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"Stochastic Analysis", 25.10. – 31.10.92

Self-Avoiding Walks

Gordon Slade, Dept. of Math. & Stat.,
McMaster University, Hamilton, Ont, Canada L8S4K1

This talk describes recent joint work with Takashi Hara, which studies the usual model of self-avoiding walks on the hypercubic lattice $\mathbb{Z}^d$ (with the uniform measure on the set of n-step self-avoiding walks), for dimensions $d \geq 5$. We prove that the leading asymptotic behaviour of the number of n-step walks is exponential, the mean-square displacement is to leading order linear in the number of steps, and the scaling limit is Brownian motion. The proof uses the lace expansion, and is computer assisted.

A law of large numbers for upcrossing measures

Motivated by applications in fatigue analysis we define for a regular function $f: I \to \mathbb{R}$ (I a compact interval) the (discrete) "upcrossing measure" $\overset{\circ}{\nu}_I(f,.)$ on $\Delta = \{(x,y) \in \mathbb{R}^2 : x < y\}$ by $\overset{\circ}{\nu}_I(f, \Delta_{xy}) = \lim_{\varepsilon \downarrow 0}$ (# upcrossings of f from $x+\varepsilon$ to $y-\varepsilon$) where $\Delta_{xy} = \{(v,w) : v \leq x < y \leq w\}$. We also define a closely related measure $\mu_I(f,.)$ on Δ which counts excursions of f and which is the continuous analog of the "rainflow matrix" used in fatigue analysis. If X is a real-valued stationary process with cadlag paths satisfying a mild integrability assumption then $t^{-1}\overset{\circ}{\nu}_{[0,t]}(X,.)$ and $t^{-1}\mu_{[0,t]}(X,.)$ both converge a.s. vaguely to the same limit $\tilde{\mu}$. In the ergodic case $\tilde{\mu}$ is deterministic and may be contrasted with the spectral measure. We compute $\tilde{\mu}$ explicitly e.g. for onedimensional diffusions and functions of Markov chains. If B is standard Brownian motion then both $\overset{\circ}{\nu}_{[0,t]}(B,.)$ and $\mu_{[0,t]}(B,.)$ divided by the local time at t converges a.s. vaguely to a deterministic limit which we identify explicitly.

Michael Scheutzow, TU Berlin

Recurrence for semimartingale reflecting Brownian motions

R. J. Williams, University of California, San Diego
(joint work with Paul Dupuis)

We prove that a sufficient condition for a semimartingale reflecting Brownian motion in an orthant (SRBM) to be positive recurrent is that all solutions of an associated deterministic Skorokhod problem are attracted to the origin. To prove this result, we construct a Lyapunov function for the SRBM. Conditions for positive recurrence obtained by Hobson and Rogers (1992) in dimension two and by Harrison and Williams (1987) for reflection matrices arising from single class queueing network models can be verified using our results.

Discretization of positive harmonic functions on Riemannian manifolds and Martin boundary

W. Ballmann, U Bonn
(joint work with F. Ledrappier)

Given a discrete subset X of a connected Riemannian manifold M, the aim is to construct a Markov chain on X with similar potential theory as the Laplacian on M. Such a construction has been given by Furstenberg and, more recently, by Lyons and Sullivan. Under certain assumptions on X they obtain a family of probabilities μ_y, $y \in M$, on X and they establish interesting properties of these measures. These results have been extended by Ancona and Kaimanovich.
The main new result about the LS-measures reads as follows:

if the geometry of M is bounded, if X is separated and if the ε-neighborhood of X is recurrent for some $\varepsilon > 0$, then the data involved in the construction of the LS-measures can be chosen such that $G(x,y) = \varkappa g(x,y)$, where $x + y$ are in X, G is the Green function on M, g is the Green function associated to the measures μ_y on X and $\varkappa > 0$ is a constant independent of x, y. Obviously, this implies inclusion of the Martin boundary of X into the Martin boundary of M.

Markov uniqueness and applications

Michael Röckner, Bonn
(joint work with T.S. Zhang)

We prove that for $\varphi \in H^{1,2}_{loc}(\mathbb{R}^d; dx)$ ~~$\varphi \neq$~~ the generalized Schrödinger operator $S = \Delta + 2\varphi^{-1}\nabla\varphi\cdot\nabla$, $Dom(S) = C_0^\infty(\mathbb{R}^d)$, has exactly one self-adjoint extension on $L^2(\mathbb{R}^d; \varphi^2 dx)$ which generates a (sub-) Markovian semigroup on $L^2(\mathbb{R}^d; \varphi^2 dx)$ ("Markov uniqueness"). This is based on our previous work where a necessary and sufficient condition on φ for this to hold was proved, but which was only verified to always hold if $d = 1$. Then we derive that Markov uniqueness also holds when $\mathbb{R}^d$ is replaced by an infinite dimensional space E and Lebesgue measure dx by a finite positive measure μ on E in a certain class, but not necessarily absolutely continuous w.r.t. a Gaussian. In this case $S = \Delta_\infty + \beta\cdot\nabla$ on bounded smooth cylinder functions where β is the logarithmic derivative of μ w.r.t. a rigging $E' \subset H \subset E$ for some Hilbert space H. As a consequence we prove that we have Markov uniqueness for all positive finite measures μ on E admitting an integration by parts formula provided H is chosen properly. We also discuss applications to show uniqueness of the corresponding martingale problem (under symmetry assumptions) and of the stochastic quantization of infinite and finite volume quantum fields.

Double intersection local times and chaos expansions

Peter Imkeller, München (with V. Perez-Abreu, J. Vives)

Double intersection local times $\alpha(x,\cdot)$ of Brownian motion W in $\mathbb{R}^d$ which measures the intersections of $W_t + x$ and W_s, $s \neq t$, can be expanded by series of multiple Wiener-Ito integrals. This way they are seen to belong to Sobolev spaces of any order less than $(4-d)/2$ with respect to the canonical Dirichlet structure on Wiener space, as long as $x \neq 0$. As $|x| \to 0$, relevant renormalizations of $\alpha(x,\cdot)$ can be read off the expansions.

Spectral estimates for non-symmetric Markov chains

J-D Deuschel, ETH Zürich (with C. Mazza)

Consider an Markov chain generated by an irreducible transition matrix $L_\beta = \{ q_\beta(x,y) \}$ of the form $q_\beta(x,y) \simeq \exp(-\beta\, V(x \to y))$ for some non negative rates $V(x \to y)$. Let $C(\beta)$ be the spectral gap associated with L_β and $\tilde{C}(\beta)$ the spectral gap associated with the symmetrized $\tilde{L}_\beta = 1/2\, (L_\beta + L_\beta^*)$, where L_β^* denotes the adjoint of L_β. We show that although in general $\tilde{c}(\beta) \le c(\beta)$, both $\tilde{c}(\beta)$ and $c(\beta)$ ~~behave~~ are logarithmically equivalent as $\beta \to \infty$. We apply the result to the convergence of time non-homogeneous processes of the annealing type.

Gaps in the spectrum of the Dirichlet and Neumann Laplacian

Pawel Kröger, Grenoble

Let $\Omega \subseteq \mathbb{R}^n$ be a bounded domain with piecewise smooth boundary. It was proved by Li and Yau in 1983 that the Dirichlet eigenvalues λ_j of the Laplacian satisfy the inequality

$$\sum_{j=1}^{k} \lambda_j \geq \frac{n}{n+2} (2\pi)^2 (\mathrm{Vol}\, B_1^n \, \mathrm{Vol}\, \Omega)^{-\frac{2}{n}} k^{\frac{2+n}{n}};$$

B_1^n stands for the unit ball in $\mathbb{R}^n$. We show that the Neumann eigenvalues μ_j satisfy the opposite inequality, i.e.,

$$\sum_{j=1}^{k} \mu_j \leq \frac{n}{n+2} (2\pi)^2 (\mathrm{Vol}\, B_1^n \, \mathrm{Vol}\, \Omega)^{-\frac{2}{n}} k^{\frac{2+n}{n}}.$$

In conjunction with heat kernel estimates we obtain the following results on the possible size of gaps in the spectrum: $\lambda_{k+1} - \lambda_k \le C \cdot \lambda_k^{5/6}$ and $\mu_{k+1} - \mu_k \le C \cdot (\mu_k^{5/6} + 1)$. The constants can be given explicitly in terms of geometric invariants of Ω. Finally, we obtain that the multiplicities of λ_k and μ_k are smaller than $C \cdot \lambda_k^{\frac{n-1}{2} + \frac{5}{6}}$ and $C \cdot (\mu_k^{\frac{n-1}{2} + \frac{5}{6}} + 1)$, resp.

On Chavel's conjecture

Krzysztof Burdzy (joint work with R. Bass)

Let $p^D(t,x,y)$ denote the transition density for reflected Brownian motion in a domain $D \subset \mathbb{R}^n$. A conjecture of Chavel says that

$$p^{D_1}(t,x,y) \geq p^{D_2}(t,x,y)$$

for all $t>0$ and $x,y \in D_1$ provided $D_1 \subset D_2$ and both domains D_1 and D_2 are convex. The conjecture is false.

Reflected diffusions and Lévy processes

Jean Bertoin (Paris VI)

Let $s: [0,\infty) \to [0,\infty)$ be a convex increasing function, and m the measure on $[0,\infty)$ given by $dm(x) = 2/s'(x)\,dx$. Consider (X,P), the regular honest diffusion process with scale function s and speed measure m, the boundary condition being that of instantaneous reflection. Then X is a Dirichlet process with decomposition $X = N - B$, where B is a standard BM and N a locally 0-energy additive functional. Moreover, N decreases when $X>0$, and it follows that N and B have the same supremum process a.s. In this sense, the canonical decomposition of X resembles that of a reflected BM. To study the pair (X,N), it is useful to introduce the underlying Lévy process $\mathcal{N} = N \circ \tau$, where τ is the inverse local time at 0 of X. The strategy is to try to shift properties of the Lévy process to the diffusion. We are led to introduce the law of X conditioned on "$\mathcal{N} \geq 0$", respectively on "$\mathcal{N} \leq 0$", and we present several connections between the initial diffusion and the conditioned ones. In particular, we get stochastic differential equations for the conditioned diffusions, where the 3-dimensional Bessel process plays an important role.

Brownian motion among decaying traps

Erwin Bolthausen, Universität Zürich

(joint work with Frank den Hollander, Utrecht)

Let Γ be the random subset of $\mathbb{R}^d$ obtained as the union of the unit balls whose centers are the points of a Poisson point process with intensity $\nu > 0$, and let β_t, $t \geq 0$, be an independent Brownian motion. The individual balls are interpreted as traps for the Brownian motion which, however, have only a finite lifetime. The lifetimes τ_i of the traps are assumed to be independent with $P(\tau_i > t) = \xi(t) \sim a t^{-\gamma}$. The union of the traps present at time t is denoted by Γ_t. σ is the trapping time $\inf\{t \geq 0 : \beta_t \in \Gamma_t\}$. The decay property of $P(\sigma > t)$ for $t \to \infty$ is investigated and on a heuristic level, the conditioned law of the Brownian path given that $\sigma > t$ is discussed. There is a critical value for γ at $2/d$ $(d \geq 3)$ for the surviving strategy.

Self attracting diffusions

Y. Le Jan. M. Cranston.

If $V(r^2)$ is a positive function on $\mathbb{R}^+$, with $V'(0) > 0$, we study the solution of the equation

$$\frac{dx_t}{dt} = v_t + \varepsilon \frac{dw_t}{dt} - \int_0^t V'(\|x_t - x_s\|^2)(x_t - x_s)\, ds \quad \text{where}$$

w_t is a Wiener process in $\mathbb{R}^d$. We show that it converges when $V(r^2) = \alpha r^2$ and apply this observation to see that $\frac{x_t}{t} \to 0$ when V has finite range. x_t is also expected to converge in that case.

Higher Order Weak Approximation of Ito Processes with Jump Component

Eckhard Platen, Canberra/Berlin

In the talk we consider discrete time approximations of Ito processes with Poissonian jump component. These approximations can be applied as numerical methods for solving corresponding SDE's. They are based on a stochastic Taylor expansion with respect to multiple Ito integrals with constant integrands. The higher order weak convergence of the proposed approximations is proved using the smoothness of the solutions of related integro partial differential equations. A short application is discussed.

Generating random trees from point processes

Götz Kersting (Frankfurt/Main)

It is well-known that one can generate a critical, binary Galton-Watson-tree by means of an exponential random walk. We present a method to produce non-binary Galton-Watson-trees from a Poisson-process in $\mathbb{R}^{+} \times \mathbb{R}^{+}$ with a suitable intensity measure. As an application we discuss the contour of the tree in case that the offspring distribution has long tails (it is no longer a Brownian excursion).

(joint work with J. Geiger, Frankfurt)

Differential Inequalities and Random-Cluster Processes

Geoffrey Grimmett (Cambridge)

Consider an interacting process on $\mathbb{Z}^d$, with parameters $\underline{J} = (J_1, J_2, \dots, J_r)$ and order parameter $\theta(\underline{J})$. If one can derive differential inequalities of the form

$$\frac{\partial \theta}{\partial J_i} \leq \alpha(\underline{J}) \frac{\partial \theta}{\partial J_j} \quad \text{for all } i,j,$$

where α is finite on (at least) the interior of the parameter space, then such inequalities may be used to obtain

(a) equality of certain critical exponents, and

(b) certain information about the shape of the critical surface.

In particular, one obtains that the critical "inverse-temperature" is a strictly monotonic function of the interaction vector $\underline{J}$.

Such differential inequalities may be proved for any random-cluster (Fortuin-Kasteleyn) process with pair or many-body interactions, so long as the parameter q satisfies $q \geq 1$. This implies, in the usual way, information about the phase transitions of Potts models in arbitrary dimensions and with general many-body interactions.

Some of this work is joint with Carol Bezuidenhout and Harry Kesten.

On the expected volume of the Wiener Sausage for a brownian bridge

M. van den Berg, Heriot-Watt University, Edinburgh

Let K be a compact set in $\mathbb{R}^2$ with non-empty interior, and let $S_K(t) = \{x \in \mathbb{R}^2 : x = B(s)+k,\ 0 \le s \le t,\ k \in K\}$ be the Wiener sausage for the standard planar brownian bridge $B(\cdot)$ with $B(0) = B(t) = 0$.
The asymptotic behaviour of the expected volume of $S_K(t)$ is investigated for $t \to \infty$. Upper and lower bounds have been obtained for the corresponding as. problem in $\mathbb{R}^m$, $m \geq 3$.

Statistics of Markov step processes observed up to suitable random times

Reinhard Höpfner, Freiburg

Consider a Markov step process $X = (X_t)_{t \geq 0}$ whose generator depends on an unknown one-dimensional parameter $\vartheta \in \Theta$. Consider all possible triplets $(\vartheta, (u_n)_n, (U_n(\cdot))_n)$ where ϑ is a point in Θ, $u_n \downarrow 0$ a sequence of real numbers, $(U_n(\cdot))_n$ an increasing sequence of random time changes, these three components being linked together by a condition $(*)$:

$(*)$ if ϑ is the true parameter, then the observation scheme $(U_n(\cdot))_n$ "stabilizes information about ϑ" at rate $(u_n)_n$.

Under weak conditions, log-likelihood ratio processes in filtered local models $(\Omega, \mathcal{F}_\infty, (\mathcal{F}_{U_n(t)})_{t \geq 0}, \{P_{\vartheta + u_n h} : h \in \mathbb{R}\})$ corresponding to triplets $(*)$ admit a decomposition (relative to P_ϑ)

$$\Lambda^{(\vartheta + u_n h)/\vartheta}(U_n(\cdot)) = h(u_n M_\vartheta(U_n(\cdot))) - \frac{1}{2} h^2 (u_n^2 \langle M_\vartheta \rangle (U_n(\cdot))) + \ldots$$

as $n \to \infty$, where M_ϑ is a locally square integrable local martingale relative to P_ϑ and $(\mathcal{F}_t)_{t \geq 0}$. Under a "homogeneity" assumption on the model, we can construct one such observation scheme with the property

$\forall \vartheta \in \Theta$, the above decomposition holds with $u_n = 1/\sqrt{n}$ and $\frac{1}{\sqrt{n}} M_\vartheta(U_n(\cdot))$ converging weakly to Brownian motion.

This is the statistical property LAN, allowing for asymptotically efficient estimation of the unknown parameters as well as for asymptotically optimal tests, based on observation of a trajectory of X over the random time interval $[\![0, U_n(1)]\!]$ as $n \to \infty$.

Schrödinger equations and diffusion theory

Masao Nagasawa (Zürich)

§1. General Theory. Schrödinger's conjecture (1932) claims

"Non-relativistic quantum mechanics must be a diffusion theory".

The conjecture has been solved in Nagasawa (1989):

"Schrödinger and diffusion equations are equivalent"

§2. Schrödinger equations with severely singular potentials

We consider the harmonic oscillator in one-dimension and analyse the influence of an additional singular potential $V(x)$.

(i) The ground state turns out to be degenerated with

$$V(x) = \frac{1}{2}\frac{r}{|x|^{2(1+r)}} - \frac{r+2}{2}\frac{1}{|x|^{2+r}} + \frac{r}{|x|^r}, \quad r > 0,$$

(ii) Add

$$V(x) = \frac{\varepsilon}{8}\frac{1}{x^2}, \quad -1 \leqq \varepsilon < 3$$

If $-1 \leqq \varepsilon < 0$, the additional potential $V(x)$ pushes up the ground state eigenvalue, and pulls down the first excited state eigenvalue.

If $0 < \varepsilon < 3$, the $V(x)$ pushes down the ground state eigenvalue, and lifts up the first excited state eigenvalue.

These astonishing phenomena are analyzed in terms of singular diffusion processes.

The cocycle property

by

Ludwig Arnold (Universität Bremen)

Let $f_0 \in C_b^{k,\delta}$, $f_1, \dots, f_m \in C_b^{k+1}$, $k \geq 1$, $0 < \delta \leq 1$. Then

$$dx = f_0(x)\,dt + \sum_{j=1}^{m} f_j(x) \circ dB^j =: \sum_{j=0}^{m} f_j(x) \circ dB^j$$

generates a C^k-flow of diffeomorphisms of $\mathbb{R}^d$, i.e. there is a version of the solution which satisfies

$$\varphi_{s,t}(\omega)x = x + \int_s^t \sum_{j=0}^{m} f_j(\varphi_{s,u}(\omega)x) \circ dB_u^j$$

and has the property that for all $\omega \in \Omega$ $\quad \varphi_{s,s}(\omega) = id$ and

$$\varphi_{s,t}(\omega) = \varphi_{u,t}(\omega) \circ \varphi_{s,u}(\omega), \qquad s,u,t \in \mathbb{R}^+$$

If $(\Omega, \mathcal{F}, \mathbb{P})$ is the canonical Wiener space and $\theta_t\omega(\cdot) := \omega(\cdot + t) - \omega(t)$ the $\mathbb{P}$-preserving flow, we can ask whether $\varphi(t,\omega) := \varphi_{0,t}(\omega)$ is a <u>cocycle</u> over θ, i.e. satisfies

$$\varphi(t+s,\omega) = \varphi(t, \theta_s\omega) \circ \varphi(s,\omega) \quad \text{identically} \qquad (*)$$

Uniqueness of solutions immediately gives $(*)$, but on a set Ω_s of full measure which depends on $s \in \mathbb{R}^+$. Can this crude cocycle be perfected? M. Bismut (1981) said 'NO'.

<u>Theorem:</u> If $k \geq 2$ then φ can be perfected.

The proof follows the one for linear vector fields given by S. Mohammed & M. Scheutzow (1990) and uses the approximation of φ by perfect cocycles φ_ε and a strong flow convergence theorem by Kunita (1990) giving $\varphi_\varepsilon \to \varphi$, hence inheriting the cocycle property from φ_ε to the limit φ.

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Foundations of Quasi Sure Analysis

Given $\mathbb{R}^N$, gaussian measure

$\mathbb{D}_\infty(\mathbb{R}^N)$ Then taking on $\mathbb{R}^N$ the product topology

① $c_{p,r}(O) := \inf \|u\|_{\mathbb{D}^p_r}$ $\quad u \geq 1\!\!1_O$ (O open

② $c_{p,r}(A) := \inf c_{p,r}(O)$ $\quad O \supset A$

(3) $\gamma(A) = \sup \theta(A)$ $\quad \|\theta\|_{(\mathbb{D}^p_r)'} \leq 1, \ \theta > 0$

Th A Borelian $\Longleftrightarrow$ $\gamma(A) = c_{p,r}(A)$

Th: $\Phi^i(\xi) := \sum_k \alpha^i_k \xi^k$ $\quad \xi \in \mathbb{R}^N$

α orthogonal mat

Then Φ^* is a quasi continuous ~~isomorphism~~ bijection, of $\mathbb{R}^N \rightarrow \mathbb{R}^N$

Th $X^* \subset H \subset X$ an abstract Wiener sp

e_k o.n basis of H, $e_k \in X^*$

Then $\psi(x) = \{\langle x, e_k \rangle_{X \times X^*}\}_{k \in N}$

$$\Psi : X \longrightarrow \mathbb{R}^N$$

Ψ^* is $qr(p, r)$ - continuous

bijection of its domain on its image

$$c_{p,r}\left((\Psi(X))^c \right) = 0$$

Paul Malliavin, Paris

See Albeverio, Fukushima, Röckner, Hansen, Ma
J.FA 1992 for related results

Wiener-Hopf factorisation of generators + applications

In fluid models of queues, as used in telecommunications applications, one describes the net workload offered to the queue by a fluctuating additive functional $\varphi_t = \int_0^t v(X_s)\,ds$ of a finite Markov chain X. In this way, the content of the queue at time t is simply $\varphi_t - \inf\{\varphi_u : u \le t\}$. To find the equilibrium distribution of the content, one needs to obtain the Wiener-Hopf factorisation of (X, φ), or, more exactly, its time-reversal. This notion was first studied by David Williams + others; one defines

$$\tau_t^{\pm} = \inf\{u : \pm\varphi_u > t\}$$

$$Y_t^{\pm} = X(\tau_t^{\pm}),$$

and seeks the generators of $Y^{\pm}$. Barlow, Rogers & Williams found a simple algebraic characterisation of these.

More recently, J.E. Kennedy + D Williams have investigated the situation where φ is perturbed by the addition of εB_t, a small Brownian noise; they obtain an algebraic characterisation of the generators. Although in the noiseless case the generators of the $Y^\pm$ processes of the reversed chain are simply related to the generators of $Y^\pm$, it appears to be different for the noisy problem. We give an expression of the generators of the $Y^\pm$ processes of the reversed process in the noisy case; the proof is probabilistic, and no algebraic proof is yet known.

L.C.G. Rogers, London.

Parabolic Martin Boundaries (joint work with K. Burdzy)

Thomas S. Salisbury
York University, Canada

Martin boundaries parametrize weak limits of conditioned Brownian motion. For elementary domains D, the parabolic Martin boundary (PMB) of Brownian motion ~~(in a Euclidean domain D)~~ may be written down, and one can ask to what extent the PMB of more complicated domains has a similar description. In all domains we have $PMB \supset \mathbb{R}\times Acc$, where Acc is the set of accessible points of the (harmonic) Martin boundary MB. If D is intrinsically ultracontractive, or is a planar domain of finite area, then $PMB = (\mathbb{R}\times MB) \cup \{\pm\infty\}$ where the additional points $\pm\infty$ close up each end of $\mathbb{R}\times MB$. Using moment estimates for the lifetimes of conditioned Brownian motion in thin Lipschitz tubes, one may however build a large catalogue of examples, illustrating ways this factorization can fail. Yet another type of behaviour, arising e.g. with $D=(0,\infty)$, is that minimal parabolic functions $h(t,x)>0$ have the form $e^{-\lambda t}\varphi(x)$ where $\Delta\varphi=\lambda\varphi$. For domains of the form $D_f=\{\underline{x};\ |(x_1,\dots,x_n)|\le f(x_0)\}$, f Lipschitz with $\int_0^\infty f(x)dx=\infty$, we conjecture that this factorization holds iff $\int_0^\infty f(x)^3dx=\infty$ too. Partial results in this direction are given

H-transform of superbrownian motion

Ludger Overbeck (Universität Bonn)

It is shown that H-transforms of superbrownian motions are the unique solutions of martingale problems of interacting superbrownian motions, with non-criticality parameter $H_x'(s,\mu)/H(s,\mu)$.

Two classes of H-transforms were identified as superbrownian motion conditioned on an event in the remote future.
Firstly, space-time harmonic functions H, which depends only on the total mass, i.e. $H(s,\mu) = \eta(s,\mu(1))$. The extremals η's are parametrized by $c \in [0,\infty) \cup \{\theta\}$ and P^{η^c} a.s. we have $\frac{X_t(1)}{t^2} \to c \in [0,\infty)$ and $\eta^\theta \equiv 1$.
Secondly, the class of additive H-transforms, $H(s,\mu) = \mu(h(s))$, h parabolic, was investigated. If $h^a(s,x) = e^{ax - \frac{1}{2}a^2 s}$ is extremal the H^a-transform is the superbrownian motion ~~which an ine~~ with an particle which moves as the h^a-transform of the Brownian motion.

On the use of Palm trees in time-stationary branching populations

Conditions are given on the decay resp. growth of the branching rate $V(x)$ (as $|x| \to \infty$) of critical binary branching Brownian particles in $\mathbb{R}^d$, which guarantee resp. prohibit the existence of a nontrivial first order equilibrium state (the moral of the story being that too quick branching far out in dimensions ≥ 3, not slow enough branching far out in dimension 2, and some branching at all in dimension 1 leads to local extinction.)
The existence parts of the result are obtained by analyzing the equilibrium Laplace transforms, whereas the non-existence parts are proved by the backward-tree representation of the Palm population

(i.e. the population of relatives) of an individual at late time. (These results were stimulated by a recent discussion with Ted Cox)

A similar "Palm tree" technique can be used to show (and this is joint with A. Stöckl, Universität Linz) that each clan in a time-stationary (constant rate) branching Brownian particle system in $\mathbb{R}^d$ $(d \geq 3)$ populates a fixed ball B in $\mathbb{R}^d$ at arbitrarily early and late times iff $d=3$ or 4; for $d \geq 5$ the first immigration and last emigration times for B are finite a.s.

Anton Wakolbinger, Universität Frankfurt

A Concise Account of the Thermodynamic Formalism

J.T. Lewis (Dublin)

This is a report on joint work with Ch. Pfister (Lausanne). We describe the thermodynamic formalism of Ruelle [1] and Lanford [2] in a general setting: E is a topological space, $\{\mathbb{K}_n\}_{n\geq 1}$ is a sequence of positive measures on the Borel subsets of E, $\{V_n\}_{n\geq 1}$ is a divergent sequence of positive real numbers; we define the Lanford entropy function $\lambda : E \to \bar{\mathbb{R}}$ for the pair $(\{\mathbb{K}_n\}, \{V_n\})$. If the Lanford entropy exists, then a vague large deviation principle holds with rate-function $-\lambda$.

When E is a Banach space, we give a condition for the Lanford entropy to exist and be given by $\lambda = -c^*$, where c^* is the Legendre-Fenchel transform of the scaled cummulant generating function $c : E^* \to \bar{\mathbb{R}}$. Motivated by Gibbs' axiomatization of thermodynamics (see Gross [3] for a modern account), we say that $s : E \to \bar{\mathbb{R}}$ is a Gibbs entropy function if s is concave and s is C^1 on the interior of dom s. We have the following, which is the basis of the thermodynamic formalism:

if $\lambda = -c^*$ and c is strictly convex, then the Lanford entropy is a Gibbs entropy.

An application to risk theory is to be found in [4].

[1] Ruelle, D.: JMP 6, 201-219 (1965) [3] Gross, L.: LNM 929 (1982)
[2] Lanford, D.E.: LNP 20 (1973) [4] Martin-Löf, A.: Scand. Actuarial J. (19..)

Passages and local time for branching Brownian motions

Ingemar Kaj, Uppsala University (joint w. P. Salminen)

Consider a realization of a one-dimensional branching Brownian motion generated by a single particle at the origin. Fix $x>0$ and count any particle visiting x as the first among the members of its line of descent from the initial particle. The resulting first-passage process indexed by $x \geq 0$ is a Markov branching process which can be characterized in terms of the original branching mechanism. In the superprocess rescaling scheme we obtain weak limits and derive the law of some local time functionals.

Schrödinger Bridges, Entropy Minimization and Markov Property

N. Gantert, ETH Zürich (joint work H. Föllmer)

Let P_x^y be Brownian bridge from x to y, ν be a prob. distr. on $\mathbb{R} \times \mathbb{R}$ and $Q = \int P_x^y \, \nu(dx, dy)$ be the mixture of the bridges with joint endpoint distribution ν. Then Q is a "reciprocal process" which has a two-sided Markov property (see, e.g. B. Jamison). If we fix two marginal distr. ν_0 and ν_1, there is a unique ν with ν_0, ν_1 s.t. $Q = \int P_x^y \, \nu(dx, dy)$ has the Markov property. In this case, we can get the Markovian Q as the solution of an Entropy-minimization problem. This variational problem comes up, if we look at large deviations from the marginals of the empirical distribution of a sequence of i.i.d. Brownian motions. In our case, the Markovian Q is even a h-path process, since the Radon-Nikodym of Q w.r.t. P splits in a factor depending only on X_0 and a factor depending only on X_1.

We replace BM with a Markov process $(X_t)_{0 \le t \le 1}$, with values in a standard Borel space X, distr. P. We show that the prob. distr. Q which minimizes $H(\cdot | P)$ with fixed initial distr. ν_0 and terminal distr. ν_1, has the Markov property. If $(X_t)_{0 \le t \le 1}$ is right-cont., the same is true if we fix, instead of ν_0 and ν_1, the whole flow of marginals $(\nu_t)_{0 \le t \le 1}$ – provided always there is Q with ν_0 and ν_1 (resp. $(\nu_t)_{0 \le t \le 1}$) and finite entropy w.r.t. P.

However, the Markovian reciprocal process need, in general, not be a h-path process. If $Q \ll P$ and $\frac{dQ}{dP} = \varphi(X_0, X_1)$, a necessary and sufficient condition for Q to have the Markov property is that for each t there are fcts. φ_0^t, φ_1^t s.t.

$$\varphi(X_0, X_1) = \varphi_0^t(X_0, X_t)\, \varphi_1^t(X_t, X_1) \quad P\text{-a.s.}$$

This weaker factorisation is, in general, not equivalent to the usual one.

M. Yor — Some recent developments in enlargements of filtrations.

Abstract: Consider $(B_t, t \le 1)$ a 1-dim BM, and define $g = \sup\{t<1 : B_t = 0\}$, and $\mathcal{F}_t = \sigma\{B_s, s \le t\}$.

One can prove (to appear in Sém. Prob. XXVI) that if (X_t) is a square integrable Brownian martingale, $X_g = 0$ iff $E[X_1 | \mathcal{F}_g] = 0$.
I devoted my lecture to finding all the random variables $X \in L^2$ such that $X_g = E[X | \mathcal{F}_g]$. This raises some interesting questions about the enlargement of $(\mathcal{F}_t)$ which makes g a stopping time.

D. Bakry — From Gross's logarithmic Sobolev inequality to Babenko's inequality.

Let $Ff(x) = \int f(y)\, e^{ixy} \frac{dy}{\sqrt{2\pi}}$ denotes the Fourier transform in $\mathbb{R}^n$.
Babenko's inequality asserts that $\|Ff\|_q \le \left(\frac{p^{1/p}}{q^{1/q}}\right)^{n/2} \|f\|_p$, where $\frac{1}{p} + \frac{1}{q} = 1$. Beckner (75) proved that this is equivalent to the inequality $\|P_{-i\pi/2 - \frac{1}{2}\log(p-1)} f\|_{q,N} \le \|f\|_{p,N}$, where P_t denotes the Hermite semigroup and $\| \ \|_{p,N}$ the L^p-norm in the standard Gauss measure.
We show that any symmetric diffusion semigroup P_t ~~with~~ wich satisfies a Gross's logarithmic Sobolev inequality c ($c = 2$ in case of Hermite semigroup) then
$\|P_{-ic\pi/4 - \frac{c}{4}\log(p-1)} f\|_{q,\nu} \le \|f\|_{p,\nu}$ with $\frac{1}{p} + \frac{1}{q} = 1$
and $\| \ \|_{p,\nu}$ is the L^p-norm for the invariant measure ν of P_t.

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M. Cranston   On Isotropic Flows

$$\text{Let} \quad C^{ij}(x-y) = \int_0^\infty \int_{S^{d-1}} e^{i\rho\langle u, x-y\rangle} (\delta^{ij} - u^i u^j)\, \sigma(du)\, F(d\rho)$$

be a covariance function where F has moments of all orders. Take $\{A_t(x)\}$, $x \in \mathbb{R}^d$ be a field of Brownian motions satisfying $E\, A_t^i(x) A_s^j(y) = t \wedge s\, C^{ij}(x-y)$. Then $\Phi_t(x) = x + \int_0^t dA_s(\Phi_s(x))$ defines a flow on $\mathbb{R}^d$. This process has been studied by Baxendale, Harris and Le Jan, among others. In my lecture I describe the behavior of geometric quantities; curvature and torsion of a curve, scalar curvature and gauss curvature of a manifold when these objects are acted on by the flow.

R. Leandre. Bismut measure, Dirac operator over the free loop space and elliptic cohomology.

That talk is a survey over 4 papers, one with J. Jones, two alone and one in preparation with J. Jones.

In the first part of this talk, I do a L^p theory of Chen-forms over the free loop space endowed with the Bismut measure. We do integration by parts over the free loop space (based over the Bismut's representation of grad $\log p_t(x,y)$) and establish a Sobolev-calculus over the free loop space. We establish a link between the Hochshild cohomology of the manifold and the exterior derivative of Chen forms.

In the second part, we propose a formalism which allows to define a "normalised" De Rham operator over the free loop space. We goes back to a limit operator over the loop space and shows that is index is the Euler-Poincaré characteristic of the Manifold. We do the same for the Dirac operator, but we cannot construct it over a neighborhood of the constant loop. The index of its limiting operator is related to the elliptic cohomology, following an idea of Taubes. (But we change the measure which allows us to extend the limiting operator)

Some results on stochastic holonomy

Classical holonomy operators are given by multiplicative integrals of connection one forms. We discuss the problem of constructing stochastic connections A and associated holonomy operators $m^{(A)}$ (in the abelian case: $m^{(A)}(C) = \int_C A$, C a ~~curve~~ curve), with distribution of A such that A is a homogeneous Markov field. We first recall the general concept of homogeneous Markov field and illustrate it by the case of scalar Euclidean homogeneous Markov fields with Gauss distribution (Nelson's free Markov field) over $\mathbb{R}^d$, and, for d=2, also with non Gaussian distribution given in terms of perturbations by additive functionals given by some "interaction density function" v.
We then describe Gaussian Markov homogeneous vector fields, like those describing the (quantized Euclidean) free electromagnetic potential field A over $\mathbb{R}^d$, d=2,4. A solves a stochastic p.d.e. which can be simply written as $\partial A = F$, exploiting the isomorphisms (as vector spaces) $\mathbb{R}^2 \cong \mathbb{C}$ resp. $\mathbb{R}^4 \cong \mathbb{H}$ ($\equiv$ the quaternions), ∂ being the conjugate of the respective Cauchy-Riemann operator. F is a $\mathbb{R}^d$-valued Gaussian white noise over $\mathbb{R}^d$. Call μ_G this distribution of A. $e^{i\int_C A}$ exists then as a non trivial element in $L^p(\mu_G)$ (for smooth ~~curves~~ loops C), for d=2, in fact a stochastic Stokes formula holds, for d=2, $\int_C A = F(n_C)$; $n_C(x)$ being the winding number of F (here we look at F as a random measure). (C. Becker '91).
Two main extensions should be mentioned: 1) For d=2 in the case of non abelian Yang-Mills fields A and smooth curves C (not necessarily closed). One has then an identification of (quantized) Yang-Mills fields with stochastic homogeneous Markov cosurfaces and homogeneous multiplicative stochastic integrals in the sense of Albeverio, Høegh-Krohn, Holden, Kolsrud ('85-'89), further studied by L. Gross, King, Sengupta, Driver ...
2) For d=4 there exists an extension to the case where μ_G is replaced by $\mu = \alpha\mu_G + \beta\mu_P$, $\alpha,\beta \geq 0$; μ_P being a suitable probability measure on $\mathcal{S}'(\mathbb{R}^4; \mathbb{R}^4)$ describing a random field of Poisson type (independent at every point), with suitable invariant Lévy-Khinchine measure. A with distribution μ is still

homogeneous and Markov and, for $\mu = \mu_p$, $e^{i\int_C A}$ exists in $L^p(\mu_p)$, $\forall 1 \le p < \infty$ (this is work by Albeverio, Høegh-Krohn, Iwata, Kolsrud, Tamura).

We close by presenting three new results concerning "Higgs random fields" i.e. pairs (φ, A), φ a vector (V-valued) random field, A a stochastic connection described heuristically by "$\exp\left(-\int_{\mathbb{R}^d} [|D_A\varphi|^2 + m^2|\varphi|^2]dx\right) d\mu(A) \prod_{x \in \mathbb{R}^d} d\varphi(x)$" with D_A the covariant derivative (A has values in the Lie algebra of some compact Lie group, represented unitary by a representation ρ in V). 1) For $d = 2$, $\mu = \mu_p$ there is a "non commutative Feynman-Kac formula" expressing $e^{-tD_A^* D_A}(x,y) = E_{0,x}^{t,y}\, tr[\rho(m_b^{(A)})]$ ($m_b^{(A)}$: multiplicative integral of A against the brownian bridge b from $(0,x)$ to (t,y), $x, y \in \mathbb{R}^2$). This then yields a representation of objects like $E(\langle\varphi(x), \rho(m_C)\varphi(y)\rangle_V)$, $x, y \in \mathbb{R}^2$, $C \in C^1([0,1]; \mathbb{R}^2)$ by brownian bridges.

2) For $d = 2$, $\mu = \mu_G$: objects like the above are ~~most~~ trivial, for interesting objects one has to renormalize, as shown in recent work by Kusuoka and myself. We prove in particular that $\beta^{t\pi} E_{0,x}^{t,y}\left(e^{i\int_{bC^{-1}} A_{\alpha,\beta}}\right)$ converges in $L^p(\mu)$ as $\alpha \downarrow 0$, $\beta \uparrow \infty$ to a nontrivial element (with $\hat{A}_{\alpha,\beta}(k) = \chi_{[\alpha \nmid \beta]}(k)\,\hat{A}(k)$, $\hat{A} \equiv$ Fourier transform of A), for all $1 < p < \infty$. The method of proof uses Malliavin calculus (fractional order Sobolev spaces on abstract Wiener spaces) and martingale methods (Burkholder, Lyons-Zheng ...)

3) $d = 4$, $\mu = \mu_p$: H. Tamura and myself showed recently the existence of $E_\mu \int_0^\infty e^{-m^2 t} E_{0,x}^{t,y}\left(e^{i\int_b A}\right)dt$ and found the behavior as $|x-y| \to \infty$. Methods include fine behavior of brownian bridges and exploitation of a Lemma of Nakao (comparing brownian bridge volumes with Wiener measure volumes).

S. Albeverio

On One-Dimensional Stochastic Differential Equations

H.J. Engelbert, Friedrich-Schiller-Universität Jena

We consider the one-dimensional stochastic differential equation without drift

(1) $$X_t = X_0 + \int_0^t b(s, X_s)\, dB_s \quad , t < S(X),$$

where $(B, \mathbb{F})$ is a Brownian motion and $S(X)$ the explosion time of X. The diffusion coefficient b is a measurable function on $[0,+\infty) \times \mathbb{R}$. We ask for conditions for existence and uniqueness of (weak) solutions. There are two types of existence results:

I. If $b(t,x)$ is a continuous function in t and $b^{-2}(t,x) \le h(x)$ where h is locally integrable then there exists a nonexploding solution $(X, \mathbb{F})$ such that the inverse of the square variation is adapted to the associated Brownian motion.

II. If $b^2(t,x)$ and $b^{-2}(t,x)$ are locally integrable then there exists an, in general, exploding solution to Equ. (1). If, moreover, the set $\{x : \sup_{0\le t\le N} b^2(t,x)\}$ has positive Lebesgue-measure then the solution does not explode. This result is due to T. Senf (Jena) and an earlier version was proven by A. Rozkosz and L. Słomiński (Toruń). The method is time change and weak convergence.

For the question of uniqueness in law we state the following conjecture: If the assumptions of I or II hold then uniqueness in law is satisfied.

For proving uniqueness in law of the solution of Equ. (1) there are two ingredients:

1) There exists a solution of the equation

(2) $T_t = \int_0^t b^{-2}(T_s, W_s)\,ds \;, \; T_0 = 0,$

which is adapted to the Brownian motion W.

2) The ordinary stochastic differential equation (2) has a solution unique in law.

Under the conditions of I and II, the first problem is solved. The second problem is left open.

Approximate travelling waves for noisy K.P.P. Equations

K. D. Elworthy (Warwick University)

Consider the generalized KPP equation with small parameter μ, as described by Freidlin:

$$\frac{\partial u^\mu}{\partial t} = \frac{1}{2}\mu^2 \Delta u_t^\mu + \frac{1}{\mu^2} c(x, u_t^\mu) u_t^\mu \qquad x \in \mathbb{R}^n$$

but with a possibly μ-dependent initial condition

$$u_0^\mu(x) = e^{-S_0(x)/\mu^2} T_0(x)$$

Freidlin showed the existence of travelling waves as $\mu \downarrow 0$ under certain conditions on c and on u_0^μ. In this report of work with H. Zhao (Academica Sinica) and A. Truman (Swansea) we will exploit the underlying classical mechanics to obtain simple proofs of some of these results and to obtain sharp estimates on the behaviour at the 'trough' of the wave. We consider various situations

(i) the basic equation, sharp estimates at the trough

(ii) superposition of several initial sources

(iii) $u_0(x) = \chi_A(x)$ (Freidlin's case)

(iv) the equation perturbed by the addition of a white noise term $\quad + \frac{\epsilon}{\mu} u_t^\mu(x)\, dW_t$

Finally, in a different direction we consider a simple case of a situation where the classical mechanics itself has to be random: this part comes from a project with R. Sowers (U.S.C.).

The solution of differential equations driven by rough signals
Terry J. Lyons (University of Edinburgh)

If X_t is a piecewise smooth path in a vector space V then the sequence $1 \oplus \int_{t>s_1>s} dX_{s_1} \oplus \cdots \int\cdots\int_{t>s_1>\dots s_n>s} dX_{s_n} \otimes \cdots \otimes dX_{s_1}$ in the associative algebra of tensors of degree at most n is in fact an element of the n-step free nilpotent group G_n over V. In general defining a choice of nth order iterated integrals corresponds to lifting the path X from V up through the projective family $V \leftarrow G_2 \leftarrow \cdots G_n \leftarrow \dots$ to G_n.

Consider the space $\Omega_\alpha^{(n)}(V)$ of α-Hölder continuous paths in G_n (where any of the natural homothetic metrics is used). We have the following
THEOREM: If $\alpha > 1/(n+1)$ then there is a unique lift of paths in $\Omega_\alpha^{(n)}(V)$ to G_j for all $j > n$ so that the paths remain α-Hölder continuous. $\Omega_\alpha^{(n)} \cong \Omega_\alpha^{(m)}$, $m > n$.
THEOREM: A C^r map $f: V \to V$ induces a continuous map of $\Omega_\alpha^{(n)}(V)$ to itself formally taking $\dots\iint dX \otimes \dots dX \to \dots \iint d f(X) \otimes \dots d f(X)$ providing that $\alpha > 1/n+1$.
Both theorems fail completely if $\alpha \leq 1/(n+1)$.
An easy lemma shows that if Brownian paths are enhanced by Levy area than with probability one they are in Ω_α^2 for $\frac{1}{3} < \alpha < \frac{1}{2}$. The above theorems apply and in particular, all iterated integrals of Brownian paths are continuous functionals of the Brownian path as an element of $\Omega_\alpha^{(2)}$.

The theorems allow one to make precise the definition of an equation such as $dY_t = f(Y_t)\, dX_t, Y_0 = y$, $y \in U$, X the projection to V of a path $\hat{X}$ in $\Omega_\alpha^{(n)}$ and $\alpha > 1/(n+1)$.

Definition. A solution to $dY_t = f(Y_t)\,dX_t$ is a lift of $\hat{X}$ to a path $\hat{Z}$ "$=\widehat{(Y,X)}$" in $\Omega_\alpha^{(n)}(U \oplus V)$ such that $(y_0 + \int_0^t f(Y_t)\,dX_t,\ X_t)$ with its iterated integrals (defined by the theorems) is $\widehat{(Y_t, X_t)}$.

The same approach allows one to perform Picard iteration. But as yet the question of convergence is in general open.

KOMBINATORIK

(1.–7. November 1992)

Some problems in combinatorial number theory

Béla Bollobás (Cambridge)

(joint work with Paul Erdős and Guoping Jin (Cambridge))

The results below are from our work in progress on Ramsey-type problems on integers.

For $A \subset \mathbb{N}$, set $\sigma(A) = \sum_{a \in A} a$ and $\Sigma(A) = \{\sigma(B) : B \subset A\}$.

Define $f_k(n) = \min\{m \in \mathbb{N} : [m] = A_1 \cup \ldots \cup A_k \Rightarrow n \in \bigcup_1^k \Sigma(A_i)\}$.

It is shown that if n is sufficiently large then

$$\lfloor 2\sqrt{n} \rfloor + 2 \le f_2(n) \le 2\sqrt{n} + 2\log_2 n.$$

We believe that there is a function $\omega(n) \to \infty$ such that $f_2(n) \ge 2\sqrt{n} + \omega(n)$; perhaps even $f_2(n) - 2\sqrt{n} = \Theta(\log n)$.

For $k \ge 3$ our results on $f_k(n)$ are not very sharp, but we can determine the asymptotic size of $f_k(n)$ for every fixed k.

We have also investigated some other functions, including

$$g_k(n) = \min\{\sigma(A) : A \subset [n-1];\ A = \bigcup_1^k A_i \Rightarrow n \in \bigcup_1^k \Sigma(A_i)\}.$$

It is easily seen that if $k \ge 2$ is fixed and n is sufficiently large then $g_k(n)$ is well defined. Furthermore, if $n \ge 12$ then, trivially, $g_2(n) \le \sum^{f_2(n)} i \le 2n + 5(\log_2 n)\sqrt{n}$. We have proved that, if n is sufficiently large,

$$\sqrt{2n}/8 \le g_2(n) - 2n \le 5(\log_2 n)\sqrt{n},$$

but we could not determine the proper order of $g_2(n) - 2n$.

It would be of interest to determine what happens if in the definitions of f_k and g_k we replace $\mathbb{N}$ be a sequence S, say by the sequence of primes.

The solution of the knight's Hamiltonian path problem

(joint work with Axel Conrad, Tanja Hindrichs and Hussein Morsy)
The following results are proved. There exists a Hamiltonian path on the $n \times n$ chessboard for the knight iff $n \geq 5$ and a Hamiltonian circuit iff $n \geq 6$ and n is even. For a given source s and a given terminal t there exists for $n \geq 6$ an s-t-Hamiltonian path iff the obvious color criterion is fulfilled. The paths can be computed in optimal linear time. The proof technique is a divide-and-conquer technique partitioning the board into small quadratic and rectangular boards. Solutions for these subboards are concatenated to a solution of the large problem. For boards up to size 11×11 several hundred s-t-problems have been solved with the help of a computer and these solutions are used by table-look-up. This approach may also explain why this problem (400 years old) has not been solved earlier.

Ingo Wegener (Univ. Dortmund)

A combinatorial identity – proofs and variations

The identity $\sum_k \binom{n}{k}^2\binom{n+k}{k}^2 = \sum_k \binom{n}{k}\binom{n+k}{k}\sum_j \binom{k}{j}^3$, which relates the Apéry-numbers on the l.h.s. with the Franel numbers, is interesting not only for its application in number theory, but also for the variety of proof techniques that may be used for approaching it: techniques from hypergeometric functions (indeed, it is a consequence of Bailey's bilinear generating function for the Jacobi polynomials), combinatorial interpretation in a non-obvious way, "mechanical" or "automatized" proofs in the sense of D. Zeilberger, but also standard use of computer algebra methods. Each proof has its own perspective and potential for generalisation and variation. Important for application, however, is the fact that the classical recurrences due to Franel (1895!) and Apéry (1978) turn out to be "legendre conjugates", which leads to a simultaneous approximation of $\zeta(3)$ and $\pi^2/8$, as observed by A. Schmidt (Copenhagen).

On some theoretical aspects of VLSI routing

In this talk I report about various aspects of local and global routing problems in VLSI design. I concentrate on mathematical models that lead to Steiner tree and Steiner tree packing problems. I discuss the polyhedral approach to these combinatorial optimization problems and explain some of the results on the facial structure of these polytopes. I conclude with a few remarks on computational experiences with this method. This is joint work with Alexander Martin and Robert Weismantel.

Martin Grötschel (ZIB and TU Berlin)

Scale isometric subgraphs of hypercubes

Call a graph G on $\{1,\dots,n\}$ l_1-graph (or scale isometric subgraph of hypercube) if $\lambda \cdot d(G) = \lambda((d_{path}(i,j)))$ isometrically embedds into a $d(H(m,2))$ for some m, λ. The min λ is the scale $\lambda(G)$, the min λ/m is the size $s(G)$. Call G l_1-rigid if above embedding is essentially unique. G is l_1-graph iff it is isometric subgraph of the direct product of copies of $\frac{1}{2}H(m,2)$ and $K_{m\times 2}$. Scale 1, 2 correspond to G be an isometric subgraph of $H(m,2)$ or $\frac{1}{2}H(m,2)$. I give some partial classifications of l_1-graphs, operations on them, bounds for $s(G)$, study l_1-rigidity etc.

