

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 13/1981

Stochastische Analysis

15.3. bis 21.3.1981

Die Tagung fand unter der Leitung von Herrn J. Jacod (Rennes) und Herrn K. Krickeberg (Paris) statt. Die Vorträge überdeckten einen weiten Bereich der Theorie der stochastischen Prozesse, von Zusammenhängen mit der Analysis bis zu Anwendungen wie der Konstruktion stochastischer Modelle und der Theorie der stochastischen Filtrierung und Kontrolle. Im Mittelpunkt des Interesses standen jedoch zwei besonders aktuelle Themen: einerseits stochastische Differentialgleichungen, andererseits stochastische Prozesse mit zwei Indizes ("zweidimensionale Zeit").

Vortragsauszüge

S. ALBEVERIO (Bielefeld)

Brownian motion, random fields and representation of group of mappings

We discuss the "Markov unitary representation"  $U$ , of the groups  $G^X$  of mappings from a Riemannian manifold  $X$  into a Lie group  $G$ , as a non commutative extension of the concept of markov processes and fields.

- 1)  $G = \mathbb{R}$  :  $U$  is given by left translations using the quasi invariant Gaussian measure with covariance the potential kernel of Brownian motion.
- 2)  $X = \mathbb{R}^1$ ,  $S^1$ ;  $G =$  compact Lie group :  $U$  is given by right translations using the quasi invariance of Wiener measure on  $C(\mathbb{R}; G)$

3) X Riemannian manifold, G = compact semisimple Lie group: U is given by

$$\psi \in G^X \longrightarrow U(\psi) \exp \omega \equiv e^{-\frac{(\beta(\psi), \beta(\psi))^2}{2}} e^{-(\beta(\psi), \text{Ad} \psi \omega)} \exp(\text{Ad} \psi \omega + \beta(\psi))$$

where  $\omega$  is in the Hilbert space  $H$  of 1-forms from the tangent bundle  $TX$  into the Lie algebra  $\mathfrak{g}$ ,  $\beta(\psi)$  being the Maurer-Cartan cocycle, and  $\exp \omega \equiv 1 + \frac{d\omega}{2} + \dots \in \exp H$

Th. 1 : U is irreducible if  $d = 2$  and  $|\epsilon| > \inf(\mathcal{J}2TP(X))^{\frac{1}{2}}$  for all roots  $\epsilon$  of  $\mathfrak{g}$ , or if  $d \geq 3$

Th. 2 : If  $d=1$ , U is reducible.

Th. 1, 2 are proved in a paper by S. Albeverio, R. Höegh-Krohn, D. Testard (J. Funct. Anal.). Work by the same authors with A. Ventrik on the structure of the irreducible components for  $d=1$  and on the remaining cases for  $d=2$  is in progress. The results are proved by exploiting properties of Brownian motion on a group and of the Gaussian generalised random fields associated with commutative subgroups of  $G^X$

D. BAKRY (Stasbourg)

A remark on two indices semimartingales

We study processes of the form  $X_{s,t} = E(A_t / \mathbb{F}_s)$  where  $A_t$  is a bounded variation process. If  $dA_t$  is absolutely continuous with respect to a deterministic bounded measure  $\mu$ , with  $E \int (\frac{dA}{d\mu})^2 d\mu$ , Wong and Zakai have shown that  $X_{s,t}$  is then a two indices semimartingale.

(i.e., we can define stochastic integrals  $\int \int H_{s,t} dX_{s,t}$ , where  $H(\omega, s, t)$  is a bounded measurable process, previsible in  $(\omega, s)$ ).

We show that it is no longer true when  $dA_t$  does not verify the above domination hypothesis.

M.T. BARLOW (Cambridge)

Strong and weak solutions to Stochastic Differential Equations

We consider the one-dimensional S.D.E. :

(\*) {  $dX_t = a(X_t)dB_t$  ;  $X_0 = x$  } , where B is a Brownian motion.

