \|f\|_\alpha < \infty\}$ of arithmetical functions f with bounded (semi-) norm $\|f\|_\alpha := \{\limsup_{x\to\infty} x^{-1} \sum_{n\le x} |f(n)|^\alpha\}^{1/\alpha}$ ($\alpha \ge 1$) resp. bounded (semi-) norm $\|f\|_\alpha := \limsup_{x\to\infty} x^{-1} \sum_{n\le x} |f(n)|^\alpha$ ($0 < \alpha \le 1$) are considered. Defining the space L^* of uniformly summable functions by $f \in L^* :\Leftrightarrow \lim_{K\to\infty} \sup_{x\ge 1} x^{-1} \sum_{\substack{n\le x \\ |f(n)|\ge K}} |f(n)| = 0$,

the author gives a complete characterization of additive functions $f \in L_\alpha$ ($\alpha > 0$) and of multiplicative functions $g \in L^*$. As an application of these results the asymptotic behaviour of $\sum_{n\le x} f(n)$ (f additive $\in L_1$) and of $\sum_{n\le x} g(n)$ (g multiplicative $\in L^*$) is described. Furthermore, additive and multiplicative functions which are almost-periodic resp. almost-even, are completely characterized.

A.IVIC: On some problems concerning the number of non isomorphic abelian groups of finite order. Several problems concerning $a(n)$, the number of non-isomorphic abelian groups with n elements are discussed. It is indicated how a result of the type $\sum_{\substack{x < n \le x+h \\ a(n)=k}} 1 = (d_k + o(1))h$ can be obtained by estimates

for the error term $\Delta(1,2,2;x)$ in the asymptotic formula for $\sum_{kl^2m^2\le x} 1$. The best result that I am able to obtain at present is $h \ge x^\theta$, $\theta = 0.3305\ldots$ by B.R. Srinivasan's method of two-dimensional exponent pairs.

Formulas for the iterated function $a(a(n))$ are also derived, e.g. $\sum_{n\le x} a(a(n)) = Cx + O(x^{1/2}(\log x)^4)$.

The values of the function $a(n)$ can be naturally compared to the values of some other arithmetical functions such as $d(n)$, $\omega(n)$, $\Omega(n)$ etc.. In the first case I can prove

$$\sum_{\substack{n \leq x \\ d(n) > a(n)}} 1 = x + O(x(\log x)^{\varepsilon-1}), \quad \frac{x \log\log x}{\log x} \ll \sum_{\substack{n \leq x \\ d(n)=a(n)}} 1$$

$$\ll x(\log x)^{\varepsilon-1}.$$

H. IWANIEC: Kloosterman sums in analytic number theory.

The Linnik - Selberg conjecture on average is proved. Let $\Gamma_0(Q)$ be the Hecke congruence subgroup, $\alpha = \frac{h}{k}$, $(h,k)=1$, $k|Q$ be a cusp of $\Gamma_0(Q)$ and let $\rho_{j\alpha}(n)$ be the n-th Fourier - Bessel expansion of Maass wave $u_j(z)$ around α, i.e.

$$u_j(\sigma_\alpha z) = \sqrt{y} \sum_{n \neq 0} \rho_{j\alpha}(n) K_{i\varkappa_j}(2\pi|n|y)e(nx)$$

where $\sigma_\alpha \infty = \alpha$, $\sigma_\alpha^{-1} \Gamma_\alpha \sigma_\alpha = G$, Γ_α - stabilizer of α in $\Gamma_0(Q)$ and G - stabilizer of ∞ in PSL(2,Z). Then

Theorem 1. (Deshouillers and speaker) For $K \geq 1$, $N \geq 1$ and any complex numbers we have

$$\sum_{\varkappa_j \leq K} \frac{1}{\operatorname{ch} \pi\varkappa_j} \Big| \sum_{N < n \leq 2N} a_n \rho_{j\alpha}(n) \Big|^2 \ll (K^2 + \mu N^{1+\varepsilon}) \sum_{N < n \leq 2N} |a_n|^2,$$

$\mu = (k,Q/k)/Q$.

An analogous result holds for Fourier coefficients of holomorphic cusp forms. Theorem 1 and a summation formula of Kusnietsov and are used to prove the following result (Deshouillers and speaker) Theorem 2. Let $g(n,m,c)$ be a function of C^2 class such that $\operatorname{supp} g(n,m,c) \subset [N,2N]\times[M,2M]\times[C,2C]$

$N,M,C \geq 1$ and $\left| \frac{\partial^{p_1+p_2+p_3}}{\partial n^{p_1}\partial m^{p_2}\partial c^{p_3}} g(n,m,c) \right| \leq N^{-p_1} M^{-p_2} C^{-p_3}$, $0 \leq p_1 p_2 p_3 \leq 2$.

Let $S(n,m,c) = \sum^{*}_{d \bmod c} e(n\frac{d}{c} + m\frac{\bar{d}}{c})$ be Kloosterman sum where $\bar{d}d \equiv 1 \bmod c$. Then for $Q \geq 1$

$$\sum_n \sum_m \sum_c a_n b_m g(n,m,c) S(n\bar{Q},m,c) \ll C^{\varepsilon} \frac{(C\sqrt{Q} + \sqrt{MN} + C\sqrt{M})(C\sqrt{Q} + \sqrt{MN}\, C\sqrt{M})}{C\sqrt{Q} + \sqrt{MN}}$$

$$\left(\sum |a_n|^2\right)^{1/2} \left(\sum |b_n|^2\right)^{1/2} .$$

Three applications of theorem 1 and theorem 2 were presented.

I) Let p_n be the greatest prime factor of $n^2 + 1$. If $\varepsilon > 0$ then there are infinitely many n such that $p_n \geq n^{6/5 - \varepsilon}$.

II) Let d(n) be the number of positive divisors of n. Then

$$\sum_{n \leq x} d(n)d(n+1) = xP(\log x) + O(x^{1/2 + \varepsilon})$$ wher P is a quadratic polynomial with leading coefficient $6/\pi^2$.

III) Let $T \geq 1$, $1 \leq N \leq T^{1/5}$. For any complex numbers a_n we have

$$\int_0^T |\zeta(1/2 + it)|^4 \left| \sum_{n \leq N} a_n n^{it} \right|^2 dt \ll T^{1+\epsilon} \sum_{n \leq N} |a_n|^2 .$$

M. JUTILA: On the mean value of $L(1/2,\chi)$ for real characters.
Let $\chi_d(n) = (\frac{d}{n})$ (Kronecker's symbol) and $L(s,\chi_d)$ the corresponding Dirichlet L-function. In a recent paper, D. Goldfeld and C. Viola (J. Number Theory 11 (1979), 305-320) conjectured asymptotic formulae for the sums $\sum L(\frac{1}{2},\chi_d)$, where d runs over the fundamental discriminants either in the interval $(0,D]$ or in the interval $[-D,0)$, and also formulae for similar sums with χ_d replaced by the Legendre symbol $(\frac{n}{p})$, with $p \equiv v$ mod 4, $v = 1$ or 3. Such formulae, as well as an asymptotic formula for the mean sqare of $L(\frac{1}{2},\chi_d)$, are proved in a forthcoming paper of the author. The talk deals with results and methods of this paper.

H.-J. KANOLD: Zur elementaren Abschätzung von $\pi(x)$.
Wie üblich sei $\pi(x) = \sum_{p \leq x} 1$; für $x > 0$ definieren wir $\eta_x = \frac{\pi(x)\log x}{x}$. Mit elementaren Methoden werden für η_x Abschätzungen nach unten und nach oben hergeleitet. Die Ergebnisse sind in den folgenden drei Sätzen formuliert.
Satz 1. Für $10 < n \in \mathbb{N}$ gilt $\eta_x > \eta = \frac{1}{30}\log(6^9 10^5) > 0{,}921292$.
Satz 2. Für reelles $x > 0$ gilt $\eta_x \leq \eta_{113} = 30\frac{\log(113)}{113} < 1{,}255059$
Satz 3. Es ist $\eta \leq \varliminf_{x\to\infty} \eta_x \leq \varlimsup_{x\to\infty} \eta_x \leq \frac{6}{5}\eta < 1{,}105551$.
Diese Ergebnisse sind bekannt, aber die hier verwendeten Beweise weichen von den bekannten ab, sie sind ohne Computer nachzuvollziehen und erfordern nur eine Primzahltabelle, in der die Primzahlen bis 380 000 angegeben sind. Die Untersuchungen wurden in Zusammenarbeit mit Herrn Heiko Harborth und Herrn Arnfried Kemnitz durchgeführt.

I. KÁTAI: On arithmetical functions defined by some expansions of integers. Let q (>1) be a fixed integer; $N = \bigcup_{k=1}^{\infty} N_k$; N_k be sets of natural numbers, with the following property: for every nonnegative integer there exists one and only one k and $m_k \in N_k$ such that: $n \equiv m_k \bmod q^k$, $m_k \leq n$. Set $N_k = \{m_k\}$, m_k denotes the general element of N_k. If N is given so, then every n has a well-defined decomposition: $n = m_{k_1} + 2^{k_1}[m_{k_2} + \ldots [m_{k_{v-1}} + [2^{k_{v-1}} m_{k_v}] \ldots]$. We call a function $f(n)$ to be N-additive if it can be written in the form (1) $f(n) = \sum_{j=1}^{v(n)} H(m_{k_j}, j)$.

