MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 27/1981

Maßtheorie

21.6. bis 27.6.1981

Die Tagung, an der 51 Wissenschaftler aus 17 Ländern teilnahmen, stand unter der Leitung von D. Kölzow (Erlangen) und D. Maharam-Stone (Rochester). In ihrem Verlauf wurden insgesamt 36 Vorträge gehalten; abgeschlossen wurde sie mit einer 'Problem Session'.

Es ist geplant, einen Tagungsbericht zu veröffentlichen, wenn möglich wieder in den 'Lecture Notes in Mathematics' des Springer Verlages.

Die Tagungsteilnehmer möchten sich an dieser Stelle beim Direktor des Mathematischen Forschungsinstituts, Herrn Professor Dr. Barner, und seinen Mitarbeitern für die große Unterstützung bedanken, die den erfolgreichen Verlauf der Tagung möglich machte.

Vortragsauszüge

Allgemeine Maßtheorie

W. ADAMSKI:

On tight set functions

Let $\mathcal{K}$ and $\mathcal{L}$ be lattices of subsets of a set X such that $\mathcal{L}$ dominates $\mathcal{K}$ and $K \setminus L \in \mathcal{K}$, $L \setminus K \in \mathcal{L}$ for all $K \in \mathcal{K}$ and $L \in \mathcal{L}$. Let $\lambda : \mathcal{K} \to [0,\infty]$ be semifinite, tight and σ-smooth at $\emptyset$. Then $\gamma := \lambda_{*}|_{\mathcal{L}}$ (i.e. $\gamma(L) := \sup\{\lambda(K) : K \in \mathcal{K}, K \subseteq L\}$ for $L \in \mathcal{L}$) is compementary tight and σ-smooth from below. For a subset Q of X define $\lambda^s(Q) := \sup\{\lambda^{*}(\bar{K}) : \bar{K} \in \mathcal{K}_\delta,\ \bar{K} \subseteq Q,\ \lambda^{*}(K) < \infty\}$,

$$\gamma^1(Q) := \inf\{\textstyle\sum_n \gamma(L_n) : (L_n) \subseteq \mathcal{L},\ Q \subseteq \bigcup_n L_n\}.$$

Then $(X, \mathcal{M}(\lambda^s), \lambda^s)$ and $(X, \mathcal{M}(\gamma^1), \gamma^1)$ are complete measure spaces, where $\mathcal{M}(\lambda^s)$ and $\mathcal{M}(\gamma^1)$ denote the collections of λ^s- and γ^1-measurable subsets of X respectively. For every σ-ring $\mathcal{A}$ with $\mathcal{K} \subseteq \mathcal{A} \subseteq \mathcal{M}(\lambda^s)$, λ^s is the smallest measure extension of λ to $\mathcal{A}$ and for every σ-ring $\mathcal{B}$ with $\mathcal{L} \subseteq \mathcal{B} \subseteq \mathcal{M}(\gamma^1)$, γ^1 is the largest measure extension of γ to $\mathcal{B}$.

The main result was the following:

<u>Theorem</u>: If $\lambda(K) = \inf\{\gamma(L) : K \subseteq L,\ L \in \mathcal{L}\}$ for all $K \in \mathcal{K}$ then $\mathcal{M}(\lambda^s) = \mathcal{M}(\gamma^1)$ and for every paving $\mathcal{C}$ with $\mathcal{K}_\delta \subseteq \mathcal{C} \subseteq \mathcal{M}(\gamma^1)$ we have $\lambda^s(Q) = \sup\{\gamma^1(C) : C \in \mathcal{C},\ C \subseteq Q,\ \gamma^1(C) < \infty\}$ for all $Q \subseteq X$. In particular, $\lambda^s|_{\mathcal{A}}$ is the essential measure associated to $\gamma^1|_{\mathcal{A}}$ for every σ-ring $\mathcal{A}$ with $\mathcal{K} \subseteq \mathcal{A} \subseteq \mathcal{M}(\gamma^1)$.

J.P.R. CHRISTENSEN:

Some open problems in the classical theory of moments and ...

Let u be a positive measure on $\mathbb{R}$ having moments of all orders. Let $\mathcal{P}$ be the space of all polynomials in one variable and $1 \leq p < \infty$; when do we have $\bar{\mathcal{P}} = L^p(u)$? The classical cases p = 1, 2 and some preliminary results are discussed in a forthcoming paper (jointly with CH. BERG) in Ann. Inst. Four..

Conjecture 1: If $1 < p < 2$ and $\overline{\mathcal{P}} = L^p(u)$, then $\overline{\mathcal{P}} = L^{p'}(u)$ for $1 \leq p' < 2$.

Conjecture 2: If $2 < p < \infty$ and $\overline{\mathcal{P}} = L^p(u)$, then for all $1 \leq p' < \infty$ we have $\overline{\mathcal{P}} = L^{p'}(u)$

Conjecture 3: If $1 < p < \infty$ and $\overline{\mathcal{P}} = L^p(u)$ but $\overline{\mathcal{P}} \neq L^{p'}(u)$ for some $p' > p$, then u is discrete and supp(u) consists of the zeroes of an entire function of order at most 1.

The following is Streutel's conjecture on infinite divisibility:

Let for $1 \leq \nu \leq n$ positive numbers p_ν, r_ν be given, then the function on $]0,\infty[$ given by $\sum_{\nu=1}^{n} p_\nu/(r_\nu + x)^2$ is an inf. div. Laplace transform.

Assuming this, it was shown that there exists a very strange pair of measures, namely two positive measures u, v on $\mathbb{N}_0$, infinitely divisible in the usual (convolution) sense such that (1) $\forall\, n \in \mathbb{N}_0$: $u(\{n\}) + v(\{n\}) = 1$ and (2) $(1/N) \sum_{i=0}^{N-1} u(\{i\})$ oscillates between o and 1 as N tends to infinity.

The speaker expressed the opinion that such a pair cannot exist (and therefore Streutel's conjecture is false).