Theorem Let  $0 < \delta < \infty$  . Then there exists a continuous function

a:  $\mathbb{R} \rightarrow \mathbb{R}$  with  $0 < \delta < a(x) < K < \infty$  for  $x \in \mathbb{R}$  , some  $\delta, K$ , such that

- (i)  $|a(x) - a(y)| \leq C|x - y|^\alpha$
- (ii) (\*) has no strong solution.

The method of the proof is to construct two different solutions to (\*) : no strong solution can then exist. Let X be one solution, a perturbation method gives a process  $Y^\epsilon$  , which is a solution to (\*) on  $\{ |X - Y^\epsilon| > \epsilon \}$  . The pair  $(X, Y^\epsilon)$  induces a measure  $Q^\epsilon$  on  $C(\mathbb{R}_+, \mathbb{R}^2)$  , and  $Q^\epsilon$  converges weakly to some measure Q . The co-ordinate processes are both solutions to (\*) under Q , and for suitable a(.) it is possible to show that  $Q(X_t = Y_t) < 1$  .

TH. BARTH (Zürich)

Stochastic Differential Equations with Infinite-dimensional Brownian

Motion.

For a infinite dimensional Brownian motion  $W = (w_t^1, w_t^2, \dots)_{t \geq 0}$  the stochastic integral defines an isometry from the space  $\mathbb{L}^2$  of non-anticipating  $\mathbb{N} \times \mathbb{N}$ -matrix valued processes X such that

$\|X\|_2 = \{ \int_0^\infty \text{tr} |X_t|^2 ds \}^{\frac{1}{2}} < \infty$  into the space  $\mathbb{H}^2$  of  $l^2$ -valued processes Y such that  $\|Y\| = \{ \sup_{t \geq 0} \text{tr} |X_t|^2 \}^{\frac{1}{2}} < \infty$  . Moreover, the integral has the same continuity properties as in the finite dimensional case. The Banach fixed point method applies to solve the equation

$X_t = C_t + \int_0^t f(s, X_s) ds + \int_0^t G(s, X_s) dW_s$  , where the initial process C



is continuous adapted with values in  $\mathbb{R}^2$ , the coefficients are measurable and satisfy a Lipschitz condition

$|f(s,x)-f(s,y)| \leq l(t)|x-y|$ ,  $|G(s,x)-G(s,y)| \leq l(t)|x-y|$ , where  $l$  belongs to  $L^2_{loc}(\lambda, \mathbb{R}_+)$ , and a growth condition:  $f(\cdot, x)$ ,  $G(\cdot, x)$  belong to  $L^2_{loc}(\lambda, \mathbb{R}^2)$  for some  $x$  in  $\mathbb{R}^2$  to prevent explosion. The continuous global solution is locally in  $\mathbb{H}^2$ . By a change of the norm of  $\mathbb{H}^2$  the equation is solved on any interval  $[[0, T]]$ , where the stopping time  $T$  localizes  $X$  in  $\mathbb{H}^2$ , i. e.  $X^T_{T>0}$  is in  $\mathbb{H}^2$ . The conditions correspond to those introduced by C. Carathéodory for ordinary differential equations, where also the method of changing of norm is used.

J. BROSSARD (Grenoble)

Inequalities for Martingales with Two Indices and Moderate Functions.

Let  $M_{t_1, t_2} = \int_{s_1}^{t_1} \int_{s_2}^{t_2} dB_{s_1} dB_{s_2}$  be a bi-Brownian (local) martingale. Define  $M^* = \sup_{t_1, t_2} |M_{t_1, t_2}|$  (maximal variable) and  $S^2 = \iint \varphi_{s_1, s_2}^2 ds_1 ds_2$  (quadratic variation).