Michel Deza (CNRS, Ecole Normale Sup. Paris)

The k-Satisfiability Problem and Ramsey Theory

We consider the k-Satisfiability Problem: Given a family F of r clauses $C_1, C_2, \ldots, C_r$ in conjunctive normal form of k literals corresponding to k different boolean variables of a set of n variables. Is F satisfiable?

For two given graphs G_1 and G_2 we show how the computation of the Ramsey number $R(G_1, G_2)$ can be formulated as a Satisfiability problem. For every n (number of vertices) we construct a formula F_n such that F_n is satisfiable if and only if $n < R(G_1, G_2)$.

In a similar way the computation of Van der Waerden numbers for two colours can be formulated as a Satisfiability Problem.

Ingo Schiermeyer (RWTH Aachen)

Some results on degree sequences of graphs

In the first part of the talk, bounds on the independence number of a graph are given in terms of the degree sequence. We give a new proof of a lower bound due to Favaron, Maheo and Saclé (1991) and the best possible low upper bound (in terms of the degree sequence), namely

$$\alpha(G) \leq n-k \quad \text{, where}$$

$$k := \min \{ l : (d_{l+1} \geq \ldots \geq d_n) \preceq (d_1 \geq \ldots \geq d_l)^* \}$$

where „$\preceq$" denotes dominance order and „$*$" the conjugate partition.

The second part reports on some joint work with M. Aigner (FU Berlin). Denote by $m_B(G)$ (resp. $m_M(G)$) the least number m such that there exists a weighting $f : E \to \{0,1\}^m$ of the edges of the graph G such that all the sums $\sum_{e: v \in e} f(e)$ are different (v a vertex of G, i.e. $v \in V$, $e \in E$). Here the sum is interpreted as the Boolean sum (resp. mod 2 - sum). Then for all graphs without isolated vertices and edges, the inequalities

$$m_B(G) \leq \lceil \log_2 |V| \rceil + 1 \quad , \quad m_\mu(G) \leq \lceil \log_2 |V| \rceil + 4$$

hold.

Eberhard Triesch (Bonn)

SETS OF TYPE (s,t) AND CODEWORDS FOR A PROJECTIVE PLANE (I)

Part I Let π^* be a projective plane of order n^2 containing a Baer subplane π. If π contains a set K of type (s,t), i.e. every line of π meets K in either s or t points, then a set with four intersection numbers can be constructed in π^*. More precisely, this set consists of the pts in $\pi^* \setminus \pi$ lying on the t-secants (s-secants) to K. These lines are (n^2-n)-secants to the constructed set and partition it. The s-secants (t-secants) are exterior lines. And the remaining lines of π^*, i.e. the tgts to π, meet the set in two possible numbers of pts. The known examples of sets with two intersection numbers in a projective plane (hyperovals, Baer subplanes, unitals, unions of mutually disjoint Baers, max'l arcs etc.) provide examples of the constructed sets, which can be used to constructed codewords in the code for π^* (see part II).

Marialuisa J. de Resmini (Rome)

Part II The code C of $\Pi^* = (\mathcal{P}^*, \mathcal{L}^*)$ over $\mathbb{F}_p$, where p is a prime dividing the order n^2 of the plane, is the subspace of $\mathbb{F}_p^{\mathcal{P}^*}$ spanned by the incidence vectors v^L of the L of Π^*.

The (s,t)-set $\mathcal{K}$ in the Baer subplane $\Pi = (\mathcal{P}, \mathcal{L})$ of Π^* defines two incidence vectors $v^{\mathcal{P}_s}$ and $v^{\mathcal{P}_t}$ in $F_p^{\mathcal{P}^*}$, where $\mathcal{P}_s$ is the set of points in $\mathcal{P}^* - \mathcal{P}$ that are on the lines of Π^* that meet $\mathcal{K}$ in s points, and similarly for $\mathcal{P}_t$.
Then (i) if $\frac{n}{t-s} \equiv 0 \pmod p$, both $v^{\mathcal{P}_s}$ and $v^{\mathcal{P}_t}$ are in $C^\perp$;

(ii) if $\frac{n}{t-s} \equiv \frac{k-t}{s-t} \equiv 0 \pmod p$ then $v^{\mathcal{P}_s} \in C \cap C^\perp$, with a similar statement for $v^{\mathcal{P}_t}$.

<u>Note</u>: if $v^{\mathcal{P}}$, the incidence vector of the Baer subplane, $\notin C$ (as is the case for desarguesian planes, and possibly for all others), then $v^{\mathcal{P}_t} \in C \cap C^\perp \Rightarrow v^{\mathcal{P}_s} \notin C \cap C^\perp$.

J. D. Key (Clemson, SC)

The WATERLOO-Problem (joint work with K.T. Arasu, J.F. Dillon, D. Jungnickel, S.L. Ma)

Let D be an abelian (v,k,λ)-difference set in G and let R be an abelian $(v,2,k,\lambda/2)$-relative difference set in $G \times N$, where $N = \{1, i\}$. Then R is a "lifting" or "extension" of D. The "Waterloo-Problem" is the problem to decide which (v,k,λ)-difference sets admit such liftings to relative difference sets.

<u>Main Theorem</u>: Let D be the classical Singer difference set with parameters $((q^{d+1}-1)/(q-1), (q^d-1)/(q-1), (q^{d-1}-1)/(q-1))$ that describes $PG(d,q)$. Then D is <u>never</u> extendable. The complement of D is liftable if and only if d is even. □

Parametrically, no other liftings of difference sets are known. We can prove for many of the existing series of difference sets that no liftings are possible. However, the case of the complements of the Paley-difference sets is still undecided.

Alexander Pott (Gießen)

On Symmetry Classes of Plane Partitions

A plane partition is a subset π of $\mathbb{N}^3$ s.t. $(i,j,k)\in\pi$, $i'\le i$, $j'\le j$, $k'\le k$ implies $(i',j',k')\in\pi$. The group S_3 acts on $\mathbb{N}^3$, hence also on the set of plane partitions. Fix $a,b,c\ge 0$ and define $B:=\{(i,j,k)\in\mathbb{N}^3 : i<a,\ j<b,\ k<c\}$. For $G\le S_3$, let $N_G(B):=\#$ G-invariant plane partitions $\pi\subseteq B$. [Assume B is G-invariant; otherwise $N_G(B)=$ undefined.]

<u>Theorem 1</u>: If $G\le S_3$, then $N_G(B)=\prod_{x\in B/G}\frac{r(x)+1}{r(x)}$,

where $B/G=$ a set of orbit representatives for G on B, and $r(i,j,k)=i+j+k+1$.

<u>Remark</u>: The cases $G=\{1\}$, $G=S_2$, $G=C_3$ are due to MacMahon, Andrews-Gordon-Macdonald, and Andrews, resp. The case S_3 is new.

Following Macdonald and Stanley, we define, for each $G\le S_3$:

$$N_G(B,q)=\sum_{\pi\subseteq B/G} q^{|\pi|},\qquad P_G(B,q)=\prod_{x\in B/G}\frac{1-q^{|Gx|\cdot(r(x)+1)}}{1-q^{|Gx|\cdot r(x)}},$$

$$N'_G(B,q)=\sum_{\pi\subseteq B/G} q^{|\pi/G|},\qquad P'_G(B,q)=\prod_{x\in B/G}\frac{1-q^{r(x)+1}}{1-q^{r(x)}},$$

where both sums range over all G-invariant $\pi\subseteq B$, $|\pi/G|=\#$ of G-orbits of π, and $|Gx|=$ size of the G-orbit of x. Obviously, $N_G(B,1)=N'_G(B,1)=N_G(B)$ and $P_G(B,1)=P'_G(B,1)$. Theorem 1 asserts that $N_G(B,1)=P_G(B,1)$. It is known that

$N_G(B,q)=P_G(B,q)$ for $G=\{1\}, S_2, C_3$

$N'_G(B,q)=P'_G(B,q)$ for $G=\{1\}, S_2$, and conjecturally S_3.

It is also known that $N'_{C_3}(B,q)\ne P'_{C_3}(B,q)$, $N_{S_3}(B,q)\ne P_{S_3}(B,q)$.

For $\pi\subseteq B$, the complement of π is the plane partition $\pi^c:=\{(i,j,k)\in B : (a-i-1, b-j-1, c-k-1)\notin\pi\}$. The map $\pi\mapsto\pi^c$ defines an additional symmetry of the set of plane partitions $\subseteq B$. Let $\Gamma=S_3\times K=S_3\times\{1,c\}$, a symmetry group of order 12. Following Stanley, it is natural to consider the number of G-invariant plane partitions $N_G(B)$ for every subgroup G of Γ.

<u>Theorem 2</u>: If $G\le S_3$, then $N_{G\times K}(B)=N'_G(B,-1)=P'_G(B,-1)$.

If $G \leq \Gamma$ is isomorphic to a subgroup H of S_3, but is not itself contained in $\{(g,1) : g \in S_3\} \subseteq \Gamma$, we say that G is a 'twist' of H.

Theorem 3: If G^* is a twist of $G \leq S_3$, then

$$N_{G^*}(B) = N_G(B,-1) = P_G(B,-1).$$

Remarks. 1) Every subgroup of Γ is covered by Theorems 1-3.

2) The cases $G = \{1\}$, S_2, C_3 of Theorems 2 and 3, as well as the equalities $N_{G\times K}(B) = P_G'(B,-1)$ and $N_{G^*}(B) = P_G(B,-1)$ in the case $G = S_3$ are a synthesis of previous work by Andrews, Gordon, Kuperberg, Macdonald, Mills-Robbins-Rumsey, Proctor and Stanley, but the unified presentation we give here is new.

3) The equalities $\cancel{N_{G\times K}(B) =}$ $N_G(B,-1) = P_G(B,-1)$ and $N_G'(B,-1) = P_G'(B,-1)$ for $G = S_3$ are new.

John Stembridge

University of Michigan

Finite Ramsey Problems

For the classical Ramsey numbers $r(G,H)$ and for the following variations it is reported on all exact values: (1) Ramsey numbers for sets of graphs. - (2) Ramsey numbers for two-colored complete bipartite graphs. - (3) Ramsey numbers for two-colored cube graphs. - (4) Ramsey numbers for two-colorings of diagonals of convex n-gons. - (5) The Esther Klein problem in the plane, in the projective plane, for pseudolines, and for drawings of complete graphs in the plane.

Heiko Harborth

(BRAUNSCHWEIG)

Combinatorics, Determinants and Addition Theorems
(on joint work with W.H. Burge)

Recently we have discovered a number of identities of which the following are typical example

$$\det\left(\binom{x+i+j}{2i-j}+\binom{y+i+j}{2i-j}\right)_{0\le i,j\le n-1}$$

$$=\det\left(\frac{2}{x+1-y}\left\{\binom{i+j+x+1}{2i-j+1}-\binom{i+j+y}{2i-j+1}\right\}\right)_{0\le i,j\le n-1}$$

$$=\prod_{k=0}^{n-1}\Delta_{2k}(x+y),$$

where $\Delta_{2k}(u)$ is a simple quotient of various rising factorials in u. Our results can be used (among other things) to provide new proofs of the Totally Symmetric Self-Complementary Plane Partitions Theorem.

The proofs of such results depend in the final analysis on hypergeometric series identities for a balanced ${}_4F_3$.

When $x=y$, Mills-Robbins-Rumsey have nice combinatorial interpretations of these identities.

Is there a combinatorial explanation of the fact that these determinants are, in fact, polynomials in $x+y$?

This work will appear under the title Determinant Identities, Pac. J. Math., late 1992 or 1993.

George E. Andrews
Penn State Univ.

TOTAL POSITIVITY AND GENERALIZED SYMMETRIC FUNCTIONS

We point out that totally positive matrices arise often in combinatorics and that there is an intimate connection between them and some generalizations of the classical symmetric functions. More precisely, we show that many of the familiar matrices arising in combinatorics, as well as in the theory of symmetric functions, and many of their generalizations, have remarkable total positivity properties, and that, conversely, any totally positive matrix can be realized as a matrix of (suitably defined) generalized complete homogeneous symmetric functions evaluated at nonnegative real numbers. We also obtain a characterization of totally positive matrices in terms of planar digraphs. The method that we use to prove these results is completely combinatorial and has its origin in a technique for counting non-intersecting paths in directed graphs first used by Lindström (though he used it for completely different purposes). We use some variations and generalizations of it.

Francesco Brenti
(PERUGIA)

PLANAR GRAPH COLORING WITH AN UNCOOPERATIVE PARTNER

(Tom Trotter - ASU and Bellcore   Hal Kierstead ASU)

A graph G is to be colored with colors from a finite set $\{1, 2, \ldots, t\}$. On each round, a partner (enemy) may interfere and color (legitimately) a previously uncolored vertex. If the partner colors, then we get to color the next vertex. The <u>game</u> <u>chromatic</u> <u>number</u> $\chi_g(G)$ is the least t for which we are guaranteed to be able to color the graph regardless of how and when our uncooperative partner acts.

<u>Theorem 1</u>. The game chromatic number of a planar graph is at most 33.

<u>Theorem 2</u>. There exists a planar graph with game chromatic number 8.

<u>Theorem 3</u>. For every proper minor closed class Γ of graphs there exists a constant $c = c_\Gamma$ so that $\chi_g(G) \le c \;\; \forall\, G \in \Gamma$

The proofs of Theorems 1 and 3 use the following graph theoretic terminology. Given a linear order L on the vertex set V of G, define $bd(x; L) = |\{y \in V : y < x$ in L and $xy \in E\}|$; $bd(L) = \max_{x \in V} bd(x; L)$ and $bd(G) = \min_L bd(L)$. Erdős and Burr (1976) conjectured that $\forall\, d \;\; \exists\, c$ so that if $bd(G) \le d$, then every red/blue coloring of the edges of K_{cn} yields a monochromatic copy of G. Chvatal, Rödl Szemerédi and T proved this when $\Delta(G) \le d$ and Chen and Schelp extended it to the class of planar graphs. We modified their results as follows.

Define admissibility $ad(x; L) = \max\{S \subseteq V :$ $y \in S \Rightarrow y \le x \in S$, if $S' = S - N(x) \ne \emptyset$, then $\exists\, |S'|$ neighbors of x $u_1, u_2, \ldots, u_{|S'|}$ with $u_i > x$ in L and a labelling $y_1, y_2, \ldots, y_{|S'|}$ of S' so that $u_i y_i \in E$. Admissibility of $L = \max ad(x; L)$ and admissibility of $G = \min_L$ admissibility of L.

<u>Theorem 4</u>. $\chi_g(G) \le 1 + \chi(G)$ admissibility of G

<u>Theorem 5</u>. ~~The~~ Admissibility of a planar graph is at mo[illegible] (T16[illegible])

Maximal partial planes

A partial plane $(\mathcal{P}, \mathcal{L})$ of order $n \geq 1$ consists of a set $\mathcal{P}$ of n^2+n+1 points and a set $\mathcal{L}$ of subsets of $\mathcal{P}$, called lines, such that
(1) every line has precisely $n+1$ points, and
(2) distinct lines have at most one line in common.
The partial plane $(\mathcal{P}, \mathcal{L})$ is called maximal if $\mathcal{L}$ can not be extended, that is, if there is no partial plane $(\mathcal{P}, \mathcal{L}')$ with $\mathcal{L} \subsetneq \mathcal{L}'$. It is know that a maximal partial plane $(\mathcal{P}, \mathcal{L})$ satisfies

$$\lfloor \tfrac{3}{2}n+2 \rfloor \leq |\mathcal{L}| \leq n^2+n+1$$

The lower bound was proved by Füredi + Spissich and can always be achieved. Partial planes with n^2+n+1 lines are projective planes. In this talk the following result is presented

For $n \geq 15$, a maximal partial plane that is not a projective plane has at most n^2+1 lines.

Moreover the maximal partial planes with n^2+1 lines fall into three classes. The partial planes of two of these classes are related to projective planes, and those of the third class are related to signed Biplanes.

Klaus Metsch
TU Eindhoven

Applications of the Theory of Topological Semigroups to combinatorics. (In collaboration with Vitaly Bergelson.)

It has been known since 1976 that whenever $\mathbb{N} = \bigcup_{i=1}^{r} A_i$, there is some i such that A_i contains $FS(\langle x_n \rangle_{n=1}^{\infty}) \cup FP(\langle y_n \rangle_{n=1}^{\infty})$ for some sequences $\langle x_n \rangle_{n=1}^{\infty}$ and $\langle y_n \rangle_{n=1}^{\infty}$. (Here $FS(\langle x_n \rangle_{n=1}^{\infty}) = \{\sum_{n \in F} x_n : F$ is a finite nonempty subset of $\mathbb{N}\}$, and $FP(\langle y_n \rangle_{n=1}^{\infty}) = \{\prod_{n \in F} y_n : F$ is a finite nonempty subset of $\mathbb{N}\}$.) However the only proof has utilized the algebraic structure of $\beta\mathbb{N}$, the Stone-Čech compactification of $\mathbb{N}$. We provide now an elementary proof of this and several stronger facts.

ON THE MATCHING COMPLEXES OF BIPARTITE GRAPHS

The matchings in a bipartite graph form a simplicial complex, which in many cases has strong structural properties. We use an equivalent description as "chessboard complexes": the simplicial complexes of all non-taking rook-positions on chessboards of various shapes.

In this talk we present a (simple) proof that the chessboard complexes $\Delta_{m,n} \cong M(K_{m,n})$ are vertex decomposable (in the sense of Billera & Provan), hence shellable, for $n \geq 2m-1$. More generally, the $\left\lfloor \frac{m+n-2}{3} \right\rfloor$-skeleton of $\Delta_{m,n}$ is always vertex decomposable. This sharpens results and answers a question of Björner, Lovász, Vrećica & Živaljević (1991).

Günter M. Ziegler
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Order Series of Partially Ordered Sets

The order polynomial of a poset is an enumerative invariant introduced by Stanley, which has many connections with enumeration problems in combinatorics, algebra, and geometry. We introduce a "multi-analogue" of the order polynomial with some interesting combinatorial properties. The series $E_P(t,\underline{X})$ in variables t and $\underline{X} := \{X_a : a \in P\}$ contains the order polynomial essentially as its multilinear term $[\underline{X}^P]\, E_P(t,\underline{X})$. Given a poset P and a collection $\underline{Q} := \{Q_a : a \in P\}$ of pairwise disjoint posets indexed by P, the E-series of the composite poset $P[\underline{Q}]$ can be calculated (by an explicit formula) from the corresponding series for P and each Q_a $(a \in P)$. Two posets P and Q have isomorphic comparability graphs if and only if $E_P(t,\underline{X}) = E_Q(t,\underline{X})$ up to relabelling the $\underline{X}$ variables. For any poset P, the series $E_P(t,\underline{X})$ is a rational power series in $\mathbb{Z}(t,\underline{X})$. If P is known to have $\mathrm{width}(P) \le k$ then this series can be computed in polynomial time as a function of $|P|$. Finally, for an interval poset P we give an explicit product formula for $E_P(t,\underline{X})$.

David G. Wagner
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(P.S. See "Order Series of Labelled Posets", to appear sooner or later at a Journal near you.)

Some infinite families of (new) large sets of t-designs.

(Kramer, Magliveras, O'Brien)

Let t, v, k and λ be the parameters of a t-(v,k,λ) design $(X, \mathcal{B})$, so that $|X| = v$. Let $\binom{X}{k}$ denote the collection of k-subsets of X. A large set $LS[n](t,k,v)$ is a partition of $\binom{X}{k}$ into n desjoint t-(v,k,λ) designs. A recent result of Q.R. Wu generalizing results of L. Teirlinck and of Khosrovshahi, Ajoodani-Namini states: Thm $LS[n](t,k,v_1)$, $LS[n](t,k,v_2)$, $LS[n](k-2,k-1,v_1-1)$, $LS[n](k-2,k-1,v_2-1) \Rightarrow$ the existence of a large set $LS[n](t,k,v_1+v_2-k+1)$

A corollary of this theorem is of course:

Corol. $LS[n](t,k,v)$, $LS[n](k-2,k-1,v-1) \Rightarrow LS[n](t,k,v+m(v-k+1))$.

Joint work with E.S. Kramer and O'Brien (Eamonn) results in the construction of the following large sets $LS[11](2,4,14)$; $LS[5](3,4,13)$ and $LS[3](4,5,13)$. Since an $LS[3](4,6,14)$ is previously known, These "small" large sets imply the existence of $LS[11](2,4,14+11m)$; $LS[5](3,4,3+10m)$; $LS[3](4,6,5+9m)$ and $LS[3](4,5,4+9m)$. Among the 4-$(v,6,\lambda)$ designs all are new except for the 4-$(23,6,57)$.

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A Public key cryptosystem from logarithmic signatures of finite permutation groups. (Magliveras, Stinson, Tran van Trung)

A symmetric key cryptosystem based on logarithmic signatures for finite permutation groups was described by S. Magliveras in the late 1970's. We presently describe how in principle logarithmic signatures can be used to construct a new type of public key cryptosystem. The new system relies on the fact that there exist non-transversal logarithmic signatures which can be written as the functional composition of a small number of transversal logarithmic signatures.

Since transversal logarithmic signatures can be inverted efficiently, while non-transversal ones can not, a new type of a trap door system can be constructed.

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Finite group actions, subgroups of finite index in free products and asymptotic expansion of $e^{P(z)}$

The solution of several problems centering around finite group actions and arithmetic properties of subgroup counting functions was announced.

i) Given any finite group G one has a full asymptotic expansion for the number of G-actions on an n-set which is explicit in the order $m = |G|$ and the number $s_G(d)$ of subgroups of index d in G, $d | m$. The asymptotic behavior of these functions has been studied quite intensively since the early 50's and the best result so far was Wilf's (rather complicated) asymptotic formula for cyclic G [BAMS 86]. In fact, our result for finite group actions is a special case of a more general expansion for the coefficients α_n of entire functions $\exp(P(z)) = \sum \alpha_n z^n$, where $P(z)$ is a real polynomial.

ii) Until recently not much was known about arithmetic properties of the function $s_\Gamma(n)$ counting the subgroups of index n in a fin. gen. virtually free group Γ. The asymptotic expansion described above in conjunction with other techniques now allows to obtain a full asympt. expansion of the fct. $s_\Gamma(n)$ and an explicit growth estimate on the difference function $s_\Gamma(n+1) - s_\Gamma(n)$, at least in the case when Γ decomposes as a free product. Moreover, it is possible to describe in terms of structural invariants exactly when two groups Γ_1 and Γ_2 have (asymptotically) the same number of subgroups of index n and one can show that these combinatorial data (given by the asymptotics of $s_p(n)$) already determine Γ up to finitely many isomorphism classes.

Thomas Müller
(Bielefeld)

On the Regularity Lemma

The Regularity Lemma of Szemerédi is a result that asserts that every graph can be partitioned in a certain regular way. This result has numerous applications, but its known proof is not algorithmic. Here we first demonstrate the computational difficulty of finding a regular partition: Deciding if a given partition of an input graph satisfies the properties guaranteed by the lemma is co-NP complete. However, despite this difficulty, the lemma can be made constructive. For any n-vertex graph a partition with the properties guaranteed by the lemma can be found in time $O(M(n)) = O(n^{2.376})$ ($M(n)$ is the time required to multiply two $n \times n$ 0,1-matrices over $\mathbb{N}$). The algorithm can be parallized and implemented in NC^1. This algorithm supplies efficient sequential and parallel algorithms for many problems. (joint work with N. Alon, R.A. Duke, V. Rödl and R. Yuster).

Hanno Lefmann
(Univ. Dortmund)

More Designs in Codes and in Cosets, too.
(George Kennedy, NSA, Maryland & Vera Pless, Univ of ILL-Chicago)

Our aim is to find more designs in codes. A code C is called formally self-dual (f.s.d.) if the weight distribution of C equals the weight distribution of $C^\perp$. The only f.s.d. codes with all non-zero weights divisible by a number $\delta > 1$ are binary f.s.d. codes with $\delta = 2$.

Theorem. If C is an extremal f.s.d. even $[2n,n]$ code, then vectors of a fixed weight in $C \cup C^\perp$ hold a 3-design whenever $2n \equiv 2 \pmod 8$ and a 1-design whenever $2n \equiv 6 \pmod 8$.

Examples of such codes are $(10,5,4)$ and $(18,9,6)$ f.s.d. even codes. We show, using the designs contained in these codes and their duals, that any two $(10,5,4)$ codes are equivalent and that any two $(18,9,6)$ codes are equivalent.

Using the Pascal triangle for designs we are able to demonstrate the following.

Theorem: Let C be a $[2n,k]$ even code with $2n \equiv 2 \pmod 4$ such that the vectors of weights $n-1$ and $n+1$ hold complementary t-designs. Then the vectors of weight n in a coset of weight 1 hold a t-design when t is odd and a $(t-1)$-design when t is even.

We are able to use this design in the coset to extend the design in the code.

We are also able to find t-designs in the shadow of a type I self-dual code when vectors of a fixed weight in the code hold designs. These can also be used to extend the designs "held" by vectors in the code.

Both types of designs in codes are illustrated for the "baby Golay" $(22,11,6)$ code.

Vera Pless, Chicago, U.S.A.

A Counterexample to Borsuk's Conjecture

Jeff Kahn & Gil Kalai

Define $f(d)$ to be the least s such that any bounded set in $\mathbb{R}^d$ can be covered by s sets of smaller diameter. "Borsuk's Conjecture" of 1933 was that $f(d) = d+1$. We show $f(d) > (1.1)^{\sqrt{d}}$.

Was machte Nicolo Tartaglia in der Nacht zum Aschermittwoch des Jahres 1523 in Verona?

Seinem „General Trattato" (Teil 2, Venedig 1556, folio 17 recto) zufolge löste er in dieser Nacht ein kombinatorisches Problem. Er beobachtete nämlich in der Karnevalszeit einen Schwarm Jugendlicher sowie Leute reiferen Alters, wie sie durch das Werfen von drei Würfeln das sogenannte Buch des Glücks des Lorenzo Spirito (il libro (detto) della ventura di Lorenzo Spirito) befragten über solche Dinge, über die das Buch vorgibt, Auskunft zu geben. In diesem Buch fand er die 56 Wurfmöglichkeiten aufgelistet, von denen er sagt, daß sein Autor sie durch Experimentieren herausgefunden hatte. Er, Tartaglia, wollte nun mathematisch beweisen, daß 56 die korrekte Anzahl sei. Dabei verallgemeinerte er sogleich und fragte nach der Anzahl der Würfe, die man mit k Würfeln machen könne. Über dieses Problem dachte er die ganze Nacht zum Aschermittwoch über nach. Am Morgen hatte er die Lösung. Er gibt sie in Form einer Tabelle, die er kommentiert, wobei ich seine Argumente nicht ganz verstanden habe. Die Tabelle aber zeigt deutlich sein Vorgehen. Er fragt nämlich, so sage ich hier, nach der Anzahl der Würfe, die man mit k Würfeln machen kann, so daß die höchste Augenzahl gleich $i \in \{1, 2, \ldots, 6\}$ ist. Natürlich soviele, wie man mit $k-1$ Würfeln machen kann, so daß die höchste Augenzahl kleiner oder gleich i ist. Dies ergibt dann folgende Tabelle.

per 1 dato	1	1	1	1	1	1
per 2 dati	1	2	3	4	5	6
per 3 dati	1	3	6	10	15	21
per 4 dati	1	4	10	20	35	56
per 5 dati	1	5	15	35	70	126
per 6 dati	1	6	21	56	126	252
per 7 dati	1	7	28	84	210	462
per 8 dati	1	8	36	120	330	792

Es ist klar, daß hier ein Streifen des arithmetischen Dreiecks der Binomialkoeffizienten entsteht. Das arithmetische Dreieck kommt in Tartaglias Buch 100 Seiten später im Zusammenhang mit dem binomischen Lehrsatz noch einmal vor, ohne daß der Autor einen Zusammenhang herstellt.

Dem libro (detto) della ventura bin ich auch nachgegangen. Das einzige auf der Welt noch nachweisbare Exemplar liegt in der Stadtbibliothek von Ulm. Wer mehr über diese Geschichte wissen möchte, konsultiere mein Buch: „Leonardi Pisani liber abbaci oder Lesevergnügen eines Mathematikers." B.-I. Wissenschaftsverlag, Mannheim 1992.

Heinz Lüneburg (Kaiserslautern)
5.11.1992

On the variance of a partially ordered set

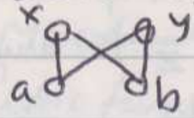
Let P be a finite poset. A function $x: P \to \mathbb{R}$ is called an optimal representation if $\frac{1}{|P|} \sum_{p \in P} x^2(p) - \left(\frac{1}{|P|} \sum_{p \in P} x(p) \right)^2$ is minimal under the conditions $x(q) - x(p) \geq 1$ whenever $q > p$. The minimal value of the objective function is called variance of P. We present a polynomial algorithm having a Max-Flow-Algorithm as a procedure which determines an optimal representation.
The poset P is said to be rank compressed if it is ranked and the rank function is an optimal representation. We give a lattice-theoretic overview on rank compressed posets and prove (joint work with S.L. Bezrukov) that the ordinal sum, direct product and ordinal product of rank compressed posets is rank compressed. Finally we provide examples which show that in general rank compression is not preserved by the direct sum, rankwise direct product, and exponentiation.

Konrad Engel
(Universität Rostock)

Möbius Function Identities and Planar Posets

Let P be a finite poset with n elements, and let $\mathbb{Q}(P)$ be the field of rational functions in P, regarded as a set of indeterminates. Let $\mathcal{L}(P)$ denote the set of linear extensions of P, and for $\varphi \in \mathcal{L}(P)$ define $\omega(\varphi) = \prod_{i=2}^{n} (\varphi^{-1}(i) - \varphi^{-1}(i-1))^{-1} \in \mathbb{Q}(P)$.
If P is connected and planar, then

$$\sum_{\varphi \in \mathcal{L}(P)} \omega(\varphi) = \prod_{a < b} (b-a)^{\mu(a,b)} .$$

where μ denotes the Möbius function of P. If P is disconnected, the corresponding sum is zero. If P is not planar, the result is false; for example, taking $P =$  we have

$$\sum_{\varphi \in \mathcal{L}(P)} w(\varphi) = \frac{x+y-a-b}{(x-a)(x-b)(y-a)(y-b)}$$

(Here a poset is defined to be planar if, after adjoining a 0 and 1 the Hasse diagram is planar.) We raise two questions: (1) is there a proof (or explanation) based on the classical theory of Möbius functions; and (2) is there an extension to non-planar posets. The problem arose from a question involving characters of S_n and Young's seminormal representations.

Curtis Greene
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Schur functions: theme and variations

An (incomplete) survey of various generalizations, analogues and deformations of Schur functions: in particular:–

(a) Let $(a_n)_{n\in\mathbb{Z}}$ be any sequence of elements of a commutative ring R, and let

$$(x|a)^r = (x+a_1)(x+a_2)\dots(x+a_r)$$

for each $r \geq 0$. Let $x_1, \dots, x_n$ be independent indeterminates over R, and for each partition $\lambda = (\lambda_1, \dots, \lambda_n)$ of length $\leq n$ let

$$S_\lambda(x|a) = \det\left((x_i|a)^{\lambda_j+n-j}\right) / \det\left((x_i|a)^{n-j}\right)$$

($n \times n$ determinants). These "Schur functions" are (non-homogeneous) symmetric polynomials in $x_1, \dots, x_n$ (with coefficients in R), which include as special cases the factorial Schur functions of Biedenharn and Louck (take $a_r = 1-r$), the α-paired factorial Schur functions of the same authors (take $a_r = (r-1)\alpha + (r-1)^2$), and the classical Schur functions (take $a_r = 0$). All the determinantal formulas (Jacobi–Trudi, Giambelli, etc.) for classical Schur functions have their analogues in the present context.

(b) Let F be a finite field of q elements, let $x_1, \dots, x_n$ be independent indeterminates over F and let λ as before be a partition of length $\leq n$. Define

$$S_\lambda = \det\left(x_i^{q^{\lambda_j+n-j}}\right) / \det\left(x_i^{q^{n-j}}\right)$$

which is a polynomial in $x_1, \dots, x_n$ stable under the group $GL_n(F)$. Again the determinant formulas for Schur functions have their analogues for these S_λ.

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Connection coefficients for the symmetric group and symmetric functions

In unpublished work, I. G. Macdonald has considered a new basis for the algebra of symmetric functions which is related by Lagrange inversion to the complete symmetric functions. The connection coefficients for this basis are equal to the connection coefficients for conjugacy classes in the symmetric group in an extreme case. We (this work is joint with D.M. Jackson) give a constructive proof of this equality which makes clear the role of Lagrange inversion, based on the set of edge-rooted two-coloured plane trees with given degree distributions in the two colour classes of vertices.

This result allows us to transform combinatorial factorization problems in the symmetric group to algebraic problems of expanding symmetric functions with respect to Macdonald's basis. A theorem is proved which allows us to carry out such expansions for some general classes of symmetric functions. This procedure is applied to solve a number of combinatorial factorization problems.

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The Complexity of the Permanent

An old question of Polya (1913) is, when is it possible to compute the permanent of a 0-1 matrix A by changing some of the $+1$ to -1 giving matrix B so that $\det B = \operatorname{per} A$. Call such a ~~mod~~ change a modification. Valiant (1979) shows computing per A is #P-hard. Thomassen (1992) shows no matrix with just 4 entries in each row and column has a modification. I show a connection between the two results by proving

Theorem: Computing the permanent of a matrix with entries from $\{0, 1, -1\}$ and exactly 4 non zero entries in each row and column is #P-hard.

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THE FIBONACCI INFINITE WORD

The Fibonacci (finite) words are given by:

$f_1 = a$, $f_2 = ab$, $f_{n+2} = f_{n+1} f_n$.

The Fibonacci infinite word f is the "limit" of this family of finite words: $f = aba\,ab\,aba\,aba\,ab\,aba\,ab\,aba$ ----

Let $F(f)$ be the set of factors of f; let ϕ be the golden ratio $\frac{\sqrt{5}+1}{2}$; let $\{a,b\}^+$ denote the free semigroup on $\{a,b\}$ and let $|w|$ denote the length of a word $w \in \{a,b\}^+$.

PROPOSITION (PIRILLO). $\forall w \in \{a,b\}^+$ $\forall r > \frac{2+\phi}{2}$ $(w^r \in F(f) \Rightarrow \exists n \ |w| = |f_n|)$.

Several known results on the word f are consequences of the above proposition.

(GIUSEPPE PIRILLO) Giuseppe Pirillo
IAMI CNR
FIRENZE (ITALIA)

Waterfalls, antichains and capacities of digraphs

We investigate the asymptotic growth of the largest induced subgraph — in co-normal powers of a simple graph — on which non-adjacency is an equivalence relation (equivalence graphs). For directed graphs we introduce an analogous graph product and analyse the directed version of the problem, the asymptotic growth of induced subgraphs corresponding to a particular orientation of the edges, called waterfalls. A waterfall is an equivalence graph with linearly ordered equivalence classes in which every edge is present with an orientation pointing away from the class which comes first in the order. We give an upper bound on the corresponding graph capacity value and apply it to show that the cardinality of the largest waterfall is multiplicative for several classes of graphs. These include bipartite graphs and all cycles.
The behaviour of another new digraph capacity concept, applicable as a common framework of some extremal set theory questions, is also discussed.

This is joint work with Anna Galluccio, Luisa Gargano and János Körner.

Gábor Simonyi
Mathematical Institute, Budapest
and
DIMACS Center, Rutgers University

On a Combinatorial Problem in Group Theory with Applications in Finite Geometries and Design Theory

Let G be a group of order $s^2 > 1$. A set $\mathcal{P}$ of subgroups of order $s = \sqrt{|G|}$ is called an (s,r) - Partial Congruence Partition if $|\mathcal{P}| = r$ and $U \cap V = \{1\}$ for any two different members U and V of $\mathcal{P}$.

Given a partial congruence partition $\mathcal{P}$ in G, then ~~die~~ the incidence structure

$$\mathcal{N}(\mathcal{P}) := (G, \{Ug \mid U \in \mathcal{P}, g \in G\}, \in)$$

is an (s,r) - translation net, i.e. an (s,r) - net admitting G as translation group acting sharply transitively on the set of points. In this generality, these structures were first studied by Alan Sprague and Dieter Jungnickel in the early eighties.

A further application was given by John Dillon in 1975: If s is even and $r = \frac{1}{2}s$, then

$$\mathcal{D}(\mathcal{P}) := \bigcup_{U \in \mathcal{P}} U - \{1\}$$

is a certain difference set in G and therefore the incidence structure

$$\mathcal{B}(\mathcal{P}) := (G, \{Dg \mid g \in G\}, \in)$$

is a symmetric 2-design admitting G as a Singer group, i.e. acting sharply transitively on the set of points and lines.

We are interested in the number $T(G) := \max\{r \mid$ there exists an (s,r) - partial congruence partition$\}$ for certain classes of groups G and survey some results. Furthermore, we describe methods which allow to construct partial congruence partitions containing subgroups which are normal in G.

We refer to D. Jungnickel / D. Hachenberger, "Translation Nets. A Survey", a paper recently published in "Discrete Mathematics".

Dirk Hachenberger
Fachbereich Mathematik der Universität Kaiserslautern
Erwin-Schrödinger-Straße
6750 Kaiserslautern

The combinatorics of the recursive S_ω.

I gave a conspectus of the content of

KOMBINATORIK DER KOMPOSITION UND ANDERER VERKNÜPFUNGEN

REGULÄRER UND SINGULÄRER MORPHISMEN

KLASSISCH, REKURSIV, TOPOLOGISCH

(Eine Erweiterung des März 1990 herausgegebenen gelben Bandes von 54 Seiten)

KLAUS LEEB

Universität Erlangen

Februar 1992

, which is available in the library, and in particular pointed out the problem on p. 16 ibid..
My conjecture is that if $I(f; a, b, c, \ldots) = 1$, where I is a free group word balanced in all variables but f, and each $(f; a, b, \ldots)$ variable is substituted by a simple (in the sense of Tschirnhaus) permutation, then all slopes of f are $\underline{1}$.

Klaus Leeb

Random l-colorable graphs

In this talk we investigate properties of the class of all l-colorable graphs on n vertices, where $l = l(n)$ may depend on n. Let G_n^l denote a uniformly chosen element of this class, i.e., a random l-colorable graph. For a random graph G_n^l we study in particular the property of being uniquely l-colorable. We show that not only there exists a threshold function $l = l(n)$ for this property, but this threshold corresponds to the chromatic number of a random graph. We also prove similar results for the class of all l-colorable graphs on n vertices with $m = m(n)$ edges.
This is joint work with A. Steger, Bonn.

Hans Jürgen Prömel
(Universität Bonn)

How to share a secret in large hierarchies

An (s,t)-multilevel shared secret scheme consists of two sets A and B of "partial secrets" such that a secret datum K can be retrieved, if
- at least s partial secrets of A or
- at least t partial secrets of $A \cup B$

are present.
The fundamental structure for constructing such schemes can be described as follows. In a projective plane consider a line l through the secret point K and a conic C through K such that l is a tangent to C. Consider a set A of points $\neq K$ on l

and a set B of points $\neq K$ on C such that no line joining two points of B intersects l in a point of A.
We investigate the problem of maximizing $|A|$ and $|B|$. (Joint work with F. Wettl, Budapest.)

Albrecht Beutelspacher
(Universität Gießen)

The asymptotic structure of H-free graphs

Let H be a fixed finite graph. In this talk we consider properties of a 'typical' H-free graph. More precisely, we investigate the structure of a random H-free graph, assuming equal distribution on the set of all H-free graphs.
For the special case that H is a clique the problem was solved by Kolaitis, Prömel, Rothschild (1987), improving an earlier result of Erdős, Kleitman, Rothschild (1976). They showed that a random $K_{\ell+1}$-free graph is with probability one ℓ-colorable. A complete characterization of those graphs H sharing the property that a random H-free graph is ℓ-colorable with probability one was given by Prömel, Steger (1992): These graphs H are exactly those graphs with chromatic number $\ell+1$ which contain a color-critical edge.
As a first step in understanding this problem for general graphs H we consider in this talk those graphs which contain a color-critical vertex. This is joint work with Chr. Hundack and H.J. Prömel.

Angelika Steger
(Universität Bonn)

Symmetric chain decompositions for linear lattices

This is joint work with B. F. Voigt from Darmstadt. We present a uniform and explicit construction for symmetric chain decompositions for linear lattices, that is valid also in case of 'real' lattices. This answers a question of Greene and Kleitman (1976) and completes a previous description of a result of Loeb (Fleischer).

B. Voigt
[illegible]

The core of a poset.

The passing through- or PT-order on a poset is the quasi-order $\trianglelefteq$ in which $a \trianglelefteq b$ holds iff every maximal chain through a also passes through b. Call a subset X of the poset P a $\trianglelefteq$-good set if X is a $\trianglelefteq$-antichain which is $\trianglelefteq$-dominating. In general such a subset does not exist, but if P is chain-complete (CC) (ie. every non-empty chain in P has both an inf & a sup), then $\exists$ a $\trianglelefteq$-good subset $X \subseteq P$. Moreover, X is unique up to isomorphism and is a retract of P which reflects the fixed-point property (fpp), ie. P has fpp iff X has fpp. (P has fpp if every order preserving map $f: P \to P$ has a fixed point).

Since a retract of a CC poset is also CC, and the intersection of CC posets is CC, this suggest we iterate the above construction and define a strictly decreasing sequence $P = P_0 \supset P_1 \supset \cdots \supset P_\lambda = \Phi$, where (i) $P_{\alpha+1}$ is a $\trianglelefteq_{P_\alpha}$-good subset of P_α, and (ii) $P_\alpha = \bigcap_{\beta<\alpha} P_\beta$ if α is a limit, (iii) $\Phi = P_\lambda$ is a $\trianglelefteq_\Phi$-good subset of itself. In general the intersection of retracts is not a retract but we have the following result: Theorem Let $\mathcal{F}^*$ denote the class of CC posets which have no infinite antichain. If $P \in \mathcal{F}^*$ then the sequence $P = P_0 \supset P_1 \supset \cdots \supset P_\lambda = \Phi$ defined above is finite in length and the terminal set Φ is also finite.

In the case of finite posets the terminal set Φ is just the core of P obtained by dismantling. The core is a retract which reflects the fpp.

Thus the above result says that a poset $P \in \mathcal{F}^*$ has a finite core. This fact enables us to extend to $\mathcal{F}^*$ several results which are known for finite posets. This work was done jointly with LI Boyu (Xiàn).

F. Chulner, Calgary.

The height of a random tree - a quick estimate

In the talk a natural greedy algorithm for estimating the height of a random tree is proposed and studied. In particular, it is shown that for the expected length EH_n of a path find by the algorithm in a random tree on n vertices we have

$$\lim_{n\to\infty} EH_n/\sqrt{n} = \sqrt{\pi}\,\frac{\sqrt{2}\ln(\sqrt{2}+1)-1}{2-\sqrt{2}\ln(\sqrt{2}+1)} = 0.579686\ldots$$

Tomasz Łuczak, Poznań

On a constructive approach to well order types

Using the concept of coding functions ($\varphi(x,y) > x$, monotone with respect to both arguments) we code combinatorial objects into ordinals and obtain lower and upper bounds for sequences of binary trees, which show unprovability in formals systems such as Peano Arithmetics. The methods also allow for determining the maximal well order type whose existence is garantied by the Theorem of De Jong and Parikh.

Wolfgang ~~Thumser~~, Bielefeld

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Subspace Arrangements and Decision Trees

For $\lambda = (\lambda_1, \dots, \lambda_p)$ a partition of n, consider first the p-dimensional subspace $K_\lambda = \{x \in \mathbb{R}^n \mid x_1 = \dots = x_{\lambda_1}, x_{\lambda_1+1} = \dots = x_{\lambda_1+\lambda_2}, \dots, x_{\lambda_1+\dots+\lambda_{p-1}+1} = \dots = x_n\}$, and then the subspace arrangement $\mathcal{A}_\lambda = \{\sigma(K_\lambda) \mid \sigma \in S_n\}$. Here S_n acts on $\mathbb{R}^n$ by permuting coordinates. Some general properties of such arrangements and their complements $M_\lambda = \mathbb{R}^n - \bigcup_{\sigma \in S_n} \sigma(K_\lambda)$ were discussed. The following results were presented:

~~(1) M_λ~~ for the case $\lambda = (k, 1, \dots, 1)$:

(1) M_λ has ~~tors~~ torsion-free singular cohomology (both over $\mathbb{R}$ and $\mathbb{C}$), which (in $\mathbb{R}$-case) is non-zero if and only if the dimension is a multiple of $k-2$.
(Joint work with V. Welker)

(2) The sum of Betti numbers of M_λ is a lower bound for the number of leaves on any linear decision tree testing points $x \in \mathbb{R}^n$ for membership in M_λ. This leads to the following result: The asymptotic worst-case complexity for deciding whether some k members of a string $x_1, \dots, x_n$ of real numbers $x_i \in \mathbb{R}$ are equal, is $\Theta(n \log \frac{2n}{k})$.
(Joint work with L. Lovász and A. Yao)

Anders Björner

Young symmetrizers for spin representations of symmetric groups

There is a classical method due to A. Young (1901/2) which yields, through combinatorial consideration of what are now called 'Young tableau', primitive idempotents called 'Young symmetrizers' which themselves lead to irreducible representations of S_n. Although work on projective (spin) representations of S_n goes back to I. Schur (1911), it is only recently that M. Nazarov (1988/90) showed how to construct the irreducible representations explicitly. However, in that construction, Young symmetrizers were not involved. Recently, Nazarov (September 1992) has shown in a remarkable piece of work how Young symmetrizers can be described in this context. His approach is totally different from the classical approach, his theory being based on an alternative description of the Young symmetrizers due to I. Cherednik which involved a certain degenerate affine Hecke algebra and ideas originating in Yang-Baxter theory and quantum groups.

This lecture was a brief report on Nazarov's work which appeared as a Preprint 900 (RIMS, Kyoto). Here, Young tableaux are replaced by shifted Young tableaux (as expected) but more surprisingly, the work is simplified by considering the covering group $S_n \ltimes Cl_n$ of the hyperoctahedral group $S_n \ltimes \mathbb{Z}_2^n$ rather than the covering group T_n of S_n. This work should open the door to many new problems and developments.

Alun O. Morris, Aberystwyth.

On orthogonal double coverings of the K_n

An orthogonal double coverings of the complete graph K_n is a collection of n spanning subgraphs $G_1, G_2, \dots, G_n$ of the K_n such that
- every edge of the K_n belongs to exactly 2 of the G_i's and
- every pair of G_i's intersect in exactly on edge.

It is proven that an orthogonal double covering exists for all $n \geq 2$ (with the only undecided exeption $n = 18$) if the G_i's have maximum degree 2, which proves a conjecture of CHUNG & WEST (communicated at Oberwolfach October '81 with solutions for 6 of 12 residue classes).

Hans-Dietrich Gronau, Rostock

Chemical isomerism, a basic problem of algebraic combinatorics

This problem means to construct, for a given chemical formula, prescribed and forbidden substructures, all the corresponding connected multigraphs. This problem dates back to the 18.th century (A. v. Humboldt), and it is solved (among others) by our program system MOLGEN which was briefly described. It is the outcome of a joint project with R. Laue and R. Grund.
Afterwards a few basic principles were mentioned which are used in it, and applications (by B. Schmalz) to the constructive theory of t-designs were described.

A. Kerber (Bayreuth)

Partitioning the hypercube in sets with mutual distance one.

Let Q_n be the graph on $\{0,1\}^n$ defined by $x \sim y$ if x and y differ in exactly one coordinate. If we partition the vertex set of Q_n in m parts, such that between two different parts there is always an edge somewhere, then, since the total number of edges in Q_n equals $n2^{n-1}$ we obviously get $\binom{m}{2} \leq n2^{n-1}$. We show that it is possible to realize a partition with m parts such that $\binom{m}{2} = cn2^{n-1}$ for some constant c between $\frac{1}{4}$ and $\frac{1}{2}$, depending on how close n is to the nearest smaller power of 2. This answers a question of R. Ahlswede

Aart Blokhuis
Eindhoven (NL)

Enclosing Latin Squares and Triple Systems

A triple system $(V, \mathcal{B})$ of order v and index λ, denoted $TS(v,\lambda)$ is a v-set V together with a multiset $\mathcal{B}$ of 3-element subsets of V (called triples) with the property that every 2-element subset occurs in exactly λ of the triples. A $TS(w,\mu)$ $(W, \mathcal{D})$ is an enclosing of a $TS(v,\lambda)$ $(V,\mathcal{B})$ if $V \subseteq W$ and $\mathcal{B} \subseteq \mathcal{D}$ (as multisets). The enclosing is faithful if all triples of $\mathcal{D}$ lying entirely on V are from $\mathcal{B}$. Necessary conditions for such a faithful enclosing are derived, and shown to be sufficient when $w \geq 2v+1$, when $\mu = 2\lambda$ and for many small cases. Related problems on latin squares are outlined.

Charles Colbourn.
Waterloo, Canada.

Homology of Partition Lattices

We consider the homology of the order complex of various subposets of the lattice of partitions of the set $\{1,2,\ldots,n\}$. One such subposet is the d-divisible partition lattice which consists of partitions all of whose block sizes are divisible by d. We construct an explicit natural basis for the top homology group. Each basis element turns out to be a fundamental cycle of the barycentric subdivision of the boundary of an $(\frac{n}{d}-1)$-cube. Moreover the cycles correspond in a natural way to permutations in S_{n-1} whose descent set is $\{d, 2d, \ldots, n-d\}$. The basis yields a direct combinatorial derivation of a result, conjectured by Stanley and proved by Calderbank, Hanlon and Robinson, relating the character of S_n acting on the top homology to a certain skew character.

Michelle Wachs

Point line configurations, hermitian polarities and an observation of M. O'Nan

Aiden A Bruen (with J.C. Fisher)

Let $\pi = PG(2,F)$ the projective plane over the commutative field F. Let S be a set of points in π and assume that there is a 1-1 (injective) mapping f from S into the set of lines of π satisfying the following 2 properties.

(A) if P is in S then f(P) does not contain P :

(B) if P_1, P_2 are distinct points in S then P_1, P_2 and $f(P_1) \cap f(P_2)$ are collinear

We then refer to S as a co tangency set

Our main observation is as follows.

