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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 30/1981

Nichtstandard-Analysis und Anwendungen

5. bis 11. Juli 1981

Zum vierten Male seit 1970 fand unter der Leitung von W.A.J. Luxemburg (Pasadena) und D. Laugwitz (Darmstadt) eine Tagung über die Methoden und Anwendungen der Nichtstandard-Analysis statt, wieder als ein internationales Symposium. Besondere Gewichte lagen bei der Methode der Nichtstandard-Hüllen in der Funktionalanalysis, bei der Behandlung von Differentialgleichungen mit infinitesimalen Parametern und in der Stochastik.
Zur Ergänzung wurden Abendsitzungen veranstaltet, bei denen historische, didaktische und philosophische Fragen im Anschluß an Vorträge behandelt wurden. Auch bei diesen Diskussionen wurden Erfahrungen aus vielen Ländern ausgetauscht und vertieft.

Vortragsauszüge

S. Albeverio:
POINT INTERACTIONS, LOCAL TIMES, QUANTUM FIELDS

We discuss partial differential operators with singular coefficients that have a natural nonstandard description but can only be described indirectly by standard means. Such operators have been introduced by physicists in nuclear and solid state physics. Two examples are

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a) $-\Delta + \sum_{y\in Y} \lambda_y^\varepsilon \, \delta_\varepsilon(x-y)$

b) $-\Delta + \lambda^\varepsilon \int_0^t \delta_\varepsilon(x-b(\sigma))d\sigma,$

Δ being the Laplacian in $\mathbb{R}^d$, Y a discrete subset of $\mathbb{R}^d$, b(σ) Brownian motion on $\mathbb{R}^d$, δ_ε a nonstandard realization of Dirac measure, with infinitesimal parameter ε. For d = 2,3 in a) and d = 4 in b) a suitable choice of λ_y^ε resp. λ^ε infinitesimal is necessary and sufficient for getting non trivial standard part of the resolvent kernels to a), b), thus yielding standard operators associated with a), b). Hyperfinite methods are also discussed in this connection and in relation with Dirichlet forms and symmetric Markov processes in quantum mechanics and quantum field theory. The relation of the above operators with Edwards-Westwater's models of polymers is mentioned, as well as with the problem of the construction of relativistic quantum fields. These results will appear in a forthcoming book in collaboration with J.E. Fenstad, R. Høegh-Krohn and T. Lindstrøm.

José M. Bayod:

A SPHERICAL COMPLETION RELATED TO THE INFINITESIMAL HULL OF AN ULTRAMETRIC SPACE

A well-known property of the infinitesimal (or non-standard) hull $\hat{X}$ of a metrix space X is that it is complete whenever the non-standard model where it is built is $\aleph_1$-saturated; and the completion of X is the infinitesimal hull of the set pns $^*X$ of approachable points of $^*X$. In case X is an ultrametric space, it was proved by the author in another paper that $\hat{X}$ is spherically complete, but the exploration of a space related to $\hat{X}$ that could be called a spherical completion of X was left open. In this note we identify a subset of fin $^*X$ greater than pns $^*X$ and define an equivalence relation on it, which extends the relation of infinite nearness in pns $^*X$ and such that the quotient set is a spherically complete ultrametric space containing X and with no intermediate spherically complete space.

The problem of extending the notion of spherical completeness to (non-ultrametric) metric spaces will also be discussed.

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Steven F. Bellenot:

NONSTANDARD HULLS AND TRANSFINITE DUALS OF BANACH AND FRÉCHET SPACES

The nonstandard hull of certain Banach and Fréchet spaces are used as a setting to construct isometric concrete representations of their separable transfinite duals. Done in detail is J the James Banach space, but the construction works for spaces with a neighborly basis. It is shown that the ω-th dual of a non-reflexive space contains a neighborly basic sequence. Applications include showing X^{ω^2+1} is non-separable if X is non-reflexive and if $X^2/X^0 \neq \{0\}$ but is reflexive, then X has uniformly complemented l_p^n's for p = 1,2 or ∞.

I.P. van den Berg:

CONVEX GALAXIES IN $\mathbb{R}$

Working in Hrbacek's axiomatical system we define a convex galaxy G in $\mathbb{R}$ which is symmetrical as regards to zero in two equivalent ways: (i) $G = \bigcup_{stn} [-a_n, a_n]$, where $(a_n)_{n\in\mathbb{N}}$ is an internal strictly increasing function and (ii) $G = f(\mathfrak{C})$, where $\mathfrak{C}$ is the principal galaxy and f is an internal strictly increasing odd C^∞-function. We call non-linear those convex galaxies which are additive groups non-isomorphic to $\mathfrak{C}$. We have the following characterization: If $(a_n)_{n\in\mathbb{N}}$ and f are as above, then G is non-linear iff $\frac{a_{n+1}}{a_n}$ is infinite for standard n and $\frac{f'(t)}{f(t)}$ is infinite for t in $\mathfrak{C}\setminus\{0\}$. We may consider all convex galaxies in $\mathbb{R}$ as being built up of (possibly) and interval and (translations of) groups in the light of the following theorem:

Theorem: Let (G,H) be a cut of $\mathbb{R}$ in a galaxy G and a halo H. Then there is a real number a and a unique convex group K such that either K is a galaxy and $G =]-\infty,a] \cup a+K$ or K is a halo and $G =]-\infty,a]\setminus a+K$.