It is obvious that the N-additivity is a generalization of the q-additivity that was introduced by A.O. Gelfond. There are a lot of open questions concerning the distribution of N-additive functions. Assume that $\sum \operatorname{card}(N_l)/q^l = 1$ holds. I guess that the convergence of

$$\text{(a)} \quad \sum_{j=1}^{\infty} \sum_{m_k \in N} H(m_k, j)/q^k \quad , \quad \text{(b)} \quad \sum_{j=1}^{\infty} \sum_{m_k \in N} H^2(m_k, j)/q^k$$

is sufficient and necessary for the existence of a limit distribution of $f(n)$. I can prove that it is sufficient if $m_k > q^{k-t}$ for all $m_k \in N$ with a suitable t. If $H(m,j)$ does not depend on j, and $\sum \operatorname{card}(N_k) k^2/q^k < \infty$ then

$$\frac{f(n) - a \log n}{\sigma\sqrt{\log n}}$$ has the Gaussian limit distribution.

G. KOLESNIK: On the order of $\Delta_3(x)$.
Let $\Delta_3(x)$ be the error term in the asymptotic formula for $D_3(x) = \sum_{nmk \leq x} 1$. Chen proved that $\Delta_3(x) \ll x^{5/11 + \varepsilon}$. We improved this result to $\Delta_3(x) \ll x^{43/96 + \varepsilon}$. The result is obtained by using the improved estimates of double exponential sums.

J. KUBILIUS: On an inequality for additive arithmetical functions.
Let $f(m)$ be a real-valued strongly additive arithmetical function. Denote

$$A_n(f) = \sum_{p \leq n} \frac{f(p)}{p}, \quad B_n^2(f) = \sum_{p \leq n} \frac{f^2(p)}{p}, \quad S_n(f) = \sum_{m=1}^{n} (f(m)-A_n(f))^2.$$

In 1954 I proved that the inequality $S_n(f) \leq c\, n\, B_n^2(f)$ is true with some absolute constant c. The aim of this talk is to evaluate the constant c. Let $\tau_n = \sup S_n(f)/(nB_n^2(f))$.

Then the inequalities $1.5 + o(1) \leq \tau_n \leq 1 + \sqrt[k]{T_k - 1} + o(1)$ hold for all even positive k's. Here

$$T_k = \int\limits_{\substack{0 \leq u_j \leq 1 \\ u_j + u_{j+1} > 1 \\ j = 1,\dots,k \\ (u_{k+1} = u_1)}} \cdots \int \frac{du_1 \cdots du_k}{u_1 \cdots u_k}.$$

For $k = 2,3,6,8$ $T_k = \zeta(k)$. Hence it follows that $\tau_n \leq 1.502\ldots + o(1)$. I guess that $T_k = \zeta(k)$ for all even positive k's. If this is true then $\lim \tau_n = 1.5$. Similar results are true for arbitrary complex-valued additive functions.

D. LEITMANN: On the prime number theorem of Pjateckij - Shapiro.
In 1953 Pjateckij - Shapiro proved his famous theorem

$$\sum_{\substack{p \leq x \\ p = [n^c],\, n \in \mathbb{N}}} 1 \sim x^{\gamma}/(\log x) \quad (\gamma = 1/c) \text{ for } 1 < c < 12/11.$$

The range for those c for which this asymptotic relation holds was widened to $1 < c < 10/9$ by Kolesnik in 1967. Now the upper bound of this interval can be improved to 69/62. This is an immediate corollary of the following

Theorem. $0 < \gamma < 1$, $1 \leq k \leq N^{1-\gamma}\log^2 N$. Then $\sum_{n \leq N} \Lambda(n) e^{2\pi i n^{\gamma} k}$

$$\ll_{\varepsilon} \left(\frac{N^{1-\gamma}}{k} + 1\right) N^{62/69} \log^7 N.$$

The proof depends on the following tools: 1. Vaughan's identity,

2. Kolesnik's estimate for $\sum_{x \leq X} \sum_{\substack{y \leq Y \\ xy \leq N}} e^{2\pi i k x^{\gamma}(y^{\gamma} - (y+k)^{\gamma})}$

3. Estimation of $\sum_{X < x < X' \leq 2X} e^{2\pi i \eta x^{\gamma}}$ $(\eta > 0)$ by iteration of van der Corput's method.

M.-C. LIU: Some results in Diophantine approximation.
Let λ_j , $j = 1,\dots,8$, be any non-zero real numbers such that not all λ_j are of the same sign and not all ratios λ_j/λ_k are rational. In this talk the following result by M.C. LIU, S.M. NG, and K.M. TSANG is given.
If η, σ are any real numbers with $0 < \sigma < 3/70$ then the inequality $\left|\eta + \sum_{j=1}^{8} \lambda_j n_j^3\right| < (\max_{1\le j\le 8} n_j)^{-\sigma}$ has infinitely many solutions in positive integers n_j. The result gives a better error term than an estimate in (H.Davenport and K.F. Roth, Mathematika 1955).

J. LOXTON: Irregularities of distribution.
Let $z_1, z_2, \dots$ be an infinite sequence of points on the unit circle and set $f_n(z) = \prod_{j=1}^{n} (z-z_j)$ and $A_n = \sup_{|z|=1} \log|f_n(z)|$. Erdös asked whether it is possible to find a sequence so that A_n is bounded. Last year, Wagner showed that this is impossible and, in fact, $A_n \gg \log\log n$ infinitely often, for any sequence of points. Wagner obtained his result by adapting the method used by Schmidt in his work on irregularities of distribution for sequences of points in the unit interval. Halász has recently shown how to obtain Schmidt's results by a modification of the earlier work of Roth. The same idea can be used to discuss the polynomial discrepancy and yields $A_n \gg (\log n)^{1-\epsilon}$ infinitely often, for any $\epsilon > 0$. The best possible lower bound would be $A_n \gg \log n$ infinitely often; indeed, it is probably true that $A_n \gg \log n$ for almost all n.

L. LUCHT: Natural bounderies of power series with multiplicative coefficients. Denote by K the set of multiplicative functions $f: \mathbb{N} \to \mathbb{C}$ with the following properties:

a) There exists a constant $s = s_f \in \mathbb{C}$ with $\sigma \geq \mathrm{Re}\ s \geq 0$ and a slowly oscilating function L such that for each $q \in \mathbb{N}$ there is a constant c_q with

$$\sum_{n \leq x} f(n)\chi(n) = \begin{cases} (c_q+o(1))x^s L(x) & \text{if } \chi=\chi_0 \bmod q \\ o(x^\sigma |L(x)|) & \text{if } \chi \neq \chi_0 \bmod q \end{cases} \quad (x \to \infty),$$

χ running through the characters mod q, and χ_0 denoting the principal character mod q.

b) There is a $q^* \in \mathbb{N}$ with $c_{q^*} \neq 0$.

c) For each prime p there is an $\epsilon > 0$ such that $\sum\limits_{v \geq 2} \frac{f(p^v)}{p^{v(\sigma-\epsilon)}}$

d) The limit $\lim\limits_{x \to \infty} x^s L(x)$ does not exist.

The multiplicative functions investigated by Wirsing, by Halász, and by Elliott substantially belong to the set K. This is also true for suitable convolution products of these functions. The following theorem answers (to a certain extend) a question posed by W. Schwarz at the Oberwolfach Meeting in Number Theory, 1978.

Theorem. Let $f \in K$, $s = s_f$. Then the following assertions are equivalent.

A) $\sum\limits_{n=1}^{\infty} f(n)z^n$ has the unit circle $|z| = 1$ as a natural boundery.

B) There are infinitely many primes powers p^β $(\beta \in \mathbb{N})$ such that $\sum\limits_{v \geq \beta} \frac{f(p^v)}{p^{vs}} \neq \frac{1}{\varphi(p)} \frac{f(p^{\beta-1})}{p^{(\beta-1)s}}$.

M. MENDES FRANCE: <u>Integral geometry and uniform distribution mod 1.</u> Let Γ be a bounded plane curve of length $|\Gamma|$. Let D be a line and $N(D) = \mathrm{card}(D \cap \Gamma)$. When D runs through the set Ω of straight lines which intersect Γ, the average (expectation) of the number of intersection points is $E(N) = \int_\Omega N(D)\, dD \,/\, \int_\Omega dD$.

Theorem (Steinhaus). Let K be the convex hull of Γ and let $|\partial K|$ be the length of the boundery of K. Then $E(N) = 2|\Gamma|/|\partial K|$.

Suppose now that Γ is a curve of infinite length. For every $t > 0$, define Γ_t as the beginning portion of Γ of length t. The average number of intersection points of D with Γ_t is $E(N_t)$.

Definition. Γ is said to be superficial if $\lim_{t\to\infty} E(N_t) = \infty$.

Let $u = (u_1, u_2, \ldots)$ be a sequence of real numbers. Put $w_0 = 0$, $w_n = \sum_{k=0}^{n-1} e^{2\pi i u_k}$ and consider the infinite polygonal line the summits of which are $w_0, w_1, \ldots$. Call $\Gamma(u)$ the polygonal line.

Theorem. The sequence u is uniformly distributed mod 1 if and only if the curves $\Gamma(u)$, $\Gamma(2u)$, ... are all superficial. The proof uses Steinhaus' result. The above result is part of joint work in progress with M. DEKKING.

H. MÖLLER: Fundamental units of real quadratic fields.

Let K be a real quadratic field. A number γ of K with conjugate γ' is called reduced if $\gamma > 1$ and $-1 < \gamma' < 0$. If γ is reduced, then the continued fraction algorithm $\gamma \to (\gamma - [\gamma])^{-1}$ generates a purely periodic sequence of reduced numbers of K.