Z. FROLIK:

On measure-fine spaces II

If X is a uniform space denote by $\check{X}$ the Samuel compactification of X (i.e. the structure space of the algebra $U_b(X)$ of bounded uniformly continuous functions on X), hence $U_b(X) \sim C(\check{X})$. For each $\mu \in U_b(X)'$ denote by $\check{\mu}$ the Riesz representation of the corresponding functional on $C(\check{X})$, i.e. $\mu(f) = \int \check{f}\, d\check{\mu}$, where $\check{f}$ is the continuous extension of f to $\check{X}$. Denote by $\mathcal{M}_u(X)$ the space of uniform measures on X. For a uniform space X and a complete metric space S define $\mathcal{M}(X,S) := \{f: X \to S$: the domain of the largest continuous extension of f to a mapping of a subspace of $\check{X}$ into S supports $\check{\mu}$, $\mu \in \mathcal{M}_u(X)\}$,

$\mathcal{N}(X,S) := \{f: X \to S$: $\mathrm{Mol}^+(f): \mathrm{Mol}^+(X) \to \mathrm{Mol}^+(S)$ extends to a continuous mapping from $\mathcal{M}_u^+(X)$ into $\mathcal{M}_u^+(S)$

Then $\mathcal{M} = \mathcal{N}$, and if $FX = \varprojlim_{S \text{ compl. metr.}} (X,S)$, then $\mathcal{U}(FX,S) = \mathcal{M}(FX,S) = \mathcal{M}(X,S)$ for each X,S.

E. GRZEGOREK:

Symmetric σ-fields of sets and universal null sets

I.) Let $\mathcal{A}$ be a σ-field on a set X and let $\mathcal{A}^n$ be the product σ-field on X^n.

The symmetric sets in $\mathcal{A}^n$ (i.e. those invariant under permutations of the coordinates) form a sub-σ-field of $\mathcal{A}^n$. It was shown by B.V. RAO (Coll. Math. 1970) that for $n = 2$ the symmetric σ-field on X^n is generated by the symmetric rectangles. For $n > 2$ this need not be true. In this talk a set-theoretical proof of the following generalization of RAO's result was given.

Theorem: The symmetric σ-field on X^n is generated by the family $\{\bigcup_{s\in S_n} A_{s(1)} \times \dots \times A_{s(n)} : A_1, \dots, A_n \in \mathcal{A}\}$, where S_n denotes the permutation group on $\{1, \dots, n\}$.

II.) Let $\mathcal{N}$ be the σ-ideal of universal null-subsets of $\mathbb{R}$. Under (CH), SIERPINSKI proved in 1935 that there is $N \in \mathcal{N}$ such that $N + N = \mathbb{R}$. The same holds under (MA). LAVER (1976) showed that ZFC + (all $N \in \mathcal{N}$ have cardinality at most $\aleph_1$) is consistent if ZFC is consistent, hence SIERPINSKI's result is unprovable in ZFC. Here it was shown that ($\exists N \in \mathcal{N}$: $N + N \notin \mathcal{N}$) can be proved in ZFC. Question: Can one prove in ZFC that there is an $N \in \mathcal{N}$ with $N + N$ not a Lebesgue null-set?

J. LEMBCKE:

Measures with a finite number of given image measures

The following problem was considered:

Given a set $I \neq \emptyset$, Hausdorff spaces X, Y_i ($i \in I$), continuous mappings $f_i : X \to Y_i$, and regular Borel measures μ_i on Y_i ($i \in I$), under which conditions does the existence of a finitely additive Borel measure ν on X with $f_i(\nu) = \mu_i$ ($i \in I$) imply the existence of a regular (countably additive) Borel measure on X with the same property?

It was shown that for I finite, X Souslin or σ-compact and bounded μ_i's the following condition is sufficient:

The set $\{(f_i(x))_{i\in I} : x \in X\}$ is a countable union of measurable rectangles in $\prod_{i\in I} Y_i$.

An example of FREMLIN shows that this condition cannot be dropped.

P. MASANI:

The regularization of finitely additive measures over normal topological spaces

Let (i) Ω be a normal Hausdorff space, (ii) $\mathcal{T}$ be the family of all open subsets of Ω, (iii) $\mathfrak{A}$ be a ring such that $\mathcal{T} \subseteq \mathfrak{A} \subseteq 2^{\Omega}$, (iv) $\mathfrak{A}_o$ and $\mathfrak{B}_o$ be the ring and the σ-ring generated by $\mathcal{T}$, and (v) FA($\mathfrak{A}$, $\mathbb{R}_{o+}$) and CA($\mathfrak{A}$, $\mathbb{R}_{o+}$) be the classes of finitely additive and countably additive measures on $\mathfrak{A}$ to $\mathbb{R}_{o+}$. The following theorem was established:

Theorem: (a) For $\nu \in$ FA($\mathfrak{A}$, $\mathbb{R}_{o+}$) there exists a unique μ such that

(α) $\mu \in$ FA($\mathfrak{A}_o$, $\mathbb{R}_{o+}$) and μ is outer regular on $\mathfrak{A}_o$,

(β) $(A \in \mathfrak{A}_o \;\&\; \mu(\delta A) = o) \Rightarrow \mu(A) = \nu(A)$.

(b) Let Ω be compact and $\nu \in$ FA($\mathfrak{A}$, $\mathbb{R}_{o+}$). Then there exists a unique μ such that

(α) $\mu \in$ CA($\mathfrak{B}_o$, $\mathbb{R}_{o+}$) and μ is regular on $\mathfrak{B}_o$,

(β) $(A \in \mathfrak{A} \cap \mathfrak{B}_o \;\&\; \mu(\delta A) = o) \Rightarrow \mu(A) = \nu(A)$.

(δA denotes the boundary of A)

This theorem simplifies appreciably the study of the compactness properties of the Lévy topology for the cone Y_{o+} of outer regular measures in FA($\mathfrak{A}$,$\mathbb{R}_{o+}$).

V.S. PRASAD:

A survey of homeomorphic measures

Given a topological measure space (X,λ) with a "distinguished" Borel measure λ on X, we ask what conditions are necessary for a Borel measure μ on X, to assure the existence of a homeomorphism h of X onto itself such that $\mu(E) = \lambda(h(E))$ for all Borel sets E in X, (In this case we say that μ is homeomorphic to λ). The results obtained by OXTOBY and his colleagues for special pairs (X,λ) were surveyed. Specifically the folloeing was considered:

(i) $X = I^n$, the n-dimensional cube and λ the product Lebesgue measure,

(ii) $X = I^\infty$, the Hilbert cube and λ the product Lebesgue measure,

- extensions of (i) and (ii) to compact connected manifolds modelled on I^n and I^∞ were also considered -

(iii) $X = \mathfrak{N}$, the irrationals in $[0,1]$ and λ the restriction of the Lebesgue measure,

(iv) $X = \prod_1^\infty \{0,1\}$, the Cantor set and λ the product measure $\prod_1^\infty p$ where $p(\{0\}) = r$ and $p(\{1\}) = 1-r$.