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Bent Birkeland:

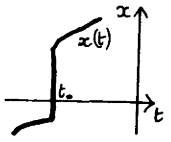
A NON-STANDARD TREATMENT OF THE EQUATION $y' = f(y,t)$

We consider the initial value problem $y' = f(y,t)$, $y(0) = 0$, with f continuous, but without any condition which ensures uniqueness. The non-standard version of the classical Cauchy-Peano approximation is used to deduce some properties of the set of solutions of this problem, including an old result of H. Kneser (1923).

Francine Diener:

SAUTS DES SOLUTIONS D'EQUATIONS $\varepsilon\ddot{x} = \phi(x,\dot{x})$

L'une des principales difficultés dans l'étude d'équations différentielles du type $\varepsilon\ddot{x} = \phi(x,\dot{x})$, est la présence de sauts, c'est à dire de portions de solutions parcourus à vitesse infiniment grande. Ces sauts (ou couches limites ou libres) correspondent à des portions quasi verticales du graphe $(t,x(t))$ des solutions. Pour étudier ces sauts, on introduit un changement d'échelle du temps, par exemple $\tau = (t-t_o)/l(\varepsilon)$, qui "étire" le saut dans le temps. Le problème est de determiner, lorsqu'on ne connait pas les solutions un changement d'échelle convenable. J'étudie ce problème à l'aide de trois exemples intégrables, présentant un saut en $t_o = 0$:



(1) $\varepsilon\ddot{x} = (1/2)x^3$, $x_1(t) = \sqrt{\varepsilon}/(\sqrt{\varepsilon}-t)$ $t\in[-1,0]$

(2) $\varepsilon\ddot{x} = \dot{x}$, $x_2(t) = \exp(t/\varepsilon)$ $t\in[-1,0]$

(3) $\varepsilon x = \dot{x}^2$, $x_3(t) = -\varepsilon \log(e^{-1/\varepsilon}-t)$ $t\in[-1,0]$

On définit l'épaisseur du saut $G = \{t\leq 0 \mid x(t) \neq 0\}$ qui est une galaxie. Dans les deux premiers exemples, G est une galaxie linéaire et dans le troisième elle est non linéaire. On peut voir facilement qu'il est alors impossible d'"étire" le saut par un changement d'échelle du type $\tau = \dfrac{t-t_o}{l(\varepsilon)}$. Par la méthode du plan d'observabilité, qui consiste en une étude géométrique des trajectoires dans un rapetissement convenable du plan de phase, on peut calculer, dans la plupart des cas la galaxie épaisseur en fonction de l'équation différentielle uniquement.

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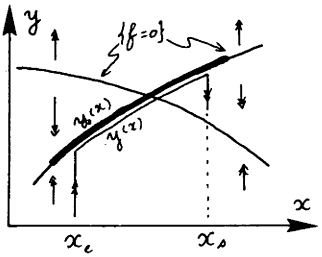
Marc Diener:

LEMME DE ROBINSON APPLIQUÉ AUX ÉQUATIONS DIFFÉRENTIELLES

Un halo (resp. une galaxie) est par définition un ensemble strictement externe qui est l'intersection (resp. la réunion) des élements d'indice standard d'une suite interne strictement décroissante (resp. croissante) de parties internes d'un ensemble interne E. L'ensemble des points à distance infiniment petite (resp. distance finie devant ε) d'une courbe $\{f=0\}$ de $\mathbb{R}^2$ constitue un exemple de halo (resp. galaxie) de $\mathbb{R}^2$. En utilisant l'idée du lemme de Robinson on montre facilement le pricipe (dit de Fehrele) suivant: aucun halo n'est une galaxie. Ce principe, avec le principe de Cauchy (qui peut se formuler "aucun ensemble strictement externe n'est interne") constituent deux principes de permanence essentiels à l'étude des perturbations singulières d'équations différentielles.



On applique ces principes à l'étude du comportement des solutions des équations differentielles $dy/dx = f(x,y,a)$ $(\varepsilon \simeq 0,\ \varepsilon>0)$, où f est C^∞ standard et $a \simeq a_o$, a_o standard, le long d'une solution-canard $y_o(x)$. On montre à l'aide de la loupe $Y = (y-y_o(x))^{[\varepsilon]}$, que $\int_{x_e}^{x_s} f'_y(x, {}^o y_o(x), a_o)dx = 0$.

Max Dresden:

PROBLEMS IN THEORETICAL PHYSICS AMENABLE TO NON STANDARD ANALYSIS

It was recognized early, that the statistical description via the canonical ensemble does not yield phase transitions for finite systems. (Finite Volume V, finite number N). It is therefore necessary to perform a limiting process. It had been demonstrated earlier that it is very natural to employ an infinite volume ${}^*V$ and an infinite number of particles ${}^*N$ to replace this limiting process. By using various inequalities, it can be proven that for infinite volumes ${}^*V$, no matter which one, the thermodynamic functions exist and are finite.