Theorem If S is a cotangency set then it cannot contain a quadrangle.

We obtain some consequences of this result including a well-known result on hermitian curves due to M. O'Nan. Extensions to non-hermitian unitals are sketched

Aiden B

(Aiden A Bruen)

Circular systems of splits
(joint work with Andreas Dress)

Every metric d on a finite set X admits an additive decomposition into a "totally decomposable" metric d_0 and a metric residue d_1, where d_0 is a sum of split (alias cut) metrics that are weakly compatible in the following sense: any three of the corresponding splits, $\{A_i, X-A_i\}$ $(i=0,1,2)$, with $A_i \cap A_{i+1} \overset{-A_{i+2}}{\neq} \emptyset$ $\underset{\text{modulo } 3}{(i=0,1,2)}$ satisfy $A_0 \cup A_1 \cup A_2 = X$ (see Advances Math., vol. 92, 1992, pp. 47–105). In most applications the system of splits giving the totally decomposable part d_0 is even circular in the sense that the points of X can be represented on a circle such that the splits correspond to some lines. Such circular systems can be characterized by forbidden minors, and the associated metrics can be recognized in polynomial time.

hans-jürgen bandelt

Distinguishing Ramsey Patterns

Imre Leader (Cambridge).
Joint work with Walter Deuber, Neil Hindman and Hanno Lefmann.

Hindman's Theorem states that whenever we partition $\mathbb{N}$ as $C_1 \cup \ldots \cup C_k$ there exists a sequence $x_1, x_2, \ldots$ in $\mathbb{N}$ such that $FS((x_n)_1^\infty) \subseteq C_i$ for some i, where as usual $FS((x_n)_1^\infty)$ denotes $\{\sum_{n \in A} x_n : A \subseteq \mathbb{N}, A$ finite non-empty$\}$.

Recently, this was extended to show the following. For any $a_1, \ldots, a_s \in \mathbb{N}$, whenever we partition $\mathbb{N}$ as $C_1 \cup \ldots \cup C_k$ there exists a sequence $x_1, x_2, \ldots$ in $\mathbb{N}$ such that $FS((a_n)_1^s, (x_n)_1^\infty) \subseteq C_i$ for some i, where $FS((a_n)_1^s, (x_n)_1^\infty)$ denotes $\{\sum_{n \in A_1} a_1 x_n + \sum_{n \in A_2} a_2 x_n + \ldots + \sum_{n \in A_s} a_s x_n :$ $A_1, \ldots, A_s$ finite non-empty, $\max A_i < \min A_{i+1}\ \forall i\}$.

Here we show that these are pairwise incompatible, in the following sense. Take $(a_n)_1^s$, $(b_n)_1^t$ such that $a_i \neq a_{i+1}$, $b_i \neq b_{i+1}$ (all i), with $(a_n)_1^s$ not a rational multiple of $(b_n)_1^t$. Then there is a colouring $\mathbb{N} = C_1 \cup \ldots \cup C_k$ such that no colour class contains both an $FS((a_n)_1^s, (x_n)_1^\infty)$ and an $FS((b_n)_1^t, (x_n)_1^\infty)$. This very strange phenomenon is in complete contrast to the finite case.

THE POWER OF NON-COMMUTATIVITY

In 1976 Diffie and Hellman proposed their now famous key exchange method which exploits exponentiation in a finite cyclic group. In this talk we describe a method for key exchange which makes use of the non-commutativity of elements in a non-abelian group. Protocols for a full public key scheme are described and various groups are discussed as possible candidates to implement the system.

S. Wanstone
Waterloo.

Numerische Integration

8. – 14. Nov. 92

Positivity and Monotonicity in Quadrature

Given a quadrature method, it is not only of theoretical interest but also of practical importance to know sufficient conditions on the integrand which guarantee

(a) a one-sided approximation of the integral (POSITIVITY)

(b) monotone convergence of the remainders (MONOTONICITY).

For most of the familiar quadrature methods these problems have been considered and sign restrictions on certain higher order derivatives have been given as sufficient conditions. Examples indicate, however, that these conditions seem to be far from being necessary. Using the concept of positive definite functions, we specify in the case of the trapezoidal method wider classes of functions that guarantee positivity and monotonicity. Extensions to related quadrature methods are also discussed.

Gerhard Schmeißer
(Erlangen)

ERROR in DE: GRID and BOGS

Plum '90 treated differential operators by a numerical homotopy method. He used Simpson's quadrature in case $f=pq$ (where p fixed, q: computed polynomials), estimating the error via Leibniz' formula and grid bounds for polynomials (Ehlich). We discuss other approaches, based on BOGS (biorthogonal systems: Chebyshev expansions): General estimator, decomposition $f=pq$, Fourier differentiation. Further we mention variants and extensions.

Karl Zeller (Tübingen).

A New Variable Transformation for Numerical Integration

Presently variable transformations are used to enhance the performance of lattice rules for multidimensional integration. The transformations that are used in the literature so far are of either polynomial or exponential nature.
We propose yet a new transformation that is trigonometric in nature that we designate the $\sin^m$-transformation. We analyze its effect within the framework of one-dimensional integration and show that it has some very interesting mathematical properties. We demonstrate its numerical efficiency by applying it to various one-dimensional integrals of smooth and singular functions. Present results indicate that the new transformation is more advantageous than the known polynomial transformations, and has less underflow and overflow problems than exponential transformations.

Avram Sidi (Technion)

Quadrature formulas for convex classes of functions

We study the problem of optimal recovery in the case of a nonsymmetric convex class of functions. We begin with the problem of optimal integration of convex functions.

We prove that adaption does not help in the worst case but considerably helps in the case of Monte Carlo methods.

Then we study more generally certain new Gelfand-type n-widths that are useful for nonsymmetric classes.
We give examples for linear problems on a convex set of functions where adaptive methods are much better than nonadaptive ones.

10.11.1992

Erich Novak (Erlangen)

"On integration of singular functions;

A probabilistic approach"

We discuss integration of scalar functions that are regular everywhere but at some singular points.
In the worst case setting, due to unknown location of singularities, any quadrature requires a substantial

number of function values in order to approximate the integral with small error. The situation is drastically different in a probabilistic setting. Indeed, under some stochastic assumptions, we provide an adaptive quadrature that, modulo a small probability, behaves as well as an optimal quadrature for functions without singularities.

10.11.1992

Grey Wasilkowski
(University of Kentucky, USA)

"Multivariate Integration in Various Settings
(joint work with Klaus Ritter & Greg Wasilkowski)

We begin with multivariate integration in the average case setting. We show estimates on the nth minimal average error in terms of the smoothness of a covariance function of stochastic process. The proof technique is based on relations between the average and worst case settings. We illustrate these estimates for isotropic Wiener and Wiener sheet stochastic processes.

We briefly indicate extensions of our estimates for nonlinear quadrature formulas, adaptive information, different error criteria and for the probabilistic setting.

10.11.92

Henryk Woźniakowski
(Columbia University & University of Warsaw)

COLLOCATING CONVOLUTIONS

An explicit method is derived for collocating either of the convolution integrals $p(x) = \int_a^x f(x-t)\, g(t)\, dt$, or $q(x) = \int_x^b f(t-x)\, g(t)\, dt$, where $x \in (a,b)$, a subinterval of $\mathbb{R}$. The collocation formulas take the form $\underline{p} = F(A_m)\underline{g}$, or $\underline{q} = F(B_m)\underline{g}$ where $\underline{g}$ is an m-vector of values of g evaluated at the "Sinc points", A_m and B_m are explicitly described square matrices of order m, and $F(s) = \int_0^c e^{-t/s} f(t)\, dt$, for arbitrary $c \in [(b-a), \infty]$. The components of the vectors $\underline{p}$ (and $\underline{q}$) approximate the values of p (q) at the Sinc points, and may then be used to approximate p (and q) via Sinc interpolation. If u is the solution of a PDE expressed in terms of a ν-dimensional convolution integral over a rectangular region $\underline{B}$, and if u is analytic and of class Lip_α on the interior of each line segment in $\underline{B}$, then the complexity of computing an ε-approximation to u by the above method is $O([\log \varepsilon]^{2\nu+2})$.

10.11.92 (Frank Stenger)

Univ of Utah

The error of positive quadrature formulas

Let $u(x) = (1-x)^\alpha (1+x)^\beta \log^{\Gamma_0} \frac{e}{1-x^2} \prod_{1,k}^{r} |x-t_k|^{\gamma_k} \log^{\Gamma_k} \frac{e}{|x-t_k|}$

where $\alpha, \beta, \gamma_k > -1$, $-1 < t_1 < \dots < t_r < 1$ and Γ_k nonne-gative integers. Let further

$$e_m(f)_u = \int_{-1}^{1} f(x)\, u(x)\, dx - \sum_{1,k}^{m} w_k f(x_k), \quad w_k > 0,$$

$f^{(r-1)} \in AC_{loc}$ be the error of a p.q.r. with al gebraic degree $\geq m-1$. We prove

$$|e_{2m-1}(f)_u| \leq C \, \| f^{(r)} \varphi^{r} u_m \|_{L_1([x_1, x_m])} + C \, \| f^{(r)} \varphi^{2r} u \|_{L_1(I_m)},$$

where $I_m = [-1, x_1] \cup [x_m, 1]$, $u_m(x) = (1-x)^{\alpha}(1+x)^{\beta} \log^{\Gamma_0} \frac{e}{1-x^2} \; \times$

$$\times \prod (|x-t_k| + m^{-1})^{\gamma_k} \log^{\Gamma_k} \frac{e}{|x-t_k| + m^{-1}}$$

and C is a positive constant independent of m, f and t_k. We consider some special cases and, as an application, we estimate the coefficients w_k of the formula.

Giuseppe Mastroianni
Univ. della Basilicata
Via N. Sauro
(Italia) Potenza

Convergence Properties of Adaptive Integration Processes.

We derive convergence results of local adaptive and global region-size adaptive integration algorithms. An adaptive integration process is associated with a tree of regions where each node corresponds to a region and its children nodes to its subregions. The depth of subtrees at distinguished nodes is delimited according to convergence characteristics of the adaptive strategy for the considered function class.

The approach by Rice (1974, 1975) is extended to handle multivariate integration over the N-dimensional cube and simplex, where the integrand function may have a vertex singularity. Extensions to cover some other types of singular behaviour follow naturally.

As an application, speedup results can be derived for parallel versions of the algorithms, assuming a model on the processing of the region trees in parallel.

Nov. 10, 1992 Elise de Doncker (co-authored by J. Vakalis)

Western Michigan University

Some problems involving orthogonal polynomials.

Let a weight-function w be positive and continuous over $(-1, 1)$, and suppose that

$$\int_{-1}^{1} w(x)\, dx$$

exists. Also let p_n be the polynomial of degree n in the corresponding orthogonal sequence. Some properties of the coefficients in the expansions

$$1/p_n(z) = \sum_{j=0}^{\infty} b_{nj} z^{-n-j}$$

and

$$q_n(z) = \int_{-1}^{1} \frac{w(x) p_n(x)}{z-x}\, dx = \sum_{j=0}^{\infty} c_{nj} z^{-n-j-1}$$

were reviewed, and an application to the error analysis of Gaussian quadrature was described.

The rest of the talk was concerned with the function U_n for which

$$U_n^{(r)}(1) = U_n^{(r)}(-1) = 0, \qquad (r = 0, 1, ---, n-1),$$

$$U_n^{(n)}(x) = w(x) p_n(x), \qquad (-1 < x < 1).$$

A generalization of a theorem of Markoff (1886) on the variation of the zeros of p_n when w involves a parameter was conjectured, and the conjecture was applied to determine the form of U_n

when $w(x)/w(-x)$ is strictly increasing

November 10th, 1992

D. B. Hunter, University of Bradford.
Harry V. Smith Leeds Metropolitan University, England.

A stability test for linear difference forms

The stability of linear multistep method or of a LTI-system (e.g. digital filter) depends on the fact whether the characteristic polynomial p_n, $p_n(z) = \sum_{\nu=0}^{n} a_\nu z^\nu$, has all its zeros in the unit circle. We consider the polynomials q_n, q_{n-1} where $q_n := \sum_{\nu=0}^{n} a_\nu T_\nu$, $q_{n-1} := \sum_{\nu=0}^{n-1} a_{\nu+1} U_\nu$ with the Chebyshev polynomials T_ν, U_ν of the first resp. second kind. Then the common zeros of q_n and q_{n-1} in the interval $(-1,1)$ determine the unimodular zeros of p_n. All zeros of p_n lie in the interior of the unit circle iff by Euclidean division, started with q_n and q_{n-1}, one can build up a Sturm sequence of maximal length $n+1$ and there occur n sign changes at -1.

Franz Locher (Hagen)

Error bounds using total variation

Let R denote the error of a positive quadrature rule for a weighted integral. We are interested in bounds of the type

$$|R[f]| \leq \beta_s \operatorname{Var} f^{(s-1)},$$

that is in the determination of the constants

$$\beta_s(R) := \sup \{ |R[f]| ; \operatorname{Var} f^{(s-1)} \leq 1 \}.$$

Under some assumptions on the weight function the asymptotics of $\sup \{ \beta_s(R) : R[\mathbb{P}_n] = 0 \}$ for $n \to \infty$ ($\mathbb{P}$: Polynomials of degree n) is determined and it is proved, that the supremum is attained (asymptotically) for the Gaussian rules. As a by-product pointwise bounds for Peano kernels are obtained.

Helmut Braß (Braunschweig)

Quadrature theory of convex functions

This talk begins with a short overview of some known results on the quadrature of convex functions. Midpoint and trapezoidal formulae have been investigated frequently, but it was shown that they are not able to compete in many situations. However, in those classes of functions which have caused some interest, the Gaussian rule is not far behind the respective optimal rule. We therefore investigate the Gaussian error for convex functions more detailed, in particular under additional assumptions (for instance, continuous differentiability in the interior of the basic interval) on the integrand. Similarities to the size of Fourier coefficients are remarkable.

Knut Petras (Braunschweig)

Numerical Integration of Nearly Singular Functions

We propose an automatic integration for approximating the integral $\int_{-1}^{1} f(x)/(x-c)dx$ for a given smooth function $f(x)$, where c is outside, but close to the integration interval $[-1,1]$. Approximating the integral is more difficult problem, if c is closer to either end of the interval [-1, 1], although the integrand is not a singular function. The present scheme is an extension of the Clenshaw-Curtis method. The function $f(x)$ is approximated by the finite sum of the Chebyshev polynomials and some extrapolation method is made use of to evaluate the integral. Numerical examples are also included.

Takemitsu Hasegawa (Fukui University
Japan)

Integrating Singularities using Non-uniform Subdivision and Extrapolation

A new approach to the computation of approximations to multidimensional integrals over an n-dimensional hyper-rectangular region, when the integrand is singular, is described. This new approach is based on a non-uniform subdivision of the region of integration and the technique fits well to the subdivision strategy used in many adaptive algorithms. The strategy can be applied to vertex singularities, line singularities and more general subregion singularities. The technique turns out to have good numerical stability properties.

11.11.92 Terje O. Espelid (University of Bergen
Norway).

Lattice Rules for Nonperiodic Integrands I

Lattice rules were originally designed for the numerical integration of periodic functions over their period interval, which is assumed to be the s-dimensional unit cube $[0,1]^s$. We introduce a slightly modified integration rule associated with an integration lattice L and show that for nonperiodic integrands it performs better than the standard lattice rule Q_L. For instance, T_L integrates all linear functions exactly, but this is not true for Q_L. Furthermore, if $s=2$ and L has a square unit cell, then T_L integrates all polynomials $a+bx+cy+dxy$ exactly. Again for $s=2$, it is shown that the Fibonacci lattice L_k obtained from the kth Fibonacci number F_k has a square unit cell if and only if k is odd. Thus, for $s=2$ there exist excellent integration lattices with square unit cell. These results were obtained jointly with Ian Sloan.

Harald Niederreiter (Vienna)

Rational Hermite Interpolation and Quadrature

We shortly introduce the barycentric representation of a rational function r_n interpolating f and its derivative f' as well at knots t_i. The remainder term is derived. By integrating r_n we construct quadrature rules of Gaussian type, i.e., quadratures with vanishing weights for $f'(t_i)$. If the denominator q_n of r_n is prescribed this means that the nodal polynomial has to be orthogonal to $\mathbb{P}_{n-1}$ with respect to w/q_n which could lead to implicitly defined orthogonal polynomials. On the other hand, rational interpolation leaves the freedom of choosing the knots. Now q_n has to be fixed appropriately. Another approach, namely interpolating a primitive of f and f itselves by r_n and using the integral over r_n' as quadrature rule, leads to dual quadratures which were introduced by Engels in 1981. They are really easy to construct if the weighting function w is constant. Furthermore, starting with a given quadrature Q_n, we can construct a rational Hermite interpolant r_n from the nodes and weights of Q_n. Integrating this r_n' then recovers Q_n. Hence, every quadrature is interpolatory in the setting of rational Hermite interpolation and dual quadratures

Nov. 9 /92 Claus Schneider

Johannes Gutenberg-Universität Mainz

Gaussian Cubature Rules

In a joint paper with Yuan Xu new results about Gaussian cubature are derived.

For two classes of integrals, introduced by Karlin/Macgregor and Koornwinder, minimal cubature formulae of an arbitrary degree are constructed.

For even degree there is an infinite number of such formulae, for odd degree there is one formula of this type.

As in the one-dimensional case, the number of nodes is $\mathbb{P}_{[m/2]}$ for degree m of exactness. The nodes are the common zeros of quasi-orthogonal polynomials.

11. Nov. 1992

Hans Joachim Schmid

Math. Inst. Erlangen

Construction of fully symmetrical cubature rules of very high degree for the square

A new method for the construction of fully symmetrical cubature formulae for the square is proposed.
Using a transformation of variables and choosing an appropriate basis for the invariant polynomials of the square, the nonlinear system that one has to solve with classical methods breaks down in three systems that one has to solve successively.
Each of these systems can be solved easily. Even for high degrees, the formulae are surprisingly easy to compute: the hardest computation to perform is the solution of a system of two polynomial equations of lower degree in two unknowns

11/11/92

Ann Haegemans
University of Leuven, Belgium

The Canonical Forms of Lattice Rules
Much of the elementary theory of lattice rules may be presented as an elegant application of classical results. These include the Kronecker group representation theorem and the Hermite and Smith normal forms of integer matrices. The theory of the canonical form ~~A~~ is a case in point. In this paper some of this theory is treated in a constructive rather than abstract manner. A step-by-step approach that parallels the group theory is described, leading to an algorithm to obtain the canonical form of a rule of prime power order. The number of possible distinct canonical forms of the same rule is derived and this is used to determine the number of rules having specified invariants

11/11/92 James Lyness (Argonne)

Lattice rules for non-periodic integrands II

This paper describes further joint work with H. Niederreiter on lattice rules in the situation in which the integrand is continuous on $[0,1]^s$ but does not have a continuous periodic extension. Now s is allowed to be arbitrary. The main focus is on a quadrature rule Bf in which the weight $1/N$ (where N is the number of points in the original rule) associated with the origin is redistributed over all the vertices, with the vertex weights chosen so that Bf integrates exactly all bilinear functions (in the case $s=2$), or all multilinear functions for general s. It is shown that Bf is optimal among modified-vertex-weight rules, in that it minimises the L_2-discrepancy error bound. Numerical experiments and further problems are discussed.

11/11/92

Ian H. Sloan, Univ. of N.S.W.

Gauss-type quadrature rules for rational functions

When integrating functions that have poles outside the interval of integration but are regular otherwise, it is suggested that the quadrature rule in question ought to integrate exactly not only polynomials (if any), but also suitable rational functions. The latter are to be chosen so as to match the most important poles of the integrand. We describe two methods for generating such quadrature rules numerically and report on computational experience with them, in particular on the evaluation of Fermi-Dirac integrals to high accuracy.

Walter Gautschi (Purdue University)

What Distributions of Points are Possible for Convergent Sequences of Interpolatory Integration Rules?

Suppose that $w > 0$ a.e. on $[-1,1]$ and is integrable. Suppose that we are given interpolatory integration rules

$$I_n[f] = \sum_{j=1}^{n} \lambda_{jn} f(x_{jn}),$$

with points x_{jn} in $(-1,1)$, so

$$I_n[P] = \int_{-1}^{1} Pw, \quad P \text{ a poly. of deg.} \leq n-1.$$

Suppose that

$$\lim_{n\to\infty} I_n[f] = \int_{-1}^{1} fw$$

$\forall$ continuous $f : (-1,1) \to \mathbb{R}$. What can we say about the distribution of the points x_{jn}? We show that half the points have arcsin distribution (i.e. behave like zeros of Chebyshev polynomials) and half may be arbitrarily distributed. Moreover, we can find interpolatory rules with all $\lambda_{jn} > 0$ in which half the points attain a given arbitrary distribution.

12 Nov 92

(D. Lubinsky, Witwatersrand University, Jhb. South Africa; + Tom Bloom, Toronto + Herbert Stahl, Berlin)

A relation between cubature formulae of trigonometric degree and lattice rules.

In the recent past, several articles on cubature formulae of trigonometric degree appeared in the Russian literature. In the first part of this talk, we give an overview of the results we know about. These include a lower bound for the number of points

and some minimal formulae.
In the second part of the talk we will generalise an earlier result on cubature formulae of algebraic degree, to prove that minimal formulae of trigonometric degree must have equal weights. This and the fact that all formulae of trigonometric degree in the literature are lattice rules, motivates us to have a closer look at the relation between „classical" lattice rules and formulae of trigonometric degree.
In the third part of the talk we construct minimal cubature formulae of arbitrary odd trigonometric degree. We compare these with Fibonacci lattice rules, which are optimal w.r.t. the Zaremba index. The comparison is made by the two construction criteria used in this talk: the trigonometric degree and the Zaremba index.

Ronald Cools, K.U.Leuven, Belgium
Nov. 12, 1992.

On the Construction of Gaussian Quadrature Formulae containing preassigned Nodes,

A general iterative method is presented for the computation of Gaussian quadrature formulae $Q^{G}_{n,m}$ that contain m preassigned Nodes. The method is locally convergent of the order two. It can be implemented in a way that every iteration step involves $O((2n+m)^2)$ arithmetical operations, and it provides simple and reliable a posteriori error estimates. A comparison is made to methods that are already in use. Numerical examples show the suitability of the method for some of the possible applications.

Nov. 12, 1992 Sven Ehrich
Universität Hildesheim

Adaptive Numerical Integration over Hyper-Spherical Regions

We describe an adaptive algorithm for integration over hyperspheres. The algorithm subdivides the hypersphere in a sequence of stages, into a collection of subregions, each of which is a product of a radial interval and a spherical simplex. The algorithm can then use appropriately transformed integration rules from simpler regions for approximation of the subregion integrals, and error estimation. Several tranformations are considered and efficient methods for computing the corresponding Jacobians are developed. The algorithm uses a subdivision strategy that chooses at each stage the subregion of the input hypershere with the largest estimated error. This subregion is divided in half along a direction that is chosen either from the set of $n(n-1)/2$ possible edge directions, or the radial direction, by using smoothness of the integrand. We focus on mathematical problems associated with the officient implementation of the algorithm.

Alan Genz, Washington State University, Pullman, WA, USA 11 Nov. 1992

"A trivariate Boolean midpoint rule"
Boolean sums have been used to generate cubature formulas which are comparable with good lattice rules. These formulas have been extended to bivariate product midpoint rules and yield cubature schemes of similar efficiency and greater simplicity (Math. Comp. 55(1990)). The objective of the lecture is to use formulas of trivariate Boolean interpolation to construct trivariate Boolean

midpoint rules which are also comparable with good lattice rules in three dimensions,

Franz-Jürgen Delvos, Univ. Siegen, Germany

Characterization of positive quadrature formulas

We say that an interpolatory quadrature formula

$$\int_{-1}^{+1} f(x)w(x)\,dx = \sum_{j=1}^{n} \beta_j f(x_j) + R_n(f),$$

$-1 < x_1 < x_2 < \ldots < x_n < 1$, is a positive $(2n-1-m, n, w)$ q.f. if $\beta_j > 0$ for $j=1,\ldots,n$ and $R_n(f)=0$ for $f \in \mathbb{P}_{2n-1-m}$. Various characterizations of positive $(2n-1-m, n, w)$ q.f., old and new, are given. So it is demonstrated that the positivity of a $(2n-1-m, n, w)$ q.f. is equivalent to the fact that there exists a sequence of positive definite numbers $c_0, c_1, \ldots, c_{2n-1}$ such that $c_j = \int_{-1}^{1} x^j w(x)\,dx$ for $j=0,\ldots,2n-1-m$. Then it is shown that the last statement is equivalent to the fact that $\prod_{j=1}^{n}(x-x_j)$ can be generated by a recurrence relation the recurrence coefficients of which coincide up to $j=0,\ldots,n-[\frac{m}{2}]$. Furthermore, as consequences of the above characterizations, a simple characterization of positive quadrature formulas with respect to the Chebyshev weight $(1-x^2)^{\pm 1/2}$ and an extension of a Theorem of Bernstein on the distribution of the nodes of a positive quadrature formula is given.

Franz Peherstorfer, Universität Linz, Österreich. 11. Nov. 1992

Intermediate Error Estimates for Quadrature Formulas

Let R be a functional which admits estimates of the form $|R[f]| \le e_i \|f^{(i)}\|$ for $i = 0, \ldots, r$. If e_0 and e_r are known, then estimates for the intermediate error constants $e_1, \ldots, e_{r-1}$ can be obtained in terms of e_0 and e_r. This was proven by A. A. Ligun for 2π-periodic functions. The estimates hold for nonperiodic functions, too, and various generalizations and specializations of these estimates are derived, with special emphasis on the estimation of quadrature errors.

Peter Köhler, TU Braunschweig, Germany 12. 11. 1992

Gaussian quadrature for splines

Explicit quadrature formulae of Gauss, Radau and Lobatto type are found for spaces of polynomial splines of degree 1 (arbitrary knots) and 2 (the case of equidistant knots). It turns out that Gaussian quadrature formulae for continuous piecewise linear functions are almost optimal in the Sobolev space

$$W_\infty^2 = \{ f \in C'[0,1],\ f' \in AC[0,1],\ \underset{t \in [0,1]}{\mathrm{vraisup}} |f''(t)| \le M \},$$

being in the same time as simple as the classical midlepoint and trapezium rule.

The investigation of Gauss type quadrature formulae for splines of degree 2 suggest that they might be also near the best quadrature formulae in W_∞^3.

Geno Nikolov, University of Sofia 12.11.1992.

Quadrature rules derived from linear convergence acceleration schemes.

We consider integrals of one of the two forms

$$\int_{-\infty}^{\infty} e^{it} f(t)\,dt \quad \text{and} \quad \int_{-\infty}^{\infty} g(t)\,dt \;, \quad g(t) = \begin{cases} O(e^{-t\alpha}) & t \to \infty \\ O(e^{t\beta}) & t \to -\infty \end{cases}$$

These integrals are approximated by means of the trapezoidal rule. The resulting infinite series are calculated using linear ~~cg~~ convergence acceleration schemes. These latters approximate the sum by a linear combination of a finite numbers of terms. The efficiency of this scheme was demonstrated on numerical examples

Sven-Åke Gustafson, HSR, Stavanger, Norway. 13.11.92

Komplexitätstheorie 15.-21.11.'92

Multiparty communication complexity, ~~and~~ circuit size and ranks of tensors

Pavel Pudlák

For $i, j \in [0, n-1]$, $\bar{x} \in \{0,1\}^n$ let f be the function
$$f(i,j,\bar{x}) = x_{i+j \pmod n}.$$
Suppose f should be computed by three players where
Player 0 knows i, j
Player 1 knows $j, \bar{x}$
Player 2 knows $i, \bar{x}$.
Players 1 and 2 send independently messages to Player 0 and he gives the value of $f(i,j,\bar{x})$.

Theorem [joint result with V. Rödl]
They need only $O(n \log\log n / \log n)$ to communicate.

Graph driven BDD's - a new data structure for Boolean functions

Ingo Wegener, Univ. Dortmund, joint work with Detlef Sieling

For many areas in hardware design like synthesis, formal verification or test pattern generation or as part of CAD tools one needs data structures for Boolean functions fulfilling the following properties.

Many (important) functions in P/poly can be represented in (small) polynomial size and the following operations can be performed by efficient algorithms: evaluation, satisfiability test, satisfiability count, satisfiability all, minimization, synthesis, equality test, redundancy test, replacement by a constant or by a function. Binary decision diagrams (BDD's) due to Bryant are the most often used data structures for this purpose. We significantly generalize this model to the so-called graph driven BDD's. Many functions of exponential BDD size can be represented in small polynomial size (like ACU's or the indirect storage access function) and all operations have efficient polynomial algorithms.

Fast Computation of Numerical Partial Fraction Decompositions and Contour Integrals of Rational Functions.

Peter Kirrinnis, Univ. Bonn

The problem of computing the integral $\int_\Gamma \frac{q(z)}{p(z)} dz$, where q and p are polynomials, given by their (complex) coefficients, and Γ is a curve in the complex plane, is investigated from the point of view of (serial) bit complexity.
Two algorithms are presented: The first computes the integral in the special case that the zeros of p lie in a small circle not intersected by Γ. The second algorithm computes a special type of partial fraction decomposition especially well suited for this application, but also of interest by itself. Combining these algorithms yields an algorithm for the computation of contour integrals in the general case.
It turns out that under reasonable rounding conditions, the integral can be computed up to an error of 2^{-s} in time $\tilde{O}(n^3(1+\tau) + n^2 s))$, bit

operations if for every zero z of p and every point y on Γ the estimates $|z| \le 1$ and $|y-z| \ge 2^{-\tau}$ hold.

Computations with Integer Division

Friedhelm Meyer auf der Heide, Paderborn

Computation trees with integer inputs and operation set $S \subseteq \{+, -, *, DIV, DIV_c\}$ are considered; DIV denotes integer division, DIV_c integer division by constants. It is shown that the expressive powers of different operation sets S are all different for at least two inputs. For computation trees with one input, only two classes exist, those with and those without DIV or DIV_c.

For this purpose the expressive powers of such operation sets are characterized.

Furthermore, lower bounds for such computation trees are shown, including the first nontrivial lower bounds for the powerful operation set $\{+, -, *, DIV\}$.

This is joint work with

Katharina Lürwer-Brüggemeier

Fair Public-Key Cryptosystems

Silvio Micali, MIT, Cambridge, USA

We show that the secret decryption key of a Public-Key Cryptosystem can be shared among several Trustees so that no minority of the Trustees can reconstruct the secret key, while any majority of the Trustees can easily compute it. Furthermore, upon receiving his own piece of the secret key, each Trustee can verify ~~any~~ (without any interaction with other Trustees or with the owner of the public/secret key pair) that he indeed has a correct piece of the secret key. That is, each Trustee can verify that, given any majority of shares that have ~~correct~~ satisfied a check similar to his own, the secret key of the given public key can be reconstructed. This scheme can be used to achieve private (encrypted) communication among citizens of a democratic country while permitting court-authorized line tapping under the circumstances envisaged by the law.

Bounds for the Computational Power and Learning Complexity of Analog Neural Nets

Wolfgang Maass, Graz

We introduce two new methods for reducing nonlinear problems about weights in multi-layer neural nets to linear problems for a transformed set of parameters. As a consequence one gets the first upper bounds for the Vapnik - Chervonenkis dimension and the computational power (for boolean inputs and outputs) of analog neural nets, i.e. nets with ~~arbitrary piecewise polynomial~~ activation functions whose output may be non-boolean (e.g. piecewise linear functions). One also gets a positive result about learning on multi-layer neural nets with a constant number of real valued inputs (in Valiant's model for probably approximately correct learning).

Finally we improve the best ~~kn~~ known lower bound for the Vapnik - Chervonenkis dimension of a neural net with w weights (and any common activation function) from $\Omega(w)$ to $\Omega(w \log w)$. This implies the somewhat surprising fact that the best known upper bound for nets with linear threshold gates (due to Cover, Baum, Haussler) is asymptotically optimal.

Models of Parallel Computation

Leslie Valiant, Harvard, Cambridge, USA

Two aspects of the bulk-synchronous parallel (BSP) model of computation are described. First it is argued that this is an appropriate pragmatic model for expressing the parallel complexity of algorithms in a machine-independent manner. For problems such as sorting and Gauss-Jordan elimination, such transportable algorithms can be developed that are efficient to within a factor of 1 (asymptotically as the problem size increases), when compared with a corresponding sequential algorithm, for wide ranges of the parameters of the model. (Joint work with A. Gerbessiotis)

Second, an algorithm for performing combining for arbitrary concurrent access patterns is described. The algorithm requires no combining within the router. It recirculates the requests through the router a small number, m, of times and performs the necessary combining at the processor nodes. For any $\varepsilon > 0$, if there are at most p^{ε} requests from each of the p nodes, and if the requests are to an appropriately hashed address space, then the algorithm takes time $(1 + o(1))\, m g p^{\varepsilon}$, ~~time~~ where $m = 1 + \lfloor \varepsilon^{-1} \rfloor$ and g is time per message that is charged. This is a factor of about m more than would be required on this model for access patterns requiring no combining.

Power sums mod p and a generalized Padé approximation problem

A. Schönhage

Over fields of characteristic zero parallel computation of matrix inverse A^{-1} or $\det A = \sigma_n$ (in the eigenvalues α_i of A) is easily done by computing $s_j = \mathrm{tr}(A^j)$ for $1 \le j \le n$ and then using Newton identities. Here this approach is adapted to fields of characteristic $p \le n$. One computes some extra power sums s_j for $j \in J(p,n)$ = first n elements of $\mathbb{N} \setminus p\mathbb{N}$;

from these sufficiently many coefficients $q_{k,i}$ in

$$\frac{a_k + a_{k+p} z + a_{k+2p} z^2 + \cdots}{1 + a_p z + a_{2p} z^2 + \cdots} = \sum_{i=0}^{\infty} q_{k,i} z^i \quad (1 \le k \le p-1)$$

are obtained; then the coefficients of the characteristic polynomial $f(t) = 1 + a_1 t + a_2 t^2 + \cdots + a_n t^n$ are determined by solving this Padé approximation problem.

Statistical Evidence for Small Generating Sets

For $n \in \mathbb{Z}^+$, let $G(n) = \min\{x : (\mathbb{Z}/n\mathbb{Z})^* \text{ is generated by primes } \le x\}$.

We present heuristic arguments and numerical data supporting the conjecture that

$$\limsup_{n \to \infty} \frac{G(n)}{\log n \log\log n} = \frac{1}{\log 2}.$$

We prove that $\frac{1}{N} \sum_{n=1}^{N} G(n) \gtrsim \log\log N \log\log\log N$.

If our conjecture is true, ~~[illegible]~~ there is a deterministic primality test using $O(\log n)^2$ multiplications mod n.

Eric BACH

Univ. Wisconsin (Madison)

Towards a Computational Theory of Statistical Tests

We initiate a computational theory of statistical tests. Loosely speaking, an algorithm is said to be a statistical test if it rejects ~~all~~ only a negligible fraction of all possible strings. We say that a test is universal for a class of tests iff it rejects all (but ~~a~~ finitely many) strings which are rejected by each test in the class.

We consider the existence of efficiency of universal statistical tests for various complexity classes. We also

(to be cont.)

(cont.) consider the relation between ensembles ~~accepted~~ passing statistical tests of a particular complexity and ensembles which are pseudorandom w.r.t. this complexity class.

Some of our results refer to particular relatively simple complexity classes such as finite state machines and counter machines. We hope that those results will stimulate investigations directed towards results concerning logspace machines.

– Joint work with Manuel Blum of UC-Berkeley

Oded Goldreich
CS Dept. TECHNION
HAIFA ISRAEL

Quadratic Dynamical Systems

Alistair Sinclair, University of Edinburgh & ICSI, Berkeley

The purpose of this talk is to promote the study of computational aspects of nonlinear systems from a combinatorial perspective. We identify the class of symmetric quadratic systems. Such systems have been widely used to model phenomena in the natural sciences, and also provide an appropriate framework for the study of genetic algorithms in combinatorial optimisation. We prove several fundamental properties of these systems, notably that every trajectory satisfying a certain condition converges to a fixed point.

We go on to give a detailed analysis of a quadratic system defined in a natural way on probability distributions over the set of matchings in a graph. In particular, we prove that convergence to the limit requires only polynomial time in the case that the graph is a tree. This result demonstrates that such systems, though nonlinear, are amenable to quantitative analysis.

Joint work with Yuri Rabinovich and Avi Wigderson.

Decision complexity of generic complete intersections

We study the complexity of algebraic decision trees that decide membership in an algebraic subset $X \subseteq R^m$ where R is a real (or algebraically) closed field. We prove a general lower bound on the verification complexity of the vanishing ideal of an irreducible algebraic subset $X \subseteq R^m$ in terms of the degree of transcendency of the minimal field of definition. As an application we determine exactly the number of additions, subtractions and comparisons that are needed to test membership in a generic complete intersection $X = Z(f_1, \dots, f_r) \subseteq R^m$; for the number of multiplications, divisions and comparisons needed we obtain an asymptotically optimal lower bound as $\max \deg f_i \to \infty$. A further application is given to test problems related to partial or continued fractions.

Peter Bürgisser, University of Bonn

Computing Irreducible Representations of Supersolvable Groups

Ulrich Baum, Bonn (joint work with Michael Clausen) 17. Nov. 92

We present an algorithm that, given a power-commutator-presentation of a supersolvable group G, computes a full set of inequivalent irreducible and monomial ordinary matrix representations of G in time $O(|G| \log |G|)$. The algorithm is based on Clifford Theory and adapting the representations to a chief series of G. Only symbolic calculations in a suitable group of roots of unity are required; no field arithmetic is needed at all. The output is valid over any field containing a suitable primitive root of unity (a primitive exp(G)-th root of unity will do).

Efficient construction of a small hitting set for Combinatorial Rectangles

Michael Luby, International Computer Science Institute
(joint work with Nati Linial, Michael Saks and David Zuckerman)

We consider a natural class of witness sets and design a polynomial time deterministic algorithm for the associated witness problem. Let d and m be positive integers, and let U be the universe that is the d-dimensional finite lattice with m ~~values~~ points per dimension, i.e. $U = [m]^d$ and thus $|U| = m^d$. We consider witness sets that are combinatorial rectangles, i.e. sets of the form $R = R_1 \times \dots \times R_d$, where, for all $i \in \{1, \dots, d\}$, $R_i \subseteq [m]$. The volume of R, $vol(R)$, is defined as $|R|/m^d$, i.e. it is the fraction of points in U that lie in R. The algorithm produces a set of points $S \subseteq U$ such that, for any R where $vol(R) \geq \varepsilon$, $S \cap R \neq \emptyset$. Both the running time of the algorithm and $|S|$ are polynomial in $\log(d)$, m and $\frac{1}{\varepsilon}$.

It is easy to see that a uniformly selected random set of points S in U of size polynomial in $\log(d)$, m and $\frac{1}{\varepsilon}$ hits every rectangle of volume at least ε with high probability. However, this does not provide a solution to the problem we consider: the crucial property missing from this proof of existence is efficient constructibility.

This work was motivated by the problem of finding efficient constructions of small sample spaces that approximate the independent uniform distribution on many multivalued random variables. The set S can be viewed as a sample space for random variables $X = X_1, \ldots, X_d$. The property that S has is that if the event $X \in R$ has probability at least ε of occurring with respect to the independent distribution then $X \in R$ has non-zero probability of occurring with respect to S.

The construction we present can also be viewed as an efficient deterministic solution to the generalization of the battleship game in d dimensions. We can view a "battleship" as a combinatorial rectangle R, and we can view S as defining a deterministic, efficiently constructible, short probe sequence that hits all battleships of size at least ε.

A Sharp Worst-Case-Analysis of the Gaussian Lattice Basis Reduction Algorithm for any Norm

Michael Kaib, Universität Frankfurt (joint work with Claus Schnorr)

We study the reduction of 2-dimensional lattices in a real vector space with arbitrary norm. We prove for any norm that the Gauß reduction algorithm terminates after at most $\log_{1+\sqrt{2}}(2\sqrt{2}\,B/\lambda_2) + o(1)$ many iterations, where B is the maximum of the norms of the two input vectors and λ_2 is the second successive minimum of the lattice with respect to the given norm. This bound is sharp for any norm and any lattice.

Precise Average Case Complexity Measures

To measure the complexity in the average case Levin has proposed a modification of the classical measure, which is obtained by taking the expectation. His motivation was to overcome problems with the expectation when trying to set up a theory of average case complexity classes. But this new measure basically can only differenciate between polynomial and super polynomial complexity.

We define and analyse a new measure obtained from monotone transformations of the probability distributions. It is shown that in this case only the ranking of the inputs by decreasing probabilities matters. As a main result we obtain tight time hierarchy results for average case complexity classes comparable to those for worst case classes. Thus, this measure turns out to be very precise. Also, a tight separation with respect to the complexity of the distributions involved – their rankability – can be established. Finally, we consider reductions and completeness in this new approach and propose a classification of NP-problems with respect to their average case behaviour.

joint work with Christian Schindelhauer

Rüdiger Reischuk
TH Darmstadt

Minimum Degree Steiner Tree Approximation

Martin Fürer Pennsylvania State University
(joint work with Balaji Raghavachari)

There is a polynomial time deterministic algorithm to compute a spanning tree of degree at most $\Delta+1$ for every graph for which a spanning tree of degree Δ exists. The same result holds for Steiner trees, whereas the directed version of the minimum degree spanning tree problem can be approximated by a spanning tree of degree $O(\Delta + \log n)$. To compute the minimum degree is well known to be NP-hard in all of these three cases.

An Approximation Algorithm for Counting the Number of Zeros of Polynomials over GF[q]

Marek Karpinski, Univ. Bonn.

We design the first polynomial time (for an arbitrary and fixed field GF[q]) (ε, δ)-approximation algorithm for the number of zeros of an arbitrary polynomial $f(x_1, \ldots, x_n)$ over GF[q]. This extends the recent approximation algorithms for the case of GF[2] (Karpinski Luby '91), and gives the first efficient method for estimating the number of zeros and nonzeros of multivariate polynomials over small finite fields other than GF[2].

The algorithm is based on the tight upper bounds proved on the sampling ratios for the number of zeros
now

of certain polynomials over $GF[q]$ in the function of the number m of terms only. The bound is proven to be $m^{\log_2 q}$, sharply. (Joint work with D. Grigoriev)

The shrinkage constant is 2.

Johan Håstad, Royal Institute of Technology, Stockholm

Suppose we have a Boolean formula of ~~a~~ size L, and that we hit this formula with a random restriction from R_p i.e. for each variable x_i we independently

keep as variable		with probability	p
replace by	0	-"-	$\frac{1-p}{2}$
-"-	1	-"-	$\frac{1-p}{2}$

After this we do the following simplifications of the formula:

At an $\vee$-gate

1) If one input is 1, replace by 1.
2) If both inputs are 0, replace by 0
3) If one input 0, use other as output
4) If one input reduces to the variable x_i ($\bar{x}_i$) substitute $x_i = 0$ (1) in subformula giving other input.

We have similar rules at $\wedge$-gates.

We prove that the expected size of the reduced formula is bounded by $O(p^2 (\log p^{-1})^{3/2} L + p\sqrt{L})$. This is optimal up to the factor $(\log p^{-1})^{3/2}$. As a corollary we obtain a formula size lower bound $\Omega(n^{3-o(1)})$ for a simple explicit formula

Complexity of effective Nullstellensätze (joint work with J. Heintz)

Marc Giusti, CNRS / Ecole Polytechnique

We consider the following problems: given $f_1, \dots, f_s$ in $k[x_1, \dots, x_n]$ (k infinite perfect field), let $V = \{ f_1 = f_2 = \dots = f_s = 0 \}$ the variety they define in $\mathbb{A}^n(\bar{k})$.

(i) Decide whether V is empty and if so, find $p_1, \dots, p_s$ in $k[x_1, \dots, x_n]$ such that $1 = p_1 f_1 + \dots + p_s f_s$ (Nullstellensatz)

(ii) Decide whether some given F vanishes on V, and if so, find $p_1, \dots, p_s$ s.t. $F = p_1 f_1 + \dots + p_s f_s$ (Nullstellensatz)

(iii) Suppose V is a reduced complete intersection. Same problem as (ii)

(iv) Compute the dimension of V, and if it is zero, given a linear form $\ell \neq 0$ in $k[x_1, \dots, x_n]$, find a non trivial polynomial p in $k[\ell]$ and $p_1, \dots, p_s$ in $k[x_1, \dots, x_n]$ such that $p(\ell) = p_1 f_1 + \dots + p_s f_s$. (Zero-dimensional elimination problem)

(v) Find the set-theoretic equidimensional (or irreducible) components of V.

Let d be the maximum of n and the degrees of the f_i's and F. After recalling from previous works that all these problems can be solved by uniform algorithms in sequential time $s^{O(1)} d^{O(n^2)}$ and parallel time $O(n^4 \log^2 sd)$, we explain how a change of data structure for representing the multivariate polynomials allows to solve some of them with well parallelizable probabilistic (randomized) algorithms in polynomial sequential time $s^{O(1)} d^{O(n)}$ (= polynomial w.r.t. dense repr. of the input)

Namely we give up the dense representation to use a mixed representation involving straight-line programs, for coding intermediate and output polynomials)

A important application is an effective Nullstellensatz (i) of optimal complexity.

We quoted also two examples due to Heintz-Morgenstern which show that the existence of uniform algorithm solving (iv) in sequential polynomial time of the input, represented only with straight-line programs, will imply $P = NP$ and a polynomial time computation of the $\mathbb{Q}$-permanent.

On randomized semialgebraic decision complexity

Thomas Lickteig, Universität Bonn

We study the impact of randomization on deciding membership in an (semi)algebraic subset of $\mathbb{R}^n$. Examples are exhibited where randomization definitely reduces the decision complexity. We also show that randomization cannot help in certain cases.

(joint work with Peter Bürgisser and Marek Karpinski)

Models for Average Case Complexity

Ingrid Biehl, Universität Saarbrücken

L. Levin developed a definition for "a function f is polynomial on average with respect to a given distribution μ".
We study the question, whether there are other ~~m~~ reasonable average-case models, and develop a definition for "average-case model", which has some good properties and allows to prove ~~a lot of~~ ~~similar~~ some results about time and space, determinism and nondeterminism similar to well-known results from worst-case complexity theory.

Quantum Complexity Theory

Umesh Vazirani, UC Berkeley

In its modern form, the Church-Turing thesis asserts that any reasonable (physically realizable) ~~model~~ computing device can be simulated with at most polynomial slowdown by a probabilistic Turing Machine. About a decade ago, Feynman pointed out that no straightforward simulation of a quantum physical system appeared possible without an exponential slowdown. A precise model of a quantum physical computer — the quantum Turing Machine — was formulated by Deutsch.

Although the QTM is ~~a~~ finitely specified by its state transition diagram, it is well-formed (realizable) only if it is time-reversible. We give a completely local criterion for checking whether a QTM is well-formed. This is the starting point of our first result - the existence of a universal QTM. What distinguishes the construction of a universal QTM from the classical case are the conflicting requirements of preserving both reversibility & quantum interference.

We also give the first evidence that QTMs might be more powerful than classical probabilistic TMs. We show how to sample according to the Fourier power spectrum of a n-bit boolean function in polynomial time on a QTM. This problem is not known to be polynomially solvable on a PTM. By specifying the ~~sampling problem~~ function by an oracle, and building on the sampling problem using recursion, we show that relativized QP is not even contained in one round Arthur-Merlin with $o(n^{\log n})$ time verifiers.

(Joint work with Ethan Bernstein)

<u>Complexity of propositional logic</u>

Jan Krajíček, Prague, Mathematical Institute

The following is a combinatorial situation encountered in lower bounds to the size of constant-depth propositional proofs.

Let $\varepsilon > 0$, n large and $k \approx n^{\varepsilon}$. M is a set of partial partitions of $2n+1 = \{0, \dots, 2n\}$ into 2-element classes. For $h_1, h_2 \in M$, h_1 and h_2 are <u>compatible</u> iff $h_1 \cup h_2 \in M$, and $|h_1|$ is the number of elements covered by h_1.

A <u>k-complete system</u> is any $S \subseteq M$ such that: (i) $\forall h \in S$, $|h| \leq k$, (ii) $\forall h_1, h_2 \in S$, $h_1 = h_2$ or h_1 and h_2 incompatible, (iii) $\forall f \in M$, $|f| + k \leq 2n \rightarrow \exists h \in S$, h and f compatible.

<u>A k-evaluation</u> of a formula φ (built from atoms p_{ij}, $0 \leq i,j < 2n+1$, $0, 1, \neg$ and $\vee$) is a pair of maps H, S assigning to any subformula ψ of φ a k-complete system S_ψ and $H_\psi \subseteq S_\psi$

such that:

(i) $S_0 = S_1 = S_{p_{ii}} = \{\emptyset\}$, $H_0 = \emptyset$, $H_1 = H_{p_{ii}} = \{\emptyset\}$,

(ii) for $i_0 \neq j_0$, $S_{p_{i_0j_0}} = \{p_{i_0j_0}\} \cup \{p_{i_0j}\, p_{ij_0} \mid$ all i, i_0, j, j_0 different$\}$

and $H_{p_{i_0j_0}} = \{p_{i_0j_0}\}$,

(iii) $S_{\neg\varphi} = S_\varphi$ and $H_{\neg\varphi} = S_\varphi \setminus H_\varphi$,

(iv) if $\varphi = \bigvee_u \varphi_u$ and none of φ_u starts with $\bigvee$, then

(a) $\forall h \in S_\varphi$, either h is incompatible with all $f \in \bigcup_u H_{\varphi_u}$ or h contains some $f \in \bigcup_u H_{\varphi_u}$,

(b) $H_\varphi = \{h \in S_\varphi \mid h$ contains some $f \in \bigcup_u H_{\varphi_u}\}$.