Theorem. The product of all numbers in the primitive period of any reduced number of K equals the fundamental unit of K.

As a consequence we get the following result, where χ is the character and D the discriminant of K :

$$\sum_{n=1}^{\infty} \frac{\chi(n)}{n} = \frac{2}{\sqrt{D}} \sum_{\substack{b=1 \\ b\equiv D \bmod 2}}^{[\sqrt{D}]} \sum_{\substack{a=\left[\frac{\sqrt{D}-b}{2}\right]+1 \\ a \mid \frac{D-b^2}{4}}}^{[(\sqrt{D}+b)/2]} \log\left(\frac{\sqrt{D}+b}{2a}\right) .$$

H.L. MONTGOMERY: The error term in the prime number theorem.

We assume the Riemann hypothesis, and enquire about the gap between the two estimates $\psi(x) = x + O(x^{1/2}\log^2 x)$, $\psi(x) - x = \Omega_{\pm}(x^{1/2}\log\log\log x)$. In proving the Ω estimate, Littlewood used Dirichlet's theorem, which is a tool of homogeneous Diophantine approximation, to attack a non-homogeneous question.

His partial success was due to the fact that the problem is almost homogeneous. One could obtain a sharper result from a strong quantitative form of Kronecker's theorem, provided that we had good information concerning linear forms in the imaginary parts γ of the zeros of the zeta function. We formulate the following

Conjecture 1. Foør every $\varepsilon > 0$ and every $K > 0$ there is a $T_0(K,\varepsilon)$ such that if $T \geq T_0(K,\varepsilon)$ then $\left|\sum_{0<\gamma\leq T} k_\gamma \gamma\right| \geq \exp(-T^{1+\varepsilon})$ whenever the k_γ are integers, not all 0, such that $|k_\gamma| \leq K$.

From conjecture 1 and RH we can deduce that

$$\overline{\underline{\lim}}\ \frac{\psi(x) - x}{x^{1/2}(\log\log\log\, x)^2} \ \begin{matrix}\geq\\ \leq\end{matrix}\ \pm \frac{1}{2\pi}\ .$$

In the proof we use the following lemma which is of independent interest.

Lemma. Let $h(y) = \sum_{n=1}^{N} c_n \cos(2\pi(\lambda_n y + \rho_n))$ where the c_n are non-negative real numbers, the λ_n and ρ_n are real. Suppose that $\left|\sum_{n=1}^{N} k_n \lambda_n\right| \geq \delta$ whenever the k_n are integers, not all 0, with $|k_n| \leq R$. Then in any interval $[a,b]$ of length $\geq R/\delta$, there is a y for which $h(y) \geq (1 - 5/R) \sum_{n=1}^{N} c_n$.

On the basis of these and other probabilistic results, I am led to formulate

Conjecture 2. $\quad \overline{\underline{\lim}}\ \dfrac{\psi(x) - x}{x^{1/2}(\log\log\log\, x)^2} = \pm \dfrac{1}{2\pi}$.

M. NAIR: <u>On distinct values of the divisor function.</u>

Let $D(x)$ be the number of distinct values assumed by the divisor function $d(n)$ for $1 \leq n \leq x$. P. Erdös and L.Mirsky (1952) estimated $D(x)$ by studyind a related function $B(x)$ and showed that $B(x) \sim D(x)$. The actual asymptotic formulae for either function is still unknown. Write $E(x) = D(x) - B(x)$. Erdös and Mirsky proved that $E(x) > c_1 \log\log\log x$. This was improved by Shiu (1978) to $E(x) > \exp(c_2(\log\log x)^{1/3}/\log\log\log x)$. We have now managed to show that $E(x) > \exp(c_3(\log x)^{1/2}/\log\log x)$ which is best possible apart from the value of c_3. (Joint work with P. SHIU)

W. NARKIEWCZ: Distributions of values of multiplicative functions in residue classes. A function $f(n)$ (integer - valued) is called WUD (mod N) (weakly uniformly distributed (mod N)) provided for all j_1, j_2 with $(j_1,N)=(j_2,N)=$; one has

$$\frac{\#\{n \leq x : f(n) \equiv j_1 \pmod N\}}{\#\{n \leq x : f(n) \equiv j_2 \pmod N\}} \to 1.$$

Theorem. Let f be multiplicative, integer-valued and let $f(p) = V(p)$ ($V \in \mathbb{Z}[X]$) for all primes p. If $V = cW^d$ ($d \geq 2$, $W \in \mathbb{Z}[X]$) then there exists a finite and effectively determinable set E of primes such that if N has no prime factors from E, then f is WUD (mod N).

Corollary. If f is as above, then it is WUD (mod p) for every prime $p \gg 1$. (This answers a question of Erdös.)

J.L. NICOLAS: Big values of arithmetic functions.

Let p_k be the k-th prime, $\theta(x) = \sum_{p \leq x} \log p$ and $N_k = 2 \cdot 3 \cdot \ldots \cdot p_k$, so that $\theta(p_k) = \log N_k$. In Math. of Comp. 29, 1975, B. Rosser and L. Schönfeld announce as a result of their extended computations that they can prove $\theta(p_k) \geq k \log k$ for $k \geq 13$. Actually this result does depend only on Chebychev's result $\pi(x) \asymp x/\log x$, because it follows from :

$$\theta(p_k) = \int_2^{p_k} \log x \, d(\pi(x)) = k \log p_k - \int_2^{p_k} (\pi(x)/x)\, dx \quad \text{that}$$

(*) $\pi(x) \asymp x/\log x \Rightarrow \theta(p_k) = k(\log k + \log\log k + O(1))$.

It is well known that the maximal order of $\omega(n) = \sum_{p \mid n} 1$ is $\log n/\log\log n$. As N_k is the smallest integer such that $\omega(N_k) = k$, the maximal order of ω is given by estimating $\omega(N_k) = k$ in terms of $\log N_k = \theta(p_k)$. So, using (*), G. Robin (Univ. of Limoges) proves the following inequalities, with $l = \log n$, $l_2 = \log\log n$

(1) $\forall n \geq 3$, $\omega(n) \leq 1.38\, l/l_2$ with equality for $n = N_9$

(2) $\forall n \geq 3$, $\omega(n) \leq (1 + 1.46/l_2)(l/l_2)$ with equality for $n=N_{47}$

(3) $\forall n \geq 267\exp(\exp(1.17))$, $\omega(n) \leq l/(l_2 - 1.17)$ with equality for $n = N_{189}$.

This last inequality improves a result of Norton (Mem. of the

A.M.S. n^{o} 106). Using sharper estimates than (*) given by Rosser and Schönfeld, G. Robin proves also that if $f(n) = \frac{\omega(n)\log\log n}{\log n}$, then $f(N_k)$ is decreasing for $k \geq 9$. This property is equivalent to $\pi(x)\log x - \theta(x) > \theta(x)/(\log x - 1)$ for $x \geq 9$ and implies (4) $\forall n \geq 3,\ \omega(n) \dot{\leq} (1 + 1/l_2 + 2.9/l_2^2)(1/l_2)$ with equality for $n = N_{442}$. The same work can be done replacing $\omega(n)$ by $\log d(n) / \log 2$, and inequalities 1,2,3,4 are obtained with other constants. The numbers N_k have to be replaced by the so called superior highly composite numbers N_ε of Ramanujan. Such a number maximises the function $n \mapsto d(n)/n^{\varepsilon}$ and has a parametric representation in terms of $x = 2^{1/\varepsilon}$. It is possible to extend these results to other additive functions f such that $f(p^a) = g(a)$ does not depend on p but only on a.

Let $\sigma(n)$ be the sum of the divisors of n. We have $\sigma(n)/n \dot{\leq} n/\varphi(n)$ where φ is Euler's function. G. Robin deduces (5) $\forall\ n \geq 3,\ \sigma(n)/n \dot{\leq} e^{\gamma} l_2 + 0.65/l_2$ with equality for $n = 12$ (γ Euler's constant) from a similar inequality for $n/\varphi(n)$ given by Rosser and Schönfeld and from the behaviour of the colossally abundant numbers (which maximises $\sigma(n)/n^{1+\varepsilon}$). Rosser and Schönfeld asked whether $n/\varphi(n)$ $e^{\gamma}\log\log n$ for all but a finite number of n's. I think that I have proved during this stay, with the help of H.L. Montgomery that this property is equivalent to Riemann's hypothesis.

H. NIEDERREITER: <u>Complete mappings and equations over finite fields</u>. This talk is about a class of mappings on finite fields that arise from combinatorics and nonassociative algebra and lead to interesting connections with the Stepanov - Schmidt method for equations over finite fields. A bijection $\theta: \mathbb{F}_q \to \mathbb{F}_q$ is called a complete mapping if $c \mapsto \theta(c) + c$ is also a bijection. Since every mapping from a finite field into itself is represented by a polynomial, we can also speak more conveniently of a complete mapping polynomial (c.m.p.). Note that $f(x) \in \mathbb{F}_q[x]$ is a c.m.p. $\Leftrightarrow$ $f(x)$ and $f(x) + x$ are permutation polynomials (p.p.). For monomials, we have the following: to fixed $k \geq 1$ $\exists$ inf. many q such that x^k is p.p. of $\mathbb{F}_q$. The study of which binomials are p.p.'s or c.m.p.'s leads to certain absolutely irreducible equations over $\mathbb{F}_q$. W.l.o.g., assume $k > 2$.