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A new result is the non standard proof of the theorem of Yang and Lu concerning the zero's of the grand partition function. This proof makes extensive use of Robinson's lemma. In so doing a good deal of analytical work can be avoided.
Another circle of problems to which non standard analysis appears applicable are the Casimir effect type problems. This necessitates algebraic and analytical manipulations with infinite objects. There are a variety of different approaches (and different results) in the literature, due to the various limiting procedures involved. In the non standard treatment, a systematic adherence to the fact that the equations and the quantities are all to be interpreted in ${}^*R$, while it is only necessary that the final physical entity shall possess a standard part, allows an unambiguous treatment of this effect.
It as suggested to investigate whether soliton phenomena (as for example described by the sine-Gordon equation) could be described by non standard procedure.
It is finally interesting to speculate on the possibility that the renormalized (standard) Hilbert space of quantum field theory is, or is included in the non standard hull of the original Hilbert space.

Max Dresden:

NON STANDARD STATISTICAL KAC MODELS

Kac constructed a number of statistical models in which the temporal evolution could be explicitly exhibited and calculated. These models were analyzed in some detail by the author, who also provided a quantum version of these moduls.
In the moduls one deals with a lattice of lattice distance a, n points, m of which are marked. The unit of time is τ_o. In the usual standard discussion all numbers are in R, τ_o is taken to be unity, so is a.
The main results is that the averaged Liouville description is - apart from the one particle case - not identical with the stochastic description by means of a Master equation. In the proof one has to take various limits, which although simple are not altogether straightforward. It is better to give a treatment where n, m are non standard, while t is kept standard. That means that the complete discussion can be given using

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one ensemble. In fact it means that for this case the relationship between the canonical and grand canonical ensemble can be analyzed precisely as a function of time.

It is especially important to study the continuous limit where $a \approx 0$, $\tau_o \approx 0$, and $t \in {}^*\mathbb{N}/N$.

In that case too, the Master equation description is not identical with the averaged Liouville equation, so that this lack of approach to equilibrium is not a pathology of the discrete nature of the model.

Bernd Eifrig:

BEMERKUNGEN ZUR MECHANIK

Lagrange-Funktionen vom Typ $L(q,\dot{q},\ldots,q^{(n)})$ geben zu verschiedenen Modellen der Mechanik Anlaß, z.B. liefert $L(q,\dot{q},\ddot{q}) = \alpha\dot{q}^2 + \beta\dot{q}\ddot{q}$ eine klassische gleichförmige Bewegung. Ein Problem der Mechanik besteht darin, $q(t)$ aus $q(t-l_1),\ldots,q(t-l_n)$ vorherzusagen. Dazu sei $L(q_{t-l_1},\ldots,q_{t-l_n})$ eine Funktion. Es wird gefordert $\int_{t_1}^{t_2} L(q_{t-l_1},\ldots,q_{t-l_n})dt$ sei stationär.

Als Konkurrenzfunktionen dienen $q_t + \varepsilon\eta_t$, $q_t^{(k)}(t_i) = 0, i = 1,2, \quad k = 0,1,2,\ldots$

Für $n=2$ liefert das $\frac{\partial L}{\partial u_1}(q_t, q_{t-h}) + \frac{\partial L}{\partial u_2}(q_{t+h}, q_t) = 0$. Formal erhält man das Resultat ebenfalls, wenn man fordert:

$L(q_{t+u} + \varepsilon\delta(u), q_{t+u-h} + \varepsilon\delta(u-h))$ ist in erster Näherung unabhängig von ε.

Beispiel: $L(q_t,q_{t-h}) = \frac{-(\varepsilon_t-\varepsilon_{t-h})^2}{2h^2}$ führt auf die Lösung:

$q_{nh+\tau} = nJ_\tau + q_\tau, \quad \tau_\varepsilon$ [periodisch mit h], $0 \le \tau < h$. Falls $h \approx 0$ und $y_{\tau/h}$ endlich, ergibt sich eine lineare, von τ abhängige Bewegung. Hängt τ vom Zufall ab, wird $X_t(\omega) = \mathrm{st}\, q_{nh+\tau(\omega)}$ beobachtet. Sei X ein lokalkonvexer Raum, $\Omega = X^{\mathbb{R}}$ der Raum der Trajektorien, $f_i = \frac{\partial L}{\partial u_i}$ ergibt sich als Bewegungsgleichung $\langle \Sigma f_i(T_{t+l_i}(\omega)), x\rangle = 0$, $x \in X$. Für beliebig viele Teilchen - mit I indiziert - wird X durch X^I ersetzt. Die erhaltenen Gleichungen sind kovariant. Ist I unendlich, bette man I in *-endliche Mengen $\hat{I}$ ein. Die korrespondierenden Bewegungen seien $q_{\hat{I}}(t)$. Als thermodynamischer Limes wird $\underset{I\to I}{\mathrm{LIM}}\ \mathrm{st}\, q_{\hat{I}}(x)$ definiert.