Lemma: If $H_\varphi \neq \emptyset$ in ~~any~~ all k-evaluations of φ then the parity principle requires exponential size constant-depth proofs from φ.

Parity principle says that the relation $\{(i,j) \mid p_{ij} = 1\}$ is not a total partition of $2n+1$ into 2-element classes. It is open whether the hypotheses of the lemma is satisfied when φ is an instance of MOD_3-principle (saying that $3l+1$ cannot be partitioned into 3-element classes) for formulas built from atoms p_{ij}.

Euclid, Gauss, LLL

Average-case analyses of three algorithms

Brigitte Vallée, Université de Caen

First, we begin by recalling some results on the average-case analysis of the Euclidean Algorithm. These results describe the behaviour of the length of the fundamental intervals associated to the continued fraction expansion;
In the second part, we describe a version of the Gauss Algorithm, which is the "central part" of the usual one. The average-case analysis is based upon the area of fundamental disks built on fundamental intervals of Euclid's algorithm. We prove that

(i) the number of iterations of Gauss's algorithm is asymptotically constant [i.e independent of the length of the vectors of the basis]; this constant is explicit and equal to

$$\beta = \frac{\pi}{\zeta(4)} \sum_{m \geq 1} \frac{1}{m^2} \sum_{n \in I_m} \frac{1}{n^2}$$

$$I_m = \{ n \mid \frac{m}{\Phi^2} \leq n \leq \frac{m}{\Phi} \}$$

where Φ is the golden ratio. $[\beta \simeq 1.08]$.
[Flajolet, V. 90].

ii) $\Pr[K \geq k]$ has a "geometrical" behaviour, i.e
$\exists a, b$ such that

$a \simeq 0.06$ $\quad a^k \leq \Pr[K \geq k] \leq b^k$.

$b \simeq 1.01$

...

We ask also some open questions about the existence of

i) the limit of $\log \Pr [K \geq k]$ when k tends to infinity.

ii) the limit of the distribution after the n^{th} step of the algorithm.

In the third part, we obtain

i) An upper bound for the average number L_n of iterations of the LLL algorithm with the parameter t $(t>1)$ in n dimensions.

$$L_n \leq n^2 \log_t n$$

[Daudé, V. 91]

ii). We describe a variant of the LLL algorithm, which is called "the gram Algorithm", for which we can obtain an upper bound for L_n, linear in the dimension n; This result is obtained under a "natural hypothesis" not yet proved

On the realization complexity of iterations of Boolean maps

O.B. Lupanov (Moscow State University, Russia)

Let $\mathfrak{M} = \{\tilde{\sigma}_1, ..., \tilde{\sigma}_M\}$ be a set of binary strings of length n; let $\mathfrak{S}_{\mathfrak{M}}$ be the set of all one-to-one maps $\mathfrak{M} \leftrightarrow \mathfrak{M}$. For any F from $\mathfrak{S}_{\mathfrak{M}}$ let $A_F(\tilde{x}, \tilde{y})$ be the following function:
$A_F(\tilde{\sigma}, \tilde{\tau}) = \underbrace{F(F(\ldots F(\tilde{\sigma})\ldots))}_{|\tilde{\tau}| \text{ times}}$ ($|\tilde{\tau}|$ denotes the number, the binary notation of which is $\tilde{\tau}$). Let us consider

all possible extensions of F and A_F to the outside of $\mathfrak{M}$. The complexity of a function f is defined to equal to the minimal number of elements which is sufficient for realization of f by a scheme of functional elements over the basis $\{\&, \vee, ^-\}$. Let $L^*(F)$ denote the complexity of the simplest extension of A_F to the outside of $\mathfrak{M}$, and let

$$L^*(\mathfrak{M}) = \max_{F \in \mathfrak{S}_{\mathfrak{M}}} L^*(F), \quad L^*(n, M) = \max_{\mathfrak{M}} L^*(\mathfrak{M})$$

($\mathfrak{M}$ has M strings of length n).

<u>Theorem</u>. If $\frac{M}{\log n} \to \infty$ then

$$L^*(n, M) \sim \frac{Mn}{\log_2 (Mn)}.$$

The proof of the theorem is based on the principle of local coding of the author, along with certain version of the result of D. Uhlig on the simultaneous realization of a function on several strings (mass-production), some modifications of certain theorems on the complexity of partial functions (E. I. Nechiporuk, N. P. Red'kin, A. E. Andreev) and some bounds of formula depth of certain functions (V. M. Khrapchenko); there is also a certain amount of „programming" in the terms of circuits.

Parallel Sparse Linear System Solving

Erich Kaltofen Rensselaer Polytechnic Institute
Troy, New York

In our algorithms, a sparse matrix is a matrix that has an efficient algorithm for multiplying it ~~with~~ a by a vector.
D. Wiedemann in 1986 invented an algorithm that can find the solution of a non-singular linear system with a sparse coefficient matrix in $O(N)$ matrix times vector operations and additionally $O(N^2)$ arithmetic operations in the coefficient field; here N is the dimension of the (square) matrix. D. Coppersmith in 1992 showed how this approach could be parallelized. With n processors, the parallel time is then $O(N/n)$ matrix times vector operations, and an additional $O(n N^2)$ sequential field operations. Both algorithms are randomized.

We show that if the matrix has the property that the degree of the minimum polynomial is equal to the rank plus 1, the parallel algorithm has a high probability of finding such a solution. This condition can be also enforced by pre- and postmultiplying by random triangular Toeplitz matrices and then post-multiplying by a random diagonal matrix. We have also implemented the method on a network of 8 Sun Sparc workstations. A system of dimension 10,000 with 300,000 non-zero entries over $GF(2^{15}-29)$ can be solved in two days.

Learning & Cryptography

We show that if a family of circuits does ~~A class of circuits~~ not contain a pseudo-random number generator, then circuits in that class are weakly learnable. As a corollary, if depth 2, poly-size circuits do not contain a pseudorandom number generator, then polynomial-size DNF is weakly learnable. If constant depth, poly-size circuits do not contain pseudo-random number generators, then there is a function f, on log n bits which can "hide" its inputs among n bits – after a problem suggested by Avrim Blum. (Joint work w/ R. Lipton)

Merrick L. Furst
Carnegie Mellon University

Topological Buildings

22.11 – 28.11.1992

Topological buildings – an approach via projections

Martina Jäger, Christian-Albrechts-Universität Kiel

Let Δ be a spherical building with vertex set $V = \bigcup_{i=1}^{n} V_i$ such that each V_i consists of all vertices of a fixed type; let each V_i carry a topology. This also provides a topology on Δ.

For each $k \in \{1, \dots, n\}$, consider the set

$$D^k := \{ (u,v) \in V^2 \setminus I \mid \mathrm{proj}_u v \text{ has a vertex of type } k \}$$

(where I denotes the incidence relation and proj the projection mapping on Δ)

and the function

$$p^k : D^k \to V^k : (u,v) \mapsto \text{vertex of type } k \text{ of } \mathrm{proj}_u v .$$

Definition. Δ is a topological building, if all p^k are continuous.

Comparing topological buildings with topological projective spaces and topological generalized n-gons, we have the following results:

A projective space of any dimension is a topological building, if and only if it is a topological projective space.

A generalized quadrangle is a topological building, if and only if it is a topological generalized quadrangle.

In the case of generalized n-gons such that $n \geq 5$ we only have:

A generalized n-gon, which is a topological building, is a top. gen. n-gon.

If S is a simplex in Δ, the set $\mathrm{Link}\, S := \{ X \in \Delta \mid X \, I \, S, X \not\ni S \}$ is a building as well.

Considering the question, whether Link S is a top. building, provided Δ is a top. building, one needs to calculate the projection mapping in Link S, denoted by ${}^{L.S}\mathrm{proj}$. By induction, it suffices to look at the case where S is a vertex.

Let S be a vertex of Δ and let u, v be vertices of Link S. It is easy to prove that $(\mathrm{proj}_u v) \setminus \{S\} \subseteq {}^{L.S}\mathrm{proj}_u v$. The other inclusion can be proved by considering a geometrical realization of an apartment of Δ in the vector space $\mathbb{R}^n$ and a geometrical realization of an apartment of Link S in a hyperplane of $\mathbb{R}^n$.

This shows that Links in top. buildings are again top. buildings.

Topological buildings - an approach via convex hulls

I defined a topological spherical building by requiring continuity of the map sending a pair of opposite chambers to the apartment they span, where this apartment is regarded as the set of its chambers. In addition, I need the following two conditions: the set of pairs of opposite chambers is open and the canonical projection from chambers to vertices is open for each type. For technical reasons the chambers obtain their topology ~~as~~ as a subspace of the product of the T_2-topologies of the vertices. It is an important consequence ~~of~~ this definition is equivalent to the usual one of a topological projective space in the case of ~~a~~ building of type A_n. For buildings of type C_2 my definition implies the one of a topological generalised quadrangle, but the converse probably is not true. So my approach seems to be somewhat stronger. In case of buildings of type C_n I deduce several continuous maps between vertices ~~of~~ in certain distance. Here the main result yields a residue that is a topological projective plane, hence by the result for buildings of type A_n a topological A_2-building.

Regina Kühne

Technische Universität Braunschweig

Topological buildings, the approach of Burns-Spatzier

The concepts and results in a paper by Burns-Spatzier (Publ. IHES 65, 1987, 5-34) are discussed. Their definitions appear to be appropriate in the compact case only. Another definition of topological buildings, due to Linus Kramer, requires continuity of projections of the building on suitable domains. For projective spaces and generalized polygons, this definition reduces to the usual one, and it is compatible with the definition of Burns–Spatzier in the compact case.

Moduli spaces of compact projective planes

For a compact topological space X, the set of all isomorphism classes of topological projective planes with point space X can be endowed with a natural topology. We dare to call this the moduli space $\mathcal{M}_X$ of topological projective planes on X. The planes with large automorphism groups are expected to be singular points of $\mathcal{M}_X$.

Theo Grundhöfer (Univ. Tübingen)

Coordinates in spherical buildings

A spherical building admits a decomposition into Schubert cells, and these cells are products of punctured panels in a rather natural way. If the building carries a good topology (e.g. compact Hausdorff), then this decomposition is very similar to a CW-complex decomposition. This has the following applications: (i) the underlying field becomes a topological field, and the correspondence between the building topology and the field topology is unique. (ii) root collineations are automatically continuous (iii) The cohomology of the space of J-flags Δ_J is $\mathbb{Z}_2^{|W/W_J|}$, if Δ_J is connected and finite dimensional.

Polygons with flag transitive automorphism groups.

Theorem: Let $\mathfrak{P}$ be a compact connected flag homogeneous polygon. If the parameters dim Line and dim pencil are equal, then $\mathfrak{P}$ is classical, i.e. $\mathfrak{P}$ arises from the canonical BN-pair of a simple Lie group.

Linus Kramer, Universität Tübingen

Slanted symplectic quadrangles

A simple modification of symplectic generalized quadrangles yields examples of non-classical generalized quadrangles. The finite quadrangles of this type are well known.

With the exception of the two ~~sm~~ smallest thick examples, all those quadrangles inherit their group of automorphisms from the classical symplectic quadrangle (as a stabilizer of a point). These are not Moufang.

Furthermore, it turns out that all even finitary permutations are projectivities of the slanted symplectic quadrangles. As a consequence, the slanted quadrangles cannot be turned into topological quadrangles.

Michael Joswig, Uni Tübingen

Geometries fixed by an automorphism of a building

The diagrams used by Tits in order to classify the semi-simple algebraic groups have a natural geometric interpretation. They provide Coxeter complexes which are "weak subcomplexes" of a given one. These subcomplexes consist of simplices fixed by a group of automorphisms of the Coxeter complex in question. This observation can be easily generalized to spherical buildings, which provides a geometric version of the theory of forms of simple algebraic groups.

Bernhard Mühlherr (Uni Tübingen)

Half regular and regular points in compact polygons

The notion of half regular and regular points is introduced. (Half) regularity allows one to define derived structure. For compact ~~hexagon~~ polygons the derived structure yields a projective plane if linepencils and ~~line~~pointrows are homeomorphic. The derived structure of a ~~qua~~ compact quadrangle is a topological projective plane and the derived structure of a compact hexagon is a compact quadrangle, if the point is regular and if linepencils and pointrows are homeomorphic. These results are put together to give a characterization of the symplectic quadrangle over $\mathbb{R}$ or $\mathbb{C}$ and the split Cayley hexagon over $\mathbb{R}$ or $\mathbb{C}$.

Andreas Schroth; TU Braunschweig

The commutativity of the ground division ring of a D_n-geometry

If Γ is a thick and residually connected D_n-geometry, $n \geq 4$, it is well known that Γ is defined over a unique ground division ring which is commutative. I give an elementary proof of the commutativity based on the construction of null polarities in the projective subspaces of Γ, for $n=4$.

Cécile Huybrechts, Bruxelles.

(For more information, see p. 209)

Line-spaces and buildings.

I explained how to make a connection between buildings and line-spaces. In the way buildings to line-spaces, I used the concept of shadow-space; in the other way, a "right" choise of types.

The concept of space is used to gain insight into other structures. An essential aspect of linear-spaces theory is concerned with axioms; that is given a set S of algebraic structure, the goal is to list a set of propertiess of line-spaces such that any space with those properties corresponds to a member of S.

An axioms system in terms of points and lines is well known for the shadow-spaces $C_{n,k}$ where $k \neq n-1$ by example polar spaces if $k=1$ and dual polar spaces if $k=n$.

I gave an axiom system in term of points and lines for suboctaedric spaces that is the case $C_{3,2}$.

LEHMAN, Serge. BRUXELLES (ULB)

AMALGONS, OVOIDS AND UNITALS

(I) Let $S = (P, L, I)$ be a generalised quadrangle or hexagon containing a flag $\{p, L\}$ with p regular and L regular. The derived geometries are respectively S_p and S_L. I describe a method to reconstruct the polygon S from the two polygons S_p and S_L. This procedure is called "Amalgamation" and S the Amalgon.

(II) Self polar generalised n-gons contain ovoids (also called unitals for $n=6$). We give a construction of the "classical" ovoids in $Q(4, \mathbb{F})$ and $H(\mathbb{F})^{dual}$, and raise the question of finding ~~ovoids~~ and other ovoids in $Q(4, \mathbb{R})$, $H(\mathbb{R})$, $W(\mathbb{R})$, $H(\mathbb{R})^{dual}$ which are compact and connected.

We also give a geometric construction of the Ree-Tits unital in $G_2(3^{2e+1})$ (joined work with Hendrik De Smet)

H. Van Maldeghem
State University Gent.

Large Deviations and Applications

Exponential large deviations of the mean under spherical distributions

Consider an n-dimensional sample $X = (X_1, \dots, X_n)'$ under a spherical distribution, i.e. $X = \mu + e$, $\mu \in \mathbb{R}^n$, where the distribution of the error vector e has a λ^n-density

(1) $$f(x;g) = c(n,g)\, g(\|x\|^2), \quad x \in \mathbb{R}^n,$$

generated by a nonnegative measurable function g with positive normalization $c(n;g)$.

We are interested in convergence rates of the least squares estimate of a possible common mean of the X_i's, that is, we want to investigate the large deviations

$$P(A_n) = P(|\bar{X}_n - \bar{\mu}_.| > \varepsilon) = 2P\Big(\sum_{i=1}^n (X_i - \mu_i) > \varepsilon\Big),$$

where $\varepsilon > 0$, $\bar{X}_n = \frac{1}{n}\sum_{i=1}^n X_i$, $\bar{\mu}_. = \frac{1}{n}\sum_{i=1}^n \mu_i$.

For a class of spherical distributions generated by a function g of type

(2) $$g(r^2) = a r^b e^{-cr^d}, \quad r > 0,$$

(a, b, c, d positive constants), the following large deviation results can be established:

THEOREM. Under spherical distribution of X according to (1) with g as in (2), we have as $n \to \infty$,

(i) if $d \geq 2$, then $\log P(A_n) \sim -c(n\varepsilon^2)^{d/2}$

(ii) if $1 < d < 2$, then $\log P(A_n) \sim -\frac{1}{2}(cd)^{2/d}\, n^{2(1-\frac{1}{d})}\, \varepsilon^2$,

(iii) if $0 < d \leq 1$, then $P(A_n)/e^{-\alpha n^\beta} \to +\infty$ for all $\alpha, \beta > 0$
['no exponential rate'].

Josef Steinebach, MARBURG

Large scale dynamics in stochastic models for interfaces

The statistical mechanics of surfaces is modelled conveniently in terms of effective interface models. They are given by a real valued field, ϕ, over the lattice $\mathbb{Z}^d$. The surface is the graph of this function. The field has the energy

$$H = \sum_{\langle x,y \rangle} V(\phi(x) - \phi(y)) \quad ,$$

where $\langle x,y \rangle$ is a pair of nearest neighbors and V is convex and bounded as $V(\phi) \geq c\,|\phi|^{1+\delta}$, $\delta > 0$. Clearly H is invariant under the global shift $\phi(x) \rightsquigarrow \phi(x) + a$, which is needed to have the interpretation of a surface energy. To H there corresponds a d-parameter family of Gibbs measures. They should be thought of being defined on the difference variables $\phi(y) - \phi(x)$, $|x-y| = 1$. They are defined by taking the infinite volume limit at fixed tilt, $\phi(x) = u \cdot x$ for $x \in \partial\Lambda$.

We consider pure relaxational dynamics

$$d\,\phi_t(x) = -\frac{\partial H}{\partial \phi(x)}(\phi_t)\,dt + dW_t(x)$$

with independent Brownian motions at each site. The goal is to prove a law of large numbers in the form

$$\lim_{\varepsilon \to 0} \varepsilon^d \sum_x f(\varepsilon x)\, \varepsilon\, \phi_{\varepsilon^{-2}t}(x) = \int d^d r\, f(r)\, h(r,t). \qquad (*)$$

The macroscopic height profile should satisfy

$$\frac{\partial}{\partial t} h_t = \mu \sum_{\alpha=1}^{d} \frac{\partial}{\partial r_\alpha} \sigma_\alpha(\nabla h_t) \, .$$

with μ the mobility and σ the surface tension, $\sigma_\alpha(u) = \frac{\partial}{\partial u_\alpha}\sigma$. Elements of the proof of $(*)$ are discussed.

H. Spohn, München

Erdös-Rényi laws and Gibbs measures

Erdös-Rényi type of laws state that in a given sample of size n, one will observe in subsamples of size $\frac{1}{t}\log n$ all deviations with rate of decay less or equal to t $(t>0)$, w.p.1 as $n \longrightarrow \infty$.

1) we give general formulations of this result, for the empirical field or process under the condition of uniform large deviation estimate (or hypermixing processes).

2) we give applications to Gibbs measures, and we study in this case the limit $t \longrightarrow 0$. The result then yields positive answers to questions like:

Can we detect phase transition from a single (but large) sample?

Can we learn some information on the other Gibbs measures?

Francis COMETS, Paris 7.

large deviations for a random walk in random environment

Let $\omega = (p_x)_{x\in\mathbb{Z}}$ be an i.i.d. collection of $(0,1)$-valued random variables. Given ω, let $(X_n)_{n\geq 0}$ be the Markov chain on $\mathbb{Z}$ defined by $X_0 = 0$ and $X_{n+1} = X_n \pm 1$ with probability p_{X_n} resp. $1-p_{X_n}$. It is shown that X_n/n satisfies a large deviation principle, i.e.,

$$\lim_{n\to\infty} \frac{1}{n} \log P_\omega(X_n = \lfloor \theta_n n \rfloor) = -I(\theta) \quad \omega\text{-a.s. for any } \theta_n \to \theta \in [-1,1].$$

First we derive a representation of the rate function I in terms of a variational problem. Second we solve the latter explicitly in terms of random continued fractions. This leads to a classification and qualitative description of the shape of I. In the recurrent case I is non-analytic at $\theta = 0$. In the transient case I is non-analytic at $\theta = -\theta_c, 0, \theta_c$ for some $\theta_c > 0$, with linear pieces in between.

This is joint work with A. Greven (Göttingen).

Frank den Hollander (Utrecht)

<u>Large deviation probabilities for some rescaled superprocesses</u>

Large deviations are discussed for the continuous super-Brownian motion in $\mathbb{R}^d$ in the case of an asymptotically small branching rate. Based on a complete blow-up property for the related cumulant equation some L^2-formula for the rate functional is derived. This formula might have some application, as well as might give some hints concerning an eventual general theory for large deviations for measure-valued diffusions behind this particular example of a super-Brownian motion.

Klaus Fleischmann, IAAS Berlin
(with Ingemar Kaj, Uppsala University)

Large deviations for Gibbsian point random fields

We present a large deviation principle for the stationary empirical fields for systems of marked point particles in boxes $\Lambda_n \uparrow \mathbb{R}^d$. The particle distributions are Gibbsian relative to one of the following types of interaction: 1) interactions of possibly infinite range with hard-core repulsion, 2) superstable pair interactions of finite range, 3) interactions of mean-field type depending on the particle marks, 4) for $d=1$: nearest-particle interactions. (In cases 2) and 4) we impose periodic boundary conditions.) Since the underlying topology is chosen fine enough, the contraction principle then gives us an LDP for the "individual empirical fields" defined by averaging over the particle positions. We also present a maximum entropy principle implying a general version of the equivalence of ensembles.

This is (in part) joint work with H. Zessin (Bielefeld).

Hans-Otto Georgii (München)

Thermodynamics of the Hopfield model

We consider the Hopfield model with $m \equiv \alpha N$ stored patterns where N is the number of neurons. We show that:

(i) in the case $\alpha = \alpha(N) \downarrow 0$ as $N \uparrow \infty$, then the free energy $f_{N,\beta} \to f_{C.W.,\beta}$ as $N \uparrow \infty$, a.s.

(ii) for each stored pattern ξ^μ, there ~~is a local measure a~~ exist Gibbs measures = the infinite volume limit s.t. the measures induced by the maps $m^\mu[\sigma] = \frac{1}{N}\sum_i \xi_i^\mu \sigma_i$ ~~on the~~ converge to delta measures on $\delta_{am^*(\beta)}$.

(iii) in the case $\alpha \neq 0$, with α sufficiently small we show ~~that existence of~~ the infinite volume measures are "close" to those in (ii), if $\beta > \beta_\alpha$, where $\beta_\alpha \downarrow 1$, also.

Joint work with Véronique Gayrard (Marseille)
Pierre Picco (Marseille)

Anton Bovier (Berlin)

A Stochastic Optimal Control Approach to the Theory of Large Deviations by Richard S. Ellis, Univ. of Mass.

This is joint work with Paul Dupuis of Brown University. We present a new and widely applicable approach to the theory of large deviations which is based on stochastic optimal control theory. In our opinion, this approach reduces many aspects of the theory of large deviations to the theory of weak convergence of probability measures. We demonstrate the versatility of the approach by applying it to three diverse large deviation problems:

(a) Small random perturbations of dynamical systems with continuous statistics.

(b) Small random perturbations of dynamical systems with discontinuous statistics.

(c) The empirical measures of Markov chains with continuous statistics and with discontinuous statistics.

While our main goal is to exhibit a general methodology, the technique allows, in the examples considered, a weakening of the assumptions that have previously been used in proving the large deviation principle. We also obtain a number of new results.

Large deviations and the isoperimetric problem in Ising model

The rate function of the empirical magnetization is computed explicitely in the case of coexistence of phases. The rate function is given by the minimum of a variant of the classical isoperimetric problem. The computation is done in two dimensions. If $\tau(n)$ is the surface tension in the direction $n \in \mathbb{R}^2$,

$\|h\|=1$, then for $-m^* < x < m^*$

$$\lim_{L\to\infty} -\frac{1}{L} \ln \mathrm{Prob}_{\mu^+_{\Lambda_L}}\Big(\frac{1}{|\Lambda_L|} \sum_{t\in\Lambda_L} \sigma(t) \sim x \Big) =$$

$$\min\Big\{ \int_\gamma \tau(n) : \gamma \text{ closed simple curve s.t. } \gamma\subset\Lambda_L \text{ and int.}\,\gamma \text{ has volume } \frac{m^*-x}{2m^*} \Big\}$$

where $\mu^+_{\Lambda_L}$ is the Gibbs measure in a square box Λ_L of volume L^2, with + boundary condition and $m^* = \lim_{L\to\infty} \mathbb{E}_{\mu^+_{\Lambda_L}}(\sigma(t))$.

Ch. Pfister (Lausanne)

<u>Brownian motion in a Poissonian potential</u>

A. S. Sznitman, ETH Zürich

I describe in this talk certain large deviation principles which govern the behavior of Brownian motion moving in a typical Poissonian potential. These large deviation principles involve the construction, via a shape theorem, very much in the spirit of first passage percolation, of certain constants which generalize the Lyapounov exponents in one dimension. These large deviation results then enable to study Brownian motion with a constant drift h moving in the same potential and describe the transition of regime which occurs between small h and large h,

Critical large deviations

Let P_0 be a product measure on $\Omega = E^{\mathbb{Z}^d}$ and denote by $R_N(\omega) = \frac{1}{|V_N|} \sum_{x \in V_N} \delta_{\theta_x \omega}$ the empirical field of the box $V_N = [1, N]^d$.
For a given interaction potential $\mathcal{J}$, define the approximate microcanonical distribution $\mu_{N,\delta}(\cdot) = P_0 \,|\, (|\mathcal{U}_N - u| \leq \delta)$, where $\mathcal{U}_N$ is the average energy of V_N. Large deviations show that the law of the empirical field R_N converges at a volume exponential rate on the set of Gibbs distribution at an appropriate inverse temperature $\beta = \beta(u)$. In case of phase transition, we expect that R_N concentrates on the extremal Gibbs states. We show that a surface exponential rate occurs for the Ising model. The critical estimate is a surface-order large deviations for the empirical magnetization of the free boundary Gibbs distribution. The method uses F-K percolation at sufficiently small temperature and the isoperimetric estimate.

J-D Deuschel ETH Zürich, (joint work with A. Pisztora and C. Newman)

The Stabilization of Statistics in Wave Equations with Mixing

A. Kometch (Moscow Univ.), E. Kopylova (Vladimir Polytech. Inst.), N. Ratanov (Chelyabinsk Univ.)

Thereexists many statistical equilibrium phenomena in physics, related to hamiltonian infinite dim. systems of math. phys. For ex., Gibbs meas-s in statistical mechanics, black-body emission low in electrodynamics. These phenomena lead us to a problem of "Statistical stabilization". This means that these statistics appear as $t \to \infty$ for the solutions of equations considered when the initial statistics at $t=0$ is "almost arbitrary".

We prove such stabilization for linear wave equation and also for the Klein-Gordon equation with constant or variable coefficients in $\mathbb{R}^n$, $n \geq 2$.

We assume initial statistics fit Rosenblatt-Ibragimov mixing condition and is homogeneous in $x \in \mathbb{R}^n$. In the case of constant coefficients we use the explicite formulas for solution and apply the extention of the "rooms-corridors" method of S.N. Bernstein, M. Rosenblatt, Ibragimov-Linnik. In the case of variable coefficients there is no of explicite formula. We reduce the case to constant coefficients case by the scattering theory. But the total energy of solutions considered is ∞ a.s. because of homogeneous initial data (and solutions). Then we must construct the scattering theorie for solutions of infinite energy.

The result is: the statistics of solutions at time t converge to some gaussian measure as $t \to \infty$. This is the analog of CLT for hamiltonian systems considered.*

This means the large deviations for solutions, considered in each bounded region of space $\mathbb{R}^n$: we can take the initial data very small bounded functions in $\mathbb{R}^n$ a.s. But, as $t \to \infty$, the solution at point considered (or the energy in region considered) may be arbitrary large.

A Kometch

*Let's note that the Gibbs measures for our linear equations "must" be gaussian, because their hamilton functions are the quadratic forms.

Finite and Infinite Systems of Interacting Diffusions

Ted Cox, Syracuse University

The subject of this talk is a theorem relating the asymptotic behavior of large finite systems of interacting diffusions and the corresponding infinite system. The infinite system $x(t) = \{x_i(t), i \in \mathbb{Z}^d\}$ is the Markov process determined by

$$(*) \quad dx_i(t) = \Big[\sum_{j \in \mathbb{Z}^d} a(i,j)\, x_j(t) - x_i(t)\Big]dt + \sqrt{g(x_i(t))}\, dW_i(t)$$

where $a(i,j)$ is an irreducible random walk kernel on $\mathbb{Z}^d$, $g: [0,1] \to \mathbb{R}^+$ is Lipschitz, $g(0) = g(1) = 0$, $g > 0$ on $(0,1)$, and $\{W_i(t)\}$ is a family of independent Brownian motions. There is a family $\{\nu_\theta, \theta \in [0,1]\}$ of invariant measures for $x(t)$, $E^{\nu_\theta} x_i = \theta$. The finite systems $x^N(t) = \{x_i^N(t), i \in (-N,N]^d\}$ are defined by an equation like $(*)$ treating $(-N,N]^d$ as a torous. The main result is that under some conditions, for $t_N \uparrow \infty$ with N, $t_N/(2N)^d \to s \in [0,1]$,

$$\mathcal{L}(x^N(t_N)) \Rightarrow \int_{[0,1]} Q(\rho, d\theta)\, \nu_\theta$$

where $Q(\rho, \cdot)$ is the transition of a certain diffusion on $[0,1]$. In particular, we see that if $t_N = o(N^d)$ as $N \to \infty$, $\mathcal{L}(x^N(t_N)) \Rightarrow \nu_\rho$, so that the invariant measures of the infinite system describe the behavior of the finite systems for times up to a certain order.

This work is joint with Andreas Greven and Tokuzo Shiga

Random Perturbations of Dynamical Systems with Conservation Laws.

Mark Freidlin, Univ. of Maryland at College Park, USA.

The evolution of first integrals along the trajectories of the perturbed system is considered. After proper rescaling of time the first integral converges to a diffusion process on a graph corresponding to the conservation Law. Under certain assumptions concerning the non-perturbed system on the level set of the first integral the limiting process turns out Markovian. The limiting process is defined by a family of second order differential operators and by a collection of gluing conditions in the vertices. The operators are the result of averaging over the ~~level~~ connected components of the level sets. The gluing conditions are calculated in the vertices of the graph corresponding to the saddle points of the first integral (if it is defined by a smooth function). Extremal points of the first integral corresponds to the vertices which are inaccessable for limiting process, and no boundary conditions should be given at these points.

Large deviations for Interacting Particle Systems

We study the large deviations of the space-time empirical averages of a d-dimensional stochastic spin system whose Markov semigroup is generated by the operator

$$Lf(\sigma) = \sum_{i \in \mathbb{Z}^d} c(\vartheta_i \sigma)\left[f(\sigma^i) - f(\sigma)\right]$$

where ϑ_i is the shift on $\{-1,1\}^{\mathbb{Z}^d}$ and $\sigma^i(j) = (-1)^{\delta_{ij}}\sigma(j)$.
We prove a n^{d+1}-large deviation principle for the empirical process

$$R_{n,\omega} = \frac{1}{n^{d+1}} \sum_{\substack{i \in \mathbb{Z}^d \\ i_j = 0,1,\dots,n-1}} \int_0^n \delta_{\vartheta_{t,i}\,\omega}\, dt ,$$

where $\omega \in \Omega = \mathbb{D}(\mathbb{R}, \{-1,1\}^{\mathbb{Z}^d})$ and $\vartheta_{t,i}$ are the space-time shift maps on Ω, and we identify the rate function. Moreover we prove that the zeroes of the rate function correspond to the invariant measures for the system. We also give results on some related problems, as the "contraction" to deviations of lower level and critical large deviations for non-ergodic systems.

P. Dai Pra, Università di Padova, Italy

Maximum Entropy Principle for Uniformly Ergodic Markov Chains

I extended results of Bolthausen and Schmock (1989) about the maximum entropy principle for the empirical process of strongly ergodic discrete-time Markov chains to more general empirical processes by putting more restrictive assumptions on the functional H of the empirical process $\{L_n\}_{n \in \mathbb{N}}$. Using a special construction, multivariate empirical processes and certain continuous-time Markov processes with continuous paths can be treated. The weak accumulation points of the transformed path measures

$$\hat{\mathbb{P}}_n(A) := \frac{\mathbb{E}[1_A \exp(nH(L_n))]}{\mathbb{E}[\exp(nH(L_n))]} + 1, \quad A \subset \Omega \text{ measurable},$$

are mixtures of Markov chains minimizing a certain free energy. The proof relies on large deviation results in the τ-topology for Markov processes, which are due to Bolthausen (1987)

Uwe Schmock,
Institut für Angewandte Mathematik
der Universität Zürich, Switzerland

Large deviations theorems for likelihood estimators

By A. A. Borovkov, A. A. Mogulskii (Inst. Math., Novosibirsk, Russia)

Let $a_1(\theta), a_2(\theta), \ldots$ be i.i.d. random fields in $(C(\Theta), \mathcal{B})$, where $C(\Theta)$ is linear space of continuous functions $f(\theta)$, $\theta \in \Theta$, and Θ is closed bounded subset of R^k.

We call a vector $\theta_n^+ \in \Theta$ at which $A_n(\theta) = a_1(\theta) + \cdots + a_n(\theta)$ attains its maximum a maximum point of $A_n(\theta)$:

$$A_n(\theta_n^+) = \max_{\theta \in \Theta} A_n(\theta).$$

The vector θ_n^+ is not uniquely defined. So we define „upper" and „lower" distributions of θ_n^+ by formulaes:

$$P_+(\theta_n^+ \in B) \equiv P\left(\max_{\theta \in B} A_n(\theta) \geq \max_{\theta \in \Theta \setminus B} A_n(\theta)\right),$$

$$P_-(\theta_n^+ \in B) \equiv P\left(\max_{\theta \in B} A_n(\theta) > \max_{\theta \in \Theta \setminus B} A_n(\theta)\right).$$

In this talk the object of study is the „fine" asymptotics of the sequence

$$P_\pm(\theta_n^+ \in B).$$

Spectral gap and logarithmic Sobolev inequality for Glauber and Kawasaki dynamics.

By Shenglin Lu and H. T. YAU

We prove that there is a spectral gap uniformly w.r.t. the volume and boundary condition for the Glauber dynamics. If the Glauber dynamics ~~are~~ is replaced by the Kawasaki dynamics then the spectral gap is proved to shrink by $1/L^2$. We assume some mixing conditions for the Gibbs state is ~~assumed~~ hold. Furthermore, we prove similar result for the logarithmic sobolev inequality except for the Kawasaki dynamics for dimension $d > 1$.

Courant Inst. of Math Sciences
New York University

Action functional for dynamical systems with discontinuities

A. Korostelev, Institute for System Analysis, Moscow

A well-known "continuous mapping" method is applied to a piecewise smooth dynamical systems having a surface of "stable discontinuity". For such a system disturbed by a standard white gaussian noise of a small intensity ε, i.e. for the solution of the stochastic equation

$$\dot{X}^{\varepsilon}(t) = b(X^{\varepsilon}(t)) + \varepsilon \dot{W}(t), \quad 0 \le t \le T, \quad \varepsilon \to 0, \quad X^{\varepsilon}(0) = 0,$$

the action functional (i.e. the rate function governing the L.D.) is obtained. The basic idea is that there exists a continuous mapping $F: C_{0,T} \mapsto C_{0,T}$, which is Lipschitz in the space of continuous functions $C_{0,T}$, and satisfies: $X^{\varepsilon}(t) = F(\varepsilon W)$. Moreover, there exists another mapping $G: C_{0,T} \mapsto C_{0,T}$ such that $G(\varepsilon W) = \pi^{\varepsilon}(t)$ where $\pi^{\varepsilon}(t) = \int_0^t I(X_1^{\varepsilon}(s) > 0)\,ds$, i.e. $\pi^{\varepsilon}(t)$ is the staying-time of $X^{\varepsilon}(t)$ in the positive half-space (we assume w.l.o.g. that the surface of discontinuity is described by $x_1 = 0$). If $\varphi \in C_{0,T}$, and $\psi = F\varphi$, $\mu = G\varphi$, then the inverse mapping has an explicit expression: $\varphi = \psi - \int b_+(\psi)\,d\mu - \int b_-(\psi)\,d(t-\mu)$ where $b_\pm$ are one-sided limits of b on the surface of discontinuity. Thus, appling the L.D.P for the Wiener process one gets without any cumbersome calculation the action functional for the joint process $(X^{\varepsilon}, \pi^{\varepsilon})$: $\quad I(\psi, \mu) = \frac{1}{2} \int_0^T \| \dot{\psi} - b_+ \dot{\mu} - b_-(1-\dot{\mu}) \|^2$.

The extensions are discussed to the jumping processes. It is known that if one of the three staying-times (in positive, negative, half-spaces, and that on the surface of discontinuity) is vanishing, then the approach applies. In particular, the LD's for the solution of $\dot{X}^{\varepsilon} = -c\,\mathrm{sgn}(X^{\varepsilon}) + \dot{\xi}^{\varepsilon}$, where ξ^{ε} is the rescaled Poisson process, are governed by the action functional

$$I(\psi, \mu) = \int_0^T L_0(\dot{\psi} + c\dot{\mu}) \quad \text{where } L_0(u) = 1 + u\log(u/e).$$

But the same equation noised by the two-sided Poisson process (jumps ± 1 w.p. 1/2) leads to a problem that has no simple solution.

Large deviations for branching diffusions

(T. Y. Lee)

For branching (multiplication rate $= \varepsilon^{-1} C$) Brownian motion (diffusivity $= \varepsilon D$) starting from the origin, write P^ε for probab. measure (and E^ε for expectation). We ask

1. $P^\varepsilon\{$ sample tree has at least one 1-branch in a tiny "neighborhood" of $\varphi(s)$, $0 \le s \le 1 \} \asymp$?

2. $P^\varepsilon\{$ sample tree has at least one 2-branch in a tiny nbd of $(\varphi_1, \varphi_2)\} \asymp$?

3. $P^\varepsilon\{ R_1 \sim b_1, R_2 \sim b_2 \} \asymp$?

, where $\asymp$ means logarithmic equivalence as $\varepsilon \to 0$,

$R_t \equiv$ the position of the rightmost particle at time t.

Problems 1 and 2 are answered, problem 3 is given partial solution.

Large Deviations in $\mathbb{R}^d$.

P. Ney, Madison, WI, USA

Let $X_1, X_2, \cdots$ be i.i.d. r.v.'s taking values in $\mathbb{R}^d$, $S_n = \sum^n X_i$, $\Lambda(\alpha) = E e^{\langle \alpha, X_1 \rangle}$, $\alpha \in \mathbb{R}^d$, $\mathcal{D}(\Lambda) = \{\alpha : \Lambda(\alpha) < \infty\}$. If $\mathcal{D}(\Lambda)$ does not contain a nghd. of the origin, then the level sets of $\Lambda^*(x) = \sup_\alpha [\langle \alpha, x \rangle - \Lambda(\alpha)]$, $x \in \mathbb{R}^d$, will not be compact, and the LDP upper bound may fail. However, if the level sets of Λ^* can be suitably approximated by half-spaces, then an upper bound can be proved. Conditions are given (nec. + suff.) for such an approximation to be possible. They boil down to the property that the generating functions of certain marginal r.v.'s should not be degenerate (i.e. $\not\equiv \infty$ away from 0).

The above results are extended to approximation and separation theorems for the conjugate f^* of an essentially arbitrary convex ftn. f. The hypotheses are expressed in terms of the domain $\mathcal{D}(f)$. This leads to a classification of the sections of f^* into "elliptic", "parabolic" and "hyperbolic" classes, which are natural extensions of the conic sections.

Large Deviations for the Occupation time functional of a Poisson system of Independent Brownian Particles.

Let $\{N_s, s \geq 0\}$ be the evolution system starting from N_0, a Poisson point process with intensity dx, where each particles independently follows the law of a d-dimensional BM. Take $\varphi \in L^1(\mathbb{R}^d)$ with compact support, and let $N_s(\varphi) = \sum_{x \in supp(N_0)} \varphi(B_s^x)$ and $L_T(\varphi)(t) = \int_0^t N_{Ts}(\varphi)\, ds$.

We study the large deviations and central limit theorems for $L_T(\varphi)(t)$, $t\in[0,1]$. In the lower (recurrent) dimensions $d=1,2$ we have critical orders $T^{1/2}$ and $T/\log(T)$, whereas in higher (transient) dimensions we have the usual order T. We give explicit expressions for the corresponding rate functions and covariance functionals and derive some asymptotic microcanonical distributions.

J.D. Deuschel (ETH), KM Wang (Univ. of Zürich).

A matrix representation for the 1-dimensional transfer operator

We consider the transfer operator $\mathcal{L}$ defined by

$$\mathcal{L}f(i_{-\infty}^{0}) = \sum_{i_1} \ell(i_{-\infty}^{1}) f(i_{-\infty}^{1})$$

where $i_{-\infty}^{0} \in A^{\times \mathbb{N}_-}$, A is a finite alphabet, ℓ and f are lower semicontinuous non-negative functions on $A^{\times \mathbb{N}_-}$. We construct a non-negative matrix Q with index set S = the collection of finite sequences of A-symbols, and such that

$$\mathcal{L} I_{j^*} = \sum_{i^*} Q(i^*, j^*) I_{i^*}, \quad (I_{i^*} = \text{the indicator of a cylinder } i^* \in S)$$

Under the usual variation conditions we establish positive/geometric recurrence properties of Q. These are related to the eigenvalue problem for the transfer operator $\mathcal{L}$ (Ruelle's Perron-Frobenius theorem).

Esa Nummelin (University of Helsinki)

Large deviations techniques in analysis of monomolecular layers.

W. A. Woyczynski (Cleveland, Ohio)

The partition function of a statistical mechanical system of hard, oval shaped molecules moving on real line and with rotational degree of freedom, is replaced by its Poissonized version, which, in turn, can be analyzed via large deviation techniques when considered in the thermodynamic limit. Join work with J. Szulga and J.A. Mann.

Some Comments on the Hierarchical Mean-field Limit

We begin with a system of a large number of components whose interactions are organized in a hierarchical manner. The kth level of the hierarchy is comprised of N objects of the (k-1)st level and the strength of the interaction decreases as a function of the hierarchical distance (and also as a function of N). The single level hierarchy in the limit $N \to \infty$ is known as the mean field limit. The case in which N is fixed and $k \to \infty$ corresponds to the thermodynamic limit. The hierarchical mean field limit corresponds to the finite or infinite hierarchy in the $N \to \infty$ limit. The effect of taking the limit $N \to \infty$ is to separate the natural time scales or spatial scales relevant to the different levels of the hierarchy. To illustrate we consider two examples. The first is the continuous spin ferromagnetic model. In joint work with Jürgen Gärtner the hierarchical mean field limit of this ferromagnetic model is analysed using multilevel large deviation theory as $N \to \infty$. This analysis leads to a pattern of discrete symmetry breaking in the mean field limit. The second model considered is the stepping stone model arising in population genetics. This model has been analysed in joint work with Andreas Greven using multiple time scale analysis. This work shows that the criteria for continuous symmetry breaking in this model in the hierarchical mean field and in the thermodynamic limit sense are in fact equivalent for a large family of interaction strengths.

Donald Dawson (Carleton University, Ottawa)

Second order large deviations

We start with a family of distributions $\{\mathcal{L}(X_\varepsilon) : \varepsilon \to 0\} =$ $\{p_\varepsilon(x)dx : \varepsilon \to 0\}$ on $U \subseteq \mathbb{R}^d$ of the form (uniformly on compacts)

$$p_\varepsilon(x)dx = (2\pi\varepsilon)^{-d/2} \exp\left(-\tfrac{1}{\varepsilon} K(x) - K_0(x) - \varepsilon K_1(x) + o(\varepsilon)\right) dx_1 \ldots dx_d$$

$K(\,)$ is not necessarily the Legendre-Transform of a cumulant generating function. Just $K(\cdot)$ smooth and $K(x^*) = 0$ for some x^*, $K(x) > 0$ for $x \neq x^*$, $K''(\cdot)$ positive definite ($K_0(\cdot)$ and $K_1(\cdot)$ are required to satisfy ~~also~~ certain smoothness conditions as well)

For nice sets $A = \{x : F(x) \leq \text{const}\}$ we find an asymptotic expansion ($\Lambda(p) = \frac{1}{2}[\Phi^{-1}(p)]^2$)

$$\Lambda\, P_r(X_\varepsilon \in A) = \frac{1}{\varepsilon} K(\hat{x}) + \left(H_0(\hat{x}) + \delta(\hat{x})\right) + O(\varepsilon)$$

$K(\hat{x}) = \inf\{K(x) : x \in \partial A\}$

$\delta(\hat{x})$ vanishes when A is a halfspace

$$H_0(\hat{x}) = \frac{1}{2} \ln\left[\frac{K' \cdot (K'')^{-1} \cdot K'}{2K}(\hat{x})\right] + K_0(\hat{x}) + \frac{1}{2} \ln|\det K''(\hat{x})|$$

In the second part of the lecture such an asymptotic expansion was given explicitly in a particular case; we studied an approximation of the so-called noncentral t-distribution.

$$T^{(n)} := \frac{\overline{Y}}{\sqrt{\frac{1}{n-1} \cdot \sum (Y_i - \overline{Y})^2}} \quad \text{in the general case}$$

and in the special gaussian case

$$\tilde{T}^{(n)} = \frac{\vartheta + \frac{1}{\sqrt{n}} Z_0}{\sqrt{\frac{1}{n} \sum_1^n Z_i^2}} \quad \text{with } Z_0, Z_1, \ldots, Z_n \text{ independent standard normal.}$$

$$P_\vartheta\left(\tilde{T}^{(n)} \le \frac{\vartheta + a}{\sqrt{1 - a(a+\vartheta)}} \right) \approx$$

$$\approx \Phi\left(\pm\sqrt{2} \sqrt{n \cdot K(\vartheta, a) - \frac{1}{2} \ln\left[\frac{2K(\vartheta,a)}{a^2 (1 + \frac{1}{2}\vartheta(a+\vartheta))} \right] + \text{rest}} \right)$$

where $2K(\vartheta, a) = a^2 - a(a+\vartheta) - \ln[1 - a(a+\vartheta)]$ for $a \in (-\infty, +\infty)$

4. Dez. 92 Hermann Dinges (Frankfurt a. M.)

A variational characterisation of the speed of a one-dimensional self-repellent random walk.

Let Q_n^α be the probability measure for an n-step random walk $(0, S_1, \ldots, S_n)$ on $\mathbb{Z}$ obtained by weighting simple random walk with factor $(1-\alpha)$ for every selfintersection. This is a model for a one-dimensional polymer. We prove that for every $\alpha \in (0,1)$ there exists $\theta^*(\alpha) \in (0,1)$ such that

$$\lim_{n \to \infty} Q_n^\alpha \left(\frac{|S_n|}{n} \in [\theta^*(\alpha) - \varepsilon, \theta^*(\alpha) + \varepsilon] \right) = 1 \quad \text{for every } \varepsilon > 0.$$

We give a characterisation of $\theta^*(\alpha)$ in terms of the largest eigenvalue of a 1-parameter family $\mathbb{N} \times \mathbb{N}$ matrices, which allows us to prove that θ^* is an analytic function, $\theta^*(0) = 0$, $\theta^*(1) = 1$, $\theta^*(x) \in (0,1)$ for $x \in (0,1)$.

Besides for the speed we prove a limit law for the local times of the walk. The techniques used enable us to treat more general forms of self-repellence involving multiple intersections.
This is joint work with Frank den Hollander (Utrecht).

Dec/14/92

A. Greven (Göttingen)

Weak convergence theory approach to large deviations

A Puhalskii

We use ideas and methods of weak convergence theory to establish for large deviations results analogous to those in weak convergence.

The main result is an analogue of Prohorov's theorem. Say that a sequence (P_n) of probability measures on the Borel σ-field of a topological space is large deviation (l.d.) relatively compact if any its subsequence contains a further subsequence obeying the l.d.p with some function.

Then the theorem states that for a Tihonov space exponential tightness of (P_n) implies l.d. relative compactness. For a Polish space the converse is also true.

The theorem is applied to study large deviations of semimartingales. To this end, we introduce for large deviations analogues of the methods of finite dimensional convergence and of martingale problems in weak convergence. This allows us to attain new results on large deviations of semimartingales with paths in the Skorokhod space.

4.11.92

Large deviations for U-statistics

Let $(X_i)_i$ be a sequence of i.i.d. random variables taking place in some Polish probability space X with common law π. It is well known that

$$U_n := \frac{1}{\binom{n}{m}} \sum_{1 \le i_1 < \dots < i_m \le n} h(X_{i_1}, \dots, X_{i_m}) \quad \text{and}$$

$$V_n := \frac{1}{n^m} \sum_{1 \le i_1, \dots, i_m \le n} h(X_{i_1}, \dots, X_{i_m})$$

are "good" estimators for $E(h)$, for some integrable function $h: X^m \to \mathbb{R}^{d\pi^m}$.
Under the condition that the moment generating function of h and every "diagonal" of h exists, we derive a large deviation principle for the distribution of U_n and V_n. In both of the cases the rate function is given by

$$I(y) = \inf_{\substack{\rho \in M(X) \\ \int h \, d\rho^m = y}} H(\rho | \pi)$$

where $H(\cdot | \pi)$ denotes the usual entropy wrt. π. Our key tools are the contraction principle, Sanov's theorem and a graph-theoretic result on the factorization of complete ~~#~~ hypergraphs due to Baranyai.

4-12-92 Matthias Löwe

Phase transition in random external magnetic field - A ~~con~~ ~~ge~~ conjecture

We discussed a one-dimensional long-range interaction model with a random external ~~field~~ magnetic field. Our conjecture is that there is a phase transition in this model at low temperatures. This conjecture follows from a large deviation result about the distribution of the average spin in this model. We claim that the rate function appearing in in this result is not convex in a certain region. This convexity is the cause of the phase transition, and its appearance is closely related to the long range interaction of the model

Peter Major (Budapest)

The large deviations for a simple information network.