Theorem 1. Let $k > 2$. Then (i) if k is not a prime power, then $\forall$ $\mathbb{F}_q$ with $q \geq (k^2 - 2k + 2)^2$ there is no p.p. of $\mathbb{F}_q$ of form $ax^k + bx$ with $ab \neq 0$; (ii) if $k = p^t$, then $\forall$ $\mathbb{F}_q$ with $q \geq (k^2 - 2k + 2)^2$, char $\mathbb{F}_q \neq p$, there is no p.p. of $\mathbb{F}_q$ of form $ax^k + bx$ with $ab \neq 0$.

Corollary. If k and q are as in Th. 1, then there is no c.m.p. of $\mathbb{F}_q$ of form $ax^k + bx$ with $a \neq 0$.

More generally, one can study the question of finding p.p.'s of $\mathbb{F}_q$ of the form $ax^k + bx^j$ with $ab \neq 0$, $1 \leq j < k$. If k may depend on q, then we can find c.m.p.'s of form $ax^k + bx$ for inf. many $\mathbb{F}_q$, even when k is not a power of char $\mathbb{F}_q$. Using a Weil estimate for quadratic character sums, one shows the following

Theorem 2. Let q be odd. Then $N = \operatorname{card}\{b \in \mathbb{F}_q : x^{(q+1)/2} + bx$ is c.m.p. of $\mathbb{F}_q\}$ satisfies $N \geq q/4 - 5/2 - (3/4)q^{1/2}$ if char $\mathbb{F}_q > 3$, and for char $F_q = 3$ we have $N = (q-9)/4$ if $q \equiv 1 \bmod 4$, $N = (q-3)/4$ if $q \equiv 3 \bmod 4$.

Corollary. C.m.p.'s of form $x^{(q+1)/2} + bx$ exist exactly for all odd $q \geq 13$ and $q = 7$.

B. NOVÁK: Lattice points in many-dimensional ellipsoids.

Let $Q(u_j) = Q(u_1, \dots, u_r)$, $r > 4$, be a positive definite quadratic form with a symmetric integral coefficient matrix. Let $\alpha_1, \alpha_2, \dots, \alpha_r$ be real numbers. Let us put

$$a_n = \sum e^{2\pi i \sum_{j=1}^{r} \alpha_j m_j}, \text{ where the summation runs over}$$

all systems $m_1, m_2, \dots, m_r$ of integers such that $Q(m_j) = n$. For the sequence a_n an asymptotic formula can be derived (involving Bessel's function, Kloosterman and Gaussian sums) with remainder term $O(n^{r/4 - 1/4 + \epsilon})$. For the exact order f of a_n the following inequalities hold: $r/2 - 1 - r/(4(\gamma+1)) \leq f \leq \max(r/2 - 1 - (r-1)/(4(\gamma+1)), r/4 - 1/4)$, where γ is the supremum of all β for which the inequality $k^\beta \max_j \| \alpha_j k \| \ll 1$ has infinitely many solutions.

J. PINTZ: On Heilbronn's triangle problem.
Let $P_1, P_2, \ldots, P_n$ be a distribution of n points (where $n \geq 3$) in a closed disc of unit area such that the minimum of areas of the triangles $P_i P_j P_k$ (taken over all selections of three out of n points) assumes its maximum possible value $\Delta(n)$. Heilbronn conjectured over 30 years ago $\Delta(n) \ll n^{-2}$. It was proved by P. Erdös in 1950 that $\Delta(n) \gg n^{-2}$. K.F. Roth proved the first non-trivial estimate $\Delta(n) \ll n^{-1}(\log\log n)^{-1/2}$ in the same year. About twenty years later this was improved by W.M. Schmidt to $\Delta(n) \ll n^{-1}(\log n)^{-1/2}$ making use of a different method. Soon after this, using an entirely new method Roth proved $\Delta(n) \ll n^{-\mu+\epsilon}$ with $\mu = 2 - 2/\sqrt{5} = 1.105\ldots$ and somewhat later he refined his method to yield to $\mu = (17 - \sqrt{65})/8 = 1.117\ldots$. Very recently the following theorem was proved in a joint work with J. KOMLÓS and E. SZEMERÉDY:
Theorem. $c_1 n^{-2} \log n \leq \Delta(n) \leq n^{-8/7} \exp(c_2 \sqrt{\log n})$ with explicitly calculable positiv absolute constants c_1 and c_2.
The upper bound was achieved by a further refinement of the method of Roth, the lower bound - which disproves Heilbronn's conjecture - by combinatorical methods.

S. PORUBSKÝ: On Voronoi's congruence.
The following extension of Voronoi's congruence involving Bernoulli numbers B_k (in the even index notation) is proved via non-archimedean Bernoulli distributions:

$$(c^{2k}-1)\frac{B_{2k}}{2k}+\frac{2k-1}{6}c^{2k-2}(c^2-1)N^2B_{2k-2}+\binom{2k-1}{3}\frac{c^{2k-2}(c-1)(c-2)}{24}$$

$$\cdot N^4B_{2k-4}\equiv c^{2k-1}\sum_{x=1}^{N-1}x^{2k-1}\left[\frac{cx}{N}\right]-\frac{2k-1}{2}c^{2k-2}N\sum_{x=1}^{N-1}x^{2k-2}\left[\frac{cx}{N}\right]^2$$

mod N^2 for $k\geq 4$ and $\frac{c^{2k+1}-1}{2}NB_{2k}+\frac{c^{2k-1}(c-1)(2c+3)}{12}\binom{2k}{2}N^3B_{2k-2}$

$$\equiv c^{2k}\sum_{x=1}^{N-1}x^{2k}\left[\frac{cx}{N}\right]-kc^{2k-1}N\sum_{x=1}^{N-1}x^{2k-2}\left[\frac{cx}{N}\right]^2 \bmod N^2 \text{ for}$$

$k\geq 3$, where N is a positive integer and c a rational number prime to N ($[x]$ stands for the greatest integer in x).

K. RAMACHANDRA: Some problems of analytic number theory.
I give a brief report of the work done in colloboration with R. BALASUBRAMANIAN. Here I combine to a special case of more general results which will appear in Hardy - Ramanujan Journal 4 (1981). Let $F(s) = \prod_{k=1}^{\infty}\zeta(ks) = \sum_{n=1}^{\infty}a_n n^{-s}$. $F(s)$ is defined by the series in $\sigma>1$. But it can be continued analytically in $\sigma>0$. It is regular except at simple poles at $\beta = 1, 1/2, 1/3, \ldots$. It may be remarked that $\sigma = 0$ is a natural boundery for $F(s)$. Put $A(x) = \sum_{n\leq x} a_n$,

$$A(x) = \frac{1}{2\pi i}\int_{|s-1|=1-\frac{2}{101}} F(s)\,x^s s^{-1}\,ds + E(x),$$

$$M(a,b) = \max_{a\leq x\leq b}\left(E(x)/(x^{1/10}\exp((1/20)(\log x/\log\log x)^{1/2})\right),$$

where $0<a<b$, $m(a,b) = \min_{a\leq x\leq b}\left(E(x)/(x^{1/10}\exp((1/20)(\log x/\log\log x)^{1/2})\right)$, and $\mu(a,b) = \max_{a\leq x\leq b}(|E(x)|x^{-1/6})$.

Then we prove the following theorem

Theorem. For all $y\geq 100$, we have

$M(e^y,e^{1000y}) > 10^{-800}$, $m(e^y,e^{1000y}) < -10^{-800}$, and $\mu(e^y,e^{1000y}) > 10^{-800}$.

The result on μ follows from some work starting from our result $\frac{1}{T}\int_T^{2T} |F(1/6 + it)|^2\, dt \gg T \log T$ and the other two results follow from some work starting from our earlier result that $\max_{T \leq t \leq 2T} |F(1/10 + it)| > T \exp((3/5)(\log T/\log\log T)^{1/2})$ for all T.

R.A. RANKIN: Recent work on modular forms.

The following topics were discussed:

(i) Newforms. The work of Atkins and Lehner (Math. Ann. 185 (1970), 134 - 160) and Li (ibid. 212 (1975)) extending earlier work of E. Hecke and M. Petersson and so showing that the space of cuspforms $\{\Gamma_0(N),k,\chi\}_0$, with old form removed has an orthogonal basis of newforms that are eigenforms for all the Hecke operators T_n was briefly discribed.

(ii) Order of Fourier coefficients. Deligne's work has proved that for a cuspform coefficient $a_n = O(n^{(k-1)/2 + \varepsilon})$, or for a newform coefficient $|\lambda(p)| \leq 2p^{(k-1)/2}$ but other problems remain, such as the correct order of $T(n) = \sum_{m \leq n} a(m)$ which is $O(n^{k/2 - 1/6 + \varepsilon})$ but not $O(n^{k/2 - 1/4})$. Results of H. Joris (Mathematika 22 (1975), 12 - 19) mentioned.

(iii) Divisibility properties. Work of Swinnerton - Dyer and Serre. Results holding for all, and almost all n.

iv) Poincaré series. Identical vanishing and non-vanishing of $G_k(z,m) = \frac{1}{2}\sum_{c,d} e^{2\pi i m T z}(cz+d)^{-k}$ for $m > 0$. Distribution of zeros for $m \in \mathbb{Z}\setminus\mathbb{N}$ and for Poincaré series of more general type.