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Walter Felscher:

ON THE HISTORY AND PHILOSOPHY OF INFINITESIMALS

An interval, formed by two different points on a line, is a *finite geometrical quantity;* such quantities may be subdivided into smaller and smaller ones. The *Classical Continuum Problem* asks whether the points themselves are also geometrical quantities, *constituting* the line in an inherent fashion (in which case they necessarily are *indivisible* quantities) or whether the relationship between the line and the points is purely external (as in the modern notion that the line is the *set* of its points, viz. the *extension* of the concept of being a point on the line). Examples of the former point of view in Renaissance and Baroque mathematics are pointed out (e.g. Cavalieri); in particular, we quote Pascal's observation that indivisibilia do form an own Archimedian class below that of the finite quantitites, and we also quote the non-committal discussions of various possible points of view offered by Leibniz (for details cf. the author's "Naive Mengen und Abstrakte Zahlen", vol.II, p. 161 ff; Mannheim 1978). The fact that proofs must proceed in a finite number of steps (implicit in Euclid, explicit in Archimedes) could not be accommodated with the use of indivisibles and led, historically, to their abandonment. The proof-technical achievement of the 19th century analysis consists in the *finitarization* of mental experiments referring to infinite processes; it was obtained by the use of *higher order notions* (sequences, functions) as mathematical objects. It is emphasized that the methods of non-standard analysis are based on this same conceptual frame, extending it by introducing further higher order notions and making explicit use of the relationship between syntax (language) and semantics (mathematical structures) - which, in traditional mathematics, may usually be safely forgotten. Non-standard analysis, in its *methods of proof*, does *not* return to those of the 18th century but continues (and extends) the standard methods; at the same time, however, this very extension allows for a close imitation of the *modus inveniendi* as well as the *modus calculationis* of authors as Euler and Cauchy.

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Jens Erik Fenstad:

APPLICATIONS OF HYPERFINITE LATTICES

The idea of "hyperfinite discretization" has proved to be an efficient method in many areas of mathematics (e.g. stochastic integration, the theory of Dirichlet forms, differential equations (ordinary, partial, stochastic), etc.). We begin by discussing an example of singular perturbations of the Schrödinger equation, viz. $H = -\Delta - \sum_{y \in I} \lambda_y \delta(x-y)$, where Y is a discrete set of points in $\mathbb{R}^s$.

In the second part of the talk we gave examples of the use of hyperfinite lattices in statistical mechanics, in particular we proved the following theorem: Let ϕ be a nearest-neighbor interaction and let $\mathbb{Z}^\nu$ be written as a disjoint union $\mathbb{Z}^\nu = \Lambda_1 \cup C \cup \Lambda_2$, where C is the common boundary of Λ_1 and Λ_2. Let f_1, f_2 be tame functions such that the support of f_1 is disjoint from Λ_2 and the support of f_2 is disjoint from Λ_1. For a pure state of the system, we have

$$E(f_1 f_2 | C) = E(f_1 | C) \cdot E(f_2 | C).$$

Jacques Harthong:

LA PROPAGATION DES ONDES

Nous étudions la mécanique ondulatoire d'un point de vue macroscopique, c'est à dire en considérant la constante de Planck $\hbar$ comme un infiniment petit. La mécanique classique est alors obtenue comme une "trace macroscopique", c'est à dire que les objets mathématiques qui lui sont attachés

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apparaissent en prenant la partie standard d'objets de nature ondulatoire.

Notre principal instrument est le paquet d'ondes, qui est défini comme un élément u de ${}^*L^2(\mathbb{R}^n) \cap {}^*C^\infty(\mathbb{R}^n)$ de la forme $u(x) = c(x_o - x)e^{i/\hbar \cdot p_o \cdot x}$, où c vérifie les conditions siuvantes:

(i) $\|c\| = (\int |c(x)|^2 \, dx)^{1/2} \sim 1$

(ii) $\forall \alpha \neq (0,\ldots,0) \quad \|(i\hbar \frac{\partial}{\partial x})^\alpha c\| \simeq 0$

(iii) $\forall \alpha \neq (0,\ldots,0) \quad \|x^\alpha c\| \simeq 0.$

On dit que u est concentré en (x_o, p_o). Nous avons alors le théorème suivant:

Théorème: Soit ψ une fonction élément de ${}^*L^2(\mathbb{R}^n) \cap {}^*C^\infty$, vérifiant:

(i) $(\int |\psi(y)|^2 \, dy)^{1/2} = \|\psi\| \sim 1$

(ii) $\forall x \in {}^*\mathbb{R}^n, \quad x \simeq 0 \Rightarrow \int |\psi(y+\hbar x) - \psi(y)|^2 \, dy \simeq 0$