Theorem : If  $\psi$  is a moderate function, there exist  $c$  and  $C$  (depending only on  $\psi$ ) such that  $cE \psi(M^*) \leq E \psi(S) \leq CE \psi(M^*)$

A moderate function is a right continuous increasing function such that  $\psi(0) = 0$  and  $\psi(2x) \leq k \psi(x)$ . The theorem is a consequence of the two following inequalities, obtained for all positive  $n$  :

- (1)  $P(M^* \geq u) \leq C_n \{P(S \geq u) + \frac{1}{u^n} E(S^n; S < u)\}$
- (2)  $P(S \geq u) \leq C_n \{P(M^* \geq u) + \frac{1}{u^n} E(M^{*n}; M^* < u)\}$

(The inequality (2) is much more difficult than (1))

P. CHEN (Pekin)

Stochastic Processes Indexed by Riesz Spaces

Let  $T$  be an Archimedean Riesz space with an order unit  $e$ . Strengthening order is defined by  $0 < t$  iff  $T = \bigcup_{n=1}^{\infty} [-nt, nt]$ ,  $s < t$  iff  $0 < t-s$ . Let  $\mathcal{B}$  denotes the Borel  $\sigma$ -field relative to the order topology and  $\mathcal{V}$  the family of all closed left filterable subsets

of  $T$ . Let  $(\Omega, \mathbb{F}, \mathbb{P})$  be a complete probability space,  $\{\mathbb{F}_G\}_{G \in V}$  be a family of complete sub- $\sigma$ -fields of  $\mathbb{F}$  consistent with set operations.

The general theory of stochastic processes indexed by  $T$  contains two sorts of progressive, optional, predictable processes as well as two sorts of stopping introduced by  $\{\mathbb{F}_t = \mathbb{F}_{[0,t]}\}_{t \in T}$  and  $\{\mathbb{F}_t^* = \mathbb{F}(t, \cdot)^c\}_{t \in T}$  respectively. We have section theorems. When  $T$  is locally compact and  $\{\mathbb{F}_G\}_{G \in V}$  satisfies the hypothesis of conditional independence, we can define stochastic integrals of predictable processes with respect to square integrable martingales, though the projection and dual projection theories are not clear yet at the moment.

L. CHEVALIER (Grenoble)

Stochastic Calculus for Continuous Two Parameter Martingales

The talk is devoted to the proof of the following "Ito formula": if  $M$  is a continuous local (locally  $L^2$ , for example) martingale and  $f: \mathbb{R} \rightarrow \mathbb{R}$  is 4 times continuously differentiable, then, for all  $t \in \mathbb{R}_+^2$ ,

$$\begin{aligned} f(M_t) = & \iint_{[0,t]} f'(M_s) dM_s + \iint_{[0,t]} f''(M_s) d\bar{M}_s + \\ & + \frac{1}{2} \int_0^t \int_{s_1}^{s_2} f'''(M_{s_1, t_2}) d[M, M]_{s_1, t_2}^1 + \frac{1}{2} \int_0^{t_2} \int_{t_1}^{s_2} f'''(M_{t_1, s_2}) d[\bar{M}, \bar{M}]_{t_1, s_2}^2 \\ & - \frac{1}{2} \iint_{[0,t]} f'''(M_s) d[M, \bar{M}]_s - \iint_{[0,t]} f'''(M_s) d[\bar{M}, M]_s \\ & - \frac{1}{4} \iint_{[0,t]} f''''(M_s) d[\bar{M}, \bar{M}]_s \quad \text{a.s.} \end{aligned}$$

(the only non self-explanatory notation is  $\bar{M}$ , which denotes the "sum over  $\mathbb{R}_+^2$  of the products of horizontal and vertical increments of  $M$  at time  $t$ ", a martingale again). This result holds under natural conditions on the filtration  $(\mathbb{F}_t)$ ; they are fulfilled, in particular, when  $(\mathbb{F}_t)$  is associated with either the tensor product of two independent Brownian motions or the two parameter Wiener process. The proof is based upon a concrete construction of processes  $[M, M]$  and  $\bar{M}$ , and a suitable "discrete Taylor formula".