W. SCHACHERMAYER:

Measurable functionals on spaces of uniformly continuous functions

First, the following theorem due to J.P.R. CHRISTENSEN was recalled:

Theorem: Let $f: l^\infty \to \mathbb{R}$ be linear and such that the restriction of f to the unit ball is σ^*-Baire-property-measurable. Then $f \in l^1$.

From this CHRISTENSEN and PACHL deduced:

Theorem: Let M be a closed subset of $\mathbb{R}^n$ and $f: U_b(M) \to R$ linear and strongly Baire-property-measurable. Then f is induced by a Radon measure on M.

Here $U_b(M)$ is the space of bounded uniformly continuous functions on M equipped with the finest locally convex topology agreeing with the topology of pointwise convergence on bounded uniformly equicontinuous sets.

It is an open question, whether this theorem generalizes to arbitrary complete metric spaces. It was shown:

Proposition: Let M be complete metric and $f: U_b(M) \to \mathbb{R}$ be of first Baire class. Then f is induced by a Radon measure.

Corollary: Let M be complete meric. The space $M_R(M)$ of bounded signed Radon measures on M is sequentially $\sigma(M_R(M), U_b(M))$-complete.

Finally the following example was given: Let $M = c_0$ and denote for $0 < \eta < \infty$ $Lip_\eta(M) := \{f \in U_b(M): \|f\|_\infty \leq 1, \|f\|_{Lip} \leq \eta^{-1}\}$. There is a multiplicative linear functional f on $U_b(M)$ which is not induced by a Radon measure, such that its restriction to $Lip_\eta(M)$ is BP-measurable, for $0 < \eta < \infty$.

A.H. STONE:

(reporting on joint work with R.D. MAULDIN)

Realization of maps

Consider a product measure space $(X, \mathcal{M}, \lambda) \times (Y, \mathcal{L}, \mu)$, where all measures are assumed finite and complete. The measure algebra $\mathcal{E}$ of Y is a metric space, with distance $\varsigma([A],[B]) = \mu(A \triangle B)$. A map $\Phi: X \to \mathcal{E}$ is called "realizable" if there is a measurable subset $E \subseteq X \times Y$ such that, for each $x \in X$, $\Phi(x) = [E_x]$, the measure class of the section of E over x. If instead this holds for almost all $x \in X$, Φ is "almost realizable". Finally, Φ is "essentially separable-valued" provided for some λ-null set N, $\Phi(X \setminus N)$ is a separable subset of $\mathcal{E}$.

Theorem: If $\Phi: X \to \mathcal{E}$ is a map, the following are equivalent. (a) Φ is realizable, (b) Φ is almost realizable, (c) Φ is measurable and essentially separable-valued.

If there are no real-valued measurable cardinals, then Φ is realizable if and only if Φ is measurable. But if there is a real-valued measurable cardinal, then there exist probability measure spaces $(X, \mathcal{M}, \lambda)$ and $(Y, \mathcal{L}, \mu)$ and a measurable $\Phi: X \to \mathcal{E}$ that is not realizable. The analogous situation for Borel measurability was also considered; here $\mathcal{M}$ and $\mathcal{L}$ are the Borel sets of metrizable spaces X and Y, the measures are Borel measures and the realizing set E is to be Borel. If Y is separable, then $\Phi: X \to \mathcal{E}$ has a Borel realization if and only if Φ is Borel measurable. The extension to nonseparable Y remains open.

H. VON WEIZSÄCKER:

The dying witness

The talk gave a slight improvement of a result reported during an earlier conference.

Theorem: Consider a probability space $(X \times Y, \mathcal{F} \otimes \mathcal{G}, P)$ and a decreasing sequence $(\mathcal{G}_n)$ of sub-σ-fields of $\mathcal{G}$. Assume there is a conditional probability kernel (P_x) on $\mathcal{G}$ given $\mathcal{F}$. If the $\mathcal{G}_n$ are countably generated, then the following are equivalent: $(\mathcal{G}_\infty = \bigcap_n \mathcal{G}_n)$

1) $\mathcal{F} \otimes \bigcap_n \mathcal{G}_n = \bigcap_n \mathcal{F} \otimes \mathcal{G}_n$ mod P.

2) There is a countably generated σ-field $\mathcal{H} \subseteq \mathcal{G}_\infty$ such that $\mathcal{H} = \mathcal{G}_\infty$ mod P_x for P-almost all x.

3) For every $a \in \mathcal{F} \otimes \mathcal{G}$ there is some $h \in \mathcal{F} \otimes \mathcal{G}_\infty$ such that $h(x, \cdot) = E_{P_x}^{\mathcal{G}_\infty}(a(x, \cdot))(\cdot)$ for P-almost all x.

4) $\forall a \in \mathcal{F} \otimes \mathcal{G}$ $\forall \varepsilon > 0$ $\exists \mathcal{R}$ finite $\subseteq \mathcal{F}$ $\forall n$ $\exists g_n$ such that $\forall y$ $g_n(\cdot, y) \in \mathcal{R}$ and $\int |g_n - E^{\mathcal{F} \times \mathcal{G}_n}(a)| \, dP < \infty$.

($\mathcal{G}_n$ is the memory of a witness of some mathematical talk).