(iii) $\forall P \in {}^*\mathbb{R}^n, \quad P \simeq 0 \Rightarrow \int |e^{iP \cdot y} - 1|^2 \, |\psi(y)|^2 \, dy \simeq 0$

et posons $f_{\psi,c}(x,p) = (2\pi\hbar)^{-n/2} \int \psi(z)\, c\,(x-z)e^{-i/\hbar \cdot p \cdot z} dz$, ou c est un paquet d'ondes concentré en $(0,0)$.
Alors la densité $|f_{\psi,c}(x,p)|^2$ sur ${}^*(\mathbb{R}^n \times \mathbb{R}^n)$ posside une partie standard qui est une mesure de Radon sur $\mathbb{R}^n \times \mathbb{R}^n$, et qui ne dépend pas du choix de c.
Il est possible de donner aussi des résultats de propagation.
(Article à paraitre aux Annales de l'Institut Fourier, Grenoble, France).

C. Ward Henson:

LIPSCHITZ, UNIFORM AND LINEAR HOMEOMORPHISM OF NONSTANDARD HULLS OF BANACH SPACES

In the first part we will summarize the recent joint work by S. Heinrich and P. Mankiewicz studying uniform and Lipschitz equivalence of Banach spaces. They observe that if ϕ is a uniform homeomorphism from X onto Y and if H is an infinite number, then $\psi(p) = \frac{1}{H} \cdot {}^*\phi(Hp)$ induces a Lipschitz homeomorphism from $\hat{X}$ onto $\hat{Y}$. (Here $\hat{X}$, $\hat{Y}$ denote nonstandard hulls or ultrapowers of X, Y respectively.) They use this and ideas of Rademacher's classical theorem, that Lipschitz functions on $\mathbb{R}^n$ are a.e. differentiable, to prove new standard theorems about uniform homeomorphism

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and to give easy proofs of the results of Ribe.
In the second part we will discuss recent joint work of Heinrich, Henson and Moore concerning linear isomorphism of nonstandard hulls. This will include examples, general results and applications.

Detlef Laugwitz:

OMEGA -CALCULUS AS A GENERALIZATION OF FIELD EXTENSION

Let K be an ordered field $(= \mathbb{Q}, \mathbb{R})$. Then the non-archimedean extension can be obtained by adjoining a new constant Ω which behaves like all sufficiently large natural numbers with respect to the order relation and the four rational operations. Now let T be any theory, that is a collection of theorems containing some elementary number theory. T is formulated in a usual formal language. Add a new symbol Ω to the set of extra logical constants of the language of T. If $A(.)$ is a predicate such that $A(n)$ belongs to T for almost all n then, by definition, $A(\Omega)$ belongs to $T\langle\Omega\rangle$. Examples:

$\Omega > q_o$ for each fixed $q_o \in K$; $(-1)^\Omega = 1 \vee (-1)^\Omega = -1$;

$\left|(1 + \frac{1}{\Omega})^\Omega - \sum_{k=o}^{\Omega} \frac{1}{k!}\right| < a$ for each $0 < a \in K$.

(The numbers $\alpha,\ldots$ of $T(\Omega)$ are defined by sequences $a(n) \in K$, $a(\Omega) = b(\Omega)$ if $a(n) = b(n)$ for almost all n.)
It can be shown that this gives an approach to Nonstandard Analysis.

Tom L. Lindström:

A HYPERFINITE APPROACH TO MARKOV-PROCESSES AND DIRICHLET-FORMS

Let $S = \{1,2,\ldots,N\}$ be a hyperfinite set, and Δt a positive infinitesimal. We study stationary Markov-processes $X: \Omega \times \{0, \Delta t, 2\Delta t,\ldots\} \to {}^*\mathbb{R}$ given by a transition matrix $P^{\Delta t} = \{p_{ij}\}$. Assume that there exists an internal measure m on S such that $p_{ij}m_i = p_{ji}m_j$, and let $\langle,\rangle$ be the inner product $\langle f,g\rangle = \sum_{i=1}^{N} f(i)g(i)m_i$. The Dirichlet-form ξ associated to this Markov-chain is the nonnegative, symmetric form

$$\xi(f,g) = \frac{1}{\Delta t} \langle (I - P^{\Delta t})f,g\rangle .$$

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If $t = k\,\Delta t$, let $P^t = (P^{\Delta t})^k$ and $\xi^{(t)}(f,g) = \frac{1}{t}\langle (I-P^t)f, g\rangle$. The domain $\mathcal{D}[\xi]$ of ξ is defined to be all functions $f: S \to {}^*\mathbb{R}$ such that (i) $\xi_1(f,f) = \xi(f,f) + \langle f,f\rangle < \infty$ and (ii) $\xi(f,f) \approx \xi(P^t f, P^t f)$ for all positive infinitesimals t. We characterize and describe the domain - and study applications to the path properties of the process X - using the approximating forms $\xi^{(t)}$, $t >> 0$.