M.H.A. DAVIS (London)

Extended Generators and Multiplicative Functionals of Hunt Processes

In this talk a simple proof is presented of the fact that if  $(X_t)$  is a hunt process on a finite dimensional  $C^\infty$  manifold and the domain  $D(A)$  of its extended generator  $A$  includes all  $C^\infty$  functions, then  $A$  is in local coordinates, a "Lévy generator". The key observation is this: let  $f$  belongs to  $D(A)$  and  $M_t^f$  be the continuous local martingale part of  $M_t^f$ ,  $M_t^f = f(X_t) - f(X_0) - \int_0^t Af(X_s)ds$ , then  $\langle M^f, M^g \rangle_t = \int_0^t D(f,g)(X_s)ds$ , where  $D(f,g)$  can be expressed explicitly in terms of  $A$  and the Lévy system of  $(X_t)$ . A little stochastic calculus shows that for each  $x,g$  the map  $f \rightarrow D(f,g)(x)$  is a derivation, i.e. a linear functional satisfying the Leibnitz rule. This shows immediately that it is a first order differential operator, so that  $D$  takes the form  $D(f,g) = \sum_{i,j} a_{i,j}(x) D_i f D_j g$ , ( $D_i$  is the derivation in the  $i^{th}$  direction), the coefficients  $a_{i,j}$  essentially give us the second order part of  $A$ . The rest is plain sailing.

This argument arose in analysing the multiplicative functional

$\exp(g(X_0) - g(X_t))$ . The generator of the corresponding semigroup is  $e^g A e^{-g}$  and this can be expanded in the "Baker-Campbell-Hausdorff" formula:  $e^g A e^{-g} = \sum_{n=0}^\infty \frac{1}{n!} ad_g^n A$ , where  $ad_g A$  is the formal "Lie derivative" of  $A$  with respect to  $g$ , i.e.  $(ad_g A)f = gAf - A(gf)$

D. DURR (Bochum)

A Mechanical Model for Brownian Motion

We consider the motion of a massive convex body immersed in an ideal gas of point particles of mass  $m$  in the limit  $m \rightarrow 0$ , while the density of the point particles increases like  $m^{-\frac{1}{2}}$  and their velocity distribution is scaled like  $f_m(v) = m^{\frac{3}{2}} f(m^{\frac{1}{2}}v)$  in dim. 3. We prove an extension of Holley's one dimensional result: the process

describing the dynamical state of the convex body, converges in distribution to an appropriate Ornstein-Uhlenbeck process as  $m \rightarrow 0$

This was done in cooperation with S. Goldstein and J. Lebowitz.

R. SURRETT (Ithaca)

Splitting Intervals

In this work we consider a model in which the unit interval undergoes random subdivision. If at time  $n$  there are  $n+1$  pieces with lengths  $l_1, l_2, \dots, l_{n+1}$  we pick the  $i^{\text{th}}$  with probability proportional to  $l_i^\epsilon$  and split it into pieces of length  $l_i V$  and  $l_i(1-V)$  where  $V$  has distribution  $F$  and is independent of process at time  $n$ . The problem we will consider is the limiting behaviour of  $\frac{1}{n} N_n(x)$  where  $N_n(x)$  is the number of intervals in  $[0, x]$ . This problem has been solved when  $\epsilon = \infty$  (split the longest interval),  $\epsilon = 1$ ,  $\epsilon = 0$ , and  $\epsilon = -\infty$  (split the shortest interval). The limits of  $\frac{1}{n} N_n(x)$  are resp. uniform, uniform, random singular distribution, random point mass. In this talk, we give approach for the other values of  $\epsilon$ .