Deskriptive Mengenlehre und meßbare Selektionen

D.H. FREMLIN:

(reporting on joint work with R.W. HANSELL)

Borel isomorphisms of bounded class

If X and Y are metric spaces and $f: X \to Y$ is a Borel isomorphism, there exists a $j < \omega_1$ such that the Borel class of $f^{-1}(H)$ is at most j for all open subsets H of Y. The proof depends on the following

Lemma: Let I be any set, $\langle \mathcal{A}_\xi \rangle_{\xi < \omega_1}$ a family of subsets of $\mathcal{P}(I)$ such that (i) for any even $\xi < \omega_1$, the union of any sequence in $\bigcup_{\eta < \xi} \mathcal{A}_\eta$ belongs to $\mathcal{A}_\xi$ and the intersection of any sequence in $\mathcal{A}_\xi$ belongs to $\mathcal{A}_{\xi+1}$,
(ii) $\bigcup_{\eta < \omega_1} \mathcal{A}_\eta = \mathcal{P}(I)$. Set $\mathcal{J}_\xi = \{A : \mathcal{P}(A) \subseteq \mathcal{A}_\xi\}$, $\mathcal{J} = \bigcup_{\xi < \omega_1} \mathcal{J}_\xi$.
Then (a) if κ is a cardinal such that $\mathcal{P}(\omega_1 \times \kappa) = \mathcal{P}(\omega_1) \hat{\otimes} \mathcal{P}(\kappa)$, and $\mathcal{A} \subseteq \mathcal{P}(I)$ is a disjoint set of cardinality at most κ, there is a $j < \omega_1$ such that $\bigcup \mathcal{A}' \in \mathcal{A}_j$ for every $\mathcal{A}' \subseteq \mathcal{A}$ (b) if $\mathcal{A} \subseteq \mathcal{P}(I)$ is disjoint, there is a $j < \omega_1$ such that $\mathcal{A} \setminus \mathcal{J}_j$ is countable (c) $\mathcal{J}$ is an ω_1-saturated σ-ideal in $\mathcal{P}(I)$.

If every cardinal κ satisfies (a) of the lemma, there is now a fairly quick proof of the theorem; otherwise a more delicate argument is necessary.

Problem: It remains unknown whether the theorem is valid for Borel measurable functions in general; this is a problem of A.H. STONE.

S. GRAF:

(reporting on joint work with G. MÄGERL)

Baire sections for group homomorphisms

As a partial answer to a question of KUPKA - which in turn was triggered by a paper of RIEFFEL (Am. J. Math. 88(1966)) - the following result was proved: Let X and Y be compact topological groups and p be a continuous group homomorphism from Y onto X. Then there exists a Baire measurable map $f: X \to Y$ such that $p \circ f = id_X$ and $f(e_X) = e_Y$ (e_X, e_Y denoting the neutral elements). The main tool in the proof of this result is the following selection lemma: Let X and Z be compact spaces and assume, in addition, that Z is metrizable and that any two disjoint open subsets of X can be separated by open $\tilde{\mathfrak{F}}_\sigma$-sets (this is the socalled Bockstein Separation Property). Then every compact-valued upper semicontinuous correspondence from X to Z has a Baire measurable selection.

R.D. MAULDIN:

Measurable marriages

Let R be a Lebesgue measurable subset of $[0,1] \times [0,1]$ such that the Lebesgue measure of the set $\{x: \lambda(R_x) > 0 \text{ and } \lambda(R^x) > 0\}$ equals 1, where R_x is the set of all y such that $(x,y) \in R$. Then there are Borel subsets D, E of $[0,1]$ with measure 1 and a Borel measurable isomorphism of D onto E whose graph is a subset of R.

S.M. SRIVASTAVA:

Some measurable selection theorems

Theorem: Let T and X be Polish spaces and $F: T \to X$ be a multifunction such that (i) F(t) is a G_δ in X for all $t \in T$, (ii) $G(F) := \{(t,x): x \in F(t)\}$ is a Borel set in $T \times X$, and (iii) $\{t \in T: F(t) \cap V \neq \emptyset\}$ is a Borel set in T for all open subsets V of X. Then F admits a Borel measurable selector.

A proof of this theorem, recently discovered by R. BAMA and V.V. SRIVATSA (c.f. Effective selections and parametrizations, to app. in J. Symb. Logic), was presented and it was shown that the same idea can be used to prove (under the axiom of Projective Determinacy) the above result for higher projective classes.

Liftings

V. LOSERT:

Strong liftings for a certain class of compact spaces

Let X be a compact space and μ be a Radon probability measure on X. Let $\mathcal{E}$ be the measure algebra of μ and $\mathcal{A}$ be the subalgebra of $\mathcal{E}$ consisting of the equivalence classes of all sets A with $\mu(\partial A) = 0$. If Y is the representation space of $\mathcal{A}$ and $\bar{\mu}$ is the measure on Y induced by μ, it was proved that $(Y,\bar{\mu})$ always has a strong lifting. One can construct a strong lifting l such that l(U) is open if U is open and the existence of such a lifting characterizes spaces Y of the type described above. If $\mu(\{x\}) = 0$ for all $x \in X$ and $\mathcal{A} \neq \mathcal{E}$ then it can be shown that Y has no strong lifting such that l(A) belongs to the completion of $\mathfrak{B}_0(Y)$, the σ-algebra of Baire measurable sets.

A. VOLČIČ:

Liftings and Daniell integrals

If I is a Daniell integral, the function space $\mathcal{L}^{\infty}$ is defined as follows: $k \in \mathcal{L}^{\infty}$ iff $k-f \in \mathcal{L}^1$ for all $f \in \mathcal{L}^1$. This space turns out to be a vector lattice, an algebra containing the constants and a Banach space under the norm defined by $\|k\|_{\infty} := \inf\{\lambda : x\,|k(x)| > \lambda\}$ is a local null set . A set T is locally null if $I(f \cdot \chi_T) = 0$ for all $f \in \mathcal{L}^1$. The integral I is said to be essential if the locally null sets are null; it is said to be localizable if it is essential and $\mathcal{L}^{\infty}$/(null sets) is complete with respect to uniformly bounded sets; it is said to be strictly localizable if it is essential and if there exists a family $\{f_{\alpha}\}$ of integrable functions such that (1) for all $\alpha \neq \beta$, $\{x: f_{\alpha}(x) \neq 0\} \cap \{x: f_{\beta}(x) \neq 0\} = \emptyset$ and (2) for all positive f in $\mathcal{L}^1$ there is an α with $I(|f_\alpha| \wedge f) > o$.

<u>Theorem:</u> I has a lifting iff I is strictly localizable.

The proof used the fact that for a strictly localizable integral there is a weak unit, i.e. a real-valued Stone-measurable function which is strictly positive on X. This fact permits one to reduce the problem to the measure theoretic situation.