Peter Loeb:

SOME RESULTS ON NONSTANDARD MEASURE THEORY

Let Z be an internal set in an κ_1-saturated enlargement, and let L be an internal lattice of ${}^*\mathbb{R}$-valued functions defined on Z. Let I be an internal positive linear functional on L. For this talk we assume that $1 \in L$ and ${}^\circ I(1) < +\infty$. Let L_o be the set of ${}^*\mathbb{R}$-valued functions g on Z such that $\forall \varepsilon > 0$ in $\mathbb{R}$, $\exists\, f \in L$ with $|g| \le f$ and $I(f) < \varepsilon$. Such a g is called null. Let L_1 be the set of real-valued functions h on Z such that $h = f+g$ where $f \in L$, $g \in L_o$. For such an h, ${}^\circ I(f) < +\infty$ and the positive $\mathbb{R}$-linear functional $J: J(h) = {}^\circ I(f)$ is well defined. L_1 is a vector lattice, and if $\{h_n : n \in \mathbb{N}\}$ is an increasing sequence in L_1 with real-valued upper envelope h, then $h \in L_1$ if and only if $\sup_n J(h_n) < +\infty$, in which case, $J(h) = \sup_n J(h_n)$. If $\mathcal{F} = \{A \subset Z: \chi_A \in L_1\}$, then $\mathcal{F}$ is a σ-algebra and $\mu: \mu(A) = J(\chi_A)$ is a countably additive complete real-valued measure on $\mathcal{F}$. A bounded real-valued h on Z is $\mathcal{F}$ measurable iff $h \in L_1$ in which case $J(h) = \int h\,d\mu$. (The obvious extension to an unbounded h also holds.)

Example 1: Let ν be an internal *finitely additive measure on an internal algebra $\mathcal{A}$ in Z with ${}^\circ\nu(Z) < +\infty$. Let L be the lattice of ${}^*\mathbb{R}$-valued *simple functions on Z and I the ν-integral. The usual extension from $(Z, \mathcal{A}, {}^\circ\nu)$ to $(Z, \sigma(\mathcal{A}), {}^\circ\nu)$ is immediate.

Example 2: Let $(X,\mathcal{F})$ be a compact Hausdorff space and I an internal positive linear operator on ${}^*C(X)$ with ${}^\circ I(1) < +\infty$. With saturation, the Riesz-representation theorem follows almost immediately when we consider the lattice of functions in L_1 constant on monads. Also immediate are the results on the weak*-standard part of I due to Anderson, Rashid, and Loeb, for a completely regular space. Extensions of the latter results to non-regular spaces are considered with counter-examples showing the

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need to assume that the standard part of an internal-near standard set is compact. With this assumption the results extend.

W.A.J. Luxemburg:
EULER'S DISCOVERY OF THE RELATION $\sum_{1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

Using the methods of nonstandard analysis and Euler's relation

$$(*) \quad \sin\left(\frac{1}{\sqrt{y}}\right) = {}_1 \frac{1}{y^{\omega+1/2}} \cdot \sum_{k=0}^{\omega} (-1)^k \frac{y^{\omega-k}}{(2k+1)!} ,$$

where ω is infinitely large, it is shown that if $\{\lambda_k : k = 1,2,\ldots,\omega\}$ are the roots of the polynomial on the right-hand side of (*), then

$$\sum_{1}^{\infty} \frac{1}{n^2 \pi^2} = \mathrm{st}\left(\sum_{1}^{\omega} \lambda_k\right) = \frac{1}{3!} .$$

Some other proofs will also be discussed.

L. C. Moore, Jr.:
BANACH SPACES E WITH $\hat{E}$ ISOMETRIC TO $\hat{c}_o$

(These results are joint work with S. Heinrich and W. Henson.)
Proposition 1: A real Banach space E has a nonstandard hull $\hat{E}$ linearly isometric to a nonstandard hull $\hat{c}_o$ of c_o if and only if for some infinite set Δ E is linearly isometric to a subspace F of $l_\infty(\Delta)$ such that

(1) $F \supseteq c_o(\Delta)$,
(2) no function in F is bounded away from zero ,
(3) for every f in F and $0 < \alpha < \beta$ there exists a subset D of Δ such that
  (i) $\{d : |f(x)| \geq \alpha\} \supseteq D \supseteq \{d : |f(x)| \geq \beta\}$,
  (ii) for every $h \in F$ we have $h\chi_D \in F$, and
  (iii) there exists $g \in F$ with $|g| = \chi_D$.

Proposition 2: If E is a separable Banach space and $\hat{E}$ is isometric to $\hat{c}_o$, then E is isometric to an abstract M-space.

Theorem: There exists E with $\hat{E}$ isometric to $\hat{c}_o$ but E is not isometric to any Banach lattice.

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Edward Perkins:

ON THE SMALL OSCILLATIONS OF BROWNIAN MOTION

By counting the number of times an infinitesimal random walk, X, exits from a square root boundary we construct a "local time", L. We show that the law of (st(X), st(L)) is independent of the infinitesimal step size and of the underlying distribution of the original random walk. As a result, a weak invariance principle is established. The following theorem is one application of this construction:

Theorem: If B_t is a Brownian motion, then

$$X \equiv \inf \{\limsup_{h \to o^+} |B(t+h) - B(t)|\, h^{-1/2} |\ t \geq 0\} = 1 \quad \text{a.s.} \ \square$$

This refines results of Dvoretsky (1963), who showed that $x \geq .25$ a.s. and Kahone (1974), who showed that $X < \infty$ a.s.