R.J. ELLIOTT (Hull)

A Two Parameter Exponential Formula

Consider a machine with two components, which run for different lengths of time. The probability space on which the first failure times  $(T_1, T_2)$ , of the two components, can be considered can be taken as the positive quadrant  $\Omega = [0, \infty]^2$ , with the product filtration. If the distribution  $m$  of  $(T_1, T_2)$  under which  $T_1$  and  $T_2$  are independent, is replaced by an absolutely continuous measure  $\bar{m} \ll m$ , the martingale

$L_{t_1, t_2} = \mathbb{E} \left( \frac{d\bar{m}}{dm} \mid \mathcal{F}_{t_1, t_2} \right)$  can be represented as

$$L_{t_1, t_2} = L_{t_1, 0} L_{0, t_2} \left\{ 1 + I_{t_1 \geq T_1} I_{t_2 \geq T_2} \frac{R(T_1, T_2)}{(1 + H(T_1, T_2))(1 + \bar{H}(T_1, T_2))} \right\} \times$$

$$\exp \left\{ -I_{t_1 > T_1} \int_0^{t_2} \frac{R(T_1, s_2)}{1 + H(T_1, s_2)} d\bar{p}_{s_2} - I_{t_2 > T_2} \int_0^{t_1} \frac{R(s_1, T_2)}{1 + \bar{H}(s_1, T_2)} d\bar{p}_{s_1} + \int_0^{t_1} \int_0^{t_2} R(s_1, s_2) d\bar{p}_{s_1} d\bar{p}_{s_2} \right\}$$

where  $H, \bar{H}, R$  can all be expressed in terms of  $L_{\omega, \omega}$ .

H. FOLLMER (Zürich)

On Dirichlet Processes and Pathwise Itô Calculus

Motivated by results of Fukushima on Dirichlet spaces, we introduce Dirichlet processes as "cadlag" processes  $(X_t)_{t \geq 0}$  in  $\mathbb{L}^2$  with "zero conditional energy" and show that this is equivalent to a decomposition  $X = M + A$  where  $M$  is a martingale in  $\mathbb{L}^2$  and  $A$  is a process "of zero energy", i.e.,  $E[\sum_{t_i \in T, t_i < t} (A_{t_{i+1}} - A_{t_i})^2] \rightarrow 0$  as the step  $|T|$  of the partition  $T$  goes to 0.

For Dirichlet processes, semimartingales, ... there is a partition scheme  $(T_n)_{n \geq 0}$  with  $|T_n| \rightarrow 0$  such that almost all paths  $x: [0, \infty] \rightarrow \mathbb{R}^1$  have the property that the point measures

$\sum_{t_i \in T_n} \epsilon_{t_i} (x_{t_{i+1}} - x_{t_i})^2$  converge to a measure with distribution function  $[x, x]_t^c = [x, x]_t^c + \sum_{s < t} \Delta x_s^2$ . For such a path, Itô's formula follows as an exercise in real analysis.

J. FRANKE (Heidelberg)

Euler Lagrange Equation for a Stochastic Optimisation Problem

We consider the stochastic optimisation problem :

$$E\{R(z - X_T) + \int_0^T S(v(t, X_t))dt\} = \min! , \quad \text{under the constraint } v \in K$$

Here,  $\{X_t, 0 \leq t \leq T\}$  solves

$$dX_t = v(t, X_t)dt + H(X_t, v(t, X_t))N(dt) , \quad X_0 = a$$

$N$  is a Poisson measure,  $R, S, H$  are given "smooth" functions,  $T$  is a fixed terminal time;  $v(t, x)$  can be chosen from the set of admissible controls  $K$ , which is a convex, closed set with non-empty interior in the space of measurable functions  $v(t, \cdot)$  on  $[0, T] \times \mathbb{R}$ , which are continuously differentiable with respect to  $x$  and for which

$$\|v\| = \text{esssup}_t \sup_x |v(t, x)| + \text{esssup}_t \sup_x |v_x(t, x)| < \infty$$

This optimisation problem serves as an illustration for the application of Dubovitskij's and Milyutin's theory in stochastic setting. As a first step, we derive the Euler-Lagrange equation as necessary condition for a  $v^0$  in  $K$  to be a local minimum of  $L$  under the



constraint  $K$ . Using theorems of Gikhman and Skorohod, we derive therefore the directional derivative of  $L$  at  $v^0$  in direction  $v$ .