Differentiation von Maßen und Integralen

B. BONGIORNO:

<u>Essential variations</u>

Let Ω be an open subset of $\mathbb{R}^n$, $[x,x+h]$ be the interval having x and x+h as points of extreme coordinates, $\{e_i\}$ be the canonical basis of $\mathbb{R}^n$. For $f:\Omega \to \mathbb{R}$ we set $F_f[x,x+h] := \sum_{k=1}^{n} \Big| \sum_{s=o}^{k} (-1)^s \sum_{J\in C_{k,s}} f(x + \sum_{i\in J} h_i e_i) \Big|^n$,

where $h = (h_1, \dots , h_n)$ and $C_{k,s}$ is the set of all combinations of k-elements of class s. Denote by $\mathcal{D}_\alpha$ the differentiation basis of all α-regular intervals contained in Ω and by S the set of all full subbbases of $\mathcal{D}_\alpha$. Then the essential variation of F_f with respect to $\mathcal{D}_\alpha$ is defined as follows:

$${}^{ess}V(F_f) := \inf_{\mathcal{S}\in S} \sup_{\{I_i\}\subseteq \mathcal{S}} \sum_i |F_f(I_i)| \quad , \text{ where } \{I_i\} \text{ is a disjoint family of}$$

$\mathcal{D}_\alpha$-intervals. The following generalization of a Cacciopoli theorem was proved:

<u>Theorem:</u> If f is Lebesgue measurable, then for each $o<\alpha<1$ the essential variation of F_f with respect to $\mathcal{D}_\alpha$ is σ-finite iff f is a.e. differentiable. In a similar way one can get a generalization of the Khintchine theorem on a.e. approximate differentiability of functions of one real variable.

D. PREISS:

<u>Differentiation of measures in Hilbert spaces</u>

If X is a finite dimensional Banach space, a result of BESICOVITCH and MORSE states that for each finite Borel measure μ and each $f\in L^1(u)$ we have $1/(\mu(B(x,r))) \int_{B(x,r)} f(y)du(y) \to f$ μ-a.e..This type of differentiation property is, in fact, equivalent to some notion of finite dimensionality (for Banach spaces the usual one). MATTILA's results show that some differentiation theorems hold for some "infinite dimensional" measures, but they cannot be used for a large class of interesting cases. Two examples were given showing that even the weakest forms of the differentiation theorem are false for Gaussian measures in Hilbert spaces. Further two positive results were presented: (i) For Gaussian measures on Hilbert spaces the differentiation theorem holds if the covariance behaves well. (ii) If the measures are supported by a set which can be approximated by finite dimensional sets, then an ineuality between measures of small balls implies an inequality between measures (for arbitrary measures on Hilbert spaces).

Maße mit allgemeinem Wertebereich

S.D. CHATTERJI:

Hilbert space valued measures

Let (Ω, Σ) be a space Ω equipped with an algebra of subsets Σ and let $\mathcal{H}$ be a Hilbert space (over $\mathbb{R}$ or $\mathbb{C}$). For $\mu : \Sigma \to \mathcal{H}$ we say that μ is orthogonally scattered (o.s.) if $A \cap B = \emptyset$ imlpies $\mu(A) \perp \mu(B)$. We say that $\tilde{\mu} : \Sigma \to \tilde{\mathcal{H}}$ is a dilation of μ (where $\tilde{\mathcal{H}}$ is another Hilbert space) if there is a closed linear manifold $M \subseteq \tilde{\mathcal{H}}$ which is unitarily isomorphic to $\mathcal{H}$ $(\pi : M \to \mathcal{H})$ such that $\mu(A) = \pi P(u(A))$ for all A in Σ (where P is the orthogonal projection of $\tilde{\mathcal{H}}$ onto M. The main theorem was:

Theorem:Let $\mu : \Sigma \to \mathcal{H}$ be a bounded finitely additive measure. Then there exists $\tilde{\mu} : \Sigma \to \tilde{\mathcal{H}}$, a f.a. o.s. dilation of μ. If μ is countably additive then there exists a c.a. o.s. dilation of μ.

This is a generalisation of a theorem of NIEMI (1977). A new type of proof was given for this theorem. Several remarks connecting the theorem to generalised Aronszajn kernel theorems were appended.

P. MORALES:

Extension of a tight set function with values in a uniform semigroup

Let $\mathcal{L}$ be a lattice of subsets of a set X with $\emptyset \in \mathcal{L}$, let S be a complete Hausdorff uniform semigroup and let $\lambda : \mathcal{L} \to S$ be a set function with $\lambda(\emptyset)=0$. We say that λ is increasingly convergent if, for every increasing sequence (L_n) in $\mathcal{L}$, $(\lambda(L_n))$ converges. An important fact is, that if λ is increasingly convergent, the limit of the net $\{\lambda(L): E \supseteq L, L \in \mathcal{L}\}$, which is denoted by $\lambda_-(E)$, exists for all $E \subseteq X$. We say that λ is tight if λ is increasingly convergent and $\lambda(L) = \lambda(L \cap K) + \lambda_-(L \setminus K)$ for all $K, L \in \mathcal{L}$. The properties of the extension λ_- restricted to the ring generated by $\mathcal{L}$ were studied and, using the Carathéodory approach, it was shown that earlier extension theorems due to CHOKSI, HUNEYCUTT, KELLEY - NAYAK - SRINIVASAN, KISYŃSKI, LIPECKI, NAYAK - SRINIVASAN, SION, TOPSØE and others can be unified.

Example: Theorem: If λ is tight, locally s-bounded and continuous at $\emptyset$, then λ extends uniquely to a $\mathcal{L}_{\sigma\delta}$-regular measure $\bar{\lambda}$ on the σ-ring generated by λ such that the restriction of $\bar{\lambda}$ to the δ-ring generated by is locally s-bounded.

P.K. PAVLAKOS:

On the space of lattice semigroup-valued set functions

In this talk the HEWITT-YOSIDA decomposition theorem for measures taking values in lattice semigroups was generalized. In particular, it was proved that $M_\sigma(H,X)$ is a band in $M(H,X)$, where $M(X,H)$ (resp. $M_\sigma(X,H)$) is the lattice semigroup of finitely additive (resp. σ-additive with respect to order convergence) set functions with domain a ring H and range a lower complete lattice semigroup.