A. REICH: Large values of zeta-functions.

In the case of the Riemann zeta-function $f(s) = \zeta(s)$ (also for arbitrary L-series or the Dedekind zeta-function $\zeta_K(s)$ of an algebraic number field K) we give an answer to the question, how often the function assumes large values on infinite arithmetical progressions in the critical strip: Let be $\sigma > 1/2$, $R > 0$, $\Delta > 0$, $L = L(f,\sigma,\Delta,R) = \{ n \in \mathbb{N} : |f(\sigma+i\Delta n)| > R \}$, and denote by $\underline{\rho}(L) = \underline{\lim} \frac{1}{N} \operatorname{card}\{n : n \in L\}$ the lower asymptotic density of L. For $\sigma > 1$ one has the trivial relation $L = \emptyset$ for $R > R_0$, but in the critical strip the following holds (for example $f(s) = \zeta(s)$)

Theorem. For any $\Delta > 0$, and any disc $D \subset \{ s \in \mathbb{C} : 1/2 < \operatorname{Re} s < 1 \}$, any holomorphic $g: D \to \mathbb{C}$ without zeros, any $\varepsilon > 0$ the relation $\underline{\lim} \frac{1}{N} \operatorname{card} \{ n \in \mathbb{N} : \sup_{s \in D} |f(s+i\Delta n) - g(s)| < \varepsilon \} > 0$ holds.

A similar property is shown for Dedekind's $\zeta_K(s)$ (to be published in Arch. d. Math.). Therefore one has immediately the Corollary. If $1/2 < \sigma < 1$ then $\underline{\rho}(L) > 0$.

To get an uniform upper estimate for many $\Delta > 0$ it is shown

Theorem. There exists a countable (exceptional) set $A \in \mathbb{R}^+$ such that for $\sigma > 1/2$, $\Delta \notin A$, $\lim\limits_{N \to \infty} \frac{1}{N} \sum\limits_{n=1}^{N} |f(\sigma+i\Delta n)|^2$

$= \lim\limits_{T \to \infty} \frac{1}{T} \int\limits_0^T |f(\sigma+it)|^2 \, dt$ holds.

This leads to the

Corollary. $\underline{\rho}(L) \leq R^{-2} \sum\limits_{n=1}^{\infty} n^{-2\sigma}$ holds for all $\Delta \in \mathbb{R}^+ \setminus A$ and all $\sigma > 1/2$.

G.J. RIEGER: <u>Circles, triangles, and spheres of Ford.</u>

For every rational number h/k (reduced) denote by C(h/k) the open circular disc (= Ford circle) in the cartesian plane x,y with center $h/k, 1/(2k^2)$ and radius $1/(2k^2)$. Any two different circles are disjoint; they have a point of contact if and only if they belong to neighbors in a suitable Farey sequence F_n. The neighbors and this point of contact form a right triangles. Given F_n, we denote by L_n the length of the polygon joining 0 and 1 along the legs of these triangles. Theorem 1. There exists a real number C with 1 C (1.28...) 2 and $L_n = C + O(\log^2 n/n)$.

A similar result holds if the legs are replaced by the corresponding arcs on the Ford circles. Also, remarks are made about the generalizations to $Q(\sqrt[3]{1})$ (see Crelle 303/304, 1978) and a model i presented made out of wood.

I.Z. RUZSA: Additive functions and random variables.
Let f be an additive function and let ξ_p (p prime) be independent random variables with the distribution $P(\xi_p = f(p^k)) = (1 - 1/p)p^{-k}$. Heuristically we expect that the distribution of f on the interval $[1,x]$ is more or less similar to that of the random variables $\eta_x = \sum_{p \le x} \xi_p$;

many celebrated theorems (e.g. Erdös and Wintner's), though stated in other terms, correspond to this principle. Among others, we prove the following theorem. Let F be a non-negative-valued increasing function defined on $[0,\infty)$, f a real-valued additive function and M a real number. We have $\frac{1}{x} \sum_{n \le x} F(|f(n) - M|) \le c\, E(F(3|\eta_x - M|))$, where E

denotes expectation and c is an absolute constant. For $F(x) = x^2$ this reduces to the Turán - Kubilius inequality, $F(x) = x^a$ has been recently proved by Elliott. Some results can be obtained for functions with values in a topological group G, e.g. I can solve the problem of existence of a limiting distribution when G is locally compact.

B. SAFFARI: An extremal problem for exponential sums.
Let I be a sub-interval of $[0,1]$ of length $|I| > 0$ and whose interior does not contain the mid-point 1/2 of $[0,1]$. The problem is to determine $C = \sup_f \int_I |f(x)|^2\, dx \,/ \int_0^1 |f(x)|^2\, dx$

where the supremum is taken over the sums of exponentials of the form $f(x) = \sum_{k=1}^{q} \exp(2\pi i n_k x)$ where $n_1, \dots, n_q$ are

arbitrary distinct integers. This problem arose out of a functional analysis paper "Weak restricted and very restricted operators on L^2" by J.M. Ash (to appear). Since $|f(1-x)| = |f(x)|$, it follows that $C \le 1/2$. Also, an old result of Halász - Montgomery shows that, at least for some intervals

I, one has $C \leq 1/4$. On the other hand Ash had proved, as a corollary to his paper, the very interesting result $C \geq C_0$ where C_0 is an absolute (ineffective) positive constant. Pichorides proposed an elementary argument to show that C_0 can be made effective (cf. appendix to Ash's paper). Refinements of this idea lead at least to $C \geq (2/\pi^2) \max_{\alpha > 0} \alpha^{-2}\sin^4\alpha > 1/10$. This lower bound can be improved by the same method. Several people suggested other methods.

H. SARGES: Least quadratic non-residues in algebraic number fields. (joint work with W. SCHAAL) Let $v_{\mathfrak{p}}$ denote a positive integer of an algebraic number field K such that $v_{\mathfrak{p}}$ is a least quadratic non-residue modulo a prime ideal $\mathfrak{p}$ of K, least in the sense that $N(v_{\mathfrak{p}})$ is minimal. Then the following generalization of Linnik's result is shown:
For $x \geq 2$ and $\varepsilon > 0$ $|\{\mathfrak{p} \mid N\mathfrak{p} \leq x \text{ and } N(v_{\mathfrak{p}}) > N\mathfrak{p}^{\varepsilon}\}| = O_{\varepsilon}(\log\log x)$.
The proof requires the large sieve in number fields and the relation $\sum_{\substack{\alpha \in U \\ \mathfrak{p} \mid \alpha \Rightarrow N\mathfrak{p} \leq x^{uc}}} 1 = \varphi(c)Bx^u + O(x^u/\log x)$ where $U = \{\alpha \mid \alpha \succ 0,\ |\alpha^{(l)}| \leq x,\ l=1,\dots,u\}$, $B = (2\pi)^{r_2}/|\sqrt{d}|$ and $\varphi(c) = 1 - \int_c^1 \varphi(t/(1-t))\,dt/t$, $1 \geq c > 0$.

J.P. SERRE: Selberg upper bound sieve.
This well known upper bound has nothing to do with prime numbers. It is a purely combinatorial statement on sets from which one removes subsets. Two applications were given. One, to abelian finite group, from which some classes modulo subgroups are sifted. Another one, to questions as: how many p's, with $p \leq x$, are such that the Ramanujan τ function, evaluated at p, is a square?

H.P. SCHLICKEWEI: An extension of a result of Senge and Straus.
Generalizing a result of H.G. Senge and E.G. Straus we show the following

Theorem. Let $\theta_1,\dots,\theta_s$ be natural numbers. Then the number of integral s-tuples $(n_1,\dots,n_s)$ satisfying $n_1+\dots+n_s=0$, where for each i n_i has a sum of digits in base θ_i which lies below a fixed bound M, is finite if and only if for any pair $i \neq j$ $(1 \leq i,j \leq s)$ $\log \theta_i / \log \theta_j \notin \mathbb{Q}$.

The proof of this theorem uses essentially the author's p-adic generalization of W.M. Schmidt's subspace theorem concerning the approximation of algebraic numbers by rationals.

J. SCHOISSENGEIER: Zeta-function and sequences of primes mod α.
Let $g(x) \in \mathbb{R}[x]$, $\lim\limits_{x\to\infty} g(x) = \infty$ and $m = \deg g(x)$. Define $p(x) = i\, g(xe^{-\pi i/2m})$ and $q(x) = x(xp'(x))'$. Let $w(z)$ be a holomorphic function such that $e^{w(z)}p'(e^{w(z)}) = z$ if $\operatorname{Re} z \in (0,1)$ and if $\operatorname{Im} z > K$, K large enough. Chose $\operatorname{Im} w(z)$ such that $|\operatorname{Im} w(z) - \pi/2m| \leq \pi/m$. Then $w(z)$ is determined uniquely.

The following theorem is valid

Theorem 1. Let $h \geq 1$. There exists a $K > 0$ (depending only on g) such that if $N \to \infty$ $\sum\limits_{n \leq N} \Lambda(n)\, e^{-ihg(n)} =$

$$- \sqrt{2\pi/h} \sum_{Kh \leq \gamma \leq hNg'(N)} e^{\rho w(\rho/h) - hp(e^{w(\rho/h)}) - i\pi\rho/2m} q(e^{w(\rho/h)})^{-1/2}$$

$+ O(\sqrt{N}\log^2 hN)$. $\rho = \beta + i\gamma$ runs through the non-trivial zeros of the zeta-function.

Theorem 2. Let $\alpha > 0, \sigma > 0$, $g(x) = \alpha x^{\sigma}$, $h \geq 1$. Then for some $K > 0$, $N \to \infty$, $\sum\limits_{n=1} \Lambda(n) e^{-ihg(n)} =$

$$- \sqrt{2\pi/\sigma} \sum_{Kh \leq \gamma \leq h\sigma N^{\sigma}} e^{\rho/\sigma \log \rho/(h\alpha\sigma e) - i\pi\rho/2\sigma} \rho^{-1/2}$$

$+ O(\log^2 hN\, N^{1/2})$.