We consider a tandem system as on picture

$$\longrightarrow \boxed{1} \longrightarrow \boxed{2} \longrightarrow$$

defined by i.i.d vectors $(\tau_i, \xi_i^1, \xi_i^2)$. Here τ_i are intervals between mesagess and ξ_i^j are times for transmition messages through j-th node. We assume that $P(\tau_i > x) = e^{-\lambda x}$, $E\xi_i^j = \mu_j$ and $\varphi_j(\theta) = Ee^{\theta \xi_1^j}$

then $\frac{1}{x}\log \text{Prob}(w > x) \to -\min\{\beta_1, \beta_2\}$,
where w is totale waiting time of message in tandem, and β_j are defined from equations
$\beta_j = \lambda[\varphi_j(\beta_j) - 1]$.

E. Pechersky.

Hydrodynamic limit for 1-dimensional exclusion processes

We consider the particles' system on 1-dimensional periodic lattice with hard core exclusion. The jump rate is spatially homogeneous, non-degenerate and satisfies the detailed balance condition with respect to a trivial Hamiltonian. $\mathcal{H} \equiv 0$. The Bernoulli measures are therefore reversible for the dynamics. For this model, the non-equilibrium fluctuation problem (in the gradient case by using the method of Chang-Yau) and the hydrodynamic limit (in the general non-gradient case by applying the method of Varadhan: this part is due to Uchiyama) are discussed. The basic tools are Logarithmic Sobolev inequality and Spectral gap for the exclusion process.

Tadahisa Funaki (Nagoya)

Large Deviations for Sequences of Mixtures

Say that a family $\{P^n_\theta : \theta \in \Theta, n \geq 1\}$ is exponentially continuous if when $\theta_n \to \theta$, one has that $\{P^n_{\theta_n}\}$ satisfies an LDP with rate function $\lambda(\theta, \cdot)$ for each $\theta \in \Theta$. In this case, if μ is a measure on Θ, then $P^n =: \int_\Theta P^n_\theta \, d\mu$ satisfies an LDP with rate function $\inf\{\lambda(\theta, \nu) : \theta \in \mathrm{supp}(\mu)\}$ provided Θ is compact and given weak regularity conditions; see Dinwoodie and Zabell, Annals of Probability 1992. In this talk I discuss to what extent the conditions of this theorem can be weakened; a necessary and sufficient condition for exponential continuity is given; and a relationship with epiconvergence is discussed.

This is joint work with I. H. Dinwoodie (Tulane Univ., New Orleans).

4 December 1992, S. L. Zabell.

Hydrodynamic Limit for Hamiltonian Systems with Noise.

We consider a Hamiltonian system of N particles in the phase space $(T^3 \times R^3)^N$ evolving under a short range pair potential of the form $V(\frac{x-y}{\varepsilon})$ where ε is a scale parameter related to N by $N\varepsilon^3 = 1$. We aim to establish a relationship between the Hamiltonian Dynamics and the corresponding Euler equation derived by the thermodynamic formalism. In order to achieve this some small noise is added to the velocity components in such a way as not to destroy the conservation of momenta and energy. The classical Hamiltonian is replaced by one with bounded velocities. Then in a regime where the Euler equation has a smooth solution, we show that a suitably prepared local Gibbs families of densities on the phase space constructed from the solutions of Euler equation is close to the corresponding true solution of the Hamiltonian system with noise.

(Joint work with S. Olla and H. T. Yau)

4 December 1992. SRS Varadhan

Thermodynamical Aspects of Large Deviations

J.T. Lewis (Dublin)

The use in risk theory of intensive parameters analagous to the thermodynamic temperature (Martin-Löf 1986) prompts the question:

under what conditions does the machinery of equilibrium thermodynamics apply in the theory of large deviations?

In joint work with Ch. Pfister (Lausanne), we examine the thermodynanic formalism of Ruelle (1965) and Lanford (1973) in the setting of probability measures on Banach spaces. We define a Lanford entropy function and a grand canonical pressure and give conditions for the equivalence of ensembles. Motivated by Gibbs' axiomatization of thermodynamics (Gross 1982), we define a Gibbs entropy function. We give conditions for the Lanford entropy function to exist and be a Gibbs entropy function; we examine the connection with the large deviation principle (cf. O'Brien and Vervaat 1990).

Martin-Löf, A.: Entropy: a useful tool in risk theory Scand. Actuarial J. 1986, 223-235

Ruelle, D.: J. Math. Phys. 6, 201-209 (1965)

Lanford, O.E.: 1971 Battelle Lectures LNP 20 (1973)

Gross, L.: St Flour - X 1980 LNM 929 (1982)

O'Brien, G., Vervaat, W.: Capacities, Large Deviations and Log-Log Laws York Univ. Report 90/19 (1990)

Large Deviations in Search for Significant Variables of a function

M. B. Maljutow (Moscow)

Let a function $f(x_1, \ldots, x_t)$ of a vast number of variables may be expressed in the form $g(x_{\lambda_1}, \ldots, x_{\lambda_s})$ where $\lambda_1, \ldots, \lambda_s$ is a sequence of unknown indices, s is small as compared to t. Arbitrarily choosing the sequence $\overline{x}(i) = (x_1(i), \ldots, x_t(i))$, $i = 1, \ldots, N$ we observe the values of a random variables z_i which are related to the sequence of $y_i = f(\overline{x}(i))$ via transition probabilities $T(z_i \mid y_i)$. Measurements are independent given sequence $\overline{x}(1), \ldots, \overline{x}(N)$. The main quantity of interest is the minimal sample size N_ε guarantying the correct decision on $\overline{\lambda} = \lambda_1, \ldots \lambda_s$ with probability of error not exceeding ε. Both the cases of static and sequential designs are investigated. In both cases the upper estimate for N_ε is $\lesssim$ const $\ln t$ when $t \to \infty$, $s =$ const. Of special interest is the additive smooth model $g(x_1, \ldots, x_s) = \sum_1^s g_\alpha(x_\alpha)$ disturbed by the additive noise. Under the condition of subgaussian tales of errors A. Korostelev proved LD estimate for rather unexpected statistic – inconsistent estimate of $\varsigma_\alpha^2 = \int_{-1}^{1} g_\alpha^2(x)\,dx$, assuming that all $g_\alpha(\cdot)$ and their derivatives are bounded and $\varsigma_\alpha^2 \geq \Delta > 0$. This estimate is the base for obtaining the estimate for N_ε mentioned above. For sequential design the simple lemme on large deviations for supermartingales allows us to obtain the same asymptotics in a more simple way. Some lower bounds for N_ε are reviewed and cases where estimates for N_ε are precise are mentioned.

Droplet condensation: Large and moderate deviations at phase transition.

S.B. Shlosman (Moscow & Irvine, CA).

Deviations are studied for the sum $S_N = \sigma_1 + \dots + \sigma_N$ of the random variables $\sigma_i = \pm 1$, which are distributed according to the Ising model random field with inverse temperature $\beta \gg \beta_{crit}$, on the ν-dimensional lattice. The magnetic field is zero, and (+)-phase is considered. It is shown that for deviations b such that

$$b - E(S_N) \geq -c N^{\alpha}, \quad \alpha < \frac{\nu}{\nu+1}$$

for some $c > 0$, one has

$$\Pr\{S_N = b\} = \frac{2}{\sqrt{2\pi \mathcal{D}_{N,b}}} \exp\{-I_N(b)\}(1 + o_N(1)),$$

where $\mathcal{D}_{N,b}$ is the „tilted" variance, and $I_N(\cdot)$ is the rate function. In the complementary region

$$b - E(S_N) \leq -c N^{\gamma}, \quad \gamma > \frac{\nu}{\nu+1}, \quad c > 0$$

one has

$$\ln \Pr\{S_N = b\} \Big/ (E(S_N) - b)^{\nu - 1/\nu} = O_N(1).$$

So the region of deviations around $E(S_N) - N^{\nu/\nu+1}$ contains a treshold where the condensation of microscopic ($\sim \ln N$) droplets to macroscopic droplet ($\sim N^{\varkappa}$, $\varkappa \geq 1/\nu+1$) takes place.

Large deviations from a hydrodynamic scaling limit for a nongradient system

We consider the symmetric simple exclusion process with coloured, but mechanically identical particles as a simple, but physically motivated, example of a nongradient system. Colour density profiles are shown to have a hydrodynamic scaling limit which appears as a law of large numbers for an appropriate sequence of measures. The limiting equation has the form

$$\begin{cases} \frac{\partial \vec{\rho}}{\partial t} = \frac{1}{2} \nabla A(\vec{\rho}) \chi(\vec{\rho}) \nabla \vec{\rho} \\ \vec{\rho}(0) = \vec{\rho}_0 \end{cases}$$

compressibility

where A is a matrix involving the self-diffusion constant, $D_s(\rho)$ = the limiting covariance of a test particle in density ρ. Large deviations are calculated from this scaling limit with a rate function which is approximately the H_{-1} norm with "weights" $A^{-1}(\rho)$.

Jeremy Quastel (UC Davis)

Hydrodynamics for the generalized exclusion process.

The generalized exclusion process with two-particles per site is one of the simplest infinite particle systems which is non-gradient with product form invariant measures and for which one can prove hydrodynamical limits. The limiting equation is, as expected, of the form:

$$\partial_t \rho = \partial_x(\hat{a}(\rho)\partial_x \rho)$$

where $\hat{a}$ is given by a variational formula.

C. Kipnis & S. Olla

(Joint work with C. Landim)

An Application of large deviation principles for the empirical measure of interacting particles systems. C. Landim (Courant Inst.)

We consider the symmetric simple exclusion process. for which a large deviation principle for the empirical measure was proved by Kipnis, Olla and Varadhan in finite volume and extended to infinite volume by Landim.
We obtain a large deviation principle for the occupation time of a site in this model as a consequence of the previous result in dimension 1

Theory and Numerical Methods for Initial-Boundary Value Problems

6. 12. 1992 – 12. 12. 1992

Numerical Methods for Free-Surface Flows.

Many free-surface flow problems (fingering in Hele-Shaw Cells or ground water, Rayleigh-Taylor instabilities, Kelvin-Helmholtz instabilities) are ill-posed in the sense of Hadamard. In particular, the smallest scales grow the fastest unless some physical regularization is present. In the limit of small physical regularization, the motion can be extremely difficult to compute numerically by standard methods. However, the analytic continuation of the of the equations into the complex physical plane presents a new formulation that is well-posed for numerical calculation. We describe a spectral method capable of advancing the finger in the Hele-Shaw Cell for long times and for studying the competition of fingers in a Hele-Shaw Cell with high accuracy. Our methods generalize to other free-surface flows that are ill-posed.

Greg Baker
Ohio State University, Columbus, OH.

Stable Difference Schemes for Parabolic Systems

The purpose of this talk is to discuss the employment of generalized numerical radii in order to investigate stability of implicit difference schemes for the initial-boundary value problem associated with a general, well-posed, multi-space dimensional, parabolic system of the form

$$\frac{\partial u}{\partial t} = \sum_{1\le p\le q\le d} A_{pq}\frac{\partial^2 u}{\partial x_p \partial x_q} + \sum_{1\le p\le d} B_p \frac{\partial u}{\partial x_p} + Cu \quad x_p \in \mathbb{R},\ t\ge 0,$$

where A_{pq}, B_p and C are constant matrices. Moshe Goldberg (Technion)

On a boundary layer in hypersonic reacting Euler flow, Rolf Jeltsch, ETH

We consider hypersonic flow around a blunt body of a mixture of gases which are chemically not in an equilibrium. It is shown that the chemical reactions induce an extremely thin unphysical boundary layer. A modification of the Van Leer flux vector splitting is presented which is able to indicate the presence of the boundary layer. This is joint work with M. Fey ETH Zürich and S. Müller, RWTH Aachen.

Numerical Solution of laminar Flame Problems on parallel Computers with distributed Memory

G. Bader, University of Heidelberg

We present a data decomposition algorithm for the solution of stationary 2-D-Flame Models. In particular we describe the efficient implementation of Block-ILU Preconditioning for the solution of highly unsymmetric Systems of linear equations. Further a Multigrid approach for the solution of Flame-Problems. Numerical result, which show the high efficiency, of this approach will be presented.

A Remark on Approximate Inertial Manifolds and the Navier-Stokes Equations (joint work with J.G. Heywood)

This lecture contributes to the discussion about the ability of the AIM method to model turbulent flow, and about the theoretical potential of the AIM-based "nonlinear" Galerkin method to provide a computational basis for the calculation of turbulent flow. An error analysis is presented for the NGM from which it appears clear that the reason for reported success of the NGM centers on the compatibility of the solution along the boundary, i.e., the well-known Gibb's phenomenon in Fourier approximation, and has nothing to do with "turbulence modeling".

R. Rannacher (University of Heidelberg)

Long-Time-Asymptotics of Perturbed Gravity Waves

It is well known that solitary waves exist under the influence of gravity on the surface of an inviscid fluid, when the Froude number is greater than one. However, the description of the large-time behavior of 'local' perturbations as solutions of the full 2d-Euler equations is still an open problem. In this lecture we discuss a method for its resolution.
First, it is observed that the slowest transport happens in a four-dimensional subspace of the phase space. There we construct a socalled Eichform via a Floquet-transformation. The Eichform isolates the given solitary wave in a one-parameter family of those waves,

and it reveals the space-asymptotics which a 'local' perturbation has to obey. An asymptotic Eichform is derived then, such that the full equations contain in addition only fast decaying terms in space. These terms yield decay of the solutions which is at least $O(t^{-1})$ faster than the slowest decay, which is determined by the asymptotic Eichform. The final result then shows that the long-time behavior is given by two modulated waves moving in opposite directions and decaying like $O(t^{-3/2})$.

This is joint work with Mariana Haragus from Nice.

Oberwolfach, December 8, 1992

Klaus Kirchgässner

The dynamics of crystalline microstructure and phase boundaries

The deformation $y(x,t): \Omega \times \mathbb{R}^+ \to \mathbb{R}^3$ of solid crystals where the spatial domain $\Omega \subseteq \mathbb{R}^3$ can be modeled by

$$y_{tt} = \operatorname{div} \sigma(\nabla y(x,t))$$

with appropriate boundary and initial conditions. Equilibrium solutions to these models often have highly oscillatory deformation gradients when the stress tensor $\sigma(F) = \partial\phi(F)/\partial F$ is derived from a nonconvex energy density $\phi(F)$ where $F = \nabla y(x,t)$. We describe these highly oscillatory solutions by a mathematical definition of material microstructure using the Young measure. We present numerical methods and results for the dynamical development of material microstructure and for the propagation of phase boundaries in the presence of material microstructure. Joint work with Petr Klouček.

Mitchell Luskin
Minnesota

Error Estimates for Finite Element Approximation of Degenerate Parabolic Systems

We consider the following model problem for reactive solute transport in porous media with equilibrium ("$k=\infty$") or non-equilibrium adsorption:

$$\partial_t u + \partial_t v - \Delta u = f \quad \text{in } \Omega \subset \mathbb{R}^N, t \in (0,T]$$

$$\partial_t v = k(\varphi(u) - v) \quad \text{—"—}$$

+ initial / boundary conditions, where $k \leq \infty$, i.e. for $k=\infty$ the problem reduces to the scalar equation

$$\partial_t (u + \varphi(u)) - \Delta u = f$$

The problems are degenerate, as typical non-linearities have the form $\varphi(u) = \alpha u^p$, $\alpha > 0$, $p \in (0,1)$.

We study error estimates for the semidiscrete Galerkin approximation with linear finite elements, consistent or with quadrature, and the fully discrete versions based on the backward Euler method. We derive error estimates in energy norms which partially exhibit the full approximation power of the trial space despite of the degeneration. It turns out that regularizing the problem is a useful tool in general and in case of non-degeneracy conditions, i.e. if there is a minimal growth of the solution away from the front $\partial \operatorname{supp} u$, the regularization improves on the results. Joint work with J. W. Barrett (London)

Peter Knabner
IAAS, Berlin

Computing Periodic Solutions of Parabolic Systems by Using a Singular Subspace as 'Reduced Basis'

The search for a periodic solution is reduced to a fixed point problem for the Poincaré map: $\mathcal{E}(u) = u$. Let Z and Y be orthogonally complementary subspaces built up from singular vectors of $\mathcal{E}'(u) - I$. These subspaces are mapped into the corresponding left singular subspaces $\bar{Z}, \bar{Y}$. To get back to a fixed point problem one constructs an orthogonal mapping J which map Z, Y to $\bar{Z}, \bar{Y}$. By applying J^T on $0 = \mathcal{E}(u) - u$ this FPequ splits into two locally decoupled FPequs. By selecting the 'right' singular subspace $Z(\bar{Z})$ the FPequ projected onto Y gets contractive and the (usually small) FPequ projected on Z shows up many features of the full system. One can show that derivatives up to the 2nd order coincide with the corresponding Ljapunov-Schmidt reduction. Since in many application the dimension of the 'reduced basis' Z is very low one has a useful technique to study (branches of) periodic solution with the aid of a tiny system. The computation of Z and $\bar{Z}$ leads to a Riccati eq. which can be solved using bordering techniques which are well known.

Helmut Jarausch, RWTH Aachen

Lyapunov functions and the Navier-Stokes Equations

The planar Navier-Stokes equations with periodic boundary conditions and time-independent forcing are considered. It is shown that there exists an asymptotically stable attracting set if a Galerkin approximation of sufficient high dimension has an asymptotically stable attractor which is sufficiently strongly attracting. There is no restriction on the geometric structure of the attractor, other than that it is compact. Lyapunov functions are used to characterize its stability. The result is a generalization of work of Constantin, Foias and Temam for steady state attractors

Peter Kloeden Deakin University, Australia

On approximation methods for illposed parabolic equations

Cauchy problems for parabolic initial value problems will be considered with the Inverse Heat Conduction Problem (IHCP) as a model example. Due to the illposedness of the problem, large errors in the approximating solutions may occur in the presence of small errors in the data.

Approximation methods for such problems should therefore stabilize - or regularize - this behaviour. For several methods, rules for stabilizing are well-known, however, a rigorous analysis is often not available.

In this contribution, a short overview of available approximation methods will be presented. In more detail, a sequential approximation scheme for the IHCP will be discussed including a stability and error analysis.

H.-Jürgen Reinhardt, Univ. Siegen

Dynamical Adaptive Method based on Wavelets for the Approximation of PDE

The multiresolution analysis introduced by Y. Meyer and coauthors provides a nice framework for representation of functions with local irregularity (shock of sharp gradient) with minimal number of degrees of freedom while maintaining good accuracy. This has been used widely in data compression. The idea here is to use this minimal representation of the type of functions for the approximation of the solution of PDE. The result is an adaptive method that, at each time step

infers the minimal representation of the function, solution of the equation, from the minimal representation of the solution already computed. This can be employed either for an unsteady situation or a steady one when the steady solution is computed as the limit of an unsteady solution.
We present the method in one dimension first with periodic boundary condition then extend it to more general cases (2D, Dirichlet BC)

Yvon Maday Univ Pierre et Marie Curie
Paris 6

Finite Element Discretization of Navier - Stokes Equations with Varying Density

In the incompressible Navier - Stokes equations, when the Boussinesq' approximation is not valid, the variations of the density must be taken into account. We study two variational formulations of a model where the density is assumed to be given but non constant. We compare finite discretizations which rely on these formulations. New extensions are presented.

Christine Bernardi
C.N.R.S et Université Pierre et Marie Curie
Paris

Hydrodynamic Instabilities in Systems Driven by Surface Forces

The linear stability of 2-dimensional toroidal flow in a cylindrical liquid bridge driven by thermocapillary forces is investigated by the application of spectral methods. The 2-dimensional basic flow and temperature fields are calculated by a Galerkin tau method. The neutral modes giving rise to 3-dimensional instabilities are obtained by Galerkin-collocation-tau. Although the applicability of this method is limited to moderate Marangoni numbers, the threshold value and the space-time structure of the neutral disturbances can be obtained with reasonable good accuracy. The instability mechanisms and the physical properties of the supercritical flow are discussed.

Hendrik C. Kuhlmann
ZARM - Universität Bremen

On two dimensional hyperbolic equations: the stability of difference schemes with shock tracking

Consider a shock wave in two dimensional compressible inviscid gas moving into a gas at rest. We consider the stability for the linearized equations of a class of finite difference schemes away from this shock wave coupled with a scheme explicitly tracking this moving internal boundary. We use energy methods.

Christian Klingenberg
Heidelberg

Uniqueness and Existence of a pressure field for an incompressible flow

Yann BRENIER Université Paris 6

The motion of an ideal incompressible flow obeys a well known least Action Principle: for any sufficiently short time interval, the flow minimizes its kinetic energy integrated over the time interval. This leads to the following least Action Principle:
Given $T>0$, X the physical domain and h a self diffeomorphism of X, with jacobian determinant equal to 1, Find a time dependent family $t \to g(t)$ of self diffeomorphisms of X such that $g(0,x)=x$, $g(T,x)=h(x)$, $\det \partial_x g(t,x) = 1$, minimizing $\int_0^T \int_X |\partial_t g(t,x)|^2 dx dt$.
This formulation of the Euler equations has been well known since Arnol'd's paper in the Ann. Institut Fourier (66). Local existence and uniqueness were obtained by Ebin & Marsden (70) when h lies in a small neighborhood of the identity map for a suitable Sobolev norm. In the large, uniqueness can easily break down (take X as the unit disk in $\mathbb{R}^2$, $h(x)=-x$, $T=\pi$, then $g_\pm(t,x) = xe^{\pm it}$, with complex notations, provide two different solution). In 1987, A. Shnirelman proved that existence can break down in 3 dimensions ($X=[0,1]^3$). Independently, we introduced, in 1989, a generalized framework (very much in the spirit of L.C. Young's ideas) for which global existence of solutions can be easily obtained.
In the present report, we adress the problem of finding the corresponding pressure field which can be seen as the dual unknown of the least Action Problem. The pressure turns out to be the right quantity to look at since both existence and uniqueness can be obtained, with continuous dependence with respect to h. The regularity problem for the pressure field is open.

Computing Stability Bounds for Thermocapillary Convection under Zero Gravity

In the float-zone process of crystal growth temperature-gradient induced surface tension gradients along the outer free surface drive axisymmetric convection rolls in the cylindrical float-zone. These are present even under zero gravity. For increasing temperature differences measured by the Marangoni number this convection becomes unstable leading to poor crystal quality. It is thus desirable to determine bounds for the stability limit. Both energy and linear theory results are obtained for a model problem. In addition to the solution of the underlying Boussinesq equations generalized large and sparse eigenvalue problems have to be solved. The numerical approach is outlined and results are compared to those from experiments.

Hans D. Mittelmann (Arizona State Univ, Tempe)

Stabilizing Effect of Surface Tension and Formation of Pinching Singularities in Fluid Interfaces

It is well-known that without physical regularization, vortex sheets will develop curvature singularity in finite time. Here we consider the stabilizing effect of surface tension for incompressible, inviscid fluid interfaces. We show that surface tension stabilizes high mode instabilities when the interface is linearized around any prescribed, time dependent smooth solution. Our stability estimate provides a sharp account on dynamically excited unstable modes due to the local compression of the Lagrangian fluid markers. Using the linear stability result, we're able to prove stability and convergence of a spectrally accurate boundary integral method. We then use our spectrally accurate method to study the nonlinear ~~stabilizing result~~ effect numerically. An efficient implicit scheme is designed to relax the severe stiffness of the interface problem due to the presence of surface tension. We found that if surface tension is above certain value, the interface problem has a smooth global solution. However, if surface tension is below certain critical value, the interface can form a pinching singularity in finite time. This singularity is different from the singularity in vortex sheets without surface tension.

Thomas Y. Hou
Courant Institute

Non-Reflecting Boundary Conditions for the Euler Equations

Michael Giles, Oxford University

This talk discusses the variety of far-field b.c.'s used for the solution of the Euler equations in the context of two-dimensional flows in turbomachinery and external aerodynamics. Starting from an assumption of linear perturbations to a uniform steady flow it is possible to construct exact non-local nonreflecting b.c.'s. These then lead to a number of different approaches.

- 1D b.c.'s: These assume waves leaving normal to the boundary. An improved well-posed version assumes known non-zero angle.
- steady-state: Taking the limit as frequency approaches zero gives spatially non-local b.c.'s which are easily implemented in turbomachinery. Similar b.c.'s can also be used when there is a single known non-zero frequency.
- 2D approximate: Using the ideas of Engquist & Majda produces approximate local nonreflecting b.c.'s. Using the theory of Kreiss it can be shown that the outflow b.c. is well-posed but not the inflow. A modification to the inflow b.c. makes it well-posed and fourth order nonreflecting.

Adaptive Composite Overlapping Grids for Gas Dynamics

A method under development for the numerical solution of the compressible Euler equations of gas dynamics in regions of complex geometry is presented. Regions of complex geometry in two and three space dimensions are represented by the method of composite overlapping grids, as developed by Chesshire & Henshaw (J. Comp. Physics, V. 90 p. 1). A composite overlapping grid consists of a set of logically rectangular or hexahedral non-orthogonal curvilinear grids that overlap where they meet and completely cover the computational region. PDEs are solved using standard finite difference techniques, but with additional boundary conditions that interpolate the solutions between component grids. The Adaptive Mesh Refinement (AMR) method developed by M. Berger is combined with the overlapping grid method to give adaptive resolution of complex structure in the flows. Since the AMR method is also based on logically rectangular grids, very few changes are required in the algorithm in order to use it with overlapping grids. The Euler equations are solved on the grids using a class of high-order Godunov methods of the type developed by P. Colella. A discretization for the Euler equations must have the basic properties that shock singularities propagate at the correct speed, and that the error committed in the shock is damped out extremely rapidly as it moves away from the shock. The high-order nature of the Godunov method guarantees that the smooth parts of the solutions are computed accurately, and thus the end-states for the shock and hence its speed will be correct. The upstream nature of the method ensures that shock errors are damped out quickly. Numerical examples are presented demonstrating the high-order Godunov method on overlapping grids.

David L. Brown
Los Alamos National Laboratory, USA

Evolution Galerkin methods in one & two dimensions

In discretising evolutionary problems a combination of three principal ideas has proved to be particularly useful: approximating the evolution operator; Galerkin or Petrov Galerkin projection onto a finite element trial space; and a recovery procedure to obtain higher order accuracy in an adaptive manner. Several examples of the first will be given, of which that based on tracing the characteristics will be considered in more detail. In the form of Brenier's transport collapse operator, using L_2 projection onto piecewise constants and with recovery by piecewise linears it gives a family of schemes of the form $U^{n+1} = PE_\Delta RU^n$. They are explicit, unconditionally stable, TVD and TVB and converge to the entropy-satisfying solution of a scalar conservation law. Using a Riemann-Stieltjes parametrization, several equivalent forms are given in 1D; and the importance of certain corner terms in the 2D case is pointed out. Systems of equations are dealt with by wave decomposition to approximate the evolution operator.

K.W. Morton
Oxford University Computing Laboratory

Pointwise error estimates for a streamline diffusion scheme on a Shishkin mesh for a convection-diffusion problem.

We analyse a streamline diffusion scheme on a special piecewise uniform mesh for a model time-dependent convection-diffusion problem. The method with piecewise linear elements is shown to be convergent, independently of the diffusion parameter, with a pointwise accuracy of almost order 5/4 outside the boundary layer and almost order 3/4 inside the boundary layer. Numerical results are also given.

Martin Stynes
University College Cork
Ireland

Adaptive Rothe's Method for Time-Dependent Problems

An adaptive approach for IBVPs like diffusion-dominated or Schrödinger-type equations is presented. It contains an adaptive discretization of the evolution operator in time first and viewing the spatial discretization as a perturbation. This allows time-steps which belong to the dynamics of the problem and gives an easy matching of time and space accuracies. The use of a multigrid-type algorithm on highly nonuniform grids. This provides an optimal elliptic subproblem solver, which allows to change the mesh easily from timestep to timestep. Applications to the 2D and 3D simulation of the heating of tumors (hyperthermia) in cancer therapy is given.

Folkmar A. Bornemann, Freie Universität Berlin

Numerical approximation of connecting orbits

We consider the numerical computation of orbits which connect steady states or periodic orbits in parameter-dependent dynamical systems. Typically, such problems arise when determining the shape and speed of travelling waves in parabolic systems.
Connecting orbits satisfy a boundary value problem on the real line. We show that the well-posedness of such b. v. p. is related to the geometric condition of transversal intersection of stable and unstable manifolds.
In the case of stationary to periodic connecting orbits it turns out that a crucial role is played by the property of asymptotic phase and by the induced foliations of stable and unstable manifolds.
For the numerical approximation we truncate to a finite interval and set up appropriate asymptotic boundary conditions. Some exponential error estimates are provided for the solutions on the finite and the infinite interval.

Wolf-Jürgen Beyn, Universität Bielefeld.

Analysis of Boundary Conditions at Artificial Boundaries with Applications in Fluid Mechanics

We consider the construction of boundary conditions at artificial boundaries for the solution of time dependent problems. The difficulty is to construct "consistent" but simple (time-local) approximations - i.e. conditions almost satisfied by the exact solution. Various constructions based on linear analysis are given for the (compressible and incompressible) Navier-Stokes equations. Some approximate conditions are seen to perform well in practice well beyond the validity of the linear approximation - a fact we do not yet understand. Some lower bounds on the error for solutions of the wave equation in exterior domains are also given.

Thomas Hagstrom - U. of New Mexico

Continuation of Invariant Tori

We consider a dynamical system depending on a parameter λ. Assume that for $\lambda = \lambda_0$ an invariant 2-torus M_0 is known in terms of a parametrization w_0. Under suitable assumptions, there is a branch of invariant tori $M(\lambda)$ for λ near λ_0, which we try to follow computationally. The main idea is to use a coordinate system determined by w_0 and to update the coordinates as the computation proceeds. In practice, the parametrizations can only be determined on a grid. We give an error analysis as the gridsize tends to zero.

Jens Lorenz, U. of New Mexico

Computing the Oscillations of a Free Surface Jet

A numerical method for computing the motion of an inviscid and irrotational fluid jet issuing from an elliptical orifice is described. The differential equation for the evolution of the potential on the boundary, and of the shape of the boundary is discretized by 4'th order accurate centered differences in space. The resulting system of ODEs is integrated in time by an 4'th order accurate four stage Runge-Kutta method. To evaluate the time derivatives of the potential and the shape of the boundary, it is necessary to solve Laplaces equation with Dirichlet data. The problem is transformed onto a fixed computational domain where the elliptical equation is solved by a 4'th order accurate finite difference method on a composite overlapping grid. One advantage of this approach is that the computational domain only needs to be gridded once. Instead the transformed Laplace equation will get coefficients that vary both in time and space. By studying the spectrum of the discrete linearized operator, it is found that the spatial discretization is not completely satisfactory because one pair of complex conjugated eigenvalues of the linearized operator has a small real part. However, numerical examples show that the equations still can be integrated successfully, at least until time of the order $O(1)$, if the initial cross-section is sufficiently close to a circle. For initial cross-sections with larger aspect ratio, the break down time decreases when the number of grid points increases.

N. Anders Petersson
Center for Nonlinear Studies
Los Alamos National Laboratory

Finite element methods for solving the Boussinesq-approximation of the Navier-Stokes equations

We consider stability and convergence of finite element discretizations for the incompressible Navier-Stokes equations. There are different reasons for spurious numerical oscillations of standard Galerkin finite element methods, e.g., dominance of convective terms, inappropriate pairs of finite elements for approximating the velocity and pressure field, etc. We propose to combine the stable nonconforming Crouzeix/Raviart element with an upstream technique for handling the influence of the convective terms. For solving the nonlinear system of equations a multigrid method is used. Finally, some numerical results are presented.

Lutz Tobiska
Technical University Magdeburg

Numerical treatment of initial-boundary value problems in chemical kinetics with the method of lines

A set of chemical problems is connected with reactions and diffusion of some substances in a reactor. A typical example of these diffusion-reaction processes is the radical copolymerization of two chemical species in presence of an initiator. The reactions occuring in several partial steps form chains of polymers while diffusion processes take place parallel in time.
The resulting mathematical model consists of a system of nonlinear parabolic differential equations with added conditions at the starting point and the boundaries. This initial-boundary value problem is treated by the numerical method of lines in such a way that, after semidiscretization in spatial direction, different known initial-value problem solvers for the solution of the arising ordinary differential equations (stiff systems) are used.

Some results of the numerical computations and remarks on performance of the compared solvers are given.

Peter Seifert
Technical University Dresden

Stabilized Galerkin Methods for Solving the Incompressible, Nonisothermal, Nonstationary Navier-Stokes Equations

We consider the finite element discretization of the incompressible Navier-Stokes equations with Boussinesq approximation in the nonisothermal case. Spurious numerical solutions of standard Galerkin finite element methods might be caused by dominating convective terms and/or inappropriate pairs of velocity/pressure interpolation which do not pass the discrete BABUŠKA-BREZZI condition. As a remedy we add least-squares formulations of the basic equations. It turns out that the resulting Galerkin/least-squares method stabilizes both instabilities. First we analyze the stability and convergence of the time-integration procedure for scalar problems. Secondly we consider the stability and convergence of the method for a linearized problem arising from the simple iteration procedure. In particular, we consider the parameter design problem for the given problem. We conclude with some numerical results.

Gert Lube
Magdeburg Univ. of Technology

Stability of Broadwell shocks

The discrete kinetic Broadwell model in one space dimension describes the evolution of the distribution of the three velocities $(1, 0, -1)$ in space-time. 1964 Broadwell found explicit expressions of travelling shock waves connecting equilibrium states satisfying the Rankine-Hugoniot condition.
In this talk, I presented joint work with Zhouping Xin on asymptotic stability of Broadwell shocks.

We have proved that Broadwell shocks, which initially are locally perturbed, converge time asymptotically to a superposition of a translated shock wave, ~~a~~ a diffusion wave and a linear coupled diffusion wave (with zero mass). The sum of the diffusion wave and the linear wave solves a Navier-Stokes type approximation of the Broadwell equations, with slightly modified viscosity in the shock region compared to the traditional Navier-Stokes equation obtained from the Chapman-Enskog expansion.

Anders Szepessy
Royal Inst. of Technology, Stockholm

Adaptive Techniques in the Computation of Inviscid Compressible Flow

A finite-volume scheme is used for the computation of steady and unsteady inviscid compressible flow fields in complex geometries. The method works on general triangulations and is an upwind TVD-MUSCL-type of scheme. Two adaptive techniques of point enrichment are used to re/derefine the triangulation. A finite-element-type of residual is used as an error indicator. The question of norms in which this residual can be measured is adressed.

Thomas Sonar
Institut für Theoretische Strömungsmechanik
DLR Göttingen

On entropy consistency for large time step schemes

Zur numerischen Approximation der Lösungen hyperbolischer Erhaltungsgleichungen gibt es Verfahren mit großem Zeitschritt, die auf dem Godunov und dem Glimm-Schema aufbauen. Die Entropiekonsistenz dieser Verfahren war ein offenes Problem. Resultate, die mit Wang Jinghua erzielt wurden, wurden vorgetragen.

Gerald Warnecke
Stuttgart

"Practical use of high order accurate difference methods"

We develop centered difference methods of order 4 and 8 for the solution of the compressible Navier-Stokes equations on a curvilinear grid. Stability analysis for initial-boundary value problems is performed numerically, and stable boundary operators are found.

The method is used to compute the Mach 3 flow past a disk, where the bow shock is fitted to the outer grid boundary.

A resolved solution is obtained for Re=1000 using 4th order accuracy. This solution is compared with a coarse grid shock capturing TVD solution on a highly stretched grid. Good agreement is observed between the two methods, even in the boundary layers.

Björn Sjögreen
Uppsala University

Asymptotische Statistik

13.12. – 19.12.1992

Rank Statistics Under Dependent Observations And Application To Experimental Designs

This is joint work with E. Brunner. In a general model of the form

$$\mathcal{X}_i(n) = (X_{i1}(n), \ldots, X_{im_i(n)}(n))'$$

with independent random vectors of (non-constant) dimension $m_i(n)$ we prove asymptotic normality of simple linear rank statistics under overall ranking. When the score function is of class $C^2[0,1]$, and very likely only in this case, such a statistics can be used in applications, i.e. the unknown variance can be estimated and the assumptions in the theorem be checked. Applications include multivariate test for symmetry, Kruskal-Wallis test under repeated measurements, Friedman test with overall ranking (and repeated measurements) and many more. The proof of the result follows known schemes.

M. Denker
University of Göttingen

On testing the extreme value index via the POT-method

Consider an iid sample $Y_1, \dots, Y_n$ of random variables with common distribution function F, whose upper tail belongs to a certain neighborhood of the upper tail of a generalized Pareto distribution H_β, $\beta \in \mathbb{R}$. We investigate the testing problem $\beta = \beta_0$ against a sequence $\beta = \beta_n$ of contiguous alternatives, based on the point processes N_n of the exceedances among Y_i over a sequence of thresholds t_n. It turns out that the (random) number of exceedances $\tau(n)$ over t_n is the central sequence for the log-likelihood ratio of $\log\{d\mathcal{L}_{\beta_n}(N_n)/d\mathcal{L}_{\beta_0}(N_n)\}(N_n)$, yielding its local asymptotic normality (LAN). This result implies in particular the surprising fact that $\tau(n)$ carries asymptotically all the information about the underlying parameter, which is contained in N_n. We establish sharp bounds for the rate at which $\tau(n)$ becomes asymptotically sufficient, which show however that this is quite a poor rate.

Michael Falk
Kath. Universität Eichstätt

Asymptotic expansions for the confidence intervals

An asymptotically unbiased confidence interval is constructed from an unbiased test up to the third order, where the second and third order derivatives of the log-likelihood function are used. And also its application to the location parameter case is described. Further, from the viewpoint of a posterior risk, the upper and lower confidence limits are derived and, in practice, obtained up to the second order in case of the normal, uniform and truncated normal distributions. The relationship between the loss function and a confidence level is also discussed. It is noted that the level can be determined from a shape of the loss function and is connected to the length of a confidence interval.

Masafumi Akahira
University of Tsukuba

Asymptotic normality of pseudo likelihood estimates for Gibbs point processes defined through a pair potential.

Due to the possibility of phase transitions it is not possible to prove asymptotic normality of maximum likelihood estimates in Gibbs point processes for all values of the parameters defining the process. The problem is to get a bound on the mixing coefficients of the process. Contrary to this asymptotic normality can be proved for the maximum pseudo likelihood estimates without using the mixing properties. Instead one uses ergodicity and a property that resembles that of a martingale difference scheme, i.e. the score function is a sum of terms each of which has mean zero conditioned on the surroundings. The final result says that it is possible to calculate a stochastic norming of the pseudo likelihood estimates such that the normed estimates have asymptotically a standard normal distribution for a stationary Gibbs point process.

The talk is based on joint work with Hans R. Künsch.

Jens Ledet Jensen
Aarhus University

Asymptotics for plug-in estimators with application to the bootstrap

Let $X_1, X_2, \ldots$ be i.i.d. with (unknown) common distribution $P \in \mathcal{P}$. If $\hat{P}_N = p_N(X_1, \ldots, X_N)$ is a sequence of estimators of P with values in $\mathcal{P}$, we can estimate a "parameter" sequence $\tau_N(P)$ by the plug-in estimators $\tau_N(\hat{P}_N)$. We show that under a variety of continuity conditions on τ_N and convergence assumptions on $\hat{P}_N$, the plug-in estimators are indeed consistent. These results are proven in sufficient generality to be directly applicable to the bootstrap.

W. R. van Zwet
Leiden

Asymptotics of stochastic optimization problems

We consider an optimization problem of the form

$$(P) \quad \| \; \mathbb{E}(H(x,\xi)) + \psi_C(x) = \text{min!}$$

where $\psi_C(x) = \begin{cases} 0 & x \in C \\ \infty & x \notin C \end{cases}$

and its "empirical" version

$$(P_e) \quad \| \; \frac{1}{n} \sum_{i=1}^{n} H(x,\xi_i) + \psi_C(x) = \text{min!}$$

Let x^* be the solution of (P) and $\hat{X}_n$ be the solution of (P_e). Set

$$T_n := \Gamma_n^{-1}(\hat{X}_n - x^*)$$

We want to study the asymptotic distribution of T_n for a suitably chosen sequence of regular matrices $\Gamma_n \to 0$.

By change of coordinates

$$T_n = \operatorname{argmin}\ \varsigma_n \cdot \sum_{i=1}^{n} \left[H(x^* + \Gamma_n t, \xi_i) - H(x^*; \xi_i) \right] + \psi_C(x^* + \Gamma_n t)$$

for $\varsigma_n > 0$, arbitrary.

For a variety of cases, the epi-limit in distribution of the process

$$Z_n = \varsigma_n \sum_{i=1}^{n} \left[H(x^* + \Gamma_n t, \xi_i) - H(x^*, \xi_i) \right] + \psi_C(x^* + \Gamma_n t)$$

may be found. The limit of $\psi_C(x^* + \Gamma_n t)$ depends on the local curvature of the set C near x^*. If Γ_n is a multiple of the unit matrix, then the limiting constraint set is the tangent cone. If Γ_n is different in different directions, other limiting constraint set may appear. The stochastic process

$$\varsigma_n \sum_{i=1}^{n} \left[H(x^* + \Gamma_n t, \xi_i) - H(x^*, \xi_i) \right]$$

converges typically to a process of the form $D(t) + S(t)$, where $D(t)$ is some deterministic function and $S(t)$ is a self-similar zero mean stochastic process. We discuss cases where $S(t) = t' \cdot Y$; $Y \sim N(0, \Sigma)$. as well where $S(t)$ is a generalized Wiener process or a generalized Poisson process.

G. Pflug, Wien

Estimating a parameter in a birth-and-death process model

We deal with estimation of an unknown one-dimensional parameter ϑ ranging over $\Theta = (-C, +\infty)$, in a particular birth-and-death process model where the observed process is either transient (case $\vartheta > 1$), positive recurrent (case $\vartheta < 0$) or recurrent well (case $0 \leq \vartheta \leq 1$). It is known that a certain random observation scheme establishes local asymptotic normality at all points $\vartheta \in \Theta$ of the model, everywhere with same local scale $1/\sqrt{n}$. We construct and discuss different families of estimator sequences for the unknown parameter. Some of these sequences, being regular in the sense of Hájek at all points $\vartheta \in \Theta$, fail to be efficient in the sense of the convolution theorem, on different subsets of Θ.

Reinhard Höpfner, Freiburg/Paris

Quasi-likelihood models and efficient estimation

Consider an ergodic Markov chain on the real line, with parametric models for the conditional mean and variance of the transition distribution. Such a setting is an instance of a quasi-likelihood model. The customary estimator for the parameter is the maximum quasi-likelihood estimator. We show:
1. The maximum quasi-likelihood estimator is as good as the best estimator that ignores the model for the conditional variance.
2. There is an estimator which is as good as the maximum quasi-likelihood estimator if the conditional variance is correctly specified, and strictly better, and efficient, if it is not.
3. An efficient estimator in the quasi-likelihood model is given as a weighted nonlinear one-step least squares estimator, with weights involving predictors for the third and fourth centered conditional moments.

Wolfgang Wefelmeyer, Köln

Asymptotics of Algorithms

We determine the asymptotic distribution for some examples of stochastic recursive algorithms. The proof based on a contraction technique consists of three steps. First determine the stable limiting equation of a normalized version of the recursion, Secondly choose a probability metric which leads to contraction properties of the operator describing the limiting equation. This metric has to reflect the structure of the recursion. Thirdly decompose the normalized recursion into one part which converges to zero and one part which approximates the limiting equation. The examples discussed include sorting algorithms, trie algorithms, search algorithms, the bootstrap estimator and iterated function systems. The talk is based on joint work with S.T. Rachev.

Ludger Rüschendorf, Münster

Recent results for Kolmogorov-Smirnov tests and related tests

The first part of the talk deals with two-sample goodness of fit tests of Kolmogorov-Smirnov, Cramér-von Mises and Anderson-Darling type when ties are present. Two methods are presented in order to obtain valid (asymptotically) α-similar tests. Also the power function is calculated in direction of non-parametric tangent vectors.
The second part deals with the local comparison

of different tests. Each non-parametric unbiased test has a principal component decomposition of the curvature of the power function given by a Hilbert-Schmidt operator. Thus every non-parametric test has reasonable curvature only for a finite number of orthogonal directions of alternatives. As application one obtains results about the curvature of the two-sided Kolmogorov-Smirnov tests. It is shown that these tests prefer for small α approximately the same direction as the two-sample median rank test. The results are analogous to earlier results of Hájek and Šidák for one-sided Kolmogorov-Smirnov tests.

Arnold Janssen (Düsseldorf)

Accurate test limits with estimated parameters
Joint work with W. Albers G. D. Otten (Enschede)

Due to measurement errors, producers are typically forced to set test limits well within specification limits. The methods used in practice are rather informal and usually conservative with respect to consumer loss, thus leading to unnecessary loss of yield. We present approximations for test limits which are still relative easy to evaluate and moreover very accurate. In addition, the analytical tractability of these approximations allows extension to the more realistic case where parameters are estimated.

Wilbert Kallenberg (Enschede)

Chi-squared tests with large number of degrees of freedom.

Let $x_1, \dots, x_N$ be independent normally $N(\mu_i, 1)$ distributed r.v.'s, and $H_0: \mu := (\mu_1, \dots, \mu_N) = 0$ against $H_1: \mu \neq 0$, $\Sigma \mu_i = 0$ is to be tested. It is proved that the chi-squared test based on $\chi_N^2 = \Sigma(x_i - \bar{x})^2$ is asymptotically most powerful within the class of symmetric (permutation invariant) tests, as $N \to \infty$.

Dmitrii Chibisov (Moscow)

Normal approximation for mean-value estimates in absolutely regular Voronoi tessellations

We consider a d-dimensional Voronoi tessellation $V(\Psi) = \{C_i, i \in \mathbb{N}\}$ generated by a stationary point process $\Psi = \{X_i, i \in \mathbb{N}\}$; $C_i = \{x \in \mathbb{R}^d : \|x - X_i\| \leq \|x - X_j\| \text{ for } j \neq i\}$. One of the best studied models for a random tessellation of the space $\mathbb{R}^d$ is the well-known Poisson-Voronoi tessellation which is generated by a homogeneous Poisson process Ψ. In order to measure the departure of an observed tessellation from a Poisson VT (which often serves as a kind of gauge model) one has to find suitable characteristics and corresponding test statistics with known (approximate) distribution. The question arises how to find such a distribution. To tackle this problem we assume that the generating point process Ψ satisfies a β-mixing condition (absolute regularity), e.g. in case of Poisson cluster and certain Gibbs processes, and show that the random closed set $\partial V(\Psi) = \bigcup_{i \in \mathbb{N}} \partial C_i(\Psi)$ - the skeleton of the VT - and, hence, all associated point processes (nodes

midpoints of edges, circumcentres of facets) satisfy a similar absolut regularity condition.
Finally, applying some results and techniques from the limit theory of strongly mixing random fields we obtain asymptotic normality of the proposed intensity estimators of the associated point processes. Using a suitable estimate of the asymptotic variance of these estimates (which is shown to be asymptotically unbiased and consistent) we can establish asymptotic $100(1-\alpha)\%$ confidence intervals for the intensities under consideration. Here asymptotics mean that the sampling region grows unboundedly in all directions.

Lothar Heinrich, Freiberg

Random and Incidental Nuisance Parameters:

Consider a model with structure parameter ϑ and nuisance parameter η. Suppose that $\vartheta \in \mathbb{R}$, $\eta \in (0,1)$, and that η is governed by a uniform distribution. Then for a loss function W there is a bound β_W such that the following holds: If the risks of a permutation invariant estimator sequence (ϑ_n) are asymptotically seldom worse than $\beta_W + \varepsilon$ for all $\varepsilon > 0$, then they are asymptotically seldom better than $\beta_W - \varepsilon$ for all $\varepsilon > 0$.

Helmut Strasser (Vienna).

On efficient estimation in semiparametric regression

The problem of constructing efficient estimates of the finite dimensional parameter in regular semi-parametric regression models with unknown error and covariate distributions is discussed. The efficient influence function is derived and shown to depend on the projection of characteristics of the model onto some subspace. It is then shown how to construct an efficient estimate if appropriate estimates of the regression function and this projection are available. Special care in estimating these quantities. An example is presented in which the projection can be calculated explicitly and a plug-in estimate does not work under minimal conditions. It is shown that empirical projection estimates ~~should~~ work in this case.

Anton Schick (Binghamton)

Bootstrapped nonlinear estimators

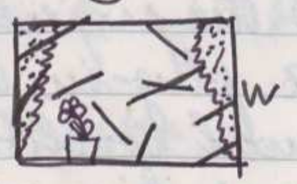
We are looking for strong LLN for bootstrapped statistics. We give conditions for the uniform a.s. convergence for bootstrapped normalized weighted sums of variables where the coefficients depend on parameters. This result we use for the proof of the strong LLN of the bootstrapped least squares estimator in nonlinear distribution families. As a second problem we use the bootstrap distribution for the BIAS reduction of nonlinear estimators. H convex parameters are to be estimated then this procedure leads to estimators improved w.r.to the mean squared error.

Henning Läuter (Potsdam)

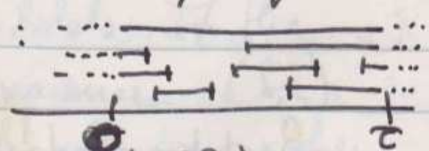
For 3 more, see pages 204-205. Jk dk

Laslett's line segment problem

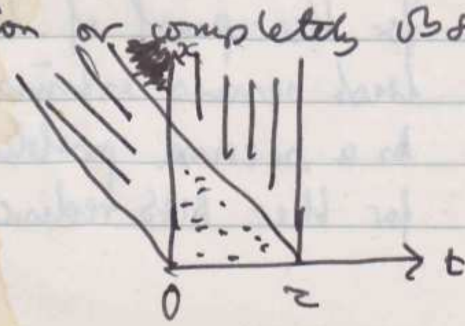
G. Laslett (Biometrika, 1981?) considered ~~the~~ a number of related nonparametric estimation problems including the following: a Poisson line segment process is observed through a bounded window W, so that some line segments are completely observed, some are 'censored' on one or both sides:



Assuming independent line segment lengths and orientations the object is to estimate the length distribution F. Following Laslett we show how the likelihood can be calculated. Computation of the NPMLE of F is feasible though to derive its asymptotic properties is an open problem. We specialize to the one-dimensional case:



Here we show the NPMLE is consistent (Wijers, 1992) using a general convexity argument and the fact that after reparametrization to a length biased version of F, we have a pure 'nonparametric missing data problem' in which underlying i.i.d. copies of X, T, where X ~ V (unknown) and $T \mid X = x \sim \text{Unif}(0,\tau)$ are grouped onto lines or into a region or completely observed according to the following figure:



The sharp point at the top of the

'completely observed' region introduces a singularity which so far has prevented us from proving (root n style) asymptotics. In fact by a result of van der Vaart (1991) root n estimation of $V(\tau)$ is impossible. Some modifications are proposed which are almost efficient.