M.M. RAO:

Domination problem for vector measures and an application

Let (Ω, Σ) be a measurable space and X a Banach space. A vector measure $\nu : \Sigma \to X$ is p-dominated by a finite positive measure $\mu : \Sigma \to \mathbb{R}^+$ if $\|\nu\|_p(\Omega) < \infty$

$\|\nu\|_p$ is the p-semivariation of ν relative to μ: $\|\nu\|_p(\Omega) :=$ $\sup\{\|\int_\Omega f(\omega)d\nu(\omega)\|_X : \|f\|_{q,\mu} \leq 1\}$, $p \geq 1$, $1/p + 1/q = 1$, the integral being taken in the Dunford-Schwartz sense. The general domination problem is to determine p (and μ) for given X and to classify X for given p and some μ. The following weaker answer to the first case was given: For general X, there exists a Young function $\Phi: \mathbb{R}^+ \to \mathbb{R}^+$, $\Phi(o) = o$, and a finite measure μ such that $\Phi(x)/x \nearrow \infty$ as $x \nearrow \infty$ and $\|\nu\|_\Phi(\Omega) < \infty$. On the other hand if X is a space of $\mathcal{L}_p$-type, $1 \leq p \leq 2$ (e.g. $X = L^p(\mu')$) then for every $\nu : \Sigma \to X$ there exists a scalar measure μ such that $\|\nu\|_2(\Omega) < \infty$ relative to μ. (This result depends crucially on the Grothendieck-Pietsch inequality).
As an application an operator representation for a certain class of stochastic processes was given.

Maße auf linearen Räumen

W. HERER:

Stochastic bases in Fréchet spaces

Let (E, τ) be a real Fréchet space and μ be a Borel probability measure on E.

Definition: A birthogonal system $\{x_k, f_k\}_{k \in \mathbb{N}}$ in (E, E^*) is called a stochastic basis iff there is a Borel subspace B of E with $\mu(B) = 1$ such that

$$\tau\text{-}\lim_{n \to \infty} \sum_{k=1}^{n} f_k(x)\, x_k = x \quad \text{for all } x \in B.$$

Theorem 1: Let E be a separable countably Hilbertian F-space and μ be a probability measure on E. Then there is a stochastic basis in (E, μ).

Theorem 2: Let E be a separable F-space and μ be a Gaussian probability on E. Then there is a stochastic basis in (E, μ).

A. HERTLE:

On the existence of Gaussian surface measures in separable Hilbert spaces

Let H be a real separable Hilbert space and μ be a Gaussian measure on H with covariance operator $A \neq o$ and mean $a \in H$. Let $\lambda_1 \geq \lambda_2 \ldots$ be the eigenvalues and $\{e_k\}$ be a basis of eigenvectors of A.

We say that μ induces a surface neasure on $M \subseteq H$ if the limit

$$\mu_M(f) = \lim_{\varepsilon \to 0} 1/2\varepsilon \int_{d(x,M) \leq \varepsilon} f(x)\, d\mu(x)$$

exists for all bounded continuous functions f on H. Essentially , to show that μ_M exists one shows that the distribution function $\varphi(r) = \int_{\vartheta(x) \leq r} f(x) d\mu(x)$ is differentiable, where $M = \{x: \vartheta(x) = 1\}$ then $\mu_M(f) = \varphi'(1)$. For this reason the growth behaviour of the Laplace transform $L\varphi$ of φ was studied. For the following types of analytic manifolds M it was shown that the Gaussian surface measure μ_M exists: Spheres, Ellipsoids and Hyperplanes. In all of theses cases an explicit formula involving A, a, the λ_k's and the e_k's was given.

H.-U. HESS:

On Wiener's constructional approach to Brownian motion

This talk dealt with Wiener's original method for constructing a measure that describes Brownian motion. The essential point in his procedure is that he provides an explicit mapping from the unit interval into the function space $\mathbb{R}^{[0,1]}$ such that the Lebesgue measure is carried over to the Wiener measure. At first sight this constructional method looks like Kuratowski's construction of a Borel isomorphism between uncountable Polish spaces. So the question arises whether the socalled quasi-intervals used by Wiener can be modified in sach a way that a Borel isomorphism from a subset of $[0,1]$ into the function space can be constructed such that Lebesgue measure is carried over to Wiener measure. That question found a positive answer through a construction that led to the following result:

Theorem: There is a zero-dimensional subset M of $[0,1]$ and a (1,1) homeomorphism ϕ from M onto the space of λ-Hölder-continuous functions such that 1) $\lambda^1(M) = 1$, 2) $\phi(\lambda^1)$, considered as a measure on $C([0,1])$ is the Wiener measure

Stochastische Prozesse

L. EGGHE:

Sub- and superpramart theory in Banach lattices

It was proved that in a Banach lattice with (RNP) every positive subpramart which is of class (B) or which has a uniformly integrable subsequence is strongly convergent a.s. (and conversely). In fact, an even more general result was proved which generalizes these results as well as the submartingale convergence theorem of H. HEINICH. The method of proof is an elaboration of the proof of the real subpramart convergence theorem of MILLET-SUCHESTON, in order to extend a result on submartingales (lemma 5-2-9 in Neveu "Discrete parameter martingales"). After this, a method of DAVIS-LINDENSTRAUSS-GHOUSSOUB is applied to prove strong convergence in the general setting.
As a second main result it was shown that in a reflexive Banach lattice every class (B) positive superpramart is weakly converging a.s. (this being new even for positive supermartingales).
In the third part sequences which are not necessarily positive were considered. A result of J. SZULGA was reproved and extended and a theorem of BLAHE extended to Banach lattices. Both of these results yield a characterization of Banach lattices isomorphic to $l^1(I)$ for some index set I.

M.P. ERSHOV:

On some questions concerning the Ionescu Tulcea theorem on constructing measures by transition kernels and the infinitesimal continuity of martingales

Two possibilities to generalize the Ionescu Tulcea theorem were discussed:
1. To construct the "ske-product" measure from a initial measure and a transition kernel one does not have to require that the latter be a measure on the whole σ-algebra in the second factor. It is sufficient to have a "weak" kernel which is σ-additive on every sequence of measurable sets in the second factor for all points of the first factor outside a measure-thin set. However to be able to prove the ITThm for weak transition problems on has to

impose an additional measurability condition on the "bad" thin set.

2. Considering the ITThm as describing the mechanism of constructing a measure by passing from the past to the immediate future, one can ask what the natural analogue in the continuous time case is. It seems that transition kernels from the past to the infinitesimal future are a natural substitute. Simple examples show, however, that even if a kernel has a representing measure it need not be unique. In the case of measures corresponding to continuous martingales with certain conditions on their diffusion functionals, it was shown that the transition probabilities from the past to the infinitesimal future do determine the measure uniquely.