Corollary 1. $\psi(N) = -\sqrt{2\pi}\, e^{-\pi i/4} \sum\limits_{0 < \gamma \leq 2\pi N} (2\pi)^{-\beta}\gamma^{\beta-1/2} e^{i\gamma\log\gamma(2\pi e)}$

$+ O(N^{1/2}\log^2 N)$.

Corollary 2. (Wolke, Stux) Let $\alpha > 0$, $0 < \sigma < 1$. Then $(p^{\sigma}\alpha)_{p \text{ prime}}$ is uniformly distributed.

Corollary 3. (Hardy, Littlewood) Let $I \subseteq [0,\infty)$, I compact, $a > 0$. Then $\sum_{0<\gamma<T} e^{a\rho\log(-i\rho)} x^{\rho} \rho^{-a/2} = O(T^{(a+1)/2})$

uniformly with respect to $x \in I$.

Hardy conjectured that one can improve this estimate to $O(T^{1/2+\epsilon})$. The editors of Hardy's collected papers mentioned that this is probably false. We can prove

Corollary 4. The estimation $O(T^{(a+1)/2})$ is sharp if and only if $1/a \in \mathbb{N}$ and $x\pi e \notin \mathbb{Q}$.

E.J. SCOURFIELD: On the property that $(\varphi(n),\varphi(n+1))$ has no odd prime divisor. For $f = \varphi$ (Euler's function) or σ (the divisor-sum function), denote by $N_f(x)$ the number of positive integers $n \le x$ such that the GCD $(f(n),f(n+1))$ has no odd prime divisor. Then we have

Theorem 1. For $f = \varphi$ or σ, $\frac{x}{\log x \text{ logloglog } x} \ll N_f(x) \ll \frac{x}{\log x} \exp(A \text{ loglog } x (\text{logloglog } x)^{-1/2})$ for any positive constant A satisfying $A > C^{1/2}$ where $C = 2e^{-\gamma} \prod_{p \ge 3} (1-(p-1)^{-2})$.

The proof depends on applications of results from sieve theory, and the upper bound result can be generalized considerably. Let $S_{\varphi}(x)$, $S_{\sigma}(x)$ denote the number of primes $p \le x$ such that there is no odd prime dividing $(p-1, \varphi(p-1))$, $(p+1, \sigma(p+1))$ respectively. Then the lower bound above is deduced from the following result, which is analogous in a sense to Erdös' estimate for the number of integers $n \le x$ such that $(n, \varphi(n))=1$:

Theorem 2. $S_f(x) \sim \frac{C\,x}{\log x \text{ loglog } x}$ as $x \to \infty$ where C is defined as above and $f = \varphi$ or σ.

G. TENENBAUM: On the divisor density of an integer sequence. Let n denote a positive integer, $\tau(n)$ the number of its divisors and $\tau(n,A)$ the number of those divisors of n which belong to a given sequence A. R.R. Hall has introduced the following definition: an integer sequence A is said to have divisor density z, and one writes $DA = z$, if $\tau(n,A) \sim z\tau(n)$ for almost all n's. It can be seen easily that the squarefree numbers or any arithmetic progression distinct from $\mathbb{N}$ fails to have divisor density. Moreover, the asymptotiv formula $\sum_{n<x} \tau(n,A) \sim x \sum_{\substack{d<x \\ d\in A}} \frac{1}{d}$ which holds if the series $\sum_{d\in A} \frac{1}{d}$ diverges, might lead to the assumption that divisor density is closely related to logarithmic density: this is not so in fact, and Hall has shown that for any given pair $(z,w) \in [0,1]^2$ there exists a sequence A with the property that $DA = z$ and $\delta A = w$ (here and in the sequel δ denotes the logarithmic density). In the opposite direction, Hall proves the following result, first conjectured by Erdös:

Theorem 1. (Hall) Let $\{b_j\}$ be a sequence of real numbers satisfying $b_{j+1} \geq cb_j$ for some $c > 1$ and all j's, and set $A = \{d : \exists\, j: b_{2j} \leq d < b_{2j+1}\}$. Then, if $\delta A = z$, one also has $DA = z$.

The concept of divisor density is also related to uniform divisor distribution, also introduced by Hall: a sequence of real numbers $\{f(n)\}$ is uniformly divisor distributed mod 1 if $\Delta(n,f) := \sup_{0\leq u<v\leq 1} \left| \frac{1}{\tau(n)} \operatorname{card}\{d: d|n,\ f(d) \in [u,v[\bmod 1 - (v-u)\right|$ tends to zero for almost all integers n.

Set $A(z,f) = \{d: f(d) < z \bmod 1\}$, then, clearly, for any u.d.d. function f, $DA(z,f) = z$ for all $z \in [0,1]$. Hall showed that the converse is also true. In the case of additive functions f, the Weyl sums $\sigma_v(n,f) = \sum_{d|n} e^{2\pi i f(d) v}$ are multiplicative for any $v \in \mathbb{Z} \setminus \{0\}$ and this may be used to prove uniform divisor distribution results. The function $F(d) = \log d$ has been studied by Hall and Erdös; it is u.d.d., and one has $\Delta(n,\log) \leq \tau(n)^{-\lambda}$ for any $\lambda < \frac{\log \pi}{\log 2} - 1$. Katâi has shown that an additive function f is u.d.d. if and only if the series $\sum_p \|vf(p)\|^2/p$ diverges for every non zero integer v, where $\|x\|$ denotes the distance of x to the nearest integer. Non additive functions are much more difficult to

deal with. Hall showed that $(\log d)^{\alpha}$ and $(\log\log d)^{\beta}$ are u.d.d. for $0<\alpha<1+\log 2$ and $\beta>1$; the restriction on β is sharp, but he conjectured that $\alpha>0$ is sufficient. This conjecture is an easy consequence of corollary 1 below.
I can prove the following results:
Theorem 2. Let A be an integer sequence with characteristic function χ. Then DA = z if and only if $\sum_{k<x} \left| \sum_{\substack{n<x \\ n\equiv 0 \bmod k}} n^{-1}(\chi(n)-z)4^{-\Omega(n)} \right|$ $= o((\log x)^{1/2})$ as $x\to\infty$, where $\Omega(n)$ denotes the number of prime factors of n counted with multiplicity.
Theorem 3. Let b_j be a sequence of real numbers satisfying $\operatorname{card}\{j: b_j<x\} = O((\log x)^{\alpha})$ as $x\to\infty$, for some $\alpha>0$, and set $A = \{d: \exists j: b_{2j} \leq d < b_{2j+1}\}$. Then if $\delta A = z$, one also has DA = z.
Corollary 1. Let f be a differentiable real function. Suppose $f'(x)\to 0$ as $x\to\infty$ and the sequence $\{f(n)\ n\in\mathbb{N}\}$ is uniformly distributed mod 1, then $f((\log d)^{\alpha})$ is u.d.d. for any positive α. Corollary 2. Let g be a differientiable function. Suppose $|x\, g'(x) \log x|\to\infty$ as $x\to\infty$ and that there exists an α such that the function $x\mapsto x\, g'(x)\, (\log x)^{-\alpha}$ is monotonic and tends to zero at infinity, then g is u.d.d..
Theorem 4. The following condition is necessary and sufficient for f to be u.d.d.

$$\sum_{k<x} \sum_{\substack{n<x \\ n\equiv 0 \bmod k}} e^{2\pi i v f(n)}/(n4^{\Omega(n)}) = o((\log x)^{1/2}),$$

$(v=\pm 1, \pm 2, \ldots)$.
This last theorem can be used to obtain Katâi's criterion for uniform divisor distribution of additive functions.

V.T. SÓS: Intersection properties os subsets of integers.
Let $A_1,\ldots,A_N$ be a family of subsets of $1,\ldots,n$. For a fixed integer k we assume that $A_i\cap A_j$ is an arithmetic progression of k elements whenever $1\leq i<j\leq N$. We would like to determine

the maximum of N. For $k = 0$ with R.L. Graham, M.S. Simonovits we proved that $N \leq \binom{n}{3}+\binom{n}{2}+\binom{n}{1}+1$. For $k \geq 2$ the asymptotically extremal system has $N = (\pi^2/24 + o(1))\, n^2$ sets. (So for $k \geq 2$ the extremum depends on k very weakly.) For $k = 1$ the maximum is between $\binom{n}{2}+1$ and $(\pi^2/24 + 1/2)\, n^2 + o(n^2)$. Probably the lower bound is sharp. These results form a part of joint work with M. SIMONOVITS.

R.C. VAUGHAN: Some remarks on Weyl sums.

Let $f(\alpha) = \sum_{x=1}^{N} e(\alpha x^k)$, $v(\beta) = \int_0^N e(\beta\gamma^k)d\gamma$, $S(q,a) = \sum_{r=1}^{q} e(\frac{a}{q}r^k)$,

$G(\alpha,q,a) = q^{-1}S(q,a)v(\alpha-\frac{a}{q})$, $E = f(\alpha) - G(\alpha,q,a)$. It is shown

that if $(a,q) = 1$ and if $|\beta| \leq N/(2kqN^k)$, then $E \ll q^{1/2+\epsilon}$.