Richard Gill. (Utrecht)

Copula Models

A parametric copula model $\{C_\theta : \theta \in \Theta\}$ is a parametric family of distribution functions on the unit square with uniform(0,1) marginal distributions: $C_\theta(u,1) = u$, $C_\theta(1,v) = v$, $u,v \in [0,1]$. The Archimedean copulas are of the form $C(u,v) = \psi^{-1}(\psi(u) + \psi(v))$, and have an interpretation in terms of frailty models when $\psi^{-1}(u) = \underline{E}\, e^{-uW}$ is completely monotone. Semiparametric copula models can be obtained from any particular parametric copula models by composition with arbitrary marginal distributions:

Model 1. $\mathcal{P}_1 = \{P_{\theta,G} : P_{\theta,G}$ has d.f. $F_{\theta,G}(s,t) = C_\theta(G(s), t)$, $\theta \in \Theta$, $G \in \mathcal{G}\}$

Model 2. $\mathcal{P}_2 = \{P_{\theta,G,H} : P_{\theta,G,H}$ has d.f. $F_{\theta,G,H}(s,t) = C_\theta(G(s), H(t))$, $\theta \in \Theta$, $G \in \mathcal{G}, H \in \mathcal{H}\}$.

In this talk I discussed information bound theory for semiparametric copula models with emphasis on "model 2", the case of two unknown marginal distributions. In this case the efficient score function l_θ^* for estimation of θ is

$l_\theta^* = \dot{l}_\theta - \dot{l}_g a_* - \dot{l}_h b_*$ where $A_*' = a_*$, $B_*' \equiv b_*$ are determined by the coupled differential equations

$$(*) \quad \begin{aligned} A_*'' - \alpha A_*(u) &= -\gamma(u) - \int_0^1 B_*(v) K(u,v)\,dv \\ B_*'' - \beta B_*(v) &= -\delta(v) - \int_0^1 A_*(u) K(u,v)\,du \end{aligned}$$

where $\alpha(u) = E(\dot{l}_u^2 \mid U=u)$, $\gamma(u) = E(\dot{l}_\theta \dot{l}_u \mid U=u)$

$\beta(v) = E(\dot{l}_v \mid V=v)$, $\delta(v) = E(\dot{l}_\theta \dot{l}_v \mid V=v)$

$$K(u,v) = \ddot{l}_{uv}(u,v)\, c_\theta(u,v),$$

$$\dot{l}_\theta(u,v) = \frac{\partial}{\partial\theta} \log c_\theta(u,v).$$

For most families C_θ the equations $(*)$ do not have an explicit solution. However, in the special case of the bivariate normal copula family

$$\{C_\theta(u,v) = \Phi_\theta(\Phi^{-1}(u), \Phi^{-1}(v)) : -1 < \theta < 1\},$$

the equations (*) have an explicit solution leading to the efficient influence function

$$l_\theta^*(s,t) = \frac{1}{(1-\theta^2)^2}\left\{xy - \frac{\theta}{2}(x^2+y^2)\right\}\Bigg|_{\substack{x = \Phi^{-1}(G(s)) \\ y = \Phi^{-1}(H(t))}}$$

and the efficient information for θ is $I_\theta^* = (1-\theta^2)^{-2}$, which is exactly the same as for estimation of θ in the bivariate normal submodel. Thus the bivariate normal submodel is least favorable. The efficient score equation $0 = \mathbb{P}_n \hat{l}_\theta^* = n^{-1}\sum_1^n \hat{l}_\theta^*(S_i, T_i)$ leads to the normal scores rank correlation coefficient

$$\hat\theta_n = \sum_1^n \Phi^{-1}\left(\tfrac{i}{n+1}\right)\Phi^{-1}(R_{ni}/n+1) \Big/ \sum_1^n \Phi^{-1}\left(\tfrac{i}{n+1}\right)^2$$

as an estimator of θ, and it follows from Ruymgaart, Shorack, and van Zwet (1972) that $\hat\theta_n$ is asymptotically efficient: $\sqrt{n}(\hat\theta_n - \theta) \to_d N(0, (1-\theta^2)^2)$.

Jon A. Wellner, Seattle

A new approach to testing goodness of fit

We study the properties of a method of testing for goodness of fit. This method can be viewed as a way of selecting the best among a family of tests corresponding to test statistics $\{T_j\}, j \geq 1$ such that the tests based on T_j are "best" against alternatives in a finite dimensional parametric family $\mathcal{F}_j$. We suppose $\mathcal{F}_j \subset \mathcal{F}_{j+1}$, $j \geq 1$. The tests we propose roughly,

(i) Are powerful against low dimensional alternatives ($\mathcal{F}_j$ with j small)

(ii) Perform as well as the best of the T_j based tests for alternatives not belonging to $\bigcup_k \mathcal{F}_k$.

(iii) Are consistent against all alternatives.

This kind of approach is, in part, proposed but not studied analytically in Rayner and Best (1989).

Peter J. Bickel, Berkeley

On the application of martingale inequalities to maximum likelihood estimation

We consider uniform exponential probability inequalities for martingales, imposing entropy conditions which are analoguous to those used in empirical process theory. These inequalities can be applied in several maximum likelihood problems.

For example, let $\{N_t\}$ be a counting process with continuous compensator $\{A_t\}$, and suppose that $dA/d\mu = a(\theta_0)$, $\theta_0 \in \Theta$. Let

$$L_T(\theta,\theta_0) = \int_0^T \log\left(\frac{a(\theta)}{a(\theta_0)}\right) dN - \int_0^T (a(\theta) - a(\theta_0))\, d\mu$$

be the log-likelihood ratio, and let

$$h_T^2(\theta,\tilde{\theta}) = \frac{1}{2}\int_0^T \left(a^{1/2}(\theta) - a^{1/2}(\tilde{\theta})\right)^2 d\mu$$

be the Hellinger process. Then one can define a function $\phi_\sigma(b)$, expressed in terms of the entropy with bracketing of $\{a(\theta) : \theta \in \Theta\}$ endowed with metric $h_T(\theta,\tilde{\theta})$, such that for $b_* \geq \phi_\sigma(b_*)$ we have

$$\mathbb{P}\left(L_T(\theta,\theta_0) \geq 0 \wedge h_T(\theta,\theta_0) > b_* \text{ for some } \theta \in \Theta\right)$$

$$\leq C_1 \exp(-C_2 b_*^2) + \mathbb{P}(A(T) > \sigma^2).$$

Sara van de Geer (Leiden)

Bootstrap Tests for Multimodality

Some results are presented for the expected number of modes of kernel density estimators. These results are applied for the study of test statistics for multimodality based on kernel density estimates. Asymptotic results are given which suggest that bootstrap can be used for achieving critical values and that these bootstrap tests are conservative.

Enno Mammen
Heidelberg

An optimum design for estimating the first derivative.

This is a joint work with Roy E. Erickson and Jan Mařík (both from East Lansing, MI). The result concerns an optimum choice of step lengths in the estimate of the first derivative, used in Fabian (Ann. Math. Statistics: 1967, 38, 191-200; 1968, 39, 457-466, 1327-1333). The optimum choice leads to the minimal expected squared error of a stochastic approximation procedure. The problem is equivalent to the minimization of Γ treated in the theorem below

Theorem. Let m be an integer, $m \geq 2$,

$$X = \{ x ;\ x \in R^m ,\ 0 < x_1 < -x_2 < \cdots < (-1)^{m-1} x_m ,\ \prod_{i=1}^{m} |x_i| = 1 \}$$

and let Γ be defined on X by

$$\Gamma(x) = \frac{\det[1, x^3, \ldots, x^{2m-1}]}{\det[x, x^3, \ldots, x^{2m-1}]}$$

($x^\lambda = \langle x_1^\lambda, x_2^\lambda, \ldots, x_m^\lambda \rangle$). Then the minimal value of Γ is m and is attained at exactly one point x, given by

$$x_i = (-1)^{i-1}\, 2\cos\left(\frac{m+1-i}{2m+1}\pi\right) \text{ for } i = 1, \ldots, m,$$

($x_i/2$ are the roots of the 2nd type Chebyshev polynomial of degree m.)

Vaclav Fabian, East Lansing, MI

Linking blocks in the bootstrap for stationary observations.

We consider the problem of estimating the distribution of a statistic when the underlying observations are stationary. A truly model free procedure for this is the blockwise bootstrap which resamples independent blocks of consecutive observations. For consistency it only requires mixing and moment conditions. Still, for finite n the bootstrap variance has a bias even in the case of the mean. In order to reduce this bias, we propose to link the blocks by choosing

their starting points according to a Markov chain. The transition probabilities are determined by assuming a partially specified model for the observations, e.g. an ARMA- or a Markov model. The analysis of this method leads to the study of U-statistics with kernels depending on the sample size.
(joint with E. Carlstein, Chapel Hill)

Hans R. Künsch
ETH Zürich

GEOMETRY IN Non-parametrics WITH CENSORING

IN THIS Talk, We USE A GEOMETRIC APPROACH

TO (1) IDENTIFY THE OPTIMAL RATES INFORMATION FOR ESTIMATING Non-parametric Functionals WITH CENSORING

(2) CONSTRUCT OPTIMAL RATE ESTIMATORS THROUGH A GEOMETRIC QUANTITY AND F/R/W PROCEDURE.

(3) CONSTRUCT Nearly Optimal (Best) Estimators which ARE WITHIN ONLY 25% TO THE BEST AMONG ALL POSSIBLE PROCEDURES IN EST. NON-PARAMETRIC FUNCTIONALS WITH CENSORING.

Simply, IN A GENERAL AND UNIFIED SET UP.

OUR CENTRAL TOOL IS A GEOMETRIC QUANTITY CALLED "Modulus OF CONTINUITY". THE ISSUES (1) and (2) WERE NOT ANSWERED COMPLETELY IN THE LITERATURE; AND (3) WAS NEVER STUDIED BEFORE.

Richard C. Liu 12/18/9
RICHARD C. LIU

Estimation of Pearson Correlation Ratio, with Applications to Nonparametric Regression Model Building.

In a nonparametric regression setting with multiple random predictor variables, we give the asymptotic distributions of estimators of global integral functionals of regression. We apply the results to the problem of obtaining reliable estimators and confidence procedures for the nonparametric coefficient of determination. This coefficient, which is also called Pearson's correlation ratio, gives the fraction of the total variability of Y that can be explained by a given set of covariates. It can be used to measure the relative importance of subsets of regressors, & nonlinearity of regression function, and a number of other restrictions on the form of regression. In addition to providing asymptotic results, we propose several methods of data-dependent bandwidth selection and use Monte Carlo simulation to ~~obtain~~ study finite sample size results.

Alexander Samarov
MIT + U. Mass
Boston, USA.

2/18/92

Geometrie und Kombinatorik

vom 13. Dez. – 19. Dez. 1992

K-Loops, eine hierher gehörende Bemerkung über unendliche Frobeniusgruppen und Alexander Kreuzers neue Beispiele

Seit Karzels Vorlesung (1965) weiß man, daß die scharf 2-fach transitiven Gruppen als Gruppe der linearen Abbildungen $x \to a + m \cdot x$ eines Fastbereichs $(F, +, \cdot)$ dargestellt werden kann. Präzisiert man die Eigenschaften der additiven Struktur $(F, +)$, so kommt man zum Begriffs der K-Loop. Jede K-Loops $(K, +)$ ist eine Bruck-Loop aber nicht umgekehrt. Falls $x \to x+x$ injektiv ist, gilt auch die Umkehrung. Das prominenteste Beispiel einer K-Loop fand A. Ungar in der Menge der relativistisch zulässigen Geschwindigkeiten $\{v \in \mathbb{R}^3 \mid |v| < c\}$ zusammen mit der relat. Addition (oder analog in der Menge der Boots in der homogenen orthochronen Lorentzgruppe). Ungar bezeichnet diese Loops jetzt als Gyrogroups.
Hier werden nun Alexander Kreuzers, auch endliche neue Beispiele vorgestellt, die durch geschickte Modifizierung einer Zassenhaus'schen Konstruktion gefunden wurden.
Jede K-Loop läßt sich in gewisser Weise quasidirekt in eine Frobeniusgruppe einbetten. Dies führt zu folgender Vermutung: Ist G eine unendliche Frobeniusgruppe und G_a ein Stabilisator, dann gibt es in der Menge N von fixpunktfreien Abbildungen plus id eine Teilmenge $N' \subseteq N$ sodaß $G = N' \cdot G_a$ eindeutig dargestellt wird und N' normal in Bezug auf G_a ist. N' wäre dann der Frob. Kern zum Frob. Komplement G_a

19. Dez. 1992 Heinrich Wefelscheid

Zum Büschelsatz in Kettengeometrien

Im Jahre 1988 hat W. Benz diejenigen unitären, assoziativen und kommutativen Algebren (A, K), deren Kettengeometrie $\Sigma(K,A)$ dem Büschelsatz genügt, unter der Voraussetzung $|K| \geq 13$ gekennzeichnet (J. of Geom. 31). In seiner Diplomarbeit hat A. Lutz gezeigt, daß diese Kennzeichnung auch für $|K| \geq 7$ gilt. Wie W. Benz angegeben hat, gilt sie dagegen nicht im Fall $|K| = 5$. Im nicht-kommutativen Fall ist über die Gültigkeit des Büschelsatzes wenig bekannt. Es wird eine Klasse von Algebren angegeben, in deren Kettengeometrie der Büschelsatz gilt.

18.12.1992 H.-J. Kroll (München)

Anzahlprobleme in Geometrie und Versuchsplanung

Der Vortrag beschäftigt sich mit dem für die Versuchsplanung fundamentalen Problem der Konstruktion aller (v, b, r, k, λ)-Blockpläne (BIBD) zu vorgegebenen Parametern v, b, r, k, λ. Eine „Übersetzung“ in die Geometrie führt auf das Problem der Konstruktion aller $S_\lambda(2, k, v)$. Die wesentlichsten Ergebnisse werden unter besonderer Berücksichtigung des Falles $\lambda = 1$, $r = k$ (projektive Ebene der Ordnung $n := k-1$) referiert.
Für die Bestimmung der Anzahl nicht-isomorpher projektiver Ebenen einer Ordnung $n \leq 7$ wird ein einheitlicher, rein inzidenzgeometrischer Beweis vorgestellt, der für die Ordnungen $n = 6, 7$ erheblich kürzer als alle bekannten Beweise ist.

18.12.1992 Helga Tecklenburg (Gießen)

Zum Problem der inneren Form des Pasch-Axioms in Anordnungsräumen.

Die Theorie der Anordnungsräume mit Hilberts Verlängerungsaxiom ist neben (1.) der linearen Anordnung jeder Geraden, ohne Randelemente der Anordnung, vor allem charakterisiert durch (2.) das allgemeine Paschaxiom. Anstelle von (2.) genügt dabei auch das „spezielle Paschaxiom" (P). Und (P) ist – aufgrund von (1.) – äquivalent zur logischen Summe seiner beiden Teilaussagen ($P_{außen}$) und (P_{innen}). E. H. Moore [1902] zeigte, daß hiervon das Axiom ($P_{außen}$) allein genügt. Zur Frage, ob auch (P_{innen}) allein anstelle von (2.) genügt – wobei man intuitiv die Unmöglichkeit vermutete – konnte meines Wissens bisher kein exaktes Gegenmodell angegeben werden.

Hier wird ein solches Gegenmodell konstruiert.

18. 12. 1992

Martin Schröder
(Duisburg)

Die Lösung des Waterloo-Problems

Sei $\mathcal{D}$ ein klassischer symmetrischer Blockplan $PG_{d-1}(d,q)$ oder das Komplement eines solchen Blockplans. Bekanntlich (Satz von Singer) hat $\mathcal{D}$ eine zirkulante Inzidenzmatrix N. Das Waterloo-Problem besteht darin, die Einträge 1 in N so mit Vorzeichen zu versehen, daß eine orthogonale zirkulante Matrix N' entsteht.

SATZ. Für $\mathcal{D} = PG_{d-1}(d,q)$ ist das Waterloo-Problem nicht lösbar. Für das Komplement von $PG_{d-1}(d,q)$ ist das Waterloo-Problem genau dann lösbar, wenn d gerade ist.

Zum Beweis wird das Problem zunächst in ein Problem für Differenzenmengen übersetzt. Danach führen Methoden aus der Geometrie (Quadriken), Darstellungstheorie, algebraischen Zahlentheorie und ein bekanntes Resultat über die diophantische Gleichung $\frac{x^k-1}{x-1}=y^2$ zum Ziel.

Dieter Jungnickel, Gießen

Über die Anzahl maximaler Untersysteme in $S(2,4,v)$

Genau für alle $v \in \{1,13\} + 12\mathbb{N}_0$ existieren Systeme $S(2,4,v)$. Wir fragen nach der Anzahl $N(v)$ von Untersystemen der maximalen Mächtigkeit $r = \frac{1}{3}(v-1)$ bei gegebener Ordnung v.

(1) $N(v \in \{16,25,28,37\} + 36\mathbb{N}_0) = \{0\}$ (de Resmini)

(2) $1 \in N(v \in \{49,40\} + 36\mathbb{N}_0)$ (Shen Hao)

(3) $2,4 \in N(v \in \{121,40\} + 108\mathbb{N}_0)$

(4) $\frac{1}{2}(3^k-1), \frac{1}{2}(5\cdot 3^{k-2}-1) \in N(v \in \{\frac{1}{2}(3^{k+2}-1), \frac{1}{2}(3^{k+3}-1)\} + 3^k\cdot 12\mathbb{N}_0)$

(5) $0 \in N(v \in \{16,25\} + 12\mathbb{N}_0)$? Conjecture

Der Beweis von (3) wird im Detail vorgeführt.

Herbert Zeitler, Bayreuth

The Helly dimension of convex bodies

Vladimir G. Boltyanski

This is my second talk on this subject (the first was in February). Some new results appear during this time.

According to classical Helly theorem, if every $d+1$ of convex bodies $M_1, \dots, M_s$ in $\mathbb{R}^d$ have a common point, then $M_1 \cap \dots \cap M_s \neq \emptyset$. In particular, let $M \subset \mathbb{R}^d$ be a compact, convex body. Then if $M_1, \dots, M_s$ are translates of M and every $d+1$ of them have a common point, then $M_1 \cap \dots \cap M_s \neq \emptyset$. But, sometimes the number $d+1$ can be decreased. If, for example, M is a parallelotope and every 2 of its translates $M_1, \dots, M_s$ have a nonempty intersection, then $M_1 \cap \dots \cap M_s \neq \emptyset$. This leads us to the notion of Helly's dimension. We say that a compact, convex body $M \subset \mathbb{R}^d$ ~~has~~ is r-Helly dimensional (we write $\operatorname{him} M = r$) if for ~~every family~~ $M_1 \dots M_s$ ~~of~~ (its translates) every $r+1$ of which have a common point the relation $M_1 \cap \dots \cap M_s$ holds. Thus $\operatorname{him} M \leq d$ for every compact, convex body in $\mathbb{R}^d$, and $\operatorname{him} M = 1$ for parallelotopes. The problem of geometrical description of bodies with $\operatorname{him} M = r$ is important in Combinatorial geometry. It is solved now for several r and several classes of convex bodies (for zonoids, for example). The Hungarian mathematician Kincses has given a geometrical description of centrally symmetric compact, convex bodies with $\operatorname{him} M \leq 4$. In the talk was formulated a conjecture which (if it is true) describes a ~~[illegible]~~ solution of Helly-dimensional classification of all centrally symmetric, compac, convex bodies. Prof Kincses and I hope to justify this hypothesis. Some other unsolved problems were indicated.

December 15, 1992. [signature]

(V. Boltyanski)

Möbiusebenen mit Kreisspiegelungen

Es wurden Beispiele von nicht miquelschen Möbiusebenen angegeben, bei denen an jedem Kreis eine Spiegelung existiert.

14.12.92 H. Mäurer (Darmstadt)

Prüfzeichensysteme

1. Ein Prüfzeichensystem über einer endlichen Gruppe G bestehe hier aus einer Permutation T von G und einer Prüfgleichung $\prod_{i=1}^{n} \delta T^{i}(a_i) = c$ (mit $c \in G$ fest), die von jedem zulässigen Wort $a_1 \dots a_n \in G^n$ erfüllt werden muß. Ein solches System erkennt alle Einzelfehler, aber Nachbartranspositionen nur, wenn gilt: $(*)$ $aT(b) \neq bT(a)$ für alle $a, b \in G$ mit $a \neq b$.
2. Im Vortrag wurden einige Beispiele für $(*)$ erfüllende T angegeben, u.a.
a) G Gruppe ungerader Ordnung, $T: x \mapsto x^{-1}$ b) G Gruppe der $m \times m$-Dreiecksmatrizen über $K = GF(q)$, $T: \begin{pmatrix} 1 & \cdot & 0 \\ a_{ij} & \cdot & 1 \end{pmatrix} \mapsto \begin{pmatrix} 1 & \cdot & 0 \\ f_{ij}(a_{ij}) & \cdot & 1 \end{pmatrix}$ mit f_{ij} Orthomorphismen von $(K,+)$ c) G Gruppe der Matrizen $[x,y,z] := \begin{pmatrix} 1 & x & y & z \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & -x \\ 0 & 0 & 0 & 1 \end{pmatrix}$ über $GF(q)$ und $T: [x,y,z] \mapsto [f(x), g_x(y), h_{xy}(z)]$ mit f, g_x, h_{xy} Orthomorphismen.
3. Ferner wurde gezeigt, daß Automorphismen T einer Gruppe G die Bedingung $(*)$ höchstens dann erfüllen, wenn sie auf den Konjugiertenklassen $\neq \{1\}$ fixpunktfrei operieren. Damit kommt z.Bsp. bei den Diedergruppen D_n mit $n \neq 2$ und bei den affinen Gruppen $A(1,q)$ kein Automorphismus als T in Frage.
4. Zwei Permutationen T und $\hat{T}$ von G heißen stark äquivalent, falls gilt: $\exists \alpha \in \mathrm{Aut}\, G: \hat{T} = \alpha^{-1} \circ T \circ \alpha$. Eine zu T stark äquivalente Abbildung $\hat{T}$ gestattet es, Einzelfehler, Nachbar-, Sprung-Transpositionen, Zwillings- bzw. Sprungzwillingsfehler genau dann zu erkennen, wenn T die entsprechende Eigenschaft hat. Zwei Abbildungen T und $\tilde{T}$ heißen schwach äquivalent, falls es eine zu T stark äquivalente Permutation $\hat{T}$ und $c, d \in G$ gibt mit $\tilde{T} = R_c \circ \hat{T} \circ L_d$; hierbei bezeichnen R_c und L_d die Links- bzw. Rechtstranslation mit d bzw. c. Wieder gilt: T läßt Einzelfehler bzw. Nachbartranspositionen erkennen genau dann, wenn $\tilde{T}$ diese Eigenschaft hat. Allerdings gehen die Prüfgleichungen nicht ineinander über, auch nicht die übrigen Fehlererkennungs-

Eigenschaften.

5. Als Beispiele wurden (bis auf schwache Äquivalenz) bestimmt:
a) die (*) erfüllenden Permutationen von C_5 und b) die (*) erfüllenden Permutationen der Diedergruppe D_5, die die Untergruppe $\langle b \rangle$ der Ordnung 5 auf sich abbilden: Man erhält nur eine Klasse und als deren Vertreter z. Bsp. T mit $T(b^i) = b^{-i}$ und $T(ab^j) = a\,b^{j+1}$.

15. 12. 1992 Ralph-Hardo Schulz

Generalized euclidean spaces and their automorphisms

Generalized euclidean spaces were introduced by Kroll-Sörensen and Karzel as incidence spaces with a congruence relation on pair of points which is compatible with the incidence structure. They can be described by means of anisotropic metric vector spaces (V, K, q) over commutative fields K with $|K| \geq 3$ and char $K \neq 2$ or $\dim(V,K) = 2$.
In such geometries, besides the congruence between pair of points, one may define other metric relations, like perpendicularity between lines and two different congruence relations between angles, and consider the corresponding automorphism groups.
We prove (together with Karzel and Stanik) that any 3-dimensional anisotropic metric vector space can be considered as the subspace J of pure quaternions in a suitable generalised quaternion field (H, K) so that the related 3-dimensional euclidean space $(J, \mathcal{L}_J, \equiv_J)$ is embedded in a 4-dimensional euclidean space $(H, \mathcal{L}_H, \equiv)$ whose congruence is given by the norm form of (H, K).
Therefore the automorphism groups of the different metric structures defined either in $(H, \mathcal{L}_H, \equiv)$ or in $(J, \mathcal{L}_J, \equiv_J)$ are related, by means of the left multiplications in H^*, to the automorphism group of the quaternion field.

16. 12. 1992 Silvia Pianta (Brescia)

Some results on Galois Geometries.

A short summary of the old developments of the theory is given. Some recent results on caps in high dimensional spaces (by P. Grandi) and on arcs with weighted points (by E. D'Agostini) are presented.

An interesting open problem is to prove the non-existence of planes $\pi(q)$, $q > 9$ in which all complete k-arcs have the same number k of points.

17. 12. 1992 Adriano Barlotti

Eine hinreichende Bedingung für die Existenz S-zyklischer $SQS(v) = S(3,4,v)$

Ausgehend von Arbeiten der Herren F. Fitting und E. Köhler, bei denen die Existenzfrage von S-zyklischen $SQS(v)$ auf die Bestimmung eines 1-Faktors eines gewissen Graphen $G_2(p)$ zurückgeführt wird, konstruieren wir den Bahnengraphen von $G(p)$ (auch Köhler-Graph $KG_2(p)$ genannt), bei dem die Ecken die Bahnen einer Automorphismengruppe U von $G_2(p)$ sind und die Kanten in naheliegender Weise bestimmt werden. Hat $KG_2(p)$ einen 1-Faktor, dann auch $G_2(p)$. Die Graphen mit $p \equiv 5\ (12)$ können in 4 Klassen eingeteilt werden nach den Ecken vom Grad < 3. Ist dieser Graph (oder ein entsprechend reduzierter Graph) brückenlos, dann kann ein Satz von Petersen herangezogen werden um die Existenz eines 1-Faktors von $KG_2(p)$ zu zeigen. Die Bedingung der Brückenlosigkeit führt auf einen zahlentheoretischen Satz, dessen Beweis in einer Vielzahl von Schritten erläutert wird. Damit ist die Existenz eines 1-Faktors von $KG_2(p)$ gesichert. Bei Verwendung eines Satzes von Piotrowski kann dann die lange offene Vermutung bewiesen werden: Ein streng zyklisches $SQS(v)$ existiert, wenn $v \geq 4$

$v \equiv 0(2)$; $v \not\equiv 0(4)$ und wenn jeder ungerade Primteiler p von v die Bedingung $p \equiv 5(12)$ erfüllt. Ist $v \equiv 0(2)$, $v \not\equiv 0(8)$, $v \geq 4$ und ist für ungerade Primteiler p von v stets $p \equiv 5(12)$ dann existiert ein S-zyklisches Steiner Quadrupel System $SQS(v)$.

17.12.1992 Helmut Siemon

Beziehungen zwischen K-Loops, der relativ. Vektoraddition und der hyperb. Geometrie.

Bis heute wissen wir nicht, ob es Fastbereiche gibt, die keine Fastkörper sind. Die Eigenschaften des additiven Loops eines Fastbereichs veranlaßten 1972 W. KERBY und H. WEFELSCHEID den Begriff des K-Loops einzuführen. 1980 bemerkte G. KIST, daß gewisse (von G. GLAUBERMAN 1966 angegebene) Bol-Loops auch K-Loops sind, und 1988 stellte A. UNGAR im Rahmen der speziellen Relativitätstheorie eine binäre Operation (Addition von Geschwindigkeiten) vor, von der H. WEFELSCHEID feststellte, daß sie ein Beispiel eines echten K-Loops ergibt. Gerade vor wenigen Tagen gelang es A. KREUZER eine ganze Klasse endlicher und unendlicher K-Loops anzugeben. In dem Vortrag wird gezeigt: Unter Benutzung 2-reihiger Hermite-scher Matrizen läßt sich auch die Punktmenge des hyperbolischen Raumes in einer eleganten Weise in einen K-Loop verwandeln und gleichzeitig auch die relativistische Geschwindigkeitsaddition beschreiben.

17.12.1992 Helmut Karzel

Über Extremaleigenschaften in der Banach-Minkowskischen Ebene

Die Eichfigur einer Banach-Minkowskischen Ebene sei eine zentralsymmetrische, konvexe und kompakte Menge. Es wird gezeigt, daß in einer solchen Ebene, in der die Eichfigur k der Flächeninhalt $A_M(k) = \pi$ zugeordnet ist, für gleichseitige Dreiecke Δ der Seitenlänge 1 stets

$$\frac{1}{8}\pi \leqq A_M(\Delta) \leqq \frac{1}{6}\pi ,$$

bzw. für Reuleaux-Dreiecke Δ_R der Breite 1 stets

$$\frac{1}{6}\pi \leqq A_M(\Delta_R) \leqq \frac{1}{4}\pi$$ gilt.

Der Inhalt $\frac{1}{6}\pi$ für ein Δ_R tritt nur in einer Ebene mit einem affinregulären Sechseck als Eichfigur auf, und zwar fällt Δ_R mit dem erzeugenden gleichseitigen Dreieck Δ zusammen.
Der Inhalt $\frac{1}{4}\pi$ kommt nur Reuleaux-Dreiecken der Breite 1 zu, die zugleich in der Ebene Kreise und entweder Parallelogramme oder zentralsymmetrische Sechsecke sind. Weitere Reuleaux-Dreiecke, die minkowskische Kreise sind, gibt es nicht.

Bernd Wernicke, Erfurt

Kombinatorische und strukturelle Fragestellungen zu OSTWALD-Mustern

Der bekannte Chemiker W. OSTWALD hat im Zusammenhang mit seiner Farbforschung Ideen zu einer „gesetzlichen Lehre über Formen" und [illegible] eine

auf eine praktische Nutzung ausgerichtete Mustererzeugung dargelegt (Welt der Formen, Leipzig 1922–1926).
Ausgehend von dieser Idee wird ein Begriff der (elementaren) OSTWALD-Muster eingeführt. Auf der Grundlage eines hexagonalen bzw. quadratischen Gitters wird eine Parkettierung $\mathfrak{P}$ n-ter Ordnung betrachtet und hinsichtlich einer Untergruppe G von S(B) und eines Themas $\hat{T}$ die Form $F = F(T, G)$ gebildet. Das Muster $M = M(F, E)$ wird durch eine Untergruppe E von S(P) erzeugt, die transitiv über den durch $\mathfrak{P}$ bestimmten Pikeln operiert.
Von Interesse sind eine Reihe von Anzahlbestimmungen und Symmetrieeigenschaften von Formen und Formstrukturen sowie vielfältige künstlerische Aspekte an den Mustern hinsichtlich diskreter Raumgruppen und ihrer verfeinerten Klassifizierungen. Es ist bemerkenswert, daß sich verschiedenartige Verfeinerungen der Klassifizierungen weitgehend schon mit elementaren OSTWALD-Mustern repräsentieren lassen. Dabei ergeben sich eine Reihe weiterer Fragestellungen.

17.12.92 Eckhard Quaisser

[Ende Geometrie u. Kombinatorik]

Asymptotische Statistik
↓ Fortsetzung

Asymptotic expansions for L^p-norms of empirical processes

Let B be a real Banach space. Let $X, X_1, X_2, \ldots \in B$ denote a sequence of i.i.d. r.v. with values in B. Denote $S_n = n^{-1/2}(X_1 + \ldots + X_n)$. Let $\varpi: B \to R$ be a polynomial. We consider Edgworth expansions for the distribution function of $\varpi(S_n)$, as well as for the derivatives of this distribution function. As an application we get asymptotic expansions in the integral and local limit theorems for the general ω-statistic $\omega_n^p(q) = n^{p/2} \int_0^1 (F_n(x) - x)^p q(x)\,dx$. Here $p = 2, 4, \ldots$, $q: [0,1] \to [0,\infty]$ is a weight function, and F_n is a empirical distribution function.

17.12.92 V. Bentkus

Estimating the expectation of a discount reward process

~~Suppose~~ Let $\theta = E\sum \lambda^{t-1} R_t$ for unknown random process. We look for unbiased estimator efficien under white noise models and show that under some model, geometric stopping time and equals weights yield the efficient design. On strange things happen under other models.

17.12.92 Yacov Ritov

Transformation Models

A general class of semiparametric transformation models is considered. A second order differential equation is derived which determines a semiparametric information bound for estimation of the real parameter involved. A frailty model of Clayton and Cuzick for survival data is studied in some detail. It is shown that the information bound is sharp by constructing an estimator attaining it.

17.12.92 Chris Klaassen

ON the Calculation of Bartlett correction in time series models:

Computing the Bartlett correction for the likelihood ratio statistic involves the computation of approximate bias of the likelihood ratio. This may be difficult in time series models. An alternative approach to the calculation is suggested. The approach is related to the one that uses a formula for the conditional distribution of MLE given the ancillaries, in the case of transformation models.

18/12/92

P. Jeganathan

Saddlepoint and Laplace-type expansions – an overview.

Saddlepoint approximations are known as a highly accurate form of asymptotic expansions of a density (Daniels, 1954), or of a distribution function (Esscher, 1932, Lugannani and Rice, 1980). Basic results concern sums of i.i.d. random variables, but applications often go further.

An overview of the possibilities of deriving saddlepoint expansions for various statistics is given. This starts from the basic derivation of such an expansion, for example for sums of independent random variables, and continues with the possible operations leading from one such expansion to another. These operations include conditioning, marginalization, non-linear transformation, and one-dimensional integration.

Applications in statistics are discussed, in particular the difficulties related to expansions of distributions of test statistics.

18/12/92 U. Skovgaard

Asymptotic behaviour of L-statistics.

The aim of our talk was to present some results of our attempts to apply Probability in Banach spaces techniques to the investigation of an asymptotic behaviour of L-statistics. More specifically we considered asymptotic normality of a linear combination of a function of order statistics. Namely we commented two improved variants of known sufficient conditions for asymptotic normality. The first one concerned the case when the weight constants have specific form and the second one allowed the weight constants to be arbitrary. To obtain our results we used differentiability of superposition (or Nemytskii) operators induced by an integral representation of the L-statistics and a central limit theorem for distributions of the empirical process with a.a. paths in a Banach function space.

18/12/92 Norvaiša

Some Aspects of Random Trees

A survey was given of some recent results concerning binary trees with stochastic structures. More specifically, models for river networks and for disordered electrical networks were considered.

The stochastic and statistical analysis of river networks has, within the last ten years, grown into a very substantial and exciting subject area. This is however not so widely recognized among probabilists and statisticians because most of the important papers have been published in Earth Sciences journals, particularly Water Resources Research. Much of the work relate to Horton's Laws, which are empirically determined regularities pertaining to order, length and drainage area of stream links (or sections). Of some special interest are a series of recent papers that establish a connection between fractality and Horton ratios. Other results describe the limiting behaviour, in stochastic river networks, of the main channel length, the width of the network, etc., the statements being conditional on the number of sources or on the order of the network.

For electrical networks with random ~~networks~~ resistances one of the questions of interest is to characterize the distributional properties of the total resistance R of the network. The distribution function of R can generally not be found explicitly. However, certain specifications in terms of inverse Gaussian and reciprocal inverse Gaussian distributions do allow an explicit solution.

18 December 1992

Ole E. Barndorff-Nielsen

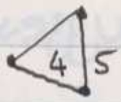
Extensions of buildings & generalisations, 03 – 09 January, 1993

The commutativity of the ground division ring of a D_n-geometry

If Γ is a thick and residually connected D_n-geometry, $n \geq 4$, it is well known that Γ is defined over a unique ground division ring which is commutative.
A footnote of Tits (1964) suggests to give an elementary proof of this fact.
It is easy to show that it suffices to treat the case of D_4-geometries.
The main step of the proof is to build a null polarity in the 3-dimensional projective subspaces of Γ (i.e. a polarity π such that any point p is incident with $\pi(p)$).
Here is how to construct the null polarities. Consider two end nodes of the diagram D_4 and call elements of these types respectively points and blocks. A block has a structure of 3-dimensional projective geometry. For any disjoint blocks B and B', we find a natural duality $\delta_{BB'}$ of the residue Γ_B onto $\Gamma_{B'}$. Next, we show that for every block B, there are blocks B' and B'' such that B, B' and B'' are pairwise disjoint.
The mapping $\delta_{B'B} \circ \delta_{B'B''} \circ \delta_{BB'}$ is a null polarity in Γ_B.

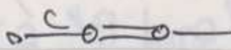
Cécile Huybrechts, Bruxelles

CHANGING GENERATORS OF REFLEXION GROUPS

If $\Gamma = \langle \rho_0, \rho_1, \dots, \rho_{n-1} \rangle$ is a group generated by involutions ρ_j, which satisfies the intersection property $\langle \rho_i \mid i \in I \rangle \cap \langle \rho_i \mid i \in J \rangle = \langle \rho_i \mid i \in I \cap J \rangle$ for all $I, J \subseteq N := \{0, 1, \dots, n-1\}$, then Γ underlies a flag-transitive thin geometry. (Its elements are the cosets $\Gamma_j \alpha$ with $\alpha \in \Gamma$, where $\Gamma_j := \langle \rho_i \mid i \neq j \rangle$, with incidence given by $\Gamma_j \alpha \cap \Gamma_k \beta \neq \emptyset$.) Now Γ may underlie different geometries, obtained from alternative choices of generators. In this talk, various systematic ways of changing generators were considered, with particular reference paid to triangle groups ($n = 3$) generated by hyperplane reflexions in unitary space. As an example, there are 13 different ways of generating the group with diagram  (the inner mark stands for the relation $(\rho_0 \rho_1 \rho_2 \rho_1)^4 = \varepsilon$), of order 2160.

Peter McMullen, University College London.

SOME NON-SPLIT EXTENSIONS OF ORTHGONAL GROUPS.

My purpose is to give explicite constructions of the non-split extensions $3^7 \cdot O(7.3)$, $3^8 \cdot O^-(8.3)$, which are maximal subgroups of the Fischer group F_{24} and the Fischer-Griess Monster F_1, and these subgroups are the point-stabilizer of their 3-local geometries belonging to the diagram . This purpose is not established yet, but some subgroups of $3^7 \cdot O(7.3)$ has been constructed.

For $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3) \in \mathbb{F}_3^3$, we define $g(x.y) = (x_1 - y_1)(x_2 y_3 - x_3 y_2)$. [This is due to R.L. Griess] We set $M = \mathbb{F}_3 \times \mathbb{F}_3^3$, and define some permutations $P(\alpha . x)$

on $\mathcal{M}$ by

$$p_{(\alpha, x)} : (\beta, y) \mapsto (g(x,y) - a - \beta, -x-y)$$

where the subscript (α, x) is also in $\mathcal{M}$.

Then the group ~~is isomorphic to~~ $\langle p_{(\alpha,x)} \mid (\alpha, x) \in \mathcal{M} \rangle$ is isomorphic modulo center to some subgroup of $3^7 O(7,3)$. Moreover we have

$$p_{(\delta, w)}^{\left(p_{(\alpha,x)} p_{(\beta,y)} p_{(\gamma,z)}\right)^2} = p_{(\delta+\Delta, w)}, \quad \Delta = \begin{vmatrix} x & \vdots \\ y & \vdots \\ z & \vdots \\ w & \vdots \end{vmatrix},$$

and this equation gives a simple proof of the non-splitness of the group $3^7 O(7,3)$.

Similarly a subgroup $3^6 \cdot 3^5 . O(5,3)$ of $3^7 O(7,3)$ can be constructed explicitly. This group is corresponding to the stabilizer of a isotropic vector.

MASAAKI, KITAZUME
(Chiba University)

Some extended generalized hexagons

A few years ago R. Weiss studied geometries of type $\circ\overset{c}{—}\circ\equiv\circ$, where the generalized hexagon is known and either thick or point-thin, under the condition that there exists 3 mutually collinear points that are not on a circle. In the case where the generalized hexagon is the one associated with $PSL_3(2):2$ he also assumed that the stabilizer of a point p, G_p, induces this group on the residue of p. In this talk we discuss the situation where G_p induces ~~a~~ $7:6$ on the residue of p.

John van Bon, Tufts University

CP^* Geometries

Γ is a geometry with diagram $\underset{0}{\circ}\overset{c}{-}\underset{1}{\circ}\overset{P^*}{-}\underset{2}{\circ}$, assumed to be residually connected, to satisfy (LL) (i.e. 2 points lie on at most 1 line) and to have a flag-transitive group of automorphisms G. Such a geometry is one type belonging to the diagram $\circ\overset{c}{-}\circ\overset{P^*}{-}\circ$, the other type being in essence $\circ\overset{P}{-}\circ-\circ$ of which there are only two examples. G must act imprimitively on the 2-elements.
The Petersen graph may be embedded in vector spaces of dimension 4, 5 or 6 over $GF(2)$ with edges as non-zero vectors and points as 2-dimensional subspaces. Geometries (with diagram $\circ\overset{c}{-}\circ\overset{P^*}{-}\circ$) may be constructed as follows: consider the vector space as an affine space and take the points of the geometry to be the points of the affine space; take as lines the 15 affine lines through the origin that correspond to the edges of the Petersen graph together with all translates; take as planes the 10 affine planes through the origin corresponding to the points of ~~the~~ the Petersen graph together with all translates. The geometries so constructed have 16, 32, 32 and 64 pts with full automorphism groups $2^4{:}S_5$, $2^5{:}S_5$, $2^5{:}A_5$ and $2^6{:}S_5$ respectively.
An alternative viewpoint is to consider the collinearity graph $\bar{\Gamma}$ of Γ: the neighbourhood graphs must be regular of valency 4, 6, 8, 12 or 14 on 15 vertices. There appear to be very few possibilities except in the case of valency 4. In the latter case $\bar{\Gamma}$ is locally the line graph of the Petersen graph; also Γ has the property that any triangle lies in a circle (2-element).

Oliver King, Newcastle University U.K.

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Chiral polytopes

Abstract polytopes are discrete geometric structures which generalize the classical notion of a convex polytope. Chiral polytopes are those abstract polytopes which have maximal symmetry by rotation, in contrast to the abstract regular polytopes which have maximal symmetry by reflection. Chirality is a fascinating phenomenon which does not occur in the classical theory. In ~~a~~ joint work with E. Schulte we give the basic theory of chiral polytopes. We present some general results and discuss the existence of chiral polytopes in higher dimensions.

Asia Ivić Weiss, York University, Canada

Generalizing the Alexandrov theorem on spacetime in special relativity.

The Alexandrov theorem for a Minkowski space M_n of dimension n states that a permutation of the points preserving 0-distances is an automorphism of M_n. The result has been extended to arbitrary fields, the metric being defined by a quadratic form of index ≥ 1 and infinite dimensional spaces being allowed (J. Lester (1985), E. Schröder (1990)). Our viewpoint departs from metric and from automorphisms. We consider any affine space A and a set B of points at infinity generating the hyperplane at infinity. The Alexandrov space Alex (A, B) is the set of points of A equipped with the collection of sets $C(p, B)$ with $p \in A$ and $C(p, B)$ the union of all lines of A containing p and a point $b \in B$. We get sufficient conditions on B, in order that Alex (A, B) does uniquely determine all lines of A and the parallelism. One condition like this is to ask that B is a nondegenerate polar space of index ≥ 1, embedded in the hyperplane at infinity, with no nucleus and different from a symplectic polar space. Such a structure, for index ≥ 2, provides a geometry of type $\circ\overset{Af}{—}\circ—\circ\cdots\circ—\circ=\circ$.

Francis Buekenhout, Bruxelles

Classification of the tilde geometries of symplectic type

Tilde geometries are the geometries belonging to the diagram $\circ\overset{\sim}{—}\circ—\circ\ldots\circ—\circ$, where $\circ\overset{\sim}{=}\circ$ stands for the triple cover of the generalized quadrangle of order $(2,2)$ (so that $Aut(\circ\overset{\sim}{=}\circ) \cong 3\cdot S_6$). Nice sporadic examples ~~of tilde~~ for the groups M_{24}, He, Co_2, M are known due to Ronan-Smith. An infinite series of examples arising as 1-covers of symplectic geometries over $GF(2)$, was constructed by Ivanov-Shpectorov. By now a complete classification of flag-transitive tilde geometries is available and the classification (or rather a characterization) in the title, is a part of the general classification belonging to (Shpectorov-Stroth) several authors. As a by-product of the classification, all the natural embeddings of the tilde geometries are known.

Sergey Shpectorov, VNIISI, Moscow
temporarily TUE Eindhoven

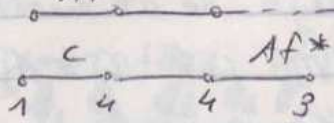
Linear-dual-linear extensions of projective geometries

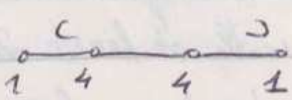
I consider flag-transitive geometries belonging to the following diagram of rank $n \geq 4$

$(r \leq t < s < \infty)$

By a Theorem of Delandtsheer, the possibilities ~~the~~ that can occur are the following:

(1)


(2)


(3)


It is known that (3) characterizes a unique geometry for HS. We prove that (2) is impossible. We also obtain some partial results on (1), towards a proof of the following conjecture: (1) characterizes geometries obtained from PG(n, q) (for some prime power q) by deleting a hyperplane and the star of a point and, possibly, by taking quotients.

Antonio Pasini (*)
University of Naples.

(*) Joint work with Alberto Del Fra, University of Rome.

Flagtransitive rank 3 geometries, which are locally complete graphs

Let G be a flagtransitive group on a geometry Γ of type $\underset{P}{\bullet}\overset{c}{-\!-}\underset{L}{\bullet}\overset{c^*}{-\!-}\underset{F}{\bullet}$ (P: point, L: line, F: plane), supposing that the stabilizer of a point has no regular normal subgroup.
It is allready known that G is finite. Furthermore we have $K_P = K_F = 1$ and the groups G_P and G_F are isomorphic (also as 2-transitive permutation groups on Res(P) ~~Res~~, Res(F) respectively).
We get the nice criterion: If $Z(B) = 1$ and $G_P \neq L_2(11), A_7, L_2(8)Z_3$ of degree 11, 15, 28, then Γ is a hypercube.
With the help of this criterion and the Todd-Coxeter-algorithm we are able to show: If one point is incident to at most 20 planes then Γ and G are known or $G \cong Z_2^{\cdot} M_{22}$ with $G_P \cong A_7$. We can give a simple geometrical description of this example.
Finally assuming that one point is incident with at least 20 planes we get Γ is a hypercube or G_P is a group of Lie-type of rank 1.

Barbara Baumeister, Freie Universität Berlin.

The affine plane $AG(2,q)$, q odd, has a unique one point extension

Let I be a finite inversive plane of odd order q. If for at least one point p of I the internal affine plane I_p is Desarguesian, then I is Miquelian. Other formulation: the finite Desarguesian affine plane of odd order q has a unique one point extension; this extension is the Miquelian inversive plane of order q. It follows that there is a unique inversive plane of order q, with $q \in \{3, 5, 7\}$.

J. A. Thas, Univ. of Gent, Belgium

Bounding the diameter of locally Γ graphs.

<u>Thm</u> Let Γ be a Taylor graph, not the hexagon. Then locally Γ graphs have bounded diameter.

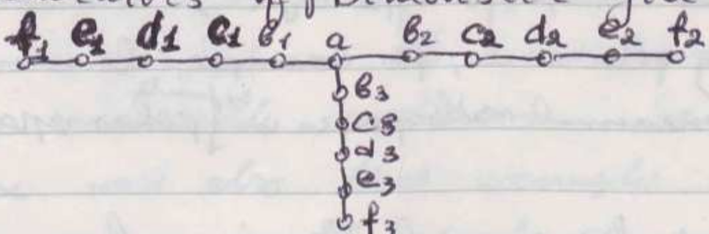
<u>Examples</u> 1. There are precisely three locally Γ graphs, when Γ is the incidence graph of the 7-point biplane. But when $\Gamma' = \Gamma -$ edge, then there are infinitely many locally Γ' graphs.

2. There are infinitely many graphs locally the line graph of the Petersen graph.

A.E. Brouwer, Amsterdam/Eindhoven

A Coxeter subsystem for ${}^3D_4(2)$ of the Coxeter system $\circ \overset{6}{—} \circ — \circ — \circ — \circ — \circ$ for the Thompson group.

Y-generators of the Bimonster give rise to a Coxeter group



admitting the automorphism π of order 3 that rotates 3 branchs. But the additional relation $(ab_1c_1ab_2c_2ab_3c_3)^{10}=1$ which defines the Bimonster doesn't admit π. Nevertheless in the Monster π can be interpreted as an inner automorphism induced by 3C-element of the Monster. Its centralizer in the Monster is the Thompson group so that it may be considered as a quotient of

$$\langle x_1 \overset{6}{—} x_2 — x_3 — x_4 — x_5 — x_6 \rangle^+$$

We prove the following results.

<u>Theorem 1</u> The subgroup $\langle x_1 \overset{6}{—} x_2 — x_3 — x_4 \quad x_6 \rangle^+$ is isomorphic to ${}^3D_4(2)$.

Denote $y_i = x_i x_6$, $1 \le i \le 4$.