B. GRIGELIONIS (Vilnius)

On Multiple Random Time Change of Semimartingales

On the probability space  $(\Omega, \mathbb{F}, P)$  with the filtration  $(\mathbb{F}_t, t \geq 0)$  of subalgebras let us consider a  $m$ -dimensional  $(P, \mathbb{F}_t)$ -semimartingale  $X_t = (X_t^1, \dots, X_t^m)_{t \geq 0}$ , having a triplet  $(\alpha, \beta, \Pi)$ , of the predictable characteristics. Conditions are found when there exists a multiple random time change  $(T_t^1, \dots, T_t^m)_{t \geq 0}$ , such that  $Y_t = (X_{T_t^1}^1, \dots, X_{T_t^m}^m)_{t \geq 0}$  has independent locally infinitely divisible components.

X. GUYON (Orsay)

A Girsanov Theorem for Two-parameter Brownian Semimartingale

We give some results on identification of the weak martingale part  $Y$  of a representable Brownian semimartingale  $Z$ . We then study the estimation of the bounded variation process  $B = Z - Y$  of  $Z$ : for this we study exponential martingales, and conformal changes of probability (i.e. that preserves F.4.). When the martingale part of  $Z$  is strong, we give a result of absolute continuity of the measures associated to  $Z$  and  $Y$ .

J. JACOD (Rennes)

Central Limit Theorem, Martingales and Semimartingales

We present an extension of the method of accompanying laws that is fitted to convergence of a sequence  $(X_n)$  of semimartingales to a process with independent increments  $X$ , in both of the following cases: either the functional convergence (i.e. convergence in distribution for Skorohod topology), or the finite dimensional convergence along a subset  $D$  of  $\mathbb{R}_+$ . In particular, if  $D$  contains only one point, this gives results on convergence of random variables. The criteria are obtained in terms of the local characteristics of the  $X_n$ 's.

We also try to examine to which extent these criteria are

necessary for obtaining functional convergence of the  $X_n$ 's . We have only very preliminary results, precisely that if the sequence  $(X_n)$  goes in distribution to  $X$  and under an additional mild assumption, then the quadratic variations  $[X_n, X_n]$  also converge in distribution to  $[X, X]$  . Most of these results are in a joint paper with A.Ktopotowski and J. Memin.

F. JONDRAŁ (Ulm)

On the Functional Derivatives of Generalized Complex Brownian Functionals

The complex white noise is interpreted as a measure space constructed over the space of tempered distributions. Starting with the Wiener-Itô decomposition of the space of functionals of complex white noise having finite variances and using the integral representation of Brownian functionals as well as the Sobolev spaces of fractional order, generalized functionals of complex white noise (also called generalized complex Brownian functionals) may be defined. By the help of the notion of functional derivatives in P. Levy's sense, we are able to compute functional derivatives of (generalized) complex Brownian functionals.

M. KOHLMANN (Bonn)

Approximation of a Partially Observed Control Problem

We describe an approximation of a partially observed control problem

$$dx_t = f(t, x_t, u(t, y))dt + g(t, x_t)dw_t \quad (\text{system's dynamics})$$

$$dy_t = h(t, x_t)dt + k(t, x_t)dw_t \quad (\text{observation})$$

$$J(u) = \min! \quad (\text{cost})$$

The approximation bases on a discretization of the available information, on which admissible controls are to depend. Using Skorohod imbedding and convergence leads to an existence result for the above problem as the limit of the discretized problems.