L. SUCHESTON:

<u>On two-parameter stochastic processes</u>

Let Y_i be exchangeable random variables in L_p^+, $o < p < 1$, $X_{-n} = ((1/n)(Y_1+\dots+Y_n))^p$. Then X_{-n} is a descending submartingale (supermartingale if $p \geqslant 1$) which converges to zero a.s. and in L_1. In two dimensions, if the columns and rows of Y_{ij} can be exchanged, then $X_{-m,-n} = ((1/mn)(\sum_{(i,j)\leq(m,n)} Y_{ij}))^p$ is a descending 1-submartingale for the filtration $\mathcal{F}_{-m,-n} = \sigma(X_{-r,-s}: r \geqslant m,\ s \geqslant n)$ provided that Y_{11}^p L logL. (A 1-submartingale is a submartingale such that $X_{-m,-n} \leq E[X_{-k,-n} \mid \mathcal{F}_{-m,-1}]$ if $k \leq m$.) L logL bounded 1-submartingales upper demi-converge: $\overline{\lim}\, X_t$ = stoch lim X_t. These results were obtained with G.A. EDGAR. The theory of demi-convergence was more fully developped with A. MILLET. Thus the submartingale result extends to the descending case (with a more difficult proof), to the continuous parameter case (existence of upper demi-continuous modifications), and to Banach lattices. There is also a martingale version: Let $(X_t, \mathcal{F}_t)$, $t \in \mathbb{N}^2$ (or $t \in -\mathbb{N}^2$) be a positive 1-martingale, then $\underline{\lim}\, X_t$ = stoch lim X_t.

Ergodentheorie

A. BELLOW:

On "bad" universal sequences in ergodic theory

Let (Ω,α,μ) be the Lebesgue space and $\mathcal{T}$ the group of all bijections of Ω which are bimeasurable and measure-preserving (i.e. all automorphisms of (Ω,α,μ)) Let $\mathfrak{n} = (n_k)$ be an increasing sequence of positive integers. We say that the I.E.Th. (Individual Ergodic Theorem) holds for T along $\mathfrak{n}$ if for all $f \in L^1$ the sequence $(1/p) \sum_{k \leq p} f(T^{n_k}(\omega))$ converges a.s.. The sequence $\mathfrak{n}$ is called "bad" universal (for the I.E.Th.) if for each ergodic or aperiodic $T \in \mathcal{T}$, the I.E.Th. fails for T along $\mathfrak{n}$.

Theorem: Given any increasing sequence $\mathfrak{m}$, there is a subsequence $\mathfrak{n}$ of $\mathfrak{m}$ which is "bad" universal. Furthermore $\mathfrak{n}$ can be so chosen that every subsequenc $\mathfrak{n}'$ of $\mathfrak{n}$ is "bad" universal.

Applications of the above Theorem and techniques to "sweeping out" the whole space with arbitrarily small sets: I) Over an arbitrary increasing sequence with $T \in \mathcal{T}$ ergodic or aperiodic; II) Over an infinite sequence of translations in the case of the circle group (I) strengthens a result of M. ELLIS and N. FRIEDMAN). There are also applications to uniformly distributed (mod 1) sequences, suggesting some interesting open problems.

J.R. CHOKSI:

(reporting on joint work with V.S. PRASAD)

Ergodic theory on homogeneous measure algebras

Many results in ergodic theory are proved only for transformations on a Lebesgue space. In this talk a survey was given of how most of these results go over to automorphisms of a homogeneous measure algebra: some in a straightforward fashion, some involving genuine new difficulties. The few exception are all open problems. Results of J.C. OXTOBY and S. EIGEN were included in the talk.

Feynman - Integral

G.W. JOHNSON:

The equivalence of two approaches to the Feynman integral

Feynman's two "definitions" of the Feynman integral were discussed as well as the contribution of KAC. CAMERON and STORVICK have recently introduced a Banach algebra S of (equivalence classes of) functions on Wiener space $C_o([o,t])$ on which the Feynman integral is relatively well behaved. These functions are of the form $\tilde{\sigma}(x) = \int_{L_2([o,t])} \exp\{ i \int_o^t v(s)\tilde{d}x(s)\} d\sigma(v)$ where $\sigma \in M(L_2([o,t]))$ and $\int_o^t v(s)d\tilde{x}(s)$ is a stochastic integral. Some of the results about S which have been obtained by CAMERON-STORVICK and JOHNSON-SKOUGH were presented. Finally a result of JOHNSON was given which shows that the study of the Banach algebra S is essentially equivalent to the study of the Banach algebra of Fresnel integrable functions introduced by ALBEVERIO and HØEGH-KROHN. (More details can be found in a paper by the speaker which has the same title as this talk.)

G. KALLIANPUR:

Cameron - Feynman integrals in definite metric spaces

Let E be a Krein space with inner product $(.,.)$ and $E=E_+ x E_-$ a fundamental decomposition with associated fundamental symmetry J. Then $(x,y)_J:=(Jx,y)$ is a usual inner product, under which E, E_+ and E_- become Hilbert spaces (assumed separable) denoted by H, H_+ and H_- such that $H=H_+ \otimes H_-$. Introduce a measurable norm (in the sense of GROSS) such that it defines abstract Wiener spaces (H,B,P) and $H_\pm, B_\pm, P_\pm$). Let F be a functional on $B_+ x B_-$ such that (1) The integral $H(\lambda_1,\lambda_2) := \int_{B_+ x B_-} F(\lambda_1^{-1/2}x_1, \lambda_2^{-1/2}x_2) d(P_+ x P_-)(x_1,x_2)$ exists and is finite for all $\lambda_1, \lambda_2 > o$, (2) There exists a function H^* of the complex variables z_1, z_2, holomorphic in $\Delta = \{(z_1,z_2)\colon \text{Re } z_i > o\}$, such that $H^*(z_1,z_2)=H(z_1,z_2)$ for real z_1, z_2, (3) Assume that for some $q = (q_1,q_2) \in \mathbb{R}^2$ the limit $\lim_{(z_1,z_2)\to(-iq_1,iq_2),\ (z_1,z_2)\in\Delta} H^*(z_1,z_2)$ exists.

Then this (unique) limit is defined to be the Cameron-Feynman integral of the second kind, denoted $\mathcal{J}_q^2$, and contains CAMERON's definition of the analytic Feynman integral. $\mathcal{J}_q^2$ is a generalisation of the analytic Feynman integral of CAMERON-STORVICK. It can be applied to certain problems in Quantum Mechanics (e.g. the anharmonic oscillator with n degrees of freedom) which cannot be handled with the analytic Feynman integral of CAMERON-STORVICK. It also provides an integral (by the analytic continuation approach) which gives the same results as the extended Fresnel integral of ALBEVERIO and HOEGH-KROHN.