<u>Theorem 2.</u> ${}^3D_4(2)$ has the following presentation:

$y_1 \overset{6}{—} y_2 — y_3 — y_4$ $\quad ((y_1y_2)^2y_3)^6 = 1$, $y_5 = (y_4(y_1y_2)^3y_3)^7$, $(y_4y_5)^4 = 1$, $y_5^2 = y_1$.

S.V. Tsaranov /Moscow/

Ovoids, spreads, and flat geometries

In an incidence structure (P, L), an <u>ovoid</u> is a set of points meeting every line in a unique point – this is the dual concept to that of a <u>spread</u>. It is shown by transfinite induction that, in any incidence structure (P, L) with infinitely many lines which satisfies

(a) # points/line = # lines (= n, say),

(b) if n is regular then $\alpha(p, \ell) < n$ for all non-incident (p, ℓ); a slightly stronger condition if n is singular.

then P can be partitioned into ovoids.

The hypotheses are satisfied, for example, in all infinite generalized polygons with $s = t$, and in all classical dual polar spaces over infinite fields except for type $O_4^+(F)$. Similar techniques handle many other cases, for example the incidence structure of points and maximal subspaces in infinite polar spaces of finite rank.

The results show that, for example, there is no hope of listing all geometric hyperplanes in infinite dual polar spaces. Also, with a technique due to Kantor, a plethora of flat geometries with diagrams like o=o—o can be constructed.

Peter J. Cameron (London)

Folding diagrams & filling apartments

Consider a building with certain diagram Δ. Certain foldings of Δ give rise to lower rank buildings. A lot of examples related to generalised polygons are known (see Mühlherr's talk) We present a heuristic argument for the converse: given a (next abstract) generalised polygon S under which conditions can one reconstruct the building it is supposed to live in naturally? This can be answered in many cases by considering the apartment of S and appropriately filling it. This way, for example, we construct the 24-cell out of a customary octagon.

H. Van Maldeghem (Gent, Belgium)

Embeddings of buildings

Polarities of projective spaces provide a polar spaces embedded in these spaces. This situation is also well known for other buildings. Examples are the hexagons fixed by a triality in D_4 or the octagons fixed by a polarity in F_4. In the case of thin buildings - the Coxeter complexes - one gets also other embeddings which do not come from an automorphism of a diagram. These are provided by admissible partitions. So the question is, whether there exist also thick examples of such embeddings. The main result about this question is the following:

Theorem: Every n-gon $(3 \le n < \infty)$ embedded in a building of rank at least 3 of irreducible type is Moufang.

B. Mühlherr (Tübingen)

Nonabelian representations of geometries

Let $S = (P, L)$ be a point-line incidence system with 3 points on a line. A group H is a <u>representation group</u> of S if it is generated by a set of (non-necessary distinct) elements x_p indexed by points $p \in P$ such that (i) $x_p^2 = 1$; (ii) $x_p x_q x_r = 1$ if p, q, r are distinct and collinear. If in addition (iii) $[x_p, x_q] = 1$

for all $p, q \in P$ then H is a <u>representation module</u> of S. It is clear how to define the universal representation group and the universal representation module in terms of generators and relations. Let $G \cong J_4$, F_2 or F_1 and $\mathcal{Y}(G)$ be the 2-local geometry having the central involutions as points. It is known that $\mathcal{Y}(G)$ does not have a nontrivial representation module, but G is obviously a representation group of $\mathcal{Y}(G)$.

<u>Conjecture</u>. The universal representation groups of $\mathcal{Y}(G)$ for $G \cong J_4$, F_2 and F_1 are isomorphic to J_4, $2 \cdot F_2$ and F_1, respectively.

A considerable progress in proving the conjecture was recently done ~~#~~ in my joint work with D.V. Pasechnik and S.V. Shpectorov.

A. A. Ivanov (Moscow & Ann Arbor)

<u>Some flagtransitive extensions of polarspaces of non-classical type</u>

<u>Theorem 1.</u> Let $\mathcal{G}$ be a residually connected thick geometry belonging to the diagram $\circ\overset{c}{-\!-}\circ=\!=\circ-\!-\circ$ admitting a flag-transitive automorphism group G. If the residue $\mathcal{G}_p$ at a point p is not the dual of a classical polar space, then one of the following occurs (it is conjectured the case (2) does not hold).

(1) $\mathcal{G}_p$ is the sporadic A_7-geometry, $G = \mathrm{Aut}(\mathcal{G}) \cong 2^4 : A_7$ and $\mathcal{G}$ is isomorphic to a geometry on 16 points, which can be described in terms of Steiner System $S(24, 8, 5)$. There is a unique circular extension $\mathcal{G}'$ of $\mathcal{G}_p$ belonging to the diagram $\circ\overset{c}{-\!-}\circ=\!=\circ-\!-\circ\overset{c^*}{-\!-}\circ$ and there is no extension of ~~the~~ $\mathcal{G}'$.

(2) $G = \mathrm{Aut}(\mathcal{G})$ acts regularly on chambers and $y+2 = 2^e$ or 3^e for $e \geq 1$.

Constructions and Characterization of some $c.C_2$-geometries with non-classical point residues are also given.

Satoshi Yoshiara (Hirosaki Univ.)

Extensions of semiaffine linear spaces

For a non-incident point-line pair (P,l) in a linear space S we denote by $\pi(P,l)$ the number of lines on P that are disjoint to l. We call a linear space H-semiaffine, if H is a set of integers containing all $\pi(P,l)$. The finite $\{0,1\}$- and $\{0,1,2\}$-semiaffine linear spaces have been determined by Dembowski and Hauptmann, Olanda-Lo Re; apart from few exceptions they are all in a natural way embeddable in a projective plane.

Here we discuss finite incidence structures S with the property that for any point P, the derivation S_P is a $\{0,1,2\}$-semiaffine linear space.

It turns out that any such finite structure is (apart from few exceptions with very few points) embeddable in an inversive plane or in an extension of a projective plane. (Joint work with D. Olanda, Napoli).

Alfred Beutelspacher (Gießen).

Some flag-transitive extensions of polar spaces of non-classical type

Theorem. Let $\mathcal{G}$ be a [illegible] geometry belonging to the diagram [illegible] admitting a flag-transitive automorphism group G. If the residue $\mathcal{G}_p$ at a point p is not the dual of a classical polar space, then one of the following occurs (it is conjectured the case (2) does not hold).

(1) $\mathcal{G}_p$ is the sporadic A_7-geometry, $G = \mathrm{Aut}(\mathcal{G}) \cong 2^4 \cdot A_7$ and $\mathcal{G}$ is isomorphic to a geometry on 16 points, which can be described in terms of Steiner system [illegible]. There is a unique [illegible] extension of the [illegible] belonging to the diagram [illegible] with [illegible]

(2) $G = \mathrm{Aut}(\mathcal{G})$ acts regularly on chambers and [illegible]

Constructions and characterizations of some C_n-geometries and non-classical polar spaces are also given.

Satoshi Yoshiara (Hirosaki Univ.)

GRUNDLAGEN DER GEOMETRIE

3.1.1993 – 9.1.1993

Examples of K-loops

In order to describe sharply 2-transitively groups, H. Karzel introduced the notion of a neardomain $(F, \oplus, \cdot)$. W. Kerby and H. Wefelscheid called the additive structure $(F, \oplus)$ a K-loop, which is a loop ~~the~~ with the properties:
For every $a, b, c \in F$ there exists an automorphism $\delta_{a,b} \in Aut(F, \oplus)$ with $a \oplus (b \oplus c) = (a \oplus b) \oplus \delta_{a,b}(c)$, $\delta_{a, b\oplus a} = \delta_{a,b}$, $\delta_{a,b} = id$ if $a \oplus b = 0$, and $\ominus(a \oplus b) = (\ominus a) \oplus (\ominus b)$.
A. Ungar shows that for $\mathbb{R}^3_c := \{v \in \mathbb{R}^3 : |v| < c\}$ (c = speed of light), $(\mathbb{R}^3_c, \oplus)$ with the relativistic velocity addition is a K-loop. Further examples are:

1. For a field $(K, +, \cdot)$ let $F := K^4$ and for $x, y \in F$ let $(x_1, x_2, x_3, x_4) \oplus (y_1, y_2, y_3, y_4) := (r, x_2 + y_2, x_3 + y_3, x_4 + y_4)$ with $r := x_1 + y_1 + (x_2 + 2y_2)(x_3 y_4 - x_4 y_3)$. Then $(F, \oplus)$ is a K-loop.
2. For a map $\lambda : \mathbb{R} \to \mathbb{R}^*$, $x \to \lambda(x) := \lambda_x$ with $\lambda(x)\lambda(y) = \lambda(x+y) + \lambda(x-y)$ let for $(x,y), (z,w) \in \mathbb{R}^2$: $(x,y) \oplus (z,w) := (x+z, \varphi(x,z)y + \psi(x,z)w)$ with $\varphi(x,z) = \lambda_{2x} \lambda_x^{-1} \lambda_{x+2z} \lambda_{2x+2z}^{-1}$, $\psi(x,z) := \lambda_{2z} \lambda_{2z+2x}^{-1}$. Then $(\mathbb{R}^2, \oplus)$ is a K-loop.
3. Furthermore every Bruck-loop with the property "$x \oplus x = 0 \Rightarrow x = 0$" ~~[illegible]~~ is a K-loop, and the finite examples for Bruck-loops of H. Niederreiter and K.H. Robinson turn out to be K-loops. Hence for odd primes p, q with q dividing $(p^2 - 1)$ there exists a K-loop $(L, \oplus)$ with $|L| = pq$.

4.1.93

Alexander Kreuzer
Technische Universität München

A characterization of miquelian Minkowski planes

An automorphism α of a Minkowski plane M is called a (p,q)-homothety if the restriction of α to the affine derivation $A(q)$ at the point q is a dilatation with fixed point p. The Minkowski plane M is called (p,q)-transitive, if the group $\Gamma(p,q)$ of all (p,q)-homotheties of M acts transitively on $C\setminus\{p,q\}$ where C is a cycle with $p,q\in C$. M is called strongly q-transitive if it is (p,q)-transitive for every point p of $A(q)$. In 1982 E. Hartmann charactized the miquelian Minkowski planes M by the property: M is strongly q-transitive for all points q. We weaken this condition and obtain the Theorem: A Minkowski plane M is miquelian if and only if ther is a generator G such that M is stro q-transitive for all points $q\in G$.

4.1.1993 Hans-Joachim Kroll (München)

Generalized kinematic groups.

A generalized kinematic group is a kinematic space $(G,\cdot,F)$ together with an involutory antiautomorphism $*: G\to G$ of the group $(G,\cdot)$ and a subset $P\subset Fix(*)$ such that $\forall x\in G: xPx^*=P$. G can be provided with a linear incidence-structure $\mathcal{L}$ and a congruence $\equiv$. With respect to the trace structure of $\mathcal{L}$ and $\equiv$, P becomes an „absolute space" and $G^\square :=$ $=\{g^\square := g_\ell\circ(g^*)_r \mid g\in G\}$ a motion group of $(P,\mathcal{L}_P,\equiv_P)$. In this way one can obtain all absolute planes and the 3-dim. hyperbolic space. For special generalized kinematic groups P can be turned in a K-Loop $(P, \oplus)$ and $G^{\square}$ has the representation as a quasi-direct product $G^{\square} = P^{\oplus} \rtimes_Q Q^{\square}$ where $Q := \{ x \in G \mid x^* = x^{-1} \}$. This result can be applied on the proper orthochronous linear Lorentz-group G where P is a set of pure boosts and Q the group of rotations of the 3-dim. euclidean space.

4.1.1993 Helmut Karzel (TUM)

Möbius planes and geometry of reflections

At the beginning we refered to results of P. Dembowski, H. Mäurer and H. Karzel. Geometry of reflections in Möbius planes was also investigated by E. Molnár (over pythagoreian fields) and by K. Lang (over fields of char ≠ 2). We have given a group theoretical system of axioms for a miquelian Möbius plane (independent of the characteristic). We need for the proof of the „Berührsatz" (Touch axiom) one of the two orthogonality axioms. So we have a system of three simple „incidence" axioms and one weak „orthogonality" axiom for a group so that the group plane is a Möbius plane with a reflection in each circle.

4. 1. 1993 Bernd Wennicke (Erfurt)

Investigations to the mapping geometrical representation of geometries

At first we refered to conditions at the conception of the mapping geometrical representation. Then is given some results of the „complete" representability of the plane affine geometry (and the affine geometry with arbitrary dimension ≥ 2) on basis of affine reflections resp. non involutory shears resp. axiale dilatations resp. skew reflections resp. m-reflections. In last case the necessary condition for the representability results from the reflection geometrical language.
In second part is given a characterization theorem for the representability of the plane parabolic geometry: This geometry is representable iff all models are translation planes with char $\neq 2$ and (L,L)-transitivity (with regard to incidence structure)

4.1.93 E. Quaisser (Potsdam)

Spiegelungsgeometrie unendlich dimensionaler Räume (Reflection Geometry of infinite dimensional spaces)

Nachdem in mehreren Jahrzehnten bedeutende Beiträge zur Spiegelungsgeometrie metrischer Ebenen erschienen waren und von F. Bachmann in einer Monographie zusammengefaßt wurden, begann die Übertragung der Methoden auf den n-dimensionalen Fall (Ahrens '59, Kinder '65). Besondere Probleme bestanden bei der Einbeziehung unendlich dimensionaler Räume, zu deren Lösung von mir zwei Varianten vorgestellt wurden: 1) Zulassung gewisser "zulässiger" unendlicher Produkte von Erzeugenden – 2) Verwendung geteilter Erzeugenden-Systeme (Anwendung auf affine Räume beliebiger Dimension '71).

Daran schloß sich die entsprechende Kennzeichnung euklidischer und nichteuklidischer Räume beliebiger Dimension mittels eines geteilten Erzeugenden-Systems (Ewald '74; vgl. auch J.T. Smith '74/'75) sowie metrisch-euklidischer und pseudoeuklidischer Räume (Klotzek/Stamm '81; Klotzek '83); weitere modelltheoretische Fragen wurden von E. Quaisser '82 beantwortet. Die verallgemeinerte Erzeugung wurde von Kiy auf (verallgemeinerte) euklidische und pseudoeuklidische Räume mit unterschiedlichem Erfolg angewandt ('81 und '82). Abschließend wurde eine Liste untersuchungswürdiger Probleme vorgelegt.

4.1.1993 Bruno Klotzek (Potsdam)

Spreads of right quadratic skew field extensions

Let L/K be a right quadratic (skew) field extension and let $\tilde{\mathcal{P}}$ be a 3-dimensional projective space over K which is embedded in a 3-dimensional projective space $\mathcal{P}$ over L. Moreover let $\mathcal{I}$ be a line of $\mathcal{P}$ which carries no point of $\tilde{\mathcal{P}}$. The main result is that – even when L or K is a skew field – the following holds true: A desarguesian spread $\bar{\mathcal{P}}$ is given by the set of all lines of $\tilde{\mathcal{P}}$ which are indicated by the points of $\mathcal{I}$. (The case $L = \mathbb{C}$, $K = \mathbb{R}$ is due to Corrado Segre (1891) the cases when L and K are commutative or even finite have been discussed by Beutelspacher–Ueberberg, Bruck, Lunardon, B. Segre.) A spread of $\tilde{\mathcal{P}}$ arises in this way iff there exists an isomorphism of L onto the kernel of the spread such that K is elementwise invariant. Furthermore a geometric characterization of right quadratic extensions with a left degree other than two and of quadratic Galois extensions is given.

H. Havlicek – Wien

Characterizations of distances in Einstein's Cylinder Universe

Let C^n be the set of points of n-dim ECU and let d be a function from $C^n \times C^n$ into the set $R_{\geq 0}$ of non-negative reals such that

(i) d is a 2-point-invariant of the group of motions of C^n,

(ii) d is additive on admissible point triples,

(iii) d is locally Lorentz-Minkowskian

(iv) a normalization condition: $d\big((1,0,\dots,0),(1,0,\dots,0,1)\big) = 1$

Then $d^2(x,y) = \left| (\arccos x_0 y_0)^2 - (x_{n+1} - y_{n+1})^2 \right|$ for all $x, y \in C^n$ such that the angle between x_0, y_0 is less than π, z_0 being the projection of $z \in C^n$ into the hyperplane $\xi_{n+1} = 0$! arc cos has to be chosen in $[0, \pi]$.

6. 1. 1993 Walter Benz (Hamburg)

(n)-varieties in linear spaces

G. Tallini (Roma)

A (n)-variety H in a linear space (P, L) is a subset of P such that:
$\forall l \in L$, either $l \subset H$ or $|l \cap H| \in \{0, 1, n\}$, $n \geq 2$
and n-secant lines do exist.
H is called non-singular quasiregular if $\forall x \in H$ the union of the lines through x either tangent or contained in H is a subspace $\tau(x) \neq P$ (regular, strongly regular respectively if $\tau(x)$ is either a hyperplane or a prime). Here we outline the foundations of the theory of (n)-varieties, in particular of quasiregular ones, in a linear space (P, L) pointing out the properties of a linear space containing a (n)-variety and conversely. We explain several results and open problems on this subject.

6-1-1993 Giuseppe Tallini (Roma)

A conjecture of A. ROSA, concerning S(2,4,v).

A report about joint works with SHEN HAO (Shanghai) is given.
(1) Exactly $\forall v \in \{40, 49\} + 36\mathbb{N}_0$ there exist $S(2,4,v)$ with exactly one subsystem $S(2,4,r = \frac{1}{3}(v-1))$.
(2) It's possible that within the complement of $S(2,4,r)$ there always exists an affine plane $AG(2,3)$. Adding a line from $S(2,4,r)$ to $AG(2,3)$ in the usual way a projective

plane $PG(2,3)$ is obtained.
Some results of REES–STINSON are used.
(3) For all admissible numbers v, $v \neq 13$ there exists a system $S(2,4,v)$ without any subsystem $S(2,4,r)$. (The conjecture of A. ROSA).

H. Zeitler (Bayreuth)

Topics on hypervector spaces

M. Scafati Tallini (Rome)

We define hypervector space over a field K the quadruplet $(V,+,\circ,K)$, where $(V,+)$ is an abelian group and

$$\circ : K \times V \to P'(V)$$

is a mapping of $K \times V$ into the set $P'(V)$ of non-empty subsets of V, such that the following conditions hold:

(1) $\forall a,b \in K, \forall x \in V, (a+b)\circ x \subseteq (a\circ x)+(b\circ x)$,

(2) $\forall a \in K, \forall x,y \in V, a\circ(x+y) \subseteq (a\circ x)+(a\circ y)$,

(3) $\forall a,b \in K, \forall x \in V, a\circ(b\circ x) = (ab)\circ x$,

(4) $\forall a \in K, \forall x \in V, a\circ(-x) = (-a)\circ x = -(a\circ x)$,

(5) $\forall x \in V, x \in 1\circ x$.

Here we explain various properties of such spaces, the geometric structures that we associate with them and the cathegory of such spaces, once given in a suitable way the notions of factor space and homomorphism.

On 0-distance-preserving permutations of affine and projective quadrics

Let $(V, \mathbb{F}, q)$ be a metric vector space. The set $\mathcal{F}_\Pi := \{\mathbb{F}x \mid x \in V \setminus \{0\} \wedge q(x) = 0\}$ is the quadric w.r.t. q of the projective space $\Pi := \Pi(V, \mathbb{F})$. In the affine space $A := A(V, \mathbb{F})$, the set $\mathcal{F}_A := \{x \in V \mid q(x) + \varkappa(x) + \alpha = 0\}$ ($\varkappa : V \to \mathbb{F}$ linear, $\alpha \in \mathbb{F}$) is an affine quadric w.r.t. q, and $\mathcal{F}_A^\pi := \mathcal{F}_A \cup \{(\mathbb{F}x)\| \mid x \in V \setminus \{0\} \wedge q(x) = 0\}$ is a quadric of the projective closure $\Pi(A)$ of A.

Assume $\mathcal{F}_\pi$ resp. $\mathcal{F}_A^\pi$ contains lines, but no double points. For $\mathcal{F} := \mathcal{F}_\pi$ resp. $\mathcal{F} := \mathcal{F}_A$, points $X, Y \in \mathcal{F}$ are called 0-distant (or conjugate or parallel), written $X \approx Y$, iff the connecting line $\overline{X,Y}$ is contained in $\mathcal{F}$. Now assume $4 \le \dim \Pi \le \infty$ in the projective space resp. $4 \le \dim A \le \infty \wedge |\mathbb{F}| \geqslant 4$ in the affine case.

Then the following theorem holds true:

If φ is a permutation of $\mathcal{F}$ with the property

$$(*) \qquad X \approx Y \iff X^\varphi \approx Y^\varphi \quad \forall X, Y \in \mathcal{F},$$

then there exists an $\alpha \in \mathbb{F} \setminus \{0\}$ and a semi-linear bijection (σ, ρ) of $(V, \mathbb{F})$ such that $q \circ \sigma = \alpha \cdot \rho \circ q$ and

$X^\varphi = X^\sigma \;\; \forall X \in \mathcal{F}_\pi$ resp. $X^\varphi = 0^\varphi + X^\sigma \;\; \forall X \in \mathcal{F}_A$.

7.1.93 Eberhard Schröder (Hamburg)

Are there fractal structures in circle geometries?

For $z_n, w \in \mathbb{C}$, the iteration $z_{n+1} = z_n^2 + w$ leads to the well known Mandelbrot set $\mathcal{M}_\mathbb{C}$ and filled in Julia set $\mathcal{J}_\mathbb{C}(w)$. From a geometrical point of view $\mathbb{C}$ fixes the miquelian Möbius plane. There are further circle geometries over $\mathbb{R}$ of dimension 2 and 3; to each of them belongs an algebraic structure. These structures are $\mathbb{D} := \{a + b\varepsilon \mid a, b \in \mathbb{R} \not\ni \varepsilon, \varepsilon^2 = 0\}$, $\mathbb{R} \times \mathbb{R}$, $\mathbb{C} \times \mathbb{R}$, $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$, $\mathbb{D} \times \mathbb{R}$, $\mathbb{R}[X]/\langle X^3 \rangle$, $\mathbb{R}[X,Y]/\langle X^2, XY, Y^2 \rangle$.

<u>Problem</u>: What are the shapes of $\mathcal{M}_L$, $\mathcal{J}_L(w)$, if $L \in \{\mathbb{D}, \mathbb{R} \times \mathbb{R}, \mathbb{D} \times \mathbb{R}, \dots\}$

<u>Answer</u> (roughly): Look at a homeomorphic image of the "Feigenbaum"-

function $f(x) = x^2 + c$ in the right way and you know (nearly) everything you want!
In detail, all 2- and 3-dimensional Mandelbrot sets and all 2-dimensional Julia sets are illustrated by theorems and/or (computer) graphics.

7. 1. 93

Hans-Joachim Sauvage (Hamburg)

Centre Functions of Triangles

Identify the Euclidean plane with the complex numbers $\mathbb{C}$; then for any triangle Δabc, the cross ratio $[\infty, a; b, c]$ is called the shape of Δabc, and determines Δabc up to (direct) similarity. Shapes may be used to provide simple analytic proofs of many theorems about analytic triangles.

With respect to a fixed reference triangle Δabc, the triangle coordinate of any point p is $p_\Delta := [p, a; b, c]$. If p is a special point of the triangle (i.e. if it is defined in terms of the vertices and is similarly situated in similar triangles) then (Theorem) p_Δ is a function of the shape. If the special point is a centre, then its function satisfies several symmetry conditions.

These functions may be used to prove theorems about special points. Example: the circumcentre, the nine-point centre, and the two isogonic centres lie on a circle.

Complex triangle centres can be used to develop a theory of centres: the problem of finding all reduces to the problem of solving a certain system of functional equations.

June Lester, U. New Brunswick (Fredericton)

Automorphisms of Incidence Loops

A FIBERED INCIDENCE LOOP with PARALLELISM $(\mathcal{P},\mathcal{L},\parallel,\cdot)$ is nothing else then an incidence space with parallelism $(\mathcal{P},\mathcal{L},\parallel)$ together with a set $\mathcal{C}$ of translations of $(\mathcal{P},\mathcal{L},\parallel)$ acting transitively on $\mathcal{P}$ defined as the left mappings, for any $a\in\mathcal{P}$: $a^{\cdot}:\mathcal{P}\to\mathcal{P};\ x\to ax$. Furthermore a family $\mathcal{F}$ of subsets of, called STRINGS, namely $\mathcal{F}:=\{Mx : M\in\mathcal{L}(1), x\in\mathcal{P}\}$, can be defined under suitable conditions such that any two distinct points belong to exactly one string and an equivalence relation $\hat{\parallel}$, called string-parallelism and fulfilling the Euclidean axiom, is allowed.
This fact lead us to consider also the right mappings, for any $a\in\mathcal{P}$: ${}^{\cdot}a:\mathcal{P}\to\mathcal{P};\ x\to xa$.
Then properties fulfilled by the set $\mathcal{P}^{\cdot}:=\{a^{\cdot} : a\in\mathcal{P}\}=\mathcal{C}$ as a subset of $Aut(\mathcal{P},\mathcal{L},\parallel)$ and by the set ${}^{\cdot}\mathcal{P}:=\{{}^{\cdot}a : a\in\mathcal{P}\}$ as a subset of $Aut(\mathcal{P},\mathcal{F})$ are studied.

Mario Marchi (Brescia, Italy).

Characterization of isometric and similarity mappings, and fractals with paradoxical geometric properties

1) Let Ω be a countable subset of the set of positive numbers, $a, b\in\Omega$, $a\neq b$; and let $f:\mathbb{R}^n\to\mathbb{R}^n$, $n\geqslant 3$, be an injective mapping such that the condition

$d(X,Y)\in\{a,b\}$ (where $X,Y\in\mathbb{R}^n$, d - Euclidean metric) always implies that $d(f(X),f(Y))\in\Omega$. Then the mapping f is a similarity. (If Ω is the set of all prime numbers, then f is an isometry.)

2) There is a curve $\mathfrak{M} \subset \mathbb{R}^n$ (i.e., a subset of $\mathbb{R}^n$ which is homeomorphic to a straight line) which has the following property: for every straight line $\ell \subset \mathbb{R}^n$ the intersection $\ell \cap \mathfrak{M}$ has cardinality of the continuum.

3) There is a connection between the characterization of isometric (and similarity) mappings (see 1) and fractals which are analoges to the curve $\mathfrak{M}$ (see 2).

A.V. Kuz'minykh

7.1.93 (Novosibirsk, Russia)

Low order projective planes

It is well-known that there exists (up to isomorphism) exactly one projective plane of order n for $n \in \{2, 3, 4, 5, 7\}$, while a projective plane of order 6 does not exist. With short uniform proofs we show the uniqueness and existence or non-existence resp. of these planes.

7.1.93 Helga Tecklenburg (Gießen)

Examples of spherically complete spaces
(joint work with Paulo Ribenboim)

Let $(\Gamma, \leq)$ be a partially ordered set with $0 < \gamma$ for every $\gamma \in \Gamma$ and let $X \neq \emptyset$ be a set. A mapping $d: X \times X \to \Gamma \cup \{0\}$ is called an ultrametric, if it has formally the same properties as a metric, but instead of the triangle inequality the following one for all $x, y \in X$, $\gamma \in \Gamma$: if $d(x,y) \leq \gamma$, $d(y,z) \leq \gamma$, then also $d(x,z) \leq \gamma$. The concept of spherically completeness for such a space is a strengthening of completeness. Because one has a Banach-like Fixed Point Theorem for spherically complete spaces (cp. Priess-Crampe, Ribenboim, Abh. Math. Sem. Hamburg 1993), one is interested in examples of such spaces. Outside of valuation theory, complete Boolean algebras (with the symmetric difference as a distance) and function spaces R^X (with $d(f,g) = \{x \in X \mid f(x) \neq g(x)\}$ are examples of spherically complete spaces.

7. 1. 93 Priess-Crampe (München)

FOUNDATIONS OF A GENERALIZED AFFINE GEOMETRY

(A) CONCEPT : We develop an axiomatic structure theory of affine geometry which induces the geometry of all affine submodules of unitary modules (over associative rings). We point out that it is not intended to include "weak affine" geometries (as introduced by E. Sperner, H.-J. Arnold), "quasi-affine" geometries (as studied by J. André) or "generalized partial affine" geometries (as investigated by W. Leißner, F. Radó) — nevertheless the latter occur as partial substructures in our set-up.

(B) MAIN RESULTS: (1) Completion of "affine line systems" to "affine closure systems".
(2) Algebraization of affine spaces (in our sense) by unitary modules.

(C) BACKGROUND: The foundations of affine geometry which are presented here derive from algebraization questions in projective lattice geometry (which is a synthetic concept of (a complete) projective geometry on unitary modules): Algebraization theorems in this field (as J. von Neumann's well-known coordinatization of complemented modular lattices by regular rings, the characterization of primary lattices by Artinian Hjelmslev rings (R. Baer, E. Inaba, B. Jónsson and G. Monk) and K. Faltings' fundamental algebraization of modular lattices with "point system" possessing a "regular hyperplane") are proved first by a deep analysis of the implicitely given affine geometry associated with and second by an extension to the whole geometry. With respect to the latter, the leading question was whether there does exist a concept of affine geometry on its own which allows a structure theorem. In particular, the presented foundations yield a positive answer to this question.

(Stefan E. Schmidt, Mainz)

Non miquelian inversive planes with inversions for all circles

Let K be a field, $\sigma_1, \sigma_2 \in \mathrm{Aut}\, K$ with $\langle \sigma_1, \sigma_2 \rangle \cong \mathbb{Z}_2 \times \mathbb{Z}_2$, and $F_i := \mathrm{Fix}\, \sigma_i$ $(i = 1,2)$. If $U := F_1^\times \cdot F_2^\times \cdot \{k^2 \mid k \in K^\times\}$ is a proper subgroup of $K^\times$, then
$\mathfrak{F} := \{F_1 \cdot u \mid u \in U\} \cup \{F_2 \cdot u' \mid u' \in K^\times \setminus U\}$

is a spread of K and
$(K\cup\{\infty\}, \{\sigma(f\cup\{\infty\}) \mid f\in\mathfrak{F}, \sigma\in PSL(2,K)\}, \in)$ is a non miquelian inversive plane with inversions for all circles.

H. Mäurer (Darmstadt)

Remarks on Bodenmiller's Theorem

The original version of Bodenmiller's Theorem states that the three circles with the diagonals of a complete quadrilateral as diameters intersect in the same two points. We provide a simple proof of an abstract version of this theorem containing several variations of the classical result as special cases. The radical axes of the three Bodenmiller spheres of a quadrilateral in a (pseudo-) euclidean space coincide.

Rudolf Fritsch (München)

On the Hanner numbers

A compact, convex body $M\subset\mathbb{R}^d$ is said to possess the (n,k)-Intersection Property, $n>k\geq 2$, if the following assertion is true: For every collection $M_1,\dots,M_s$ of translates of M, if every k of them have a common point, then every n of them have a common point as well. Hanner has found all the bodies with $(3,2)$IP. For example, in $\mathbb{R}^3$ only affine images of the cube and the regular octahedron possess this property. Some other results in this direction were obtained by Lindenstrauss, Hansen, Lima. So, Lima has established that if $M\subset\mathbb{R}^d$ is centrally symmetric, compact, convex body with $(4,3)$IP, then it has $(\infty,3)$IP, i.e, in accordance with a result of the talker, M is a direct vector sum of two-dimensional sets and one-dimensional ones. In the talk a general necessary and sufficien condition of $(k+1,k)$IP was formulated. It follows that Lima's result, i.e., $(4,3)\text{IP} \Rightarrow (\infty,3)\text{IP}$ takes place for arbitrary bodies (not only centrally symmetric). All bodies of such a kind were described in a (1992) paper of the talker.

08.01.93

(Vladimir G. Boltyanski)

The lattice of connected subgroups of an algebraic group.

In joint work with K. STRAMBACH and G. ZACHER we have studied the lattice $\Lambda(G)$ of all closed connected subgroups of an algebraic group G over some algebraically closed field. Lattice theoretical properties considered by us were those which are shared by projective spaces, like modularity, atomicity, the "Jordan-Dedekind"-condition, complementarity etc. We classify the groups for which $\Lambda(G)$ satisfies these conditions; in prime characteristics some interesting phenomena arise. Here I want to mention the following

Theorem: For a connected algebraic group G over an algebraically closed field the lattice $\Lambda(G)$ is complementary if and only if the solvable radical R of G is a vector group and G splits over R.

P. Plaumann (Erlangen)

"Computational Methods for Nonlinear Phenomena"

10. 1. – 16. 1. 1993

Test function for Hopf points

Test functions for certain bifurcation points are scalar functions being monitored during pathfollowing of parameter dependent equilibria and strictly changing sign at the bifurcation point of interest. Test functions can be used for detection, computation and pathfollowing auf ~~equilibria~~ bifurcation points (also of higher codimension).

Our test function for Hopf bifurcation points are based on the solution of bordered linear systems

$$(1)\quad \begin{pmatrix} A^2+\nu I & A\underline{r} & \underline{r} \\ \underline{\ell}^T A & 0 & 0 \\ \underline{\ell}^T & 0 & 0 \end{pmatrix} \begin{pmatrix} \underline{u} \\ \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \underline{0} \\ 1 \\ 0 \end{pmatrix}$$

with Jacobian $A := g_x(\underline{x},\lambda) \in \mathbb{R}^{n,n}$ of the dynamical system $\dot{\underline{x}} = \underline{g}(\underline{x},\lambda)$ and certain bordering vectors $\underline{r}, \underline{\ell}$.

By $M_{H,\nu} := \{ (A,\nu) \in \mathbb{R}^{n,n} \times \mathbb{R} : \alpha(A,\nu) = 0,\ \beta(A,\nu) = 0 \}$ a codimension-2-manifold is defined. A is called a Hopf matrix with Hopf number ν iff $(A,\nu) \in M_{H,\nu}$. ($\nu > 0$ corresponds to real, $\nu < 0$ to imaginary Hopf matrices and $\nu = 0$ to Takens-Bogdanov matrices).

Eliminating $\nu = \nu(A)$ for instance from $\beta(A,\nu) = 0$, we obtain a test function $T_H(A) := \alpha(A, \nu(A))$ for Hopf points. We give analytical expressions for the derivatives of α, β and T_H which can be cheaply evaluated as a byproduct of the numerical solution of (1). This is important for the efficiency of the computation of Hopf curves which may smoothly connect real and imaginary Hopf points via Takens Bogdanov points.

Examples of Hopf curves in applications are given.

Bodo Werner (Hamburg)

Bifurkation des Konfigurationsraumes am Beispiel des Vielgelenkbogens

The plane five-hinged arch is a simple mechanism with two degrees of freedom. Depending on the distance of the two base joints, its configuration space is a sphere or a closed surface of genus four. Near the critical parameter value, strong singularities of the curvature occur. The orbits are deflected by an angle depending on the discrete curvature.

Hans Brunelli Zürich

Nonlinear Stability Analysis of a Low Plateform Railway Car

To calculate the turning point of the amplitude curve of a periodic solution in R^{28} for a system of stiff nonlinear differential equations is a challenging problem. Instead of using simulation of path following methods multiparameter bifurcation theory is used to obtain from studying still the steady state the turning point of the periodic solution. The obtained results agree well with experimental results.

(Joint work with: G. Xu, A. Steindl) Hans Troger (Wien)

DFG Deutsche Forschungsgemeinschaft

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The Numerical Detection of Hopf Bifurcations in large Systems arising in Fluid Dynamics

Mixed finite element discretizations of the Navier-Stokes equations for incompressible viscous flow and related equations produce large nonlinear systems where Jacobian matrices are nonsymmetric, sparse, but exhibit a special block structure. One approach to determine the linearised stability of a steady state is to compute the "leftmost" (or "dangerous") eigenvalue of a generalised eigenvalue problem $Aw = \mu Bw$, where $A = \begin{bmatrix} K & C \\ C^T & 0 \end{bmatrix}$, $B = \begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix}$ with K nonsymmetric.

The talk described a technique for finding the (possibly complex) dangerous eigenvalue based on (i) a preconditioning step using a Modified Cayley transform, and (ii) the application of standard iterative solvers, like Arnoldi's method.

Numerical results to illustrate the technique and its application to the Hopf detection problem were given for four problems, viz., double diffusion convecting flow past a circular cylinder, flow over a backward-facing step, and the Taylor problem.

Alastair Spence (Bath, UK)

Computation and Parametrisation of Invariant Manifolds

The computation of invariant manifolds for the autonomous system

$$(+) \qquad \dot{\underline{u}} = \underline{F}(\underline{u}) \qquad \underline{F}: R^n \to R^n$$

is often attempted by utilising the dynamics on the manifold itself, eg. a Poincaré map. We would prefer to view the problem geometrically and divorce the computation from these dynamics.

If $\mathcal{M}$ is a k-dimensional C^1 sub-manifold of R^n then $\mathcal{M}$ is invariant for (+) iff

$$P^{\perp}_{T_x\mathcal{M}} \underline{F}(\underline{x}) = \underline{0} \qquad \forall \underline{x} \in \mathcal{M}$$

($T_x\mathcal{M}$ being the k-dimensional tangent space of $\mathcal{M}$ at $\underline{x}$ and P_S the orthogonal projection onto a subspace S of R^n), which provides n-k equations at each point of $\mathcal{M}$. The remaining k equations necessary represent a choice of parametrisation of $\mathcal{M}$. For example, in the common continuation framework where a nearby manifold $\mathcal{M}^0$ is already known, this can completely provide the parametrisation: ie. the additional k equations may be taken as

$$P_{T_{x^0}\mathcal{M}^0}(\underline{x}-\underline{x}^0) = \underline{0} \qquad \forall \underline{x}^0 \in \mathcal{M}^0.$$

It is often better, however, to impose at least part of the parametrisation directly.

i) For periodic solutions, $\mathcal{M}$ is an embedding of S^1 in R^n and we choose the canonical arc-length parametrisation. $\mathcal{M}^0$ only enters via a single phase condition. The final BVP to be solved is analogous to the usual time parametrisation but with vector field $\underline{F}/\|\underline{F}\|$ and parameter L (length of curve) rather than period.

ii) For homoclinic/heteroclinic orbits, $\mathcal{M}$ is an embedding of $(0,1)$ in R^n and we again choose arc-length parametrisation. $\mathcal{M}^0$ now plays no role. The final BVP is over $(0,1)$ with singularities at the end points rather than over $(-\infty,\infty)$ as with time parametrisation.

iii) For invariant tori, $\mathcal{M}$ is an embedding of $S^1 \times S^1$ in R^n. Parametrisations defined by conditions on the first fundamental form, eg. orthogonality & constant surface area, are being investigated.

iv) With stable/unstable manifolds of fixed points, we end up with a quasi-linear hyperbolic system extending out from the singular fixed point. In this case an adaptively changing parametrisation is indicated.

Gerald Moore (Imperial College, London)

Test functions and augmented systems

For the numerical detection and computation of singular points of a nonlinear system of equations

$$f(x,\tau,\alpha)=0\ , \quad f: \mathbb{R}^{n+\ell}\times\mathbb{R}^{p}\times\mathbb{R}^{q}\to\mathbb{R}^{n} \text{ smooth},$$

x - state variables,

τ - distinguished bifurcation parameters,

α - generic unfolding parameters,

i.e. solutions (x^*,τ^*,α^*) with

$$\operatorname{rank} f_x(x^*,\tau^*,\alpha^*) = n-k\ , \quad k\geq 1,$$

one is interested in the construction of so-called test functions. These are functions $b: \mathbb{R}^{n+\ell}\times\mathbb{R}^{p}\times\mathbb{R}^{q}\to\mathbb{R}^{\ell+p+q}$ such that (x^*,τ^*,α^*) is an isolated solution of the augmented system

$$f(x,\tau,\alpha)=0$$
$$b(x,\tau,\alpha)=0\ .$$

For a wide range of types of singular points, test functions are developed within the so-called singularity theory for the case $n=k$, i.e. when techniques like Lyapunov-Schmidt reduction have already been applied. But only partial results were known how to extend these test functions for the reduced problem to the general setting. In the talk, a construction process was presented, how one can transcribe such test functions of the reduced problem into the desired test functions for the general problem.

Peter Kunkel (Oldenburg)

Hopf/Steady-State Mode Interactions of a fluid conveying Tube with D_n-symmetric elastic support

We consider an elastic tube, which is elastically supported by n identical springs. By increasing the flow rate the tube looses stability, either by a zero eigenvalue (stiff support) or by a Hopf bifurcation (soft support).
For a certain value of the stiffness a zero eigenvalue and a pair of imaginary eigenvalues occur simultaneously.
Due to the symmetry these eigenvalues occur in pairs.
By projecting the partial differential equations onto the six-dimensional eigenspace and applying Normal Form simplification we obtain a stationary system of equations with $D_n \times S^1$-symmetry.
The symmetric solutions can be classified by their isotropy subgroups.
Also asymmetric solution branches are known to possibly branch off from the trivial state.

Alois Steindl

Scientific Software aspects in Bifurcation Analysis

In the first part of the talk, the current status of software for bifurcation analysis is briefly reviewed. Attention is paid to the requirements for a (standard) interactive environment and for reliable numerical algorithms.
In the second part, an algorithm is presented for determining the steplength used in a continuation procedure. The algorithm takes into account the behaviour of testfunctions for bifurcations to increase the robustness of continuation w.r.t. undesired branch switching. This approach also reduces the possibility to overlook important bifurcations.

Dirk Roose, K.U. Leuven

Direct Methods for Computing Bifurcation Points

A nonlinear system of n algebraic equations of the form $F(y, \lambda, \alpha) = 0$ depending on n state variables y, the control parameter λ and l unfolding parameters α is considered. It is assumed that there exist simple singular solution points $(y^*, \lambda^*, \alpha^*)$ at which the Jacobian $\partial F(y^*, \lambda^*, \alpha^*)$ has rank $n-1$. Simple singular points $(y^*, \lambda^*, \alpha^*)$ of this kind are for example turning points, simple bifurcation points, hysteresis points, pitchfork bifurcation points etc. They can be characterized by appending the original problem $F(y, \lambda, \alpha) = 0$ by $l+1$ equations $f(y, \lambda, \alpha) = 0$ such that $(y^*, \lambda^*, \alpha^*)$ is a regular solution of the combined system. For simple singular points with codimension ≤ 2 we constructed robust functions f_i that can be evaluated and differentiated cheaply. By exploiting the special structure of the defining equations $f_i(y, \lambda, \alpha) = 0$ we developed Newton-type methods for computing the desired simple singular point $(y^*, \lambda^*, \alpha^*)$. In this way Q-quadratically convergent are obtained whose efficiency is demonstrated by a numerical example.

Gerd Pönisch, Dresden University of Technology

A global analysis of Newton's iterations for determining turning points

The global convergence of a direct method for determining turning (limit) points of a parameter-dependent mapping is analysed. It is assumed that the relevant extended system has a singular root for a special parameter value. The singular root is classified as a bifurcation singularity. Then, the Theory for Imperfect Bifurcation offers a particular scenario for the split of the singular root into a finite number of regular roots due to a given parameter imperfection. The relationship between the scenario and the actual performance of Newton method is studied. Both theoretical and experimental arguments are presented in order to question the claim that a particular bifurcation singularity organizes the Newton method assuming small parameter perturbations

Vladimír Janovský (Prague)

Hamiltonian Perturbation Theory — from Poincaré to Nekhoroshev!

The purpose of this talk is to review Hamiltonian Perturbation Theory as it evolved during the last hundred years since Poincaré's famous "Les méthodes nouvelles de la mécanique céleste" was published. The central theme dealt with is the stability of elliptic equilibria. The construction of formal integrals, Siegel's theorem on the divergence of the Birkhoff normal form, the non-existence of integrals, a KAM-result of J. Pöschel, an example of Arnold diffusion due to R. Douady, the Zehnder-Genecand result on the existence of infinitely many transversal homoclinic points are described. The main emphasis is on a Nekhoroshev-type estimate of A. Giorgilli and its application to the Trojan asteroids

Urs Kirchgraber, Zürich

On the Reduced Basis Method

The reduced basis method can be used to reduce the size of approximating systems without significantly increasing the error inherent in the underlying discretization. Given a system of equations $F(z)=0$, $F: \mathbb{R}^{m+1} \to \mathbb{R}^m$, The reduced basis method solves $PF(x_0 + V_d\tau + V_R y) = 0$ where $P^2 = P$ and $V_d \oplus V_R$ is a subspace of a regular splitting $V_d \oplus V$ of R^{m+d}. A method is given for constructing projections that minimize constants in standard error estimates and make the method more robust. It is shown how to select P to achieve additional accuracy in the parameter τ at simple turning points.

G. Reddien, Dallas

Theoretical and computational aspects of normal forms

The normal form reduction of a vectorfield at one of its singularities is usually done by an order by order approach, where for each order k one looks for an appropriate transformation which simplifies as much as possible the k-th order term in the Taylor expansion of the vectorfield. Such an approach leads at each order to a so-called splitting problem, which in fact consists in finding an appropriate pseudo-inverse of a linear operator whose dimension increases rapidly with the dimension of the phase space and the order k. In practice this has till now put severe restrictions on normal form calculations, certainly when one wants to work symbolically, keeping the dependence on the parameters of the original vectorfield. In this talk we discuss an approach, based on former work of Cushman and Sanders, which seems to be able to handle these problems rather efficiently. We describe in particular a splitting algorithm which avoids the explicit calculation of inverses. We also discuss a number of other aspects of normal form calculations, some of which are still unsolved.
(Joint work with Jan Sanders and Jan-Cees van der Meer).

André Vanderbauwhede Gent, Belgium

Numerical analysis of tertiary and quaternary solutions and their stability in cases of fluids flows in plane layers and in the problem of the geodynamo

Many fluid systems with simple geometrical configurations of the boundaries reach a turbulent state through a sequence of supercritical (or weakly subcritical bifurcations. Rayleigh-Bénard convection in a layer heated from below and the Taylor-Couette system are the most prominent examples. Generically the secondary state reached after the first bifurcation from the basic state assumes the form of roll-like motions superimposed onto the basic flow. A large variety of different tertiary states are found in different systems or as a function of parameters for a given system after the second bifurcation has occurred. The Galerkin method in which the dependent variables are expanded in systems of functions satisfying all boundary conditions is an ideal tool to investigate steady states and their instabilities (=bifurcation points). Here we demonstrate in particular that the wavy roll state in the Taylor-Couette-System and in the Bénard-System with imposed shear represent the same state of motion except for minor quantitative differences. The "sequence of bifurcation approach" can also be applied to the problem of the geodynamo, which is realized in the liquid iron core of the Earth. The axial symmetry of the basic static state is broken first by the onset of convection in the form of columns with azimuthal wavenumber m. Depending on which further symmetries are broken, different tertiary solutions can be obtained with a magnetic field as additional degree of freedom of the system. See joint papers in the Journal of Fluid Mechanics with R.M. Clever for the first kind of problems and joint paper with K.K. Zhang for the geodynamo problem which have been published in the Journal "Geophysical and Astrophysical Fluid Dynamics" in the years 1987-89.

F. Busse, Universität Bayreuth, Theor. Physik

Bifurcations in singularly perturbed problems

In this talk we consider singularly perturbed ordinary differential equations of the form:

$$(1)\quad \begin{array}{l} \dot{x} = f(x,y,\varepsilon,\mu) \\ \varepsilon\dot{y} = g(x,y,\varepsilon,\mu) \end{array}, \quad x \in \mathbb{R}^n,\ y \in \mathbb{R}^k,\ \mu \in \mathbb{R}^p,\ 0<\varepsilon<<1$$

We demonstrate how Fenichel's invariant manifold theory (JDE 79) for problem (1) can be used to understand certain global aspects of the problem. In particular we show that the existence of transversal singular ($\varepsilon = 0$) heteroclinic and homoclinic orbits implies the existence of these orbits for small positive values of ε. The method is applied to the travelling wave problem of the Fitzhugh-Nagumo equations. As a second application we briefly discuss the bifurcation of heteroclinic orbits in the problem of viscous profiles for magnetohydrodynamic shock waves (joint work with H. Freistühler, RWTH). We pose some open questions which could be investigated numerically

Peter Szmolyan, Technische Universität Wien, Inst. f. Ang. Math.

Stable Capillary Surfaces under Zero Gravity

Symmetric capillary surfaces in a cube are considered under zero gravity conditions. Only local minima of the total energy are computed for contact angles of 70, 50, and 40 degrees. Graphs of the energy, the area, and the pressure versus the volume reveal several interesting facts. Particular attention is paid to nonlinear phenomena including hysteresis and that for angles below a bound, here 45 degrees, the liquid creeps along the edges without limit.

This latter effect can be observed for arbitrarily small volumes in the case of a 40 degree contact angle. Other observations include that of up to ten local minima for certain volumes and that the volume ranges for which various shapes are minimizing the energy depend strongly on the contact angle.

Hans D Mittelmann (Arizona State University)

Discretization of autonomous equations and homoclinic orbits
(joint with Bernold Fiedler, Stuttgart)

One-step discretizations of order p and step size ε of autonomous ordinary differential equations $\dot{x} = f(\lambda, x)$ can be viewed as time-ε maps of rapidly forced, non-autonomous equations

$$\dot{x} = f(\lambda, x) + \varepsilon^p g(\varepsilon, \lambda, t/\varepsilon, x), \quad x \in \mathbb{R}^n, \ \lambda \in \mathbb{R}, \ \text{"}\cdot\text{"} = \frac{d}{dt},$$

where g has period 1 in t/ε. We study the behaviour of a homoclinic orbit $\Gamma = \Gamma(t)$, $\varepsilon = 0$, $\lambda = 0$ $(t \in \mathbb{R})$ under discretization. Under generic assumptions, Γ turns out to break, and the perturbed stable and unstable invariant manifolds intersect each other transversally for small positive ε which implies chaotic behaviour. However, the transversality effects are estimated from above to be exponentially small in ε. For example, the length $\ell(\varepsilon)$ of the parameter interval of λ-values for which the intersection of the local invariant manifolds is non-empty can be estimated by $\ell(\varepsilon) \leq C e^{-2\pi\eta/\varepsilon}$, where C and η are positive constants. The factor η is related to the minimal distance from the real axis of the poles of $\Gamma(t)$ in the complex t-plane.