Georges Reeb:

ON INTERNAL SET THEORY

B. Benninghofen and M.M. Richter:

TOPOLOGICAL FORMULAS IN TOPOLOGY

We start with a non-standard model for set theory which satisfies the axioms of Nelson. The topological formulas are those external formulas in which the predicate "standard" occurs at most negatively and all external quantifiers are bounded.

Proposition: A non-empty external set M is the monad of some standard filter iff M can be defined by some topological formula.

Some examples are given: The Krull-topology, the Scott-topology (in computer sciences), various uniformities in distribution theory. In particular those pre-distributions are characterized which are distributions. χ_o-monads are those monads which can be defined by topological formulas where the external quantifiers range over at most countable sets. For those monads there is an extended transfer principle which gives a generalized bounded convergence theorem.

The notion of a monad is extended to non-standard filters; an application is the characterization of locally connected spaces.

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Hermann Rodenhausen:

NONSTANDARD TOOLS IN ADAPTED PROBABILITY LOGIC

A formal language is presented to talk about stochastic processes on adapted probability spaces, i.e. structures of the form $(\Lambda, (F_t)_{t\in[0,1]}, \mu, x)$ where $(F_t)_{t\in[0,1]}$ is a monotonously increasing family of σ-algebras on Λ, μ is a probability measure on F_1, and x is a measurable mapping $\Lambda \times [0,1] \to \mathbb{R}$. Notions such as stopping time, adapted process, and martingale can be expressed in this logic.
The logic was introduced by H.J. Keisler and D. Hoover who used it to prove results in nonstandard stochastic analysis. We show how nonstandard methods can be used to prove a completeness theorem for the logic.

Bernd Rodewald:

LEONHARD EULERS ENTDECKUNG DER GAMMAFUNKTION

Es wird eine Methode Leonhard Eulers zur Interpolation von Reihen vorgestellt (vgl. Euler, Institutiones Calculi Differentialis, partis posterioris, caput 16 und 17 = Opera omnia I, 10, S. 588-647). Diese Methode liefert in gewissen Fällen eine Fortsetzung der zunächst nur für ganze x definierten Funktion $y(x) = \sum_{k=1}^{x} f(k)$ (f sei bekannt). Die wesentliche Idee des Verfahrens besteht in folgendem: Wenn die n-te Differenz der Funktion f im Unendlichen infinitesimal wird, $\Delta^n f(\Omega) \approx 0$, ($1 << \Omega \in {}^*\mathbb{N}$, n endlich), so approximiere man y zunächst in der Nähe von Ω durch ein Polynom n-ten Grades: $y(\Omega+z) := \sum_{k=0}^{n} \binom{z}{k} \Delta^k y(\Omega)$ (z endlich). Dann wird diese Interpolation mit Hilfe der "Rekursionsformel" $y(x+1) = y(x) + f(x+1)$ für endliche Argumente nutzbar gemacht. Logarithmieren ergibt: $\ln n! = \sum_{k=1}^{n} \ln k$, so daß diese Methode zur Interpolation der Fakultät geeignet ist. Euler erhält so die Produktdarstellung der Gammafunktion:

$$\Gamma(x+1) = \frac{1\cdot 2^x}{1+x} \frac{2^{1-x}\cdot 3^x}{2+x} \frac{3^{1-x}\cdot 4^x}{3+x} \cdot \ldots \cdot \frac{\Omega^{1-x}\cdot(\Omega+1)^x}{\Omega+x}$$

Bemerkungen: Diese Methode kann auch in der Theorie der endlichen Differenzen zur Lösung von Differenzengleichungen der Form $u(x+1) - u(x) = B(z)$

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(B eine analytische Funktion) dienen. Offen bleibt die Frage, für welche Funktionen f genau Eulers Ansatz zu sinnvollen Ergebnissen führt.

T. Sari:

APPLICATIONS OF NONSTANDARD-ANALYSIS TO BOUNDARY VALUE PROBLEMS

A few years ago, G. Reeb suggested to use N.S.A. in perturbation problems. The method was successfully applied by several authors in numerous problems which involve singular and regular perturbations (see the work of the non standard minded people at Oran-Mulhouse-Strasbourg-Tlemcen). Our scope is to show how N.S.A. can be also used in singularly perturbed second order two point boundary value problems of type

$$\begin{cases} \varepsilon\ddot{x} = f(x,\dot{x},t,\varepsilon) \quad a < t < b, \quad \varepsilon > 0 \\ x(a) = A \quad \text{and} \quad x(b) = B \quad \text{prescribed.} \end{cases}$$

Such problems, of great interest in applied mathematics, lead to a wealth of technics - "the perturbation method" - but the user of these technics usually draws on a backlog of experience rather than some general theory. For the problems discussed, our method works more satisfactory: the important facts are that we have a secure geometrical picture which gives all possible behaviours and that the asymptotic behaviour may be easily formulated in N.S.A. We treat some examples which have often been considered in the literature, provided their wide variety of interesting behaviour. Of course, A. Robinson had in mind the use of N.S.A. as a tool in layer theory, what is clear from his chapter on the boundary layer concept in hydrodynamics and elasticity.All the material presented here is contained in the book: N.S.A. - a practical guide with applications by R. Lutz and M. Goze (Lecture notes in Mathematics, Springer 1981).