M. METIVIER (Palaiseau)

$\lambda$ -spaces and Strong Solutions of Differential Equations

The construction of a stochastic integral (with respect to a semimartingale, a random measure, a generalized functional, etc...) consists essentially in putting into evidence a space  $\Lambda$  of  $\mathcal{L}(\mathbb{B}; \mathbb{H})$ -valued processes ( $\mathbb{B}$  : Banach;  $\mathbb{H}$  : Hilbert) and a functional  $(\lambda_s^{(\phi)})_{s \geq 0}$  mapping  $\Lambda$  into positive progressively measurable processes, such that,  $A$  and  $\bar{A}$  being two suitable increasing positive processes,  $\Lambda$  is complete for the family of seminorms  $\phi \rightarrow [E(\bar{A}_T - \int_{[0, T[} \lambda_s^{(\phi)} dA_s)]^2$ , where  $T$  is any stopping time, and admits a dense subset of simple predictable processes  $\sum_{i=1}^n 1_{]s_i, t_i]} \times F_i a_i$  where  $a_i$  belongs to  $\mathbb{L}$ , where  $\mathbb{L}$  is a given linear subspace of  $\mathcal{L}(\mathbb{B}; \mathbb{H})$ . The quintuple  $(\Lambda; \mathbb{L}; A; \bar{A}; \lambda)$  is called a  $\lambda$ -space. For every process  $Z$  with values in  $\mathbb{B}$  such that for every  $\mathbb{L}$ -valued simple predictable  $Y$  and every stopping time  $T$  :  $E(\|\sup_{s < t} \int_{]0, s]} Y dZ\|^2) \leq E(\bar{A}_T - \int_{[0, T[} \lambda_s(Y) dA_s)$ , we consider the stochastic equation :

$$X_t = V_t + \int_{]0, t]} a_s(X) dZ_s \quad ; \text{ under a classical Lipschitz hypothesis on the functional } a \text{ (associating a process } a(X) \text{ in } \Lambda \text{ to every cadlag } \mathbb{H}\text{-valued adapted } X)$$

we prove existence and uniqueness of the strong solution on a maximal stochastic interval  $]0, T[$ , which is an explosion-time when finite. A simple condition of non explosion is given.

P.A. MEYLER (Strasbourg)

Riesz Transforms on Wiener Space

Let  $(P_t)$  be the Ornstein Uhlenbeck semigroup considered by Malliavin and Strock :  $\mathcal{E}$  being  $C(\mathbb{R}_+, \mathbb{R})$  and  $\mu$  being Wiener measure on  $\mathcal{E}$ ,  $(P_t)$  is self adjoint on  $\mathcal{E}$  with respect to  $\mu$  and satisfies  $P_t(\int_0^\infty u_s dB_s) = e^{-\frac{1}{2}t} \int_0^\infty P_t u_s dB_s$  where  $(B_s)$  is the coordinate process (i.e. standard Brownian motion) and  $(u_s)$  is predictable. Let  $L$  be the

generator of  $(P_t)$ ,  $C$  be the self adjoint operator  $-(-L)^{\frac{1}{2}}$ ,  $\Gamma$  be the square of the field operator ( $\Gamma(f,f) = Lf^2 - 2fLf$ ). It is shown that there is a norm equivalence in  $L^p$  between  $[\Gamma(f,f)]^+$  and  $Cf$ , for  $1 < p < \infty$ .

E. PARDOUX (Marseille)

### Non Linear Filtering and Smoothing

Let  $X_t$  be an unobserved Markov diffusion process, with generator  $L$ . Suppose we observe the process  $Y_t = \int_0^t h(X_s) ds + W_t$ , the Wiener process  $W$  being possibly correlated with  $X$ .

We characterize the conditional density of  $X_s$ , given  $\{Y_\theta, \theta \leq t\}$  ( $s < t$ ) via the values at time  $s$  of the solutions of two Stochastic Partial Differential Equations: a forward one, starting at time 0, and a backward one, starting at time  $t$ . In the case  $s \geq t$ , we need only the forward SPDE. The results generalize to the case where  $Y_t$  enters the coefficients of both the  $X$  and the  $Y$  equations.