Our results are visualized by high precision numerical experiments. The experiments show that, due to exponential smallness, homoclinic transversality becomes practically invisible under normal circumstances, already for only moderately small step size.

Jürgen Scheurle, Hamburg

Stable Computation of Simple Bifurcation Points and Emanating Branches

For determining simple bifurcation points of nonlinear Systems $F(y) = F(x,\lambda) = 0$, an extended system $F(y) + \mu d = 0$, $w^{iT} v^i(y) = 0$ $(i = 1,2)$ is used where

$$v^i(y) := B_i(y)^{-1} e^{n+1} \quad , \quad B_i(y) := \begin{bmatrix} \partial F(y) + d^i w^{iT} \\ r^{iT} \end{bmatrix} .$$

This is a generalization of Pönisch's [85] system where only row replacements in ∂F are used. The general rank-1-regularization used here allows to choose the parameters $\{d, d^i, r^i, v^i\}$ such that a bound of $\|B_i^{-1}\|$ is small (which increases robustness), that $B_2 - B_1$ has rank 1 (which saves computational costs), and that also simple ~~turning~~ bifurcation points which are turning points w.r.t. e^{n+1} can be computed. An implementation is described which, from the viewpoint of computational costs, is comparable to Janovský's modification of Pönisch's method. Moreover, the quantities computed during the iteration allow to determine the tangents of the emanating branches.

Hubert Schwetlick (Technical University of Dresden), joint with E.L. Allgower (Colorado State University, Fort Collins)

Computational Methods for Symmetry Creation

We consider Dynamical Systems which possess symmetry – PDEs on the line with periodic boundary conditions, for instance. An attractor of such a system might have less symmetry than the full symmetry of the problem. In this case there always exist several attractors which are related by symmetry transformations and, varying a parameter, it might happen that those conjugate attractors collide. Then the resulting attractor has more symmetry and this phenomenon is called Symmetry Creation. During the last two years Symmetry Creation has frequently been found in the numerical simulation of several PDEs (eg. in the Ginzburg-Landau equation or in the Kuramoto-Sivashinsky equation). It also has been observed in physical experiments like the Taylor-Couette apparatus or the Faraday experiment. Based on the

Karhunen-Loève decomposition we will present a numerical method for the detection of Symmetry Creation in Dynamical Systems.

Michael Dellnitz (Universität Hamburg)

Invariant Manifolds in Control Theory

We present some examples how the theory of invariant manifolds has been recently used in the study of affine nonlinear control problems. First, we investigate the problem of local asymptotic stabilization for systems having a well-defined relative degree. Secondly we deal with local qualitative control problems including various classical control tasks. Finally, for systems with disturbances we investigate the error feedback regulator problem and present a state space approach to nonlinear H_∞-control

Dietrich Flockerzi (Würzburg)

Friction-induced Vibrations in Mechanical Systems

We consider a multibody system with f degrees of freedom and an arbitrary number of contacts ($n_F < f$) where friction with stick and slip might occur. If these contacts are not decoupled by some force law, a sudden change of the contact situation in one contact influences the state in all other contacts. This generates a new type of motion (and a new set of eqns. of motion) which must be compatible with all relevant constraints. The problem can be formulated as a system suitable for an application of the so-called complementarity problem known from optimization theory (optimizing some criterion with inequality conditions). As for dynamical contact problems in each point of contact either relative kinematical magnitudes are zero and at the time the corresponding constraint are not zero (or vice versa), the scalar products of both are always zero. The complementarity formulation solves the problem of finding the correct type of motion after a contact event compatible with all constraints. For numerical purposes the Lemke-algorithm is used. Some practible examples show the efficiency of the procedure.

Friedrich Pfeiffer (TU-München)

Bifurcation Phenomena in a Model of Neural Dynamics.

We present a mathematical model, which describes the dynamics of a simple neural net consisting of an excitatory and inhibitory neuron. The mathematical model consists of a system of two differential equations. We show, that the system possesses characteristical nonlinear properties such as hysteresis and bifurcations of periodic solutions. Examples of hysteresis and Hopf curves and global bifurcations are given.

Fotios Giannakopoulos (Köln)

Numerical approximation of connecting orbits

Orbits which connect steady states or periodic orbits typically arise when determining the shape and speed of travelling waves in parabolic systems.
For the numerical approximation one has to solve a boundary value problem on the real line. We show that the well posedness of such boundary value problems can be characterized by the transversal intersection of certain stable and unstable manifolds.
In the case of stationary to periodic connecting orbits it turns out that a crucial role is played by the property of asymptotic phase and by the induced foliations of stable and unstable manifolds.
The numerical method consists in truncation to a finite interval with appropriate boundary conditions on both ends. The error induced by this truncation is shown to decay exponentially with the size of the interval.

Wolf-Jürgen Beyn (Bielefeld)

A new principle for constructing tensor methods

Tensor methods converge local superlinearly towards regular as well as singular solutions of nonlinear equations. This property makes them attractive for solving nonlinear equations. The new construction principle leads to a special tensor method which is characterized by the following features:

- local cubic convergence towards regular solutions
- local superlinear convergence with Q-order 1.5 towards regular singularities
- use of second order directional derivatives which keep computational costs at a low level.

This set of useful features can be viewed as a further

development of the methods due to Schnabel / Frank and Griewank, respectively.
Interesting points for further discussion may be: irregular singularities and variants for globalization.

Annegret Hoy (Friedberg)

Derivative Convergence for Iterative Equation Solvers

When parameter dependent nonlinear equations are solved by iterative methods the iterates may be viewed as functions of the parameters. We examine under which conditions the derivatives of these functions converge to the derivatives of the limiting implicit functions defined by the nonlinear system. For Newton's method and Q-superlinearly convergent secant updating methods quadratic and linear convergence of the derivatives is obtained, respectively. While not guaranteed theoretically derivative convergence is observed and verified for a multigrid code applied to transsonic flow over a wing. Finally it is shown that if the system Jacobian can be formed and factorised higher derivatives can be obtained at a complexity that grows only quadratically with their degree.

Designing software for bifurcation problems: phylosophy and practical experience.
(joint with Yu. Kuznetsov, V. Levitin, E. Nikolaev)

We discuss different aspects of LOCBIF software project. The applied fields we are experienced with (ecology, biochemistry, chemistry, electrical circuits etc.) motivate the concentration on generic vector fields and diffeomorphisms with relatively low number of variables and relatively high number of parameters (3 or more). The graph of adjacency of singularities up to codimension three represents the concentrated view on that how different singularities are organized with respect to their unfoldings. In numerical approach, this graph is reformulated using the language of (scalar) test bifurcation functions. Then continuation techniques is applied, which allows us to end up with the code supporting „travelling" along a solution manifold and exploring its stratification. Computer science aspects of the project (tool for defining models, graphic user interface) are discussed as well as directions of farther developments.

Alexander Khibnik (Pushchino)

MODEL REDUCTION & BIFURCATION CALCULATIONS FOR NONLINEAR PDE

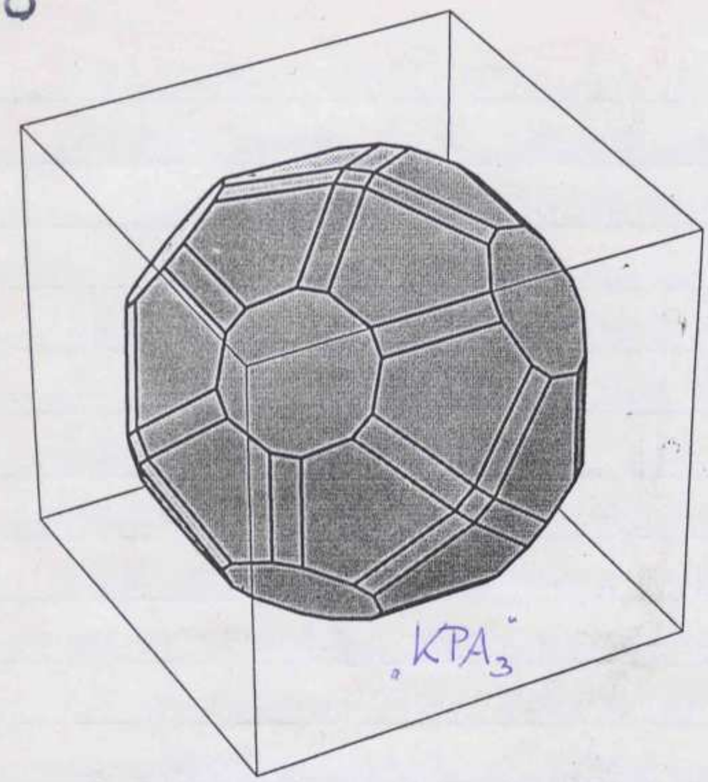
We study low-dimensional dynamics and instabilities in pattern forming systems with complex geometries (e.g. transitional flows). We use techniques inspired from the theory of inertial manifolds as well as image processing (Karhunen-Loève, SVD) to obtain reduced dynamic models on which the bifurcation calculations can be performed. In particular, we introduce a combination of the two, yielding low-dimensional, approximately invariant nonlinear subspaces on which the long-term dynamics lie. Our test examples include flows in complex geometries, reaction-diffusion & amplitude equations.

Yannis Kevrekidis (Princeton)

Computational Methods in High-T_c Superconductivity

We survey problems of current interest in high-temperature superconductivity research and highlight their mathematical and computational components. We emphasize problems related to the vortex state and describe some phenomenological models that have been (and are being) used to study the dynamics and structure of vortices.

Hans G. Kaper (Argonne Nat. Lab)



Combinatorial Optimization

~~January~~ 17-23, 1993

Constructing the Permuto-Associahedra

Victor Reiner (Minneapolis) and Günter M. Ziegler (ZIB-Berlin)

We construct a family of polytopes KPA_{n-1}, the "Permuto-Associahedra". Here KPA_{n-1} is an $(n-1)$-dimensional polytope whose vertices correspond to the complete bracketings of permutations of $\{1, 2, \ldots, n\}$, with a natural (combinatorially defined) notion of adjacency. Our proofs yield integral coordinates, with all vertices on a sphere, and include a complete description of the facet-defining inequalities.

This solves a problem of M.M. Kapranov (Northwestern U.), who had defined KPA_{n-1} as a combinatorial object and showed that it corresponds to a cellular ball.

Günter M. Ziegler

A Survivable Network Design Problem Involving Multicommodity Flows

Mechthild Stoer (ZIB-Berlin), Geir Dahl (Norwegian Telecom)

The problem is to extend the capacities of a given network such that the traffic demands can be met in each failure situation (single edge and single node failure), and such that the extension cost is minimal. A cutting plane algorithm using Benders decomposition, and some polyhedral results are presented.

M. Stoer

Bottleneck Monge Matrices and their Role in Combinatorial Optimization

Rainer E. Burkard, TU Graz (A)

A matrix $C = (c_{ij})$ is said to be a bottleneck-Monge matrix, if

(1) $\max(c_{ip}, c_{jq}) \leq \max(c_{iq}, c_{jp})$ for all $1 \leq i < j \leq m$, $1 \leq p < q \leq n$.

If the cost matrix of a time transportation problem fulfills (1), the North West Corner Rule yields an optimal solution.

In particular, every $(2 \times n)$ matrix can columnwise be reordered, such that (1) is fulfilled. Thus any such transportation problem is solvable by a greedy-like algorithm. The same reordering is used in Johnson's Rule for 2-machine flow shop problems. More general, if C is the matrix of processing times of an m machine flow shop problem, where $-C$ fulfills (1), then the job sequence $(n, n-1, \dots, 2, 1)$ minimizes the makespan.

Finally we point out that a bottleneck TSP is optimally solvable by an $O(n^2)$ algorithm if its cost matrix fulfills (1).

Klinz, Rudolf and Wöginger (TU Graz) have recently investigated the recognition of Bottleneck Monge properties. In particular a $(0,1)$-matrix without equal rows and columns or rows and columns of constant 1's fulfills (1), if and only if it is a double staircase matrix of the form

$$\begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$$

Further results concern the recognition of matrices which fulfill (1) after an appropriate permutation of its rows and columns.

R. Burkard

The cocycle lattice of binary matroids
László Lovász and Ákos Seress

We consider the lattice (grid) generated by the incidence vectors of cocycles. ~~We~~ It is shown that in the dual of this lattice, every denominator is a power of 2. Matroids for which these dual lattice is $\frac{1}{2}$ integral are characterized: they are those matroids that, when embedded in the projective space over GF(2), contain a member of the (punctured) Reed-Muller code. This gives a polynomial time algorithm to recognize such matroids.

Optimierung mit ~~Mem~~ Mitgliedschaft Orakel:
Ravi Kannan (mit Hilfe von: M. Padberg, M. Jünger, R. Bixby, B. Cook, S. Thienel, G. Rote, J. Fücks bzw G. Reinelt)

Sei K ein oben Monotoner Körper in $\mathbb{R}^n$, und $c \cdot x$ eine Linearform von $\mathbb{R}^n \to \mathbb{R}_+$, mit $c \geq 0$. Das Problem ~~ein~~ besteht darin eine fast optimale lösung x in K zu finden zum ~~de~~ Minimierungsproblem

Min $c \cdot x$, $x \in K$, K obenmonontone.

La soluzione è trovata per un algoritmo ~~di~~ $O(\frac{n^6}{\epsilon^2} \log \ldots)$ dove ϵ, e la tolleranza del approximatione.

Der Algorithmus ist ein "Randomitierter Algorithmus" und functioniert miestens so: gestern ginge er mit Wahrscheinlichkeit fast 1 zu einer zulässigen lösung x^t und einer Unterschranken ~~L~~ damit mglW $K \subseteq \{x : c \cdot x \geq L\}$. ~~In~~ Auf der nächsten Stufe verbesseren wir die Löche von L und $c \cdot x^t$ um einem Factor von 4/3 mit einem Random Walk, ~~der Rand~~ dessen Konvergenz ~~des~~ Haupt Satz ist. SZERVOS

THE MICE-COLLECTING TRAVELING SALESCAT PROBLEM

In the traveling salesman problem for moving points, we are given n objects moving at given constant speeds in fixed directions, in some Euclidean space $\mathbb{R}^d$, and a salesperson with a bound on her own speed wants to visit all objects as fast as possible from a given starting point.
Possible applications (beside the one suggested by the metaphor in the title) include in-flight refuelling of planes — a problem in $\mathbb{R}^2$, suggested by D. Bertsimas — and a man who wants to look at each of n ladies strolling along a street — a one-dimensional problem, suggested by L. Lovász.
We present a dynamic programming algorithm that solves the one-dimensional version of the problem (all points move on a single line) in $O(n^4)$ time and $O(n^3)$ space. The crucial property that the algorithm exploits is a certain convexity property of the optimal solution.
I also show some ideas that might lead to a heuristic approximation algorithm for the problem in two dimensions.
This problem was originally posed by Mordecai Golin, now at Hong Kong University.

Günter Rote (TU Graz)

Depot Scheduling

We describe a model of the food distribution system of a large department store chain and its solution. This involved parallel MIP on ~~an~~ ~~R~~ cluster of RS/6000's

Bill Pulleyblank - Yorktown Heights

Polyhedra for Lot-Sizing with Wagner-Whitin Costs

Yves Pochet & Laurence Wolsey, CORE, Louvain-la-Neuve

We examine the single-item lot-sizing problem with Wagner-Whitin costs i.e. $p_{t-1} + h_{t-1} \geq p_t$, $p_{t+1} + g_t \geq p_t$ for $t = 1, \ldots, n$, where p_t, h_t, g_t are the unit production, storage and backlog costs. For the uncapacitated problem with backlogging (BLS) and the constant capacity problem (CLS), an explicit description of the convex hull of solutions in the basic stock, backlog and set-up variables is unknown. Here we describe integral polyhedra which solve the two problems with Wagner-Whitin costs. We obtain $O(n^2)/O(n^3)$ separation algorithms for CLS and BLS respectively, and extended formulations with $O(n^2)$ constraints and variables in both cases.

LAWolsey

An approximation algorithm for the generalized assignment problem

David Shmoys and Éva Tardos (Cornell)

We consider the generalized assignment problem, which can be viewed as the following problem of scheduling unrelated parallel machines: each of n jobs is to be assigned to exactly one of m machines; if job j is assigned to machine i, it requires p_{ij} units of processing and incurs a cost of c_{ij}; if each machine is available for T time units, find a feasible assignment of minimum total cost. Our main theorem is then as follows: given values C and T, we can ~~decide~~, in polynomial time, either prove that no feasible schedule of length T and total cost C exists, or else find a schedule of total cost at most C and length at most $2T$.

A Linear-Time Algorithm for Edge-Disjoint Paths in Planar Graphs

Dorothea Wagner, Technische Universität Berlin
joint work with Karsten Weihe

We consider the problem of finding edge-disjoint paths in a planar graph, s.t. each path connects two specified vertices on the outer face boundary. We focus on the case where the evenness condition is satisfied. The "classical" result for that problem is the Theorem of Okamura & Seymour which says that a problem is solvable iff the cut condition is fulfilled. Several algorithms solving this problem which are based on this result are known from the literature. The best one receives a running time of $\mathcal{O}(n^{5/3}(\log\log n)^{1/3})$.
In this talk a new algorithm is presented which requires only $\mathcal{O}(n)$ time. The approach also yields an alternative proof for the Theorem of Okamura & Seymour.

Fun with Random Walks

Prabhakar Raghavan, IBM Yorktown Heights
work with Peter Doyle, Don Coppersmith & Marc Snir.

We describe a cat-and-mouse game played on a graph with positive real weights on its edges. We give a tight characterization of the value of the game, developing a synthesis problem for random walks. This synthesis problem is solved using random walks and their connection with electric networks. We use these results to give optimal algorithms for a server problem in a class of metric spaces.

Combinatorial classification of polymatroids

E. Girlich

We investigate the combinatorial structure of polymatroids $P(r) = \{ x \in R^+_n : \sum_{i \in I} x_i \leq r(I),\ I \subseteq N \}$, $N = \{1, 2, \dots, n\}$ where the rank-function $r(I) : 2^N \to R^+$, is a submodular, isotone and normalized function. Edmonds describes the minimal systems of facets of a n-dimensional polymatroid. By $R(r)$ we denote the facet-poset, where the sets $I \in R(r)$ are r-closed and r-inseperable.

Two polytopes P_1, P_2 are combinatorial equivalent, iff there exists an isomorphism μ with

$$F_1 \subseteq F_2 \subseteq P_1 \iff \mu(F_1) \subseteq \mu(F_2) \subseteq P_2 .$$

Combinatorial equivalent polytopes have the same structural vectors $f(P) = (f_0(P), f_1(P), \dots, f_{n-1}(P))$, where $f_i(P)$ denote the number of i-faces of the polytope P. By $H(n)$ we denote the cone of all rank-functions $r(I)$, which are a positive combination of Boolean rank-functions. $H(n)$ is a simplicial cone, All Polymatroids $P(r)$ where r is an element of a faces of $H(n)$, are combinatorial equivalent. For all $r \in \operatorname{int} H(n)$ the Polymatroid $P(r)$ is combinatorial equivalent with the permutation-polymatroid $P(r_a)$, where $r_a(I) = \sum_{i=1}^{|I|} a_i$, $a = (n, n-1, \dots, 2, 1)$.

E. Girlich

Technical University
Otto von Guericke
Magdeburg.

On the Monotonization of Polyhedra

Egon Balas, Carnegie Mellon University, Pittsburgh, PA 15213, USA
and Matteo Fischetti, University of Padua, Padua, Italy

We define a generalized monotonization of a polyhedron P, g-mon(P), that subsumes both the submissive (downward monotonization) and the dominant (upward monotonization) of P; and give a broad sufficient condition for an inequality that defines a facet of g-mon(P) to also define a facet of P. For the case of the traveling salesman polytope P, both in its symmetric and asymmetric variants, we give sufficient conditions trivially easy to verify, for a facet of the monotone completion of the TS polytope to define a facet of the TS polytope itself. The upshot of this research is that all cases in which facets of the monotonized polyhedron do not correspond to facets of the polyhedron itself are pathological in a well defined sense.

Egon Balas

Implementation of a branch and cut algorithm for the traveling salesman problem

Stefan Thienel, Universität zu Köln
joint work with
Michael Jünger, Universität zu Köln
Gerhard Reinelt, Universität Heidelberg

We resemble the implementation of Padberg and Rinaldi (1991), yet there are some differences. Fractional LP-solutions are not only used for the computation of lower bounds but also to compute good tours. The set of active variables is generated and managed differently. The computation of the reduced costs is done in a hierarchical fashion. Since our separation algorithms are less sophisticated, we ~~cannot~~ cannot solve so large problems to optimality as they could. Our implementation contains the feature to compute a tour and a lower bound for any prespecified guaranteed quality. The integrated computation of upper and lower bounds may be also useful if only a limited amount of cpu-time is available. Finally we show how this implementation for the TSP can be used as framework for other combinatorial optimization problems.

Stefan Thienel

Balanced 0 ± 1 Matrices

by Gerard Cornuéjols, GSIA, Carnegie Mellon Univ
Pittsburgh Pa 15213

Some problems of propositional logic, such as satisfiability, MAXSAT and logical inference can be formulated as integer programs, where the constraint matrix has coefficients $0, \pm 1$. In this talk, we consider such logic problems when the resulting integer programs can be solved as linear programs. This is the case when the constraint matrix is a $0, \pm 1$ balanced matrix, namely when, in every submatrix with exactly two nonzero entries per row and column, the sum of the entries is a multiple of four.

Gerard Cornuéjols

Approximation through uncrossing
by Michel Goemans, MIT, Cambridge, MA 02139 USA.

We consider the class of combinatorial optimization problems which can be formulated by an integer program of the form

$$\text{Min} \sum_{e\in E} c_e x_e$$
$$\text{s.t.} \sum_{e\in \delta(S)} x_e \geq f(S) \qquad \forall S \subseteq V$$
$$x_e \in \{0,1\} \qquad e \in E$$

under some restrictions of $f: 2^V \rightarrow \mathbb{N}$. This class includes classical problems such as the Steiner tree, T-join or survivable network design problems. We present a primal-dual algorithm with a worst case performance guarantee. In the case of uncrossable functions with range $\{0,1\}$, the ratio between the values of the primal integral solution and the dual feasible solution is at most 2. In many cases, the algorithm can be efficiently implemented.
The algorithm and the structure of the IP raise a number of interesting issues. For example, we have derived a generalization of Padberg & Rao's characterization of minimum odd cuts to minimum uncrossable cuts.

Michel Goemans

Reliability, Covering and Balanced Matrices
by Michael Ball, U. of NC, Chapel Hill, NC and U. of MD, College Park, MD, USA

The problem of designing a system subject to a reliability constraint is of practical significance. Yet, it has received little research attention since evaluating system reliability is, itself, an NP-hard problem for most interesting classes of systems. By taking logarithms constraints on cutset failure probabilities can be modeled as linear constraints. This fact leads to the construction of generalized covering models which produce upper and lower bounds for the aforementioned system design problem. For cases with cutsets of small cardinality, the general covering problem can be converted to a set covering problem. We show that in special cases the associated matrix is balanced and thus the set covering problem can be solved using linear programming techniques. We give computational evidence which shows that general problems with small cutsets can be easily solved with branch and bound.

Michael Bell

Finding Minimum Cost-to-Time Ratio Cycles with Small Integral Transit Times

Mark Hartmann, Dept. Operations Research, University of North Carolina, Chapel Hill, NC 27599 USA

Let $G=(V,E)$ be a digraph; for each $e \in E$ there is an associated integral cost c_e and a non-negative integral transit time t_e. The minimum cost-to-time ratio cycle problem is to find λ^*, the minimum of $c(\Gamma)/t(\Gamma)$ over all directed cycles Γ of G. We present a new algorithm for finding λ^* whose running time is dominated by $O(\Sigma_{u\in V} \max\{t_{uv} : uv \in E\})$ minimum cost paths calculations on a digraph with non-negative arc costs. Further, we consider early termination of the algorithm and a faster implementation in case $t(\Gamma) > 0$ for all directed cycles Γ. The algorithm can be viewed as an extension of the $O(|V||E|)$ algorithm of

Karp for the minimum cycle mean problem. Our algorithm can also be modified to solve the related parametric minimum cost paths problem with the same bound on the running time.

Walter Hunt

Two frameworks for optimization problems.

Emo Welzl, Freie Universität Berlin

We present two frameworks for solving optimization problems including Linear Programming (n constraints, d variables), computing the smallest enclosing ball of n points in $\mathbb{R}^d$, or computing the distance between two n-vertex (or n-facet) d-polytopes. The algorithm developed in the framework can be shown to use a subexponential number of arithmetic operations in the unit cost model:

expected $O(nd^2 + e^{O(\sqrt{d \log d})})$, randomized.

(This bound relies on work by Clarkson, Kalai, Matoušek/Sharir/Welzl, and Gärtner).

Here is Gärtner's framework. Suppose we are given an n-element set H with a linear ordering $\prec$ on 2^H. Our goal is to find the minimal element in 2^H; $(H, \prec)$ is given implicitly by the following oracle: for $F \subseteq G \subseteq H$, the oracle either reports that F is optimal in 2^G (i.e. $F = \min(2^G)$), or it provides a set F', $F' \subseteq G$, $F' \prec F$. (No conditions on $(H, \prec)$ are required!) Every deterministic algorithm requires $2^n - 1$ oracle queries in the worst case, while a randomized algorithm can solve every such problem with expected $e^{O(\sqrt{n})}$ oracle queries.

Emo Welzl

Projected Faces Property

F. Margot, Ecole polytechnique fédérale de Lausanne, Switzerland

A new method for proving integrality of polytopes corresponding to the convex hull of the characteristic vectors of solutions to some combinatorial problems on graphs definable by compositions is presented. The method is based on a relation called "projected faces property" between a polytope Q and one of its projection Q': The pair (Q, Q') has the projected faces property if each face of Q projects onto a face of Q'.
The method is illustrated with the Maximum Cut on 2-trees and has been used to derive complete linear characterizations for the Independent Set, Strong Connectivity Orientation and Strong Connectivity Augmentation on series-parallel graphs.

Online Hamiltonian Path Problems

Norbert Ascheuer, Konrad-Zuse-Zentrum für Informationstechnik, Berlin

We present the results of a joint project with industry that had the goal to optimize the movements of a stacker crane of an automatic storage system. This question leads to the solution of online Hamiltonian Path problems. The comparison of the online behaviour of several heuristics is presented. The question of determining a lower bound for a good online strategy then leads to offline Combinatorial Optimization problems such as Hamiltonian Path Problems with and without additional constraints (time windows, precedence constraints). Modellings and preliminary computational results are presented.

Generalizing the All-pairs Min Cut Problem

David Hartvigsen

(APMC)

The all-pairs min cut problem on a nonnegative edge weighted graph is to find, for each pair of nodes, a min cut that separates the pair. Gomory and Hu (1961) presented a structural characterization of collections of cuts that solve the APMC problem. We show how the APMC problem can be generalized to matroids and we present several theorems that characterize the structure of solutions to this more general problem. The result of Gomory and Hu is a special case of one of these theorems. In particular we find that the APMC problem is a matroid optimization problem.

Area Minimization of Simple Polygon

Sándor Fekete (Stony Brook)

The "Minimum Area Polygon" problem (MAP) asks for a simple polygon with a given planar set of vertices for which the enclosed area attains the minimum. Pick's theorem provides a relation between the area of a simple polygon and the number of grid points it meets; this yields a combinatorial lower bound for the area of a polygon. Considering this lower bound leads to the "Grid Avoiding Polygon" problem (GAP), which asks for a simple polygon with a given set of (grid) vertices that does not meet any additional grid points. We prove that GAP is NP-complete, implying NP-completeness of MAP. This result answers a question stated by Suri in 1989 We also show that the respective maximization problems are NP-complete and closely related to MAP and GAP. We give upper and lower bounds on approximation factors for MAP. Finally, we show that it is NP-hard to minimize the volume of the k-dimensional faces of a simple d-dimensional polyhedron with a given set of vertices, answering a generalization of a question of O'Rourke.

Odd Cycles and $0\text{-}\frac{1}{2}$ Cuts.

Matteo Fischetti, DEI, University of Padova
(joint work with Alberto Caprara, DEIS, University of Bologna)

Given the polyhedron $P := \operatorname{conv}\{x \in \mathbb{Z}^n : Ax \le b,\ x \ge 0\}$, where A is a $m \times n$ integer matrix and $b \in \mathbb{Z}^m$, a (Chvátal–Gomory) cut is a valid inequality for P of the form $\lfloor \lambda^T A \rfloor x \le \lfloor \lambda^T b \rfloor$, where $\lambda \in \mathbb{R}^m_+$. We study (Chvátal–Gomory) $0\text{-}\frac{1}{2}$ cuts, arising for $\lambda \in \{0, \frac{1}{2}\}^m$. We show that the associated separation problem, $0\text{-}\frac{1}{2}$ SEP, has a pleasant combinatorial structure, and can be solved efficiently when A has, at most, 2 odd coefficients per row, or at most 2 odd coefficients per column. We also outline a heuristic algorithm for $0\text{-}\frac{1}{2}$ SEP which turns out to be exact for a subclass of $0\text{-}\frac{1}{2}$ cuts that often contains wide families of strong (facet-inducing) inequalities for P. Applications are briefly outlined. In particular, we show how the heuristic we propose is capable of finding violated simple Möbius ladder inequalities when applied to the Acyclic Subgraph (or Linear Ordering) problem.

Rounding Proportions.

Michel Balinski, CNRS and Ecole Polytechnique, Paris
(joint work with S.T. Rachev, Univ. of California, Santa Barbara)

It is a widely appreciated fact that the sums of rounded proportions often fail to add to 100. The original data comes from some distribution and are rounded to numbers according to some procedure that determines the distribution they inherit. How to do it "best"? This extends work of Mosteller, Youtz and Zahn and of Diaconis and Freedman who assessed the probability that conventionally rounded proportions add to 100. It is shown that other rules of rounding can improve upon the conventional rule in various contexts that depend upon the distribution of the original data and on the measure of "best".

Network design using multicommodity flows

Sunil Chopra (Northwestern University)

Given a graph $G = (V, A)$, a set of commodities K (each with a source and sink) consider the problem of designing a minimum cost network where there are flow costs on each arc and capacity on each arc can be purchased in batches of size C at a cost w_a. This problem is shown to be NP-hard even on series-parallel graphs. For the case with a single commodity (single source and sink) we give a polynomial time algorithm and a complete inequality description for the case where flow required is at most the batch size. For the general case we describe families of valid inequalities and use them in a branch-and-cut procedure to solve the problem.

Sunil Chopra

Monique LAURENT (LIENS - ENS Paris).
joined work partly with Bert Gerards (CWI, Amsterdam)
and partly Svata Poljak (Charles U, Prague).

Let $\mathcal{L}$ be a clutter on E. $\mathcal{L}$ is said to be $1/d$-integral if for all $1/d$-integral bounds a, b, all vertices of the polyhedron $Q(\mathcal{L}) \cap \{x : a \le x \le b\}$ are $1/d$-integral. The case $d=1$ corresponds to the classical notion of weak Max-Flow-Min-Cut property (or $\mathbb{Q}_+$-MFMC, or ideal). A binary clutter is the port of a binary matroid. We (with B. Gerards) prove that a binary clutter $\mathcal{L}$ is $1/d$-integral for some (or for all) $d \geq 2$ iff $\mathcal{L}$ has no Q_6, no Q_7 minor. (Q_6 is the port of F_7^*, Q_7 is the port of a series extension of F_7).

Given a graph $G=(V,E)$, the polytope $S(G) = \{x : x(F) - x(C-F) \le |F|-1$ for $F \subseteq C$, C cycle of G, $|F|$ odd$\}$ is a linear relaxation. As a consequence of the above result, $S(G)$ is box $1/d$-integral for $d \geq 2$ if and only if G is not contractible to K_4. We also have some results on $1/3$-integral graphs G, ie. such that all vertices of $S(G)$ are $1/3$-integral (with S. Poljak). Namely, the 1-sum of two $1/3$-integral graphs is $1/3$-integral; the 2-sum of an integral graph and a $1/3$-integral graph is $1/3$-integral. Also, the 3-sum of an integral graph with a $1/3$-integral graph such that the common triangle carries at least one equality for each vertex, is $1/3$-integral. Several minimal excluded minors for $1/3$-integrality are known. They suffice for characterizing $1/3$-graphs on $n \le 7$ nodes.

Matchings, Matchable Sets, Hilbert Bases

A *matchable set* of a graph $G=(V,E)$ is the set of end nodes of some matching of G. If $P(G)$ is the matching polytope of G and $Q(G)$ is the matchable set polytope, then $Q(G)=\{Ay : y \in P(G)\}$, where A is the incidence vector of G. A rational polytope P has the *$\frac{1}{m}$ Hilbert Property* if each $x \in P$ has an expression as a convex combination of vertices of P with coefficients λ such that λ is $\frac{1}{m}$ as discrete as x. (That is, qx integral $\Rightarrow$ $mq\lambda$ integral.) For $m=1$, this is equivalent to $\{(v,1) : v \text{ a vertex of } P\}$ being a Hilbert basis. We prove that for any G, $Q(G)$ has the $\frac{1}{2}$ HP, and moreover, λ can be chosen to have at most $3(|V|-1)/2$ non-zeroes. A general result on the $\frac{1}{m}$ Hilbert property implies only that λ can be required to have at most $3|V|$ nonzeroes. We conjecture that the correct number should be $|V|+1$. The proof uses the result that for any $x \in Q(G)$ there exists $y \in P(G)$ with y $\frac{1}{2}$ as discrete as x, and with y having at most $3(|V|-1)/2$ non zeroes. Part of this is joint with J. Green-Krótki. (W.H. Cunningham, Waterloo)

Improvements in chromosome classification

The process of classifying the chromosomes of a human cell into one of 24 classes can be modelled by a transportation problem whose costs measure the distance between a digital image of an individual chromosome and a "typical" image of a member of a class. Using the logarithm of the Mahalanobis distance as such a measure we eliminate classification errors which come from noisy data. Further improvements in classification come from detailed analysis of the digital chromosome images. Using some region growing process we produce data which encode the banding structure of chromosomes and yield further improvements in classification and in automatic identification of chromosomes as image objects.

P. Kleinschmidt, Passau

A polyhedral approach to stochastic optimization problems, ..., by Dimitris Bertsimas, MIT.

We formulate classical stochastic optimization problems (multi-armed bandits, branching bandits, multiclass queueing systems) as linear programming problems over extended polymatroids, polyhedra that strictly extend polymatroids. Optimization of a linear objective over an extended polymatroid is solved by an adaptive greedy algorithm and leads to optimal solutions with an indexability property. Interesting consequences of our new characterization include a deeper understanding of Gittins indices for multi-armed bandit problems, the fastest known algorithm to compute these indices, unexpected connections between different problems, ability to perform sensitivity analysis to name a few. Our approach also addresses in a unified way various issues: discounted or average cost, rewards or taxes, pre-emption or nonpre-emption discrete or continous time, work conserving or idling policies (joint work with J. Niño-Mora).

We then provide a new method based on potential functions to obtain polyhedral and nonlinear characterizations (relaxations) of more complicated (typically PSPACE-complete) stochastic optimization problems (for example, sequencing and routing of multiclass queueing networks), we Optimize over these relaxations in polynomial time using techniques from semi-definite programming and thus obtain lower bounds to the optimal solution value. In the class of systems which can be formulated as extended polymatroids our linear characterizations are exact. Moreover, in this case our approach leads to reformulations of extended polymatroids using a polynomial number of variables and constraints. (joint work with J. Paschalidis and J. Tsitsiklis)

January, 21, 1993. Dimitris Bertsimas.

Research problems in molecular bioinformatics
Thomas Lengauer, GMD & Univ. Bonn

Molecular bioinformatics is a termed coined for an area that aims at supporting the design and analysis of large biomolecules such as proteins and nucleic acid with the computer. The talk first gives a quick introduction into the structure of proteins and the three structural aspects of proteins that determine their chemical function

- sequence
- 3d structure
- dynamics

Then the development of the problem of aligning a protein sequence into a known protein structure is described. Finally other algorithmic problem areas in molecular bioinformatics such as molecular surface recognition, computational support of genome sequencing and protein structure prediction are mentioned.

The p-Median polytope
by Antonio Sassano, Univ. di Roma "La Sapienza"

A spanning p-star of a digraph $G=(V,E)$ is a subset F of E with $|F|=|V|-p$ and with the property that each arc of F goes from a node in a set $S \subseteq V$ with p elements to a distinct node of the set $V-S$. The p-Median polytope $M_K(G)$ is the convex hull of the incidence vectors of the spanning p-stars of G. We show that $M_K(G)$ has dimension $|V|-1$ and it is a "slice" of the Vertex Packing polytope associated with a suitable graph H_G derived from G ($M_k(G) = \mathrm{conv}\left(\mathcal{P}(H_G) \cap \{x \in \{0,1\}^{|E|} : \sum_{e \in E} x_e = k\}\right)$). In addition, we exhibit some basic classes of facet-defining inequalities for $M_k(G)$. Finally, for two of such classes we describe exact separation algorithms with a polynomial running time —

Antonio Sassano

Placement of Telecom satellites in the GSO

Thomas M. Liebling, EPF Lausanne

Placing telecom satellites in the Geostationary orbit (GSO) has become an increasingly sensitive issue at the International Telecommunications Union. In fact the allotment of service arcs, which takes place in worldwide conferences is based on the solution of this nonlinear, nonconvex optimization problem.

One of its possible formulations is

$$\max\left(z \mid W_i \le \vartheta_i \le E_i \ \forall i;\ \min\left(|\vartheta_j - \vartheta_i|,\ 2\pi - |\vartheta_j - \vartheta_i|\right) \ge z\, d_{ij} \ \forall i,j\right)$$

This problem is solved using a partial enumeration heuristic, since for a fixed order of the satellites on the GSO, it becomes the dual of the LP consisting in finding a circuit of minimum cost to weight ratio on a graph, where each edge is associated with a cost and a weight.

This LP is efficiently solved using a version of the simplex method "à la Cunningham".

This ~~talk~~ is based on Susan E. Bickerstaff-Spälti' PhD thesis

Th. M. Liebling

Integer linear programs for local maximum cuts

Svatopluk Poljak, Charles University (Prague) and
Academia Sinica (Taipei)

Let $G=(V,E)$ be a cubic graph, and $x=(x_e)$, $e\in E$, variables associated with the edges. Consider a system of inequalities consisting of $|V|$ blocks, where each block corresponds to a vertex, and the nonnegativity constraint $x\geq 0$. Every block consists of either one inequality

$$x_e > x_f + x_g$$

or three inequalities

$$x_e \leq x_f + x_g$$
$$x_f \leq x_e + x_g$$
$$x_g \leq x_e + x_f$$

where e, f, g are the edges incident to a common vertex. We prove that the system of inequalities is feasible if and only if it has an integer solution bounded by $1\leq x_e\leq 2|V|$, $e\in E$, and give a combinatorial characterisation of solvability.

As a corollary, we prove that any local search for max-cut in a weighted cubic graph requires at most $O(n^2)$ steps.

PRESERVING AND INCREASING EDGE-CONNECTIVITY

András Frank (Budapest + Bonn)

This is a joint work with J. Bang-Jensen (Odense) and B. Jackson (London).

Generalizing a theorem of J. Edmonds, we prove the following:

THM. 1. Let $D=(V,A)$ be a directed graph with a special node r, called root, and a subset T of target nodes so that the in-degree of each other node is at least its out-degree. There is a family $\mathcal{F}$ of k edge-disjoint r-arborescences in D so that each member of $\mathcal{F}$ contains every element of T if and only if $d_{in}(X) \geq k$ holds for every $X \subseteq V-r$ for which $X \cap T \neq \emptyset$.

THM. 2 If D is a pre-flow digraph (i.e. $d_{in}(v) \geq d_{out}(v)$ for every $v \in V-r$) then for any k there are k edge-disjoint arborescences rooted at r so that each vertex v belongs to $\min(k, \lambda(r,v))$ of them where $\lambda(r,v)$ denotes the maximum number of edge-disjoint paths from r to v.

These results are derived with the help of a splitting theorem that generalizes an earlier result of W. Mader.

András Frank

Path- and treewidth of some perfect graphs

We show that the pathwidth of a cocomparability graph equals its treewidth. The proof is based on a new notion, called interval width, for a partial order P, which is the smallest width of an interval order contained in P, and which is shown to be equal to the treewidth of its cocomparability graph G (plus 1). We observe that determining any of these parameters is NP-hard and develop approximation

algorithms for interval width of P whose performance ratios depend on the dimension of P. Applying similar proof techniques, we also show that the treewidth of a graph without asteroidal triples equals its pathwidth.
[Some of the results are joint work with Michel Habib]

Rolf Möhring, TU Berlin

Solving TSP's - Applegate, Bixby, Chvátal, Cook

The result of an approximately five year computational study of the traveling salesman problem were reported. The results of this work include the solution of 16 previously unsolved real-world instances from "TSPLIB", among them an example with 3038 nodes. The computational effort to solve this problem was estimated as equivalent to approximately 1½ years of CPU time on a SPARC station 2. This instance is by far the most difficult solved to date and demanded numerous theoretical and practical improvements in the methods used for obtaining exact solutions to NP-hard combinatorial optimization problems. Parallelism was crucial to the computation as were major improvements in our ability to solve hard linear programming problems. However, undoubtedly the most important new developments were theoretical and involved new separation routines. In one of these, we used the PQ-tree data structure to examine the collection of all tight subtour constraints for the current solution vector. In the case that this collection does not have the consecutive ones property, we can derive a violated "comb inequality" from a theorem of Tucker.

R.E. Bixby 22. Jan 93

Minimally non-greedy structures

joint work with Y. CARO, M. TARSI Tel-Aviv
András SEBŐ
ARTEMIS Grenoble and Bonn

In a common paper with CARO and TARSI we study when various classes of problems can be solved in a greedy or – with the terminology of some predecessors – in a "random" way.

The results we give are sometimes polynomial algorithms to recognize greedy instances, for other problems we also give structural characterizations, for yet others we provide NP-completeness proofs.

In the talk I speak in details about a structural characterization of minimal hypergraphs in which the greedy algorithm does not necessarily lead to a maximum matching, and about minimal "not-jump-systems". I give in the talk the full proof of (an easy) special case of the former result concerning perfectly matchable hypergraphs, implying the polynomial recognizability of greedy graph decomposition and factorization problems.

Computing Image distances by network flows.

F. Barahona, C. Cabrelli, U. Molter

We present experiments with the Kantorovitch / Hutchinson metric for images. We use network flow methods to compute it in the case of digital images. We show examples when this metric gives the right answer, we also show cases when a different metric is needed.

Heuristics, LPs, and Trees on Trees

We study a class of models, known as overlay optimization problems, with two sets of variables x and y, related by linking constraints $x \leq y$. For example, in some telecommunication settings y corresponds to a spanning tree and x to an embedded Steiner tree (or an embedded path). For the general problem, we describe a heuristic solution procedure and establish a worst-case performance guarantee for this heuristic as well as for a linear programming relaxation of the model. For certain models, these performance guarantees are 33%. We also develop heuristic and linear programming performance guarantees for specialized models, a dual path connectivity model with a worst-case guarantee of 25% and an uncapacitated network design model with a worst-case guarantee ~~of~~ (approximately) proportional to the square of the number of commodities.

This is joint work with A. Balakrishnan and P. Mirchandani.

Thomas Magnanti, MIT, Jan 22, 1993

The Inverse Shortest Path Problem

joint work with A. Bachem S. Fekete W. Hochstättler
Christoph Moll (Universität Köln)

We consider the following problem: Given a graph $G(V,E)$, some pairs of nodes $E' \subset V \times V$, distances $D(v,w)$ for every pair $(v,v) \in E'$. Are there weights for the edges $e \in E$, such that the induced distances d_w on the graph correspond to the given distances ($d_w(v,w) = D(v,w)\ \forall (v,w) \in E'$).
We show that this problem is NP-complete even in very restricted cases (Planarity of $G(V, E \cup E')$, $E' \subset (v_0, v_1) \times V$) On the other hand, we present polynomial time algorithms for the following cases ($E' = \{v_0, .., v_k\} \times V$, $E' \subset \{v_0\} \times V$)
If we fix pathes for every pair in E', we have to consider the problem of finding weights such that these paths are shortest paths of given distances. This problem can be formulated as a linear ~~g~~ program. We use this observation for heuristical approaches.

The Fleet Assignment Problem: Solving a Large-Scale Integer Program

Given a flight schedule and a set of aircraft, the fleet assignment problem is to determine which type of aircraft should fly each flight segment. This talk describes a daily, domestic fleet assignment problem and then presents chronologically the steps taken to solve it efficiently. Our model of the fleet assignment problem is a large multi-commodity flow problem with side constraints defined on a time-expanded network. These problems are often severely degenerate, which leads to poor performance of standard LP techniques. The methods used to attack this problem include an interior point algorithm, cost perturbation, model aggregation, branching on set-partitioning constraints and prioritizing the order of branching. The computational results show that the algorithm finds solutions with a maximum optimality gap of 0.22% and is more than 2 orders of magnitude faster than using default options of a standard LP-based branch-and-bound code.

George Nemhauser
Georgia Inst. of Technology
Atlanta GA
USA

A parallel implementation of an interior point method

(joint work with M. Streizel, M. Wattauer)

In this talk we report on a parallel implementation of a primal-dual interior point method for linear programming. We started with the sequential implementation of the OB1-code of Lustig, Marsten and Shanno and used a fan-in algorithm to parallelize the numerical factorization of the Cholesky factor. We show that a balanced subtree mapping yields a good load balancing on a message passing computer. For a transputer cluster speedups of 3 to 5 on 6 processors give the best relative efficiency. Numerical results on the Netlib examples are presented.

J. Rocher, Köln

Optimum Crew Pairing

The problem considered is to partition the flight legs into pairings (tours of duty beginning at a crew base and returning) for a monthly airline problem. Work has been done with Ranga Anbil, Rajan Tanga, and K.S. Ramakrishnan at American Airlines Decision Technologies on the domestic daily problem. First, a linear program is solved over several million columns and then an integer program is solved over a subset of 15,000 columns. The new methodology has been in use for 1½ years with a resulting savings of about $2 million/year. A long-haul problem has been solved in joint work with Lavent Hatay and Cynthia Barkhart for a small package carriage. Exact linear programming solutions are obtained using column generation techniques combined with shortest path. Better deadheads are chosen using dual information

Ellis Johnson
Yorktown Hts and Atlanta

Using Path inequalities in a Branch and Cut code for the Symmetric Traveling Salesman Problem

In this talk we show how path inequalities can be effectively used to solve TSP instances. They are also used in connection with a new branching rule, that of using subtour eliminations to separate the solution set. Computational results are given which show the efficiency of this approach. The problem of the density of these inequalities is also addressed.

Denis Naddef
ARTEMIS-IMAG – Grenoble

Two-Connected subgraphs and polyhedra

In this talk we consider the problems of finding a two-edge (two-node) connected spanning subgraph of minimum weight. These problems are closely related to the widely studied traveling salesman problem and have applications to the design of reliable communication and transportation networks. We discuss the polytopes associated with the solutions to these problems. We give complete descriptions of these polytopes for the class of Halin graphs. We show that when the graph is series-parallel, the polytope associated with the two-edge connected spanning subgraphs is completely described by the trivial constraints and the so-called cut constraints. We also discuss some classes of facet defining inequalities of these polytopes and other polyhedral aspects when the graph is general

Ali Ridha Mahjoub
Laboratoire d'informatique (LIBr), Brest, France

Graph 5-colouring.

H. Hadwiger conjectured in about 1940 that every graph not contractible to K_{p+1} is p-colourable. For $p \leq 3$ this is easy, but for $p=4$ it is extremely difficult, for K. Wagner showed this was equivalent to the four-colour conjecture (in 1937). In joint work with N. Robertson and R. Thomas, we showed that Hadwiger's conjecture for $p = 5$ is also equivalent to the four-colour conjecture; indeed, that any minimal counterexample has a vertex whose deletion leaves a planar graph.

Paul Seymour
Bellcore, Morristown NJ, USA

Lehman's Theorem and Stable Set Polyhedra

WE CONSIDER A TRANSFORMATION OF COVERING POLYHEDRA TO PACKING POLYHEDRA WHICH PRESERVES FACE DIMENSIONS. THIS IS USED TOGETHER WITH RESULTS OF LEHMAN ABOUT CONTRACTION-MINIMAL MATRICES TO GIVE A LINEAR DESCRIPTION FOR THE STABLE SET POLYTOPES OF NEAR-BIPARTITE GRAPHS. THESE ARE GRAPHS FOR WHICH $G-N(v)$ IS BIPARTITE FOR EACH NODE v; this CLASS CONTAINS THE COMPLEMENTS OF LINE GRAPHS. THE RESULTS ALSO LEAD TO SEVERAL NEW CLASSES OF MINIMALLY NON-t-PERFECT GRAPHS.

Bruce Shepherd
London School of Economics

Disjoint Paths on the Möbius Strip.

Theorem: Let G be a graph embedded on the Klein Bottle such that all faces are bounded by even circuits. For each $w \in \mathbb{Z}_+^E$ with $w(\delta(v))$ even for each $v \in V$ ~~we have th it holds~~ the following holds:

$$\min\{w^T x \mid x(C) \geq 1 \text{ (C odd circuit)}, x \in \mathbb{Z}_+^E\} =$$

$$\min\Big\{\sum_{C \text{ odd circuit}} y_C \;\Big|\; \sum_{\substack{C \text{ odd circuit} \\ C \ni e}} y_C \leq w_e \ (e \in E);\ y_C \in \mathbb{Z}_+ \text{ (C odd circuit)}\Big\} \quad \square$$

The result is a consequence of the disjoint paths theorems by Okamura & Seymour, ~~and~~ Okamura and Schrijver for graphs on the plane disk or the annulus, and of a similar, new result on disjoint paths on graphs embedded on the Möbius strip.

Bert Gerards CWI Amsterdam