Verschiedenes

R.J. GARDNER:

(reporting on joint work with R.D. MAULDIN)

The Hausdorff dimension of a set of complex continued fractions

For real continued fractions, $\frac{1}{a_1+}\frac{1}{a_2+}\cdots$, $a_k \in \mathbb{N}$, whose values are the irrationals $I \subseteq (0,1)$, Gauss's measure is the unique probability which is (i) invariant under the shift $S: I \to I$ defined by $S(\frac{1}{a_1+}\ \cdots) = \frac{1}{a_2+}\ \cdots$ and (ii) equivalent to Lebesgue measure λ. (Gauss's measure has $x \mapsto (1/\log 2)(1/1+x)$ as Radon-Nikodym derivative w.r.t. λ) Suppose J is the set of values of the complex continued fractions $\frac{1}{b_1}+\frac{1}{b_2}+\cdots$ where $b_k \in \mathbb{N} \times \mathbb{Z}$ (to guarantee convergence). Can a measure on J be found with properties similar to Gauss' measure? One approach is to replace λ above by Hausdorff s-measure for a suitable $s \in [1,2]$ (though one cannot expect the measure of J to become positive and finite). A first step would be to estimate the Hausdorff dimension of J. It was shown that $1 <$ Hausdorff $\dim(J) < 2$ holds. The method used is due to V. JARNIK; the novel features were certain estimates for complex continued fractions previously only obtained for real continued fractions, and the delicacy of the calculations, particularly for the upper bound. In fact, a computer was used at one point.

P. MATTILA:

Integralgeometric properties of measures and capacities

Let $A(n,m)$ be the set of m-dimensional affine subspaces of $\mathbb{R}^n$, and let $\lambda_{n,m}$ be the orthogonally invariant measure on $A(n,m)$. If μ is a positive Radon measure on $\mathbb{R}^n$ one can use differentiation theory to define Radon measures μ_A, $A \in A(n,m)$, such that $\operatorname{spt} \mu_A \subseteq A \cap \operatorname{spt} \mu$ and for any non-negative Borel function f on R^n $\int f\, d\mu_A = \lim_{\delta \downarrow 0} \delta^{m-n} \int_{\{x:\operatorname{dist}(x,A) \leq \delta\}} f\, d\mu$ for $\lambda_{n,m}$-almost all $A \in A(n,m)$.

For a lower semicontinuous non-negative function K on $\mathbb{R}^n$ the K-energy of μ is $I_K(\mu) = \int K\, d(\mu \times \mu)$. If $H(x,y) = K(x,y) \cdot |x-y|^{n-m} \geq b > 0$ for $|x-y| \leq 1$, then $\int I_H(\mu_A) d\lambda_{n,m} \leq c(n,m) I_K(\mu)$.

For a compact subset F of $\mathbb{R}^n$ the K-capacity of F was defined by $C_K(F) = \sup\{I_K(\mu)^{-1} : \operatorname{spt} \mu \subseteq F,\ \mu(F) = 1\}$. Then $C_K(F) \leq c(n,m) \int C_H(F \cap A) d\lambda_{n,m} A$. Similar integralgeometric inequalities were given for the capacities of orthogonal projections. These results on capacities can be used to obtain information on the integralgeometric behavior of the Hausdorff dimension.

R.F. WHEELER:

Some topological questions related to the Pettis integral

Let X be a real Banach space, B_X its closed unit ball. Topological properties of B_X with the relative weak topology $\sigma(X,X^*)$ were investigated. In particular, say that X has the retraction property (RP) if there is a weakly continuous retraction of X onto B_X. Also, X has the CCC property if (B_X, weak) satisfies the countable chain condition for open sets; X has the Dunford-Pettis-Phillips property (DP^3) if every weakly compact operator has separable range. Then $RP \rightarrow CCC \rightarrow DP^3$.

If X has a separable dual, $X = c_0(\Gamma)$, or $X = C(K)$, K scattered, then X has RP. For reflexive spaces, all three properties are equivalent to separability. If $X = L^1(\mu)$, then $CCC \leftrightarrow DP^3 \leftrightarrow$ (μ is σ-finite). If $X = C(K)$, then $CCC \leftrightarrow DP^3 \leftrightarrow$ $\leftrightarrow$ (K is measure separable, i.e., $L^1(\mu)$ is separable for all $\mu \in M(K)$). An example of a non-separable dual with DP^3 was given.

A.C. ZAANEN:

Some remarks and a question about the definition of Orlicz spaces

In 191o F. RIESZ (Math. Ann.) defined L_p-spaces ($1 < p < \infty$) for Lebesgue measure and he established some of the main properties of these spaces. In 1912 W.H. YOUNG (Proc. Royal Soc.) defined the function classes $Y_\Phi = \{f : \int \Phi(|f|)dx < \infty\}$, where $\Phi(u) = \int_0^u \varphi(t)dt$ for some continuous and strictly increasing function with $\varphi(o) = o$. If Φ increases fast (e.g. $\Phi(u) = e^u - u$), then Y_Φ is not a vector space. If Φ satisfies the "Δ_2-condition" (i.e. there exists $C > o$ such that $\Phi(2u) \leq C\Phi(u)$ for all $u \geq o$), then Y_Φ is a vector space. The natural question whether in this case $\Phi^{-1}(\int \Phi(|f|)dx)$ is always a norm in Y_Φ is not mentioned by YOUNG nor by later authors (although in 1928 R. COOPER and G.H. HARDY (J. London Math. Soc.) gave a negative answer to a related question). In 1932 W. ORLICZ (Bull. Acad. Polon.) showed that Y_Φ (with the Δ_2-condition) can be normed in such a way that Y_Φ becomes a Banach space, but ORLICZ's definition is not the expression $\varrho(f) = \Phi^{-1}(\int \Phi(|f|)dx)$ mentioned above. In the talk a simple proof was indicated that $\varrho(f)$ is a norm on Y_Φ if and only if $\Phi(u)/\Phi(1) = u^p$ for some $p > 1$. In other words, $\varrho(f)$ is a norm only for L_p-spaces. The question referred to in the title was whether there are any references to this fact to be found in the literature.

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