H. PINSKY (Evanston)

### Stochastic Taylor Formulas and Riemannian Geometry

We consider mean-value formulas on a Riemannian manifold. In the non stochastic approach of Gray-Willmore, the mean value of a function over a geodesic sphere is defined using the exponential mapping and integration on the tangent space of the manifold. In the case of Euclidian space this formula agrees with Pizetti's formula, a power series in the radius of the geodesic sphere. Using Brownian motion on the manifold, we define a new mean-value by the exit distribution from a geodesic ball. Using an iterated version of Dynkin's identity, we generate an expansion of our mean-value, which agrees with the Gray-Willmore formula in case of constant curvature. For a surface of variable curvature, the two mean-value are different.

P. PROTTER (West Lafayette)

On the Equality of Certain Filtrations

For a Brownian motion  $B$  with  $L$  its local time at zero, let  $\mathcal{F}^\varepsilon = B + \varepsilon L$ . The minimal completed filtration of  $\mathcal{F}^\varepsilon$  equals that of  $B$  for all  $\varepsilon$  sufficiently large (e.g.,  $|\varepsilon| \geq 16$ ).

D. SURGAILIS (Vilnius)

Multiple Itô-Wiener Integrals and Self-similar Random Fields

A large new class of self-similar stationary random fields in  $\mathbb{R}^d$  is constructed via multiple Itô-Wiener integrals with respect to Poisson and general infinitely divisible random measure in  $\mathbb{R}^{d+1}$ . Various properties of such fields including ergodicity, the existence of higher order moments and the uniqueness of stochastic integral representation are investigated. New results are given about the domain of attraction of self-similar random fields subordinated to Gaussian white noise (constructed by R.L. Dobrushin) as well as of random fields introduced above. Other properties and (open) problems related to Itô-Wiener integrals are discussed.

H. WALK (Giessen)

A Stochastic Remes Algorithm

There is treated the Chebyshev approximation problem of minimizing  $\|f - (a_0 h_0 + \dots + a_N h_N)\|$  with respect to  $(a_0, \dots, a_N)$  in  $\mathbb{R}^{N+1}$ ; where  $\|\cdot\|$  denotes the max-norm on  $C[0,1]$  and where the function values of  $f$  in  $C^2[0,1]$  are observable only with random noise and the given functions  $h_0, \dots, h_N$  in  $C^2[0,1]$  satisfy the Haar condition.

A stochastic Remes algorithm, without use of an estimation of second derivatives, is proposed which yields a recursive estimation of the alternant and thus an estimation of the optimal coefficients  $a_0, \dots, a_N$  and of the minimal value which plays a key role in the investigation. For arbitrary starting points one obtains strong consistency of the estimation sequences and convergence in distribution with norming

factor  $n^{\frac{1}{2}}$  in the case of the alternant and  $n^{\frac{1}{2}}$  in the case of the coefficients and the minimal value (invariance principles).

H. ZESSIN (Bielefeld)

Remarks on Bogoljubov's Hierarchy Equations

Gradient systems in the sense of Lang are considered :

- a) At time 0 the particles will be distributed in  $\mathbb{R}^n$ ,  $n \geq 1$ , according to some suitable initial state;
- b) The motion of the particles after time 0 is completely deterministic in nature : for a particle with initial condition  $Z(0, a, \mu) = a \in \mathbb{R}^n$  acting via a two-body force  $-D\Phi$  with other particles, having initial conditions distributed over a simple Radon point measure  $\mu$ , the motion is formerly described by

$$(*) \quad \frac{d}{dt} Z(t, a, \mu) = \int_{a \neq b \in \mu} -D\Phi [Z(t, a, \mu) - Z(t, b, \mu)]$$

A detailed analysis of the corresponding BBGKY hierarchy equations, which is based mainly on the theory of Palm measures yields : the equilibrium states of (\*) are characterized as the so called rigid states. Furthermore we prove that the time evolution of a spatially homogeneous initial state converges weakly to the set of rigid states, if  $\Phi$  is a non negative, decreasing, convex pair potential.

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