Moreover, if the condition on β is relieved then one still has $E \ll q^{\epsilon}(q + qN^k|\beta|)^{1/2}$. One consequence of this is that when $k \geq 3$ and either $|\alpha - \frac{a}{q}| \leq q^{-1}N^{-3/2}$ with $q > N$ or $|\alpha - \frac{a}{q}| > q^{-1}N^{-3/2}$,

then $E \ll N^{3/4+\epsilon}$. This gives a new and completely different proof of Weyl's inequality when $k = 3$. Also the relationship between $f(\underline{\alpha}) = \sum_{x \leq X} e(\alpha_1 x+\ldots+\alpha_k x^k)$ and $J_s^{(k)} = \int_{[0,1]^k} |f(\underline{\alpha})|^{2s}\, d\underline{\alpha}$

was discussed and an outline of the proof of the statement

If $\exists$ j,a,q, s.t. $2 \leq j \leq k$, $|\alpha_j - \frac{a}{q}| \leq q^{-2}$, $(a,q) = 1$, $q \leq N^j$,

then $f(\underline{\alpha}) \ll (J_s^{(k-1)}\,(2N)\, N^{k(k-1)/2}\,(q^{-1} + N^{-1} + qN^{-j}))^{1/2s} \log N$

was given.

B. VOLKMANN: On Strassen's law of the iterated logarithm.

(joint work with P. SZÜSZ) Kolmogorov's celebrated law of the iterated logarithm (1929) has been generalized in a number of directions, notably by Strassen (1964) who proved the following theorem: Let $\xi_1, \xi_2, \ldots$ be a sequence of independent identically distributed random variables with $E(\xi_i) = 0$ and $D^2(\xi_i) = 1 + o(1)$; consider their partial sums $S_n = \xi_1 + \dots + \xi_n$, let $T_n(i) = S_i(2n\log\log n)^{-1/2}$ $(i = 1,\dots,n)$ and define, for each n, the function φ_n on $[0,1]$ as linear interpolation of the values $\varphi_n(i/n) = T_n(i)$. Then the set of limit points of $\varphi_3, \varphi_4, \dots$, under uniform convergence, is almost certainly equal to the space γ of all absolutely continuous functions on $[0,1]$ with $x(0) = 0$ and $\int_0^1 x^2(t)\,dt \leq 1$. The authors have, instead of Strassen's methods (Brownian k-dimensional motion and functional analysis), used the classical approach of Kolmogorov in order to prove a generalized version of Strassen's theorem. The condition of identical distribution could be dispensed with, and Kolmogorov's condition $|\xi_n| = o(n(\log\log n)^{-1/2})$ could be replaced by a much weaker one which is less stringent than requiring the existence of some fractional moment $E(\xi_i^{2+\varepsilon})$, $\varepsilon > 0$, $i=1,2,\dots$. Furhtermore, a Strassen - type theorem for weakly dependent random variables was established which is applicable to continued fractions, thus generalizing the Kolmogorov type law of the iterated logarithm due to Szüsz (1971).

D. WOLKE: <u>On the explicit formula of Riemann - von Mangoldt</u>. By means of mean value theorems for Dirichlet polynomials and zero density results for the Riemann zeta - function the following slightly improved version of the classical Riemann - von Mangoldt formula is discussed: Let $\delta > 0$, $x^c < T \leq x^{1-c}$. Let $\rho = \beta + i\gamma$ denote non trivial zeros of

$\zeta(s)$. Then there exists a $\tau \in [T/2, 3T/2]$ such that $\psi(x)$ $= \sum_{n \le x} \Lambda(n) = x - \sum_{\rho, |\gamma| \le \tau} x^{\rho}/\rho + O(xT^{-1}(\log x)^{1/2})$. It is highly probable that this formula holds without the factor $(\log x)^{1/2}$. A further improvement would give a better upper bound for the difference of consecutive primes (if RH assumed).

Because of shortage of time the following two lectures could not be given.

M. HUXLEY: Two remarks on the λ^2 sieve.

1. Selberg's upper bound for a sievable sequence A with

$$\sum_{\substack{a \in A \\ a \equiv 0 \bmod d}} 1 = x\, \rho(d)/d + R(d) \quad \text{(where } \rho \text{ is multiplicative)}$$

states that the number of members of A with no prime factor $\le z$ is at most $\le x/G(z) + \sum_{d_1} \sum_{d_2} \lambda(d_1)\lambda(d_2)G^{-2}(z)R([d_1, d_2])$ where $\lambda(d)$, $G(z)$ are constructed from $\lambda(d)$ (Halberstam + Richert in "Sieve methods"). If $\rho(p)$ is k on average, $G(z) \sim H \log^k z$. Under the one - side condition on $\rho(p)$

$$\sum_{p \le u} \frac{\rho(p)\log p}{p - \rho(p)} = k \log u + E(u) \text{ where } E(v) \le E(u) + K$$

for $u \le v$ one has $G(z) \ge H \log^k z \left(\frac{\log z - 2^{k+1}K}{\log z + K}\right)$.

The error term can also be estimated as

$$O(e^k z^2 \{\max(1, \log K)\}^2 \max_d |R(d)|/\rho(d))$$

2. In the sieve of Jurkat and Richert the error term $O(L/(\log y)^{1/14}$ can be replaced by $O(L/(\log y)^{\theta})$, $\theta = (\log 3 - 1)/(\log 9/2 - 1)$ $< 1/5$. (jointly wrought with GREAVES)

W. SCHWARZ: Power - series with multiplicative coefficients.

The author reproves a result of Lucht and Tuttas: If f is a multiplicative function with mean - value $M(f) \neq 0$ and $\| f \|_2 < \infty$, then the power - series $\sum f(n) \cdot z^n$ either represents a rational function or it is non - continuable beyond $|z| = 1$. As an application, following Rubel and Stolarsky, all possible functions $F(z) = \sum \frac{1}{n!} f(n) z^n$ are determined, which are bounded on the negative real axis (the assumptions on f are as above).

The following problems were posed :

J.-M. DESHOUILLERS: If you shake an additive basis, will it remain an additive basis?

Question: Let $f\colon \mathbb{N} \to \mathbb{N}$ be such that for every additive basis B, the sequence $f(B)$ is an additive basis; does there exist a positive constant α such that $f(x) - \alpha x$ is bounded? To avoid trivial cases, assume $f(0) = 0$, $f(1) = 1$,

$$\underline{\lim} \frac{\log f(n)}{\log n} > 0, \quad \Delta f \geq 0 \quad (\text{where } \Delta f(n) = f(n+1) - f(n)).$$

Comments. For some cases cf. Deshouillers - Erdös - Sårközy (A.A. 30 (1976), 121 - 132) and Deshouillers - Fouvry (J.L.M.S. (2), 14 (1976), 413 - 422).

I know that the conjecture is true in the two extreme cases: $f(x) = x + o(x)$ or $\Delta^2 f > 0$.

P. ERDÖS:

1. Denote by $1 = d_1 < d_2 < \dots < d_{\tau(n)} = n$ the consecutive divisors of n, $f(n) = \sum_{i=1}^{\tau(n)-1} ((d_{i+1}/d_i) - 1)^2$. Is it true that for infinitely many n $f(n) < C$ holds?

2. Let $f(n) = \pm 1$ be a numbertheoretic function. Is it true that for every C there exist d and m such that

$$(1) \quad \left| \sum_{k=1}^{m} f(dk) \right| > C.$$

(1) gehört offenbar zum van der Waerdenschen Ideenkreis. I conjectured (1) nearly 50 years ago and I give 500 Mark for a proof or a counterexample. Perhaps it is true that

$$(2) \quad \max_{md \leq x} \left| \sum_{k=1}^{m} f(dk) \right| > C \log x .$$

It is easy to see that (2) is best possible if it is true.

A. IVIĆ:

Let a(n) denote the number of non-isomorphic abelian groups with n elements; let $C_1, C_2, \dots$ denote positive, absolute constants. Let N be a-highly composite if $a(n) < a(N)$ for $n < N$, and let $G(x) = \sum_{N \leq x} 1$, $H(x) = \sum_{N \leq x} a(N)$.

1. Is it true that $\log G(x) = (C_1 + o(1)) \log\log x$, $\log H(x) = (C_2 + o(1)) \frac{\log x}{\log\log x}$?

2. Is it true that for $n \geq n_0(\epsilon)$ $\log a(n) + \log a(n+1) < \left(\frac{\log 5}{4} + \epsilon \right) \frac{\log n}{\log\log n}$?

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3. Find the maximal order of magnitude of $a(a(n))$; it can be shown that it is at least $\exp(C_3(\log n/\log\log n)^{1/2})$ and that it is not greater than $\exp(C_4(\log n/(\log\log n)^2)$.

4. Let $a^{(j)}(n) = \underbrace{a(\ldots(a(n))\ldots)}_{j\ \text{times}}$. It can be shown that uniformly in j $\sum_{n \leq x} a^{(j)}(n) = K(j)x + O(x^{1/2}\log^4 x)$ with a suitable constant $K(j) > 0$. Is it true that $\sum_{n \leq x} K(n) = C_5 x + O(x^{1-\epsilon})$?

M. MENDES FRANCE:

1. A set $H \subset \mathbb{N}$ is called a Van der Corput set if the following implication holds: $(u_{n+h} - u_n)$ is equidistributed mod 1 for all $h \in H \Rightarrow (u_n)$ is equidistributed mod 1.
Examples: $\mathbb{N}$, $a\mathbb{N}$ $(a \in \mathbb{N})$, $P-1$ (P set of primes), $P + 1$, the set of squares,... (see Kamae - Mendes France, Israel J. of Math. 1978) Question 1: Is it true that if H is a Van der Corput set then $aH = \{ah \mid h \in H\}$ is a Van der Corput set $(a \in \mathbb{N})$? (RUZSA answered affirmatively)
Question 2: Suppose $h_0 \in H$. Is it true that if H is a Van der Corput set, then $H \setminus \{h_0\}$ is again a Van der Corput set. (RUZSA answered affirmatively)

2. Let $n \in \mathbb{N}$ and let $s(n)$ be the sum of the digits of n in basis g ($g \geq 2$ is a given integer). Given $m \geq 2$ and $a \in \{0,1,\ldots,m-1\}$, $(m,g-1) = 1$. Consider the set $S(a,m) = \{n \mid s(n^2) \equiv a \bmod m\}$. Is it true that $S(a,m)$ has density $1/m$?