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Walter Schnitzspan:

NICHTSTANDARD-ANALYSIS IN DER SCHULE

These: Die Nichtstandard-Analysis läßt sich mit Erfolg in der Schule unterrichten.

Es wird von Unterrichtsversuchen berichtet, die in normalen Grundkursen und Leistungskursen Mathematik in der Sekundarstufe II durchgeführt wurden. Hierbei ergab sich zwangsläufig ein konstruktiver Aufbau, der die Infinitesimalrechnung als Rechnung mit dem "unendlich kleinen bzw. großen" ernst nahm und der zum besseren Verständnis der Methoden und Geschichte der Mathematik einiges beitrug.

Detlef D. Spalt:

ZUR KORREKTHEIT DER INFINITESIMALIEN

Die Objektivität des mathematischen Wissens ist ein Produkt dogmatischen Denkens. Dem nämlich geht es um Wahrheit und Autorität. Skeptisches Denken dagegen handelt von Kritik und Geschichte. Geschichte dabei genommen als Realgeschichte - in der Mathematik als im Kontext ernstzunehmende Originalliteratur. Kritik zersetzt Wahrheit - zum Chaos? Die Methodologie wissenschaftlicher Forschungsprogramme von Imre Lakatos ist hier ein Kompromißangebot an den Rationalismus: sie ersetzt die eine Vernunft ("Wahrheit") durch mehrere ("Forschungsprogramme"). So vermag sie einen wesentlich größeren Teil der (Real-)Geschichte zu erfassen als der Dogmatismus. Die Geschichte des Cauchyschen Summensatzes ("Eine konvergente Reihe stetiger Funktionen einer Veränderlichen ist stetig.") etwa wird hier zur vielschichtigen, abwechslungsreichen, spannenden Entwicklung - gegenüber der trivialen, langweiligen Darstellung der strukturalistischen Geschichtsschreibung (Bourbaki, Kline).

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K.D. Stroyan:

PREVISIBLE SETS IN THE HYPERFINITE SETTING

(Some Fine Points of Stochastic Integration)

In Keisler's hyperfinite evolution scheme the measurability condition needed for stochastic integrands (previsibility) can be characterized in terms of arbitrarily large infinitesimal prediction.

M.E. Szabó:

STANDARD DETOURS

In this paper we compare the usual standard and nonstandard proofs of some basic theorems in calculus, functional analysis, and topology and give an account of the extent to which the standard proofs contain detours (in the sense of Gentzen) without counterparts in the corresponding nonstandard proofs. We discuss the relevance of this fact for translation procedures between standard and nonstandard proofs.

Frank Wattenberg:

A NONSTANDARD APPROACH TO THE TANGENT BUNDLE

We give a natural nonstandard construction of the tangent bundle. For finite dimensional differentiable manifolds this tangent bundle is isomorphic to the usual one. For infinite dimensional manifold the fiber is the nonstandard hull of the underlying normed linear space. At singular points the tangent bundle is defined in a natural way. As an application we consider the problem of finding geodesics.

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Manfred Wolff:

ON THE NONSTANDARD HULL OF GROUP ACTIONS ON BANACH SPACES

Let G be a locally compact abelian group acting on a Banach space E via the strongly continuous homomorphism U. The spectrum $\sigma(U)$ consists of all continuous characters χ on G for which there exists a net (x_α) of normalized vectors such that $\|U_s x_\alpha - \chi(s)x_\alpha\|$ converges to 0 uniformly on compact subsets of G. $P\sigma(U)$ is the set of all χ which are eigenvalues (in the obvious sense).

We construct in a natural way a closed subspace $\hat{E}^G$ of the nonstandard hull $\hat{E}$ of E such that (i) U can be extended to a strongly continuous homomorphism $\hat{U}$ on $\hat{E}^G$, (ii) $\sigma(U) = P\sigma(U)$, (iii) $\hat{E}^G$ is maximal with respect to (i) and (ii). This nonstandard hull $(\hat{E}^G, \hat{U})$ is again a Banach lattice with $U(G)$ a group of lattice automorphisms whenever this was true for (E,G).

Application: Let E be a Banach lattice and let $U(G)$ consist of positive operators. In addition let U be bounded. Then $1_G \in \sigma(U)$ and moreover if $\chi \in \sigma(U)$ then the closed group generated by χ is also in $\sigma(U)$.

Berichterstatter: Bernd Arnold (Darmstadt)

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