This problem was actually posed by Gelfond (Acta Arith. 1968)

Commentary: It is well known that the sequence $s(n) \bmod m$ can be generated by a finite automata and this implies a "good" behaviour of the sequence. Recently, J.P. Allouche observed that the sequence $s(n^2) \bmod m$ cannot be generated by a finite automata. This result measures to some extent the depth of the question whether $s(n^2) \bmod m$ has a density.

H.L. MONTGOMERY:

1. Let $f(x) = \inf\limits_{\delta} \sup\limits_{\mathfrak{N}} \int_{x-\delta}^{x+\delta} \left|\sum\limits_{n\in\mathfrak{N}} e(nx)\right|^2 dx \,/\, \operatorname{card} \mathfrak{N}$,

where $\mathfrak{N}$ is an arbitrary finite set of integers. Saffari has shown that $c = \inf\limits_{x} f(x) > 0$; he obtained the bound $c \geq \pi^{-1} \max\limits_{\gamma} \gamma^{-1} \sin^2 \gamma = 0.2306\ldots$. He also wrote that $f(0) = 1$ $f(1/2) = 1/2$, $f(1/3) = 1/3$, $\overline{\lim\limits_{x\to 1/2}}\ f(x) \leq 1/4$. Thus $c \leq 1/4$.

We would like to know the value of c, and likewise the value of f(x) for all x.

Related to the above, let C(a) be the inf of those constants C such that $\int_{-aT}^{aT} \left|\sum\limits_{n} a_n e^{i\lambda_n t}\right|^2 dt \leq C \int_{-T}^{T} \left| b_n e^{i\lambda_n t}\right|^2 dt$

whenever the λ_n are real, and $|a_n| < b_n$ for all n. Halász, Wirsing and I have shown that $1 \leq C(a) \leq 2$ for $0 < a \leq 1/2$, $2 \leq C(a) \leq 3$ for $1/2 < a \leq 1$, $3 < C(a) \leq 4$ for $1 < a \leq 3/2$, ... , and that $\lim\limits_{a\to 0^+} C(a) = 1$.

2. We wish to show that log 3 / log 2 is not a badly approximable number. That is, we wish to show that the continued fraction coefficients of log 3/ log 2 are unbounded.

I suggest this problem because of the following possible approach: Let $M = \{2^a 3^b : a \geq 0,\ b \geq 0\}$, and write the members of M as $1 = m_1 < m_2 < m_3 < \dots$. By Feldman's theorem it can be shown that $m_i(\log m_i)^{-\Delta} \leq m_{i+1} - m_i < m_i(\log m_i)^{-\delta}$, where $0 < \delta < 1 < \Delta$. It can also be shown that $\log 3/\log 2$ is badly approximable if and only if $m_{i+1} - m_i \gtrsim m_i(\log m_i)^{-1}$. We seek to receive a contradiction from this by constructing a generating function for M, say $\sum_{m \in M} m^{-s} = (1-2^{-s})^{-1}(1-3^{-s})^{-1}$.

Note, however, that the elements of the set $M' = \{e^a e^{\omega b} : a \geq 0,\ b \geq 0\}$, $\omega = (1+\sqrt{5})/2$ are well spaced. Thus it is important to make use of the additive structure of M, namely that $M \subseteq \mathbb{Z}$.

J.L. NICOLAS:

P. Erdös asked for the following problem (Colloq. Math. t. 42, 1979, Problem 1162, p. 399): Denote by $l(x)$ the maximal length of a sequence $0 < a_1 < a_2 < \dots < a_{l(x)} \leq x$ such that for all i,j, $a_i + a_j$ is never a square. Choosing $a_i = 3i - 2$ proves $l(x) \geq x/3$ because $a_i + a_j \equiv 2 \bmod 3$. In the other way, considering the set $\{a_i\} \cup \{[\sqrt{x}]^2 - a_i\}$ gives that $l(x) \leq \frac{x}{2}(1 + o(1))$. J.P. Massias (Univ. of Limoges) has found in June 1980 that the two following sets of 11 numbers : $\{1,5,9,13,14,17,21,25,29,30\} \cup \{10 \text{ or } 26\}$ verify that mod 32, $a_i + a_j$ is never a square. So this proves, considering the set of a's congruent mod 32 to one of these 11 numbers that $l(x) \geq \frac{11}{32}x$. Odlyzko and Lagarias have a proof of the following:

Theorem 1. Let N and $0 < a_1 < \ldots < a_n < N$ such that $a_i + a_j$ is never a square mod N, then $n \leq \frac{11}{32}$ N, so the counterexample of Massias is optimal.

Theorem 2. $l(x) \geq 0.46\ x\ (1 + o(1))$.

Problem : Compute $\overline{\underline{\lim}}\ l(x)/x$?

K. RAMACHANDRA:

1. Let $a_1, a_2, \ldots$ be a sequence of complex numbers with $\sum_{n \leq x} a_n = x + O(1)$. Put $f(s) = \sum_{n=1}^{\infty} a_n n^{-s}$, where $s = \sigma + it$, $\sigma > 1$. Then $f(s) - (s-1)^{-1}$ is regular in $\sigma > 0$. Let $N(\sigma,T)$ denote the number of zeros ρ of $f(s)$ counted with multiplicity satisfying $\mathrm{Re}\ \rho \geq \sigma$, $|\mathrm{Im}\ \rho| \leq T$, where $T \geq 10$. In a recent paper published in Crelle J. I proved that, for any constant δ with $0 < \delta < 1/2$, $N(1/2 - \delta, T) \gg T \log T$. Improve this result to $N(1/2, T) \gg T \log T$.

2. Let $f_1(z) = 2^{az}$, $f_2(z) = 2^{bz}$ where $a = t^l$, $b = t^{l+k}$, t being a fixed transcendental number, and l and k being any two natural numbers. It follows from a result of Siegel and Schneider which was rediscovered by me that one at least of the six numbers $f_1(z)$, $f_2(z)$, with $z = 1, t^k, t^{2k}$ must be transcendental, i.e. one at least of the four numbers $2^{t^l}, 2^{t^{l+k}}, 2^{t^{l+2k}}, 2^{t^{l+3k}}$ must be transcendental. Let A denote the set of natural numbers n for which 2^{t^n} is algebraic. Then A cannot contain any arithmetic progression l, l+k, l+2k, l+3k of four terms and hence by Szemeredy's theorem

$\sum_{\substack{n \leq x \\ n \notin A}} 1 = o(x)$. Improve this to $O(x^{1-\varepsilon})$ for some $\varepsilon > 0$.

I.Z. RUZSA:

To every $\delta > 0$ an $\varepsilon > 0$ can be found with the following property: If f is a real-valued additive arithmetical function, satisfying $f(n) \in [a,a+1]$ for all but εx numbers $n \leq x$, then there is a decomposition of f into two additive functions $f = f_1 + f_2$ such that $f_1(n) \in [b,b+1+\delta]$ for all $n \leq x$ and $|f_2(n) - c| < \delta$ for all but δx $n \leq x$ with suitable b and c. Probably the existing methods enable us to solve this with $\varepsilon = \delta^K$ for some large constant K; it would be nice to obtain $\varepsilon = \alpha\delta$ with some fixed $\alpha > 0$.

G. TENENBAUM: Let f be a multiplicative function satisfying $|f(n)| \leq 1$ for every positive integer n. Then Daboussi proved that $\sum_{n < x} f(n) e^{2\pi i \alpha n} = o(x)$ for any real non rational α.

Under which conditions on the (positive) multiplicative function f can one prove that, for every $\alpha \in \mathbb{R} \setminus \mathbb{Q}$,

$$\sum_{n < x} f(n) e^{2\pi i \alpha n} = o\left(\sum_{n < x} |f(n)| \right) ?$$

R. TIJDEMAN:

1. Let A and B denote monotonic increasing sequences of positive integers. $A = \{a_n\}_{n=1}^{\infty}$ is said to have bounded gaps if there exists a constant k such that $a_{i+1} - a_i \leq k$ for all i. $A - A = \{d: d = a_i - a_j \; , \; a_i, a_j \in A, \; a_i > a_j\}$ is called

difference set of A. Is it true that for every sequence A with $\overline{d}(A) > 0$ there exists a sequence B with bounded gaps such that $B - B \subseteq A - A$?

2. Let $\overline{b}_j = (b_{j1}, b_{j2}, \ldots, b_{jm}) \in \mathbb{R}^m$ for $j = 1, \ldots, k$ denote vectors with the property that the origin is in the convex hull of these vectors. Let $|b_{ji}| \leq 1$ for all j and i. It is true that for every positive integer n there exist non-negative integers $r_1, r_2, \ldots, r_k$ with sum n such that

(*) $| r_1 b_{1i} + \ldots + r_k b_{ki} | \leq m$ for $i = 1, \ldots, m$.

Is it true that for every $\varepsilon > 0$ the upper bound in (*) can be replaced by $m^{1/2 + \varepsilon}$ for m sufficiently large? It can be shown that the right-hand side of (*) cannot be replaced by $m^{1/2}/4$.

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